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CHANNEL STRUCTURE OF E-PLATFORM ENCROACHMENT

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Abstract. This paper focuses on the supply chain consisting of a manufacturer, a third-party (3P) seller, and an e-platform, while the 3P seller sells products through the e-platform. If the e-platform not only provides a sales channel for the 3P seller, but also determines the retail price and then sells products directly, we call it e-platform encroachment, which creates price competition between the 3P seller and e-platform. We find that the encroachment benefits the manufacturer and e-platform but hurts the 3P seller. And both the manufacturer and 3P seller prefer the price discrimination strategy to the uniform pricing strategy, while the e-platform does not necessarily. The price discrimination strategy encourages the manufacturer to offer the 3P seller a preferential wholesale price as the compensation and to curb the competitive advantage of the e-platform. The two pricing strategies have the same total sales quantity but different allocations across channels. Moreover, we propose a new coordination mechanism to reduce the double marginalization effect and improve supply chain performance. In extensions, we demonstrate numerically how the e-platform should decide the commission rate and compare the e-platform encroachment model with the manufacturer encroachment model to reveal the impact of different encroachment roles on all parties in the supply chain.

Mathematics Subject Classification. 91A40, 91B50.

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1. Introduction

E-platforms allow transactions to occur beyond the limitation of time and space. Due to their convenience and the impact of the COVID-19 pandemic on offline sales, global e-commerce sales have reached over $2 trillion annually.

With such huge dividends, e-platforms adopt different operation formats according to their conditions and market environment. Some e-platforms offer agency-only services for 3P sellers, such as AliExpress and Tmall. In the e-platform agency business, the 3P seller wholesales products from the manufacturer and delivers them to customers through the e-platform. The 3P sellers are flocking to large e-platforms to take advantage of the web traffic and high product exposure available through such e-platforms; in turn, e-platforms charge commission fees. In this cooperation relationship, the decision goal is to maximize their total profit.

Some other e-platforms encroach on the retail market while offering agent services for 3P sellers, such as Amazon and JD. They encroach by opening a self-support store or introducing a private brand, wholesaling

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from manufacturers, and then delivering the products to customers directly [3]. As an encroacher, the e-platform has the right to decide the products' retail price in the self-support channel (encroaching channel). Therefore, there is price competition between the e-platform and 3P seller. In this cooperation and competition relationship, each party makes decisions to maximize its own profit.

The above business diversity of large e-platforms provides material for this research. E-platform encroachment significantly impacts the supply chain structure, thus forming an online dual-channel supply chain. It is imperative for the upstream manufacturers to decide on whether to treat the downstream 3P seller and encroaching e-platform equally or differently with respect to the wholesale price. The pricing strategy is generally based on the product type. For example, in different channels, the price of electronic products is the same, but food and apparel prices differ. Some countries or regions also have relevant laws and regulations on the pricing mechanism. For instance, American Robinson-Patman act [5] and Indian competition act [28] prohibit differential pricing of goods or services.

Motivated by the e-commerce trends, we put forward the first research question of this study: Is the e-platform willing to encroach on the retail market?

As a follow-up, we ask: If the e-platform encroaches on the retail market, how should the manufacturer set wholesale prices and control the competition between the two sales channels: by a uniform pricing strategy or a price discrimination strategy?

If the e-platform encroaches, it obtains commission profit and sales profit. This study also aims to answer the third research question: How should the e-platform balance the two sources of profit under different pricing strategies?

In the wholesale price model, the efficiency of the supply chain is often low because of the double marginalization effect. Therefore, we propose the fourth research question: How can an effective contract be designed to improve the performance of the online dual-channel supply chain?

Based on the above motives, we establish a Stackelberg game model and use backward induction to derive the equilibrium decisions. The e-platform determines whether to encroach on the 3P seller. If it does not encroach, then as an agent, the e-platform does not participate in pricing. Otherwise, the manufacturer determines the wholesale prices as the leader, and then the 3P seller and e-platform determine retail prices simultaneously as followers. Our analysis shows that encroaching always benefits the e-platform and manufacturer, but it hurts the 3P seller. The price discrimination strategy is better for the manufacturer and 3P seller than the uniform pricing strategy, but this is not necessarily true for the e-platform. Under the price discrimination strategy, the manufacturer charges the e-platform a higher wholesale price, intending to reduce the competitiveness of the encroaching e-platform and compensate the 3P seller; this is called the "compensation effect". In the encroachment model, the sum of the two channels' sales quantity under the two strategies is the same. Under the uniform pricing strategy, the e-platform has a higher sales quantity than the 3P seller. Therefore, the e-platform's sales profit is always higher than its commission profit. While under the price discrimination strategy, the relationship between the e-platform and 3P seller's sales quantities is uncertain, as are the two sources of the e-platform's profit. These results reveal that the relationship between the two channels changes subtly, and the e-platform must carefully consider the balance between its profit sources. Later, we demonstrate that a dual-channel two-part tariff contract can coordinate our online dual-channel supply chain, and the manufacturer compensates the 3P seller with a lower wholesale price and a lower fixed fee.

In extensions, we reveal the following discoveries by numerical analysis.

(i) In the base model, the e-platform commission rate is exogenous. If the e-platform decides the commission rate endogenously, what impact will this have on the supply chain?
We explore how the e-platform determines the optimal commission rate, finding that when facing more fierce channel competition, the e-platform will be more exploitative toward the 3P seller but not drive it out of the market.

(ii) Besides the e-platform, the manufacturer also has an incentive to encroach on the retail market. If the manufacturer decides to encroach, it has the power to decide the retail price, and the e-platform only acts as an agent. Which model do the participants prefer, e-platform encroachment or manufacturer encroachment?

For the 3P seller, the e-platform encroachment model is always better than the manufacturer encroachment model, but not necessarily for the manufacturer and e-platform, and the conditions for them to choose a particular model are opposite but not complementary.

This study investigates the motivation for e-platform encroachment. In addition, pricing strategies, channel competition, and dual-channel supply chain contract design are studied to provide a reference for upstream manufacturers. Finally, this study makes a horizontal comparison of encroachment models to complement the research on supply chain encroachment and provide guidance for supply chain channel expansion.

The remainder of this paper is organized as follows. In the next section, we review the relevant literature. Section 3 details our model, including different manufacturer pricing strategies. Then, we propose a contract to improve the performance of the supply chain in Section 4. In Section 5, we consider the endogenous commission rate in one extension and the manufacturer encroachment patterns in the other. In Section 6, we conclude the paper with a summary of the key findings, insights, and future research directions. All of the proofs are presented in the Appendix.

2. Literature review

This study draws background information from three streams of related literature: encroachment, pricing strategy, and contract design in dual-channel supply chains.

Initially, Chiang *et al.* [11] use a mathematical model to study the impact of manufacturer encroachment, overturning the common intuition that adding a direct channel would damage the retailer. Later literature increasingly focused on the effect of encroachment on the strategic mode choice between the manufacturers and retailers. Cattani *et al.* [8] prove that the threat of the retailer switching to another manufacturer affects the manufacturer's encroachment decision. Niu *et al.* [31] and Li *et al.* [23] investigate that the encroachment of a direct online channel may lead to revenge by the retailer on the manufacturer, but fairness concerns and quality improvements can avoid this problem.

Unlike the above traditional online and offline channel supply chains, this study focuses on online channels, and the e-platform competes and cooperates with the 3P seller. This sales mode is most related to the "Plan of Open Platform" (POP), which allows third-party sellers to sell on the e-platform. Chen and Guo [9] reveal the interaction of 3P sellers' advertising strategy and the e-platform's opening strategy. In their study, the 3P seller can improve consumer awareness through joining the e-platform or advertising by itself. Mantin *et al.* [27] analyze how the e-platform opening strategy affects bargaining between the e-platform and manufacturer. Song *et al.* [36] study the selling strategy of the 3P seller in an open e-platform considering the effect of increased exposure of one product on the sales of the other. Sun and Liu [37] detail whether a retailer with a physical store should also join the e-platform to participate in online sales. The biggest difference between our study and the above studies is that we consider the e-platform's encroaching decision rather than its opening decision. Encroaching brings sales revenue to the e-platform, while opening brings commission revenue. Moreover, the decision-making of the upstream manufacturer and the coordination of the whole supply chain are considered in this study.

Most research on dual-channel pricing strategy is based on the traditional model, often incorporating the following factors: delivery lead time of online channel [17], demand disruptions [18, 44], risk aversion [24], supply shortage [40], cash-strapped suppliers [43], green supply chain [21, 32], reverse supply chain [38], closed-loop supply chain [15, 32], and showrooming effect [25]. Lou *et al.* [26] is closely related to our motif, but they respectively consider how pricing is affected by the competition or cooperation of the three participants, and the cooperation between any two sides is meant to achieve maximum aggregate revenue. In contrast, the encroachment model in our paper involves both competition and cooperation, and one side in the cooperation provides agency to the other through revenue sharing, but each still makes decisions to maximize its own profit. The model structures of Fu *et al.* [13] and Huang *et al.* [19] are related to ours. In addition to the difference in supply chain contract, neither of them distinguishes the roles of the manufacturer and 3P seller, and the manufacturer only determines the wholesale price for only one of the two channels. Tian *et al.* [39] also study

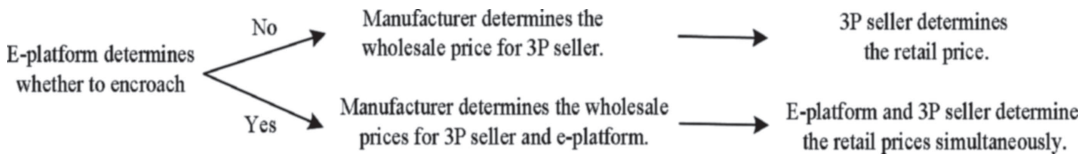

FIGURE 1. The sequence of events in the game.

a similar structure containing two competing suppliers. However, there is only cooperation and no competition between the supplier and e-platform. Our study analyzes the manufacturer's choice of pricing strategy based on the competition and cooperation between the manufacturer and e-platform, which fills the gap in dual-channel supply chain management.

Price discrimination strategy has been well studied in general but less so in the dual-channel supply chain context. Chun and Kim [12] were the first to study this mechanism in dual-channel supply chains and investigate how factors such as market size and product characteristics impact the degree of price discrimination. Zhou *et al.* [50] study the mutual restriction between manufacturer price discrimination and retailer information disclosure using a two-stage dual-channel model. Brunner [4] and Roffin and Mahanty [33] compare the manufacturer's discrimination pricing strategy and uniform pricing strategy in a dual-channel model. The two channels in their models compete with each other, but in our model, the two channels compete while also cooperating. Because of the different model structures, we have a different price discrimination strategy than the previous studies.

A contract is a standard tool to improve supply chain efficiency. Early work by Cai [6] proves that a revenue-sharing contract can coordinate the dual-channel supply chain, and the revenue sharing rate in the online and offline dual-channel model is higher than that in offline dual-channel. Chen *et al.* [10] propose the two-part tariff contract and revenue sharing contract to coordinate the dual-channel supply chain, and they achieve a win-win situation for the manufacturer and retailer. The subsequent literature studies the contract design of various types of dual-channel supply chains, such as a wholesale price contract in a supply chain with asymmetric cost information [7], a two-way revenue contract in a supply chain with risk-aversion [41], a lump sum fee contract in a supply chain with demand disruptions and production cost disruptions [47], a price discount contract or two-part tariff contract in a supply chain with emission reduction [14, 42], and a revenue-sharing contract in a dual-channel fresh agricultural product supply chain [45]. In this paper, we use the two-part tariff contract in both channels, which is effective for coordinating the online dual channel supply chain.

In summary, this study differs from the above literature in two aspects. First, our focus on the e-platform's decision to encroach distinguishes this study from previous research on e-platform openness. Second, the analysis of different pricing strategies and the design of a coordination contract in this study are based on a new channel structure.

3. MODEL

We establish a Stackelberg game model of an online dual-channel supply chain consisting of a manufacturer, a 3P seller, and an e-platform. The manufacturer produces a single product at a unit cost c. We consider whether the e-platform encroaches on the market by selling the product itself while also supporting the 3P seller's sales.

If the e-platform chooses to encroach, the manufacturer will distribute products through the channels of the 3P seller and e-platform at wholesale prices w^S and w^P, shown as Figure 2a. In reality, manufacturers may supply products to retailers at the same wholesale price (uniform pricing strategy, $w^S = w^P$) or different wholesale prices (price discrimination strategy, $w^S \neq w^P$), which are applicable to different scenarios. Apple and Huawei sell a electronic product at the same price in different channels. Some studies compare the two strategies [4, 33]. After the manufacturer determines wholesale prices, the 3P seller and e-platform determine their retail prices p^S and p^P. The two channels are superscribed as S and P, respectively.

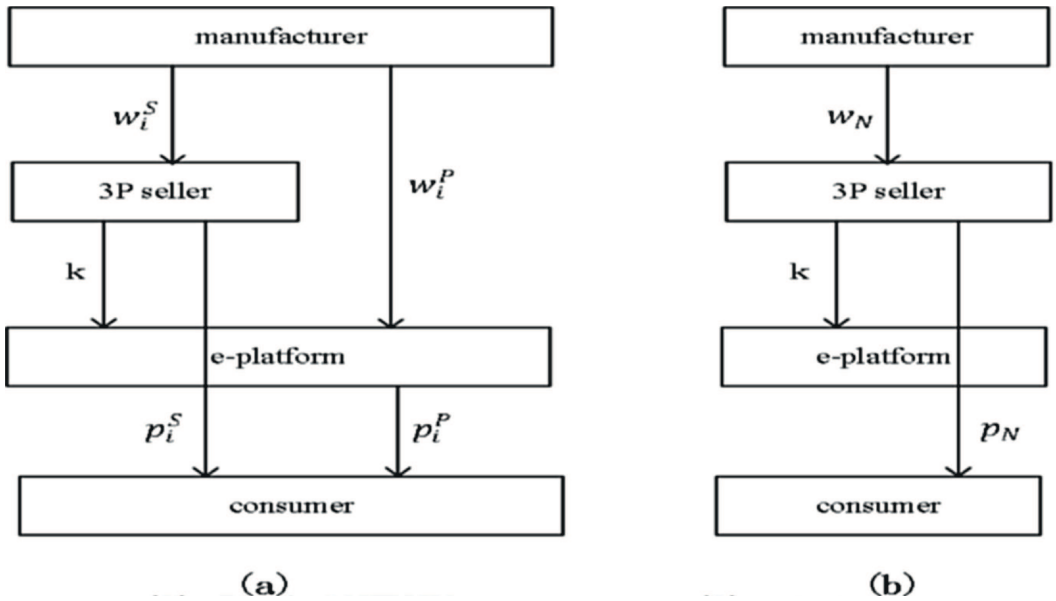

FIGURE 2. Model framework of e-platform encroachment. (a) Encroachment. (b) No encroachment.

The sales quantity of the two channels is based on Bertrand competition as in many other studies [20,29,34]:

$$q_i^S = a_1 - p_i^S + bp_i^P \tag{1}$$
$$q_i^P = a_2 - p_i^P + bp_i^S \tag{2}$$

The two channels' base demands are a_1 and a_2. For ease of exposition, we assume that $a_1 = a_2 = a$ [22,30]. This assumption is reliable because the two channels in this study face the same market and have the same customer group. To facilitate calculation and to avoid losing generality, we standardize a_1 and a_2 as 1 [16]. In addition, b is cross-price sensitivity of the two channels, reflecting the degree of competition between them and the migration rate when customers realize the price difference.

For notational convenience, $i = N, U, D, C, CO$, and DS, the subscript N refers to no encroachment, U to uniform pricing, D to price discrimination, C to a centralized system, and CO to a dual-channel two-part tariff contract.

If the manufacturer does not encroach on the retail market, there is a single-channel supply chain shown in Figure 2b. The manufacturer distributes products to the 3P seller, and the 3P seller delivers to customers through the e-platform by paying commission fees. In this case, the e-platform only provides agency services for the 3P seller and does not determine the retail price.

Referring to Abhishek *et al.* [1] and Shen *et al.* [34], we set equation (1) $q_N^S = 1 - p_N^S + bp_N^P = 0$, which gives $p_N^S = 1 + bp_N^P$. We substitute p_N^S into equation (2) to yield the sales quantity of single-channel $q_N^P = 1 + b - (1 - b^2)p_N^P$. Because there is only one channel, we rewrite it as:

$$q_N = 1 + b - (1 - b^2)p_N. \tag{3}$$

This quantity function also applies to the case where the e-platform only operates its self-support store and does not act as an agent for a 3P seller.

The e-platform provides traffic access to the 3P seller and receives a revenue cut in the form of a commission. Consistent with Zennyo [46], we assume that the e-platform charges commission fees in the form of revenue

sharing. That is, the 3P seller pays the e-platform k proportional revenue for each unit of product sold. In practice, most famous e-platforms, such as Tmall, JD, and Amazon, adopt a revenue-sharing contract, and Zhang *et al.* [48] prove that the revenue-sharing contract is better than the fixed-fee contract.

3.1. No encroachment

In this section, we discuss the non-encroachment case in which the manufacturer only wholesales products to the 3P seller, and the e-platform charges commission fees as an agent, thus, forming a single-channel supply chain with three parties (Fig. 2b).

The problems of the manufacturer and 3P seller are as follows:

$$\max_{w_N} \Pi_N^M = (w_N - c)q_N, \tag{4}$$

$$\max_{p_N} \Pi_N^S = ((1-k)p_N - w_N)q_N. \tag{5}$$

The profit of the e-platform is

$$\Pi_N^P = kp_N q_N. \tag{6}$$

By simple calculation, we get the profits of the three parties as below:

$$\Pi_N^{M*} = \frac{(b+1)(1-k-(1-b)c)^2}{8(1-b)(1-k)},$$

$$\Pi_N^{S*} = \frac{(1+b)(1-k-(1-b)c)^2}{16(1-b)(1-k)},$$

$$\Pi_N^{P*} = \frac{k(1+b)(1-k-(1-b)c)(3(1-k)+(1-b)c)}{16(1-b)(1-k)^2}.$$

We have $c < \frac{1-k}{1-b}$ in this sale model, and the calculation is given in the appendix.

3.2. Encroaching under the uniform pricing strategy

We discuss the game equilibrium under the uniform pricing strategy in this section. A manufacturer provides products to a 3P seller and an e-platform with the same wholesale price w_U. Subsequently, the 3P seller and e-platform determine the two channels' retail prices p_U^S and p_U^P in response to the manufacturer's decisions. Replace i with U in Figure 2a to obtain the framework in this case.

We use backward induction to solve this game model. Given the wholesale price of w_U, the problem of the 3P seller can be formulated as follows:

$$\max_{p_U^S} \Pi_U^S = ((1-k)p_U^S - w_U)q_U^S. \tag{7}$$

The profit of the e-platform consists of sales profit $(p_U^P - w_U)q_U^P$ and commission profit $kp_U^S q_U^S$. Therefore, the problem of the e-platform is

$$\max_{p_U^P} \Pi_U^P = kp_U^S q_U^S + (p_U^P - w_U)q_U^P. \tag{8}$$

The objective function Π_U^S and Π_U^P is respectively concave in p_U^S and p_U^P. According to the first-order conditions, the 3P seller and e-platform have the following pricing decisions:

$$p_U^S(w_U) = \frac{(1-k)(b(1+w_U)+2)+2w_U}{(1-k)(4-b^2(1+k))}, \tag{9}$$

$$p_U^P(w_U) = \frac{b(1+k)(1+w_U-k)+2(1-k)(1+w_U)}{(1-k)(4-b^2(1+k))}. \tag{10}$$

The manufacturer–the Stackelberg leader–maximizes its profit by optimally determining w_U by the following problem:

$$\max_{w_U} \Pi_U^M = w_U q_U^S(w_U) + w_U q_U^P(w_U) - c(q_U^S + q_U^P). \tag{11}$$

According to equation (11), we can have w_U^* by the first-order condition. Next, we substitute it into equations (9) and (10). The optimal decisions are revealed in Theorem 3.1.

Theorem 3.1. *Under the uniform pricing strategy, the optimal prices of the manufacturer, 3P seller, and e-platform are* $w_U^* = \frac{b(1-k)(2-2c-k)+2c(2-k)+4(1-k)-b^2(2c-k^2+k)}{4(1-b)(2-k+b)}$, $p_U^{S*} = \frac{A}{4(1-b)(1-k)(2-k+b)(4-b^2(k+1))}$, *and* $p_U^{P*} = \frac{B}{4(1-b)(1-k)(2+b-k)(4-b^2(1+k))}$.

Where A and B are available in the appendix.

The unit production cost in this section has an upper bound to ensure a positive marginal profit for all participants, that is $c < \bar{c}$, which is also presented in the appendix.

Significantly, $p_U^{S*} - p_U^{P*} = \frac{(1-b)k(b(2c+(1-k)(k+2))+2c(2-k)+4(1-k))}{2(1-k)(2+b-k)(4-b^2(k+1))} > 0$. That is, the 3P seller sets a higher retail price than the e-platform. The 3P seller pays wholesale fees as well as commission fees, which leads to a higher marginal cost and a higher retail price compared to the e-platform. We address the following Corollary 3.2 to investigate the effects of specific system parameters on these decisions.

Corollary 3.2. *The effect of k and b on equilibrium:*

(i) $\frac{\partial w_U^{S*}}{\partial b} > 0$, $\frac{\partial p_U^{S*}}{\partial b} > 0$, *and* $\frac{\partial p_U^{P*}}{\partial b} > 0$. *A higher competition degree induces a more severe double marginalization effect.*

(ii) $\frac{\partial w_U^{S*}}{\partial k} < 0$, $\frac{\partial p_U^{S*}}{\partial k} > 0$, *when* $b < \breve{b}$, $\frac{\partial p_U^{P*}}{\partial k} < 0$, *and* $b > \breve{b}$, $\frac{\partial p_U^{P*}}{\partial k} > 0$, *where* $\breve{b}$ *satisfies* $\hat{b}(k) < \breve{b} < \widetilde{b}(k)$. *A higher commission rate induces the manufacturer to reduce the wholesale price and motivates the 3P seller to raise the retail price, while how the commission rate affects the e-platform's retail price depends on the competition degree.*

See the appendix for $\hat{b}(k)$, $\breve{b}$, and $\widetilde{b}(k)$. Corollary 3.2 gives the influence of k and b on the equilibrium decisions.

When the competition between the 3P seller and e-platform is fiercer, both of them are willing to spend more to compete for the market, which makes the wholesale price and retail prices higher. Undoubtedly, the double marginalization effect is increased.

If the commission rate increases, on the one hand, the 3P seller raises the retail price to avoid the excessive reduction of marginal revenue and thus achieve a new balance of marginal revenue and cost. On the other hand, the manufacturer must lower the wholesale price to keep the sales quantity of the 3P seller, even if it also benefits the e-platform. The impact of the commission rate on the e-platform's retail price is constrained by two factors in opposite directions. First, the e-platform benefits from reducing the wholesale price, and it is eager to lower the retail price for more sales. Second, a larger commission rate incentivizes the e-platform to set a higher retail price to enhance the competitiveness of the 3P seller for more commission fees. The first effect is dominant when the competition degree is low, so the commission rate negatively affects the e-platform's retail price. The opposite is true when the competition degree is high.

When the manufacturer sets the uniform wholesale prices for the encroaching e-platform and 3P seller, the e-platform has two profit sources. The first is the commission profit from the 3P seller $\Pi_U^{p1*} = kp_U^{S*}q_U^{S*}$, and the second is the sales profit from self-support store $\Pi_U^{p2*} = (p_U^{P*} - w_U^{S*})q_U^{P*}$. Comparing the two profit sources with the e-platform's profit in the non-encroachment situation, we can reveal how the encroachment affects the commission profit and total profit. We show the influence of the parameters on the profits in Figure 3.

The parameter values of c, k, and b in this figure and the below numerical analysis are as random as possible in $(0, 1)$. All the values are in the meaningful range.

Figure 3 illuminates that the encroachment decreases the commission profit. However, the sales profit brought by encroaching can cover this loss. The sales profit is always more than the commission profit. Correspondingly,

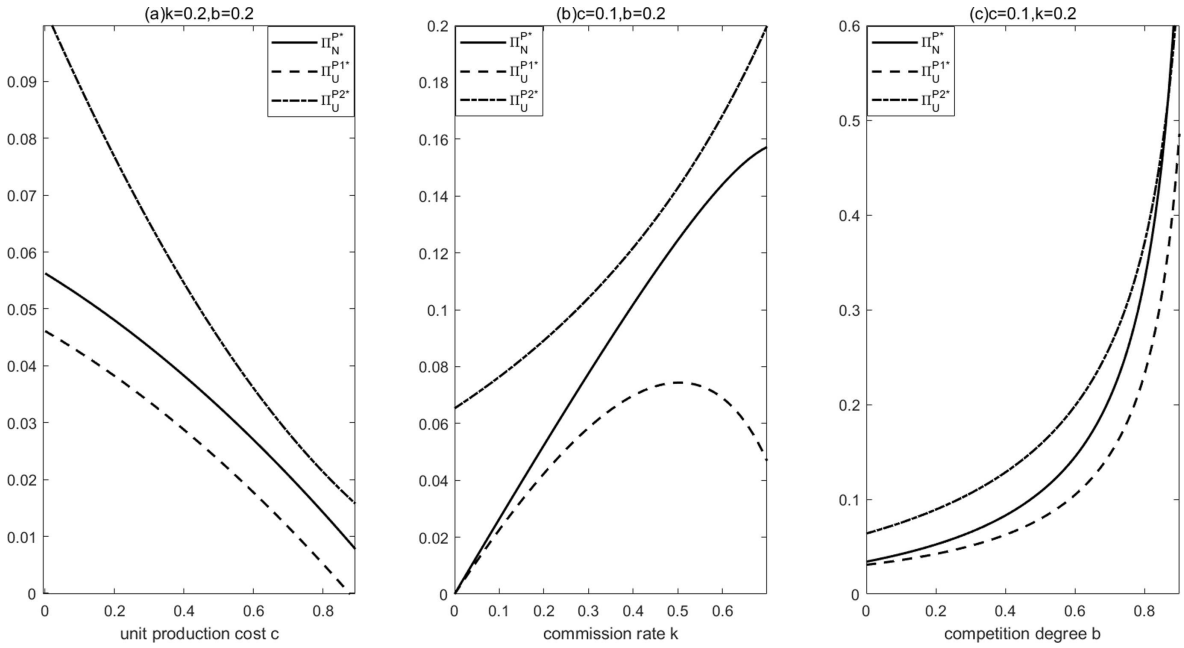

FIGURE 3. The e-platform's profit sources regarding. c, k, b under uniform pricing and non-encroachment strategy.

the e-platform should put more emphasis on the self-support business, especially when the commission rate is high.

It is easy to understand that the e-platform's profit sources decrease in unit production cost and increase in the competition degree in Figure 3a and 3c. In Figure 3b, the commission profit is quasi-concave in the commission rate, which is because the commission profit is the product of the commission rate and the 3P seller's sales revenue, and the two parts have a trade-off. We find that when the commission rate is large, the commission revenue decreases faster than the sales revenue increases. Therefore, a too small or too large commission rate is detrimental to the e-platform.

3.3. Encroaching under the price discrimination strategy

In this model, the manufacturer provides products to the 3P seller and e-platform with the different wholesale prices w_D^S and w_D^P, and then the 3P seller and e-platform determine the two channels' retail prices p_D^S and p_D^P in response to the manufacturer's decisions. A framework can be obtained by replacing i with D in Figure 2a. Therefore, the profit maximization problems of the manufacturer, 3P seller, and e-platform are given by:

$$\max_{w_D^S, w_D^P} \Pi_D^M = w_D^S q_D^S + w_D^P q_D^P - c(q_D^S + q_D^P) \tag{12}$$

$$\max_{p_D^S} \Pi_D^S = ((1-k)p_D^S - w_D^S)q_D^S, \tag{13}$$

$$\max_{p_D^P} \Pi_D^P = kp_D^S q_D^S + q_D^P(p_D^P - w_D^P). \tag{14}$$

We follow the approach in Section 3.2 to solve these problems, and the equilibrium is shown in Theorem 3.3.

Theorem 3.3. *The equilibrium of the two channels:*

(i) *The optimal wholesale prices in channel S and channel P are* $w_D^{S*} = \frac{1-k+(1-b)c}{2(1-b)}$ *and* $w_D^{P*} = \frac{1-bk+(1-b)c}{2(1-b)}$*;*

(ii) $w_D^{P*} - w_D^{S*} = \frac{k}{2}$, $|\frac{\partial w_D^{P*}}{\partial k}| < |\frac{\partial w_D^{S*}}{\partial k}|$*;*

(iii) *The optimal retail prices for the 3P seller and e-platform are* $p_D^{S*} = \frac{2(3(1-k)+c)-b^2(1-k)(c+k+2)-b((1+k)c+1-k)}{2(1-b)(1-k)(4-b^2(1+k))}$ *and* $p_D^{P*} = \frac{2(1-k)(3+c)-b^2(1+k)(2(1-k)+c)-b((1-k)^2+c(1-3k))}{2(1-b)(1-k)(4-b^2(1+k))}$*;*

(iv) *If* $b < \frac{2c}{1-k+2c}$*, then* $p_D^{P*} < p_D^{S*}$*, and if* $b > \frac{2c}{1-k+2c}$*, then* $p_D^{P*} > p_D^{S*}$*.*

Theorem 3.3(i) and (iii) show the optimal decisions in the encroachment model. Combining Theorem 3.3 with equations (1) and (2), we can get q_D^{S*} and q_D^{P*}. The unit production cost in this case has an upper bound that is $c < \bar{\bar{c}}$, which can be seen in the appendix.

Theorem 3.3(ii) indicates that the manufacturer intends to increase the e-platform's sales cost to balance the competitiveness of the two channels when the e-platform charges a large commission rate. Although the wholesale price of the e-platform also decreases in the commission rate, the degree of the decrease is lower ($|\frac{\partial w_D^{P*}}{\partial k}| < |\frac{\partial w_D^{S*}}{\partial k}|$). Since the 3P seller is given a lower wholesale price, this can be understood as a "compensation strategy".

From Theorem 3.3(iv), we can see that when the competition degree is low ($b < \frac{2c}{1-k+2c}$), the e-platform determines a lower retail price than the 3P seller, which increases the sales quantity of channel P and further moves the e-platform's commission profit and sales profit in opposite directions. However, the e-platform can obtain optimal profit. When the degree of competition is high ($b > \frac{2c}{1-k+2c}$), the change is the opposite, nonetheless, the result is the same.

Based on the optimal decisions, we further get the effect of the parameters on the decisions as shown in Corollary 3.4.

Corollary 3.4. *The effect of k and b on the equilibrium:*

(i) $\frac{\partial w_D^{S*}}{\partial b} > 0$, $\frac{\partial w_D^{P*}}{\partial b} > 0$, $\frac{\partial p_D^{S*}}{\partial b} > 0$, *and* $\frac{\partial p_D^{P*}}{\partial b} > 0$*. A higher competition degree induces a more severe double marginalization effect.*

(ii) $\frac{\partial w_D^{S*}}{\partial k} < 0$, $\frac{\partial w_D^{P*}}{\partial k} < 0$, $\frac{\partial p_D^{S*}}{\partial k} > 0$, *and* $\frac{\partial p_D^{P*}}{\partial k} > 0$*. A higher commission rate induces the decrease of wholesale prices and induces the increase of retail prices.*

Similar to Corollary 3.2, the wholesale prices and retail prices are increasing in the competition degree. Given a large commission rate, the manufacturer should decrease the wholesale price to keep the sales quantity of channel S. For the 3P seller, the negative effect of a lower revenue completely covers the positive effect of a lower wholesale cost. Thus, the 3P seller still increases the retail price. However, how the commission rate impacts the e-platform's retail price has changed because the manufacturer has more pricing freedom to manage the channels' competition and can control the sales costs of the two channels through price discrimination. The e-platform prefers to increase the retail price to get more commission revenue rather than lower the retail price to get more sales.

Given the optimal decisions of participants in Theorem 3.3, combined with equations (12), (13), and (14), we can obtain their profits, as shown in the appendix. We further analyze the two sources of the e-platform's profit. The first is the commission profit from the self-support store $\Pi_D^{p1*} = kp_D^{S*}q_D^{S*}$, and the second is the sales profit charged from the 3P seller $\Pi_D^{p2*} = (p_D^{P*} - w_D^{S*})q_D^{P*}$. Moreover, we compare them with the e-platform's profit in the non-encroachment situation.

We show the influence of the parameters on the e-platform's profit sources in Figure 4.

The functions trend in Figure 4 is similar to that in Figure 3. The encroachment also decreases the commission profit. However, when the commission rate is moderate, the commission profit is higher than the sales profit, which is the most significant difference from Figure 3. This change informs us that the price discrimination strategy allows the 3P seller to gain the compensation while giving the e-platform the opportunity to obtain a bigger share from the 3P seller.

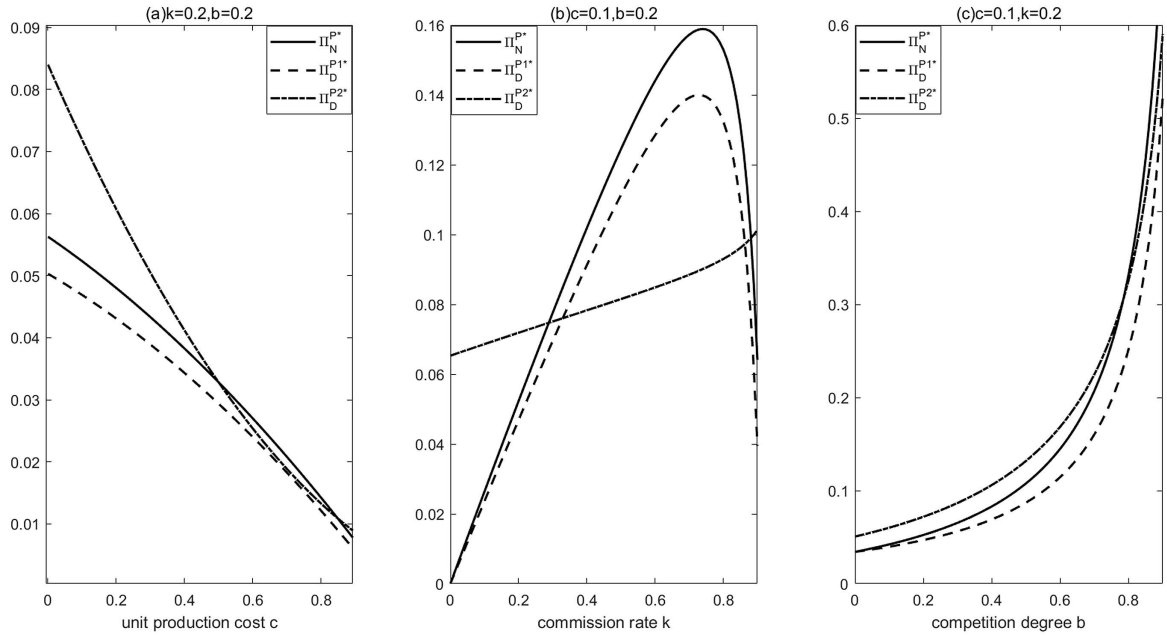

FIGURE 4. The e-platform's profit sources regarding. c, k, b under price discrimination and non-encroachment strategy.

3.4. Comparison

Comparing the encroachment and non-encroachment models, we have the following Theorem 3.5:

Theorem 3.5.

(i) $w_D^{P*} > w_U^* > w_D^{S*}$, $p_D^{P*} > p_U^{P*}$, $p_U^{S*} > p_D^{S*}$;
(ii) $q_U^{S*} < q_U^{P*}$, and the relationship of q_D^{S*} and q_D^{P*} is uncertain, $q_U^{S*} + q_U^{P*} = q_D^{S*} + q_D^{P*}$;
(iii) $\Pi_D^{M*} > \Pi_U^{M*} > \Pi_N^{M*}$; $\Pi_N^{S*} > \Pi_D^{S*} > \Pi_U^{S*}$; $\min\{\Pi_U^{P*}, \Pi_D^{P*}\} > \Pi_N^{P*}$, and if $\min\{\bar{c}, \widetilde{c}\} < c < \bar{c}$, then $\Pi_U^{P*} < \Pi_D^{P*}$, if $c < \min\{\bar{c}, \widetilde{c}\}$, then $\Pi_U^{P*} > \Pi_D^{P*}$.

Theorem 3.5(i) and (ii) compare the equilibrium between the two pricing strategies. See the appendix for $\bar{c}$ and $\widetilde{c}$. The wholesale price in the uniform pricing strategy is between the two channels' wholesale prices under the price discrimination strategy, which leads to a higher retail price for channel P and a lower retail price for channel S compared with the uniform pricing strategy. This conclusion further validates that the manufacturer manages competition through the price discrimination strategy. Interestingly, the total sales quantity under the uniform pricing and price discrimination strategy are the same, which means that the purpose of price discrimination is not to improve the total sales quantity but to control the sales cost and distribute the sales quantity across channels properly. This also explains the relationship between the two sources of the e-platform's profit in Figures 3b and 4b.

Theorem 3.5(iii) indicates that the e-platform encroachment benefits the e-platform and manufacturer, because of the sales revenue of channel P, which is in line with the fact that e-platforms such as Amazon and Taobao have opened self-support stores. However, the competition in the dual-channel model reduces the market share of the 3P seller, further reducing its profit. The price discrimination strategy benefits the manufacturer and 3P seller than the uniform pricing strategy, while not necessarily for the e-platform.

We let the price discrimination strategy be the baseline model and compare it to following models. The uniform pricing strategy is similar, and we do not repeat the analysis.

4. Full channel coordination

Supply chain performance is often significantly reduced because of the double marginalization effect. In this section, we propose a dual-channel two-part tariff contract to alleviate the problem and investigate the effects of the commission rate and competition degree on contract parameters under full channel coordination.

4.1. Centralized system

We first analyze the decisions in an integrated centralized supply chain. Recall that we use subscript C to denote the centralized system. The retail prices p_C^S and p_C^P must be determined to maximize the overall profit of the supply chain. The decision problem is formulated as follows:

$$\max_{p_C^S, p_C^P} \Pi_c = p_C^S q_C^S + p_C^P q_C^P - c(q_C^S + q_C^P). \tag{15}$$

The unique optimal solutions are presented in the following Theorem 4.1, which provides a benchmark for supply chain coordination.

Theorem 4.1. *The retail prices of the e-platform and 3P seller in the centralized system are* $p_C^{S*} = \frac{1+(1-b)c}{2(1-b)}$ *and* $p_C^{P*} = \frac{1+(1-b)c}{2(1-b)}$. *The profit of the whole supply chain is* $\Pi_C^* = \frac{(1-(1-b)c)^2}{2(1-b)}$.

Comparing the retail prices in the centralized system and price discrimination strategy, we have Corollary 4.2, as follows:

Corollary 4.2. $p_D^{S*} > p_C^{S*}$ *and* $p_D^{P*} > p_C^{P*}$.

The retail prices of the 3P seller and e-platform under the price discrimination strategy are higher than that under the centralized system, which is caused by the double marginalization effect, and therefore reduces the profit of the supply chain. Subsequently, we discuss whether a dual-channel two-part tariff contract can coordinate the online dual-channel supply chain.

4.2. Dual-channel two-part tariff contract

A two-part tariff contract has been extensively applied to coordinate supply chains in various scenarios [14, 16]. In this section, we use the two-part tariff contract in both online channels. The manufacturer offers a dual-channel two-part tariff contract $\{w_{CO}^S, w_{CO}^P, T_1, T_2\}$, where w_{CO}^S and w_{CO}^P are the wholesale prices for the 3P seller and e-platform, and T_1 and T_2 are the fixed fees charged by the manufacturer. The problems of the 3P seller and e-platform under the contract are as follows:

$$\max_{p_{CO}^S} \Pi_{CO}^S = ((1-k)p_{CO}^S - w_{CO}^S)q_{CO}^S - T_1, \tag{16}$$

$$\max_{p_{CO}^P} \Pi_{CO}^P = kp_{CO}^S q_{CO}^S + (p_{CO}^P - w_{CO}^P)q_{CO}^P - T_2. \tag{17}$$

According to equations (16) and (17), the 3P seller and e-platform will respond to the wholesale prices as:

$$p_{CO}^S(w_{CO}^S, w_{CO}^P) = \frac{b(1-k)(1+w_{CO}^P) + 2(1+w_{CO}^S - k)}{(1-k)(4-b^2(1+k)}, \tag{18}$$

$$p_{CO}^P(w_{CO}^S, w_{CO}^P) = \frac{(2(1-k)+b)(1+w_{CO}^S) - bk(k-w_{CO}^P)}{(1-k)(4-b^2(1+k)}. \tag{19}$$

Given the 3P seller and e-platform's responses, the manufacturer optimizes its decisions by solving the following problem:

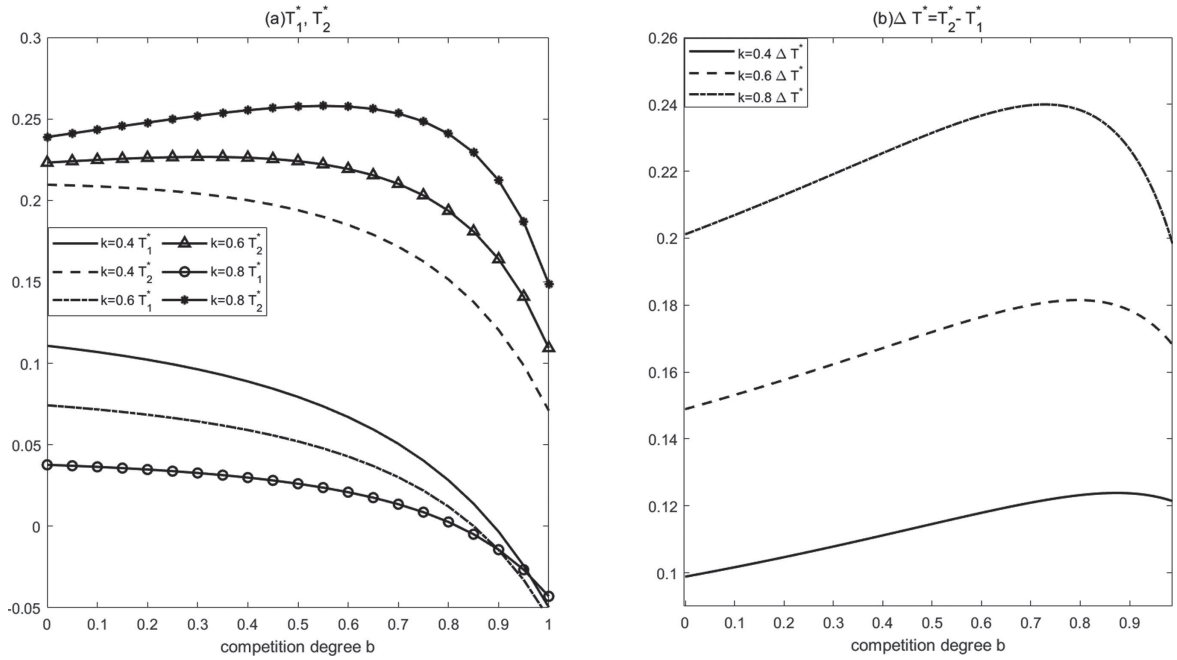

FIGURE 5. The fixed fees T_1^* and T_2^*.

$$\max_{w_{CO}^S, w_{CO}^P} \Pi_{CO}^M = q_{CO}^S w_{CO}^S + q_{CO}^P w_{CO}^P - c(q_{CO}^S + q_{CO}^P) + T_1 + T_2, \tag{20}$$

s.t.

$$q_{CO}^{S*}((1-k)p_{CO}^{S*} - w_{CO}^{S*}) - T_1 \geq \Pi_D^{S*}, \tag{21}$$

$$kp_{CO}^{S*}q_{CO}^{S*} + (p_{CO}^{P*} - w_{CO}^{P*})q_{CO}^{P*} - T_2 \geq \Pi_D^{P*}. \tag{22}$$

Equations (21) and (22) represent the individual rationality constraints of the 3P seller and e-platform, respectively. Their profits can not be lower than that in the wholesale price contract. Otherwise, it is not necessary to receive the dual-channel two-part tariff contract. By solving this problem, we have Theorem 4.3.

Theorem 4.3. *The manufacturer could offer a dual-channel two-part tariff contract* $\{w_{CO}^{S*}, w_{CO}^{P*}, T_1^*, T_2^*\}$ *to coordinate the online dual-channel supply chain.*

The value of w_{CO}^{S*}, w_{CO}^{P*}, T_1^*, and T_2^* can be see in the appendix. Given the contract in Theorem 4.3, the online dual-channel supply chain is fully coordinated, where $p_{CO}^{S*} = p_C^{S*}$ and $p_{CO}^{P*} = p_C^{P*}$. The manufacturer reduces wholesale prices to varying degrees and charges different fixed fees to stimulate the 3P seller and e-platform to determine the optimal retail prices and extracts all of the spillover profits because it sponsors the coordination contract and has substantial bargaining power. Thus, the 3P seller and e-platform obtain the same profits as in the baseline model.

It is worth noting that the manufacturer asks for different fixed fees and wholesale prices from the 3P seller and the e-platform, that is, $w_{CO}^{P*} > w_{CO}^{S*}$ and $T_2^* > T_1^*$, which indicates that under the coordination contract, the manufacturer will still demand a higher wholesale price from the e-platform to balance the competitiveness of the two channels. The e-platform has more spillover profits than the 3P seller. When b is large enough, T_1^* is

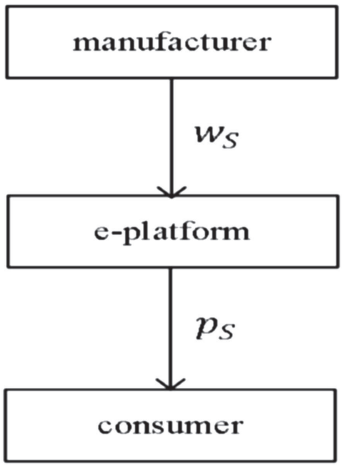

FIGURE 6. Model framework of single-channel sales model that the 3P seller exits.

negative (Fig. 5a). In other words, if the 3P seller faces fierce competition, the manufacturer will pay it a fixed fee in turn as the "compensation" besides the lower wholesale price. For example, due to the impact of COVID-19 and the market downturn, small online stores are under greater competitive pressure from big e-platforms. JD and Taobao launched subsidy support measures to support them, such as exempting their e-platform usage fees and providing low-interest and interest-free loans for a limited period.

Observation 4.4. *ΔT increases in k and is quasi-concave in b.*

The results are observed from Figure 5b. A higher commission rate increases the spillover revenue of the e-platform, thus, ΔT increases. When the competition degree is in a small value range, the increase of the competition brings more compensation to the 3P seller. But when the competition degree is in a large value range, the increased competition hurts the supply chain environment, leading to lower total sales. Hence, the manufacturer lowers subsidies.

5. Extensions

In this section, we extend our model to two related scenarios and derive meaningful managerial insights.

5.1. Endogenous commission rate

In the previous sections, we assume that the commission rate of the e-platform is exogenous. In this section, we consider how the e-platform determines the commission rate for maximum profit. In a Stackelberg game, the e-platform determines the commission rate first, then the manufacturer determines the wholesale prices, and finally, the 3P seller and e-platform determine the retail price. This game sequence is recognized in a great deal of the literature [34,35].

Here, we must consider an extreme case. If the commission rate is determined by the e-platform is too large, and the 3P seller may exit from the market because the marginal profit is negative, and there is a single-channel sales model composed of a manufacturer and an e-platform as in Figure 6.

We calculate the decisions in this case and denote it as subscript S. Similar to the calculation of equation (3), we have the sales quantity:

$$q_S = 1 + b - (1 - b^2)p_S. \tag{23}$$

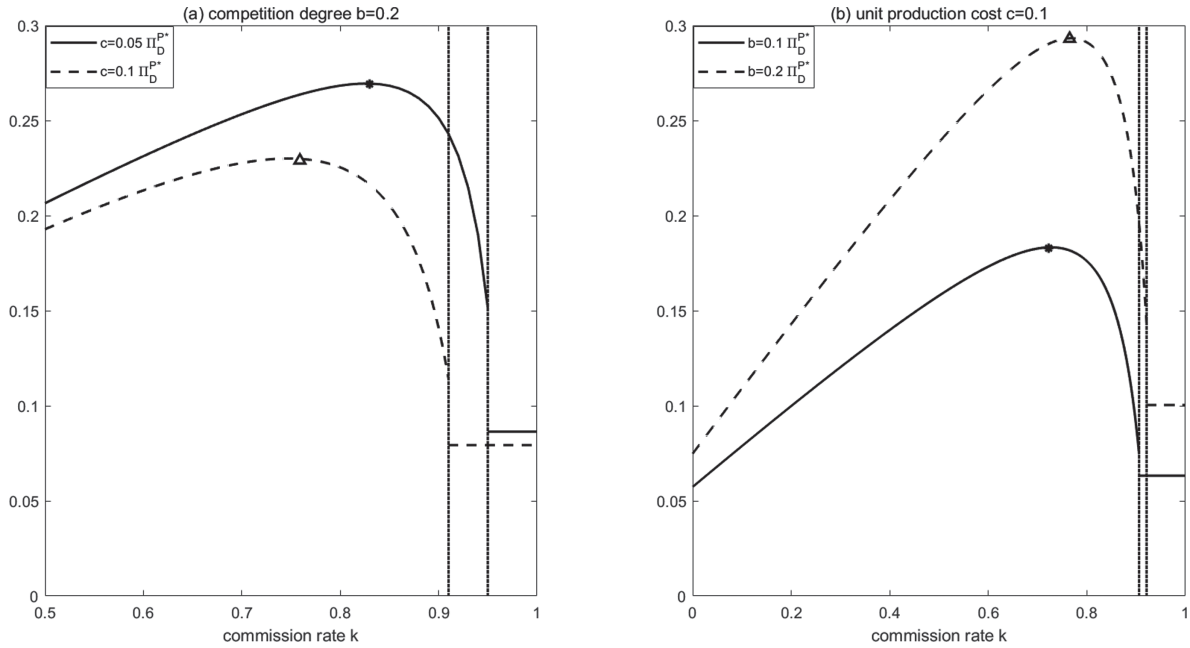

FIGURE 7. Platform's profit regarding. k.

The problems of the manufacturer and the e-platform are as follows:

$$\max_{w_S} \Pi_S^M = (w_S - c)q_S, \tag{24}$$

$$\max_{p_S} \Pi_S^P = (p_S - w_S)q_S. \tag{25}$$

We have the optimal profit of the e-platform $\Pi_S^{P*} = \frac{(1+b)(1-(1-b)c)^2}{16(1-b)}$ by a simple calculation.

We use $\bar{k}$ to represent the cut-off point between the dual-channel model and single-channel model. When $k > \bar{k}$, the 3P seller does not participant in the sale, which is case "S". When $k < \bar{k}$, the 3P seller has positive profit and participates in the supply chain. We have the e-platform's profit regarding commission rate k in Figure 7. There is an optimal commission rate k^* to maximize the profit of the e-platform. In Figure 7, we use stars and triangles for k^* and dotted lines for $\bar{k}$.

Observation 5.1. *(i) k^* decreases in c and increases in b; (ii)$k^* < \bar{k}$.*

The results can be observed from Figure 7. A higher unit production cost increases the wholesale price and the marginal cost of the 3P seller, so the e-platform will reduce the commission rate to compensate for the 3P seller (k^* on the triangle (c = 0.1) is smaller than that on the star (c = 0.05)). If the 3P seller fiercely competes with the e-platform, the e-platform will weaken the 3P seller's competitiveness by increasing the commission rate (k^* on the triangle (b = 0.2) is larger than that on the star (b = 0.1)). Interestingly, we can see that $k^* < \bar{k}$ in Figure 7, so even if the competition is fierce, the commission rate set by the e-platform still gives the 3P seller an incentive to participate in sales.

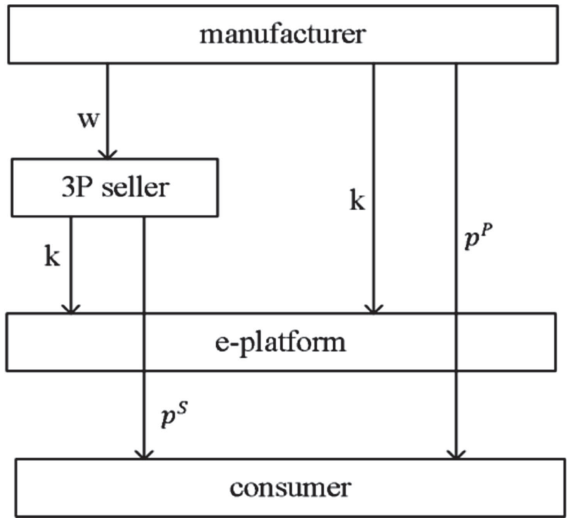

FIGURE 8. Model framework of manufacturer encroachment.

5.2. Manufacturer encroachment model

Due to the rapid development of third-party logistics and information technology, the manufacturer encroachment model is implementable. In the traditional setting, a manufacturer encroaches by introducing a direct channel and selling directly to consumers [2, 49]. In this online setting, the manufacturer encroaches through the e-platform by paying commission fees to deliver products to consumers, just like the 3P seller. The manufacturer encroachment model and e-platform encroachment model are completely different, which is mainly reflected in the ownership of products' pricing power. In the e-platform encroachment model, the e-platform determines the products' retail price. But in the manufacturer encroachment model, the e-platform acts as an agent to charge commission fees, and the manufacturer determines the products' retail price. For simplicity, we choose the e-platform encroachment model under the price discrimination strategy, which is more common than the uniform pricing strategy, compared with the manufacturer encroachment model.

The manufacturer determines the wholesale price w for the 3P seller, then the manufacturer and 3P seller determines the retail prices p^S and p^P for the consumers to:

$$\max_{w,p^P} \Pi^M = wq^S + (1-k)p^P q^P - c(q^S + q^P), \tag{26}$$

$$\max_{p^S} \Pi^S = ((1-k)p^S - w)q^S, \tag{27}$$

$$\Pi^P = kp^S q^S + kp^P q^P. \tag{28}$$

We can have equilibrium of this game by backward induction, and then substitute it into equations (26), (27), and (28) to obtain the optimal profit functions as:

$$\Pi^{M*} = \frac{(b^3 + b^2 + 4b + 12)(1 - k - (1-b)c)^2}{4(1-b)(8+b^2)(1-k)} \tag{29}$$

$$\Pi^{S*} = -\frac{(b^2+2)^2(1-k-(1-b)c)^2}{(8+b^2)^2(1-k)} \tag{30}$$

$$\Pi^{P*} = \frac{1}{4(1-b)(b^2+8)^2(1-k)^2}((112 + 16b + 36b^2 - 4b^3 + 5b^4 - 3b^5)(1-k)^2 - k(4c(1-k)$$

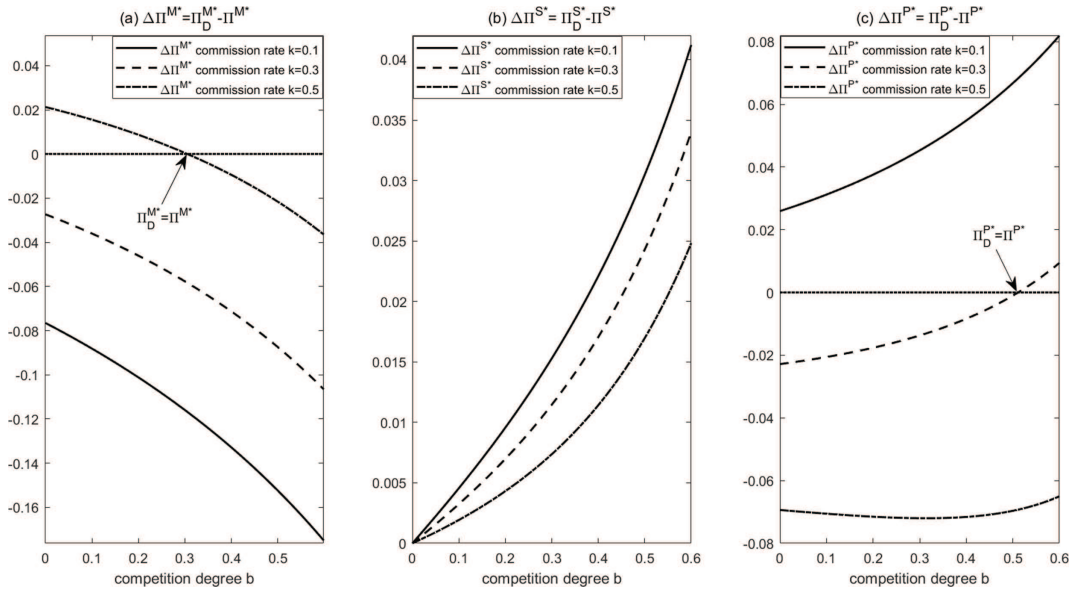

FIGURE 9. The differences of the three parties' profits between the two encroachment models.

$$(2b^4 + b^3 + 8b^2 + 8b + 8)(1-b)^2 - (5b^5 + b^4 + 24b^3 + 36b^2 + 16b + 80)(1-b)^2c^2)) \tag{31}$$

Both the e-platform encroachment model and manufacturer encroachment model add a sales channel, but in different ways. For comparing the three parties' profits in the two encroachment model, we let $\Delta\Pi^{M*} = \Pi_D^{M*} - \Pi^{M*}$, $\Delta\Pi^{S*} = \Pi_D^{S*} - \Pi^{S*}$ and $\Delta\Pi^{P*} = \Pi_D^{P*} - \Pi^{P*}$ to represent the differences. The results are represented by following Figure 9.

Observation 5.2. (i) *The 3P seller always prefers the e-platform encroachment model to manufacturer encroachment model;*

(ii) The manufacturer prefers the e-platform encroachment model when the competition degree is low, and the commission rate is high; For the e-platform, the conditions are opposite but not complementary.

The 3P seller always prefers the e-platform encroachment model because of the manufacturer's "compensation strategy". The conditions under which the manufacturer and e-platform choose the optimal encroachment model are opposite, but there is also overlap. This is because of the difference between the channel structures. The e-platform obtains two sources of commission fees in the manufacturer encroachment model (Fig. 8) but one source of that in the e-platform encroachment model (Fig. 2a). Therefore, a high commission rate will bring more benefits to the e-platform in the manufacturer encroachment model. Quite sensibly, the increase in the commission rate will damage the manufacturer's profit to a greater extent under the manufacturer encroachment model.

In the e-platform encroachment model (Fig. 2a), the e-platform has the power to determine the product price and further determine the product quantity to control the competition, and the manufacturer in the manufacturer encroachment model (Fig. 8) also has the power. Consequently, when faced a fiercer competition, the e-platform has higher resilience in the e-platform encroachment model, and the manufacturer has higher resilience in the manufacturer encroachment model.

6. Conclusion

With the increase in online market demand and the improvement of internet technology, the self-support model is the choice of many large e-platforms and the future trend of e-commerce. This paper examines the motivation of e-platform self-support and explores the channel structure under e-platform encroachment. We find that the e-platform always tends to encroach on the retail market by forming an online dual-channel supply chain with competition and cooperation. Compared to the uniform pricing strategy, the manufacturer is more inclined to use the price discrimination strategy, under which the manufacturer offers a more favourable wholesale price to the 3P seller, and the degree of price discrimination increases in the commission rate. Clearly, the price discrimination strategy benefits both the manufacturer and 3P seller. However, counterintuitively, whether it benefits the e-platform depends on the unit production cost, commission rate, and competition degree. The symmetric channels make the competing downstream parties have the same preference for the pricing mechanism [4, 33]. However, the two channels are asymmetric in our study, so the preferences of the e-platform and 3P seller are different, affected by both the commission rate and competition degree. Theoretically, the results enrich the literature of channel pricing strategy.

We show that the encroachment decreases the e-platform's commission profit but increases its total profit. The total sales quantity of the supply chain under the two pricing strategies is the same, but the distribution proportions are different, which also indirectly affects the weight of the two parts of the platform's profits. The e-platforms in Mantin *et al.* [27] and Sun and Liu [37] also obtain two parts of revenue, but they do not notice the weights of the two revenue sources. Compared to the previous literature, our research provides a more detailed theoretical study of the influence mechanism of encroachment behavior.

To avoid the double marginalization effect of the wholesale price contract and improve the performance of the supply chain, we propose a dual-channel two-part tariff contract. The contract still compensates the 3P seller by charging a lower wholesale price and a lower fixed fee, and even reverse-subsidizes the 3P seller a fixed fee when the competition is high. The manufacturer obtains the profit spillover of the contract. The new contract improves the supply chain performance while keeping the "compensation strategy", which can be used in other similar research.

After that, we study the scenario where the e-platform determines the commission rate to maximize its profit. Our analysis conclusion shows that the e-platform will always set an appropriate commission rate to prevent the 3P seller from abandoning sales, but the e-platform exploits the 3P seller more when the competition is fiercer. Further, we compare the manufacturer encroachment model with e-platform encroachment model and find the conditions of the manufacturer and e-platform choosing encroachment models are opposite but not complementary. The results provide guidance for supply chain channel expansion.

In addition to analyzing the internal influence mechanism of encroachment and pricing, our research provides a plausible explanation of why many e-platforms open self-support business and why the price discrimination strategy is more common than the uniform pricing strategy. Thus, our paper makes a novel contribution to the emerging literature on the encroaching e-platform and offers practical insights to the upstream manufacturer who need to decide on the pricing policies. We also contribute to the rich literature on dual-channel distribution by exploring e-platform encroachment.

6.1. Managerial Insights

Our model explains why numerous established e-platforms choose to start self-support businesses. JD is the most well-known example, but several prominent examples exist, such as Amazon and Walmart. We strongly recommend that e-platforms seek opportunities to compete and cooperate with 3P sellers. In addition, this paper analyzes in detail how the manufacturer should respond to the e-platform encroachment and how the e-platform should adjust its focus to the manufacturer's pricing strategy for higher profit. Finally, this paper also gives the parameters of a supply chain coordination contract to improve the performance of the supply chain. Overall, this paper provides a decision-making reference for all supply chain participants.

6.2. Limitations and future research

We propose directions for future research from the following three aspects. First, in reality, e-platforms are usually more powerful than 3P sellers, so the issue that the e-platform decides its retail price first has practical significance. Second, compared with 3P sellers, e-platforms are developing to provide better services to customers. For example, JD has established a warehousing system to respond quickly to customers' demands. Thus, the customers have different preferences for the two sales channels. How does this factor affect the equilibrium in this online dual-channel supply chain? Third, the model could consider shortages of capital for the manufacturer or 3P seller in the online dual-channel supply chain. It will be interesting and important to study the interaction of operation and financing strategy in the online dual-channel supply chain.

Appendix A.

Let us define the parameters before providing a proof. Accordingly, c, k and b are in the range of $(0,1)$. We obtain the range limits between them under the uniform pricing strategy and price discrimination strategy as:

Under the uniform pricing strategy:

1. $w_U^* > c$, we have $c < \frac{(1-k)(4+b(2-k)-b^2k)}{2(1-b)(2+b-k)}$.
2. $(1-k)p_U^{S*} > w_U^*$, we have $c < \frac{(1-k)(8+b(8-6k)-8k+b^3k(1+k)+b^2(2+k+k^2))}{2(1-b)(2+b-k)(2+bk+b)}$.
3. $p_{U^*}^{P*} > w_U^*$, which is always true.
4. $q_U^{S*} > 0$, we have $c < \frac{(1-k)(b^3k(1+k)+b^2(k^2+k+2)+b(8-6k)+8(1-k))}{2(1-b)(2+b-k)(2+bk+b)}$.
5. $q_U^{P*} > 0$, which is always true.

The intersection of the range of these five conditions in the uniform pricing strategy is $c < \frac{(1-k)(b^3k(1+k)+b^2(k^2+k+2)+b(8-6k)+8(1-k))}{2(1-b)(2+b-k)(2+bk+b)}$, and denoting $\overline{c} = \frac{(1-k)(b^3k(1+k)+b^2(k^2+k+2)+b(8-6k)+8(1-k))}{2(1-b)(2+b-k)(2+bk+b)}$.

Under the price discrimination strategy:

1. $w_D^{S*} > c$, we have $c < \frac{1-k}{1-b}$.
2. $w_D^{P*} > c$, we have $c < \frac{1-bk}{1-b}$.
3. $(1-k)p_D^{S*} - w_D^{S*} > 0$, we have $c < \frac{(2+b)(1-k)}{(1-b)(2+bk+b)}$.
4. $p_D^{P*} > w_D^{P*}$, which is always true.
4. $q_D^{S*} > 0$, that is $c < \frac{(2+b)(1-k)}{(1-b)(2+bk+b)}$.
5. $q_D^{P*} > 0$, which is always true.

The intersection of the range of these five conditions in the price discrimination strategy is $c < \frac{(2+b)(1-k)}{(1-b)(2+bk+b)}$, and denoting $\overline{\overline{c}} = \frac{(2+b)(1-k)}{(1-b)(2+bk+b)}$

The intersection of the ranges in the two strategies is $c < \overline{c}$.

Theorems 3.1 and 3.3 are the equilibrium expressions of the uniform pricing strategy and price discrimination strategy, and the calculation is simple and is practically shown in the text, so it will not be repeated here. Where $A = 2b(1-k)(4-k(1+c))+4c(2-k)+8(1-k)(3-k)-b^3(1-k)(4+2c+k-k^2)-b^2(2c(k^2-2k+3)+k^3-2k^2-9k+10)$ and $B = 2b(k^3+5ck+k+4-3(2+c)k^2)+4(1-k)(6+c(2-k)-4k)-b^3(1+k)(2c+(1-k)(4+k))-b^2(1-k)(2c(3+k)-5k^2+k+10)$.

Proof of Corollary 1.

(i) The wholesale price w_U^* regarding b:

$\frac{\partial w_U^*}{\partial b} = \frac{(1-k)(b^2(2-k^2)+2b(4+k^2-2k)+k^2+8(1-k))}{4(1-b)^2(2+b-k)^2} > 0.$

The 3P seller's retail price p_U^{S*} regarding b:

$\frac{\partial p_U^{S*}}{\partial b} = \frac{1}{4(1-b)^2(1-k)(2+b-k)^2(4-b^2(k+1))^2}(b^6(1-k^2)(2c-k^2+k+4)+2b^5(1+k)(2c(k^2-2k+3)+k^3-2k^2-9k+10)+b^4(1-k)(2c(k^3+k^2+5k+9)+2k^4-3k^3-14k^2-7k+10)-4b^3(c(-k^4+k^3-7k^2+11k+4)+k^4+$

$7k^3 - 29k^2 + k + 20) + 2b^2(1-k)(c(k^3 + 9k^2 - 8k - 24) - 15k^3 + 45k^2 + 4k - 40) - 8b(-3k^3(c+2) + k^2(11c + 7) - 2(5c-7)k + k^4 - 16) + 8(1-k)(c(2-k)^2 + 5k^2 - 22k + 20))$.

The denominator $4(1-b)^2(1-k)(2+b-k)^2(4-b^2(1+k))^2$ is positive, so we need to check the numerator, and denote it as y_4.

$\frac{\partial^5 y_4}{\partial b^5} = 720(1-k^2)(k+4+2c-k^2)b + 240(1+k)(2c(k^2-2k+3) + k^3 + 10 - 2k^2 - 9k)$ is a linear function and increases in b. When $b = 0$, $\min \frac{\partial^5 y_4}{\partial b^5} = 240(1+k)(10 + k^3 + 2c(3 + k^2 - 2k) - 2k^2 - 9k) > 0$. Thus, $\frac{\partial^5 y_4}{\partial b^5} > 0$, and $\frac{\partial^4 y_4}{\partial b^4}$ increases in b.

$\frac{\partial^4 y_4}{\partial b^4} = 360b^2(1-k^2)(2c - k^2 + k + 4) + 240b(1+k)(2c(k^2-2k+3) + k^3 - 2k^2 - 9k + 10) + 24(1-k)(2c(k^3 + k^2 + 5k + 9) + 2k^4 - 3k^3 - 14k^2 - 7k + 10)$. When $b = 0$, the positive and negative of $\frac{\partial^4 y}{\partial b^4}$ is uncertain. When $b = 1$, $\frac{\partial^4 y}{\partial b^4} = -48(c(k^4 - 10k^3 + 29k^2 - 6k - 54) + k^5 - 15k^4 + 7k^3 + 96k^2 - 4k - 85) > 0$. So $\frac{\partial^4 y_4}{\partial b^4}$ is positive or is negative first and then positive, and $\frac{\partial^3 y_4}{\partial b^3}$ is increasing or quasi-convex in b.

$\frac{\partial^3 y_4}{\partial b^3} = 120b^3(1-k^2)(2c - k^2 + k + 4) + 120b^2(1+k)(2c(k^2-2k+3) + k^3 - 2k^2 - 9k + 10) + 24b(1-k)(2c(k^3 + k^2 + 5k + 9) + 2k^4 - 3k^3 - 14k^2 - 7k + 10) - 24(c(-k^4 + k^3 - 7k^2 + 11k + 4) + k^4 + 7k^3 - 29k^2 + k + 20)$. When $b = 0$, $\frac{\partial^3 y_4}{\partial b^3} = -24(c(-k^4 + k^3 - 7k^2 + 11k + 4) + k^4 + 7k^3 - 29k^2 + k + 20) < 0$, and when $b = 1$, $\frac{\partial^3 y_4}{\partial b^3} = 24(3-k)(c(k^3 - 6k^2 + 3k + 18) + 2(k^4 - 4k^3 - 9k^2 + 2k + 10)) > 0$. Combined with $\frac{\partial^4 y_4}{\partial b^4}$ above, $\frac{\partial^3 y_4}{\partial b^3}$ is negative first and then positive, and $\frac{\partial^2 y_4}{\partial b^2}$ is quasi-convex in b.

$\frac{\partial^2 y_4}{\partial b^2} = 30b^4(1-k^2)(2c-k^2+k+4) + 40b^3(1+k)(2c(k^2-2k+3) + k^3 - 2k^2 - 9k + 10) + 12b^2(1-k)(2c(k^3+k^2+5k+9) + 2k^4 - 3k^3 - 14k^2 - 7k + 10) - 24b(c(-k^4 + k^3 - 7k^2 + 11k + 4) + k^4 + 7k^3 - 29k^2 + k + 20) + 4(1-k)(c(k^3 + 9k^2 - 8k - 24) - 15k^3 + 45k^2 + 4k - 40)$. When $b = 0$, $\frac{\partial^2 y_4}{\partial b^2} = -4(1-k)(40 + c(24 + 8k - k^3 - 9k^2) + 15k^3 - 45k^2 - 4k) < 0$, and when $b = 1$, $\frac{\partial^2 y_4}{\partial b^2} = 2(3-k)^2(2c(9-k^2) + k(1 + 11k - 12k^2)) > 0$. Combined with $\frac{\partial^3 y_4}{\partial b^3}$ above, $\frac{\partial^2 y_4}{\partial b^2}$ is negative first and then positive, and $\frac{\partial y_4}{\partial b}$ is quasi-convex in b.

$\frac{\partial y_4}{\partial b} = 6b^5(1-k^2)(2c - k^2 + k + 4) + 10b^4(1+k)(2c(k^2-2k+3) + k^3 - 2k^2 - 9k + 10) + 4b^3(1-k)(2c(k^3 + k^2 + 5k + 9) + 2k^4 - 3k^3 - 14k^2 - 7k + 10) - 12b^2(c(-k^4 + k^3 - 7k^2 + 11k + 4) + k^4 + 7k^3 - 29k^2 + k + 20) + 4b(1-k)(c(k^3 + 9k^2 - 8k - 24) - 15k^3 + 45k^2 + 4k - 40) - 8(-3k^3(c+2) + k^2(11c+7) - 2(5c-7)k + k^4 - 16)$. When $b = 0$, $8(3k^3(c+2) - k^2(11c+7) + 2(5c-7)k - k^4 + 16) > 0$ and when $b = 1$, $-4(3-k)^3(1 + k - 2k^2) < 0$. Corresponding, $\frac{\partial y_4}{\partial b}$ is positive first and then negative, and the numerator is quasi-concave in b.

When $b = 0$, $y_4 = 8(1-k)(c(2-k)^2 + 5k^2 - 22k + 20) > 0$, and when $b = 1$, $y_4 = 2(3-k)^4(1-k) > 0$. Because the numerator is quasi-concave in b, $y_4 > 0$ and $\frac{\partial p_U^{S*}}{\partial b} > 0$.

The e-platform's retail price p_U^{P*} regarding b:

$\frac{\partial p_U^{P*}}{\partial b} = \frac{1}{-4(1-b)^2(1-k)(2+b-k)^2(4-b^2(1+k))^2}(b^6(1+k)^2(-2c + k^2 + 3k - 4) - 2b^5(1-k^2)(2c(k+3) - 5k^2 + k + 10) - b^4(1-k^2)(-2c(k^2 + 10k - 9) + 4k^3 - 3k^2 - 29k + 10) + 4b^3(1-k^2)(c(-3k^2 + k + 4) + k^3 - 4k^2 - 9k + 20) - 2b^2(c(9k^4 - 22k^3 + 9k^2 + 16k - 24) - k^5 + 15k^4 - 51k^3 + 33k^2 + 44k - 40) + 8b(k-1)(k^2(1-c)(1-k) - 2k(c+5) + 16) - 8(c(2-k)^2(1+k) - k^4 + 15k^2 - 34k + 20))$.

The denominator $-4(1-b)^2(1-k)(2+b-k)^2(4-b^2(k+1))^2$ is negative, so let's check the numerator, and denote it as y_6.

$\frac{\partial^5 y_6}{\partial b^5} = 720b(1+k)^2(-2c + k^2 + 3k - 4) - 240(1-k^2)(2c(k+3) - 5k^2 + k + 10)$, which decreases in b. When $b = 0$, $\max \frac{\partial^5 y_6}{\partial b^5} = -240(1-k^2)(2c(k+3) - 5k^2 + k + 10) < 0$. Accordingly, $\frac{\partial^5 y_6}{\partial b^5} < 0$, and the fourth derivative decreases in b.

$\frac{\partial^4 y_6}{\partial b^4} = 360b^2(1+k)^2(-2c + k^2 + 3k - 4) - 240b(1-k^2)(2c(k+3) - 5k^2 + k + 10) + 24(k^2-1)(-2c(k^2 + 10k - 9) + 4k^3 - 3k^2 - 29k + 10)$. Because the fourth derivative decreases in b, when $b = 0$, $\max \frac{\partial^4 y_6}{\partial b^4} = 24(k^2-1)(-2c(k^2 + 10k - 9) + 4k^3 - 3k^2 - 29k + 10)$. We do not know if it is positive or negative. When $b = 1$, $\min \frac{\partial^4 y_6}{\partial b^4} = -48(1+k)(c(k^3 - k^2 - 24k + 54) - 2k^4 + 21k^3 - 47k^2 - 57k + 85) < 0$.Thus, the fourth derivative is negative or positive first and then negative, and the third derivative is decreasing or quasi-concave in b.

$\frac{\partial^3 y_6}{\partial b^3} = 120b^3(1+k)^2(-2c+k^2+3k-4)-120b^2(1-k^2)(2c(k+3)-5k^2+k+10)+24b(1-k^2)(2c(k^2+10k-9)-4k^3+3k^2+29k-10)+24(1-k^2)(c(-3k^2+k+4)+k^3-4k^2-9k+20)$. When $b=0$, $\frac{\partial^3 y_6}{\partial b^3} = 24(1-k^2)(c(-3k^2+k+4)+k^3-4k^2-9k+20) > 0$, and when $b=1$, $\frac{\partial^3 y_6}{\partial b^3} = -24(-k^2+2k+3)(c(k^2-9k+18)+3k^3-13k^2-10k+20) < 0$. Accordingly, the third derivative is positive first and then negative, and the second derivative is quasi-concave in b.

$\frac{\partial^2 y_6}{\partial b^2} = 30b^4(1+k)^2(-2c+k^2+3k-4)+40b^3(k^2-1)(2c(k+3)-5k^2+k+10)+12b^2(k^2-1)(-2c(k^2+10k-9)+4k^3-3k^2-29k+10)-24b(k^2-1)(c(-3k^2+k+4)+k^3-4k^2-9k+20)-4(c(9k^4-22k^3+9k^2+16k-24)-k^5+15k^4-51k^3+33k^2+44k-40)$. When $b=0$, $\frac{\partial^2 y_6}{\partial b^2} = 4(-c(9k^4-22k^3+9k^2+16k-24)+k^5-15k^4+51k^3-33k^2-44k+40) > 0$, and when $b=1$, $\frac{\partial^2 y_6}{\partial b^2} = -2(3-k)^2(6c(-k^2+2k+3)+k(-14k^2+k+13)) < 0$. Correspondingly, the second derivative is positive first and then negative, and the first derivative is quasi-concave in b.

$\frac{\partial y_6}{\partial b} = 6b^5(1+k)^2(-2c+k^2+3k-4)+10b^4(k^2-1)(2c(k+3)-5k^2+k+10)+4b^3(k^2-1)(-2c(k^2+10k-9)+4k^3-3k^2-29k+10)-12b^2(k^2-1)(c(-3k^2+k+4)+k^3-4k^2-9k+20)-4b(c(9k^4-22k^3+9k^2+16k-24)-k^5+15k^4-51k^3+33k^2+44k-40)-8(1-k)(k^2(1-c)(1-k)-2k(c+5)+16)$. When $b=0$, $\frac{\partial y_6}{\partial b} = -8(1-k)(k^2(1-c)(1-k)-2k(c+5)+16) < 0$, and when $b=1$, $4\frac{\partial y_6}{\partial b} = (3-k)^3(1+k-2k^2) > 0$. Because the first derivative is quasi-concave in b, and the first derivative is negative and then positive. Further, we can obtain the numerator is quasi-convex in b. $y_6 = -8(c(2-k)^2(1+k)-k^4+15k^2-34k+20) < 0$ when $b=0$, and $y_6 = -2(3-k)^4(1-k) < 0$ when $b=1$. So the numerator is negative, and the denominator $-4(1-b)^2(1-k)(2+b-k)^2(4-b^2(k+1))^2$ is negative too, that is, $\frac{\partial p_U^{P*}}{\partial b} > 0$.

(ii) The wholesale price w_U^* regarding k:

$\frac{\partial w_U^*}{\partial k} = \frac{(1+b)(b^2(2k-1)-b(2-k)^2-4)}{4(1-b)(b-k+2)^2}$. Let $y_1 = b^2(2k-1)-b(2-k)^2-4$, $\frac{\partial y_1}{\partial k} = 2(b^2+2b-bk) > 0$, so y_1 increases in k, when $k=1$, $\max y_1 = b^2-b-4 < 0$. Therefore, we can get $y_1 < 0$, and $\frac{\partial w_U^{S*}}{\partial k} < 0$.

The 3P seller's retail price p_U^{S*} regarding k:

$\frac{\partial p_U^{S*}}{\partial k} = \frac{1}{4(1-k)^2(2+b-k)^2(4-b^2(1+k))^2}(b^5(1-k)^2(2c+k^2+2k+3)-2b^4(2c(k^3-4k^2+7k-2)-(1-k)^2(k^2+7))+2b^3(c(k^4-6k^3+21k^2-28k+4)+(k^2-12k+8)(1-k)^2)-4b^2(2c(k^3-4k^2+4k-2)-k(k-8)(1-k)^2)+16b((1-k)^2+2c(2-k))+16(c(2-k)^2+2(1-k)^2))$.

The denominator $4(1-k)^2(2+b-k)^2(4-b^2(1+k))^2$ is positive, so let us check the numerator, and we denote it as y_3.

$\frac{\partial^3 y_3}{\partial k^3} = 4(-6b^4c-6b^4-18b^3c-42b^3-12b^2c-60b^2)+4k(6b^5+12b^4+12b^3c+12b^3+24b^2)$. The third derivative of y_3 regarding k is a first-order function of k, and the coefficient is $4(6b^5+12b^4+12b^3c+12b^3+24b^2) > 0$, so the third derivative increases in k. Corresponding, $\max \frac{\partial^3 y_3}{\partial k^3} = \frac{\partial^3 y_3}{\partial k^3} \mid_{k=1} = 24b^2(-6-b^2c-bc-5b-2c+b^3+b^2) < 0$, that is, $\frac{\partial^3 y_3}{\partial k^3} < 0$, and $\frac{\partial^2 y_3}{\partial k^2}$ decreases in k.

$\frac{\partial^2 y_3}{\partial k^2} = 4(b^5(c+3k^2)+b^4(c(8-6k)+6k^2-6k+8)+3b^3(c(2k^2-6k+7)+2k^2-14k+11)+2b^2(c(8-6k)+6k^2-30k+17)+8b+8(c+2))$. We can obtain $\min \frac{\partial^2 y_3}{\partial k^2} = \frac{\partial^2 y_3}{\partial k^2} \mid_{k=1} = 4(b^5(c+3)+b^4(2c+8)+3b^3(3c-1)+2b^2(2c-7)+8b+8(c+2))$, which is a first-order function of c and the coefficient is $4(8+b^5+2b^4+9b^3+4b^2) > 0$. Hence, when $c=0$, $\min \frac{\partial^2 y_3}{\partial k^2} = 4(b+2)((3b+2)b^3+8-7b^2) > 0$, and $\frac{\partial y_3}{\partial k}$ increases in k.

$\frac{\partial y_3}{\partial k} = 4(-b^5(1-k)(c+k^2+k+1)+b^4(c(-3k^2+8k-7)+2k^3-3k^2+8k-7)-b^3(1-k)(c(2k^2-7k+14)+2k^2-19k+14)-2b^2(c(3k^2-8k+4)-2k^3+15k^2-17k+4)-8b(1-k+c)-8(c(2-k)+2(1-k)))$. We have $\max \frac{\partial y_3}{\partial k} = \frac{\partial y_3}{\partial k} \mid_{k=1} = -8c(b^4-b^2+4b+4) < 0$, so the numerator decreases in k, and $\min y_3 = y_3 \mid_{k=1} = 8(b+1)^2(2-b^2)c > 0$. The numerator is positive, and the denominator is positive. Thus, $\frac{\partial p_U^{S*}}{\partial k} > 0$.

The e-platform's retail price p_U^{P*} regarding k:

$\frac{\partial p_U^{P*}}{\partial k} = \frac{1}{4(1-k)^2(2+b-k)^2(4-b^2(k+1))^2}(-b^5(1+k)^2(2c+(1-k)^2)+b^4(4c(k^3+k^2-5k-1)-2k^4+8k^2-8k+2)+b^3(32(1-k)^2-2c(k^4+2k^3-19k^2+24k-12))+4b^2(c(k^4-6k^3+13k^2-20k+20)+(k^2-4k+16)(1-k)^2)+8b(2c(k-2)^2+(1-k)^2(k^2-4k+2))-32(1-k)^2)$.

The denominator $4(1-k)^2(2+b-k)^2(4-b^2(k+1))^2$ is positive, so let us check the numerator, and denote it as y_5. $\frac{\partial y_5}{\partial c} = 2b(2+b-k)^2(8-b^2(1+k)^2+2b(1-k)^2) > 0$, so y_5 increases in c. If $y_5 \mid_{c=0} = -(2+b)(1-k)^2(b^4(1+k)^2-4b^3-24b^2-4b(2-k)^2+16) > 0$, we obtain $y_5 > 0$, and we denote $y_5 \mid_{c=0} = M$; If $y_5 \mid_{c=\bar{c}} = \frac{1}{4(1-b)(1-k)(2+b-k)^2(2+bk+b)(4-b^2(k+1))}(b^5(1+k)^2(2k-1)-b^4(k^4-12k^2+2k-3)+b^3(k^4-12k^3+17k^2-34k+40)+2b^2(k^4-5k^3+16k^2-54k+44)-4b(k^3-11k^2+22k-12)+16(k-1)) < 0$, we obtain $y_5 < 0$, and we denote $y_5 \mid_{c=\bar{c}} = N$. Correspondingly, we can get the conclusion: If $M > 0$, the numerator is positive, if $N > 0 > M$, the numerator is negative first and then positive, and if $N < 0$, the numerator is negative. M and N both increase in b. We denote $\widetilde{b}(k)$ as a root of $M = 0$, and $\hat{b}(k)$ as a root of $N = 0$. Thus, when $b > \widetilde{b}(k)$, $\frac{\partial p_U^{P*}}{\partial k}$ is positive, when $\hat{b}(k) < b < \widetilde{b}(k)$, $\frac{\partial p_U^{P*}}{\partial k}$ is negative and then positive, and when $b < \hat{b}(k)$, $\frac{\partial p_U^{P*}}{\partial k}$ is negative. Comprehensive, there is $\breve{b}$ that satisfies $\hat{b}(k) < \breve{b} < \widetilde{b}(k)$, when $b < \breve{b}$, we have $\frac{\partial p_U^{P*}}{\partial k} < 0$, and when $b > \breve{b}$,we have $\frac{\partial p_U^{P*}}{\partial k} > 0$.

Proof of Theorem 2.

(ii) $w_D^{P*} - w_D^{S*} = \frac{k}{2} > 0$, which is only related to the commission rate.

(iv) The difference between the retail prices is $p_D^{S*} - p_D^{P*} = \frac{k(2c-b(1-k+2c))}{2(1-k)(4-b^2(1+k))}$. If $b < \frac{2c}{1-k+2c}$, it is positive. Otherwise, it is negative.

Proof of Corollary 2.

(i) $\frac{\partial w_D^{S*}}{\partial b} = \frac{1-k}{2(1-b)^2} > 0$;

$\frac{\partial w_D^{P*}}{\partial b} = \frac{1-k}{2(1-b)^2} > 0$;

$\frac{\partial p_D^{S*}}{\partial b} = \frac{1}{-2(1-b)^2(1-k)(4-b^2(k+1))^2}(-b^4(1-k^2)(2+c+k)-2b^3(1+k)(ck+c-k+1)+b^2(c(k^2+12k+3)-15k^2+4k+11)+4b(-3ck+c+(1-k)^2)-4(5+c)(1-k))$. The denominator $-2(1-b)^2(1-k)(4-b^2(1+k))^2$ is negative, so we need to check the numerator, and denote it as y_8. $\frac{\partial y_8}{\partial c} = -b^4(1-k^2)-2b^3(k+1)^2+b^2(k^2+12k+3)+4b(1-3k)-4(1-k)$, and $\frac{\partial^2 y_8}{\partial c \partial k} = 2(1-b)^2(b^2k+2-2b) > 0$. When $k = 1$, $\max \frac{\partial y_8}{\partial c} = -8b(1-b)^2 < 0$. Therefore, $\frac{\partial y_8}{\partial c} < 0$, and the numerator decreases in c. When $c = 0$, $\max y_8 = -b^4(1-k^2)(2+k)-2b^3(1+k)(1-k)+b^2(-15k^2+4k+11)+4b(1-k)^2-20(1-k) < 0$. Accordingly, the numerator is negative and $\frac{\partial p_D^{S*}}{\partial b} > 0$;

$\frac{\partial p_D^{P*}}{\partial b} = \frac{1}{-2(1-b)^2(1-k)(4-b^2(k+1))^2}(-b^4(1+k)^2(2(1-k)+c)-2b^3(1+k)(-3ck+c+(1-k)^2)+b^2(1+k)(c(3-9k)+k^2-12k+11)+4b(1+k)(ck+c-k+1)-4(ck+c-k^2-4k+5))$. The denominator $-2(1-b)^2(1-k)(4-b^2(1+k))^2$ is negative, so let us check the numerator, and denote it as y_9. $\frac{\partial y_9}{\partial c} = -(1-b)^2(1+k)(b^2(1+k)+4b(1-k)+4) < 0$, thus, y_9 decreases in c. When $c = 0$, the maximum of the numerator is $-2b^4(1-k)(1+k)^2-2b^3(1-k)^2(1+k)+b^2(1+k)(k^2-12k+11)+4b(1-k)(1+k)-4(5-k^2-4k) < 0$, thus, the numerator is negative and $\frac{\partial p_D^{P*}}{\partial b} > 0$.

(ii) $\frac{\partial w_D^{S*}}{\partial k} = -\frac{1}{2(1-b)} < 0$;

$\frac{\partial w_D^{P*}}{\partial k} = -\frac{b}{2(1-b)} < 0$;

$\frac{\partial p_D^{S*}}{\partial k} = \frac{b^3(1+c)(1-k)^2+2b^2(1+k^2-2k(1+c))+8c}{2(1-k)^2(4-b^2(1+k))^2}$. The denominator $2(1-k)^2(4-b^2(1+k))^2$ is positive, so we need to check the numerator. Let $y_7 = b^3(1+c)(1-k)^2+2b^2(1+k^2-2k(1+c))+8c$, we have $\frac{\partial y_7}{\partial c} = b^3(1-k)^2+8-4b^2k > 0$, and y_7 increases in c. When $c = 0$, $\min y_7 = b^2(2+b)(1-k)^2 > 0$. Thus, $y_7 > 0$ and $\frac{\partial p_D^{S*}}{\partial k} > 0$;

$\frac{\partial p_D^{P*}}{\partial k} = \frac{b(2b(1+c)(1-k)^2+8c+4(1-k)^2-b^2c(1+k)^2)}{2(1-k)^2(4-b^2(1+k))^2} > 0$.

Proof of Theorem 3.

(i) $w_D^{S*} - w_U^* = -\frac{(2+b)(1-k)k}{4(2+b-k)} < 0$; $w_D^{P*} - w_U^* = \frac{k(2+b(1+k))}{4(2+b-k)} > 0$; $p_D^{S*} - p_U^{S*} = -\frac{k}{4b-4k+8} < 0$; $p_D^{P*} - p_U^{P*} = \frac{k}{4b-4k+8} > 0$.

(ii) $q_D^{S*} - q_U^{S*} = \frac{(1+b)k}{4(2+b-k)} > 0$; $q_D^{P*} - q_U^{P*} = -\frac{(1+b)k}{4(2+b-k)} < 0$; $q_U^{S*} + q_U^{P*} = q_D^{S*} + q_D^{P*} = \frac{4+b(2-k)-b^2k}{8-2b^2(1+k)}$.

(iii) Under the uniform pricing strategy, the manufacturer's profit is

$\Pi_U^{M*} = \frac{(b^2(2c-(1-k)k)+b(1-k)(2+2c-k)-2c(2-k)+4(1-k))^2}{8(1-b)(1-k)(2+b-k)(4-b^2(k+1))}$.

The 3P seller's profit is

$\Pi_U^{S*} = \frac{(b^3(1+k)(2c-k^2+k)-b^2(2c(k^2-3)+k^3+k-2)}{16(1-k)(b-k+2)^2(4-b^2(1+k))^2} + \frac{2b((3+c)k^2-(3c+7)k+4)+4c(k-2)+8(1-k)^2)^2}{16(1-k)(2+b-k)^2(4-b^2(1+k))^2}$.

The e-platform's profit is

$\Pi_U^{P*} = -\frac{1}{16(1-b)(1-k)^2(2+b-k)^2(4-b^2(k+1))^2}(b^7(1-k)^2(1+k)(4c^2-4ck(1+k)+k^2(k^2+2k-3)) - b^6(k^2-1)(4c^2(2k^2-5k+5)-4c(k^4-k^3-k^2+3k-2)+k(k^4-9k^3+27k^2-15k-4)) + b^5(4c^2(k^5-3k^4+4k^3-4k^2+3k+3)-4c(k^6-3k^5-4k^4+4k^3+11k^2+3k-12)-(1-k)^2(5k^5-9k^4+15k^3+17k^2+4k-4)) - b^4(4c^2(k^5-5k^4+4k^3+20k^2-41k+17)+4c(k^6+3k^5-10k^4+11k^2+13k-18)+(1-k)^2(3k^5-13k^4+41k^3-103k^2-56k-28)) - 4b^3(c^2(k^5-3k^4-3k^3+19k^2-22k+16)-c(k^6+2k^5-16k^4+2k^3+39k^2-12k-16)-(1-k)^2(9k^4-34k^3+34k^2+22k+16)) + 4b^2(c^2(k^5-3k^4-13k^3+67k^2-84k+24)+4c(k^5-k^4-5k^3+5k^2+12k-12)+(1-k)^2(7k^4-25k^3+23k^2-44k+8)) + 16b(c^2(k^4-6k^3+12k^2-10k+4)+ck(-k^4+2k^3+4k^2-13k+8)-2(1-k)^2(2k^3-10k^2+13k+2)) - 16(c^2(k^2-3k+1)(2-k)^2-4c(k^2-3k+2)+4(1-k)^2(k^3-4k^2+3k+1)))$.

Under the price discrimination strategy, the manufacturer's profit is

$\Pi_D^{M*} = \frac{1}{4(1-b)(1-k)(4-b^2(1+k))}(b^3(2c^2-2ck(1-k)+k^2(1-k)) + b^2(-2c^2k+4c(1-k)-k(2-k)(1-k)) - b(c^2(6-4k)+2c(k^2+k-2)-4k^2+6k-2)+2c^2(2-k)-8c(1-k)+2(2-k)(1-k))$.

The 3P seller's profit is

$\Pi_D^{S*} = \frac{(b^2c(1+k)+b(1+c)(1-k)+2(1-c-k))^2}{4(1-k)(4-b^2(1+k))^2}$.

The e-platform's profit is

$\Pi_D^{P*} = -\frac{1}{4(1-b)(1-k)^2(4-b^2(k+1))^2}(b^5(1-k)^2(1+k)(c^2-k^2)+b^4(1-k^2)(c^2(1+k)+2c-k^2(1-k))+b^3(c^2(-k^3+k^2+9k-5)+4c(1-k)+(1-k)^2(7k^2+3k+1))+b^2(c^2(k^3+k^2-5k-1)-6c(1-k^2)+(1-k)^2(7k^2+3k+3))+4b(c^2(k^2-4k+2)-c(k^2-3k+2)-3(k-1)^2k)-4(c^2(k^2-3k+1)+2c(k-1)+(1-k)^2(3k+1)))$.

First, comparing the profits of the three parties under the two pricing strategies, we have the range of unit cost $c < \bar{c}$. Accordingly, $\Pi_U^{M*} - \Pi_D^{M*} = -\frac{(1+b)k^2}{8(2+b-k)} < 0$.

Second, comparing the profits of the three parties under the uniform pricing strategy and non-encroachment case. We have the range of unit cost $c < \bar{c}$. Similarly, we can obtain $\Pi_U^{M*} - \Pi_N^{M*} = \frac{1}{8(1-b)(1-k)(b-k+2)(4-b^2(k+1))}((b+1)(b-k+2)(b^2(k+1)-4)((b-1)c-k+1)^2 + (b^2(2c+(k-1)k)-b(k-1)(2c-k+2)+2c(k-2)-4k+4)^2) > 0$.

Accordingly, $\Pi_D^{M*} > \Pi_U^{M*} > \Pi_N^{M*}$.

$\Pi_U^{S*} - \Pi_D^{S*} = \frac{1}{-16(2+b-k)^2(4-b^2(k+1))}((1+b)k(b^3(1+k)(4c-k^2+k)-b^2(4c(k^2-3)+k^3+3k-4)+4b((2+c)k^2-3(2+c)k+4)+4(-2c(2-k)+3k^2-7k+4)))$.

The denominator is negative, and denote the numerator $(1+b)k(b^3(1+k)(4c-k^2+k)-b^2(4c(k^2-3)+k^3+3k-4)+4b((2+c)k^2-3(2+c)k+4)+4(-2c(2-k)+3k^2-7k+4))$ as y_{10}. $\frac{\partial y_{10}}{\partial c} = (1+b)k(4b^3(1+k)+4b^2(3-k^2)+4b(k^2-3k)-8(2-k)) < 0$, thus, y_{10} decreases in c. When $c = \bar{c}$, $\min y_{10} = (1+b)^2(1-k)k^2(4-b^2(1+k)) > 0$, so we have $\Pi_U^{S*} < \Pi_D^{S*}$.

Similar to the above calculation, we have $\Pi_D^{S*} - \Pi_N^{S*} < 0$. Accordingly, $\Pi_N^{S*} > \Pi_D^{S*} > \Pi_U^{S*}$.

$\Pi_U^{P*} - \Pi_D^{P*} = \frac{1}{16(1-k)(2+b-k)^2(4-b^2(k+1))}(k(1+b)(b^3(1-k^2)(4c-k^2-k)+b^2(4c(k^3-k^2-k+3)-k^4+k^3+k^2-5k+4)+4b(1-k)((1+c)k^2-(c+2)k+4)-8c(2-k)+4(4-k)(1-k)^2))$.

We denote the numerator $k(1+b)(b^3(1-k^2)(4c-k^2-k)+b^2(4c(k^3-k^2-k+3)-k^4+k^3+k^2-5k+4)+4b(1-k)((1+c)k^2-(2+c)k+4)-8c(2-k)+4(4-k)(1-k)^2)$ as y_{11}. $\frac{\partial y_{11}}{\partial c} = -4k(1-b)(1+b)(b^2(1-k^2)+b(k^3-2k^2-k+4)-2k+4) < 0$, which indicates that y_{11} decreases in c. When $c = 0$, $\max y_{11} = k(1+b)(4b(1-k)(k^2-2k+4)+4(4-k)(1-k)^2-b^3k(1+k)(1-k^2)+b^2(-k^4+k^3+k^2-5k+4))$ is positive. When $c = \bar{c}$, the sign of $\min y_{11} = \frac{k^2(1+b)(1-k)}{2+bk+b}(-b^4(3-k)(1+k)^2+b^3(1+k)^2(3k-5)+b^2(-6k^2+8k+14)+b(-12k^2+8k+20)-8(1-k))$ can not be determined. When $c = \tilde{c} = \frac{-(1-k)}{4(1-b)(2+b-k)(2+b(1-k^2))}(b^3k(1+k)^2+b^2(-k^3+k-4)-4b(k^2-2k+4)-4(k^2-5k+4))$,

$\Pi_U^{P*} - \Pi_D^{P*} = 0$. Combined with $c < \bar{c}$,, we have $\Pi_U^{P*} < \Pi_D^{P*}$ when $\min\{\bar{c}, \widetilde{c}\} < c < \bar{c}$, and $\Pi_U^{P*} > \Pi_D^{P*}$ when $c < \min\{\bar{c}, \widetilde{c}\}$.

Similarly, we have $\Pi_U^{P*} - \Pi_N^{P*} > 0$ and $\Pi_D^{P*} - \Pi_N^{P*} > 0$. Therefore, $\Pi_N^{P*} < \min\{\Pi_U^{P*}, \Pi_D^{P*}\}$. In addition, when $\min\{\bar{c}, \widetilde{c}\} < c < \bar{c}$, $\Pi_U^{P*} < \Pi_D^{P*}$, and when $c < \min\{\bar{c}, \widetilde{c}\}$, $\Pi_U^{P*} > \Pi_D^{P*}$.

Proof of Corollary 3.

$p_D^{S*} - p_C^{S*} = \frac{1}{2(1-k)(4-b^2(1+k))}(b^2c(1-k^2) + b(1+c)(1-k) + c(4k-2) + 2(1-k) > 0;$

$p_D^{P*} - p_C^{P*} = \frac{1}{2(1-k)(4-b^2(1+k))}(b^2c(1-k^2) + b(1+k)(1-k+c) + 2(1-c)(1-k)) > 0.$

Proof of Theorem 5.

We have $w_{CO}^{S*} = \frac{(1-k)(b^2c-3bc+b+2c)}{2(1-b)}$ and $w_{CO}^{P*} = \frac{b^2ck+b^2c-bck-3bc-bk+b+2c}{2(1-b)}$ by $p_{CO}^S(w_{CO}^S, w_{CO}^P) = p_C^{S*}$ and $p_{CO}^P(w_{CO}^S, w_{CO}^P) = p_C^{P*}$. The 3P seller is willing to participant when its profit in this contract is not less than that in wholesale price contract. We can obtain $T_1^* = -\frac{(b^2c(1+k)+b(1+c)(1-k)+2(1-c-k))^2}{4(1-k)(4-b^2(1+k))^2} + \frac{1}{4}(1-k)(1-(1-b)c)^2$ by solving $q_{CO}^{S*}((1-k)p_{CO}^{S*} - w_{CO}^{S*}) - T_1 = \Pi_D^{S*}$ and $T_2^* = \frac{1}{4(1-k)^2(4-b^2(1+k))^2}(b^6c^2(1-k)^2(1+k)^3 + 2b^5(1-c)c(1-k^2)^2 - b^4(1-k)^2(1+k)(c^2(k^2+8k+8) + 2c(1+k) - 1) - 2b^3(1-k^2)(c^2(8k-7) + c(9-8k) - k(k-1)) + b^2(c^2(8k^4+k^3-15k^2-9k+11) + 2c(8k^3-7k^2-6k+5) - (1-k)^2(k^2+5k+9)) - 4b(1-k)(c^2(7-8k) + c(7k-8) + 2k^2-3k+1) - 4c^2(4k^3-11k^2+9k-3) - 8c(4k^2-7k+3) + 4(1-k)^2(k+3))$ by solving $q_{CO}^{P*}((1-k)p_{CO}^{P*} - w_{CO}^{P*}) - T_2 = \Pi_D^{P*}$.

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