

SUSTAINABLE INVENTORY MODEL WITH ENVIRONMENTAL IMPACT FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH COMPOSITE DEMAND

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Abstract. Global warming and climate change become a sensitive issue worldwide, and many countries try to control the CO₂ emissions by investigating in many projects. This study proposes a sustainable inventory model to reduce carbon emission. It is generally assumed that demand is increasing at starting of an inventory level and decreasing at ending for a particular newly launched product. In this situation, demand is usually represented by Normal distribution or imprecise fuzzy demand. However, in reality, those assumptions do not always hold. In most of the inventory models, it is seen that demand is a linearly increasing function of time before starting deterioration; then, the demand decreases inversely proportional to the deterioration rate after starting the deterioration. To describe the possible situations more clearly, we introduce here a new type of demand, called as composite demand. Most of the products have a fixed life span for maintaining the quality in original condition which is known as non-instantaneous deterioration. Therefore, we design an inventory model for non-instantaneous deteriorating items with composite demand function. The deterioration rate is assumed to follow an exponential distribution; the holding cost is considered as time dependent. The model is formulated based on retailer's perspective and, thereafter, the total cost for the retailer is calculated. Numerical examples are provided for better understanding of this model. The results show that the total profit for non-instantaneous deteriorating items is higher than for instantaneously deteriorating items. A sensitivity analysis is conducted based on the important parameters. Finally, the paper ends with conclusions and an outlook to future research.

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1. INTRODUCTION

The fundamental inventory model is developing day by day by several investigations done by researchers [28,34,35]. Recently, many researchers added sustainable development (SD) to the fundamental inventory model as SD is attracting the focus of today's companies. For external pressure, internal forces and market competition, companies are considering an SD as very serious issue. SD is nothing but an equity among environmental, economic and social dimensions. According to many reports, SD is "development which meets the needs of the present without compromising the ability of future generations to meet their own needs" (Brutland Commission report, 1987). The above definition adds "environmental, economic, and social resources" to the decision maker. This "three pillars" of SD are identified as "Triple Bottom Line" (TBL). Nowadays, customers are not only concerned about the quality but also aware of environment pollution. Since, global warming is becoming headline daily, so, SD is really an interesting way to maintain the "three pillars". There are mainly three types of impact for which sustainability is considered in the paper:

- Environmental impact: sustainable business practices protect nature and conserve natural resources wherever possible.
- Social impact: socially sustainable companies recognize their own importance to people and society, and work to improve their local communities.
- Business impact: consumers are more likely to support sustainable businesses. Sustainability also cuts business costs by reducing energy consumption and waste.

Generally, the word sustainability is used in global sense because the goal of sustainability is considered as global goal. Thus, we interest here to investigate the potentiality of the optimization problems with SD concerns. In a traditional inventory system, it is usually chosen that demand rate is independent of price, stock availability or time. But, after a lot of investigations, researchers found that demand is hugely dependent on stock level, price of commodities and time, too. However, in our daily marketing system, one can see that there are so many items, which people generally purchase very rapidly up to a fixed time and, after this, sales of those items are gradually decreased. As for a practical example, people prefer to buy fish or vegetables in early morning rather than in the later morning or in the afternoon, due to the freshness of the original items. So, there are some items whose demand is an increasing function of time at starting; thereafter, the demand follows a decreasing function of time. For representing this type of demand, we introduce a new demand function, termed as composite demand. Several research papers are available, where the demand function is following a Gaussian distribution function of time, or as an imprecise fuzzy demand. But, such assumptions do not always hold. It may happen that the demand rate increases exponentially but decreases linearly. In real-life situation, increasing demand may follow linear function of time, quadratic function of time, logarithmic function, exponential function, etc. However, decreasing demand always depends on the deterioration rates of the items. Thus, the demand function can be taken as a piecewise combination of two functions: an increasing function of time and a decreasing function of time. Due to the two combinations, a new demand which is composite demand appears for representing the situations.

In a traditional inventory model, it is commonly considered that deterioration starts at instant arrivals of stocks. Perishable items are mainly of two types: items which maintains constant utility throughout the lifespan *i.e.*, blood (which has a fixed lifetime of 21 days with constant utility) and medicines, whereas the other type are those which exponentially mold its utility, *i.e.*, vegetables, fruits and fish. Sustainable management of perishable products with partial lifespan is a big challenge. So, for this kind of items, the assumption that deterioration starts at an instant level of arrival of goods gives an error or vague value of total cost when it is calculated from a retailer's perspective. Therefore, it is important for an inventory model to assume non-instantaneous deterioration for items. As for example, non-instantaneous deterioration occurs in a greenhouse flower farm; the freshness of flowers is preserved in the winter season but flowers deteriorate in the summer. Reducing the deterioration of products simultaneously with carbon emissions is a burdensome task for a manager of an industry. This study provides important suggestions for industry managers to ensure better profit and provide

guidance for an appropriate investment in preservation technology. Inventory models for deteriorating products have attracted extensive interest from researchers in recent decades.

In a classical inventory system, holding cost is always assumed as constant. But in real-life situations, it is essential for the inventory manager to maintain the quality of the original goods when they are stored for future usages. Hence, for deteriorating items, consideration of variable holding cost is really relevant. As time increases, the rate of deterioration increases and the holding cost increases, too. It is naturally seen that holding cost for perishable commodities like vegetables, fruits, volatile liquids or medicines are quite high and time dependent. Hence, holding cost for such products should be taken into account and have a crucial importance in an inventory model.

In this paper, we propose an inventory model for non-instantaneous exponential deteriorating items with composite demand. The main motivations and characteristic view of this work are described below:

- Concept of sustainable inventory model is included in this study reminding the criteria of environmental pollution.
- A new type of demand function, called composite demand, is introduced here, for the first time. The characteristic of the demand function is that the demand rate is increasing at the starting time of inventory policy, and when deterioration has started, it is a decreasing function of time.
- Non-instantaneous deterioration is incorporated with composite demand function, and the decreasing demand function is proportional to the deterioration rate. This makes this model to become an attractive one and the applicability of the model works excellently.
- Time-varying holding cost is initiated to execute a realistic value, as deterioration is assumed.
- The model is based on a retailer's perspective.
- Shortages are not included in this study, since we are moving into a competitive era.
- Numerical examples illustrate that the proposed solution procedure, disclosing a reduction of the total relevant cost. This shows that the proposed model is a useful alternative for the decision makers.

The rest of the paper is organized as follows: A detailed literature survey on previous researches is provided in Section 2. Section 3 states the notations and the assumptions related with the study. A mathematical model of the proposed study is presented in Section 4. In Section 5, a solution procedure is provided for solving the proposed model. Section 6 contains a couple of numerical examples, whereas Section 7 is reserved for a sensitivity analysis with respect to the major parameters. In Section 8, different forms of composite demand function related to the inventory model is introduced. Finally, Section 9 gives an overall conclusion and future research avenues of the study.

2. LITERATURE REVIEW

The proposed inventory model is improved based on some important previous research articles. These articles are depicted as follows:

2.1. Sustainable development (SD)

SD is an economic development without destroying the environment. In recent years, SD has gained huge attention from academicians and researchers. At first, the United Nations and national governments forced to implement SD but companies were reluctant to implement SD for economical issue [5]. Later, companies were convinced that implementation of SD would destroy their competition. Bonney and Jaber [4] were the first to include SD in the fundamental inventory model. They identified inventory systems that helps to reduce the environmental pollution; these systems include solutions for handling GHGs that have already been emitted and methods to reduce the current emission of these gases. Bouchery *et al.* [5] added a warehouse-retailer supply chain policy with SD in the inventory model. They introduced a sustainability criterion to classical inventory models and formulated a multi-objective inventory model with a set of Pareto optimal solutions to provide the decision maker with better options using an interactive procedure. Shu *et al.* [48] studied a remanufacturing policy for

an inventory model. They derived the manufacturer's optimal production through the extremum method and iteration algorithm using the cap-and-trade policy. They demonstrated that remanufacturing leads to economic and environmental profits in connection with carbon emissions. Taleizadeh *et al.* [50] studied a sustainable inventory model with shortages and obsolescence in production. They developed four new sustainable EPQ models by considering different shortage scenarios and solved the models. Further, they proposed an algorithm and showed that a sustainable Economic Production Quantity (EPQ) model with partial backordering is more profitable in realistic scenarios. Sarkar and Sarkar [44] described a process to enable the industry to reduce waste disposal and consume energy with the help of a sustainable smart biofuel multi-stage production system. In this system, automated inspections were conducted using smart machines. They suggested methods to reduce the amount of waste and energy consumption. Paul *et al.* [35] measured the effect of multiple prepayments and green investment on an EPQ model. Sarkar *et al.* [45] found the combined effects of carbon emission and quality improvement for a sustainable supply chain model. The model is considered on reduction of waste for a production system. They tried to reduce the production set up cost by a discrete investment. A sustainable model with closed loop supply chain with multiple objectives was formulated by Alinezhad *et al.* [2]. They have solved their model by using Fuzzy optimization. Barman *et al.* [3] discussed the impacts of green and preservation technology investments on a sustainable EPQ model during COVID-19 pandemic. Darvazeh *et al.* [7] studied an integrated optimization model and they have solved their model by allowing sustainability criterion. In all the aforementioned models, the authors attempted to determine the effect of SD on the inventory. This study attempts to demonstrate how environmental pollution can be reduced using a sustainable inventory model and how technological improvements adversely affect the associated social costs.

2.2. Carbon emission

A major global concern for industries is related to reduce their carbon emissions. Thus, organizations have been attempting to improve the criteria by incorporating SD. Absi *et al.* [1] analyzed the carbon-cap policy with fixed and variable carbon emissions in the inventory model. They introduced four new environmental constraints in inventory problems using mathematical programming. Further, they proposed a new polynomial dynamic program for periodic carbon emission constraints with any three other constraints. Hammami *et al.* [16] investigated a multi-echelon production-inventory model with a cap-and-trade policy and lead time constraints. They adopted a general inventory policy with manufacturing facilities, external suppliers, and different distribution centers. They capitalized on the multi-echelon model to indicate the effect of carbon caps on each facility in comparison with the global cap on the entire supply chain model. Paul *et al.* [34] considered the green inventory model with the effect of carbon taxation. Tiwari *et al.* [53] investigated an inventory model with SD and the imperfect quality of items with carbon emissions. They formulated a supplier-retailer-buyer supply chain model with shortages and a partial credit policy. They evaluated the theoretical result of trade credit, replenishment, and optimal price to maximize the profit of the retailer. Das *et al.* [8] developed a multi-objective location transportation problem with variable carbon emission. They tried to reduce transportation time, transportation cost, inventory cost and carbon emission cost. They used the intuitionistic fuzzy programming to get the Pareto-optimal solution of the proposed model. Inspired by this literature review, we develop an inventory model with SD in which the emission of carbon reduction was maximized.

2.3. Non-instantaneous deterioration

Nowadays, deterioration is a common phenomenon in an inventory system. Deterioration is defined as spoilage, obsolescence or loss of marginal value of the original items. Several researches were done by scientists on deteriorating inventory. Among them, Whitin [57] was the first who studied an inventory system with deterioration. Then, Ghare and Schrader [12] designed an inventory model with exponential deterioration. Covert and Philip [6] extended Ghare's and Schrader's [12] work by using two parameters, namely, Weibull distribution and Gamma distribution. Dave and Patel [9] were the first to study deteriorating inventory with linear increasing demand. Almost all the inventory models mentioned before assumed that deterioration occurs as soon as the goods are

received in stock. However, in real-world situations, most of the goods have some life span of maintaining quality or original condition, and after that period spoilage happens. This phenomenon is termed as “non-instantaneous deterioration” by Wu *et al.* [58]. Then, Ouyang *et al.* [32] developed a non-instantaneous inventory model by allowing permissible delay in payment. Paul *et al.* [33] developed a deteriorating inventory by considering the effect of price-sensitive demand and default risk with optimal credit period and cycle time. Tayal *et al.* [51] offered and investigated an EOQ model for non-instantaneous deteriorating items with time-dependent holding cost and exponential demand. Khalilpourazari and Pasandideh [19] formulated a multi-objective EOQ model with partial backordering and defectives items. The model was based on stochastic background and the constraints of the model was solved by using MOWCA and MOGWO. Khalilpourazari *et al.* [21] studied an optimization problem with multiple items including inspection error. Maihami and Abadi [26] formulated a non-instantaneous deteriorating inventory model with permissible delay in payment and partial backlogging. Ghoreishi *et al.* [13] presented a non-instantaneous deteriorating item under inflation and customer returns with time-dependent demand. Furthermore, Geetha and Uthayakumar [11] considered price and advertisement dependent demand. In this paper, we introduce an EOQ model for non-instantaneous exponential deteriorating items with composite demand.

2.4. Composite demand

In a classical inventory model, demand was regarded as constant. But, several researchers observed in their study that variable demand is more realistic than constant. There were several forms of variable demand addressed by scientists such as linear demand, quadratic demand, declining demand, ramp type of demand, power demand, dynamic demand, parabolic demand, etc. Authors like Tripathy and Mishra [54], Khanra *et al.* [24] assumed that the demand follows a quadratic function with respect to time. Thereafter, the demand is extended up to an N -degree polynomial. Khalilpourazari *et al.* [22] designed a multi-item EOQ model with robust fuzzy optimization, random disruption and uncertainty in demand. Rajput *et al.* [42] presented an EOQ model on N -degree polynomial demand. In inventory policy, many types of demand are available other than with polynomial demand. Kumar and Singh [25] and Ukil *et al.* [55] presented a deteriorating inventory model with power demand pattern. Ouyang *et al.* [31] and Pervin *et al.* [36] worked on declining demand. Hill [17] was the first to introduce a ramp type of demand. After that, Manna and Chaudhuri [27] and Skouri *et al.* [49] also followed ramp type of demand for deteriorating items in their works. Pervin *et al.* [37] addressed stock-dependent demand, and Pervin *et al.* [38] referred to time-dependent demand in their deteriorating inventory models. Khalilpourazari *et al.* [23] described an energy efficient high precision model with robust optimization and artificial intelligent. Recently, Roy *et al.* [43] proposed a two-warehouse with price discount on backorders and trade-credit policy where the demand function was depicted in probabilistic sense. An alternative model with selling price dependent demand and quadratic varying holding cost were formulated by Porwal *et al.* [41]. Khalilpourazari and Pasandideh [20] formulated a model for emergency flood evacuation plans using robust optimization. Khalilpourazari and Doulabi [18] studied a robust model for prediction of the Covid-19 pandemic in Canada. Sarker *et al.* [46] described an order-level lot size inventory model with inventory-level dependent demand and deterioration. A multi-item deteriorating inventory model with trade-credit policy was elaborated by Pervin *et al.* [40]. An inventory model with dynamic demand over finite time horizon was stated by Dey *et al.* [10]. In this research article, the demand function is considered as a piecewise combination of two functions and name it as composite demand, which is a new contribution.

2.5. Variable holding cost

In a traditional inventory model, it is assumed that holding cost is constant. But, in real-life, holding cost may not be constant, especially when the inventory model is handling deteriorating items. As the deterioration rate increases with respect to time, it is necessary to choose the holding cost as variable to maintain the physical status of the deteriorating items. Researchers like Goh [15], Giri *et al.* [14] included variable holding cost in their models. Mohammadi and Khalilpourazari [30] designed a model on deteriorating jobs and learning effects.

Mishra [29] represented an inventory model with controllable deterioration rate and time-varying holding cost. An integrated inventory model with variable holding cost under two levels of trade-credit policy is elaborated in detail by Pervin *et al.* [39]. So, variable holding cost is an utmost important concept which necessarily leads us to imply variable holding cost into the formulated model. The achievements of various scientists connected with this study are displayed in Table 1.

3. NOTATIONS AND ASSUMPTIONS

This model is developed on the basis of the following assumptions and designations.

Notations

A	Ordering cost per order;
c_1	Unit purchasing cost per item;
d	Unit deterioration cost;
T	Cycle length;
Q	Maximum inventory level during $[0, T]$;
$P(t)$	Inventory supply rate per unit time;
t_d	Deterioration starting time;
t_1	Inventory supply time;
r	Screening rate (units/time unit);
s_c	Setup cost of repair store (\$/setup);
f	Fixed cost of transportation (\$/trip);
R	Rework rate (units/time unit);
t_s	Screening time of products (time unit);
t_R	Transportation, rework, and return time for imperfect products (time unit);
t_T	Total transportation time of imperfect products (time unit);
h_1	Holding cost of perfect items (\$/unit/time unit);
h'_1	Carbon emission cost per item on holding perfect items (\$/unit/time unit);
h_2	Holding cost of rework products (\$/unit/time unit);
h'_2	Carbon emission cost per item on holding rework item (\$/unit/time unit);
h_3	Holding cost at repair store (\$/unit/time unit);
h'_3	Carbon emission cost per item on holding item at repair store (\$/unit/time unit);
c_2	Screening cost per unit (\$/unit);
c_3	Transportation cost of the imperfect item per unit (\$/unit);
c_4	Labor and material cost required to repair a unit product (\$/unit);
c_5	Cost incurred due to a loss of sales (\$/unit/time unit);
c_6	Penalty cost incurred due to goodwill loss (\$/unit);
α	Percentage of imperfect items passed to customers (%);
β	Markup percentage by rework store (%);
c_7	Unit return cost of the imperfect product (\$/unit);
s_p	Selling price of one unit (\$/unit);
$Q_1(t)$	On-hand inventory during $[0, t_1]$;
$Q_2(t)$	On-hand inventory during $[t_1, t_d]$;
$Q_3(t)$	On-hand inventory during $[t_d, T]$;
T_{rc}	Total relevant cost;
$d_1(t)$	Deterioration rate, where $d_1(t) = e^{\theta t}$, $t \geq 0$, and $0 < \theta \leq 1$;
$D_1(t)$	Demand rate during $[0, t_d]$ is considered as a linear function of time and given as $D_1(t) = a + bt$, $t \geq 0$, where $a, b \geq 0$;

$D_2(t)$ Demand rate during $[t_d, T]$ is proportional to the deterioration rate and is assumed as $D_2(t) = \frac{k}{d_1(t)}$, where k is non-zero constant;

Assumptions

The following are the assumptions for designing the proposed model.

- (i) Single item is considered.
- (ii) Shortages are not allowed in this model, so the supply rate is always greater than the demand rate.
- (iii) Warehouse carbon emission is due to the energy consumption per unit item. Thus, according to carbon tax policy, carbon emission cost per unit item h'_1 , h'_2 and h'_3 is considered for holding perfect items, holding repair items, and holding items at the repair store, respectively.
- (iv) There is a chance to receive a lot with a fraction of defective items from a global supplier, which is from other country.
- (v) Inspection rates are considered as constant and known.
- (vi) The screening process and demand occur at the same time, but the screening rate is faster than the demand rate ($r > D_1, D_2$).
- (vii) Imperfect products have minor damage and can be repairable in a controlled system and all imperfect products are reworked.
- (viii) The percentage of imperfect products are given and known.
- (ix) The holding cost of reworked products is higher than the initial holding cost of perfect items ($h_2 > h_1$).
- (x) The reworked products are returned back when the inventory level of the system becomes zero.
- (xi) Supply rate is higher than the demand rate and assumed as $P(t) = \eta D_1(t)$, where $\eta > 1$, in $[0, t_1]$.
- (xii) Demand rate is inversely proportional to the deterioration rate in $[t_d, T]$.
- (xiii) Deterioration has not started just after stopping the supply rate; it starts later, after some time, which is assumed as $t_d = rt_1$, where $r > 1$, which reflects the model properly.

4. MATHEMATICAL MODEL

Here, the mathematical inventory system under the aforementioned assumptions is given. At the starting time, inventory level is zero and starts with a supply rate $P(t)$ and demand rate $D_1(t)$; it is continued in the time interval $[0, t_1]$. During this time interval, on-hand inventory level is given as $Q_1(t)$. After time t_1 , supply stopped but the demand rate is to be same as before, and this procedure is continued in the time interval $[t_1, t_d]$. So, on-hand inventory in this time interval is assumed as $Q_2(t)$. After time t_d , demand is changed to $D_2(t)$ as deterioration starts with a rate of $d_1(t)$. This procedure is continued during the time interval $[t_d, T]$, where T is the cycle length. During this period, on-hand inventory is regarded as $Q_3(t)$. At time t_1 , the inventory reaches its maximum level (*i.e.*, Q), whereas after the cycle length, inventory level becomes zero. After the scheduling time period, the inventory repeats itself.

Hence, the inventory level at the subsequent period is given by

$$\frac{dQ_1(t)}{dt} = P(t) - D_1(t) \text{ with } Q_1(0) = 0, \quad 0 \leq t \leq t_1, \quad (4.1)$$

$$\frac{dQ_2(t)}{dt} = -D_1(t) \text{ with } Q_1(t_1) = Q_2(t_1) = Q, \quad t_1 \leq t \leq t_d, \quad (4.2)$$

$$\text{and } \frac{dQ_3(t)}{dt} + d_1(t)Q_3(t) = -D_2(t) \text{ with } Q_2(t_d) = Q_3(t_d), Q_3(T) = 0, \quad t_d \leq t \leq T. \quad (4.3)$$

Now, the solution of equation (4.1) using the initial condition becomes

$$Q_1(t) = (\eta - 1) \left(at + \frac{bt^2}{2} \right).$$

TABLE 1. Achievements of different authors related to inventory model.

Author(s) & year	SD	Carbon emission	Instantaneous deterioration	Non-instantaneous deterioration	Demand
Shu <i>et al.</i> (2017)	Allow	Allow	Constant	–	Exponential
Taleizadeh <i>et al.</i> (2018)	Allow	Allow	Constant	–	Exponential
Ghare and Scharder (1963)	–	–	Constant	–	Exponential
Khanra <i>et al.</i> (2011)	–	–	Constant	–	Quadratic
Rajput <i>et al.</i> (2016)	–	–	Constant	–	N -th degree polynomial
Ouyang <i>et al.</i> (2005)	–	–	Constant	–	Declining
Dave and Patel (1981)	–	–	Constant	–	Time proportion
Pervin <i>et al.</i> (2017)	–	–	Constant	–	Stock-dependent
Skouri (2009)	–	–	Weibull	–	Ramp-type
Tripathy and Mishra (2010)	–	–	Weibull	–	Quadratic
Sing and Pattnayak (2013)	–	–	Time-dependent	–	Quadratic
Porwal <i>et al.</i> (2011)	–	–	Time-dependent	–	Selling price-dependent
Giri <i>et al.</i> (1996)	–	–	Time-dependent	–	Time-dependent
Manna and Chaudhuri (2006)	–	–	Time dependent	–	Ramp-type
Pervin <i>et al.</i> (2018)	–	–	Stochastic	–	Time-dependent
Ghoreishi <i>et al.</i> (2013)	–	–	–	Constant	Time-dependent
Tayal <i>et al.</i> (2015)	–	–	–	Constant	Exponential
Maihami and Abadi (2012)	–	–	–	Constant	Price and time dependent
Geetha and Uthayakumar (2016)	–	–	–	Constant	Price and advertisement dependent
Wu <i>et al.</i> (2006)	–	–	–	Constant	Stock-dependent
Kumar and singh (2011)	–	–	–	Time dependent	Power
Ouyang <i>et al.</i> (2006)	–	–	–	Probabilistic	Constant
Mishra (2014)	–	–	–	Controllable	Time-dependent
Khalilpourazari and Pasandideh (2020)	–	–	–	Constant	Stochastic
Khalilpourazari <i>et al.</i> (2019)	–	–	–	Constant	Uncertain
Mohammadi and	–	–	–	Constant	Uncertain
Khalilpourazari (2017)	–	–	–	–	–
This study	Allow	Allow	Allow	Exponential	Composite

Furthermore, from the condition $Q_1(t_1) = Q$, we get

$$Q = (\eta - 1) \left(at_1 + \frac{bt_1^2}{2} \right). \quad (4.4)$$

The solution of equation (4.2), using terminal condition, is

$$Q_2(t) = Q - a(t - t_1) + \frac{b}{2}(t^2 - t_1^2).$$

Using initial condition, the solution of equation (4.3) has the form

$$Q_3(t) = k \left[(T - t) + \frac{1}{2}(1 - 3\theta)(T^2 - t^2) + \frac{1}{6}(2 + \theta)(T^3 - t^3) + \frac{\theta}{4}(T^4 - t^4) \right] e^{-\left(t + \frac{\theta t^2}{2}\right)}.$$

Figure 1 gives a graphical representation of the proposed inventory model.

For finding the total inventory cost, the following terms are calculated.

(i) **Ordering cost** (OC) per unit time is denoted by OC and is given by

$$\text{OC} := \frac{A}{T}. \quad (4.5)$$

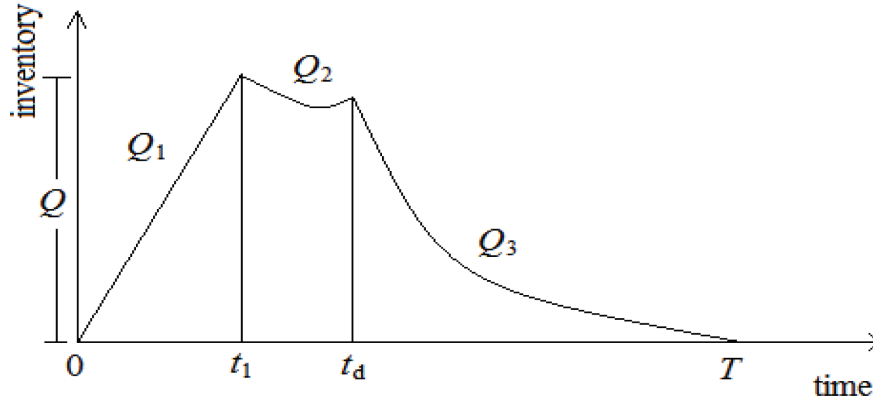


FIGURE 1. Graphical representation of the proposed inventory control model.

- (ii) The total **inspection cost** (IC) of the whole lot is denoted by IC and is given as $c_2 M(D_1 + D_2)$, where c_2 is the inspection cost of one unit, M is the duration of positive inventory time and $D_1 + D_2$ is the demand rate.
- (iii) **Production cost** (PC) per unit time PC, during interval $[0, t_1]$ be

$$\begin{aligned}
 \text{PC} &:= \frac{c}{T} \left(\int_0^{t_1} P(t) dt \right) \\
 &= \frac{c\eta}{T} \left(\int_0^{t_1} (a + bt) dt \right) \\
 &= \frac{c\eta}{T} \left(at_1 + \frac{bt_1^2}{2} \right).
 \end{aligned} \tag{4.6}$$

- (iv) The total **holding cost** (HC) per time unit is a combination of the holding cost of perfect products that are already in the system and the holding cost of repaired products, and is given as:

$$\text{HC} := (h_1 + h'_1) \left[\frac{(1 - \alpha)^2 M^2 T (D_1 + D_2)}{2} + \frac{\alpha T (M(D_1 + D_2))^2}{r} \right] + (h_2 + h'_2) \left[\frac{(\alpha M)^2 T (D_1 + D_2)}{2} \right]. \tag{4.7}$$

- (v) If a margin β is claimed as the repair charge per unit, then the **repair cost** (RC) for one unit is expressed as:

$$\text{RC} := (1 + \beta) \left[\frac{s_c + 2f}{\alpha M T (D_1 + D_2)} + c_4 + 2c_3 + (h_3 + h'_3) t_R \right]. \tag{4.8}$$

- (vi) If c_7 is the unit return cost of a product and c_6 is unit penalty cost from goodwill loss, and there is an α percentage of imperfect items that are passed to the customers, then the **goodwill penalty cost** (GPC) is given as:

$$\text{GPC} := (c_6 + c_7) \alpha M (D_1 + D_2). \tag{4.9}$$

- (vii) **Deteriorating cost** (DC) per unit time DC is defined by

$$\text{DC} := \frac{c}{T} \int_{t_d}^T [Q_3(t) d_1(t)] dt$$

$$\begin{aligned}
&= \frac{dk}{T} \left[\frac{T^2}{2} + \frac{(1-3\theta)}{3}T^3 + \frac{(2+\theta)}{8}T^4 + \frac{\theta}{5}T^5 - (\theta+1) \left\{ \frac{T^3}{6} + \frac{(1-3\theta)}{8}T^4 + \frac{(2-\theta)}{8}T^5 + \frac{\theta}{12}T^6 \right\} \right. \\
&\quad + \left(1 - \frac{\theta}{2} \right) \left\{ \frac{T^4}{12} + \frac{(1-3\theta)}{15}T^5 + \frac{(2+\theta)}{36}T^6 + \frac{\theta}{21}T^7 \right\} \left. - \frac{dk}{T} \left[rt_1 \left(T - \frac{rt_1}{2} \right) + \frac{(1-3\theta)rt_1}{2} \right] \right. \\
&\quad \times \left(T^2 - \frac{r^2t_1^2}{3} \right) + \frac{(2+\theta)rt_1}{6} \left(T^3 - \frac{r^3t_1^3}{4} \right) + \frac{\theta rt_1}{4} \left(T^4 - \frac{r^4t_1^4}{5} \right) - (\theta+1)r^2t_1^2 \left\{ \left(\frac{T}{2} - \frac{rt_1}{3} \right) \right. \\
&\quad + \frac{(1-3\theta)}{2} \left(\frac{T^2}{2} - \frac{r^2t_1^2}{4} \right) + \frac{(2+\theta)}{6} \left(\frac{T^3}{2} - \frac{r^3t_1^3}{5} \right) + \frac{\theta}{4} \left(\frac{T^4}{2} - \frac{r^4t_1^4}{6} \right) \left. \right\} + \left(1 - \frac{\theta}{2} \right) r^3t_1^3 \\
&\quad \times \left. \left\{ \left(\frac{T}{3} - \frac{rt_1}{4} \right) + \frac{(1-3\theta)}{2} \left(\frac{T^2}{3} - \frac{r^2t_1^2}{5} \right) + \frac{(2+\theta)}{6} \left(\frac{T^3}{3} - \frac{r^3t_1^3}{6} \right) + \frac{\theta}{4} \left(\frac{T^4}{4} - \frac{r^4t_1^4}{7} \right) \right\} \right]. \quad (4.10)
\end{aligned}$$

(viii) **Transportation cost (TC):** transportation cost involves the associated expenses of delivering the ready products to the customer. Here, transportation is completed by two modes, by road and by air. If d is the total distance, then d_r is completed by road and d_a is completed by air.

If x is the fixed transportation cost per trip, b the variable transportation cost for unit transportation per distance d_r , m the weight of the product, t_c the truck capacity, n_1 the number of trips and f the fixed cost per trip per truck, v the velocity of truck, then the transportation cost per cycle by road be described as:

$$= (2x + 2bd_r mQ) \frac{nQf}{t_c} + vQ.$$

If d_a is the distance travelled by air, m the weight of the product, and the cost according to weight is

$$= \begin{cases} b_1, & \text{if } 0 < m < x_1, \\ b_2, & \text{if } x_1 < m < x_2, \\ b_3, & \text{if } x_2 < m < x_3, \\ b_4, & \text{if } m > x_4, \end{cases}$$

where x_1, x_2, x_3 and x_4 are the scheduled weight chart given by air authority and $b_i, i = 0, 1, 2, 3, 4$ is the corresponding cost. Therefore, the transportation cost by air per cycle can be written as $b_i m d_a$.

Then, transportation cost per cycle is

$$TC = (2x + 2bd_r mQ) \frac{nQf}{t_c} + vQ + b_i m d_a. \quad (4.11)$$

(ix) **Carbon emission cost (CEC):** carbon emits due to lightning, heating, air conditioning and deteriorating of the product while holding the product in the warehouse. During transportation, burning fossil fuel from the truck also generates carbon emission. If c_v and c_f denote, respectively, the variable and fixed carbon emission factor per holding cost, m be the weight of product, γ the excess progressive tariff per unit carbon emission, d the distance, n_2 the number of gallons, g_t the GHG emissions amount of truck from one gallon of diesel fuel, g_m the amount of carbon emissions of air fuel for one gallon, c_a the fixed cost due to air mode, t_c the truck capacity and c_t be the carbon emission tax, therefore, per cycle carbon emission cost can be written as:

$$CEC = (c_v mQ + c_f) \gamma + \frac{2dn_2 g_t c_t}{t_c} + c_a \frac{Q}{m} g_m, \quad (4.12)$$

where $c_a \frac{Q}{m} g_m$ is the emission cost for using air mode.

Hence, the *total cost* of the system per time unit, denoted by T_{rc} , is represented as follows:

$$T_{rc} := [OC + IC + PC + HC + RC + GPC + DC + TC + CEC]. \quad (4.13)$$

Therefore, by putting the values of the parameters into equation (4.13),

$$\begin{aligned}
T_{rc} := & \frac{A}{T} + \frac{1}{T} \left[h(\eta - 1) \left(\frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) + \alpha(\eta - 1) \left(\frac{at_1^3}{3} + \frac{bt_1^4}{8} \right) + h\eta t_1 \left(at_1 + \frac{bt_1^2}{2} \right) (r - 1) \right. \\
& - h \left(\frac{at_1^2}{2} (r^2 - 1) + \frac{bt_1^3}{6} (r^3 - 1) \right) + \alpha\eta \left(at_1 + \frac{bt_1^2}{2} \right) (r^2 - 1) \frac{t_1^2}{2} - \left(\frac{at_1^3}{3} (r^3 - 1) + \frac{bt_1^4}{8} (r^4 - 1) \right) \Big] \\
& + \frac{hk}{T} \left[(T - rt_1) \left\{ T + \frac{(1 - 3\theta)}{2} T^2 + \frac{(2 + \theta)}{6} T^3 + \frac{\theta}{4} T^4 \right\} - \frac{T^2 - r^2 t_1^2}{2} \left\{ T + \frac{(1 - 3\theta)}{2} T^2 + \frac{(2 + \theta) T^3}{6} \right. \right. \\
& + \left. \frac{\theta}{4} T^4 + 1 \right\} + \frac{T^3 - r^3 t_1^3}{3} \left\{ \frac{1}{2} + \theta \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \right\} + \frac{(T^4 - r^4 t_1^4)}{24} (1 - 7\theta) + \frac{(T^5 - r^5 t_1^5)}{60} (4 + 2\theta) \\
& + \frac{5\theta}{12} (T^6 - r^6 t_1^6) \Big] + \frac{\alpha k}{T} \left[\frac{(T^2 - r^2 t_1^2)}{2} \left\{ T + \frac{(1 - 3\theta)}{2} T^2 + \frac{(2 + \theta)}{6} T^3 + \frac{\theta}{4} T^4 \right\} - \frac{(T^3 - r^3 t_1^3)}{3} \right. \\
& \times \left\{ T + \frac{(1 - 3\theta)}{2} T^2 + \frac{(2 + \theta)}{6} T^3 + \frac{\theta}{4} T^4 + 1 \right\} + \frac{(T^4 - r^4 t_1^4)}{4} \left\{ \frac{1}{2} + \theta \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \right\} \\
& + \frac{(T^5 - r^5 t_1^5)}{30} (1 - 7\theta) + \frac{(T^6 - r^6 t_1^6)}{72} (4 + 2\theta) + \frac{5\theta}{84} (T^7 - r^7 t_1^7) \Big] + \frac{c}{T} \left(at_1 + \frac{bt_1^2}{2} \right) \\
& + \frac{dk}{T} \left[\frac{T^2}{2} + \frac{(1 - 3\theta)}{3} T^3 + \frac{(2 + \theta)}{8} T^4 + \frac{\theta}{5} T^5 - (\theta + 1) \left\{ \frac{T^3}{6} + \frac{(1 - 3\theta)}{8} T^4 \right. \right. \\
& + \left. \frac{(2 - \theta)}{8} T^5 + \frac{\theta}{12} T^6 \right\} + \left(1 - \frac{\theta}{2} \right) \left(\frac{T^4}{12} + \frac{(1 - 3\theta)}{15} T^5 + \frac{(2 + \theta)}{36} T^6 + \frac{\theta}{21} T^7 \right) \Big] \\
& - \frac{dk}{T} \left[rt_1 \left(T - \frac{rt_1}{2} \right) + \frac{(1 - 3\theta) rt_1}{2} \left(T^2 - \frac{r^2 t_1^2}{3} \right) + \frac{(2 + \theta) rt_1}{6} \left(T^3 - \frac{r^3 t_1^3}{4} \right) \right. \\
& + \frac{\theta rt_1}{4} \left(T^4 - \frac{r^4 t_1^4}{5} \right) - (\theta + 1) r^2 t_1^2 \left\{ \left(\frac{T}{2} - \frac{rt_1}{3} \right) + \frac{(1 - 3\theta)}{2} \left(\frac{T^2}{2} - \frac{r^2 t_1^2}{4} \right) \right. \\
& + \frac{(2 + \theta)}{6} \left(\frac{T^3}{2} - \frac{r^3 t_1^3}{5} \right) + \frac{\theta}{4} \left(\frac{T^4}{2} - \frac{r^4 t_1^4}{6} \right) \Big\} + \left(1 - \frac{\theta}{2} \right) r^3 t_1^3 \left\{ \left(\frac{T}{3} - \frac{rt_1}{4} \right) \right. \\
& + \left. \frac{(1 - 3\theta)}{2} \left(\frac{T^2}{3} - \frac{r^2 t_1^2}{5} \right) + \frac{(2 + \theta)}{6} \left(\frac{T^3}{3} - \frac{r^3 t_1^3}{6} \right) + \frac{\theta}{4} \left(\frac{T^4}{4} - \frac{r^4 t_1^4}{7} \right) \Big\} \Big] \\
& + \frac{1}{T} \left[(2x + 2bd_r mQ) \frac{nQf}{t_c} + vQ + b_i m d_a + (c_v mQ + c_f) \gamma \right] + \frac{1}{T} \left[\frac{2dn_2 g_t c_t}{t_c} + c_a \frac{Q}{m} g_m \right] \\
& + \frac{1}{T} [(c_6 + c_7) \alpha M(D_1 + D_2)] + \frac{(1 + \beta)}{T} \left[\frac{s_c + 2f}{\alpha M T(D_1 + D_2)} + c_4 + 2c_3 + (h_3 + h'_3) t_R \right] \\
& + \frac{(h_1 + h'_1)}{T} \left[\frac{(1 - \alpha)^2 M^2 T(D_1 + D_2)}{2} + \frac{\alpha T(M(D_1 + D_2))^2}{r} \right] + (h_2 + h'_2) \left[\frac{(\alpha M)^2 T(D_1 + D_2)}{2} \right]. \quad (4.14)
\end{aligned}$$

5. SOLUTION PROCEDURE

For finding optimal value of total relevant cost T_{rc} (which is a function of T and t_1), first-order partial derivatives of T_{rc} with respect to T and t_1 have to vanish, *i.e.*,

$$\frac{\partial T_{rc}}{\partial T} = 0, \quad (5.1)$$

$$\text{and } \frac{\partial T_{rc}}{\partial t_1} = 0. \quad (5.2)$$

By solving equations (5.1) and (5.2) for T and t_1 we get a set of values of T and t_1 . Let T^* and t_1^* be the values of T and t_1 for which T_{rc} has its optimal value. Now, for ensuring minimum value of T_{rc} , i.e., for T_{rc}^* the following conditions should be satisfied by T^* and t_1^* :

$$\left(\frac{\partial^2 T_{rc}}{\partial t_1^2}\right)\left(\frac{\partial^2 T_{rc}}{\partial T^2}\right) - \left(\frac{\partial^2 T_{rc}}{\partial t_1 \partial T}\right)^2 > 0, \quad (5.3)$$

$$\text{and } \left(\frac{\partial^2 T_{rc}}{\partial t_1^2}\right) > 0, \quad \left(\frac{\partial^2 T_{rc}}{\partial T^2}\right) > 0. \quad (5.4)$$

If conditions (5.3) and (5.4) are satisfied by the point (T^*, t_1^*) , then we evaluate the minimum total cost achieved for this inventory system per unit time, namely, T_{rc}^* . Based on the above optimality conditions, two results are obtained which are stated below:

Result 1. The system of equations (5.1) and (5.2) has a unique solution.

Result 2. The solution in Result 1 satisfies the second-order optimality conditions for a minimum.

Argumentation. There is a high complexity in the system of equations. Because of this a straightforward proof does not exist but a computer proof is always ready in the modern times of high computational power. This is also reflected in Appendix A. However, on the basis of the proof and literature review, the system of equations (5.1) and (5.2) has a unique solution. Besides this, the system of solution also satisfies the system of equations (5.3) and (5.4), the second-order conditions for the minimizer.

The obtained value from Result 1, which also satisfies Result 2, is a local minimum, and up to a restriction to a local subspace of practically relevant parts of the domain where T_{rc}^* is ensured to be convex, a global solution. A global minimizer is found by a comparison between the local minimizers, taking into consideration growth and asymptotic behaviors identified. The flowchart of the solution procedure is shown in Figure 2. The convexity of T_{rc}^* is reflected in Figure 3.

Note. If one calculates the Hessian matrix for the optimal solution, it will also give the convexity of the system. But, the calculation of Hessian matrix can entangled the paper, so, it is avoided in the paper. If

$$H = \begin{pmatrix} \frac{\partial^2 T_{rc}}{\partial T^2} & \frac{\partial^2 T_{rc}}{\partial T \partial t_1} \\ \frac{\partial^2 T_{rc}}{\partial t_1 \partial T} & \frac{\partial^2 T_{rc}}{\partial t_1^2} \end{pmatrix}. \quad (5.5)$$

Here, the value of $H < 0$, hence, the total cost function T_{rc} is strictly convex function, which is matching with our assumptions.

The following stepwise procedure is described for algorithmically solving model:

Algorithm

Step 1. Start.

Step 2. Input the values of all parameters.

Step 3. If these values of parameters satisfy the proposed assumptions, then go to Step 4; otherwise go to Step 2.

Step 4. Find T^* and t_1^* using equations (5.1) and (5.2).

Step 5. If T^* and t_1^* satisfy equations (5.3) and (5.4), then go to Step 6; otherwise go to Step 4.

Step 6. Find the value of T_{rc}^* using T^* and t_1^* from equation (4.14).

Step 7. The optimal solution is (T^*, t_1^*, T_{rc}^*) , where T^* and t_1^* are the values of T and t , respectively, for which T_{rc} is at its minimum.

Step 8. Stop.

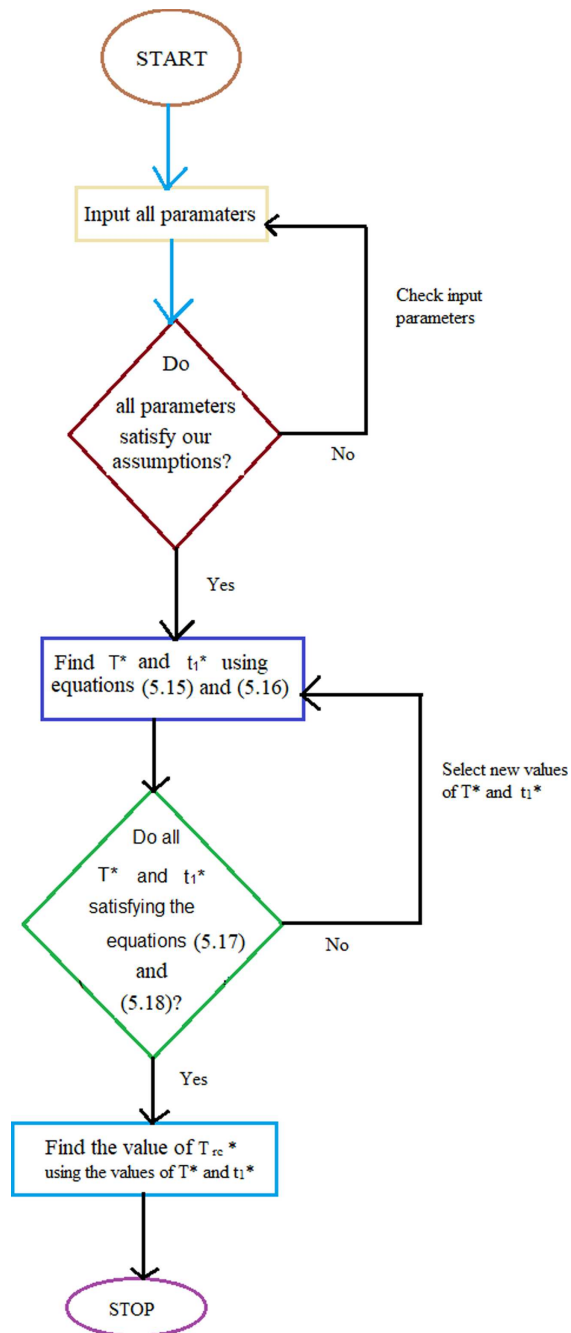


FIGURE 2. Flowchart of the solution procedure of the proposed model.

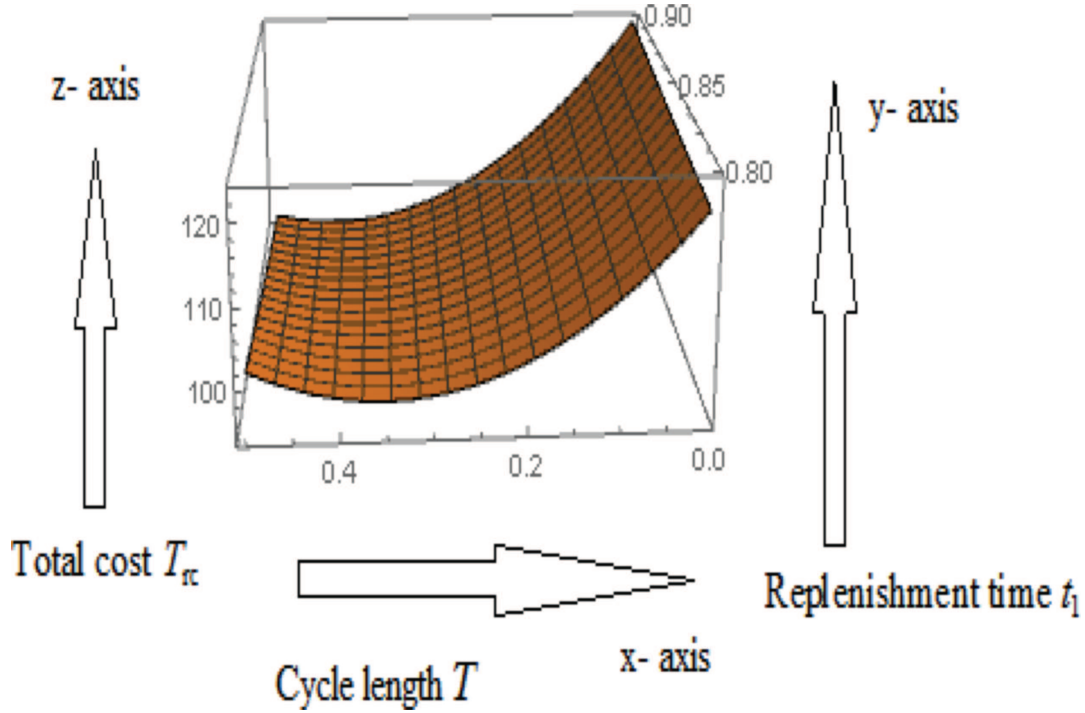


FIGURE 3. Graphical representation to show the convexity of total cost. The figure represents T , t_1 and the total cost T_{rc} , along the x -axis, the y -axis and the z -axis, respectively.

6. NUMERICAL EXAMPLES

Here, two numerical examples are assumed to justify the model under a realistic view. In both examples, an inventory system are considered with the following parameters in their proper unit.

Example 1. Let us assume: $A = 50$, $a = 7.5$, $b = 5$, $c = 5$, $h = 2.5$, $\alpha = 1.5$, $\beta = 0.09$, $\theta = 0.05$, $\eta = 2.5$, $k = 20$, $r = 1.15$, $d = 5$, $x = \$0.03/\text{trip}$, $g_t = 0.6$, $c_t = \$3/\text{ton CO}_2$, $g_m = 0.2$, $c_a = 0.4$, $\gamma = 5$, $n_2 = 30$, $c_v = 0.7$, $c_f = 0.9$, $v = 30 \text{ km/hr}$, $t_c = 30$, $m = 2 \text{ kg/unit}$, $d = 100 \text{ km}$, $b = \$0.4/\text{trip}$, $n = 4$, $f = 5$, $h_1 = 40$, $h'_1 = 50$, $h_2 = 60$, $h'_2 = 70$, $h_3 = 80$, $h'_3 = 90$, $c_6 = 0.8$, $c_7 = 0.7$.

Then, by solving equations (5.1) and (5.2), the values of T and t_1 are derived as $T^* = 0.94888$ and $t_1^* = 0.24949$, which satisfy the conditions of equations (5.3) and (5.4). Now, utilizing the values of T^* and t_1^* in equation (4.14), the total relevant cost of the inventory system are calculated with the help of Mathematica as $T_{rc}^* = 99.3213$.

Example 2. Let us choose: $A = 40$, $a = 7.5$, $b = 5$, $c = 5$, $h = 2$, $\alpha = 1.25$, $\theta = 0.05$, $\eta = 2.25$, $k = 20$, $r = 1.25$, and $d = 2$, $x = \$0.03/\text{trip}$, $g_t = 0.6$, $c_t = \$3/\text{ton CO}_2$, $g_m = 0.2$, $c_a = 0.4$, $\gamma = 5$, $n_2 = 30$, $c_v = 0.7$, $c_f = 0.9$, $v = 30 \text{ km/hr}$, $t_c = 30$, $m = 2 \text{ kg/unit}$, $d = 100 \text{ km}$, $b = \$0.4/\text{trip}$, $n = 4$, $f = 5$, $h_1 = 40$, $h'_1 = 50$, $h_2 = 60$, $h'_2 = 70$, $h_3 = 80$, $h'_3 = 90$, $c_6 = 0.8$, $c_7 = 0.7$.

Then, by solving equations (5.1) and (5.2), we obtain the values of T and t_1 as $T^* = 0.954016$ and $t_1^* = 0.233563$, which fulfil the conditions of equations (5.3) and (5.4). Now, utilizing the values of T^* and t_1^* in equation (4.14), the total relevant cost of the inventory system are calculated by employing Mathematica as $T_{rc}^* = 80.396$.

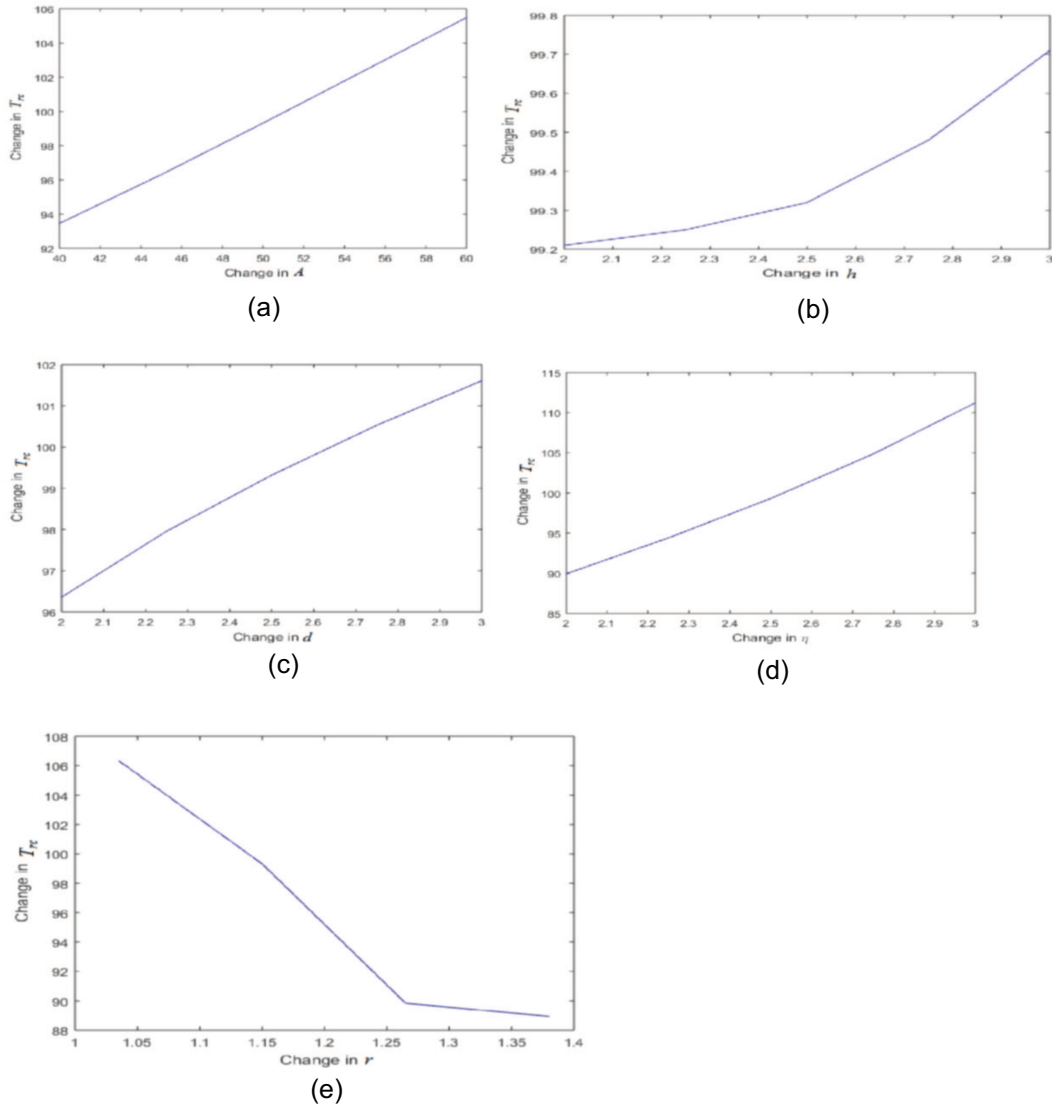


FIGURE 4. Variation of total cost with respect to different parameters. (a) Variation of total cost with respect to order cost. (b) Variation of total cost with respect to holding cost. (c) Variation of total cost with respect to deterioration cost. (d) Variation of total cost with respect to η . (e) Variation of total cost with respect to r .

7. SENSITIVITY ANALYSIS

In this section, the response of the total relevant cost are investigated to change values of major parameters A , a , b , c , d , h , α , θ , η , and r , namely, by changing of +20%, +10%, -10%, and -20% for each parameter except r . Value of r cannot decrease by more than 10%, because we assume that $r \geq 1$. Here, Example 1 is focused only in order to not exceed the length of the paper. The results are displayed in Table A.1. Based on the obtained results, Figure 3 is drawn. Here, responses of the total cost with respect to parametric variations is shown in Figure 4.

The following observations are made on the basis of Table A.1.

- (a) Total relevant cost T_{rc} grows with increasing value of A . The value of T_{rc} is diminished with decreasing value of A . Thus, for minimum T_{rc} , a small number of order is preferable.
- (b) The change of total relevant cost T_{rc} is approximately half in relative terms when compared to the change of a and c .
- (c) The change of T_{rc} is negligible with respect to the values of η and k , *i.e.*, the value of T_{rc} is not affected by a small change of η and k .
- (d) For increasing the value of deterioration cost d , the total relevant cost T_{rc} and the optimal replenishment cycle time T also grow. In this situation, the company will reduce the replenishment cycle time to diminish the total cost by lowering the deterioration of goods.
- (e) When the holding cost h and parameter α increase, the optimal replenishment cycle time T , the value of t_1 and the total relevant cost T_{rc} also increases. This implies that the company will reduce the cycle time to hold less inventories for a lowering of the corresponding cost.
- (f) When the length of the non-deteriorating period t_d increases and other parameters remain unchanged, this will reduce the total relevant cost T_{rc} and the optimal replenishment cycle time T ; therefore, the profit of the company increases. In this time period, the company will be able to store more items for possible future usages. Moreover, if the length of no deterioration time of the product can be extended for a few days or months, the total cost will be reduced effectively.
- (g) When the value of r increases, the value of optimal replenishment cycle time T and the total relevant cost T_{rc} decrease. This implies that the value of r is significant in the model.
- (h) When the value of θ increases and other parameters remain unchanged, both the total relevant cost T_{rc} and the optimal replenishment cycle time T grow. This implies that if the company can effectively reduce the deterioration of the item by improving the equipment of storehouse; then the total inventory cost be lowered.
- (i) As the distance d increases, the total cost increases. The reason behind this is to cover the increasing distance, transportation cost increases. And due to transportation, carbon emission also increases. So, the retailer needs to raise investment amount on green technology g to reduce emissions.
- (j) Total profit declines with the increase of product weight m . Because the truck cannot carry overload when the weight is increased. So, to carry all the products, a new trip n has to be added which raises transportation and carbon emission costs.
- (k) The intensification of GHG emissions causes some additional carbon emission cost. So, to manage these emissions, the manager of the firm should invest in the carbon reduction technology. Moreover, the manager must remember the fact that the investment for carbon reduction technology should be up to a certain level. Otherwise, the system may face some losses.
- (l) Product deterioration is an important issue in inventory management. Reducing the deterioration of products simultaneously with carbon emissions is an onerous task for a manager of an industry. This study provides important suggestions for industry managers to ensure a better profit and provide guidance for an appropriate investment in preservation technology.

8. DIFFERENT FORMS OF COMPOSITE DEMAND FUNCTION

It is possible to construct many composite demand functions with using two functions in a proper way. Subsequently, there are some examples of composite demand functions.

- (i) **Linear function with exponential:** it may be assumed as $D_1(t) = a + bt$, $t \geq 0$, $a \geq 0$, and $b \geq 0$ (not all 0 simultaneously), and $D_2(t) = ke^{-\theta t}$, $t \geq 0$, $0 < \theta \leq 1$ and $k \neq 0$, piecewise in their respective intervals (as used in this paper). The reverse combination may be chosen, too.
- (ii) **Quadratic function with exponential:** this may be considered as $D_1(t) = at^2 + bt + c$, $t \geq 0$, $a \neq 0$, b and c are real numbers, and $D_2(t) = ke^{\theta t}$, $t \geq 0$, $0 < \theta \leq 1$, piecewise in their respective intervals. The reverse combination also be taken.

- (iii) **Linear function with logarithmic:** it can be thought as $D_1(t) = a + bt$, $t \geq 0$, $a \geq 0$ (not all 0 simultaneously) and $b \geq 0$, and $D_2(t) = \ln(1 + kt)$, $t \geq 0$, and $k > 0$, piecewise in their respective intervals. Another reverse combination may be supposed, too.
- (iv) **Trigonometric function with linear:** this is taken here as $D_1(t) = \sin t$ where $0 \leq t \leq \frac{\pi}{4}$, and $D_2(t) = a - bt$, $t \geq 0$, $a \geq 0$ and $b \geq 0$, piecewise in their respective intervals (not all 0 simultaneously). The reverse combination can also be assumed.
- (v) **Trigonometric function with logarithmic function:** may be chosen as $D_1(t) = \tan t$ where $0 \leq t \leq \frac{\pi}{4}$, and $D_2(t) = \ln(1 - kt)$, $t \geq 0$, and $k > 0$, piecewise in their respective intervals. The reverse combination may be followed, too.

What is more, one can piece together a quadratic or cubic function with any other function, like a logarithmic, trigonometric or exponential one, etc., at the places of $D_1(t)$ and $D_2(t)$, respectively.

9. CONCLUSIONS AND POSSIBLE FUTURE DIRECTIONS

This study provides a sustainable inventory system, including CO₂ emissions. After transportation of the products, the retailer may need a huge amount of ordering cost to finish the purchase appropriately and to reduce the carbon emissions. To minimize such expenses, the retailer may orders more products with the same amount, which may as a consequence lead more carbon emission. A proper balance is required for ensuring maximum supply chain profit and sustainable development. Here, a non-instantaneous deteriorating inventory model based on real-life situations is considered. A new type of demand function, known as composite demand function, is introduced, too. Two theorems have been established to characterize the optimal solution and an algorithm is also defined to justify the validity of the model. Finally, numerical examples have been provided to illustrate the solution procedure and the algorithm.

The results from sensitivity analysis found out that the total profit is better for the non-instantaneously deteriorating items in comparison with instantaneously deteriorating items. Consideration of composite demand function showed that the value of the optimal cycle time and the optimal total cost are better for any other type of demand function. The results also revealed that the variable holding cost is more significant; it increases the optimal order quantity and decreases the total cost of the system. Furthermore, it was seen that if the ordering cost is higher, then the replenishment cycle time will be longer and the optimal order quantity will be higher.

This paper may be useful in various industries where such types of commodities are produced or businesses of such type of items are done. This paper may be useful, in general, for retail businesses such as perishable products, domestic goods, electronics components, vegetables, etc., where deterioration starting time is known to the retailer or company. This model has three special impacts as it utilizes sustainability: (i) the model can protect nature and conserve natural resources wherever possible, (ii) the model acts to improve their local communities, and (iii) the model cuts business costs by reducing energy consumption and waste. This three are the most important features of the model. There is seasonal demand for certain deteriorating products in practices. The goal of the model is to control the deterioration, reduce carbon emission rate for a non-instantaneous deteriorating item to maximize retailer profit. This study suggests that retailers must use an inventory system to control the deterioration and carbon emissions rates for maintaining sustainability.

There are some limitations that need further research in several directions. This paper assumes that carbon can be emitted, but to reduce the emission, the concept of carbon footprint and greenhouse effect is not included. However, it shall be included in future. Besides this, the paper may be extended by considering permissible delay in payment or non-zero lead time. One can add the effect of inflation to create a more realistic impact on the model. The effect of promotion efforts will be another interesting extension of this model. The intangible services, instead of (solid) goods or commodities, can be added for a future extension. Whenever after one price of the demand function follows another piece, such *switching times* could also be a stochastic extension in future research (cf. [47, 52, 56]). Finally, the various natures and formats of uncertainty of demand will eventually become a further refinement of the model.

APPENDIX A.

The necessary optimality conditions for finding the optimal values of T and t_1 are calculated by the following derivations:

$$\begin{aligned}
\frac{\partial T_{rc}}{\partial t_1} = & \frac{1}{T} \left[-h \left\{ a(-1+r^2)t_1 + \frac{b}{2}(-1+r^3)t_1^2 \right\} + \alpha \left\{ -a(-1+r^3)t_1^2 - \frac{b}{2}(-1+r^4)t_1^3 \right. \right. \\
& + \frac{1}{2}(-1+r^2)t_1^2\eta(a+bt_1) + (-1+r^2)t_1\eta\left(at_1 + \frac{bt_1^2}{2}\right) \left. \right\} + h(-1+r)t_1\eta(a+bt_1) \\
& + h\eta(-1+r)\left(at_1 + \frac{bt_1^2}{2}\right) + h\left(at_1 + \frac{bt_1^2}{2}\right)(-1+\eta) + \left(at_1^2 + \frac{bt_1^3}{3}\right)\alpha(-1+\eta) \left. \right] \\
& + \frac{c\eta}{T}(a+bt_1) + \frac{hk}{T} \left[-\frac{(1-7\theta)}{6}r^4t_1^3 - \frac{5\theta}{12}r^6t_1^5 - \frac{(2+\theta)}{6}r^5t_1^4 - r^3t_1^2 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} \right. \\
& - r \left\{ T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} + r^2t_1 \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \left. \right] \\
& + \frac{k\alpha}{T} \left[-\frac{(1-7\theta)}{6}r^5t_1^4 - \frac{5\theta}{12}r^7t_1^6 - r^6t_1^5 \frac{(2+\theta)}{6} - r^4t_1^3 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} \right. \\
& - r^2t_1 \left\{ T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} + r^3t_1^2 \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \left. \right] \\
& - \frac{dk}{T} \left[-\frac{r^2t_1}{2} + r \left(T - \frac{rt_1}{2} \right) - \frac{(1-3\theta)}{2}r^3t_1^2 + \frac{r}{2} \left(T^2 - \frac{r^2t_1^2}{2} \right) (1-3\theta) - \frac{\theta}{5}r^5t_1^4 + \frac{r}{4} \left(T^4 - \frac{r^4t_1^4}{5} \right) \theta \right. \\
& - r^4t_1^3 \frac{(2+\theta)}{8} + r \left(T^3 - \frac{r^3t_1^3}{4} \right) \frac{(2+\theta)}{6} + r^2t_1^2(1+\theta) \left\{ \frac{r}{3} + \frac{(1-3\theta)}{4}r^2t_1 + \frac{\theta}{6}r^4t_1^3 + \frac{(2+\theta)}{10}r^4t_1^3 \right\} \\
& + r^3t_1^3 \left(1 - \frac{\theta}{2} \right) \left\{ -\frac{r}{4} - r^2t_1 \frac{(1-3\theta)}{5} - r^4t_1^3 \frac{\theta}{7} - r^3t_1^2 \frac{(2+\theta)}{12} \right\} - 2r^2t_1(1+\theta) \left\{ \frac{T}{2} - \frac{rt_1}{3} \right. \\
& + \frac{1}{2} \left(\frac{T^2}{2} - \frac{r^2t_1^2}{4} \right) (1-3\theta) + \frac{1}{4} \left(\frac{T^4}{2} - \frac{r^4t_1^4}{6} \right) \theta + \frac{1}{6} \left(\frac{T^3}{2} - \frac{r^3t_1^3}{3} \right) (2+\theta) \left. \right\} + 3r^3t_1^2 \left(1 - \frac{\theta}{2} \right) \\
& \times \left\{ \frac{T}{3} - \frac{rt_1}{4} + \frac{1}{2} \left(\frac{T^2}{3} - \frac{r^2t_1^2}{5} \right) (1-3\theta) + \frac{\theta}{4} \left(\frac{T^4}{4} - \frac{r^4t_1^4}{7} \right) + \frac{(2+\theta)}{6} \left(\frac{T^3}{3} - \frac{r^3t_1^3}{6} \right) \right\} \left. \right], \tag{A.1}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 T_{rc}}{\partial t_1^2} = & \frac{1}{T} \left[-h(a(-1+r^2) + b(-1+r^3)t_1) + \alpha \left\{ -2a(-1+r^3)t_1 - \frac{3b}{2}(-1+r^4)t_1^2 \right. \right. \\
& + \frac{t_1^2 b \eta}{2}(-1+r^2) + 2t_1\eta(-1+r^2)(a+bt_1) + \eta(-1+r^2)\left(at_1 + \frac{bt_1^2}{2}\right) \left. \right\} \\
& + ht_1 b \eta(-1+r) + 2h\eta(-1+r)(a+bt_1) + h(a+bt_1)(-1+\eta) \\
& + \alpha \left(2at_1 + \frac{3bt_1^2}{2} \right) (-1+\eta) \left. \right] + \frac{bc\eta}{T} + \frac{hk}{T} \left[-r^4t_1^2 \frac{(1-7\theta)}{2} - \frac{25\theta}{12}r^6t_1^4 \right. \\
& - r^5t_1^3 \frac{(4+2\theta)}{3} - 2r^3t_1 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + r^2 \left\{ 1 + T + T^2 \frac{(1-3\theta)}{2} \right. \\
& + \frac{\theta T^4}{4} + \frac{T^3}{6}(2+\theta) \left. \right\} \left. \right] + \frac{k\alpha}{T} \left[-\frac{2}{3}r^5t_1^3(1-7\theta) - \frac{5}{2}r^7t_1^5\theta - \frac{5}{12}r^6t_1^4(4+2\theta) \right. \\
& - 3r^4t_1^2 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} - r^2 \left\{ T + \frac{T^2}{2}(1-3\theta) + \frac{\theta T^4}{4} + \frac{T^3}{6}(2+\theta) \right\} \\
& + 2r^3t_1 \left\{ 1 + T + \frac{T^2}{2}(1-3\theta) + \frac{\theta T^4}{4} + \frac{T^3}{6}(2+\theta) \right\} \left. \right] - \frac{dk}{T} \left[-r^2 - r^3t_1(1-3\theta) \right]
\end{aligned}$$

$$\begin{aligned}
& -r^5t_1^3\theta - \frac{1}{2}r^4t_1^2(2+\theta) - r^2t_1^2(1+\theta) \left\{ -\left\{ \frac{r^2}{4}(1-3\theta) - \frac{r^4t_1^2\theta}{2} - \frac{r^3t_1}{5}(2+\theta) \right\} \right. \\
& + r^3t_1^3 \left(1 - \frac{\theta}{2} \right) \left\{ -\frac{1}{2}r^2(1-3\theta) - \frac{3}{7}r^4t_1^2\theta - \frac{r^3t_1}{2}(2+\theta) \right\} - 4r^2t_1(1+\theta) \\
& \times \left\{ -\frac{r}{2} - r^2t_1(1-3\theta) - \frac{1}{6}r^4t_1^3\theta - \frac{1}{10}r^3t_1^2(2+\theta) \right\} + 6r^3t_1^2 \left(1 - \frac{\theta}{2} \right) \left\{ -\frac{r}{4} - \frac{1}{5}r^2t_1(1-3\theta) \right. \\
& - \frac{1}{7}r^4t_1^3\theta - \frac{1}{12}r^3t_1^2(2+\theta) \left. \right\} - 2r^2(1+\theta) \left\{ \frac{T}{2} - \frac{rt_1}{3} + \frac{1}{2} \left(\frac{T^2}{2} - \frac{r^2t_1^2}{4} \right) (1-3\theta) \right. \\
& + \frac{1}{4} \left(\frac{T^4}{2} - \frac{r^4t_1^4}{4} \right) \theta + \frac{1}{6} \left(\frac{T^3}{2} - \frac{r^3t_1^3}{5} \right) (2+\theta) \left. \right\} + 6r^3t_1 \left(1 - \frac{\theta}{2} \right) \left\{ \frac{T}{2} - \frac{rt_1}{4} \right. \\
& + \frac{1}{2} \left(\frac{T^2}{2} - \frac{r^2t_1^2}{5} \right) (1-3\theta) + \frac{1}{4} \left(\frac{T^4}{4} - \frac{r^4t_1^4}{7} \right) \theta + \frac{1}{6} \left(\frac{T^3}{3} - \frac{r^3t_1^3}{6} \right) (2+\theta) \left. \right\} \left. \right\}, \tag{A.2} \\
\frac{\partial T_{rc}}{\partial T} = & \frac{-A}{T^2} - \frac{1}{T^2} \left[-h \left\{ \frac{at_1^2}{2}(-1+r^2) + \frac{bt_1^3}{6}(-1+r^3) \right\} + \alpha \left\{ -\frac{at_1^3}{3}(-1+r^3) \right. \right. \\
& - \frac{bt_1^4}{8}(-1+r^4) + \frac{\eta t_1^2}{2}(-1+r^2) \left(at_1 + \frac{bt_1^2}{2} \right) \left. \right\} + h\eta t_1(-1+r) \left(at_1 + \frac{bt_1^2}{2} \right) \\
& + h \left(\frac{at_1^2}{2} + \frac{bt_1^3}{6} \right) (-1+\eta) + \alpha \left(\frac{at_1^3}{3} + \frac{bt_1^4}{8} \right) (-1+\eta) \left. \right] - \frac{c\eta}{T^2} \left(at_1 + \frac{bt_1^2}{2} \right) \\
& - \frac{dk}{T} \left[rt_1 + rTt_1(1-3\theta) + rT^3t_1\theta + \frac{rT^2t_1}{2}(2+\theta) + r^3t_1^3 \left(1 - \frac{\theta}{2} \right) \right. \\
& \times \left\{ \frac{1}{3} + \frac{(1-3\theta)}{3}T + \frac{\theta}{3}T^3 + \frac{(2+\theta)}{6}T^2 \right\} - r^2t_1^2(1+\theta) \left\{ \frac{1}{2} + T \frac{(1-3\theta)}{2} + \frac{\theta}{2}T^3 \right. \\
& + \left. \left. \frac{(2+\theta)}{4}T^2 \right\} \right] + \frac{hk}{T} \left[T + \frac{(1-7\theta)}{6}T^3 + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{5\theta}{12}T^5 \right. \\
& - \frac{1}{3} \left(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \right) (T^3 - r^3t_1^3)\theta + \frac{(2+\theta)}{6}T^3 + \frac{(2+\theta)}{6}T^4 + T^2 \left(\frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right) \\
& + (T - rt_1) \left(1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \right) + T^3 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} \\
& + \frac{1}{2}(T^2 - r^2t_1^2) \left\{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \right\} - \frac{1}{3}(T^3 - r^3t_1^3) \\
& \times \left\{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \right\} + T \left\{ T + \frac{1}{2}T^2(1-3\theta) + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \\
& - T^2 \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \left. \right] - \frac{hk}{T^2} \left[\frac{(1-7\theta)}{24}(T^4 - r^4t_1^4) + \frac{5\theta}{72}(T^6 - r^6t_1^6) \right. \\
& + \frac{(4+2\theta)}{60}(T^5 - r^5t_1^5) + \frac{1}{3}(T^3 - r^3t_1^3) \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + (T - rt_1) \left\{ T + \frac{(1-3\theta)}{2} \right. \\
& \times T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \left. \right\} - \frac{1}{2}(T^2 - r^2t_1^2) \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \left. \right] \\
& - \frac{k\alpha}{T^2} \left[\frac{(1-7\theta)}{30}(T^5 - r^5t_1^5) + \frac{5\theta}{84}(T^7 - r^7t_1^7) + \frac{(4+2\theta)}{72}(T^6 - r^6t_1^6) \right. \\
& + \frac{1}{4}(T^4 - r^4t_1^4) \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + \frac{1}{2}(T^2 - r^2t_1^2) \left\{ T + \frac{(1-3\theta)}{2}T^2 \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \Big\} - \frac{1}{3}(T^3 - r^3t_1^3) \Big\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \Big\} \Big] \\
& + \frac{dk}{T} \Big[T + T^2(1-3\theta) + T^4\theta + \frac{(2+\theta)}{2}T^3 - (1+\theta) \Big\{ \frac{T^2}{2} + \frac{(1-3\theta)}{2}T^3 \\
& + \frac{(2-\theta)}{4}T^4 + \frac{\theta}{2}T^5 \Big\} + \Big(1 - \frac{\theta}{2} \Big) \Big\{ \frac{1}{3}T^3 + \frac{(1-3\theta)}{3}T^4 + \frac{\theta}{3}T^6 + \frac{(2+\theta)}{6}T^5 \Big\} \Big] \\
& - \frac{dk}{T^2} \Big[\frac{T^2}{2} + \frac{(1-3\theta)}{3}T^3 + \frac{\theta}{5}T^5 + \frac{(2+\theta)}{8}T^4 - (1+\theta) \Big\{ \frac{T^3}{6} + \frac{(1-3\theta)}{8}T^4 \\
& + \frac{(2-\theta)}{20}T^5 + \frac{\theta}{12}T^6 \Big\} + \Big(1 - \frac{\theta}{2} \Big) \Big\{ \frac{T^4}{12} + \frac{(1-3\theta)}{15}T^5 + \frac{\theta}{12}T^7 + \frac{(2+\theta)}{36}T^6 \Big\} \Big] \\
& + \frac{dk}{T^2} \Big[rt_1 \Big(T - \frac{rt_1}{2} \Big) + \frac{(1-3\theta)}{2}rt_1 \Big(T^2 - \frac{r^2t_1^2}{3} \Big) + \frac{\theta}{4}rt_1 \Big(T^4 - \frac{r^4t_1^4}{5} \Big) \\
& + \frac{(2+\theta)}{6}rt_1 \Big(T^3 - \frac{r^3t_1^3}{4} \Big) - (1+\theta)r^2t_1^2 \Big\{ \frac{T}{2} - \frac{rt_1}{3} + \frac{(1-3\theta)}{2} \Big(\frac{T^2}{2} - \frac{r^2t_1^2}{4} \Big) \\
& + \frac{\theta}{4} \Big(\frac{T^4}{2} - \frac{r^4t_1^4}{6} \Big) + \frac{(2+\theta)}{6} \Big(\frac{T^3}{2} - \frac{r^3t_1^3}{5} \Big) \Big\} + r^3t_1^3 \Big(1 - \frac{\theta}{2} \Big) \Big\{ \frac{T}{3} - \frac{rt_1}{4} \Big\}, \tag{A.3} \\
\frac{\partial^2 T_{rc}}{\partial T^2} &= \frac{2A}{T^3} + \frac{2}{T^3} \Big[-h \Big\{ \frac{a}{2}t_1^2(-1+r^2) + \frac{b}{6t}t_1(-1+r^3) \Big\} + \alpha \Big\{ -\frac{a}{3}t_1^3(-1+r^3) \\
& - \frac{b}{8}t_1^4(-1+r^4) + \frac{\eta}{2}t_1^2(-1+r^2) \Big(at_1 + \frac{bt_1^2}{2} \Big) \Big\} + h\eta t_1(-1+r) \Big(at_1 + \frac{bt_1^2}{2} \Big) \\
& + h \Big(\frac{at_1^2}{2} + \frac{bt_1^3}{6} \Big) (-1+\eta) + \Big(\frac{at_1^3}{3} + \frac{bt_1^4}{8} \Big) \alpha (-1+\eta) \Big] + \frac{2c\eta}{T^3} \Big(at_1 + \frac{bt_1^2}{2} \Big) \\
& - \frac{dk}{T} \Big[rt_1(1-3\theta) + 3rT^2t_1\theta + rTt_1(2+\theta) + r^3t_1^3 \Big(1 - \frac{\theta}{2} \Big) \Big\{ \frac{(1-3\theta)}{3} \\
& + T^2\theta + \frac{(2+\theta)}{3}T \Big\} - r^2t_1^2(1+\theta) \Big\{ \frac{(1-3\theta)}{2} + \frac{3\theta}{2}T^2 + \frac{(2+\theta)}{2}T \Big\} \Big] \\
& + \frac{2dk}{T^2} \Big[rt_1 + rTt_1(1-3\theta) + rT^3t_1\theta + \frac{(2+\theta)}{2}rT^2t_1 + r^3t_1^3 \Big(1 - \frac{\theta}{2} \Big) \Big\{ \frac{1}{3} + \frac{(1-3\theta)}{3}T \\
& + \frac{\theta}{3}T^3 + \frac{(2+\theta)}{6}T^2 \Big\} - r^2t_1^2(1+\theta) \Big\{ \frac{1}{2} + \frac{(1-3\theta)}{2}T + \frac{T^3\theta}{2} + \frac{(2+\theta)}{4}T^2 \Big\} \Big] \\
& + \frac{hk}{T} \Big[1 - T + \frac{(1-7\theta)}{2}T^2 + 2T(1-3\theta) - \frac{(1-3\theta)}{2}T^2 + 2T^3\theta + \frac{11\theta}{6}T^4 \\
& - 2T^2 \Big(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \Big) \theta - \frac{\theta}{3} \Big(\frac{1}{2} + T \Big) (T^3 - r^3t_1^3) + T^2(2+\theta) - \frac{(2+\theta)}{6}T^3 + \frac{(4+2\theta)}{3}T^3 \\
& + 2T \Big\{ \frac{1}{2} + \Big(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \Big) \theta \Big\} + (T - rt_1) \{ 1 - 3\theta + 3T^2\theta + T(2+\theta) \} - \frac{1}{2}(T^2 - r^2t_1^2) \\
& \times \{ 1 - 3\theta + 3T^2\theta + T(2+\theta) \} - 2T \{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \} \Big] \\
& + \frac{k\alpha}{T} \Big[T + \frac{2(1-7\theta)}{3}T^3 + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{5\theta}{5}T^5 - 2T^3 \Big(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \Big) \theta \\
& - \frac{1}{4} \Big(\frac{1}{2} + T \Big) (T^4 - r^4t_1^4) \theta + \frac{(2+\theta)}{6}T^3 + \frac{5(4+2\theta)}{12}T^4 + 3T^2 \Big\{ \frac{1}{2} + \Big(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \Big) \theta \Big\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2}(T^2 - r^2 t_1^2) \{1 - 3\theta + 3T^2\theta + T(2 + \theta)\} - \frac{1}{3}(T^3 - r^3 t_1^3) \{1 - 3\theta + 3T^2\theta + T(2 + \theta)\} \\
& + 2T \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{1}{2}T^2(2 + \theta) \right\} - 2T^2 \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{1}{2}T^2(2 + \theta) \right\} \\
& - 2T \left\{ 1 + T \left(\frac{(1 - 3\theta)}{2}T^2 - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + (T - rt_1) \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{(2 + \theta)}{2}T^2 \right\} \\
& - \frac{1}{2}(T^2 - r^2 t_1^2) \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{(2 + \theta)}{2}T^2 \right\} - T \left\{ 1 + T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 \right. \\
& \left. + \frac{(2 + \theta)}{6}T^3 \right\} - \frac{2k\alpha}{T^2} \left[\frac{(1 - 7\theta)}{6}T^4 + \frac{5\theta}{12}T^6 - \frac{1}{4} \left(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \right) (T^4 - r^4 t_1^4) \theta + \frac{(4 + 2\theta)}{12}T^5 \right. \\
& \left. + T^3 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + \frac{1}{2}(T^2 - r^2 t_1^2) \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{(2 + \theta)}{2}T^2 \right\} \right. \\
& \left. - \frac{1}{3}(T^3 - r^3 t_1^3) \left\{ 1 + T(1 - 3\theta) + T^3\theta + \frac{(2 + \theta)}{2}T^2 \right\} + T \left\{ T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 \right. \right. \\
& \left. \left. + \frac{(2 + \theta)}{6}T^3 \right\} - T^2 \left\{ 1 + T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2 + \theta)}{6}T^3 \right\} \right] \\
& + \frac{2hk}{T^3} \left[\frac{(1 - 7\theta)}{24}(T^4 - r^4 t_1^4) + \frac{5\theta}{72}(T^6 - r^6 t_1^6) + \frac{(4 + 2\theta)}{60}(T^5 - r^5 t_1^5) + \frac{1}{3}(T^3 - r^3 t_1^3) \right. \\
& \times \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} + (T - rt_1) \left\{ T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2 + \theta)}{6}T^3 \right\} \\
& \left. - \frac{1}{2}(T^2 - r^2 t_1^2) \left\{ 1 + T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2 + \theta)}{6}T^3 \right\} \right] + \frac{2k\alpha}{T^3} \left[\frac{(1 - 7\theta)}{30}(T^5 - r^5 t_1^5) \right. \\
& + \frac{5\theta}{84}(T^7 - r^7 t_1^7) + \frac{(4 + 2\theta)}{72}(T^6 - r^6 t_1^6) + \frac{1}{4}(T^4 - r^4 t_1^4) \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} \\
& + \frac{1}{2}(T^2 - r^2 t_1^2) \left\{ T + \frac{(1 - 3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2 + \theta)}{6}T^3 \right\} - \frac{1}{3}(T^3 - r^3 t_1^3) \left\{ 1 + T + \frac{(1 - 3\theta)}{2}T^2 \right. \\
& \left. + \frac{\theta}{4}T^4 + \frac{(2 + \theta)}{6}T^3 \right\} \left. \right] + \frac{dk}{T} \left[1 + 2T(1 - 3\theta) + 4T^3\theta + \frac{3(2 + \theta)}{2}T^2 \right. \\
& - (1 + \theta) \left\{ T + \frac{3(1 - 3\theta)}{2}T^2 + T^3(2 - \theta) + \frac{5\theta}{2}T^4 \right\} + \left(1 - \frac{\theta}{2} \right) \left\{ T^2 + \frac{4(1 - 3\theta)}{3}T^3 + 2\theta T^5 \right. \\
& \left. + \frac{5(2 + \theta)}{6}T^4 \right\} \left. \right] - \frac{2dk}{T^2} \left[T + T^2(1 - 3\theta) + T^4\theta + \frac{(2 + \theta)}{2}T^3 - (1 + \theta) \left\{ \frac{T^2}{2} + \frac{(1 - 3\theta)}{2}T^3 \right. \right. \\
& \left. \left. + \frac{(2 - \theta)}{4}T^4 + \frac{\theta}{2}T^5 \right\} + \left(1 - \frac{\theta}{2} \right) \left\{ \frac{T^3}{3} + \frac{(1 - 3\theta)}{3}T^4 + \frac{\theta}{3}T^6 + \frac{(2 + \theta)}{6}T^5 \right\} \right] \\
& + \frac{2dk}{T^3} \left[\frac{T^2}{2} + \frac{(1 - 3\theta)}{2} \left(\frac{T^2}{3} - \frac{r^2 t_1^2}{5} \right) + \frac{\theta}{4} \left(\frac{T^4}{3} - \frac{r^4 t_1^4}{7} \right) + \frac{(2 + \theta)}{6} \left(\frac{T^3}{3} - \frac{r^3 t_1^3}{6} \right) \right], \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 T_{rc}}{\partial t_1 \partial T} &= -\frac{1}{T^2} \left[-h \left\{ at_1(-1 + r^2) + \frac{bt_1^2}{2}(-1 + r^3) \right\} + \alpha \left\{ -at_1^2(-1 + r^3) - \frac{bt_1^3}{2}(-1 + r^4) \right. \right. \\
& + \frac{\eta t_1^2}{2}(-1 + r^2)(a + bt_1) + (-1 + r^2)t_1\eta \left(at_1 + \frac{bt_1^2}{2} \right) \left. \right\} + \eta ht_1(-1 + r)(a + bt_1) \\
& + \eta h(-1 + r) \left(at_1 + \frac{bt_1^2}{2} \right) + h \left(at_1 + \frac{bt_1^2}{2} \right) (-1 + \eta) + \left(at_1^2 + \frac{bt_1^3}{2} \right) \alpha (-1 + \eta) \left. \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{c\eta}{T^2}(a+bt_1) - \frac{dk}{T} \left[r + rT(1-3\theta) + rT^3\theta + \frac{(2+\theta)}{2}rT^2 + 3r^3t_1^2 \left(1 - \frac{\theta}{2} \right) \left\{ \frac{1}{3} + \frac{(2+\theta)}{4}T^2 \right\} \right] \\
& + \frac{hk}{T} \left[r^3 \left(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \right) t_1^2\theta - r \left\{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \right\} \right. \\
& + r^2t_1 \left\{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 \right\} \left. \right] + \frac{k\alpha}{T} \left[r^4 \left(\frac{1}{2} + \frac{T}{2} + \frac{T^2}{2} \right) t_1^3\theta \right. \\
& - r^2t_1 \left\{ 1 + T(1-3\theta) + T^3\theta + \frac{(2+\theta)}{2}T^2 + r^3t_1^2(1+T(1-3\theta)+T^3\theta \right. \\
& + \left. \left. \frac{(2+\theta)}{2}T^2 \right) \right\} \left. \right] - \frac{hk}{T^2} \left[-\frac{(1-7\theta)}{6}r^4t_1^3 - \frac{5\theta}{12}r^6t_1^5 - \frac{(4+2\theta)}{12}r^5t_1^4 \right. \\
& - r^3t_1^2 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} - r \left\{ T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \\
& + r^2t_1 \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \left. \right] - \frac{k\alpha}{T^2} \left[-\frac{(1-7\theta)}{6}r^5t_1^4 - \frac{5\theta}{12}r^7t_1^6 \right. \\
& - \frac{(4+2\theta)}{12}r^6t_1^5 - r^4t_1^3 \left\{ \frac{1}{2} + \left(\frac{3}{2} - \frac{T}{2} - \frac{T^2}{4} - \frac{T^3}{6} \right) \theta \right\} - r^2t_1 \left\{ T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 \right. \\
& + \left. \left. \frac{(2+\theta)}{6}T^3 \right\} + r^3t_1^2 \left\{ 1 + T + \frac{(1-3\theta)}{2}T^2 + \frac{\theta}{4}T^4 + \frac{(2+\theta)}{6}T^3 \right\} \right] \\
& + \frac{dk}{T^2} \left[-\frac{r^2t_1}{2} + r \left(T - \frac{rt_1}{2} \right) - \frac{(1-3\theta)}{3}r^3t_1^2 + \frac{(1-3\theta)}{2}r \left(T^2 - \frac{r^2t_1^2}{3} \right) - \frac{\theta}{5}r^5t_1^4 + \frac{\theta}{4}r \left(T^4 - \frac{r^4t_1^4}{5} \right) \right. \\
& - \frac{(2+\theta)}{8}r^4t_1^3 + \frac{(2+\theta)}{6}r \left(T^3 - \frac{r^3t_1^3}{4} \right) - r^2t_1^2(1+\theta) \left\{ -\frac{r}{3} - \frac{(1-3\theta)}{4}r^2t_1 - \frac{\theta}{6}r^4t_1^3 \right. \\
& - \left. \left. \frac{(2+\theta)}{10}r^3t_1^2 \right\} + \left(1 - \frac{\theta}{2} \right) r^3t_1^3 \left\{ -\frac{r}{4} - \frac{(1-3\theta)}{5}r^2t_1 - \frac{\theta}{7}r^4t_1^3 - \frac{(2+\theta)}{12}r^3t_1^2 \right\} - 2(1+\theta)r^2t_1 \right. \\
& \times \left\{ \frac{T}{2} - \frac{rt_1}{3} + \frac{(1-3\theta)}{2} \left(\frac{T^2}{2} - \frac{r^2t_1^2}{4} \right) + \frac{1}{4} \left(\frac{T^4}{2} - \frac{r^4t_1^4}{6} \right) \theta + \frac{(2+\theta)}{6} \left(\frac{T^3}{2} - \frac{r^3t_1^3}{5} \right) \right\} + 3r^3t_1^2 \left(1 - \frac{\theta}{2} \right) \\
& \times \left\{ \frac{T}{3} - \frac{rt_1}{4} + \frac{(1-3\theta)}{2} \left(\frac{T^2}{3} - \frac{r^2t_1^2}{5} \right) + \frac{\theta}{4} \left(\frac{T^4}{3} - \frac{r^4t_1^4}{7} \right) + \frac{(2+\theta)}{6} \left(\frac{T^3}{3} - \frac{r^3t_1^3}{6} \right) \right\} \left. \right]. \quad (A.5)
\end{aligned}$$

TABLE A.1. Sensitivity analysis for various parameters involved in Example 1.

Parameter	Change (%)	Value	T	t_1	T_{rc}
A	+20	60	1.07912	0.339624	105.493
	+10	55	1.07659	0.296513	102.401
	-10	45	0.874794	0.197731	96.3063
	-20	40	0.792521	0.139994	93.4618
c	+20	6	0.80701	0.092367	111.223
	+10	5.5	0.880204	0.172222	104.852
	-10	4.5	1.01281	0.323842	94.4055
	-20	4	1.07176	0.395077	89.9469
	+20	3	0.972239	0.286605	99.7167

TABLE A.1. continued.

Parameter	Change (%)	Value	T	t_1	T_{rc}
h	+10	2.75	0.961053	0.269034	99.4813
	−10	2.25	0.935509	0.227591	99.2535
	−20	2	0.920653	0.202817	99.2995
α	+20	1.8	0.943933	0.250072	99.8935
	+10	1.65	0.946421	0.249803	99.6075
	−10	1.35	0.951326	0.249152	99.0349
	−20	1.2	0.953745	0.248772	98.7481
a	+20	9	0.810815	0.0963413	110.924
	+10	8.25	0.88324	0.174518	104.671
	−10	6.75	1.00874	0.319458	94.5186
	−20	6	1.06354	0.386035	90.0436
b	+20	6	0.92614	0.225487	100.651
	+10	5.5	0.936928	0.236892	100.087
	−10	4.5	0.962206	0.263494	98.4964
	−20	4	0.977128	0.279122	97.6063
d	+20	3	0.924575	0.262273	101.608
	+10	2.75	0.935648	0.255595	100.526
	−10	2.25	0.965275	0.244532	97.9558
	−20	2	0.986741	0.24186	96.3631
θ	+20	0.06	0.952676	0.249124	98.9993
	+10	0.055	0.950773	0.249308	99.1606
	−10	0.045	0.947011	0.249689	99.4813
	−20	0.04	0.945151	0.249886	99.6407
η	+20	3	0.797897	0.0828507	111.245
	+10	2.75	0.870852	0.162425	104.860
	−10	2.25	1.0345	0.346556	94.4054
	−20	2	1.1312	0.457251	89.9304
k	+20	24	0.955662	0.312301	107.216
	+10	22	0.952808	0.283003	100.316
	−10	18	0.943562	0.210754	98.2156
	−20	16	0.936332	0.165376	96.9806
r	+20	1.38	1.09842	0.349832	88.9505
	+10	1.265	1.02705	0.307355	93.6812
	−10	1.035	0.861307	0.168827	106.346

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