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# TRANSPORTATION PROBLEM IN FERMATEAN FUZZY ENVIRONMENT

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**Abstract.** Due to the uncertain economic and environmental situations of the society, it is impossible to quantify the supply, demand, and transportation costs of a transportation problem precisely. The purpose of this paper is to address the transportation problem where supply, demand, and transportation costs are Fermatean fuzzy numbers. Numerous approaches to addressing transportation problems with fuzzy parameters have been suggested in the literature to date, but in each of these approaches, the parameters corresponding to the transportation problems are either generalized fuzzy numbers or Pythagorean fuzzy numbers. With the help of Fermatean fuzzy sets (FFSs), a relatively new concept, one can manage ambiguous information more simply throughout the decision-making process. As a result, we have used Fermatean fuzzy parameters to solve the transportation problem in this research. Here, we have developed an algorithm to solve the transportation problem with Fermatean fuzzy parameters and have also solved the problem using the existing method. Then, the optimal value can be obtained using arithmetic operations on Fermatean fuzzy numbers. We have solved a numerical example to demonstrate the proposed methodology, and the obtained results are presented and compared with the existing literature. The importance of the research and the scope of further research are then highlighted.

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## 1. Introduction

These days, the pressure on industries/farms to perceive most effective ways to produce and deliver goods to customers has become a formidable task. The time and place of sending the products to the customers in terms of quantities, they want in a cost-effective manner, become more challenging. Therefore, transportation problems (TPs) provide a best solution to achieve this goal. This transportation problem ensures the efficient movement and timely availability of raw materials and finished products. The TP is one of the most important mathematical programming problems which arises in many real-life decision-making problems. Hence, TP has paying more attention in the literature of management or managerial decision making. The basic idea of TP is to hand out product/goods from certain places to some destinations in such a way that the total transportation cost is minimized. The transportation problem has many real-life applications, *viz.*, scheduling, production planning, location problems, inventory control etc. The basic transportation problem was first proposed by Hitchock [22]. The transportation problems can be modeled as a standard linear programming problem (LPP), which can be

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solved by the well-known simplex method. There are several methods for finding initial basic feasible solution for transportation problem. These methods are north–west corner, row minima, column minima, matrix minima and Vogel's approximation method. Also, there are several evolutionary algorithms to solve the transportation problem. Korukoglu and Balli [25] introduced an improved Vogel's approximation method for transportation problem.

In traditional transportation problems, it has been generally assumed that decision maker is sanguine about the precise values of transportation cost, supply and demand of the product. But, in fact all the parameters of the transportation problems may not be fixed/precise due to uncontrollable factors of dynamic economic environment. Therefore, parameters of the transportation problems are imprecise due to uncertainty and dearth of proper information. The uncertainty may occur due to some factors *viz.*, (i) decision maker has no idea about the transportation cost, supply and demand when goods/products are shifted to be scheduled at the beginning time. As a result some uncertainty may occur in connection with the transportation cost, supply and demand (ii) nowadays present market situation is always flooded *i.e.*, always unstable due to competitive market and hence transportation cost, supply and demand are totally unpredictable. To combat these types of situations, the problem can be modelled using the concept of uncertainty and costs, supply and demands are treated in terms of imprecise values. In such cases fuzzy set theory plays an important role to handle such situation. A pair that includes a set and a membership function is referred to as a fuzzy set. The grade of membership is the term used to describe each value in the reference set, which is known as the universe of discourse. Therefore, fuzzy decision-making method is more important here. The concept of fuzzy set was introduced by Zadeh [44] in 1965 and it dealt with imprecision, vagueness in real world situations. In the year 1970, Belmann and Zadeh [8] introduced the concept of decision making problems involving uncertainty. Zimmermann [48] showed that the solutions obtained by fuzzy linear programming are always optimum and efficient. Chanas *et al.* [13] introduced a fuzzy linear programming model for solving transportation problems with precise costs, fuzzy supply and fuzzy demand. Chanas and Kuchta [12] proposed the idea of the optimal solution for the transportation problem with fuzzy coefficients expressed in terms of fuzzy numbers, and developed an algorithm for finding the optimal solution. Since then several researchers have proposed the transportation problems in various fuzzy environments. For more details one may refer to the works of Tada and Ishii [40], Hashmi *et al.* [21], Li and Lai [29], Liu *et al.* [33], Kundu *et al.* [28], Singh and Yadav [39], Gupta and Kumari [20], Arora [2], Bharati and Singh [9], Kumar [26], Kaur and Kumar [24], Celik and Akyuz [11] and Ahmad and Adhami [1]. However, the fuzzy set takes only a membership function and the degree of non-membership function which is just a compliment of the degree of membership function. There may be a situation where the sum of the membership function and non-membership function is greater than one. Thus, orthopair fuzzy sets have been introduced in which the membership grades of an element $x \in X$ are pairs of values in the unit interval, $\langle \mu(x), \gamma(x) \rangle$, one of which indicates support of membership in the fuzzy set and other indicates support against membership. For example, Atanassov's classical intuitionistic fuzzy sets [4,5,7] and Atanassov's second kind of intuitionistic fuzzy sets [3,6]. Recently, Yager [42,43] introduced another orthopair of fuzzy set, known as Pythagorean fuzzy set (PFS), where the square sum of the support of membership and support against membership value is equal to or less than one. PFSs have attracted the attention of many researchers within a very short period of time. There are several methods in the field of PFS to solve real-life multi-criteria, decision-making problems *viz.*, extension of TOPSIS [46], similarity measure [45], alternative queuing method [19], extended TODIM methods [18], Bonferroni mean [23], improved score function [16, 18] and many others. Several researchers have also proposed real-life applications under Pythagorean fuzzy environment. For more details one may mention the works of Li *et al.* [30], Zhou *et al.* [47], Bolturk *et al.* [10], Qin [36], Wan *et al.* [41], Lin *et al.* [31] and Chen [14]. But, if orthopair fuzzy set as $\langle 0.9, 0.6 \rangle$, where 0.9 is the support of the membership of certain criteria of a parameter and 0.6 is the support against membership then it does not follow the condition of IFS as well as PFS. However, the cubic sum of the support of membership and support against membership degrees is equal to or less than one. And in this situation Senapati and Yager [38] very recently introduced Fermatean Fuzzy set (FFS). They also showed that FFSs have more uncertain than IFSs and PFSs and are capable of handling higher level of uncertainties [37].

Based on the earlier discussions on TP and recently available several research articles on TP, there are no existing methodologies which are available on TP under Fermatean fuzzy environment. Hence, there is an essential urgency to introduce a new solution methodology for solving transportation problem in the light of Fermatean fuzzy environment. To the best of our knowledge and belief, there are no optimization models in the literature for TP in Fermatean fuzzy environment. For this fact, in this paper, we have treated cost parameters, demand and supply parameters of a TP as Fermatean fuzzy as FFSs are most fruitful fuzzy sets which are more competent to manage higher level of uncertainties.

In this paper, a method is proposed for solving transportation problems in Fermatean fuzzy environment. In the proposed method transportation cost, supply and demand of the product are represented by orthopair fuzzy set $\langle \alpha, \beta \rangle$ which satisfies the relation $0 \leq \alpha^3 + \beta^3 \leq 1$ and using score function and accuracy function of the Fermatean fuzzy sets we have converted the TP into crisp TP problems. The relation $0 \leq \alpha^3 + \beta^3 \leq 1$ is taken into account because, for all TP parameters in a Fermatean fuzzy environment, the cube sum of the support for membership and the support against membership degrees is either just one or less than one.

Then the converted TP problems have been solved by excel solver to find out the optimal solutions. Solver is a Microsoft Excel add-in program for data analysis. Using solver one can determine the maximum or minimum value of one cell by changing other cells. Finally, to illustrate the proposed method we have solved a numerical example and computed results are presented and compared with the existing literature.

The rest of the paper is organized as follows: in Section 2 some basic definitions about Fermatean fuzzy sets and Pythagorean fuzzy sets are presented. The mathematical model of TP is presented in Sections 3 and 4. In Section 6, proposed solution methodology is discussed. The results of fuzzy transportation problem are discussed in Section 7. The conclusions are drawn in Section 8.

## 2. Preliminaries

In this section some basic definitions about Fermatean fuzzy sets and Pythagorean fuzzy sets are discussed. After that some score functions are proposed to implement the entire paper.

**Definition 2.1** ([38])**.** Let $X$ be a Universal set. A Fermatean fuzzy set (FFS) is an object of the form $\tilde{F} = \{\langle x, \alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x)\rangle : x \in X\}$ where $\alpha_{\tilde{F}}(x) : X \to [0,1]$ and $\beta_{\tilde{F}}(x) : X \to [0,1]$ which satisfies the relation $0 \leq (\alpha_{\tilde{F}}(x))^3 + (\beta_{\tilde{F}}(x))^3 \leq 1, \forall x \in X$. The number $\alpha_{\tilde{F}}(x)$ and $\beta_{\tilde{F}}(x)$ are the degree of membership and non-membership of the element $x \in X$ in the FFS $\tilde{F}$.

For any FFS $\tilde{F}$ and $x \in X$, the degree of indeterminacy is represented by $\pi_F(x) = \sqrt[3]{1 - (\alpha_{\tilde{F}}(x))^3 - (\beta_{\tilde{F}}(x))^3}$. It is to be noted that, for simplicity, we shall denote the object $\tilde{F} = \langle \alpha_{\tilde{F}}, \beta_{\tilde{F}} \rangle$ instead of $\tilde{F} = \{\langle x, \alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x)\rangle : x \in X\}$.

**Definition 2.2** ([38])**.** Let $\tilde{F}_1 = \left\langle \alpha_{\tilde{F}_1}, \beta_{\tilde{F}_1} \right\rangle$ and $\tilde{F}_2 = \left\langle \alpha_{\tilde{F}_2}, \beta_{\tilde{F}_2} \right\rangle$ be two FFSs. Then the basic arithmetical operations of two Fermatean fuzzy sets $\tilde{F}_1$ and $\tilde{F}_2$ are defined as follows:

(i) Addition: $\tilde{F}_1 \oplus \tilde{F}_2 = \left\langle \sqrt[3]{(\alpha_{\tilde{F}_1})^3 + (\alpha_{\tilde{F}_2})^3 - (\alpha_{\tilde{F}_1})^3(\alpha_{\tilde{F}_2})^3}, \beta_{\tilde{F}_1}\beta_{\tilde{F}_2} \right\rangle$.

(ii) Multiplication: $\tilde{F}_1 \otimes \tilde{F}_2 = \left\langle \alpha_{\tilde{F}_1}\alpha_{\tilde{F}_2}, \sqrt[3]{(\beta_{\tilde{F}_1})^3 + (\beta_{\tilde{F}_2})^3 - (\beta_{\tilde{F}_1})^3(\beta_{\tilde{F}_2})^3} \right\rangle$.

(iii) Scalar Multiplication: $\lambda \odot \tilde{F} = \left\langle \sqrt[3]{1 - (1 - (\alpha_{\tilde{F}})^3)^\lambda}, (\beta_{\tilde{F}})^\lambda \right\rangle$ provided $\lambda > 0$.

(iv) Exponent: $\tilde{F}^\lambda = \left\langle (\alpha_{\tilde{F}})^\lambda, \sqrt[3]{1 - (1 - (\beta_{\tilde{F}})^3)^\lambda}, \right\rangle$.

**Definition 2.3** ([38])**.** Let $\tilde{F}_1 = \left\langle \alpha_{\tilde{F}_1}, \beta_{\tilde{F}_1} \right\rangle$ and $\tilde{F}_2 = \left\langle \alpha_{\tilde{F}_2}, \beta_{\tilde{F}_2} \right\rangle$ be two FFSs. Then their set operations are defined as follows:

(i) Union: $\tilde{F}_1 \cup \tilde{F}_2 = \langle \max(\alpha_{\tilde{F}_1}\alpha_{\tilde{F}_2}), \min(\beta_{\tilde{F}_1}\beta_{\tilde{F}_2}) \rangle$.

(ii) Intersection: $\tilde{F}_1 \cap \tilde{F}_2 = \langle \min(\alpha_{\tilde{F}_1}\alpha_{\tilde{F}_2}), \max(\beta_{\tilde{F}_1}\beta_{\tilde{F}_2}) \rangle$.

(iii) Compliment: $(\tilde{F}_1)' = \langle \beta_{\tilde{F}_1}, \alpha_{\tilde{F}_1} \rangle$.

For more details one may refer to the works of Senapati and Yager [37, 38].

**Definition 2.4** ([37, 38])**.** Let $\tilde{F} = \langle \alpha_{\tilde{F}}, \beta_{\tilde{F}} \rangle$ be any FFS then score function of $\tilde{F}$ denoted by $S_F(\tilde{F})$ and is defined by $S_F(\tilde{F}) = (\alpha^3_{\tilde{F}} - \beta^3_{\tilde{F}})$. Here, the score function $S_F(\tilde{F}) \in [-1, 1]$.

But we have defined some score functions $S_F(\tilde{F}), S_F(\tilde{F}) \in [0, 1]$ which are as follows:

(i) $S_F(\tilde{F}) = \frac{1}{2}(1 + \alpha^3_{\tilde{F}} - \beta^3_{\tilde{F}})$.
(ii) $S_F(\tilde{F}) = \frac{1}{3}(1 + 2\alpha^3_{\tilde{F}} - \beta^3_{\tilde{F}})$.
(iii) $S_F(\tilde{F}) = \frac{1}{2}(1 + \alpha^2_{\tilde{F}} - \beta^2_{\tilde{F}})|\alpha_{\tilde{F}} - \beta_{\tilde{F}}|$.

**Definition 2.5** ([38])**.** Let $\tilde{F} = \langle \alpha_{\tilde{F}}, \beta_{\tilde{F}} \rangle$ be any FFS then accuracy function of $\tilde{F}$ denoted by $H_F(\tilde{F})$ and is defined by $H_F(\tilde{F}) = \alpha^3_{\tilde{F}} + \beta^3_{\tilde{F}}$.

**Definition 2.6.** Let $\tilde{F}_1 = \langle \alpha_{\tilde{F}_1}, \beta_{\tilde{F}_1} \rangle$ and $\tilde{F}_2 = \langle \alpha_{\tilde{F}_2}, \beta_{\tilde{F}_2} \rangle$ be two FFSs. Then ranking or order relations of $\tilde{F}_1$ and $\tilde{F}_2$ are defined as follows:

(i) $\tilde{F}_1 \succ_{\max} \tilde{F}_2$ iff either $(S_F(\tilde{F}_1) > S_F(\tilde{F}_2))$ or $(S_F(\tilde{F}_1) = S_F(\tilde{F}_2)$ and $H_F(\tilde{F}_1) > H_F(\tilde{F}_2))$.
(ii) $\tilde{F}_1 \prec_{\min} \tilde{F}_2$ iff either $(S_F(\tilde{F}_1) < S_F(\tilde{F}_2))$ or $(S_F(\tilde{F}_1) = S_F(\tilde{F}_2)$ and $H_F(\tilde{F}_1) < H_F(\tilde{F}_2))$.
(iii) $\tilde{F}_1 \simeq_{\text{equal}} \tilde{F}_2$ iff $S_F(\tilde{F}_1) = S_F(\tilde{F}_2)$ and $H_F(\tilde{F}_2) = H_F(\tilde{F}_2)$.

**Definition 2.7.** Let $X$ be a Universal set. A Pythagorean fuzzy set (PFS) is an object of the form $\tilde{P} = \{\langle x, \alpha_{\tilde{P}}(x), \beta_{\tilde{P}}(x) \rangle : x \in X\}$ where $\alpha_{\tilde{P}}(x) : X \to [0, 1]$ and $\beta_{\tilde{P}}(x) : X \to [0, 1]$ which satisfies the relation $0 \le (\alpha_{\tilde{P}}(x))^2 + (\beta_{\tilde{P}}(x))^2 \le 1, \forall x \in X$. The number $\alpha_{\tilde{P}}(x)$ and $\beta_{\tilde{P}}(x)$ are the degree of membership and non-membership of the element $x \in X$ in the PFS $\tilde{P}$.

For any PFS $\tilde{P}$ and $x \in X$, the degree of indeterminacy is represented by $\pi_P(x) = \sqrt{1 - (\alpha_{\tilde{P}}(x))^2 - (\beta_{\tilde{P}}(x))^2}$. It is to be noted that, for simplicity, we shall denote the object $\tilde{P} = \langle \alpha_{\tilde{P}}, \beta_{\tilde{P}} \rangle$ instead of $\tilde{P} = \{\langle x, \alpha_{\tilde{P}}(x), \beta_{\tilde{P}}(x) \rangle : x \in X\}$.

**Definition 2.8.** Let $\tilde{P} = \langle \alpha_{\tilde{P}}, \beta_{\tilde{P}} \rangle$ be any PFS then score function of $\tilde{P}$ denoted by $S_P(\tilde{P})$ and is defined by $S_P(\tilde{P}) = \frac{1}{2}(1 + \alpha^2_{\tilde{P}} - \beta^2_{\tilde{P}})$.

**Definition 2.9.** Let $\tilde{P} = \langle \alpha_{\tilde{P}}, \beta_{\tilde{P}} \rangle$ be any PFS then accuracy function of $\tilde{P}$ denoted by $H_P(\tilde{P})$ and is defined by $H_P(\tilde{P}) = \alpha^2_{\tilde{P}} + \beta^2_{\tilde{P}}$.

## 3. Mathematical formulation of transportation problem

Suppose a farm/company has $m$ warehouses and $n$ outlets. A single item/product is to be shifted from the warehouses to the outlets. Also assumed that each warehouse has a given amount of supply and each outlet has a given amount of demand. We have also assumed that transportation costs between every pair of warehouse and outlet is known, and these costs are assumed to be linear.

Then the mathematical formulation of the transportation problem is as follows:

$$\text{Minimize } z_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$$

$$\text{Subject to: } \sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for } i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for } j = 1, 2, \ldots, n$$
$$x_{ij} \geq 0 \qquad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \tag{1}$$

where

$m$ The total number of warehouses.
$n$ The total number of outlets.
$i$ The warehouse index for all $m$.
$j$ The outlet index for all $n$.
$a_i$ The available supply amount.
$b_j$ The market demand amount.
$x_{ij}$ The number of unit product shifted from warehouse to outlet.
$c_{ij}$ The shifted cost of one unit of product.
$z_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$ The total transportation cost.

## 4. Mathematical formulation of fuzzy transportation problems

In traditional transportation problems it is assumed that all the parameters are fixed or precise valued. But, in reality all the parameters of the transportation problems may not be fixed/precise due to uncontrollable situations of changing economic environment. Several researchers [15, 32, 35] have formulated and solved fuzzy transportation problems by considering the transportation cost, supply and demand as fuzzy numbers. The fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation parameters *i.e.*, cost, supply and demand, may be formulated as follows:

$$\text{Minimize } \tilde{z}_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \tilde{c}_{ij}$$
$$\text{Subject to: } \sum_{j=1}^{n} x_{ij} = \tilde{a}_i \qquad \text{for } i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j \qquad \text{for } j = 1, 2, \ldots, n$$
$$x_{ij} \geq 0 \qquad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \tag{2}$$

where

$\tilde{a}_i$ The available supply amount.
$\tilde{b}_j$ The market demand amount.
$x_{ij}$ The number of unit product shifted from warehouse to outlet.
$\tilde{c}_{ij}$ The shifted cost of one unit of product.
$\tilde{z}_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \tilde{c}_{ij}$ The total transportation cost.

## 5. Mathematical formulation of transportation problems in Fermatean fuzzy environment

Now, if we replace the fuzzy parameters $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ of (2) by Fermatean fuzzy parameters then the mathematical model (2) reduces to as follows:

$$\text{Minimize } \langle \alpha_{z_0}, \beta_{\tilde{z}_0} \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \odot \langle \alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}} \rangle$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = \langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i} \rangle \quad \text{for } i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = \left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j} \right\rangle \quad \text{for } j = 1, 2, \ldots, n \tag{3}$$

where

$$0 \leq (\alpha_{\tilde{z}_0})^3 + (\beta_{\tilde{z}_0})^3 \leq 1$$
$$0 \leq (\alpha_{\tilde{a}_i})^3 + (\beta_{\tilde{a}_i})^3 \leq 1 \quad \text{for } i = 1, 2, \ldots, m$$
$$0 \leq (\alpha_{\tilde{b}_j})^3 + (\beta_{\tilde{b}_j})^3 \leq 1 \quad \text{for } j = 1, 2, \ldots, n$$
$$x_{ij} \geq 0, 0 \leq (\alpha_{\tilde{c}_{ij}})^3 + (\beta_{\tilde{c}_{ij}})^3 \leq 1 \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n.$$

Now, problem (3) is a mathematical model of a transportation problem in Fermatean fuzzy environment. It is to be noted that if $\sum_{i=1}^{m} \oplus \langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i} \rangle = \sum_{j=1}^{n} \oplus \left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j} \right\rangle$ then fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem. The symbol $\sum \oplus$ denoted as summation in terms of Fermatean fuzzy addition sense.

## 6. Proposed method for solving transportation in Fermatean fuzzy environment

For finding the initial basic feasible solution and the optimal solution for the fuzzy transportation problem, there are a number of methods available in the existing literature. These methods include the north–west corner, row minima, column minima and matrix minima, as well as Vogel's approximation method. The fuzzy transportation problem can also be solved using a number of evolutionary algorithms. However, none of them have employed Fermatean fuzzy sets to address the fuzzy transportation problems. Therefore, in this research, we used the proposed method and a well-known Excel solver to solve the fuzzy transportation problem in a Fermatean fuzzy environment. The different steps of the proposed method are as follows:

**Step 1.** Calculate the score function value of each and every Fermatean fuzzy cost, Fermatean fuzzy supply and Fermatean fuzzy demand.
**Step 2.** To check the transportation problem, calculate total supply and total demand.
**Step 3.** If total supply is equal to total demand, the transportation is balanced transportation problem and go to Step 7; otherwise go to next step.
**Step 4.** If demand is not equal to supply, then add dummy variable on demand/supply and make it balance and go to Step 7.
**Step 5.** Formulate the transportation problem as linear programming problem (LPP).
**Step 6.** Solve the balanced transportation problem by using excel solver to find optimum solution.

## 7. Result and discussion with a numerical example

In this section, we provide a suitable example to illustrate our proposed solution methodology. This example has been taken from Kumar *et al.* [27]. It is important to note that, all the parameters of the fuzzy transportation problem are satisfied the Pythagorean fuzzy set property and also satisfied Fermatean fuzzy set property. Here WH1, WH2 and WH3 are three warehouses and O1, O2, O3 and O4 are four outlets/destinations. For solving the fuzzy transportation problems, we have applied our proposed algorithm. Here, we have considered two cases which are as follows:

**Case 1.** When Fermatean fuzzy costs are replaced by score function values.
**Case 2.** When Pythagorean fuzzy costs are replaced by score values (Tab. 1).

TABLE 1. Input Data for transportation problem with Fermatean fuzzy costs.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | $\langle 0.1, 0.9\rangle$ | $\langle 0.2, 0.8\rangle$ | $\langle 0.1, 0.8\rangle$ | $\langle 0.1, 0.9\rangle$ | $\langle 0.7, 0.1\rangle$ |
| WH2 | $\langle 0.01, 0.99\rangle$ | $\langle 0.3, 0.9\rangle$ | $\langle 0.3, 0.8\rangle$ | $\langle 0.1, 0.7\rangle$ | $\langle 0.8, 0.1\rangle$ |
| WH3 | $\langle 0.1, 0.8\rangle$ | $\langle 0.4, 0.8\rangle$ | $\langle 0.4, 0.9\rangle$ | $\langle 0.2, 0.9\rangle$ | $\langle 0.9, 0.1\rangle$ |
| Demand | $\langle 0.4, 0.7\rangle$ | $\langle 0.7, 0.3\rangle$ | $\langle 0.8, 0.1\rangle$ | $\langle 0.60832, 0.4\rangle$ | |

TABLE 2. Parameters for LPP (4).

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | $S_F\langle 0.1, 0.9\rangle$ | $S_F\langle 0.2, 0.8\rangle$ | $S_F\langle 0.1, 0.8\rangle$ | $S_F\langle 0.1, 0.9\rangle$ | $S_F\langle 0.7, 0.1\rangle$ |
| WH2 | $S_F\langle 0.01, 0.99\rangle$ | $S_F\langle 0.3, 0.9\rangle$ | $S_F\langle 0.3, 0.8\rangle$ | $S_F\langle 0.1, 0.7\rangle$ | $S_F\langle 0.8, 0.1\rangle$ |
| WH3 | $S_F\langle 0.1, 0.8\rangle$ | $S_F\langle 0.4, 0.8\rangle$ | $S_F\langle 0.4, 0.9\rangle$ | $S_F\langle 0.2, 0.9\rangle$ | $S_F\langle 0.9, 0.1\rangle$ |
| Demand | $S_F\langle 0.4, 0.7\rangle$ | $S_F\langle 0.7, 0.3\rangle$ | $S_F\langle 0.8, 0.1\rangle$ | $S_F\langle 0.60832, 0.4\rangle$ | |

**Case 1.** When Fermatean fuzzy costs are replaced by score function values.

To solve the above problem we have solved the following LPP (4) where all the parameters are given in Table 2.

$$\text{Minimize } S_F\langle \alpha_{z_0}, \beta_{\tilde{z}_0}\rangle = \sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij} \odot S_F\left(\langle \alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}\rangle\right)$$

$$\begin{aligned} \text{Subject to: } & \sum_{j=1}^{4} x_{ij} = S_F(\langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i}\rangle) && \text{for } i = 1,2,3 \\ & \sum_{i=1}^{3} x_{ij} = S_F\left(\left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j}\right\rangle\right) && \text{for } j = 1,2,3,4 \end{aligned} \tag{4}$$

where

$$\begin{aligned} & 0 \le (\alpha_{\tilde{z}_0})^3 + (\beta_{\tilde{z}_0})^3 \le 1 \\ & 0 \le (\alpha_{\tilde{a}_i})^3 + (\beta_{\tilde{a}_i})^3 \le 1 && \text{for } i = 1,2,3 \\ & 0 \le \left(\alpha_{\tilde{b}_j}\right)^3 + \left(\beta_{\tilde{b}_j}\right)^3 \le 1 && \text{for } j = 1,2,3,4 \\ & x_{ij} \ge 0, 0 \le \left(\alpha_{\tilde{c}_{ij}}\right)^3 + \left(\beta_{\tilde{c}_{ij}}\right)^3 \le 1 && \text{for } i = 1,2,3 \text{ and } j = 1,2,3,4. \end{aligned}$$

Now, for comparison purposes, we have taken another example of transportation problem with Pythagorean fuzzy costs which are as follows (Tabs. 3–6).

It is to be noted that, we have taken same example but transportation costs satisfies Pythagorean fuzzy set property which is defined in Definition 2.8.

**Case 2.** When Pythagorean fuzzy costs are replaced by score values.

To solve the above problem we have solved the following LPP (5) where all the parameters are given in Tables 7 and 8.

$$\text{Minimize } S_P\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = \sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij} \odot S_P\left(\langle \alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}}\rangle\right)$$

Table 3. Evaluation of score when $S_F(\tilde{F}) = \frac{1}{2}\left(1 + \alpha_{\tilde{F}}^3 - \beta_{\tilde{F}}^3\right)$.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | 0.136 | 0.248 | 0.245 | 0.136 | 0.671 |
| WH2 | 0.015 | 0.149 | 0.258 | 0.329 | 0.756 |
| WH3 | 0.245 | 0.276 | 0.168 | 0.140 | 0.864 |
| Demand | 0.361 | 0.658 | 0.756 | 0.581 | |

Table 4. Evaluation of score when $S_F(\tilde{F}) = \frac{1}{3}\left(1 + 2\alpha_{\tilde{F}}^3 - \beta_{\tilde{F}}^3\right)$.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | 0.0910 | 0.1680 | 0.1633 | 0.0910 | 0.5617 |
| WH2 | 0.0099 | 0.1083 | 0.1807 | 0.2197 | 0.6743 |
| WH3 | 0.1633 | 0.2053 | 0.1330 | 0.0957 | 0.8190 |
| Demand | 0.2617 | 0.5530 | 0.6743 | 0.4621 | |

Table 5. Evaluation of score when $S_F(\tilde{F}) = \frac{1}{2}\left(1 + \alpha_{\tilde{F}}^2 - \beta_{\tilde{F}}^2\right)|\alpha_{\tilde{F}} - \beta_{\tilde{F}}|$.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | 0.0800 | 0.1200 | 0.1295 | 0.0800 | 0.4440 |
| WH2 | 0.0098 | 0.0840 | 0.1125 | 0.1560 | 0.5705 |
| WH3 | 0.1295 | 0.1040 | 0.0875 | 0.0805 | 0.7200 |
| Demand | 0.1005 | 0.2800 | 0.5705 | 0.1260 | |

Table 6. Input Data for transportation problem with Pythagorean fuzzy costs.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | $\langle 0.1, 0.9\rangle$ | $\langle 0.2, 0.8\rangle$ | $\langle 0.1, 0.8\rangle$ | $\langle 0.1, 0.9\rangle$ | $\langle 0.7, 0.1\rangle$ |
| WH2 | $\langle 0.01, 0.99\rangle$ | $\langle 0.3, 0.9\rangle$ | $\langle 0.3, 0.8\rangle$ | $\langle 0.1, 0.7\rangle$ | $\langle 0.8, 0.1\rangle$ |
| WH3 | $\langle 0.1, 0.8\rangle$ | $\langle 0.4, 0.8\rangle$ | $\langle 0.4, 0.9\rangle$ | $\langle 0.2, 0.9\rangle$ | $\langle 0.9, 0.1\rangle$ |
| Demand | $\langle 0.4, 0.7\rangle$ | $\langle 0.7, 0.3\rangle$ | $\langle 0.8, 0.1\rangle$ | $\langle 0.60832, 0.4\rangle$ | |

$$
\begin{aligned}
\text{Subject to :} \quad & \sum_{j=1}^{4} x_{ij} = S_P(\langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i}\rangle) && \text{for } i = 1, 2, 3 \\
& \sum_{i=1}^{3} x_{ij} = S_P\left(\left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j}\right\rangle\right) && \text{for } j = 1, 2, 3, 4
\end{aligned} \tag{5}
$$

where

$$
\begin{aligned}
& 0 \le (\alpha_{\tilde{z}_0})^2 + (\beta_{\tilde{z}_0})^2 \le 1 \\
& 0 \le (\alpha_{\tilde{a}_i})^2 + (\beta_{\tilde{a}_i})^2 \le 1 && \text{for } i = 1, 2, 3 \\
& 0 \le \left(\alpha_{\tilde{b}_j}\right)^2 + \left(\beta_{\tilde{b}_j}\right)^2 \le 1 && \text{for } j = 1, 2, 3, 4
\end{aligned}
$$

TABLE 7. Parameter for LPP (6).

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | $S_P\langle 0.1, 0.9\rangle$ | $S_P\langle 0.2, 0.8\rangle$ | $S_P\langle 0.1, 0.8\rangle$ | $S_P\langle 0.1, 0.9\rangle$ | $S_P\langle 0.7, 0.1\rangle$ |
| WH2 | $S_P\langle 0.01, 0.99\rangle$ | $S_P\langle 0.3, 0.9\rangle$ | $S_P\langle 0.3, 0.8\rangle$ | $S_P\langle 0.1, 0.7\rangle$ | $S_P\langle 0.8, 0.1\rangle$ |
| WH3 | $S_P\langle 0.1, 0.8\rangle$ | $S_P\langle 0.4, 0.8\rangle$ | $S_P\langle 0.4, 0.9\rangle$ | $S_P\langle 0.2, 0.9\rangle$ | $S_P\langle 0.9, 0.1\rangle$ |
| Demand | $S_P\langle 0.4, 0.7\rangle$ | $S_P\langle 0.7, 0.3\rangle$ | $S_P\langle 0.8, 0.1\rangle$ | $S_P\langle 0.60832, 0.4\rangle$ | |

TABLE 8. Evaluation of score when $S_F(\tilde{F}) = \frac{1}{2}\Big(1 + \alpha^2_{\tilde{F}} - \beta^2_{\tilde{F}}\Big)$.

| | O1 | O2 | O3 | O4 | Supply |
|---|---|---|---|---|---|
| WH1 | 0.1000 | 0.2000 | 0.1850 | 0.1000 | 0.7400 |
| WH2 | 0.0100 | 0.1400 | 0.2250 | 0.2600 | 0.8150 |
| WH3 | 0.8150 | 0.2600 | 0.1750 | 0.1150 | 0.9000 |
| Demand | 0.3350 | 0.7000 | 0.8150 | 0.6050 | |

TABLE 9. Numerical results.

| Case | Score function $S_F(\tilde{F})$ used | Optimal solutions | Minimum cost |
|---|---|---|---|
| Case 1 | $\dfrac{1}{2}\big(1 + \alpha^3_{\tilde{F}} - \beta^3_{\tilde{F}}\big)$ | $x_{12} = 0.198,\ x_{14} = 0.473,$ $x_{21} = 0.361,\ x_{22} = 0.395,$ $x_{33} = 0.756,\ x_{34} = 0.108,$ | $S_F\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = 0.31983$ $\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = \langle 0.39526, 0.79431\rangle$ |
| Case 1 | $\dfrac{1}{3}\big(1 + 2\alpha^3_{\tilde{F}} - \beta^3_{\tilde{F}}\big)$ | $x_{12} = 0.1404,\ x_{14} = 0.4213,$ $x_{21} = 0.2617,\ x_{22} = 0.4126,$ $x_{33} = 0.6743,\ x_{34} = 0.0408$ | $H_F\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = 0.20279$ $\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = \langle 0.38228, 0.82105\rangle$ |
| Case 1 | $\dfrac{1}{2}\big(1 + \alpha^2_{\tilde{F}} - \beta^2_{\tilde{F}}\big)\lvert\alpha_{\tilde{F}} - \beta_{\tilde{F}}\rvert$ | $x_{14} = 0.1260,\ x_{21} = 0.1005,$ $x_{22} = 0.2800,\ x_{33} = 0.5705$ | $S_P\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = 0.084504$ $\langle \alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = \langle 0.35437, 0.90132\rangle$ |

$$x_{ij} \geq 0, 0 \leq \left(\alpha_{\tilde{c}_{ij}}\right)^2 + \left(\beta_{\tilde{c}_{ij}}\right)^2 \leq 1 \qquad \text{for } i = 1,2,3 \text{ and } j = 1,2,3,4.$$

LPPs is solved by using excel solver and numerical results have been shown in Table 9. Comparison results have been shown in Table 10. From Table 9, it is observed that Fermatean fuzzy cost $\langle 0.35437, 0.90132\rangle$ is less than $\langle 0.38228, 0.82105\rangle$ and $\langle 0.39526, 0.79431\rangle$. Also, $\langle 0.38228, 0.82105\rangle$ is less than $\langle 0.39526, 0.79431\rangle$.

So, sore functions $\frac{1}{2}(1+\alpha^2_{\tilde{F}}-\beta^2_{\tilde{F}})|\alpha_{\tilde{F}} - \beta_{\tilde{F}}|$ and $\frac{1}{3}(1+2\alpha^3_{\tilde{F}}-\beta^3_{\tilde{F}})$ gives the better result of a fuzzy transportation problem in Fermatean fuzzy environment. From Table 10, it seen that, the minimum cost obtained by Kumar *et al.* [27] using score function of Pythagorean fuzzy set is 0.31896. From Table 10, it also seen that, using same score function, we get same result using our proposed method. Therefore, we claimed that our method is an alternative method for solving transportation problem in fuzzy environment. It is noticed from Table 10, that Kumar *et al.* [27] have obtained minimum cost in terms of precise valued but we have obtained minimum transportation cost in terms of precise valued as well as fuzzy valued. It should be noted that no comparisons have been performed using case 1, in which the values of the score function are used in term of the Fermatean fuzzy costs, since there are no such Transportation problem exists in the Fermatean fuzzy environment. Also, further mentioned that for the purpose of numerical experiment, we have taken $3 \times 4$ size transportation problems. But we may generalize it for $m \times n$ size transportation problem and the computation of $m \times n$ size depends on the

TABLE 10. Comparison results.

| Case | Problem | Optimal solutions (our proposed method) | Minimum cost (our proposed method) | Optimal solutions by Kumar *et al.* [27] | Minimum cost by Kumar *et al.* [27] |
|---|---|---|---|---|---|
| Case 2 | LPP (6) | $x_{12} = 0.22$, $x_{14} = 0.52$ $x_{21} = 0.335$, $x_{22} = 0.48$, $x_{33} = 0.815$, $x_{34} = 0.085$ | $S_P\langle\alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = 0.31895$ $\langle\alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = \langle 0.43060, 0.77673\rangle$ | $x_{12} = 0.22$, $x_{14} = 0.52$ $x_{21} = 0.335$, $x_{22} = 0.48$, $x_{33} = 0.815$, $x_{34} = 0.085$ | $S_P\langle\alpha_{\tilde{z}_0}, \beta_{\tilde{z}_0}\rangle = 0.31895$ (Not Calculated) |

computing capacity such as memory constraint of the computing machines. Therefore, we may claim that our proposed method is a new way to tackle the uncertainty in Fermatean fuzzy environment.

## 8. Concluding remark

This article proposes a method for finding the best optimal solutions of fuzzy transportation problems, where the parameters corresponding to the transportation problem are Fermatean fuzzy sets. One of the major concerns in decision-making is the transportation problem. The costs, supplies, and demands of the transportation problem are not precise in realistic situations. Since the vast majority of real-life conditions arise from uncertain domains, the transportation problem with Fermata fuzzy parameters is more realistic than the transportation problem with exact parameters. The existing arithmetic operations of the Fermatean fuzzy sets are employed to find optimum solution. Here, we have proposed new definition of score function for FFS and for any Fermatean fuzzy set $\tilde{F}$, the sore function value lies in the unit interval *i.e.*, $0 \leq S_F(\tilde{F}) \leq 1$. From, computational results, it is mentioned that the FFSs have more appropriate than PFSs and are capable to manage higher level of uncertainties. The proposed methodology has been discussed with a numerical example. The proposed method discussed here is very simple and very easy to implement and to apply for solving realistic decision-making problems happening in the near future involving Fermatean fuzzy parameters.

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