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NEW APPROACH TO SOLVE FUZZY MULTI-OBJECTIVE MULTI-ITEM SOLID TRANSPORTATION PROBLEM

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Abstract. This paper explores the study of Multi-Objective Multi-item Solid Transportation Problem (MMSTP) under the fuzzy environment. Realizing the impact of real-life situations, here we consider MMSTP with parameters, *e.g.*, transportation cost, supply, and demand, treat as trapezoidal fuzzy numbers. Trapezoidal fuzzy numbers are then converted into nearly approximation interval numbers by using (P. Grzegorzewski, *Fuzzy Sets Syst.* **130** (2002) 321–330.) conversation rule, and we derive a new rule to convert trapezoidal fuzzy numbers into nearly approximation rough interval numbers. We derive different models of MMSTP using interval and a rough interval number. Fuzzy programming and interval programming are then applied to solve converted MMSTP. The expected value operator is used to solve MMSTP in the rough interval. Thereafter, two numerical experiments are incorporated to show the application of the proposed method. Finally, conclusions are provided with the lines of future study of this manuscript.

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1. Introduction

Transportation Problem (TP) is mainly taken into consideration at early study in Operational Research to reduce the transportation cost from sources to various destinations. Hitchcock [20] initially modelled the basic TP, by modelling it as a conventional optimization problem with two-dimensional properties, *i.e.*, supply and demand. In the classical sense of TP, the parameters, supply and demand are crisp numbers but in real situations these parameters are not always crisp. Several researchers considered TPs in a different environment and solved them by their proposed algorithm. Recently, Roy *et al.* [45] identified a new approach for solving the intuitionistic fuzzy multi-objective transportation problem. Ammar *et al.* [2] studied on multi-objective transportation problem with fuzzy numbers. Maity *et al.* [29] introduced a new approach for solving dual-hesitant fuzzy transportation problem with restrictions. Anukokila *et al.* [6] used goal programming approach for solving multi-objective fractional transportation problem with fuzzy programming. Roy *et al.* [46] analyzed the random-rough variables in a multi-objective fixed-charge transportation problem. Maity *et al.* [30] analyzed the multi-modal transportation problem and they showed its applications to artificial intelligence. Giri and

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Roy [16] designed and solved multi-objective green four-dimensional fixed-charge transportation problem under neutrosophic environment. Kaur and Kumar [23] proposed a new approach for solving fuzzy transportation problem using generalized fuzzy numbers. Mahapatra *et al.* [27] solved multi-choice stochastic transportation problem involving extreme value distribution. Mardanya *et al.* [32] analyzed how to solve JTP with multi-item under multi-objective environment. Roy *et al.* [43] solved multi-choice multi-objective transportation problem with interval goal using conic scalarization approach. Ebrahimnejad [9] simplified a new approach for solving transportation problem with generalized trapezoidal fuzzy numbers. Mardanya *et al.* [33] depicted a study on bi-level multi-objective transportation problem in fuzzy environment. A good number of researches for solving multi-objective transportation problem in different directions were developed by several researchers such as (*cf.*, [11, 28, 44–46]). Mardanya and Roy [31] studied a time-variant multi-objective linear fractional transportation problem. Alharbi *et al.* [5] provided an interactive approach to solve the multi-objective minimum cost flow problem in the fuzzy environment. Kaur *et al.* [24] solved capacitated two-stage time minimization transportation problem with restriction flow. Ammar and Khalifa [3] solved fuzzy multi-objective multi-item solid transportation problems. Mardanya *et al.* [34] solved the MMTP *via* the rough interval approach. Recently, Tanksale and Jha [48] solved a hybrid fix and then optimized heuristic for integrated inventory transportation problem in a multi-region multi-facility supply chain. However, apart from supply and demand constraints, in real-world scenarios, often we need to consider the mode of transportation (*e.g.*, goods train, cargo flights, and trucks), the kinds of goods, and so on. Under such circumstances, a TP is extended to a Solid Transportation Problem (STP), apart from source and destination constraints, additional constraints, related to the modes of transportation (conveyance) or types of goods, is chosen.

Originally the STP was stated by Schell [47]. Haley [18] showed a comparison between the STP and the classical TP, and applied Modi-method to solve the STP. If more than one objectives are to optimize in an STP, then the problem is called a multi-objective solid transportation problem (MSTP). Uncertainty arises in an STP because of imprecise data and inexact information. Some of the most relevant works related to this are as follows: Jiménez and Verdegay [21] described two forms of uncertain STP in which the considered data are interval numbers and fuzzy numbers, respectively. Ghosh *et al.* [13] derived a multi-objective solid transportation model of waste management problem in agriculture field and forest department for urban or rural development. Ida *et al.* [10] chosen multi-criteria STP with fuzzy numbers. Ammar [1] studied on multi-objective solid transportation problems. Ghosh *et al.* [12] considered a fixed-charge STP in multi-objective environment where all the data are intuitionistic fuzzy numbers with membership and non-membership function. Li *et al.* [26] presented a genetic algorithm for solving an MSTP with coefficients of the objective function as fuzzy numbers. Midya *et al.* [36] formulated and solved fuzzy multiple objective fractional optimization in rough approximation and its aptness to the fixed-charge transportation problem. Nagarjan and Jeyaraman [39] studied an MSTP with parameters as stochastic intervals. Jiménez and Verdegay [22] applied an evolutionary algorithm based on parametric approach to solve the fuzzy solid transportation problem. Ghosh *et al.* [14] solved MSTP with preservation technology using Pythagorean fuzzy sets. Yang and Liu [51] presented the expected value model, chance-constrained programming model and dependent-chance programming for fixed charge STP in a fuzzy environment. Ammar *et al.* [4] developed a fuzzy solution approach to optimize water resources management problem. Yang and Yuan [50] investigated a bicriteria STP under a stochastic environment. Ghosh *et al.* [15] solved multi-objective fixed-charge STP under type-2 zigzag uncertain environment and made a comparison between time window and preservation technology. Recently Roy and Midya [42] studied multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment and Midya *et al.* [35] solved intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. In multi-item STP (MISTP), more than one items/products is transported through the conveyances. In spite of all the developments, there are several gaps in the literature. Previous researchers investigated multi-item two dimensional TP or multi-objective STP for a single item. Few shortcomings in the existing literature studies are pointed out as follows:

1) The fuzzy STP model using expected value operator may not yield provide feasible solutions in all cases.

2) Most investigations have been studied to derive the optimal crisp solution of the fuzzy objective function that does not give a proper idea about objective value according to fuzzy penalties of the objective function. The present investigation removes the above-mentioned lacuna. There are several methods available to deal with uncertain STP in a fuzzy environment, such as chance-constrained programming, expected value operator, dependent chance constrained programming and so on. In an STP the conditions that total supply (resources) and conveyance capacities are greater than or equal to the total demands must have to be satisfied. But these conditions have not been considered in the above-mentioned methods to obtain crisp equivalent form of an STP in fuzzy environment. In this study, we assume a multi-objective multi-item solid transportation problem with fuzzy data in which more than one objective are involved, and also several items are to be transported from sources to destinations.

In this paper, we present various models of MMSTP using the parameters as trapezoidal fuzzy numbers and extend the study of MMSTP by introducing interval and rough interval (RI) in the objective functions. Finally, we analyze the results extracted from different MMSTP models with interval and RI coefficients using presented methods and existing method with respect to the same real-life problem. The main contributions of the proposed study are summarized as follows:

– Solve MMSTP with interval coefficients using fuzzy programming and compare the results with existing results mentioned in Kundu *et al.* [25].
– Investigate the solution procedures between the presented method described in Section 4 and the existing method on MMSTP.
– Design MMSTP model when the parameters in the objective functions are RIs.
– Expected value operator is introduced to tackle RI in MMSTP.
– A comparison is drawn between the solutions extracted from interval programming and RI programming for MMSTP.
– Two real-life examples are incorporated in respect to all models to illustrate the applicability of the proposed model.

The rest of the paper is sorted out in the ordered as: In Section 2, we include the basic definitions and properties of a rough interval. In Section 3, the mathematical model is proposed for multi-objective multi-item solid transportation problem. Solution procedure of the proposed method is presented in Section 4. To demonstrate the utilization of the proposed model of the MMSTP, two numerical illustrations are incorporated into Section 5. Problem formulation for first numerical example using different techniques is presented in Subsection 5.1. In Section 6, we present results and discussion of the proposed method. Finally, this paper ends with conclusions in Section 7.

2. Preliminaries

In this paper, we assume that all parameters of considered problem are expressed as trapezoidal fuzzy numbers and then we convert them into nearest interval and nearest rough interval numbers. So we need to know about trapezoidal fuzzy numbers and nearest interval and nearest rough interval numbers.

Trapezoidal fuzzy number: A fuzzy number $\tilde{F} = (p, q, r, s)$ where $p \leq q \leq r \leq s$ defined on the universal set of real numbers $\mathbb{R}$ is called a trapezoidal fuzzy number if its membership function $\mu_{\tilde{F}}(u)$ is defined as:

$$\mu_{\tilde{F}}(u) = \begin{cases} \frac{u-p}{q-p}, & \text{if } p \leq u \leq q; \\ 1, & \text{if } q \leq u \leq r; \\ \frac{s-u}{s-r}, & \text{if } r \leq u \leq s; \\ 0, & \text{otherwise.} \end{cases}$$

where $\frac{u-p}{q-p} = \mu^1_{\tilde{F}}(u)$ and $\frac{s-u}{s-r} = \mu^2_{\tilde{F}}(u)$ are left and right hand sides of the membership function $\mu_{\tilde{F}}(u)$ (see Fig. 1). When $q = r$, the trapezoidal fuzzy number becomes a triangular fuzzy number.

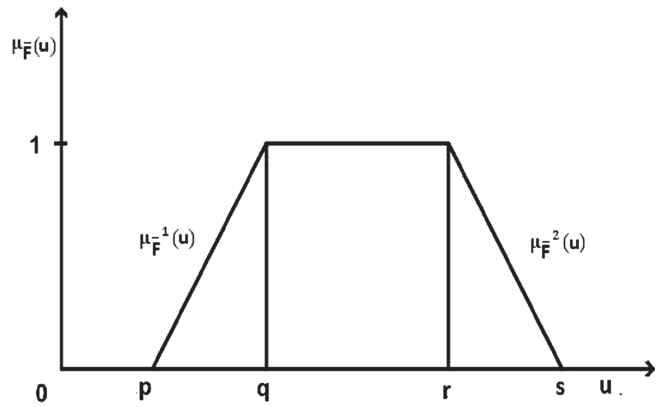

FIGURE 1. Trapezoidal fuzzy number $\tilde{F}$.

Zero trapezoidal fuzzy number: A trapezoidal fuzzy number $\tilde{F} = (p, q, r, s)$ is called zero trapezoidal fuzzy number if and only if $\tilde{F} = (0, 0, 0, 0)$.

Non-negative trapezoidal fuzzy number: A trapezoidal fuzzy number $\tilde{F} = (p, q, r, s)$ where $p \leq q \leq r \leq s$ is called non-negative trapezoidal fuzzy number if and only if $p \geq 0$.

Interval number: (Moore *et al.* [38]) An interval number is a number whose exact value is unknown distribution information, but the range of the value is known. Interval number is a number with both lower and upper bounds $U \in [\underline{u}, \overline{u}]$ where $\underline{u} \leq \overline{u}$. The main arithmetic operations can be expressed in interval numbers.

Definition 2.1. (Moore *et al.* [38]) Let $\tilde{u}_1 = [\underline{u}_1, \overline{u}_1]$ and $\tilde{u}_2 = [\underline{u}_2, \overline{u}_2]$ be closed interval numbers. The following notations can be satisfied:

(1) $\tilde{u}_1 + \tilde{u}_2 = [\underline{u}_1 + \underline{u}_2, \overline{u}_1 + \overline{u}_2]$.

(2) $\tilde{u}_1 - \tilde{u}_2 = [\underline{u}_1 - \overline{u}_2, \overline{u}_1 - \underline{u}_2]$.

(3) $\tilde{u}_1 * \tilde{u}_2 = [\min S, \max S]$, where $S = \{\underline{u}_1\underline{u}_2, \underline{u}_1\overline{u}_2, \overline{u}_1\underline{u}_2, \overline{u}_1\overline{u}_2\}$.

(4) $\tilde{u}_1\tilde{u}_2 = [\underline{u}_1, \overline{u}_1]\frac{1}{[\overline{u}_2, \underline{u}_2]}$ where $U \in [\underline{u}, \overline{u}]$ is an interval number, its absolute value is the maximum of the absolute value of its endpoints:

(5) $| u |= \max(| \underline{u} |, | \overline{u} |)$.
The center u_c and width u_w of an interval number of $U \in [\underline{u}, \overline{u}]$ are defined as follows:
$u_c = \frac{1}{2}[\underline{u} + \overline{u}]$ and $u_w = \frac{1}{2}[\overline{u} - \underline{u}]$. It is easily demonstrable that $\overline{u} = u_c + u_w$ and $\underline{u} = u_c - u_w$.

Rough set [40]: Suppose $X \neq \phi$ is a finite set of objects and we define an equivalence relation R on X that partitioned X into a family of pairwise disjoined subsets $E_1, E_2, \ldots, E_n$ each of which is an equivalence class of R and called elementary sets. The pair (X, R) is called approximation space and it is denoted by $Appr(R)$.

In the approximation space $Appr(R) = (X, R)$, given an arbitrary set $B \subseteq X$, one may represent B by a pair of lower approximation (LA) and upper approximation (UA):

$$\underline{Appr}(B) = \bigcup_{E_i \subseteq B} E_i = \{y \in X : [y]_R \subseteq B\},$$
$$\overline{Appr}(B) = \bigcup_{E_i \cap B \neq \phi} E_i = \{y \in U : [y]_R \cap B \neq \phi\}.$$

Where $[y]_R$ signifies the equivalence class containing y. The LA and UA of B can equivalently describe as:

$$\underline{Appr}(B) = \{y \in X : \forall\ z \in X,\ yRz \Rightarrow z \in X\},$$
$$\overline{Appr}(B) = \{y \in X : \exists\ z \in X \text{ such that } yRz \text{ and } z \in X\}.$$

The pair $(\underline{Appr}(B), \overline{Appr}(B))$ is called the rough set of B.

Rough interval arithmetic [41]: The RI arithmetic operations are fundamentally in view of Moore's interval arithmetic [38]. In the accompanying, some of these arithmetic operations of RIs are talked about. The detailed discussion of rough interval arithmetic (RIA) is in [41]. Presently, as indicated by Hamzehee *et al.* [19], $S = ([\underline{s}_L, \underline{s}_U], [\overline{s}_L, \overline{s}_U])$ and $T = ([\underline{t}_L, \underline{t}_U], [\overline{t}_L, \overline{t}_U])$ are two RIs. At that point, the RIA on two RIs is given by the following:

2.1) *Addition:* $S + T = ([\underline{s}_L + \underline{t}_L, \underline{s}_U + \underline{t}_U], [\overline{s}_L + \overline{t}_L, \overline{s}_U + \overline{t}_U])$.
2.2) *Subtraction:* $S - T = ([\underline{s}_L - \underline{t}_U, \underline{s}_U - \underline{t}_L], [\overline{s}_L - \overline{t}_U, \overline{s}_U - \overline{t}_L])$.
2.3) *Negation:* $-S = ([-\underline{s}_U, -\underline{s}_L], [-\overline{s}_U, -\overline{s}_L])$.
2.4) *Intersection:* $S \cap T = ([\max\{\underline{s}_L, \underline{t}_L\}, \min\{\underline{s}_U, \underline{t}_U\}], [\max\{\overline{s}_L, \overline{t}_L\}, \min\{\overline{s}_U, \overline{t}_U\}])$.
2.5) *Union:* $S \cup T = ([\min\{\underline{s}_L, \underline{t}_L\}, \max\{\underline{s}_U, \underline{t}_U\}], [\min\{\overline{s}_L, \overline{t}_L\}, \max\{\overline{s}_U, \overline{t}_U\}])$.

The set and logic operations with RIs are basic analyzed to similar operations with fuzzy set. Fuzzy sets must deal with continuous membership functions and cannot utilize Moore's interval calculus.

Order relation of a RI: Suppose S and T are any two RIs, the order relations '$\leq$' and '$<$' between S and T is characterized as $S \leq T \Leftrightarrow \frac{\underline{s}_L + \underline{s}_U}{2} \leq \frac{\underline{t}_L + \underline{t}_U}{2}$, $\frac{\overline{s}_L + \overline{s}_U}{2} \leq \frac{\overline{t}_L + \overline{t}_U}{2}$; and $S < T \Leftrightarrow S \leq T$ with $S \neq T$, respectively.

The order relation $\leq$ or $<$ demonstrates the inclination of Decision Makers (DMs) for the distinctive decisions in view of the upper most extreme midpoint in the ordinary case and in addition to the exceptional case circumstance, to concerning a maximization problem. Expected value maximization and vulnerability minimization are the choice reasoning. It is noticed that the order relations $\leq$ and $<$ so-characterized are partially ordered relation. The α-optimistic value is given by equation (2.1) and the α-pessimistic value is shown in equation (2.2) as follows:

$$\gamma_{\sup}(\alpha) = \begin{cases} (1-2\alpha)\overline{s}_U + 2\alpha\overline{s}_L, & \text{if } \alpha \leq \frac{\overline{s}_U - \underline{s}_U}{2(\overline{s}_U - \overline{s}_L)}; \\ 2(1-\alpha)\overline{s}_U + (2\alpha - 1)\overline{s}_L, & \text{if } \alpha \geq \frac{2\overline{s}_U - \underline{s}_L - \overline{s}_L}{2(\overline{s}_U - \overline{s}_L)}; \\ \frac{\overline{s}_U(\underline{s}_U - \underline{s}_L) + \underline{s}_U(\underline{s}_U - \underline{s}_L) - 2\alpha(\underline{s}_U - \underline{s}_L)(\overline{s}_U - \overline{s}_L)}{(\underline{s}_U - \underline{s}_L) + (\underline{s}_U - \overline{s}_L)}, & \text{otherwise.} \end{cases} \tag{2.1}$$

$$\gamma_{\inf}(\alpha) = \begin{cases} (1-2\alpha)\overline{s}_L + 2\alpha\overline{s}_U, & \text{if } \alpha \leq \frac{\underline{s}_L - \overline{s}_L}{2(\overline{s}_U - \overline{s}_L)}; \\ 2(1-\alpha)\overline{s}_L + (2\alpha - 1)\overline{s}_U, & \text{if } \alpha \geq \frac{\underline{s}_U + \overline{s}_U - 2\underline{s}_L}{2(\underline{s}_U - \overline{s}_L)}; \\ \frac{\overline{s}_L(\underline{s}_U - \underline{s}_L) + \underline{s}_L(\underline{s}_U - \underline{s}_L) - 2\alpha(\underline{s}_U - \underline{s}_L)(\overline{s}_U - \overline{s}_L)}{(\underline{s}_U - \underline{s}_L) + (\overline{s}_U - \overline{s}_L)}, & \text{otherwise.} \end{cases} \tag{2.2}$$

Expected value of a RI: Expected value operator is used to reduce rough interval to crisp interval. As our discussion is confined into transportation problem in rough intervals, so we have to define some important concepts on expected value operator.

Definition 2.2. Suppose $X = \{z \in X : \gamma(z) \in B\}$, where $\gamma : U \to \mathbb{R}$ is a real function, $B \subset \mathbb{R}$; and X is approximated by $(\underline{X}, \overline{X})$ according to the equivalence relation R. Then, the lower expected, upper expected and expected value of X are defined as follows:

$$\underline{E}[X] = \int_0^{+\infty} \underline{Appr}\{\gamma \geq x\}dr - \int_{-\infty}^0 \underline{Appr}\{\gamma \leq x\}dx,$$
$$\overline{E}[X] = \int_0^{+\infty} \overline{Appr}\{\gamma \geq x\}dx - \int_{-\infty}^0 \overline{Appr}\{\gamma \leq x\}dx,$$
$$E[X] = \int_0^{+\infty} Appr\{\gamma \geq x\}dx - \int_{-\infty}^0 Appr\{\gamma \leq x\}dx.$$

The correlation among the expected value $E(X)$, the lower expected value $\underline{E}(X)$, and the upper expected value $\overline{E}(X)$ is placed into the following proposition.

Proposition 2.3. *Let $X = \{z \in X : \gamma(z) \in B\}$, where $\gamma : U \to \mathbb{R}$ is a real function, $B \subset \mathbb{R}$; and X is approximated by $(\underline{X}, \overline{X})$ according to the similarity relation R, and η is a given parameter predetermied by using the DM's preference. Then expected value of X is denoted by $E(X)$ and it is defined as $E(X) = \eta\underline{E}(X) + (1-\eta)\overline{E}(X), \eta \in (0,\ 1)$.*

Proof. For proof, reader can see [49]. □

Theorem 2.4. *[49]: Suppose $B = ([a_1, b_1], [c_1, d_1])$ be a RI, where $c_1 \le a_1 \le b_1 \le d_1$. Then, the expected value of B is $E(B) = \frac{1}{2}[\eta \cdot (a_1 + b_1) + (1-\eta) \cdot (c_1 + d_1)]$.*

Proof. For proof, reader can see [49]. □

Remark 2.5. For $\eta = 0.5$ the expected value of B is $\frac{1}{4}(a_1 + b_1 + c_1 + d_1)$.

Nearest interval approximation of fuzzy number: According to Grzegorzewski [17], a fuzzy number is approximated to an equal crisp interval. The α-cut of a trapezoidal fuzzy number $F = (p, q, r, s)$ is defined as $[\underline{F}_\alpha, \overline{F}_\alpha]$. Thus, we have $\underline{F}_\alpha = p + \alpha(q-p)$ and $\overline{F}_\alpha = r + \alpha(s-r)$. We derive lower and upper bounds using the definition of nearest interval approximation as follows:

$$\underline{I} = \int_0^1 \underline{F}_\alpha(\alpha)d\alpha = [p + \alpha(q-p)]d\alpha = \frac{1}{2}(p+q),\ 0 < \alpha < 1 \tag{2.3}$$

$$\overline{I} = \int_0^1 \overline{F}_\alpha(\alpha)d\alpha = [r + \alpha(s-r)]d\alpha = \frac{1}{2}(r+s),\ 0 < \alpha < 1 \tag{2.4}$$

Here, the nearest interval approximation is used to transform a trapezoidal fuzzy number $F = (p, q, r, s)$ into a crisp closed interval as

$$[\underline{I}, \overline{I}] = \left[\frac{1}{2}(p+q), \frac{1}{2}(r+s)\right]. \tag{2.5}$$

Nearest rough interval approximation of fuzzy number: Suppose $F = (p, q, r, s)$ be a trapezoidal fuzzy number. Then α-cut of F is defined as $[\underline{F}_\alpha, \overline{F}_\alpha]$ or $[\underline{G}_\alpha, \overline{G}_\alpha]$. Therefore, we can write $\underline{F}_\alpha = p + \alpha(q-p)$ and $\overline{F}_\alpha = r+\alpha(s-r)$, $\underline{G}_\alpha = q-\alpha(q-p)$ and $\overline{G}_\alpha = s-\alpha(s-r)$. Using the definition of nearest interval approximation, we get lower and upper bounds, respectively, as

$$\underline{I_1} = \int_0^1 \underline{F}_\alpha(\alpha)d\alpha = [p + \alpha(q-p)]d\alpha = \frac{1}{2}(p+q),\ 0 < \alpha < 1,$$

$$\underline{I_2} = \int_0^1 \underline{G}_\alpha(\alpha)d\alpha = [q - \alpha(q-p)]d\alpha = \frac{1}{2}(q-p),\ 0 < \alpha < 1,$$

$$\overline{I_1} = \int_0^1 \overline{F}_\alpha(\alpha)d\alpha = [r + \alpha(s-r)]d\alpha = \frac{1}{2}(r+s),\ 0 < \alpha < 1,$$

$$\overline{I_2} = \int_0^1 \overline{G}_\alpha(\alpha)d\alpha = [s - \alpha(s-r)]d\alpha = \frac{1}{2}(s-r),\ 0 < \alpha < 1.$$

Then we define $\overline{C}_L = \underline{I_1} = \frac{1}{2}(p+q)$, $\overline{C}_U = \overline{I_1} = \frac{1}{2}(r+s)$, $\underline{C}_L = \{(\underline{I_1}^2 + \underline{I_2}^2)(\overline{I_1}^2 + \overline{I_2}^2)\}^{\frac{1}{4}} = (pqrs)^{\frac{1}{4}}$, $\underline{C}_U = \frac{1}{2}(\overline{C}_L + \overline{C}_U) = \frac{1}{4}(p+q+r+s)$. Now we define the nearest rough interval approximation of a trapezoidal fuzzy number $F = (p,q,r,s)$ as

$$[\underline{C}_L, \underline{C}_U], [\overline{C}_L, \overline{C}_U] = \left[(pqrs)^{\frac{1}{4}}, \frac{1}{4}(p+q+r+s)\right], \left[\frac{1}{2}(p+q), \frac{1}{2}(r+s)\right] \tag{2.6}$$

3. Mathematical model

The mathematical formulation of a classical TP is given as:

$$\begin{aligned} \text{minimize } & Z = \sum_{p=1}^{s}\sum_{q=1}^{t} c_{pq}x_{pq} \\ \text{subject to } & \sum_{q=1}^{t} x_{pq} \le a_p \ (p = 1,2,\dots,s), \\ & \sum_{p=1}^{s} x_{pq} \ge b_q \ (q = 1,2,\dots,t), \\ & x_{pq} \ge 0 \ \forall \ p,q. \end{aligned}$$

Here, Z stands for the objective function to manipulate total transportation cost; c_{pq} $(p = 1,2,\dots,s; q = 1,2,\dots,t)$ is considered as transportation cost per unit commodity from p^{th} origin to q^{th} destination; a_p $(p = 1,2,\dots,s)$ and b_q $(q = 1,2,\dots,t)$ are taken as availability and demand in p^{th} origin and q^{th} destination, respectively, and the feasibility condition is as follows:

$$\sum_{p=1}^{s} a_p \ge \sum_{q=1}^{t} b_q.$$

The cost c_{pq} is usually treated as being deterministic in nature. However, in real-life situations, the precise value of this transportation cost may not be known. To design the mathematical formulation of classical MMSTP, we use the following notations which are stated below:

Notations of MMSTP:

- x^l_{pqr}: Amount transported from p^{th} source to q^{th} destination for l^{th} item through r^{th} conveyance,
- $\tilde{c}^{ml}_{pqr}$: Fuzzy shipping cost per unit amount for transporting l^{th} item from p^{th} source to q^{th} destination through r^{th} conveyance for m^{th} objective function,
- $\tilde{c}^{nl}_{pqr}$: Fuzzy shipping cost per unit amount for transporting l^{th} item from p^{th} source to q^{th} destination through r^{th} conveyance for n^{th} objective function,
- $\tilde{a}^l_p$: Fuzzy capacity of p^{th} source point for l^{th} item,
- $\tilde{b}^l_q$: Fuzzy demand of q^{th} destination point for l^{th} item,
- $\tilde{e}^l_r$: Total fuzzy capacity of l^{th} item through r^{th} conveyance.

A multi-objective multi-item solid transportation problem is formulated as follows:
Model 1

$$\text{maximize } Z^m = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} \tilde{c}^{ml}_{pqr}x^l_{pqr} \ (m = 1,2,\dots,M)$$

$$\text{minimize } Z^n = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} \tilde{c}^{nl}_{pqr}x^l_{pqr} \ (n = 1,2,\dots,N)$$

$$\text{subject to } \sum_{q=1}^{t}\sum_{r=1}^{v} x_{pqr}^{l} \leq \tilde{a}_{p}^{l} \ (p=1,2,\ldots,s; l=1,2,\ldots,u),$$
$$\sum_{p=1}^{s}\sum_{r=1}^{v} x_{pqr}^{l} \geq \tilde{b}_{q}^{l} \ (q=1,2,\ldots,t; l=1,2,\ldots,u),$$
$$\sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t} x_{pqr}^{l} \leq \tilde{e}_{r}^{l} \ (r=1,2,\ldots,v),$$
$$x_{pqr}^{l} \geq 0 \ \forall \ p,q,r,l.$$

Mathematical model when parameters are interval form: If the parameters of Model 1 are considered as interval form, then the mathematical model of MMSTP is described as:

Model 2

$$\text{maximize} \quad [Z_L^m, Z_U^m] = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} [{c_L}_{pqr}^{ml}, {c_U}_{pqr}^{ml}] x_{pqr}^{l} \ (m=1,2,\ldots,M)$$
$$\text{minimize} \quad [Z_L^n, Z_U^n] = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} [{c_L}_{pqr}^{nl}, {c_U}_{pqr}^{nl}] x_{pqr}^{l} \ (n=1,2,\ldots,N)$$
$$\text{subject to } \sum_{q=1}^{t}\sum_{r=1}^{v} x_{pqr}^{l} \leq [{a_L}_{p}^{l}, {a_U}_{p}^{l}] \ (p=1,2,\ldots,s; \ l=1,2,\ldots,u),$$
$$\sum_{p=1}^{s}\sum_{r=1}^{v} x_{pqr}^{l} \geq [{b_L}_{q}^{l}, {b_U}_{q}^{l}] \ (q=1,2,\ldots,t; \ l=1,2,\ldots,u),$$
$$\sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t} x_{pqr}^{l} \leq [{e_L}_{r}^{l}, {e_U}_{r}^{l}] \ (r=1,2,\ldots,v),$$
$$x_{pqr}^{l} \geq 0 \ \forall \ p,q,r,l.$$

Model 2 is an MMSTP with interval numbers. We solve Model 2 using fuzzy programming and interval programming, and the solution procedures are given in Subsection 5.1 and subsection 5.2, respectively. To solve Model 2 by fuzzy programming and interval programming we need to reduce Model 2 into two deterministic models namely Model 3 and Model 4 which are mentioned in the solution procedure and the models are given as follows:

Model 3

$$\text{maximize} \quad Z_L^m = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} {c_L}_{pqr}^{ml} x_{pqr}^{l} \ (m=1,2,\ldots,M) \tag{3.1}$$
$$\text{minimize} \quad Z_L^n = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} {c_L}_{pqr}^{nl} x_{pqr}^{l} \ (n=1,2,\ldots,N) \tag{3.2}$$
$$\text{subject to } \sum_{q=1}^{t}\sum_{r=1}^{v} x_{pqr}^{l} \leq {a_U}_{p}^{l} \ (p=1,2,\ldots,s; \ l=1,2,\ldots,u), \tag{3.3}$$
$$\sum_{p=1}^{s}\sum_{r=1}^{v} x_{pqr}^{l} \geq {b_L}_{p}^{l} \ (q=1,2,\ldots,t; \ l=1,2,\ldots,u), \tag{3.4}$$

$$\sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t} x_{pqr}^{l} \leq {e_U}_r^l \ (r = 1, 2, \ldots, v), \tag{3.5}$$

$$x_{pqr}^{l} \geq 0 \ \forall \ p, q, r, l. \tag{3.6}$$

Model 4

$$\text{maximize} \quad Z_U^m = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} {c_U}_{pqr}^{ml} x_{pqr}^{l} \ (m = 1, 2, \ldots, M) \tag{3.7}$$

$$\text{minimize} \quad Z_U^n = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} {c_U}_{pqr}^{nl} x_{pqr}^{l} \ (n = 1, 2, \ldots, N) \tag{3.8}$$

subject to the constraints (3.3)–(3.6).

Mathematical model when parameters in RI: If the parameters of Model 1 are considered as RI, then the mathematical model of MMSTP is depicted below:
Model 5

$$\text{maximize} \quad Z^m = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} ([{d_l}_{pqr}^{ml}, {d_u}_{pqr}^{ml}], [{d_L}_{pqr}^{ml}, {d_U}_{pqr}^{ml}]) x_{pqr}^{l} \tag{3.9}$$

$$\text{minimize} \quad Z^n = \sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{v} ([{c_l}_{pqr}^{nl}, {c_u}_{pqr}^{nl}], [{c_L}_{pqr}^{nl}, {c_U}_{pqr}^{nl}]) x_{pqr}^{l} \tag{3.10}$$

$$\text{subject to} \quad \sum_{q=1}^{t}\sum_{r=1}^{v} x_{pqr}^{l} \leq ([{a_l}_p^l, {a_u}_p^l], [{a_L}_p^l, {a_U}_p^l]), \tag{3.11}$$

$$\sum_{p=1}^{s}\sum_{r=1}^{v} x_{pqr}^{l} \geq ([{b_l}_q^l, {b_u}_q^l], [{b_L}_q^l, {b_U}_q^l]), \tag{3.12}$$

$$\sum_{l=1}^{u}\sum_{p=1}^{s}\sum_{q=1}^{t} x_{pqr}^{l} \leq ([{e_l}_r^l, {e_u}_r^l], [{e_L}_r^l, {e_U}_r^l]), \tag{3.13}$$

$$x_{pqr}^{l} \geq 0 \ \forall \ p, q, r, l. \tag{3.14}$$

Model 5 is an MMSTP with parameters in RI. We solve Model 5 using expected value operator and the solution procedure is mentioned in Subsection 5.3.

4. Solution methodology

In order to solve the MMSTP with interval coefficients and with rough coefficients, we use the following techniques:

– Fuzzy programming,
– Interval programming,
– Expected value operator.

4.1. Fuzzy programming to solve Model 2

The solution of bi-objective MMSTP can be obtained by the following steps:

Step 1: Consider an MMSTP (Model 2) with M objective functions Z^m $(m = 1, 2, \ldots, M)$ of maximization type and N objective functions Z^n $(n = 1, 2, \ldots, N)$ of minimization type.

Step 2: Then reducing Model 2 into two models Model 3 and Model 4, and due to presence of multi-objective in both the models we use Step 3 to make them single objective.

Step 3: Let z_y^m $(m = 1, 2, \ldots, M)$ $(y = L, U)$ be an objective function which is of maximization type. Find the maximum value ${d_y}_{p'q'r'}^{ml}$ among all the cost parameters, then divide each of the cost parameters is by ${d_y}_{p'q'r'}^{ml}$ to obtain $\frac{{d_y}_{pqr}^{ml}}{{d_y}_{p'q'r'}^{ml}} = {r_y}_{pqr}^{ml}$ (say), where $0 < {r_y}_{pqr}^{ml} \leq 1$. In the classical procedure, for solving TP of maximization type, the allocations are made in the cells according to values of cost parameters in descending order. This suggests that the allocation probably made at the values of ${r_y}_{pqr}^{ml}$ where it is maximum. Then the cost parameters in each maximization type objective function is reduced in the same scale. Again, considering z_y^n $(n = 1, 2, \ldots, N)$ is objective function of minimization type. Find the maximum value ${c_y}_{p'q'r'}^{nl}$ among all cost parameters, then each of the cost parameters is divided by ${c_y}_{p'q'r'}^{nl}$ to obtain $\frac{{c_y}_{pqr}^{nl}}{{c_y}_{p'q'r'}^{nl}} = {s_y}_{pqr}^{nl}$ (say), where $0 > {s_y}_{pqr}^{nl} \geq -1$. Since the objective function is minimization type so the allocations are made at the nodes where values of ${s_y}_{pqr}^{nl}$ are minimum. Then each of the cost parameters in the objective functions of minimization type is reduced in the same scale.

Step 4: Thereafter considering normalize weights w_m, g_n and for the objective functions Z^m and Z^n, individually for all m and n.

Step 5: Formulating the single objective function as follows:

$$\begin{aligned}\text{maximize } Z_L &= \sum_l \sum_p \sum_q \sum_r \left[\sum_m w_m {r_L}_{pqr}^{ml} + \sum_n g_n {s_L}_{pqr}^{nl}\right] x_{pqr}^l \\ &= \sum_l \sum_p \sum_q \sum_r {u_L}_{pqr}^l x_{pqr}^l\end{aligned}$$

$$\begin{aligned}\text{maximize } Z_U &= \sum_l \sum_p \sum_q \sum_r \left[\sum_m w_m {r_U}_{pqr}^{ml} + \sum_n g_n {s_U}_{pqr}^{nl}\right] x_{pqr}^l \\ &= \sum_l \sum_p \sum_q \sum_r {v_U}_{pqr}^l x_{pqr}^l\end{aligned}$$

subject to the constraints (3.3)–(3.6).

Step 6: Solve the bi-objective MMSTP as single objective STP using each time only one objective function and ignoring other.

Step 7: We find lower bound L_p and upper bound U_p for p^{th} objective function Z^p $(p = L, U)$, where L_p is the aspired levels of achievement for p^{th} objective function, U_p is the highest acceptable level of achievement for i^{th} objective function and $d_i = [U_p - L_p]$ is degradation allowance for p^{th} objective function. When the aspiration levels and degradation allowance for each objective function are specified, a fuzzy model is formed and then it converts into a crisp model.

Step 8: From the results of Step 6, determine the corresponding values for every objective function at each solution derived.

Step 9: From Step 8, we find the best L_p and the worst U_p values for each objective function corresponding to the set of solutions. The initial fuzzy model can then be stated, in terms of the aspiration levels of each objective function, as follows:
Find x_{pqr}^l , so as to satisfy $Z_p \leq L_p : p = L, U$ with given constraints (3.3)–(3.6). For the bi-objective

FCTP, a membership function $\mu_p(x)$ corresponding to p^{th} objective function is defined as:

$$\mu_p(x) = \begin{cases} 1, & \text{if } Z_p \leq L_p, \\ 1 - \left(\frac{Z_p - L_p}{U_p - L_p}\right), & \text{if } L_p \leq Z_p \leq U_p \ (p = L, U), \\ 0, & \text{if } Z_p \geq U_p. \end{cases} \tag{4.1}$$

From the results of Step 5, determine the corresponding value for every objective function at each solution derived.
The equivalent linear programming problem for the minimization problem may then be written as:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \lambda \leq \left(\frac{Z_p - L_p}{U_p - L_p}\right) \ (p = L, U), \\ & \text{the constraints } (3.3) - (3.6), \\ & 0 \leq \lambda \leq 1. \end{aligned}$$

Here $\lambda = \min\{\mu_p(x) : p = L, U\}$. This linear programming problem can further be simplified as follows:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & Z_p + \lambda(U_p - L_p) \leq U_p \ (p = L, U), \\ & \text{the constraints } (3.3) - (3.6), \\ & 0 \leq \lambda \leq 1. \end{aligned}$$

Now we maximize λ subject to the constraints (3.3)–(3.6) using Lingo iterative scheme.

4.2. Interval programming to solve Model 2

We apply the following steps for solving Model 2 by using interval programming:

Step 1: Consider an MMSTP (Model 2) with M objective functions Z^m $(m = 1, 2, \ldots, M)$ of maximization type and N objective functions Z^n $(n = 1, 2, \ldots, N)$ of minimization type.

Step 2: Then reducing Model 2 into two models Model 3 and Model 4 ; and due to presence of multi-objective in both the models we use Step 3 to make them single objective.

Step 3: Let z_y^m $(m = 1, 2, \ldots, M)$ $(y = L, U)$ be an objective function which is of maximization type. Find the maximum value $d_{y_{p'q'r'}}^{ml}$ among all the cost parameters, then divide each of the cost parameters is by $d_{y_{p'q'r'}}^{ml}$ to obtain $\frac{d_{y_{pqr}}^{ml}}{d_{y_{p'q'r'}}^{ml}} = r_{y_{pqr}}^{ml}$(say), where $0 < r_{y_{pqr}}^{ml} \leq 1$. In the classical procedure, for solving TP of maximization type, the allocations are made in the cells according to values of cost parameters in descending order. This suggests that the allocation probably made at the values of $r_{y_{pqr}}^{ml}$ where it is maximum. Then the cost parameters in each maximization type objective function is reduced in the same scale. Again, considering z_y^n $(n = 1, 2, \ldots, N)$ is objective function of minimization type. Find the maximum value $c_{y_{p'q'r'}}^{nl}$ among all cost parameters, then divide each of the cost parameters is by $c_{y_{p'q'r'}}^{nl}$ to obtain $\frac{c_{y_{pqr}}^{nl}}{c_{y_{p'q'r'}}^{nl}} = s_{y_{pqr}}^{nl}$(say), where $0 > s_{y_{pqr}}^{nl} \geq -1$. Since the objective function is minimization type so the allocations are made at the nodes where values of $s_{y_{pqr}}^{nl}$ are minimum. Then each of the cost parameters in the objective functions of minimization type is reduced in the same scale.

Step 4: Thereafter we normalize the weights w_m, g_n and for the objective functions Z^m and Z^n, for all m and n.

Step 5: Formulating the single objective function as follows:

$$\begin{aligned}\text{maximize } \; Z_L &= \sum_{l}\sum_{p}\sum_{q}\sum_{r}\left[\sum_{m} w_m {r_L}^{ml}_{pqr} + \sum_{n} g_n {s_L}^{nl}_{pqr}\right] x^l_{pqr} \\ &= \sum_{l}\sum_{p}\sum_{q}\sum_{r} {u_L}^{l}_{pqr} x^l_{pqr}\end{aligned}$$

$$\begin{aligned}\text{maximize } \; Z_U &= \sum_{l}\sum_{p}\sum_{q}\sum_{r}\left[\sum_{m} w_m {r_U}^{ml}_{pqr} + \sum_{n} g_n {s_U}^{nl}_{pqr}\right] x^l_{pqr} \\ &= \sum_{l}\sum_{p}\sum_{q}\sum_{r} {v_U}^{l}_{pqr} x^l_{pqr}\end{aligned}$$

subject to the constraints (3.3)–(3.6).

Step 6: We maximize Z_L and Z_U under the constraints described in MMSTP *i.e.*, (3.3)–(3.6) using Lingo iterative scheme.

4.3. Expected value operator to solve Model 5

To solve Model 5 using expected value operator we go through the following steps:

Step 1: Consider an MMSTP (Model 5) with M objective functions Z^m $(m = 1, 2, \ldots, M)$ of maximization type and r objective functions Z^n $(n = 1, 2, \ldots, N)$ of minimization type.

Step 2: Convert the cost parameters of both the objective functions into crisp values by using expected value operator *i.e.*, $([{d_l}^{ml}_{pqr}, {d_u}^{ml}_{pqr}], [{d_L}^{ml}_{pqr}, {d_U}^{ml}_{pqr}]) = d^{ml}_{pqr}$ (say) and $E([{c_l}^{nl}_{pqr}, {c_u}^{nl}_{pqr}], [{c_L}^{nl}_{pqr}, {c_U}^{nl}_{pqr}]) = c^{nl}_{pqr}$ (say).

Step 3: Let Z^m $(m = 1, 2, \ldots, M)$ be an objective function which is maximization type. Find maximum value $d^{ml}_{p'q'r'}$ among all cost parameters, then divide each of the cost parameters by $d^{ml}_{p'q'r'}$ to obtain $\frac{d^{ml}_{pqr}}{d^{ml}_{p'q'r'}} = \tau^{ml}_{pqr}$(say), where $0 < \tau^{ml}_{pqr} \le 1$. In the classical procedure, for solving TP of maximization type, the allocations are made in the cells according to values of cost parameters in descending order. This suggests that the allocation probably made at the values of τ^{ml}_{pqr} where it is maximum. Then the cost parameters in each maximization type objective function are reduced in the same scale. Again, consider z^n $(n = 1, 2, \ldots, N)$ is objective function of minimization type. Find maximum value $c^{nl}_{p'q'r'}$ among all cost parameters, then divide each of the cost parameters by $c^{nl}_{p'q'r'}$ to obtain $\frac{c^{nl}_{pqr}}{c^{nl}_{p'q'r'}} = \sigma^{nl}_{pqr}$ (say), where $0 > \sigma^{nl}_{pqr} \ge -1$. Since the objective function is minimization type so the allocations are made at the nodes where values of σ^{nl}_{pqr} are minimum. Then each of the cost parameters in the objective functions of minimization type is reduced in the same scale.

Step 4: After that we consider normalize weights w_m, g_n and for the objective functions Z^m and Z^n, for all m and n, respectively.

Step 5: Formulate the single objective function

$$\begin{aligned}\text{maximize } \; Z_1 &= \sum_{l}\sum_{p}\sum_{q}\sum_{r}\left[\sum_{m} w_m \tau^{ml}_{pqr} + \sum_{n} g_n \sigma^{nl}_{pqr}\right] x^l_{pqr} \\ &= \sum_{l}\sum_{p}\sum_{q}\sum_{r} u^{l}_{pqr} x^l_{pqr}\end{aligned}$$

$$\text{maximize } \; Z_2 = \sum_{l}\sum_{p}\sum_{q}\sum_{r}\left[\sum_{m} w_m \tau^{ml}_{pqr} + \sum_{n} g_n \sigma^{nl}_{pqr}\right] x^l_{pqr}$$

Table 1. Unit transportation penalties for item 1 in the first objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(5, 8, 9, 11)$	$(4, 6, 9, 11)$	$(10, 12, 14, 16)$
S_2	$(8, 10, 13, 15)$	$(6, 7, 8, 9)$	$(11, 13, 15, 17)$
Conveyance $r = 2$			
S_1	$(9, 11, 13, 15)$	$(6, 8, 10, 12)$	$(7, 9, 12, 14)$
S_2	$(10, 11, 13, 15)$	$(6, 8, 10, 12)$	$(14, 16, 18, 20)$

Table 2. Unit transportation penalties for item 2 in the first objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(9, 10, 12, 13)$	$(5, 8, 10, 12)$	$(10, 11, 12, 13)$
S_2	$(11, 13, 14, 16)$	$(7, 9, 12, 14)$	$(12, 14, 16, 18)$
Conveyance $r = 2$			
S_1	$(11, 13, 14, 15)$	$(6, 7, 9, 11)$	$(8, 10, 11, 13)$
S_2	$(14, 16, 18, 20)$	$(9, 11, 13, 14)$	$(13, 14, 15, 16)$

$$= \sum_l \sum_p \sum_q \sum_r v^l_{pqr} x^l_{pqr}$$

subject to the constraints (3.3)–(3.6).

Step 6: We maximize Z_L and Z_U under the constraints depicted in MMSTP *i.e.*, (3.3)–(3.6) using Lingo iterative scheme.

5. Numerical experiments

We present here two examples for showing the effectiveness of the proposed methodologies. The first example is taken from Kundu *et al.* [25] and the second example is considered due to our preference to explain the applicability and effectiveness of the suggested methodology.

Example 5.1. Considering the multi-objective multi-item solid transportation problem solved by Kundu *et al.* [25]. In this problem, the number of destinations is three, while that of sources, items, conveyances, and objectives is two for each case. Authors proposed a method to find the crisp optimal compromise solution. Here we describe a new approach to find a crisp and interval solution to the same problem, we solve the same problem using the method proposed here in different approaches to extract the optimal compromise crisp solution and interval solution. The data of the problem is described in Tables 1 to 5.

5.1. Problem formulation

To make good sense of the proposed algorithms by fuzzy programming, interval programming and expected value operator here we design the problem formulation of the numerical example and solve them using LINGO iterative scheme.

Table 3. Unit transportation penalties for item 1 in the second objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(4, 5, 7, 8)$	$(3, 5, 6, 8)$	$(7, 9, 10, 12)$
S_2	$(6, 8, 9, 11)$	$(5, 6, 7, 8)$	$(6, 7, 9, 10)$
Conveyance $r = 2$			
S_1	$(6, 7, 8, 9)$	$(4, 6, 7, 9)$	$(5, 7, 9, 11)$
S_2	$(4, 6, 8, 10)$	$(7, 9, 11, 13)$	$(9, 10, 11, 12)$

Table 4. Unit transportation penalties for item 2 in the second objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(5, 7, 9, 10)$	$(4, 6, 7, 9)$	$(9, 11, 12, 13)$
S_2	$(10, 11, 13, 14)$	$(6, 7, 8, 9)$	$(7, 9, 11, 12)$
Conveyance $r = 2$			
S_1	$(7, 8, 9, 10)$	$(4, 5, 7, 8)$	$(8, 10, 11, 12)$
S_2	$(6, 8, 10, 12)$	$(5, 7, 9, 11)$	$(9, 10, 12, 14)$

5.1.1. Problem formulation by fuzzy programming

Using the steps from Step 1 to Step 5 mentioned in Section 4.1 we derive the following mathematical problem of the considered numerical example.

Model 6

$$
\begin{aligned}
\text{maximize } Z_1 = {} & 0.85x^1_{111} + 1.21x^1_{112} + 0.68x^1_{121} + 0.93x^1_{122} + 1.49x^1_{131} \\
& +1.1x^1_{132} + 1.17x^1_{212} + 0.95x^1_{221} + 1.12x^1_{222} + 1.41x^1_{231} + 1.9x^1_{232} \\
& +1.2x^2_{111} + 1.51x^2_{112} + 0.9x^2_{121} + 0.85x^2_{122} + 1.65x^2_{131} + 1.45x^2_{132} \\
& +1.8x^2_{211} + 1.6x^2_{212} + 1.14x^2_{221} + 1.17x^2_{222} + 1.62x^2_{231} + 1.8x^2_{232} \\
\text{maximize } Z_2 = {} & 1.02x^1_{111} + 1.35x^1_{112} + 1.03x^1_{121} + 1.16x^1_{122} + 1.59x^1_{131} + 1.42x^1_{132} \\
& +1.47x^1_{211} + 1.33x^1_{212} + 0.94x^1_{221} + 1.37x^1_{222} + 1.54x^1_{231} + 1.92x^1_{232} \\
& +1.35x^2_{111} + 1.46x^2_{112} + 1.16x^2_{121} + 1.02x^2_{122} + 1.57x^2_{131} + 1.48x^2_{132} \\
& +1.78x^2_{211} + 1.81x^2_{212} + 1.30x^2_{221} + 1.45x^2_{222} + 1.74x^2_{231} + 1.77x^2_{232} \\
\text{subject to } \quad & x^1_{111} + x^1_{112} + x^1_{121} + x^1_{122} + x^1_{131} + x^1_{132} \le 27 \qquad (5.1) \\
& x^2_{111} + x^2_{112} + x^2_{121} + x^2_{122} + x^2_{131} + x^2_{132} \le 38 \qquad (5.2) \\
& x^1_{211} + x^1_{212} + x^1_{221} + x^1_{222} + x^1_{231} + x^1_{232} \le 36 \qquad (5.3) \\
& x^2_{211} + x^2_{212} + x^2_{221} + x^2_{222} + x^2_{231} + x^2_{232} \le 31.5 \qquad (5.4) \\
& x^1_{111} + x^1_{112} + x^1_{211} + x^1_{212} \ge 15 \qquad (5.5) \\
& x^2_{111} + x^2_{112} + x^2_{211} + x^2_{212} \ge 21.5 \qquad (5.6)
\end{aligned}
$$

TABLE 5. Availability and demand data.

Fuzzy availability	Fuzzy demand	Conveyance capacity
$\tilde{a}_1^1 = (21, 24, 26, 28)$	$\tilde{b}_1^1 = (14, 16, 19, 22)$	$\tilde{e}_1 = (46, 49, 51, 53)$
$\tilde{a}_2^1 = (28, 32, 35, 37)$	$\tilde{b}_2^1 = (17, 20, 22, 25)$	$\tilde{e}_2 = (51, 53, 56, 59)$
$\tilde{a}_1^2 = (32, 34, 37, 39)$	$\tilde{b}_3^1 = (12, 15, 18, 21)$	
$\tilde{a}_2^2 = (25, 28, 30, 33)$	$\tilde{b}_1^2 = (20, 23, 25, 28)$	
	$\tilde{b}_2^2 = (16, 18, 19, 22)$	
	$\tilde{b}_3^2 = (15, 17, 19, 21)$	

TABLE 6. Optimal solution by fuzzy programming.

Problem	Optimal value	Optimal solution
Z_1	$[868.905, 1208.495]$	$x_{111}^1 = 13.5$, $x_{132}^1 = 13.5$, $x_{212}^1 = 1.5$, $x_{221}^1 = 18.5$, $x_{111}^2 = 13.95$,
Z_2	$[603.21, 919.585]$	$x_{122}^2 = 17$, $x_{132}^2 = 7.04$, $x_{212}^2 = 7.54$, $x_{231}^2 = 6.04$, $x_{232}^2 = 2.9$ and other variables are zero.

$$x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 \geq 18.5 \quad (5.7)$$
$$x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 \geq 17 \quad (5.8)$$
$$x_{131}^1 + x_{132}^1 + x_{231}^1 + x_{232}^1 \geq 13.5 \quad (5.9)$$
$$x_{131}^2 + x_{132}^2 + x_{231}^2 + x_{232}^2 \geq 16 \quad (5.10)$$
$$x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + \quad (5.11)$$
$$x_{111}^2 + x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \leq 52 \quad (5.12)$$
$$x_{112}^1 + x_{122}^1 + x_{132}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 + \quad (5.13)$$
$$x_{112}^2 + x_{122}^2 + x_{132}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \leq 57.5 \quad (5.14)$$
$$x_{ij} \geq 0 \ \forall \ i, j.$$

Solving Model 6 as a single objective STP using each time only one objective function and ignoring other, we get $L_1 = 113.102$, $U_1 = 119.015$, $L_2 = 127.35$, $U_2 = 132.374$ and then construct the membership function. Finally the mathematical model is designed as follows:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & Z^1 + \lambda(U_1 - L_1) \leq U_1 \\ & Z^2 + \lambda(U_2 - L_2) \leq U_2 \\ & \text{constraints (5.1)–(5.14).} \end{aligned}$$

5.1.2. Problem formulation by interval programming

Here we derive MMSTP model from the numerical example when the parameters are interval numbers.

Model 7

$$
\begin{aligned}
\text{minimize}\ \ & [Z_L^1, Z_U^1] = [6.5,10]x_{111}^1 + [10,14]x_{112}^1 + [5,10]x_{121}^1 + [7,11]x_{122}^1 + [11,15]x_{131}^1 \\
& +[8,13]x_{132}^1 + [9,14]x_{211}^1 + [10.5,14]x_{212}^1 + [6.5,8.5]x_{221}^1 + [7,11]x_{222}^1 \\
& +[12,16]x_{231}^1 + [15,19]x_{232}^1 + [9.5,12.5]x_{111}^2 + [12,14.5]x_{112}^2 + [6.5,11]x_{121}^2 \\
& +[6.5,10]x_{122}^2 + [10.5,12.5]x_{131}^2 + [9,12]x_{132}^2 + [12,15]x_{211}^2 + [15,19]x_{212}^2 \\
& +[8,13]x_{221}^2 + [10,13.5]x_{222}^2 + [13,17]x_{231}^2 + [13.5,15.5]x_{232}^2 \\
\text{minimize}\ \ & [Z_L^2, Z_U^2] = [4.5,7.5]x_{111}^1 + [6.5,8.5]x_{112}^1 + [4,7]x_{121}^1 + [5,8]x_{122}^1 + [8,11]x_{131}^1 \\
& +[6,10]x_{132}^1 + [7,10]x_{211}^1 + [5,9]x_{212}^1 + [5.5,7.5]x_{221}^1 + [8,12]x_{222}^1 \\
& +[6.5,9.5]x_{231}^1 + [9.5,12.5]x_{232}^1 + [6,9.5]x_{111}^2 + [7.5,9.5]x_{112}^2 + [5,8]x_{121}^2 \\
& +[4.5,7.5]x_{122}^2 + [10,12.5]x_{131}^2 + [9,11.5]x_{132}^2 + [10.5,13.5]x_{211}^2 \\
& +[7,11]x_{212}^2 + [6.5,8.5]x_{221}^2 + [6,10]x_{222}^2 + [8,11.5]x_{231}^2 + [9.5,13]x_{232}^2 \\
\text{subject to}\ \ & x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 + x_{131}^1 + x_{132}^1 \le [22.5,27] \\
& x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 + x_{131}^2 + x_{132}^2 \le [33,38] \\
& x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 + x_{231}^1 + x_{232}^1 \le [30,36] \\
& x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 + x_{231}^2 + x_{232}^2 \le [26.5,31.5] \\
& x_{111}^1 + x_{112}^1 + x_{211}^1 + x_{212}^1 \ge [15,20.5] \\
& x_{111}^2 + x_{112}^2 + x_{211}^2 + x_{212}^2 \ge [21.5,26.5] \\
& x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 \ge [18.5,23.5] \\
& x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 \ge [17,20.5] \\
& x_{131}^1 + x_{132}^1 + x_{231}^1 + x_{232}^1 \ge [13.5,19.5] \\
& x_{131}^2 + x_{132}^2 + x_{231}^2 + x_{232}^2 \ge [16,20] \\
& x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + \\
& x_{111}^2 + x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \le [47.5,61.5] \\
& x_{112}^1 + x_{122}^1 + x_{132}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 + \\
& x_{112}^2 + x_{122}^2 + x_{132}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \le [62,69.5] \\
& x_{ij} \ge 0\ \forall\ i,j.
\end{aligned}
$$

Now using solution procedure presented in Subsection 4.2 we derive Model 8 and Model 9 and then solve by LINGO iterative scheme.
Model 8

$$
\begin{aligned}
\text{minimize}\ \ & Z_1 = 0.85x_{111}^1 + 1.21x_{112}^1 + 0.68x_{121}^1 + 0.93x_{122}^1 + 1.49x_{131}^1 + 1.1x_{132}^1 \\
& +1.2x_{211}^1 + 1.17x_{212}^1 + 0.95x_{221}^1 + 1.12x_{222}^1 + 1.41x_{231}^1 + 1.9x_{232}^1 \\
& +1.2x_{111}^2 + 1.51x_{112}^2 + 0.9x_{121}^2 + 0.85x_{122}^2 + 1.65x_{131}^2 + 1.45x_{132}^2 \\
& +1.8x_{211}^2 + 1.6x_{212}^2 + 1.14x_{221}^2 + 1.17x_{222}^2 + 1.62x_{231}^2 + 1.8x_{232}^2 \\
\text{subject to}\ \ & \text{constraints (5.1)–(5.14).}
\end{aligned}
$$

Model 9

$$
\begin{aligned}
\text{minimize}\ \ & Z_2 = 1.02x_{111}^1 + 1.35x_{112}^1 + 1.03x_{121}^1 + 1.16x_{122}^1 + 1.59x_{131}^1 + 1.42x_{132}^1 \\
& +1.47x_{211}^1 + 1.33x_{212}^1 + 0.94x_{221}^1 + 1.37x_{222}^1 + 1.54x_{231}^1 + 1.92x_{232}^1 \\
& +1.35x_{111}^2 + 1.46x_{112}^2 + 1.16x_{121}^2 + 1.02x_{122}^2 + 1.57x_{131}^2 + 1.48x_{132}^2 \\
& +1.78x_{211}^2 + 1.81x_{212}^2 + 1.30x_{221}^2 + 1.45x_{222}^2 + 1.74x_{231}^2 + 1.77x_{232}^2 \\
\text{subject to}\ \ & \text{constraints (5.1)–(5.14).}
\end{aligned}
$$

TABLE 7. Optimal solutions by interval programming.

Problem	Optimal value	Optimal solution
(Z_1^L, Z_2^L)	$(893.625, 575)$	$x_{121}^1 = 13.5$, $x_{132}^1 = 13.5$, $x_{212}^1 = 15$, $x_{221}^1 = 5$, $x_{111}^2 = 19.25$, $x_{122}^2 = 17$, $x_{132}^2 = 1.75$, $x_{212}^2 = 2.25$, $x_{231}^2 = 14.25$, and other variables are zero.
(Z_1^U, Z_2^U)	$(1180.75, 929)$	$x_{111}^1 = 13.5$, $x_{132}^1 = 13.5$, $x_{212}^1 = 1.5$, $x_{221}^1 = 18.5$, $x_{111}^2 = 20$, $x_{112}^2 = 1$, $x_{122}^2 = 17$, $x_{212}^2 = 0.5$, $x_{232}^2 = 16$, and other variables are zero.

TABLE 8. Optimal solutions by the expected value operator.

Problem	Optimal value	Optimal solution
(Z_1, Z_2)	$(1102, 807.375)$	$x_{111}^1 = 15$, $x_{121}^1 = 12$, $x_{221}^1 = 6.5$, $x_{232}^1 = 13.5$, $x_{111}^2 = 18.5$, $x_{122}^2 = 3.5$, $x_{132}^2 = 16$, $x_{212}^2 = 3$, $x_{222}^2 = 13.5$ and other variables are zero.

5.1.3. Problem formulation by expected value operator

We solve the proposed model using the expected value operator. To do this here first we convert each of transportation cost parameter into crisp value by apply expected value operator for both the objective functions.

Model 10

$$\begin{aligned} \text{minimize} \quad Z_1 = {} & 8.25x_{111}^1 + 12x_{112}^1 + 7.5x_{121}^1 + 9x_{122}^1 + 13x_{131}^1 + 10.5x_{132}^1 + 11.5x_{211}^1 \\ & + 12.25x_{212}^1 + 7.5x_{221}^1 + 9x_{222}^1 + 14x_{231}^1 + 17x_{232}^1 + 11x_{111}^2 + 13.25x_{112}^2 \\ & + 8.75x_{121}^2 + 8.25x_{122}^2 + 11.5x_{131}^2 + 10.5x_{132}^2 + 13.5x_{211}^2 + 17x_{212}^2 \\ & + 10.5x_{221}^2 + 11.75x_{222}^2 + 15x_{231}^2 + 14.5x_{232}^2 \\ \text{minimize} \quad Z_2 = {} & 6x_{111}^1 + 7.5x_{112}^1 + 5.5x_{121}^1 + 6.5x_{122}^1 + 9.5x_{131}^1 + 8x_{132}^1 + 8.5x_{211}^1 \\ & + 7x_{212}^1 + 6.5x_{221}^1 + 10x_{222}^1 + 8x_{231}^1 + 10.5x_{232}^1 + 7.75x_{111}^2 + 8.5x_{112}^2 \\ & + 6.5x_{121}^2 + 6x_{122}^2 + 11.25x_{131}^2 + 10.25x_{132}^2 + 12x_{211}^2 + 9x_{212}^2 \\ & + 7.5x_{221}^2 + 8x_{222}^2 + 9.75x_{231}^2 + 11.25x_{232}^2 \\ \text{subject to} \quad & \text{constraints (5.1)–(5.14).} \end{aligned}$$

Model 11

$$\begin{aligned} \text{minimize} \quad Z = {} & 0.985x_{111}^1 + 1.330x_{112}^1 + 0.899x_{121}^1 + 1.070x_{122}^1 + 1.555x_{131}^1 \\ & + 1.217x_{132}^1 + 1.384x_{211}^1 + 1.303x_{212}^1 + 0.982x_{221}^1 + 1.359x_{222}^1 \\ & + 1.423x_{231}^1 + 1.286x_{232}^1 + 1.292x_{111}^2 + 1.487x_{112}^2 + 1.055x_{121}^2 \\ & + 0.985x_{122}^2 + 1.611x_{131}^2 + 1.471x_{132}^2 + 1.794x_{211}^2 + 1.75x_{212}^2 \\ & + 1.242x_{221}^2 + 1.291x_{222}^2 + 1.694x_{231}^2 + 1.789x_{232}^2 \\ \text{subject to} \quad & \text{constraints (5.1)–(5.14).} \end{aligned}$$

Example 5.2. A reputed mobile company has two production factories in two places namely S_1 and S_2 in India and distributed mobiles throughout the country. The mobiles are supplied into three markets namely D_1, D_2, and D_3. The mobiles are transported by two types of conveyances ($r = 2$), such as large trucks and freight trains. DM desires to minimize the production cost, and transportation cost by different transportation

Table 9. Production costs for item 1 in the first objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(2, 5, 6, 8)$	$(3, 5, 8, 10)$	$(6, 8, 10, 12)$
S_2	$(5, 7, 10, 12)$	$(4, 5, 6, 7)$	$(7, 9, 11, 13)$
Conveyance $r = 2$			
S_1	$(6, 8, 11, 12)$	$(4, 6, 8, 10)$	$(4, 6, 9, 11)$
S_2	$(7, 8, 10, 11)$	$(3, 5, 7, 9)$	$(10, 12, 16, 18)$

Table 10. Production costs for item 2 in the first objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(6, 7, 5, 9)$	$(3, 6, 8, 10)$	$(8, 9, 10, 11)$
S_2	$(7, 9, 11, 12)$	$(5, 7, 10, 12)$	$(9, 11, 13, 15)$
Conveyance $r = 2$			
S_1	$(6, 8, 9, 10)$	$(3, 4, 7, 9)$	$(4, 6, 7, 9)$
S_2	$(10, 12, 14, 26)$	$(7, 9, 11, 13)$	$(8, 9, 10, 11)$

Table 11. Unit transportation costs for item 1 in the second objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(6, 7, 9, 10)$	$(5, 7, 8, 10)$	$(8, 10, 11, 13)$
S_2	$(9, 11, 12, 14)$	$(6, 7, 8, 9)$	$(7, 8, 9, 10)$
Conveyance $r = 2$			
S_1	$(5, 6, 7, 8)$	$(7, 9, 10, 12)$	$(11, 13, 15, 17)$
S_2	$(7, 9, 11, 13)$	$(8, 10, 12, 14)$	$(7, 8, 9, 11)$

modes. The production costs for item 1 and item 2 are listed in Tables 9 and 10, respectively; whereas the transportation costs for item 1 and item 2 are listed in Tables 11 and 12, respectively. The supply, demand and conveyance capacity are listed in Table 13.

Now we solve Example 5.2 using fuzzy programming, interval programming and expected value operator and the solutions are listed in Tables 14, 15 and 16, respectively.

6. Results and discussion

The MMSTP was introduced by Kundu *et al.* [25], and we solve it using different techniques with new solution procedure and we achieve different compromise solutions. These results are the compromised solution of the

TABLE 12. Unit transportation costs for item 2 in the second objective.

Destinations → Sources ↓	D_1	D_2	D_3
Conveyance $r = 1$			
S_1	$(4, 6, 8, 9)$	$(7, 9, 10, 12)$	$(6, 7, 11, 12)$
S_2	$(8, 9, 12, 13)$	$(10, 11, 12, 13)$	$(9, 11, 13, 14)$
Conveyance $r = 2$			
S_1	$(4, 5, 6, 7)$	$(6, 7, 9, 10)$	$(9, 11, 12, 13)$
S_2	$(8, 10, 12, 14)$	$(5, 6, 12, 13)$	$(10, 11, 13, 15)$

TABLE 13. Availability and demand data.

Fuzzy availability	Fuzzy demand	Conveyance capacity
$\tilde{a}_1^1 = (25, 28, 30, 32)$	$\tilde{b}_1^1 = (12, 14, 17, 20)$	$\tilde{e}_1 = (48, 51, 53, 55)$
$\tilde{a}_2^1 = (32, 36, 39, 41)$	$\tilde{b}_2^1 = (15, 18, 20, 23)$	$\tilde{e}_2 = (53, 55, 58, 61)$
$\tilde{a}_1^2 = (36, 38, 41, 43)$	$\tilde{b}_3^1 = (10, 13, 16, 19)$	
$\tilde{a}_2^2 = (29, 32, 34, 37)$	$\tilde{b}_1^2 = (18, 21, 23, 26)$	
	$\tilde{b}_2^2 = (14, 16, 17, 20)$	
	$\tilde{b}_3^2 = (13, 15, 17, 19)$	

TABLE 14. Optimal solutions by fuzzy programming.

Problem	Optimal value	Optimal solution
Z^1 Z^2	$[172.25, 315.75]$ $[263.69, 423.71]$	$x_{111}^1 = 13$, $x_{132}^1 = 15$, $x_{221}^1 = 11.5$, $x_{122}^2 = 16.307$, $x_{132}^2 = 14$, $x_{211}^2 = 6.807$, $x_{221}^2 = 3.19$, and other variables are zero.

TABLE 15. Optimal solutions by interval programming.

Problem	Optimal value	Optimal solution
(Z_L^1, Z_L^2)	$(427, 452.75)$	$x_{111}^1 = 13$, $x_{121}^1 = 3$, $x_{132}^1 = 15$, $x_{222}^1 = 8.5$, $x_{111}^2 = 6.5$, $x_{122}^2 = 19.5$, $x_{132}^2 = 14$, $x_{211}^2 = 10$, and other variables are zero.
(Z_U^1, Z_U^2)	$(724.25, 764.5)$	$x_{111}^1 = 13$, $x_{132}^1 = 15$, $x_{221}^1 = 11.5$, $x_{111}^2 = 16.5$, $x_{122}^2 = 19.5$, $x_{132}^2 = 4$, $x_{232}^2 = 10$, and other variables are zero.

Table 16. Optimal solutions by the expected value operator.

Problem	Optimal value	Optimal solution
(Z^1, Z^2)	$(629, 747.25)$	$x^1_{111} = 13$, $x^1_{221} = 11.5$, $x^1_{231} = 15$, $x^2_{111} = 14.5$, $x^2_{112} = 2$, $x^2_{122} = 19.5$, $x^2_{132} = 4$, $x^2_{232} = 10$ and other variables are zero.

solution obtained by Kundu *et al.* [25]. The solutions are obtained by fuzzy programming and interval programming for the MMSTP with parameters interval numbers and the solution obtained by expected value operator for MMSTP with parameters rough interval, are summarized in Tables 6, 7 and 8, respectively. The solution is obtained by fuzzy programming for both the objective values lie within the intervals $[868, 905, 1208.495]$ and $[603.21, 919.585]$ respectively, which are better than the solution obtained by Kundu *et al.* [25]. More precisely here we use a new algorithm to convert multi-objective with coefficients interval numbers into two single objectives (due to present interval) and then we utilize fuzzy programming to solve it. The obtained solution by the proposed interval programming lies within the interval $[893.625, 1180.75]$ and $[575, 929]$, which is also another compromise solution if we do not apply fuzzy programming. We solve here MMSTP by expected value operator and we obtain the solutions for both objective functions. which are 1102, 807.375 and which strongly lie within the interval solution obtained by fuzzy programming and interval programming. Even the interval of solution for the 1st and 2nd objective functions mentioned by Kundu *et al.* [25] in method 2 was $[1039.232, 1227.548]$ and $[753.1357, 935.7518]$, respectively and the core was $[1043.019, 1233.229]$ and $[745.5609, 930.0706]$, respectively. In comparison to their solutions, the obtained solution is better, and the proposed solution procedure is easy to understand. On the other hand, using the proposed methodology, the trapezoidal fuzzy numbers are converted into the nearest intervals and rough intervals. So, for the same data, we have a conversation rule from trapezoidal fuzzy numbers to interval and rough interval numbers.

7. Conclusions and outlook

In this paper, we have incorporated a new approach for solving MMSTP involving parameters as trapezoidal fuzzy numbers. Thereafter we have converted trapezoidal fuzzy numbers into the nearest interval and nearest RI, and we have designed two models of MMSTP. The first model is on MMSTP *i.e.*, MMSTP when parameters are interval form has been solved by using fuzzy programming and interval programming. The second model is of MMSTP *i.e.*, MMSTP when parameters are RI has been solved by using the expected value operator. The superiority of the proposed approach has been also supported by a computational experiment conducted in the numerical example. We have presented the comparison of the results with the existing results mentioned in Kundu *et al.* [25] and we have found that the result has obtained by the discussed approach is better.

In this study, we have also focussed on decision making, the derived decision may fluctuate depending upon the different DM choices; as a result, the decision-making process becomes difficult. Considering this difficult decision-making process, first we have considered them as fuzzy numbers, thereafter, have been converted them into interval range and rough interval range where the DM feels relaxed to make the decision; in this paper, the range has been referred to as “solution space”.

In the future study, the contents of this paper may be extended to multi-modal solid transportation problem in the rough set environment. Also, the concept of the paper creates a wide range of uncertain MMSTP [7]. As a suggestion, one can formulate other types of uncertain MMSTP such as two-level uncertain MMSTP [8].

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References

[1] E.E. Ammar, Study on multi-objective solid transportation problems. *Eur. J. Sci. Res.* **125** (2014) 7–19.

[2] E.E. Ammar and E.A. Youness, Study on multi-objective transportation problem with fuzzy numbers. *Appl. Math. Comput.* **166** (2005) 241–253.

[3] E.E. Ammar and H.A. Khalifa, On fuzzy multi-objective multi-item solid transportation problems. *Int. J. Comput. Organ. Trends* **17** (2015) 1–19.

[4] E.E. Ammar and H. Khalifa, A fuzzy solution approach for optimizing water resources management problem. *Int. J. Ind. Eng. Prod. Res.* **30** (2019) 1–10.

[5] M.G. Alharbi, H.A.E.W. Khalifa and E.E. Ammar, An interactive approach for solving the multi-objective minimum cost flow problem in the fuzzy environment. *J. Math. Res.* (2020) **2020** 6247423.

[6] P. Anukokila, B. Radhakrishnan and A. Anju, Goal programming approach for solving multi-objective fractional transportation problem with fuzzy parameters. *RAIRO:RO* **53** (2019) 157–178.

[7] H. Dalman, Uncertain programming model for multi-item solid transportation problem. *Int. J. Mach. Learn. Cybern.* **9** (2018) 559–567.

[8] H. Dalman, N. Güzel and M. Sivri, A fuzzy set-based approach to multi-objective multi-item solid transportation problem under uncertainty. *Int. J. Fuzzy Syst.* **18** (2016) 716–729.

[9] A. Ebrahimnejad, A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. *Appl. Soft Comput.* **19** (2014) 171–176.

[10] M. Gen, K. Ida, Y. Li and E. Kubota, Solving bicriteria solid transportation problem with fuzzy numbers by a genetic algorithm. *Comput. Ind. Eng.* **29** (1995) 537–541.

[11] S. Ghosh and S.K. Roy, Fuzzy-rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station. *RAIRO:RO* **56** (2020) S2923–S2952.

[12] S. Ghosh, S.K. Roy, A. Ebrahimnejad and J.L. Verdegay, Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem. *Complex Intell. Syst.* **7** (2021) 1009–1023.

[13] S. Ghosh, K.-H. Küfer, S.K. Roy and G.-W. Weber, Carbon mechanism on sustainable multi-objective solid transportation problem for waste management in Pythagorean hesitant fuzzy environment. *Complex Intell. Syst.* **8** (2022) 4115–4143.

[14] S. Ghosh, S.K. Roy and A. Fügenschuh, The multi-objective solid transportation problem with preservation technology using Pythagorean fuzzy sets. *Int. J. Fuzzy Syst.* **24** (2022) 2687–2704.

[15] S. Ghosh, K.-H. Küfer, S.K. Roy and G.-W. Weber, Type-2 zigzag uncertain multi-objective fixed-charge solid transportation problem: time window vs. preservation technology. *Cent. Eur. J. Oper. Res.* (2022) 1–26.

[16] B. Giri and S.K. Roy, Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem. *Int. J. Mach. Learn. Cybern.* **13** (2022) 3089–3112.

[17] P. Grzegorzewski, Nearest interval approximation of a fuzzy number. *Fuzzy Sets Syst.* **130** (2002) 321–330.

[18] K.B. Haley, New methods in mathematical programming the solid transportation problem. *Oper. Res.* **10** (1962) 448–463.

[19] A. Hamzehee, M.A. Yaghoobi and M. Mashinchi, Linear programming with rough interval coefficients. *J. Intell. Fuzzy Syst.* **26** (2014) 1179–1189.

[20] F.L. Hitchcock, The distribution of a product from several sources to numerous localities. *J. Math. Phys.* **20** (1941) 224–230.

[21] F. Jiménez and J.L. Verdegay, Uncertain solid transportation problems. *Fuzzy Sets Syst.* **100** (1998) 45–57.

[22] F. Jiménez and J.L. Verdegay, Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach. *Eur. J. Oper. Res.* **117** (1999) 485–510

[23] A. Kaur and A. Kumar, A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. *Appl. Soft Comput.* **12** (2012) 1201–1213.

[24] P. Kaur, V. Verma and K. Dahiya, Capacitated two-stage time minimization transportation problem with restricted flow. *RAIRO:RO* **51** (2017) 447–467.

[25] P. Kundu, S. Kar and M. Maiti, Multi-objective multi-item solid transportation problem in fuzzy environment. *Appl. Math. Model.* **37** (2013) 2028–2038.

[26] Y.Z. Li, Improved genetic algorithm for solving multi-objective solid transportation problem with fuzzy number. *Japanese J. Fuzzy Theory Syst.* **4** (1998) 220–229.

[27] D.R. Mahapatra, S.K. Roy and M.P. Biswal, Multi-choice stochastic transportation problem involving extreme value distribution. *Appl. Math. Model.* **37** (2013) 2230–2240.

[28] G. Maity, S.K. Roy and J.L. Verdegay, Multi-objective transportation problem with cost reliability under uncertain environment. *Int. J. Comput. Intell. Syst.* **9** (2016) 839–849.

[29] G. Maity, D. Mardanya, S.K. Roy and G.W. Weber, A new approach for solving dual-hesitant fuzzy transportation problem with restrictions. *Sadhana* **44** (2019) 75.

[30] G. Maity, S.K. Roy and J.L. Verdegay, Analyzing multi-modal transportation problem and its application to artificial intelligence. *Neural Comput. Appl.* **32** (2020) 2243–2256.

[31] D. Mardanya and S.K. Roy, Time variant multi-objective linear fractional interval-valued transportation problem. *Appl. Math. J. Chin. Univ.* **37** (2022) 111–130.

[32] D. Mardanya, G. Maity and S.K. Roy, The multi-objective multi-item just-in-time transportation problem. *Optimization* (2021) 1–32.

[33] D. Mardanya, G. Maity and S.K. Roy, Solving bi-level multi-objective transportation problem under fuzziness. *Int. J. Uncertain. Fuzziness Knowlege-Based Syst.* **29** (2021) 411–433.

[34] D. Mardanya, G. Maity, S.K. Roy and V.F. Yu, Solving the multi-modal transportation problem *via* the rough interval approach. *RAIRO:RO* **56** (2022) 3155–3185.

[35] S. Midya, S.K. Roy and F.Y. Vincent, Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. *Int. J. Mach. Learn. Cybern.* **12** (2020) 699–717.

[36] S. Midya, S.K. Roy and G.W. Weber, Fuzzy multiple objective fractional optimization in rough approximation and its aptness to the fixed-charge transportation problem. *RAIRO:RO* **55** (2021) 1715–1741.

[37] R.E. Moore, Methods and applications of interval analysis. *Soc. Ind. Appl. Math.* (1979) 2.

[38] R. Moore and W. Lodwick, Interval analysis and fuzzy set theory. *Fuzzy Sets Syst.* **135** (2003) 5–9.

[39] A. Nagarjan and K. Jeyaraman, Solution of chance constrained programming problem for multi-objective interval solid transportation problem under stochastic environment using fuzzy approach. *Int. J. Comput. Appl.* **10** (2010) 19–29.

[40] Z. Pawlak, Rough sets. *Int. J. Comput. Inf. Sci.* **11** (1982) 341–356.

[41] M. Rebolledo, Rough intervals-enhancing intervals for qualitative modeling of technical systems. *Artif. Intell.* **170** (2006) 667–685.

[42] S.K. Roy and S. Midya, Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. *Appl. Intell.* **49** (2019) 3524–3538.

[43] S.K. Roy, G. Maity, G.W. Weber and S.Z. Alparslan Gök, Conic scalarization approach to solve multi-choice multi-objective transportation problem with interval goal. *Ann. Oper. Res.* **253** (2017) 599–620.

[44] S.K. Roy, G. Maity and G.W. Weber, Multi-objective two-stage grey transportation problem using utility function with goals. *Cent. Eur. J. Oper. Res.* **25** (2017) 417–439.

[45] S.K. Roy, A. Ebrahimnejad, J.L. Verdegay and S. Das, New approach for solving intuitionistic fuzzy multi-objective transportation problem. *Sadhana* **43** (2018).

[46] S.K. Roy, S. Midya and V.F. Yu, Multi-objective fixed-charge transportation problem with random rough variables. *Int. J. Uncertain. Fuzziness Knowledge-Based Syst.* **26** (2018) 971–996.

[47] E. Schell, Distributuin of s product by several properties, in *Proceedings of 2nd Symposium in Linear Programming*. DCS/comptroller, HQ US Air Force, Washington DC (1955) 615–642.

[48] A. Tanksale and J.K. Jha, A hybrid fix-and-optimize heuristic for integrated inventory-transportation problem in a multi-region multi-facility supply chain. *RAIRO:RO* **54** (2020) 749–782.

[49] S. Xiao and E.M.K. Lai, A rough programming approach to power-balanced instruction scheduling for VLIW digital signal processors. *IEEE Trans. Signal Process.* **56** (2008) 1698–1709.

[50] L. Yang and Y. Feng, A bicriteria solid transportation problem with fixed-charge under stochastic environment. *Appl. Math. Model.* **31** (2007) 2668–2683.

[51] L. Yang and L. Liu, Fuzzy fixed-charge solid transportation problem and algorithm. *Appl. Soft Comput.* **7** (2007) 879–889.