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COORDINATION EFFICIENCY FOR GENERAL TWO-STAGE NETWORK SYSTEM

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Abstract. Two-stage network data envelopment analysis (DEA) is widely used to evaluate efficiency of different organizations with multiple operations processes or hierarchical structures. Although existing two-stage network DEA assumes two-stage systems resolve the inherent conflicts between two stages, the coordination effect between the two stages is usually ignored. Recently, the relation of two-stage network DEA to traditional "black box" DEA has been studied from the perspective of system coordination. A coordination efficiency was defined and measured by a DEA-based approach based on simple two-stage network systems. In this paper, we propose an extended DEA-based approach for measuring the coordination efficiency for general two-stage network systems. The paper shows that the coordination efficiency based on the multiplier DEA and envelopment DEA approaches is equivalent to each other under both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions. The proposed approach is verified *via* two numerical examples finally.

Mathematics Subject Classification. 90C08, 90B06, 90B30.

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1. Introduction

Data envelopment analysis (DEA) is a mathematical programming approach for measuring the relative efficiency of entities using multiple inputs to produce multiple outputs [3]. The entities are usually called decision making units (DMUs). After more than 40 years of development, DEA has been significantly extended and widely applied to various areas [8, 10, 24, 25]. In recent years, DEA has been used as a data-driven tool for balanced benchmarking [27]. Most recently, it was advocated as data enabled analytics [31].

Traditional DEA models regard each DMU as a black box and the composite efficiency index is evaluated according to the inputs and outputs of the black box. This consideration makes us estimate the efficiencies of DMUs conveniently. However, the intermediate links and the coordination effects among the sub-processes are ignored. Network DEA [11, 13] opens the black boxes and estimate the overall efficiencies of the black boxes

Keywords. Data envelopment analysis (DEA), general two-stage network system, coordination efficiency, multiplier DEA model, envelopment DEA model.

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themselves and the divisional efficiencies of the sub processes embedded in them. The estimation of divisional efficiencies can help decision maker find out where the inefficiencies come from more clearly.

Two-stage network DEA [17, 22], which is a special case of network DEA, is based on the basic series network structure. Kao and Hwang [17] deemed that either constraint of the two sub-stages should also constrain the other sub-stage and the whole production system. Following this idea, they proposed a rational efficiency estimation and decomposition model that regarded the overall efficiency as the product of divisional efficiencies. Note that the weights in terms of the intermediate products were assumed the same [17, 22].

Inspired by Kao and Hwang [17] and Liang *et al.* [22], a lot of interesting studies have enriched the methodologies for two-stage network DEA. Some studies proposed to estimate the overall efficiency and divisional efficiencies based on radial measures [5, 9, 20]. Kao and Liu [18] further extended their discussions to cross efficiency measurement and decomposition. Based on non-radial measures, especially slacks-based measure (SBM) [28], some other studies developed their approaches for basic two-stage network system [7], Although the SBM-based models in Kao [15, 16] are proposed for closed series production systems and general network system, they are also suitable for two-stage network systems. Besides the efficiency measurement, a few studies investigated frontier projection and duality [7, 23]. See reviews and references by Kao [13, 14] for more comprehensive discussions on two-stage network DEA.

Chen *et al.* [6] claimed that traditional DEA and two-stage network DEA were two separate and different methods. Although two-stage network DEA solves the conflicts between two sub-stages, the relationship between traditional DEA and two-stage network DEA is usually neglected. As discussed in the existing studies [2, 26], the coordination effect widely exists in two-stage systems [12] such as supply chain. However, the coordination effect in two-stage network DEA is seldom considered although Li [19] mentioned the coordination in two-stage network DEA. Zhao *et al.* [30] firstly defined the coordination efficiency and proposed a DEA-based approach for measuring the coordination effect of a special simple two-stage DEA system, *i.e.*, two-stage supply chain system [4, 21, 29].

In this paper, we further extend the work of Zhao *et al.* [30] to general two-stage network system. Particularly, we propose both multiplier and envelopment DEA models to calculate the coordination efficiency. It shows that the multiplier DEA approach is equivalent to the envelopment DEA one for general two-stage network system. In addition, the conclusion is tenable under both constant returns to scale (CRS) and variable returns to scale (VRS) [1]. The proposed approach is illustrated by two numerical examples.

The reminder of the paper is presented as follows. Section 2 roughly introduces the proposed approach in Zhao *et al.* [30]. Section 3 proposes multiplier DEA models for estimating the coordination efficiency while Section 4 proposes envelopment DEA models for doing so. The equivalence between the multiplier and envelopment DEA approaches is discussed in Section 5. Section 6 concludes the paper.

2. The approach in Zhao *et al.* [30]

In this section, we roughly introduces the approach in Zhao *et al.* [30] to measure the coordination efficiency under basic two-stage DEA system. Assume there are n DMUs and each DMU is a simple two-stage network system as shown in Figure 1. Each DMU$_j$ uses inputs $\mathbf{X}_j(X_{1j},\ldots,X_{mj})$ to produce intermediate products $\mathbf{Z}_j(Z_{1j},\ldots,Z_{Dj})$ in the first stage, then the intermediate products $\mathbf{Z}_j(Z_{1j},\ldots,Z_{Dj})$ is used to produce outputs $\mathbf{Y}_j(Y_{1j},\ldots,Y_{sj})$ in the second stage.

In Zhao *et al.* [30], coordination efficiency is defined as the ratio of the system efficiency with coordination to the system efficiency without coordination. Under the VRS assumption, let E_o^{VRS} be the system efficiency with coordination for DMU$_o$. It can be obtained as follows:

$$
\begin{aligned}
E_o^{\mathrm{VRS}} = & \max_{u_r,w_d,v_i,u_o,w_o} \frac{\sum_{r=1}^{s} u_r Y_{ro} - w_o - u_o}{\sum_{i=1}^{m} v_i X_{io}} \\
& \text{s.t.}\ \frac{\sum_{d=1}^{D} w_d Z_{dj} - w_o}{\sum_{i=1}^{m} v_i X_{ij}} \le 1, \qquad j = 1,\ldots,n
\end{aligned}
$$

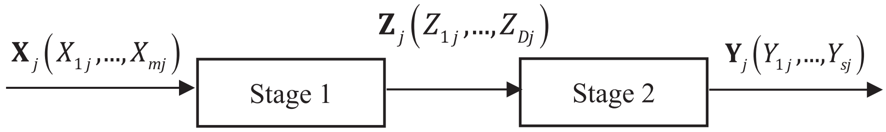

FIGURE 1. Basic two-stage network system [30].

$$\begin{aligned}
&\frac{\sum_{r=1}^{s} u_r Y_{rj} - u_o}{\sum_{d=1}^{D} w_d Z_{dj}} \le 1, && j = 1,\ldots,n\\
&u_r, w_d, v_i \ge 0, && r = 1,\ldots,s,\ d = 1,\ldots,D,\ i = 1,\ldots,m
\end{aligned} \tag{1}$$

where u_r, w_d, and v_i are weight variables (*i.e.*, multipliers) in terms of Y_{rj}, Z_{dj}, and X_{ij}, respectively. w_o and u_o are free variables for modelling VRS.

Let π_o^{VRS} be the system efficiency without coordination. It can be obtained as follows:

$$\begin{aligned}
\pi_o^{\text{VRS}} = &\max_{w_d^1, v_i, w_o} \frac{\sum_{d=1}^{D} w_d^1\left(E_o^{2\text{-VRS}} Z_{dj}\right) - w_o}{\sum_{i=1}^{m} v_i X_{io}}\\
\text{s.t. } &\frac{\sum_{d=1}^{D} w_d^1 Z_{dj} - w_o}{\sum_{i=1}^{m} v_i X_{ij}} \le 1, && j = 1,\ldots,n\\
&w_d^1, v_i \ge 0, && d = 1,\ldots,D,\ i = 1,\ldots,m
\end{aligned} \tag{2}$$

where $E_o^{2\text{-VRS}}$ is stage 2's VRS efficiency calculated as follows:

$$\begin{aligned}
E_o^{2\text{-VRS}} = &\max_{u_r, w_d^2, u_o} \frac{\sum_{r=1}^{s} u_r Y_{ro} - u_o}{\sum_{d=1}^{D} w_d^2 Z_{do}}\\
\text{s.t. } &\frac{\sum_{r=1}^{s} u_r Y_{rj} - u_o}{\sum_{d=1}^{D} w_d^2 Z_{dj}} \le 1, && j = 1,\ldots,n\\
&u_r, w_d^2 \ge 0, && r = 1,\ldots,s,\ d = 1,\ldots,D
\end{aligned} \tag{3}$$

when π_o^{VRS} is obtained, the coordination efficiency ρ_o^{VRS} of DMU_o can be calculated as follows:

$$\begin{aligned}
\rho_o^{\text{VRS}} = &\max_{u_r, w_d, v_i, u_o, w_o} \frac{\sum_{r=1}^{s} u_r Y_{ro} - w_o - u_o}{\sum_{i=1}^{m} v_i \left(\pi_o^{\text{VRS}} X_{io}\right)}\\
\text{s.t. } &\frac{\sum_{d=1}^{D} w_d Z_{dj} - w_o}{\sum_{i=1}^{m} v_i X_{ij}} \le 1, && j = 1,\ldots,n\\
&\frac{\sum_{r=1}^{s} u_r Y_{rj} - u_o}{\sum_{d=1}^{D} w_d Z_{dj}} \le 1, && j = 1,\ldots,n\\
&u_r, w_d, v_i \ge 0, && r = 1,\ldots,s,\ d = 1,\ldots,D,\ i = 1,\ldots,m.
\end{aligned} \tag{4}$$

If w_o and u_o is deleted from the above models, we can obtain the coordination efficiency under CRS. For example, if there are no variables w_o and u_o in model (1), then it measures the system efficiency with coordination under CRS, which is exactly the overall efficiency in Kao and Hwang [17]. See Zhao *et al.* [30] for more details.

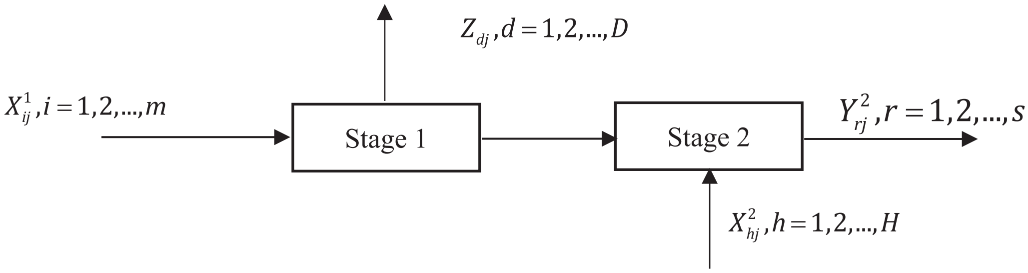

FIGURE 2. General two-stage network system.

3. Coordination efficiency for general two-stage network system based on multiplier DEA models

In this section, we investigate the coordination efficiency for general two-stage network system based on multiplier DEA model. Similarly, assume that there are n DMUs. In the first stage, each DMU$_j$ uses inputs $X_{ij}^1, i = 1, 2, \ldots, m$ to produce intermediate products $Z_{dj}, d = 1, 2, \ldots, D$ and outputs $Y_{lj}^1, l = 1, 2, \ldots, L$. In the second stage, the intermediate products $Z_{dj}, d = 1, 2, \ldots, D$ and inputs $X_{hj}^2, h = 1, 2, \ldots, H$ are used to produce the final outputs $Y_{rj}^2, r = 1, 2, \ldots, s$. Figure 2 depicts the general two-stage network system.

3.1. CRS case

In the following, we use the same definition of coordination efficiency in Zhao *et al.* [30], *i.e.*, the ratio of the system efficiency with coordination to the one without coordination. Similar to the approach for the simple two-stage network system in Section 2, the multiplier DEA model for estimating the system efficiency with coordination $\tilde{E}_o^{\text{CRS}}$ of DMU$_o$ under CRS is as follows:

$$
\begin{aligned}
\tilde{E}_o^{\text{CRS}} = &\max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \frac{\sum_{l=1}^{L} \mu_l Y_{lo}^1 + \sum_{r=1}^{s} u_r Y_{ro}^2}{\sum_{i=1}^{m} v_i X_{io}^1 + \sum_{h=1}^{H} \vartheta_h X_{ho}^2} \\
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1}{\sum_{i=1}^{m} v_i X_{ij}^1} \le 1, && j = 1, \ldots, n \\
& \frac{\sum_{r=1}^{s} u_r Y_{rj}^2}{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{h=1}^{H} \vartheta_h X_{hj}^2} \le 1, && j = 1, \ldots, n \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \ge 0, && r = 1, \ldots, s,\ d = 1, \ldots, D \\
& && i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H
\end{aligned} \tag{5}
$$

where u_r, w_d, v_i, μ_l, and ϑ_h are weight variables (*i.e.*, multipliers) in terms of Y_{ro}^2, Z_{dj}, X_{ij}^1, Y_{lj}^1, and X_{hj}^2, respectively. It should be noted that model (5) is exactly model (11.13) of Kao [14] if we neglect the non-Archimedean number constrained to the weight variables.

For the general two-stage network system, due to the existence of the outputs $(Y_{lj}^1, l = 1, 2, \ldots, L)$ in the first stage and the inputs $(X_{hj}^2, h = 1, 2, \ldots, H)$ in the second stage, we cannot obtain the system efficiency without coordination (*i.e.*, $\tilde{\pi}_o^{\text{CRS}}$) directly by following the same approach in the previous section. Hence, the coordination efficiency $\tilde{\rho}_o^{\text{CRS}}$ cannot be obtained according to the equation $\frac{\tilde{E}_o^{\text{CRS}}}{\tilde{\pi}_o^{\text{CRS}}}$. To solve this issue, we build the following model to estimate the coordination efficiency $\tilde{\rho}_o^{\mathrm{CRS}}$ directly:

$$
\begin{aligned}
\tilde{\rho}_o^{\mathrm{CRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \frac{\sum_{l=1}^{L} \mu_l Y_{lo}^1 + \sum_{r=1}^{s} u_r Y_{ro}^2}{\sum_{i=1}^{m} v_i \left(\tilde{E}_o^{1\text{-CRS}} X_{io}^1\right) + \sum_{h=1}^{H} \vartheta_h \left(\tilde{E}_o^{2\text{-CRS}} X_{ho}^2\right)} \\
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1}{\sum_{i=1}^{m} v_i X_{ij}^1} \leq 1, \qquad j = 1, \ldots, n \\
& \frac{\sum_{r=1}^{s} u_r Y_{rj}^2}{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{h=1}^{H} \vartheta_h X_{hj}^2} \leq 1, \qquad j = 1, \ldots, n \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, \qquad r = 1, \ldots, s,\ d = 1, \ldots, D \\
& \qquad i = 1, \ldots, m,\ l = 1, \ldots, L, \\
& \qquad h = 1, \ldots, H.
\end{aligned} \tag{6}
$$

The difference between models (5) and (6) is the inclusion of $\tilde{E}_o^{1\text{-CRS}}$ and $\tilde{E}_o^{2\text{-CRS}}$ in the objective function. Specifically, $\tilde{E}_o^{2\text{-CRS}}$ is the CCR efficiency of the second stage. If not considering the coordination between the two sub-stages, $\tilde{E}_o^{1\text{-CRS}}$ is the minimum reduction ratio of input X_{io}^1, which could be calculated by the following model:

$$
\begin{aligned}
\tilde{E}_o^{1\text{-CRS}} = & \max_{w_d^1, v_i, \mu_l} \frac{\sum_{d=1}^{D} w_d^1 \left(\tilde{E}_o^{2\text{-CRS}} Z_{do}\right) + \sum_{l=1}^{L} \mu_l Y_{lo}^1}{\sum_{i=1}^{m} v_i X_{io}^1} \\
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d^1 Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1}{\sum_{i=1}^{m} v_i X_{ij}^1} \leq 1, \qquad j = 1, \ldots, n \\
& w_d^1, v_i, \mu_l \geq 0, \qquad d = 1, \ldots, D,\ i = 1, \ldots, m,\ l = 1, \ldots, L
\end{aligned} \tag{7}
$$

when the system efficiency with coordination $\tilde{E}_o^{\mathrm{CRS}}$ and coordination efficiency $\tilde{\rho}_o^{\mathrm{CRS}}$ are obtained, we can get the system efficiency without coordination *via* $\tilde{\pi}_o^{\mathrm{CRS}} = \frac{\tilde{E}_o^{\mathrm{CRS}}}{\tilde{\rho}_o^{\mathrm{CRS}}}$.

3.2. VRS case

The VRS DEA model for estimating the system efficiency with coordination of DMU_o is as follows:

$$
\begin{aligned}
\tilde{E}_o^{\mathrm{VRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h, u_o, w_o} \frac{\sum_{l=1}^{L} \mu_l Y_{lo}^1 + \sum_{r=1}^{s} u_r Y_{ro}^2 - w_o - u_o}{\sum_{i=1}^{m} v_i X_{io}^1 + \sum_{h=1}^{H} \vartheta_h X_{ho}^2} \\
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - w_o}{\sum_{i=1}^{m} v_i X_{ij}^1} \leq 1, \qquad j = 1, \ldots, n \\
& \frac{\sum_{r=1}^{s} u_r Y_{rj}^2 - u_o}{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{h=1}^{H} \vartheta_h X_{hj}^2} \leq 1, \qquad j = 1, \ldots, n \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, \qquad r = 1, \ldots, s,\ d = 1, \ldots, D \\
& \qquad i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H.
\end{aligned} \tag{8}
$$

Similar to the CRS case, the coordination efficiency cannot be obtained *via* the equation $\tilde{\rho}_o^{\mathrm{VRS}} = \frac{\tilde{E}_o^{\mathrm{VRS}}}{\tilde{\pi}_o^{\mathrm{VRS}}}$ under the VRS assumption. And it is calculated by the following model:

$$
\tilde{\rho}_o^{\mathrm{VRS}} = \max_{u_r, w_d, v_i, \mu_l, \vartheta_h, u_o, w_o} \frac{\sum_{l=1}^{L} \mu_l Y_{lo}^1 + \sum_{r=1}^{s} u_r Y_{ro}^2 - w_o - u_o}{\sum_{i=1}^{m} v_i \left(\tilde{E}_o^{1\text{-VRS}} X_{io}^1\right) + \sum_{h=1}^{H} \vartheta_h \left(\tilde{E}_o^{2\text{-VRS}} X_{ho}^2\right)}
$$

$$
\begin{aligned}
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - w_o}{\sum_{i=1}^{m} v_i X_{ij}^1} \le 1, && j = 1, \ldots, n \\
& \frac{\sum_{r=1}^{s} u_r Y_{rj}^2 - u_o}{\sum_{d=1}^{D} w_d Z_{dj} + \sum_{h=1}^{H} \vartheta_h X_{hj}^2} \le 1, && j = 1, \ldots, n \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \ge 0, && r = 1, \ldots, s,\ d = 1, \ldots, D \\
& && i = 1, \ldots, m,\ l = 1, \ldots, L, \\
& && h = 1, \ldots, H
\end{aligned} \tag{9}
$$

where $\tilde{E}_o^{2\text{-VRS}}$ is the VRS efficiency of the second stage. $\tilde{E}_o^{1\text{-VRS}}$ is the minimum reduction ratio of input X_{io}^1, which could be estimated by the following model:

$$
\begin{aligned}
\tilde{E}_o^{1\text{-VRS}} = & \max_{w_d^1, v_i, \mu_l, w_o} \frac{\sum_{d=1}^{D} w_d^1 \left(\tilde{E}_o^{2\text{-VRS}} Z_{do}\right) + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o}{\sum_{i=1}^{m} v_i X_{io}^1} \\
\text{s.t. } & \frac{\sum_{d=1}^{D} w_d^1 Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - w_o}{\sum_{i=1}^{m} v_i X_{ij}^1} \le 1, && j = 1, \ldots, n \\
& w_d^1, v_i, \mu_l \ge 0, && d = 1, \ldots, D,\ i = 1, \ldots, m,\ l = 1, \ldots, L.
\end{aligned} \tag{10}
$$

Similarly, when the system efficiency with coordination $\tilde{E}_o^{\text{VRS}}$ and coordination efficiency $\tilde{\rho}_o^{\text{VRS}}$ are obtained, we can get the system efficiency without coordination under the VRS assumption *via* $\tilde{\pi}_o^{\text{VRS}} = \frac{\tilde{E}_o^{\text{VRS}}}{\tilde{\rho}_o^{\text{VRS}}}$.

4. Coordination efficiency for general two-stage network system based on envelopment DEA models

In this section, we further propose the envelopment DEA model for measuring the coordination efficiency in general two-stage network system. Likewise, both CRS and VRS cases are considered.

4.1. CRS case

Under the CRS assumption, the envelopment DEA model for estimating the system efficiency with coordination $\tilde{E}_o^{\text{CRS\_env}}$ of DMU_o is as follows:

$$
\begin{aligned}
& \min_{\eta_j, \lambda_j, \tilde{E}_o^{\text{CRS\_env}}, \tilde{Z}_{do}} \tilde{E}_o^{\text{CRS\_env}} \\
\text{s.t. } & \sum_{j=1}^{n} \eta_j X_{ij}^1 \le \tilde{E}_o^{\text{CRS\_env}} X_{io}^1, && i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \le \tilde{E}_o^{\text{CRS\_env}} X_{ho}^2, && h = 1, \ldots, H \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \ge Y_{ro}^2, && r = 1, \ldots, s \\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \ge Y_{lo}^1, && l = 1, \ldots, L \\
& \sum_{j=1}^{n} \eta_j Z_{dj} \ge \tilde{Z}_{do}, && d = 1, \ldots, D
\end{aligned}
$$

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_j Z_{dj} \le \tilde{Z}_{do}, && d = 1, \dots, D \\
&\eta_j, \lambda_j \ge 0, && j = 1, \dots, n.
\end{aligned} \tag{11}$$

Denote $\tilde{E}_o^{1\_\mathrm{CRS\_env}}$ as the minimum reduction ratio of input X_{io}^1 if not considering the coordination between the two sub-stages. It can be calculated by:

$$\begin{aligned}
\min_{\eta_j, \tilde{E}_o^{1_\mathrm{CRS_env}}} \quad & \tilde{E}_o^{1_\mathrm{CRS_env}} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \eta_j X_{ij}^1 \le \tilde{E}_o^{1_\mathrm{CRS_env}} X_{io}^1, && i = 1, \dots, m \\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \ge Y_{lo}^1, && l = 1, \dots, L \\
& \sum_{j=1}^{n} \eta_j Z_{dj} \ge \tilde{E}_o^{2_\mathrm{CRS_env}} Z_{do}, && d = 1, \dots, D \\
& \eta_j \ge 0, && j = 1, \dots, n
\end{aligned} \tag{12}$$

where $\tilde{E}_o^{2\_\mathrm{CRS\_env}}$ is the CCR efficiency of the second stage and is calculated by:

$$\begin{aligned}
\min_{\lambda_j, \tilde{E}_o^{2_\mathrm{CRS_env}}} \quad & \tilde{E}_o^{2_\mathrm{CRS_env}} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j Z_{dj} \le \tilde{E}_o^{2_\mathrm{CRS_env}} Z_{do}, && d = 1, \dots, D \\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \le \tilde{E}_o^{2_\mathrm{CRS_env}} X_{ho}^2, && h = 1, \dots, H \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \ge Y_{ro}^2, && r = 1, \dots, s \\
& \lambda_j \ge 0, && j = 1, \dots, n.
\end{aligned} \tag{13}$$

Denote $\tilde{\rho}_o^{\mathrm{CRS\_env}}$ as the coordination efficiency of DMU_o, which is calculated by:

$$\begin{aligned}
\min_{\eta_j, \lambda_j, \tilde{\rho}_o^{\mathrm{CRS_env}}, \tilde{Z}_{do}} \quad & \tilde{\rho}_o^{\mathrm{CRS_env}} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \eta_j X_{ij}^1 \le \tilde{\rho}_o^{\mathrm{CRS_env}} \left(\tilde{E}_o^{1_\mathrm{CRS_env}} X_{io}^1 \right), && i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \le \tilde{\rho}_o^{\mathrm{CRS_env}} \left(\tilde{E}_o^{2_\mathrm{CRS_env}} X_{ho}^2 \right), && h = 1, \dots, H \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \ge Y_{ro}^2, && r = 1, \dots, s \\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \ge Y_{lo}^1, && l = 1, \dots, L
\end{aligned}$$

$$
\begin{aligned}
&\sum_{j=1}^{n} \eta_j Z_{dj} \geq \tilde{Z}_{do}, && d=1,\ldots,D\\
&\sum_{j=1}^{n} \lambda_j Z_{dj} \leq \tilde{Z}_{do}, && d=1,\ldots,D\\
&\eta_j, \lambda_j \geq 0, && j=1,\ldots,n
\end{aligned} \tag{14}
$$

when the system efficiency with coordination $\tilde{E}_o^{\text{CRS\_env}}$ and coordination efficiency $\tilde{\rho}_o^{\text{CRS\_env}}$ are obtained, we can get the system efficiency without coordination *via* $\tilde{\pi}_o^{\text{CRS\_env}} = \frac{\tilde{E}_o^{\text{CRS\_env}}}{\tilde{\rho}_o^{\text{CRS\_env}}}$.

4.2. VRS case

Under the VRS assumption, the DEA model for estimating the system efficiency with coordination of DMU_o is as follows:

$$
\begin{aligned}
\min_{\eta_j,\lambda_j,\tilde{E}_o^{\text{VRS\_env}},\tilde{Z}_{do}} \quad & \tilde{E}_o^{\text{VRS\_env}}\\
\text{s.t.} \quad & \sum_{j=1}^{n} \eta_j X_{ij}^1 \leq \tilde{E}_o^{\text{VRS\_env}} X_{io}^1, && i=1,\ldots,m\\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \leq \tilde{E}_o^{\text{VRS\_env}} X_{ho}^2, && h=1,\ldots,H\\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \geq Y_{ro}^2, && r=1,\ldots,s\\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \geq Y_{lo}^1, && l=1,\ldots,L\\
& \sum_{j=1}^{n} \eta_j Z_{dj} \geq \tilde{Z}_{do}, && d=1,\ldots,D\\
& \sum_{j=1}^{n} \lambda_j Z_{dj} \leq \tilde{Z}_{do}, && d=1,\ldots,D\\
& \sum_{j=1}^{n} \eta_j = \sum_{j=1}^{n} \lambda_j = 1\\
& \eta_j, \lambda_j \geq 0, && j=1,\ldots,n.
\end{aligned} \tag{15}
$$

Denote $\tilde{E}_o^{\text{1-VRS\_env}}$ as the minimum reduction ratio of input X_{io}^1 if not considering the coordination between the two sub-stages. It can be calculated by:

$$
\begin{aligned}
\min_{\eta_j,\tilde{E}_o^{\text{1-VRS\_env}}} \quad & \tilde{E}_o^{\text{1-VRS\_env}}\\
\text{s.t.} \quad & \sum_{j=1}^{n} \eta_j X_{ij}^1 \leq \tilde{E}_o^{\text{1-VRS\_env}} X_{io}^1, && i=1,\ldots,m\\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \geq Y_{lo}^1, && l=1,\ldots,L
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{j=1}^{n} \eta_j Z_{dj} \geq \tilde{E}_o^{2\_\mathrm{VRS\_env}} Z_{do}, && d = 1, \ldots, D \\
& \sum_{j=1}^{n} \eta_j = 1 \\
& \eta_j \geq 0, && j = 1, \ldots, n
\end{aligned} \tag{16}
$$

where $\tilde{E}_o^{2\_\mathrm{VRS\_env}}$ is the BCC efficiency of the second stage and is calculated by:

$$
\begin{aligned}
\min_{\lambda_j, \tilde{E}_o^{2\_\mathrm{VRS\_env}}} \ & \tilde{E}_o^{2\_\mathrm{VRS\_env}} \\
\text{s.t.} \ & \sum_{j=1}^{n} \lambda_j Z_{dj} \leq \tilde{E}_o^{2\_\mathrm{VRS\_env}} Z_{do}, && d = 1, \ldots, D \\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \leq \tilde{E}_o^{2\_\mathrm{VRS\_env}} X_{ho}^2, && h = 1, \ldots, H \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \geq Y_{ro}^2, && r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0, && j = 1, \ldots, n.
\end{aligned} \tag{17}
$$

Denote $\tilde{\rho}_o^{\mathrm{VRS\_env}}$ as the coordination efficiency of DMU$_o$, which is calculated by:

$$
\begin{aligned}
\min_{\eta_j, \lambda_j, \tilde{\rho}_o^{\mathrm{VRS\_env}}, \tilde{Z}_{do}} \ & \tilde{\rho}_o^{\mathrm{VRS\_env}} \\
\text{s.t.} \ & \sum_{j=1}^{n} \eta_j X_{ij}^1 \leq \tilde{\rho}_o^{\mathrm{VRS\_env}} \left(\tilde{E}_o^{1\_\mathrm{VRS\_env}} X_{io}^1 \right), && i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j X_{hj}^2 \leq \tilde{\rho}_o^{\mathrm{VRS\_env}} \left(\tilde{E}_o^{2\_\mathrm{VRS\_env}} X_{ho}^2 \right), && h = 1, \ldots, H \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^2 \geq Y_{ro}^2, && r = 1, \ldots, s \\
& \sum_{j=1}^{n} \eta_j Y_{lj}^1 \geq Y_{lo}^1, && l = 1, \ldots, L \\
& \sum_{j=1}^{n} \eta_j Z_{dj} \geq \tilde{Z}_{do}, && d = 1, \ldots, D \\
& \sum_{j=1}^{n} \lambda_j Z_{dj} \leq \tilde{Z}_{do}, && d = 1, \ldots, D \\
& \sum_{j=1}^{n} \eta_j = \sum_{j=1}^{n} \lambda_j = 1 \\
& \eta_j, \lambda_j \geq 0, && j = 1, \ldots, n
\end{aligned} \tag{18}
$$

when the system efficiency with coordination $\tilde{E}_o^{\text{VRS\_env}}$ and coordination efficiency $\tilde{\rho}_o^{\text{VRS\_env}}$ are obtained, we can get the system efficiency without coordination *via* $\tilde{\pi}_o^{\text{VRS\_env}} = \frac{\tilde{E}_o^{\text{VRS\_env}}}{\tilde{\rho}_o^{\text{VRS\_env}}}$.

5. Equivalence between multiplier and envelopment DEA models

In this section, we show the equivalence between multiplier and envelopment DEA models proposed in Sections 3 and 4. Both CRS and VRS cases are considered.

5.1. CRS case

Models (5)–(7) can be converted to their linear ones *via* Charnes–Cooper transformation as follows:

$$
\begin{aligned}
\tilde{E}_o^{\text{CRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 \\
\text{s.t.} \; & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 \leq 0, && j = 1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 \leq 0, && j = 1,\ldots,n \\
& \sum_{i=1}^{m} v_i X_{io}^1 + \sum_{h=1}^{H} \vartheta_h X_{ho}^2 = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, && r = 1,\ldots,s,\; d = 1,\ldots,D \\
& && i = 1,\ldots,m,\; l = 1,\ldots,L,\; h = 1,\ldots,H
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\tilde{\rho}_o^{\text{CRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 \\
\text{s.t.} \; & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 \leq 0, && j = 1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 \leq 0, && j = 1,\ldots,n \\
& \sum_{i=1}^{m} v_i \left(\tilde{E}_o^{1\text{\_CRS}} X_{io}^1\right) + \sum_{h=1}^{H} \vartheta_h \left(\tilde{E}_o^{2\text{\_CRS}} X_{ho}^2\right) = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, && r = 1,\ldots,s,\; d = 1,\ldots,D \\
& && i = 1,\ldots,m,\; l = 1,\ldots,L,\; h = 1,\ldots,H
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\tilde{E}_o^{1\text{\_CRS}} = & \max_{w_d, v_i, \mu_l} \sum_{d=1}^{D} w_d^1 \left(\tilde{E}_o^{2\text{\_CRS}} Z_{do}\right) + \sum_{l=1}^{L} \mu_l Y_{lo}^1 \\
\text{s.t.} \; & \sum_{d=1}^{D} w_d^1 Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 \leq 0, && j = 1,\ldots,n \\
& \sum_{i=1}^{m} v_i X_{io}^1 = 1 \\
& w_d^1, v_i, \mu_l \geq 0, && d = 1,\ldots,D,\; 1,\ldots,m,\; l = 1,\ldots,L.
\end{aligned} \tag{21}
$$

It should be noted that $\tilde{E}_o^{2\_\mathrm{CRS}}$ and $\tilde{E}_o^{2\_\mathrm{CRS\_env}}$ are CCR efficiency of the second stage based on multiplier and envelopment models, respectively. Obviously, they are equal, *i.e.*, $\tilde{E}_o^{2\_\mathrm{CRS}} = \tilde{E}_o^{2\_\mathrm{CRS\_env}}$. It can be found that models (19)–(21) are equivalent dual models of model (11), model (14), and model (12), respectively. Hence, the system efficiency with coordination $\tilde{E}_o^{\mathrm{CRS\_env}}$ and coordination efficiency $\tilde{\rho}_o^{\mathrm{CRS\_env}}$ based on envelopment DEA models are equal to $\tilde{E}_o^{\mathrm{CRS}}$ and $\tilde{\rho}_o^{\mathrm{CRS}}$ based on multiplier DEA models, *i.e.*, $\tilde{E}_o^{\mathrm{CRS\_env}} = \tilde{E}_o^{\mathrm{CRS}}$ and $\tilde{\rho}_o^{\mathrm{CRS\_env}} = \tilde{\rho}_o^{\mathrm{CRS}}$. Moreover, we have $\tilde{E}_o^{1\_\mathrm{CRS\_env}} = \tilde{E}_o^{1\_\mathrm{CRS}}$. It indicates that under the CRS assumption the multiplier DEA approach is equivalent to the envelopment DEA approach for general two-stage network system.

5.2. VRS case

Similarly, models (8)–(10) can be converted to their linear ones *via* Charnes–Cooper transformation as follows:

$$
\begin{aligned}
\tilde{E}_o^{\mathrm{VRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o - u_o \\
\text{s.t.} & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 - w_o \le 0, && j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 - u_o \le 0, && j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i X_{io}^1 + \sum_{h=1}^{H} \vartheta_h X_{ho}^2 = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \ge 0, && r = 1, \ldots, s,\ d = 1, \ldots, D \\
& && i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\tilde{\rho}_o^{\mathrm{VRS}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o - u_o \\
\text{s.t.} & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 - w_o \le 0, && j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 - u_o \le 0, && j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i \left(\tilde{E}_o^{1\_\mathrm{VRS}} X_{io}^1 \right) + \sum_{h=1}^{H} \vartheta_h \left(\tilde{E}_o^{2\_\mathrm{VRS}} X_{ho}^2 \right) = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \ge 0, && r = 1, \ldots, s,\ d = 1, \ldots, D \\
& && i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
\tilde{E}_o^{1\_\mathrm{VRS}} = & \max_{w_d, v_i, \mu_l} \sum_{d=1}^{D} w_d^1 \left(\tilde{E}_o^{2\_\mathrm{VRS}} Z_{do} \right) + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o \\
\text{s.t.} & \sum_{d=1}^{D} w_d^1 Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 - w_o \le 0, && j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i X_{io}^1 = 1 \\
& w_d^1, v_i, \mu_l \ge 0, && d = 1, \ldots, D,\ i = 1, \ldots, m,\ l = 1, \ldots, L.
\end{aligned} \tag{24}
$$

Here $\tilde{E}_o^{2\text{-VRS}}$ and $\tilde{E}_o^{2\text{-VRS\_env}}$ are BCC efficiency of the second stage based on multiplier and envelopment models, respectively. Hence, we have $\tilde{E}_o^{2\text{-VRS}} = \tilde{E}_o^{2\text{-VRS\_env}}$. Besides, model (24) is the equivalent dual model of model (16). Hence, we have $\tilde{E}_o^{1\text{-VRS\_env}} = \tilde{E}_o^{1\text{-VRS}}$.

The dual model of model (15) is:

$$\begin{aligned}
\tilde{E}_o^{\text{VRS_env}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o - u_o \\
\text{s.t. } & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 - w_o \leq 0, \qquad j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 - u_o \leq 0, \qquad j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i X_{io}^1 + \sum_{h=1}^{H} \vartheta_h X_{ho}^2 = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, \qquad r = 1, \ldots, s,\ d = 1, \ldots, D \\
& \qquad i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H. \qquad (25)
\end{aligned}$$

Models (25) and (22) have the same constraints and objective function. Hence, $\tilde{E}_o^{\text{VRS\_env}}$ is equal to $\tilde{E}_o^{\text{VRS}}$.

The dual model of model (18) is:

$$\begin{aligned}
\tilde{\rho}_o^{\text{VRS_env}} = & \max_{u_r, w_d, v_i, \mu_l, \vartheta_h} \sum_{r=1}^{s} u_r Y_{ro}^2 + \sum_{l=1}^{L} \mu_l Y_{lo}^1 - w_o - u_o \\
\text{s.t. } & \sum_{d=1}^{D} w_d Z_{dj} + \sum_{l=1}^{L} \mu_l Y_{lj}^1 - \sum_{i=1}^{m} v_i X_{ij}^1 - w_o \leq 0, \qquad j = 1, \ldots, n \\
& \sum_{r=1}^{s} u_r Y_{rj}^2 - \sum_{d=1}^{D} w_d Z_{dj} - \sum_{h=1}^{H} \vartheta_h X_{hj}^2 - u_o \leq 0, \qquad j = 1, \ldots, n \\
& \sum_{i=1}^{m} v_i \left(\tilde{E}_o^{1\text{-VRS_env}} X_{io}^1 \right) + \sum_{h=1}^{H} \vartheta_h \left(\tilde{E}_o^{2\text{-VRS_env}} X_{ho}^2 \right) = 1 \\
& u_r, w_d, v_i, \mu_l, \vartheta_h \geq 0, \qquad r = 1, \ldots, s,\ d = 1, \ldots, D \\
& \qquad i = 1, \ldots, m,\ l = 1, \ldots, L,\ h = 1, \ldots, H. \qquad (26)
\end{aligned}$$

Due to $\tilde{E}_o^{1\text{-VRS\_env}} = \tilde{E}_o^{1\text{-VRS}}$, models (26) and (23) have the same constraints and objective function. Hence, $\tilde{\rho}_o^{\text{VRS\_env}}$ is equal to $\tilde{\rho}_o^{\text{VRS}}$. Based on the above findings, it shows that under the VRS assumption the multiplier DEA approach is equivalent to the envelopment DEA approach for general two-stage network system.

5.3. Numerical example

To verify the proposed approach in the previous section, we use a numerical example to illustrate in this section. Considering that the VRS case is similar to the CRS case, we only show the results under the CRS.

Particularly, we use two data sets in Liang *et al.* [21] and Chen *et al.* [4], respectively. The results are shown in Tables 1 and 2. As is shown in the tables, for general two-stage network system, the efficiency estimates (system efficiency and coordination efficiency) based on multiplier and envelopment DEA approach are totally the same.

TABLE 1. Result comparison based on data set in Liang *et al.* [21].

DMU	Envelopment DEA model				Multiplier DEA model			
	$\tilde{e}_j^{1,\text{CRS-gen-env}}$	$e_j^{2,\text{CRS-gen-env}}$	$E_j^{\text{CRS-gen-env}}$	$ce_j^{\text{CRS-gen-env}}$	$\tilde{e}_j^{1,\text{CRS-gen}}$	$e_j^{2,\text{CRS-gen}}$	$E_j^{\text{CRS-gen}}$	$ce_j^{\text{CRS-gen}}$
1	1.0000	1.000	0.902	0.902	1.0000	1.000	0.902	0.902
2	0.972	0.805	0.972	1.000	0.972	0.805	0.972	1.000
3	0.800	1.000	0.832	0.901	0.800	1.000	0.832	0.901
4	1.000	0.628	1.000	1.000	1.000	0.628	1.000	1.000
5	0.416	0.598	0.462	0.992	0.416	0.598	0.462	0.992
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	0.960	0.833	0.914	0.952	0.960	0.833	0.914	0.952
8	0.770	1.000	0.917	0.986	0.770	1.000	0.917	0.986
9	0.500	1.000	0.720	1.000	0.500	1.000	0.720	1.000
10	0.900	0.668	0.900	1.000	0.900	0.668	0.900	1.000

TABLE 2. Result comparison based on data set in Chen *et al.* [4].

DMU	Envelopment DEA model				Multiplier DEA model			
	$\tilde{e}_j^{1,\text{CRS-gen-env}}$	$e_j^{2,\text{CRS-gen-env}}$	$E_j^{\text{CRS-gen-env}}$	$ce_j^{\text{CRS-gen-env}}$	$\tilde{e}_j^{1,\text{CRS-gen}}$	$e_j^{2,\text{CRS-gen}}$	$E_j^{\text{CRS-gen}}$	$ce_j^{\text{CRS-gen}}$
1	1.000	1.000	0.902	0.902	1.000	1.000	0.902	0.902
2	0.400	0.813	0.546	0.950	0.488	0.813	0.546	0.950
3	0.800	1.000	0.832	0.901	0.800	1.000	0.832	0.901
4	0.628	0.628	0.628	1.000	1.000	0.628	0.628	1.000
5	0.404	0.597	0.452	0.959	0.676	0.597	0.452	0.959
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	0.833	0.833	0.819	0.983	1.000	0.833	0.819	0.983
8	0.771	1.000	0.917	0.986	0.771	1.000	0.917	0.986
9	0.500	1.000	0.719	1.000	0.500	1.000	0.719	1.000
10	0.6681	0.669	0.628	0.940	1.000	0.669	0.628	0.940

6. CONCLUSIONS

Although two-stage network DEA and traditional DEA are widely studied, the relation between them is seldom discussed. Zhao *et al.* [30] estimated the coordination effect for the simple two-stage network system. Their proposed coordination efficiency builds a link between two-stage network DEA and traditional DEA, which is usually neglected by the existing studies, especially supply chain DEA. In this paper, we further extend the research of Zhao *et al.* [30] on coordination efficiency to general two-stage network system.

Particularly, we develop both multiplier and envelopment DEA approaches for estimating coordination efficiency based on both CRS and VRS assumptions. As is shown in Chen *et al.* [6], for network DEA the multiplier and envelopment DEA models were two different approaches. However, we still show that under the general two-stage network system the multiplier DEA approach for measuring the coordination efficiency is equivalent to the envelopment DEA approach under both CRS and VRS cases. Two numerical examples are used to verify the proposed approach in this paper.

As pointed out by Zhao *et al.* [30], the traditional "black box" efficiency is unobtainable for a two-stage network system such as a supply chain with two members. Some endeavors or sacrifices must be made in order to coordinate the two-stage system, which inevitably gives rise to an efficiency loss [30]. As a result, the coordination efficiency defined in Zhao *et al.* [30] and this paper could be used to trace any potential efficiency losses and quantify contributions or responsibilities of each division in the two-stage network system.

For future research, it may use the proposed approach to real applications with good insights. In this paper, we define the coordination efficiency under general two-stage network system, which is still a series network structure. Future research may apply the proposed approach to general network system. Moreover, whether the finding in this paper holds for efficiency decomposition of the coordination efficiency deserves for further research. In short, we hope that our study could attract the community's more attention on the interesting characteristics of network DEA, one of the most active and exciting areas currently.

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