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MODELLING LOW-CARBON CLOSED-LOOP SUPPLY CHAIN CONSIDERING CHANNEL POWER STRUCTURES AND CROSS-SHAREHOLDING

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Abstract. Under the background of low-carbon economy, cross-shareholding is introduced into the low-carbon closed-loop supply chain (CLSC) composed of a manufacturer and a retailer, a centralized and three decentralized decision-making models with different channel power structures are constructed. The recovery, emission reduction, and pricing decisions of low-carbon CLSC are studied, and a two-part-tariff contract is proposed to coordinate the low-carbon CLSC. The results indicate that under three channel power structures, the improvement of the cross-shareholding ratio and consumers' low-carbon awareness are beneficial to improve recovery rate, carbon emission reduction and increase market demand. Under Nash Game, the profit of low-carbon CLSC can reach the centralized level which means cross-shareholding can realize the coordination of low-carbon CLSC. Under Stackelberg Game, the enterprise with larger channel power can always obtain higher profit. Cross-shareholding cannot coordinate the retailer-led low-carbon CLSC, however it can realize the partial coordination of the manufacturer-led low-carbon CLSC. Two-part-tariff contract can realize the coordination of low-carbon CLSC under both situations.

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1. Introduction

Since entering the 21st century, global resource problems and climate change have become increasingly severe, realizing the recycling of resources, reducing greenhouse gas emissions, and promoting the sustainable development of low-carbon economy has become a common concern worldwide. The UN's Global E-Waste Monitor 2020 claims, a record 53.6 million metric tonnes (Mt) of e-waste were generated globally in 2019, up 21% in five years, and only 17.4% of e-waste was recycled. This means that gold, silver, copper, and other high-value, recyclable materials (conservatively valued at $57 billion) are mostly discarded or incinerated. The report also said that a total of 98 Mt of CO_2 equivalents to 0.3% of global greenhouse gas emissions, were emitted into the atmosphere in 2019 due to the generation of e-waste, and discarded refrigerators.

According to the report Global Energy Review: CO_2 Emissions in 2020 released by the International Energy Agency (IEA) in 2021 claims, global energy-related CO_2 emissions fell by 5.8% over the past year due to the COVID-19 epidemic, which is also the largest annual decline since World War II. Global carbon emissions increased 2% to 60 million tons in December 2020 compared to the same period in 2019, driven by economic

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recovery and a lack of clean energy policies, as the recovery in economic activity raises energy demand, with major global economies being the main drivers. Emissions in many economies have surpassed pre-covid-19 levels. Emissions are now climbing in many economies. According to the IEA, the trend changes in CO_2 emissions in 2020 shows that the world still faces the challenge of curbing CO_2 emissions while ensuring economic growth and energy security. The recycling of used products can help to alleviate resource scarcity and environmental pollution. In response to these phenomena, many companies have started to implement low-carbon CLSC management, such as Caterpillar, Kodak, and Xerox [23]. Implementing low-carbon CLSC management is not about responding to policies and regulations, but about the huge economic and environmental benefits [6]. Because parts of the components in used products are reusable, moreover, it has high economic value. The remanufacturing program can save 40%−65% of production costs [25].

In addition, energy-using products (such as manufacturing machinery and equipment, network equipment, electronics, and mobile devices, etc.) generate significant carbon emissions during the manufacturing and use phases, with the majority of them occurring during the manufacturing phase. For example, Apple Environmental Process Report shows that 71% of carbon emissions in 2020 come from the manufacturing phase and 19% from the use phase [17]. In conclusion, manufacturing companies will invest a lot of money in research and development of emission reduction technologies to achieve carbon emission reduction while implementing low-carbon CLSC management. At the same time, with the development and expansion of the supply chain, Amazon, Wal-Mart, and other retailers with more say gradually emerged. The traditional manufacturer-led channel power structure of the supply chain has evolved into a retailer-led form and a balanced channel power structure of the participants.

The high barrier of green purification technology often leads to the high cost of enterprise emission reduction. High abatement costs will certainly reduce the incentive of manufacturer to reduce emissions, and further affect the long-term development of the green consumption market. Therefore, how to reduce emission reduction costs, increase the emission reductions, enhance inter-enterprise robustness, and then improve the overall competitiveness of supply chain enterprises in green development has become a practical problem to be solved urgently. To alleviate the economic pressure caused by individual enterprise emission reduction, the way of joint emission reduction of downstream enterprises has been adopted by some supply chain enterprises. Denso and Toyota have long maintained close cooperation based on cross-shareholding, and actively promote the emission reduction of the upstream and downstream enterprises in the supply chain to achieve significant results.

In summary, previous studies lack a focus on low-carbon CLSC. This paper aims to solve the following problems:

(1) How to solve the dilemma of high investment and "free-rider" of the manufacturer engaged in low-carbon production?
(2) Do different channel power structures in a low-carbon CLSC affect its members' equity strategies?
(3) How to eliminate the "Double Marginalization" and achieve the perfect coordination of the system in a decentralized low-carbon CLSC under different channel power structures and equity cooperation?

Based on this, this paper introduces cross-shareholding into the low-carbon CLSC, expanding from Stackelberg Game to three channel power structures under two Game models. Furthermore, this paper studies the emission reduction, recovery, and pricing decisions of the low-carbon CLSC, and analyzes the influence of cross-shareholding ratio, the sensitivity coefficient of consumers to carbon emission reduction and the carbon emission reduction investment cost coefficient on enterprises' decision-making and profit. At last, this paper illustrates the coordination effect of cross-holding under different channel power structures, and proposes a two-part-tariff contract to achieve coordination of low-carbon CLSC under different channel power structures.

The rest of the paper is organized as follows: Section 2 summarizes the literature in related fields. Section 3 introduces the assumptions and symbols of the decision models. Section 4 solves the equilibrium solution under different channel power structures. In Section 5, comparative analysis and the sensitivity analysis of key factors are carried out. Section 6 proposes coordination contracts to coordinate the CLSC. Section 7 analyzes the reliability of the results by numerical simulation. Section 8 summarizes the whole paper.

2. Literature review

The paper related to three streams of literature: channel power structure, carbon emission reduction in CLSC and equity cooperation.

2.1. Channel power structure

The research on channel power structure has always been a classic problem in the field of supply chain research. And this discourse is reflected in the decision-making order of supply chain participants, where the first decision-maker can influence those who make decisions later [9]. Choi *et al.* [4] investigated the pricing decisions of a low-carbon CLSC under the Stackelberg Game and the Nash Game in a low-carbon CLSC composed of two competing manufacturer and one intermediary (a common retailer). Further, Choi *et al.* [5] established a decision model of low-carbon CLSC under competing sales channels and different channel power structures, and investigated its coordination mechanisms considering consumer channel preferences. Gao *et al.* [11] investigated the impact of channel power structures on optimal decision making and performance in low-carbon CLSC, and identified the economically optimal channel power structures by considering the market demand dependent on price, recycling effort, and sales effort. Zheng *et al.* [34] explored the effects of channel competition and channel power structures on pricing, recycling decisions, and supply chain performance in a low-carbon CLSC composed of a manufacturer, a retailer, and a collector in a dual-channel context. Sahebi *et al.* [24] investigated the selection of reverse channels in a bi-level low-carbon CLSC consisting of a manufacturer and a retailer, then used the reverse induction to obtain the optimal decisions of the members. Wang *et al.* [28] analysed the impact of the reward-penalty mechanism (RPM) of government on the recovery performance of the low-carbon CLSC led by manufacturer and retailer for the low-carbon CLSC composed of the manufacturer and recycler. Research shows that the recovery rate is higher under recycler leadership than centralized decision-making and manufacturer leadership. Yan *et al.* [31] explored the manufacturer's cooperation strategy of CLSC composed of a dominant manufacturer with CSR behaviour consciousness, a retailer with CSR investment, and a third-party recycler.

2.2. Carbon emission reduction in CLSC

Savaskan *et al.* [25] proposed three recycling channels in the CLSC. Liu *et al.* [20] studied the low-carbon CLSC with different mixed recycling models led by the manufacturer, and the result showed that the manufacturer and retailer dual collecting model is the best option. Jalali *et al.* [18] explored the pricing, production, and recycling decisions of two complementary products under different recycling channels. Hong *et al.* [16] investigated the impact of value-added service on low-carbon CLSC pricing strategies under different service models. Ovchinnikov *et al.* [22] explored the impact of remanufacturing on the pricing of new products and the realization of the enterprise's economic and environmental goals through empirical analysis.

In the research of supply chain carbon emission reduction, some scholars have paid attention to the impact of carbon tax policy on supply chain emission reduction decisions. Chai *et al.* [1] investigated the favorable conditions of carbon trading policy for the manufacturer and its impact on supply chain emission reduction by building a model of monopoly manufacturer involved in both manufacturing and remanufacturing, and the study concludes that carbon trading and remanufacturing will produce dual emission reduction effects. Further, Heydari *et al.* [14] analyzed the impact of the carbon tax policy on manufacturing or remanufacturing decisions of the manufacturer. Dou *et al.* [8] studied the channel selection of low-carbon CLSC under the carbon emission tax policy, considering the carbon emission difference between new products and remanufactured products. In terms of carbon cap and trade research, Xing *et al.* [30] examined the impact of carbon emission trading price and consumers' low-carbon preferences on the decision of low-carbon CLSC members. Yang *et al.* [32] examined the low-carbon CLSC of remanufacturing under the cap-and-trade mechanism and found that remanufacturing can effectively improve the carbon emission reduction level and profits of the manufacturer and the retailer. In other research perspectives, Chai *et al.* [2] considered the manufacturing model in which the independent remanufacturer (IR) with the original equipment manufacturer (OEM) and examined the choice of carbon emission policies on the competitive strategies of OEM. The study shows that the OEM's choice of a licensed manufacturing model not only coordinates supply chain profits but also improves the environment. Taleizadeh *et al.* [27] examined the impact of carbon reduction, quality improvement, and quality of returned goods on a low-carbon CLSC remanufacturing model. Li *et al.* [19] considered both one-way and two-way cooperation between the manufacturer and retailer in a low-carbon CLSC, and found that both cooperative methods can improve recycling and carbon reduction rates. Mohajeri *et al.* [21] considered the fuzzy CSND with carbon emission as a decision variable and found that demand and recovery rate should be accurately predicted in GSC.

2.3. Equity cooperation

The implementation of carbon emission reduction by manufacturing enterprises is conducive to improving the overall performance of supply chain, but the high cost of low-carbon production and "free riding" problems reduce their enthusiasm for emission reduction. Previous studies have mostly strengthened the relationship between enterprises through contracts, while equity cooperation among enterprises in the supply chain can also strengthen the relationship between upstream and downstream enterprises, achieve revenue sharing and cost sharing, promote supply chain coordination and form enterprise strategic alliances. Serbera *et al.* [26] examined how partial vertical ownership by an upstream monopoly over one of the downstream competitive retailers can help internalize production decisions after complete divestiture. Fu *et al.* [10] compared the optimal decisions of the push-down and pull-down supply chains under cross-shareholding strategies of upstream and downstream firms in the supply chain, and put forward revenue sharing contract to coordinate. Peng *et al.* [23] found that a cross-shareholding strategy can effectively alleviate a company's capital shortage. Chen *et al.* [3] studied the impact of cross-shareholding on push-pull supply chains. Xia *et al.* [29] proposed a revenue-sharing contract for the low-carbon supply chain with two channel power structures under Stackelberg Game, and realize supply chain coordination. Zhang *et al.* [33] explored the impact of manufacturer and retailer cross-ownership strategies on value co-creation in the electronic low-carbon CLSC. Hoffler *et al.* [15] and Greenlee *et al.* [12] considered the case where a manufacturer holds shares in multiple suppliers at the same time, and explored the impact of the shareholding strategy on supply chain output. Guth *et al.* [13] found that equity cooperation among supply chain enterprises can overcome the efficiency loss caused by information asymmetry, and thus improve the overall operational efficiency of the supply chain. Dietzenbacher *et al.* [7] respectively used the Cournot and Bertrand models to study the shareholding behavior among Dutch financial firms. The results of both models show that the shareholding strategy can reduce the adverse effects of competition.

In summary, the current studies on enterprise equity cooperation in a low-carbon environment are mostly focused on the supply chain. Xia *et al.* [29] proposed a revenue sharing contract for the low-carbon supply chain with two power structures under Stackelberg game, which realized supply chain coordination. The implementation of carbon emission reduction in CLSC with remanufacturing can improve resource and environmental benefits at the same time, but there are also large emission reduction costs and "free riding" problems. This paper introduces cross shareholding into the low-carbon CLSC, verifies the coordination effect of cross shareholding under different power structures and proposes a two-part-tariff contract.

3. Problem description and notation assumptions

This paper constructs a CLSC with one manufacturer and one retailer, and the information between them is completely symmetric. The manufacturer is responsible for the production of new and remanufactured products and the used product collection effort.

The research in this paper is based on the following assumptions.

Assumption 1. *Due to advanced production technology and process, remanufactured products and new products have no difference in function appearances, such as Kodak disposable camera. This paper assumes that there is no difference between new and remanufactured products.*

Assumption 2. *Based on previous researches, it is assumed that the return rate of used products is denoted by* τ, $0 < \tau < 1$ *[19, 27],* $C(\tau) = \delta\tau^2$, $\delta > 0$, $C(\tau)$*represents the total investment in the cycle activities of manufacture, where represents remanufactured products recovery investment cost coefficient.*

Assumption 3. *With the promulgation of policies related to low-carbon cycle by the government and the improvement of consumers' environmental protection requirements, enterprises have improved their awareness of independent emission reduction. The manufacturer invests in carbon reduction (purchasing equipment, technology research, and development, etc.) in the production of new and remanufactured products. It is assumed that the manufacturer's unit carbon emission reduction is denoted by* e, $e > 0$ *[19, 27],* $I(e) = \frac{ke^2}{2}$, $k > 0$, $I(e)$ *represents the total investment in the carbon emission reduction activities of manufacture, where* k *represents green product emission reduction investment cost coefficient.*

Assumption 4. *Based on previous studies, it has been widely assumed in the literature [19, 30], the market demand function is:*

$$D = a - bp + \theta e \tag{1}$$

where a *represents the potential market size,* $a > 0$, b *represents the sensitivity coefficient of consumers to the product price,* $b > 0$, θ *represents the sensitivity coefficient of consumers to carbon emission reduction,* $\theta > 0$, p *prepresents the price of product.*

Assumption 5. *The manufacturer invests in the carbon emission reduction, which can improve the overall performance of low-carbon CLSC. For example, SAIC, BMW, and Toyota, as core enterprises in the supply chain, have invested in carbon emission reduction technologies to improve consumer satisfaction and enhance their competitiveness. However, retailer's free-riding behavior and high input of carbon emission reduction will reduce manufacturer's enthusiasm of carbon emission reduction. A stock ownership cooperation strategy can effectively enhance the relationship between enterprises in the low-carbon CLSC and eliminate the negative effects of retailer's free-riding behavior. In practice, Daimler, the world's largest maker of commercial vehicles, and Beijing Automotive share profits through stock ownership partnerships. The manufacturer and retailer form stock ownership partnerships through cross-shareholdings. The manufacturer owns* α *percentage of the retailer's shares, and the retailer owns* β *percentage of the manufacturer's shares, and* $0 < \alpha, \beta < 1$.

4. Decision model construction and solution

4.1. Model C – centralized decision-making model

In this model, the manufacturer and retailer constitute a CLSC as a whole, and the profit maximization of the low-carbon CLSC is the decision-making goal. The profit function of the centralized low-carbon CLSC is as follows:

$$\underset{(\tau,e,p)}{\text{Max}}\, \Pi_{sc}^{C} = (p - c_n + \tau\Delta)D - A\tau D - \delta\tau^2 - \frac{ke^2}{2}\cdot \tag{2}$$

Theorem 1. *In the centralized decision-making model, when* $\delta > \delta_1$*, the equilibrium decision and profit of the low-carbon CLSC are as follows:*

$$\tau^{C*} = \frac{bk(a - bc_n)(A - \Delta)}{kb(b(A-\Delta)^2 - 4\delta) + 2\delta\theta^2}, \qquad e^{C*} = \frac{-2\delta\theta(a - bc_n)}{kb(b(A-\Delta)^2 - 4\delta) + 2\delta\theta^2},$$

$$p^{C*} = \frac{kb(a(A-\Delta)^2 - 2\delta c_n) - 2ka\delta + 2\delta c_n\theta^2}{kb(b(A-\Delta)^2 - 4\delta) + 2\delta\theta^2}, \qquad D^{C*} = -\frac{2\delta bk(a - bc_n)}{kb(b(A-\Delta)^2 - 4\delta) + 2\delta\theta^2},$$

$$\Pi_{sc}^{C*} = \frac{-\delta k(a - bc_n)^2}{kb(b(A-\Delta)^2 - 4\delta) + 2\delta\theta^2}$$

TABLE 1. Notation.

Variables	Definition
w	The wholesale price of unit product, $w > 0$
m	The profit margin of unit product, $m = p - w$
p	The retail price of unit product, $p > 0$
D	Demand for product, $D > 0$
a	The potential market size, $a > 0$
b	The coefficient of consumers' price sensitivity, $b > 0$
θ	The sensitivity coefficient of consumers' carbon emission reduction, $\theta > 0$
c_n	Unit production cost of the new product, $c_n > 0$
c_r	Unit production cost of the remanufactured product, $c_n > c_r > 0$
Δ	Remanufacturing cost savings, $\Delta = c_n - c_r > 0$
A	Manufacturer's unit recovery price, $A > 0$
τ	The recovery rate, $0 \leq \tau \leq 1$
e	The carbon emission reduction, $e > 0$
α	Manufacturer's shareholding ratio, $0 < \alpha < 1$
β	Retailer's shareholding ratio, $0 < \beta < 1$
$I(e)$	Total investment in the carbon emission reduction activities of manufacturer
k	The coefficient of product emission reduction investment cost, $k > 0$
$C(\tau)$	Total investment in the cycle activities of manufacturer
δ	The coefficient of product recovery investment cost, $\delta > 0$
Π_m	Manufacturer's profit
Π_r	Retailer's profit
Π_{sc}	The CLSC's profit

where

$$\delta_1 = \max\left\{\frac{\theta^2(A-\Delta)^2}{2k}, \frac{(\Delta(\lambda-1)+A)((4kb\lambda+\theta^2)(\lambda-1)\Delta+A\theta^2)}{8k(1-\lambda)}\right\}.$$

4.2. Model M – manufacturer-led decision-making model

When there is no share-holding cooperation between the manufacturer and the retailer, the manufacturer and the retailer respectively aim to maximize their profits. The profit functions of the manufacturer and the retailer are as follows:

$$\underset{(\tau,e,w)}{\text{Max}}\ \Pi_m = (w - c_n + \tau\Delta)D - A\tau D - \delta\tau^2 - \frac{ke^2}{2} \tag{3}$$

$$\underset{(p)}{\text{Max}}\,\Pi_r = (p-w)D. \tag{4}$$

When there is share-holding cooperation between the manufacturer and the retailer, the manufacturer shares α percentage of the retailer's profit, and the retailer shares β percentage of the manufacturer's profit. The profit functions of the manufacturer and the retailer are as follows:

$$\underset{(\tau,e,w)}{\text{Max}}\ \Pi_m^M = (1-\beta)\Pi_m + \alpha\Pi_r \tag{5}$$

$$\underset{(p)}{\text{Max}}\,\Pi_r^M = \beta\Pi_m + (1-\alpha)\Pi_r. \tag{6}$$

The Game order is as follows: The leading manufacturer first determines the unit wholesale price of the product w, the collection rate of used products τ and the carbon emission reduction e. The retailer sets the unit retail price p of the product according to the manufacturer's decision. Substitute equation (1) into equations (5) and (6) by using backward induction method. The equilibrium decisions are as follows.

Theorem 2. *In the manufacturer-led decision-making model, when $\delta > \delta_1$, the equilibrium decision and profit of the low-carbon CLSC are as follows:*

$$\tau^{M*} = \frac{bk(a-bc_n)(A-\Delta)}{kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2}, \quad e^{M*} = \frac{-2\delta\theta(a-bc_n)}{kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2},$$

$$w^{M*} = (a-bc_n)\frac{2k\delta(\beta-1)(2\alpha\beta-\alpha-3\beta+1)}{(\alpha+\beta-1)(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)} - \frac{(ab(A-\Delta)^2-\delta(\beta-1)(4\alpha-6a-2bc_n))+2\delta c_n\theta^2}{kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2},$$

$$p^{M*} = -\frac{(ab(A-\Delta)^2-\delta(\beta-1)(4\alpha-6a-2bc_n))+2\delta c_n\theta^2}{kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2}, \quad D^{M*} = \frac{2\delta bk(\beta-1)(a-bc_n)}{kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2},$$

$$\Pi_m^{M*} = (\beta-1)\frac{\delta k(a-bc_n)^2}{kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2},$$

$$\Pi_r^{M*} = -\frac{\delta k(a-bc_n)^2(bk(\beta b(A-\Delta)^2+4\delta(\alpha-1)(\beta-1)^2)+2\delta\beta\theta^2)}{(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)^2}.$$

Proposition 1. (1) $\frac{\partial e^{M*}}{\partial\alpha} > 0$, $\frac{\partial\tau^{M*}}{\partial\alpha} > 0$, *when* $\beta > \frac{bk-\theta^2}{bk}$, $\frac{\partial p^{M*}}{\partial\alpha} > 0$, *on the other hand,* $\frac{\partial p^{M*}}{\partial\alpha} < 0$, $\frac{\partial D^{M*}}{\partial\alpha} > 0$, $\frac{\partial\Pi_m^{M*}}{\partial\alpha} > 0$, *when* $\alpha > \frac{(1-3\beta)(kb^2(A-\Delta)^2+2\delta\theta^2)}{4bk\delta(\beta-1)^2}$, $\frac{\partial\Pi_r^{M*}}{\partial\alpha} < 0$, *on the other hand,* $\frac{\partial\Pi_r^{M*}}{\partial\alpha} \geq 0$.
(2) $\frac{\partial e^{M*}}{\partial\beta} > 0$, $\frac{\partial\tau^{M*}}{\partial\beta} > 0$, $\frac{\partial p^{M*}}{\partial\beta} < 0$, $\frac{\partial D^{M*}}{\partial\beta} > 0$, $\frac{\partial\Pi_m^{M*}}{\partial\beta} > 0$, $\frac{\partial\Pi_r^{M*}}{\partial\beta} > 0$.

Proof. See Appendix A. □

From Proposition 1, in the manufacturer-led cross-shareholding low-carbon CLSC, an increase in the shareholding ratio of either party will benefit the manufacturer in recycling and carbon emission reduction, expand the market demand and increase the manufacturer's profit. This is similar to the research conclusion of Xia *et al.* [29], which shows that after cross-shareholding, the higher the cross-shareholding ratio, the supply chain members can reduce carbon emissions to a greater extent. On this basis, this paper expands its research conclusions. In the low-carbon CLSC, the unit retail price of the product always decreases with the increase of the retailer's shareholding ratio. The retailer's profit increases or decreases with the manufacturer's shareholding ratio depending on the size of their cross-shareholding ratios. The profits of the manufacturer and retailer always increase with the increase of the retailer's shareholding ratio.

4.3. Model R – retailer-led decision-making model

When the retailer is dominant and the manufacturer has shareholding cooperation with the retailer, the Game order is as follows: The leading retailer first decides the unit retail price of the product p, the manufacturer sets the unit wholesale price w, the collection rate τ and the carbon emission reduction e according to the retailer's decision. Substitute equation (1) into equations (5) and (6) by using the backward induction method. The equilibrium decisions are as follows.

Theorem 3. *In the retailer-led decision-making model, when* $\delta > \max\left\{\frac{akb(A-\Delta)^2}{k(a(2\alpha-2)+2bc_n(\beta-2))+2c_n\theta^2}, \frac{akb^2(\beta-2)(A-\Delta)^2}{(2bk(2\beta-3)-2\theta^2(\beta-1))-2bc_n(bk-\theta^2)}\right\}$, $k > \max\left\{\frac{\theta^2(A-\Delta)^2}{2\delta}, \frac{2\delta\theta^2(a+bc_n-a\beta)}{b((b(\beta-2)(A-\Delta)^2-4\beta\delta+6\delta)a+2b\delta c_n)}\right\}$, *the equilibrium decision and profit of the low-carbon CLSC are as follows:*

$$\tau^{R*} = \frac{-bk(a-bc_n)(A-\Delta)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)}, \quad e^{R*} = \frac{2\delta\theta(a-bc_n)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)},$$

$$w^{R*} = \frac{-(a-bc_n)(\beta-1)^2}{b(\beta-2)(\alpha+\beta-1)} + \frac{a((\beta-1)(kb(b(A-\Delta)^2-2\delta)+2\delta\theta^2((\beta-1)\theta^2-bk)))+2bc_n\delta(bk-\theta^2)}{b(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)},$$

$$p^{R*} = \frac{2bc_n\delta(bk-\theta^2)}{b(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)} + \frac{a((\beta-1)(kb(b(A-\Delta)^2-2\delta)+2\delta\theta^2((\beta-1)\theta^2-bk)))}{b(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)},$$

$$D^{R*} = \frac{2\delta bk(a-bc_n)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)}, \quad \Pi_m^{R*} = (\beta-1)\frac{\delta k(a-bc_n)^2}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)^2},$$
$$\Pi_r^{R*} = \frac{\delta k(a-bc_n)^2}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(\beta-2)}.$$

Proposition 2. (1) $\frac{\partial e^{R*}}{\partial\alpha} = \frac{\partial\tau^{R*}}{\partial\alpha} = \frac{\partial p^{R*}}{\partial\alpha} = \frac{\partial D^{R*}}{\partial\alpha} = \frac{\partial\Pi_m^{R*}}{\partial\alpha} = \frac{\partial\Pi_r^{R*}}{\partial\alpha} = 0$, $\frac{\partial w^{R*}}{\partial\alpha} < 0$.
(2) $\frac{\partial e^{R*}}{\partial\beta} > 0$, $\frac{\partial\tau^{R*}}{\partial\beta} > 0$, $0 < \theta < \sqrt{bk}$, $\frac{\partial p^{R*}}{\partial\beta} < 0$, *on the other hand,* $\frac{\partial p^{R*}}{\partial\beta} \geq 0$, $\frac{\partial D^{R*}}{\partial\beta} > 0$, $\frac{\partial\Pi_m^{R*}}{\partial\beta} > 0$, $\frac{\partial\Pi_r^{R*}}{\partial\beta} > 0$.

From Proposition 2, in the retailer-led cross-shareholding low-carbon CLSC, the wholesale price of a product is inversely proportional to the manufacturer's shareholding. However, other decisions and profits of enterprises are not affected by the proportion of the manufacturer's shareholding. This is similar to the research conclusion of Fu and Ma [10]. The research shows that the optimal profits of the upstream and downstream enterprises are not affected by the proportion of the upstream enterprises in the cross-shareholding supply chain led by the downstream enterprises. The carbon emission reduction of products, the return rate of used products, market demand, the profits of the manufacturer and retailer are increased with the increase of the retailer's share-holding. However, the retail price is affected by the sensitivity of consumers to carbon emission reduction. When consumers are not sensitive to low-carbon products, the retail price of products is inversely proportional to the retailer's shareholding ratio. As consumers become more sensitive to low-carbon products, the retailer will improve the retail price.

4.4. Model N – Nash Game decision-making model

When the manufacturer and retailer have the same market position and share the shareholding, both simultaneously make the best decision for their sides. The retailer sets the retail price p simultaneously, the manufacturer sets the wholesale price w, the collection rate of used products τ and the carbon emission reduction e. Substitute equation (1) into equations (5) and (6) by using the backward induction method. The equilibrium decisions are as follows.

Theorem 4. *In the Nash Game decision-making model, when $\delta > \delta_1$, the equilibrium decision and profit of the low-carbon CLSC are as follows:*

$$\tau^{N*} = \frac{bk(a-bc_n)(A-\Delta)}{kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2}, \quad e^{N*} = \frac{-2\delta\theta(a-bc_n)}{kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2},$$
$$w^{N*} = \frac{k((2c_n\delta(\beta-2)+a(A-\Delta)^2)^2+2a\delta(\alpha-1))+2\delta c_n\theta^2}{kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2},$$
$$p^{N*} = \frac{k(b(a(A-\Delta)^2-2c_n\delta)+2a\delta(\beta+\alpha-2))+2\delta c_n\theta^2}{kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2},$$
$$D^{N*} = \frac{-2\delta bk(a-bc_n)}{kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2}, \quad \Pi_m^{N*} = \frac{k(bk(b(A-\Delta)^2-4\delta^2)+2\delta^2\theta^2)(\beta-1)(a-bc_n)^2}{(kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2)^2},$$
$$\Pi_r^{N*} = -k(a-bc_n)^2\frac{(bk(b\alpha\beta(A-\Delta)^2+4\delta^2(\alpha-1))+2\delta\beta\theta^2)}{(kb(b(A-\Delta)^2+2\delta(\beta+\alpha-3))+2\delta\theta^2)^2}.$$

Proposition 3. (1) $\frac{\partial e^{N*}}{\partial\alpha} > 0$, $\frac{\partial\tau^{N*}}{\partial\alpha} > 0$, *when* $0 < \theta < \sqrt{bk}$, $\frac{\partial p^{N*}}{\partial\alpha} < 0$, *on the other hand,* $\frac{\partial p^{N*}}{\partial\alpha} \geq 0$, $\frac{\partial D^{N*}}{\partial\alpha} > 0$, *when* $\beta > \beta_1$, $\frac{\partial\Pi_m^{N*}}{\partial\alpha} > 0$, *on the other hand,* $\frac{\partial\Pi_m^{N*}}{\partial\alpha} \leq 0$, *when* $\alpha > \alpha_1$, $\frac{\partial\Pi_r^{N*}}{\partial\alpha} > 0$, *on the other hand,* $\alpha > \alpha_1$, $\frac{\partial\Pi_r^{N*}}{\partial\alpha} \leq 0$.
(2) $\frac{\partial e^{N*}}{\partial\beta} > 0$, $\frac{\partial\tau^{N*}}{\partial\beta} > 0$, $0 < \theta < \sqrt{bk}$, $\frac{\partial p^{N*}}{\partial\beta} < 0$, *on the other hand,* $\frac{\partial p^{N*}}{\partial\beta} \geq 0$, $\frac{\partial D^{N*}}{\partial\beta} > 0$, *when* $\beta > \beta_2$, $\frac{\partial\Pi_m^{N*}}{\partial\beta} > 0$, *on the other hand,* $\frac{\partial\Pi_m^{N*}}{\partial\beta} \leq 0$, *when* $\beta > \beta_3$, $\frac{\partial\Pi_r^{N*}}{\partial\beta} > 0$, *on the other hand,* $\frac{\partial\Pi_r^{N*}}{\partial\beta} \leq 0$. *Where* $\alpha_1 = \frac{8\delta^3(\beta-1)(bk-\theta^2)+b(A-\Delta)^2(k\beta b^2(A-\Delta)^2+2kb\delta(\beta^2-3\beta+2\delta)+2\beta\delta\theta^2)}{2kb\delta(b\beta(A-\Delta)^2+4\delta^2)}$, $\beta_1 =$

$\frac{8\delta^3(\theta^2-2bk)-b(A-\Delta)^2(kb^2(A-\Delta)^2-2kb\delta(\alpha+3)+2\delta\theta^2)}{2k\delta b^2(A-\Delta)^2}$, $\beta_2 = \frac{bk(b(A-\Delta)^2+2\delta(\alpha-1))+2\delta\theta^2}{2kb\delta}$, $\beta_3 = (A-\Delta)^2$ $\frac{kb^2((kb^2(A-\Delta)^2-6kb\delta+2\delta^2\theta^2)+2bk\delta\alpha^2+2\delta^2\theta^2)}{2kb\delta(k\alpha b^2(A-\Delta)^2+2\delta^2\theta^2)}$.

From Proposition 3, in the low-carbon CLSC with cross-shareholdings under Nash Game, the changing trend of decision-makings with the proportion of shareholding ratio is roughly similar to the result mentioned above, it will not repeat here. The increase and decrease of enterprise profits depend on the size of the cross-shareholding ratio.

5. Model comparison and analysis

Proposition 4. (1) $\frac{\partial e^{Y*}}{\partial\theta} > 0$, $\frac{\partial \tau^{Y*}}{\partial\theta} > 0$, $\frac{\partial p^{Y*}}{\partial\theta} > 0$, $\frac{\partial D^{Y*}}{\partial\theta} > 0$.

(2) $\frac{\partial\Pi_m^{M*}}{\partial\theta} > 0$, $\frac{\partial\Pi_r^{M*}}{\partial\theta} > 0$, $\frac{\partial\Pi_{sc}^{M*}}{\partial\theta} > 0$, $\frac{\partial\Pi_m^{R*}}{\partial\theta} > 0$, $\frac{\partial\Pi_r^{R*}}{\partial\theta} > 0$, $\frac{\partial\Pi_{sc}^{R*}}{\partial\theta} > 0$, *when* $\alpha > \frac{\delta(kb^2(A-\Delta)^2+2bk\delta(\beta+1)-\delta\theta^2)}{2bk(b(A-\Delta)^2-\delta^2)}$, $\frac{\partial\Pi_m^{N*}}{\partial\theta} < 0$, *on the other hand,* $\frac{\partial\Pi_m^{N*}}{\partial\theta} \geq 0$, *when* $\alpha > \frac{\delta(2bk\delta\beta^2+\beta(kb^2(A-\Delta)^2-6bk\delta-2\delta\theta^2)+8bk\delta)}{2bk(\beta(b(A-\Delta)^2-\delta^2)+4\delta^2)}$, $\frac{\partial\Pi_r^{N*}}{\partial\theta} < 0$, *on the other hand,* $\frac{\partial\Pi_r^{N*}}{\partial\theta} \geq 0$. *Where* $Y = \{C, M, R, N\}$, *the following is the same.*

From Proposition 4, in four cases, the enhanced sensitivity of consumers to carbon emission reduction is conducive to improving carbon emission reduction and recycling of used products. As the sensitivity coefficient of carbon emission reduction increases, consumers are willing to pay a higher price for carbon emission reduction products. According to the 2019 China Sustainable Consumption Report, more than 70% of consumers said they are willing to pay a higher price for environmentally friendly products, indicating increased demand in the low-carbon consumer market.

In the low-carbon CLSC dominated by the manufacturer or retailer, the profits of enterprises and low-carbon CLSC are the increasing functions of the carbon emission sensitivity coefficient of consumers. The manufacturer and retailer have the incentive to improve consumer awareness. The manufacturers are implementing carbon emission reduction, such as FAW, DFL, and Lenovo, raise consumers' awareness of environmental protection through energy-saving labels. And large retailers such as Wal-Mart, Carrefour, and Metro carry out low-carbon publicity. In the balance of power between the manufacturer and retailer, the increase or decrease of profits of enterprises and low-carbon CLSC depend on the size of the shareholding ratio of the manufacturer and retailer. In summary, when the power structure in CLSC is unbalanced, it is conducive to the CLSC to make more favorable decisions for all parties; However, when manufacturers and retailers are evenly matched, it is not conducive to the CLSC to make optimal decisions. It is necessary to adjust the power of both parties through the size of their cross-shareholding ratio to achieve a "win–win" situation.

Proposition 5. (1) $\frac{\partial e^{Y*}}{\partial k} < 0$, $\frac{\partial \tau^{Y*}}{\partial k} < 0$.

(2) $\frac{\partial\Pi_m^{M*}}{\partial k} < 0$, *when* $\alpha > \frac{-(kb^2(A-\Delta)^2+24kb\delta+2\delta\theta^2)}{4bk(3\beta^2-5\beta+2)} + \frac{16kb\delta\beta^2+8kb\delta}{4bk(3\beta^2-5\beta+2)}$, $\frac{\partial\Pi_r^{M*}}{\partial k} > 0$, *on the other hand,* $\frac{\partial\Pi_r^{N*}}{\partial k} \leq 0$.

(3) $\frac{\partial\Pi_m^{R*}}{\partial k} < 0$, $\frac{\partial\Pi_r^{R*}}{\partial k} < 0$.

(4) *When* $\alpha > \frac{\delta(kb^2(A-\Delta)^2+2bk\delta(\beta+1)-2\delta\theta^2)}{2bk(b(A-\Delta)^2-\delta^2)}$, $\frac{\partial\Pi_m^{N*}}{\partial k} > 0$, *on the other hand,* $\frac{\partial\Pi_m^{N*}}{\partial k} \leq 0$, *when* $\alpha > \frac{\delta(2bk\delta\beta^2+\beta(kb^2(A-\Delta)^2-6bk\delta-2\delta\theta^2)+8bk\delta)}{2bk(\beta(b(A-\Delta)^2-\delta^2)+4\delta^2)}$, $\frac{\partial\Pi_r^{N*}}{\partial k} > 0$, *on the other hand,* $\frac{\partial\Pi_r^{N*}}{\partial k} \leq 0$.

From Proposition 5, in four cases, the carbon emission reduction and return rate decrease with the increase of carbon emission reduction investment cost coefficient of the manufacturer. This is because when the cost of carbon emission reduction is too high, the manufacturer will reduce the investment in recycling used products, resulting in a decline in carbon emission reduction and used product recovery.

When the manufacturer is dominant, the manufacturer's profit is inversely proportional to the cost coefficient of carbon emission reduction investment. This is consistent with the fact that the profit of the retailer is affected by the size of the manufacturer and the retailer's shareholding ratio. When the retailer is dominant, the profit of the manufacturer and retailer is inversely proportional to the cost coefficient of carbon emission reduction investment. In Nash Game, the relationship between the profits of the manufacturer and retailer and the cost coefficient of carbon emission reduction investment is affected by the shareholding ratio of both sides. It can be seen that the increase of manufacturer's emission reduction costs will often damage its and retailer's profits, especially when retailers have a higher power. However, this adverse situation can be reversed by adjusting the shareholding ratio of both parties. A proper shareholding ratio allows the retailer to share the emission reduction risk of the manufacturer, which can improve the performance of both parties.

Conclusion 1. When $\beta > \beta_2 > \beta_1$, $e^{N*} > e^{M*} > e^{R*}$, $\tau^{N*} > \tau^{M*} > \tau^{R*}$. when $\beta_2 > \beta > \beta_1$, $e^{N*} > e^{R*} > e^{M*}$, $\tau^{N*} > \tau^{R*} > \tau^{M*}$. when $\beta_2 > \beta_1 > \beta$, $e^{R*} > e^{N*} > e^{M*}$, $\tau^{R*} > \tau^{N*} > \tau^{M*}$. Where $\beta_1 = \frac{kb((A-\Delta)^2-4\delta\alpha)+2\delta\theta^2}{kb((A-\Delta)^2-4\delta(\alpha-1))+2\delta\theta^2}$, $\beta_2 = \frac{kb((A-\Delta)^2-2\delta(\alpha+1))+2\delta\theta^2}{kb((A-\Delta)^2-2\delta)+2\delta\theta^2}$, and $\beta_2 > \beta_1$.

Proof. See Appendix A. □

From Conclusion 1, the changing trend of carbon emission reduction of products and returning rate of used products is always the same. When the shareholding ratio of the manufacturer and retailer satisfies certain conditions, the carbon emission reduction in Nash Game is the highest. In Stackelberg Game, the level of carbon emission reduction of enterprises is determined by the shareholding ratio of the manufacturer and retailer. The Game between the manufacturer and retailer leads to the decline in carbon emission reduction and returning rate. When the shareholding ratio of the manufacturer and retailer is less than a certain threshold, retailer-led carbon emission reduction is the highest, and manufacturer-led carbon emission reduction is the lowest. Led by the retailer and driven by the demand of low-carbon consumers, the retailer is more willing to increase the carbon emission reduction of the manufacturer to occupy the low-carbon consumption market. And because the unit profit of remanufactured products is higher than that of new ones, the manufacturer will improve the efficiency of returning used products. When the shareholding ratio of the manufacturer and retailer satisfies certain conditions, carbon emission reduction and return rate are higher in retailer-led low-carbon CLSC.

Conclusion 2. (1) When $\alpha < \alpha_1$, $p^{M*} > p^{R*} > p^{N*}$, on the other hand, $p^{R*} > p^{M*} > p^{N*}$.
(2) When $\alpha > \alpha_3$, $D^{M*} > D^{N*} > D^{R*}$, when $\alpha_3 > \alpha > \alpha_2$, $D^{N*} > D^{M*} > D^{R*}$, when $\alpha < \alpha_2$, $D^{N*} > D^{R*} > D^{M*}$. Where $\alpha_1 = \frac{kb^2(A-\Delta)^2(\theta^2(1-\beta)-bk(\beta^3-3\beta+1))+2\delta(\beta-1)(2\beta b^2k^2-bk\theta^2-\theta^4)}{4kb\delta(1-\beta)(kb-\theta^2)}$, $\alpha_2 = \frac{4kb\beta\delta(\beta-1)-(\beta^3-3\beta+1)(kb^2(A-\Delta)^2+2\delta\theta^2)}{4kb(\beta-1)}$, $\alpha_3 = \frac{2kb\beta\delta(\beta^2-1)+\beta(kb^2(A-\Delta)^2+2\delta\theta^2)}{2kb(\beta-1)}$, and $\alpha_3 > \alpha_2$.

From Conclusion 2, the retail price in Nash Game is the lowest. In the case of manufacturer-led and retailer-led, the retail price is determined by the shareholding ratios of the manufacturer and retailer. The unbalanced channel power structure always leads to higher retail prices, which harm the interests of consumers. In Nash Game, the demand of products is not always the highest. Under certain conditions, the product demand may be higher in a manufacturer-led situation. The level of demand under different channel power structures changes with the change of cross-shareholding ratio. To sum up, due to consumers' awareness of low carbon and environmental protection, the demand for products may still increase as the sales price of products increases. Enterprises should improve the consumer experience by improving the after-sales quality of products.

Conclusion 3. (1) When $\beta > \frac{3kb^2(A-\Delta)^2+2kb\delta(\alpha-7)+6\delta\theta^2}{kb^2(A-\Delta)^2-6kb\delta+2\delta\theta^2}$, $\Pi_m^{M*} > \Pi_m^{N*} > \Pi_m^{R*}$, on the other hand, $\Pi_m^{M*} > \Pi_m^{R*} > \Pi_m^{N*}$.
(2) $\Pi_m^{M*} > \Pi_r^{M*}$, $\Pi_m^{R*} < \Pi_r^{R*}$, when $\alpha > \frac{2\delta^2(2\beta(bk-\theta^2)+\theta^2)}{kb(b(2\beta-1)(A-\Delta)^2+4\delta^2)}$, $\Pi_m^{N*} > \Pi_r^{N*}$, on the other hand, $\Pi_m^{N*} \le \Pi_r^{N*}$.

From Conclusion 3, in a low-carbon CLSC, the profit is always highest when the manufacturer is in charge. This is similar to the research conclusion of Choi *et al.* [5] and Gao *et al.* [11]. This paper proves the effectiveness of this conclusion in the low-carbon CLSC with cross-shareholding. Further, this paper finds that the manufacturer's dominant power structure is the most favorable for the manufacturer; otherwise, the manufacturer will increase her own profit by adjusting the shareholding ratio of the retailer.

The profits of the manufacturer in retailer-led and Nash Game depend on the shareholding ratio of the manufacturer and the retailer. When the manufacturer and retailer play the Stackelberg Game, the side with more channel power is always more profitable. This is similar to the research conclusion of Zheng *et al.* [34] and Xia *et al.* [29]. In Nash Game, the profits of the manufacturer and retailer are affected by the shareholding ratio of both parties. In a word, in a CLSC with balanced power structure, different cross shareholding strategies can realize the redistribution of profits in the CLSC and achieve the "win–win" situation.

6. Model T – Coordination model of Low-Carbon CLSC

Conclusion 4. When $\alpha = \frac{2\beta-1}{\beta-1}$, $e^{M*} = e^{C*}$, $\tau^{M*} = \tau^{C*}$, when $\alpha = \frac{kb^2(A-\Delta)^2+4kb\delta-4\delta(bk-\theta^2)}{4\delta(\beta-1)(bk-\theta^2)} - \frac{6\delta\theta^2}{4\delta(\beta-1)(bk-\theta^2)}$, $p^{M*} = p^{C*}$, when $\alpha = \frac{(kb^2(A-\Delta)^2+4kb\delta+2\delta\theta^2)-4\delta bk}{4bk\delta(\beta-1)}$, $D^{M*} = D^{C*}$. Where $E = \beta\sqrt{(bk(b(A-\Delta)^2-4\alpha)+2\delta\theta^2)(kb^2(A-\Delta)^2+2\delta\theta^2)}$.

From Conclusion 4, when the shareholding ratios of manufacturer and retailer satisfy certain conditions, the carbon emission reduction of products, the collection rate of used products, retail price, and market demand in the manufacturer-led low-carbon CLSC can all reach the level under centralized decision-making. When $\alpha = \frac{E+F}{4kb(\beta-1)}$, $\Pi_{sc}^{M*} = \Pi_{sc}^{C*}$. However, the value of α contradicts the previous hypothesis, so it can only realize the partial coordination of the manufacturer-led low-carbon CLSC.

In order to further coordinate and improve the performance of low-carbon CLSC, according to Choi *et al.* [5], we adopt the two-tariff contract to coordinate the low-carbon CLSC. The manufacturer sets price at marginal cost and charge the retailer a fixed fee. We assume that the manufacturer charges the retailer (and the retailer charges the manufacturer) a fixed fee S.

In the manufacturer-led cross-shareholding low-carbon CLSC, this paper considers incentive compatibility and individual rational constraints between manufacturer and retailer. The profit function and constraint conditions after coordination are as follows:

$$\begin{cases} \text{Max } \Pi_m^{TM} = (1-\beta)\Pi_m + \alpha\Pi_r + S \\ \text{s.t. } \Pi_r^{TM} = \beta\Pi_m + (1-\alpha)\Pi_r - S \\ \qquad \tau^{M*} = \tau^{C*}, e^{M*} = e^{C*}, p^{M*} = p^{C*}, D^{M*} = D^{C*} \\ \qquad \Pi_m^{TM} \geq \Pi_m^{M} \\ \qquad \Pi_r^{TM} \geq \Pi_r^{M}. \end{cases} \tag{7}$$

According to the equilibrium solution in Theorems 1 and 2, equalize the retail price in manufacturer-led low-carbon CLSC with the centralized retail price ($p^{M*} = p^{C*}$). We solve for the wholesale price of the manufacturer at this time, which gets the adjusted profit of the business respectively: $\Pi_m^{TM} = \frac{k\delta(a-bc_n)^2(kb(b(A-\Delta)^2+4\alpha\delta)+2(\beta-1)\delta\theta^2)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2}$, $\Pi_r^{TM} = \frac{k\delta(a-bc_n)^2(4\delta(1-\alpha)+2\delta\beta\theta^2)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2} - \frac{k^2b^2\delta\beta(a-bc_n)^2(A-\Delta)^2}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2}$, and $\Pi_m^{TM} + \Pi_r^{TM} = \Pi_{sc}^{C}$. When $-\Delta\Pi_m^{TM} < S < \Delta\Pi_r^{TM}$ and $\Pi_m^{TM} \geq 0$, $\Pi_r^{TM} \geq 0$. The manufacturer and retailer have an incentive to adopt the coordinated contract, and the profit increment of the manufacturer and retailer after coordination is: $\Delta\Pi_m^{TM} = \frac{-4bk^2\delta^2(a-bc_n)^2((\alpha-2)\beta-2\alpha+2)(k\beta b^2(A-\Delta)^2+2\beta\delta\theta^2+4bk\delta(\beta-1)(\alpha-1)^2)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)}$, $\Delta\Pi_r^{TM} = 12bk^2\delta^2(a-bc_n)^2\frac{(\beta(kb^2(A-\Delta)^2+2\delta\theta^2)^2((\alpha-\frac{5}{3})\beta-\frac{4}{3}(\alpha-1))-\frac{16}{3}k^2b^2\delta^2(\alpha-3)(\alpha-1)^2(\beta-1)^2)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)} - (kb^2(A-\Delta)^2+2\delta\theta^2)(\beta-2)\frac{16bk^3\delta^3(a-bc_n)^2((\alpha-2)^2\beta^2+2\beta(\alpha-1)-(\alpha-1)^2)}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)^2(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)}$. Where $\Delta\Pi_m^{TM} = \Pi_m^{TM} - \Pi_m^{M}$, $\Delta\Pi_r^{TM} = \Pi_r^{TM} - \Pi_r^{M}$.

In the retailer-led low-carbon CLSC, when $\beta = 1$, $e^{R*} = e^{C*}$, $\tau^{R*} = \tau^{C*}$, $D^{R*} = D^{C*}$, $\Pi_{sc}^{R*} = \Pi_{sc}^{C*}$, the value of β contradicts the previous hypothesis, the low-carbon CLSC of cross-shareholding led by the retailer cannot realize coordination. We consider the two-tariff contract to coordinate the low-carbon CLSC. The profit function and constraint conditions after coordination are as follows:

$$\begin{cases} \text{Max } \Pi_R^{TR} = \beta\Pi_m + (1-\alpha)\Pi_r + S \\ \text{s.t. } \Pi_M^{TR} = (1-\beta)\Pi_m + \alpha\Pi_r - S \\ \qquad \tau^{R*} = \tau^{C*}, e^{R*} = e^{C*}, p^{R*} = p^{C*}, D^{R*} = D^{C*} \\ \qquad \Pi_m^{TR} \geq \Pi_m^R \\ \qquad \Pi_r^{TR} \geq \Pi_r^R. \end{cases} \tag{8}$$

According to the equilibrium solution in Theorems 1 and 3, we equalize the retail price in retailer-led low-carbon CLSC with the centralized retail price ($p^{R*} = p^{C*}$), and solve for the wholesale price of the manufacturer at this time, which gets the adjusted profit of the business respectively: $\Pi_m^{TR} = \frac{k\delta(\beta-1)(a-bc_n)^2}{kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2}$, $\Pi_r^{TR} = \frac{k\beta\delta(a-bc_n)^2}{kb(4\delta-b(A-\Delta)^2)-2\delta\theta^2}$, and $\Pi_m^{TR} + \Pi_r^{TR} = \Pi_{sc}^C$. When $-\Delta\Pi_r^{TR} < S < \Delta\Pi_m^{TR}$ and $\Pi_m^{TR} \geq 0$, $\Pi_r^{TR} \geq 0$. The manufacturer and retailer have an incentive to adopt coordinated contracts, and the profit increment of manufacturer and retailer after coordination is: $\Delta\Pi_m^{TR} = \frac{k\delta(3-\beta)(\beta-1)^2(a-bc_n)^2}{(kb(4\delta-b(A-\Delta)^2)-2\delta\theta^2)(\beta-2)^2}$, $\Delta\Pi_r^{TR} = \frac{b\delta(\beta-1)^2(a-bc_n)^2}{(kb(b(A-\Delta)^2-4\delta)+2\delta\theta^2)(2-\beta)}$. Where $\Delta\Pi_m^{TR} = \Pi_m^{TR} - \Pi_m^R$, $\Delta\Pi_r^{TR} = \Pi_r^{TR} - \Pi_r^R$.

Conclusion 5. (1) $\Pi_m^{TM} + \Pi_r^{TM} = \Pi_{sc}^C$, $\Pi_m^{TR} + \Pi_r^{TR} = \Pi_{sc}^C$.
(2) $\frac{\partial\Delta\Pi_m^{TM}}{\partial\alpha} > 0$, $\frac{\partial\Delta\Pi_r^{TM}}{\partial\beta} > 0$, $\frac{\partial\Delta\Pi_m^{TR}}{\partial\alpha} < 0$, $\frac{\partial\Delta\Pi_r^{TR}}{\partial\beta} > 0$.

From Conclusion 5, after adopting the coordination contract, when fixed fee S meets certain conditions, the profits of low-carbon CLSC led by manufacturer and retailer can reach the centralized level. When the manufacturer is dominant, the profit increment of the coordinated manufacturer increases with the increase of its shareholding ratio and the retailer's shareholding ratio. This is similar to the research conclusion [29]. This paper further illustrates its applicability in the cross-shareholding low-carbon CLSC. In the retailer-led cross-shareholding low-carbon CLSC, the manufacturer's profit is inversely proportional to the retailer's shareholding ratio, while the retailer's profit is directly proportional to its shareholding ratio.

Conclusion 6. When $\beta = \frac{\sqrt{k\delta(\alpha-\delta)(A-\Delta)^2(bk(b(A-\Delta)^2-4\alpha)+2\delta\theta^2)}+2k\delta^2(1-\alpha)}{2k\delta^2}$, $\Pi_{sc}^{N*} = \Pi_{sc}^{C*}$.

From Conclusion 6, when the shareholding ratios of manufacturer and retailer satisfy a certain condition, the profit of low-carbon CLSC in Nash Game can reach the level of centralized decision. This indicates that cross-shareholding can realize coordination of low-carbon CLSC.

7. Numerical analysis and managerial suggestions

This section will verify the above research conclusions through numerical analysis. Referring to the parameter range and Xia *et al.* [29] mention above, so we assume $a = 1$, $b = 0.5$, $A = 0.05$, $c_n = 1$, $\Delta = 0.1$, $\delta = 1(\delta > 0.3)$.

7.1. The effect of carbon emission reduction investment costs coefficient on enterprise decisions

When $\alpha = 0.25$, $\beta = 0.25$, $\theta = 0.6$, we assume $k > 0.38$. Figures 1 and 2 analyze the impact of carbon emission reduction investment cost coefficient on enterprise decisions.

Figures 1 and 2 show that, (1) With the increase of the manufacturer's carbon emission reduction investment cost coefficient, the carbon emission reduction and return rate of products in the four decision models will decrease. Some conclusions of Proposition 5 are verified. (2) The carbon emission reduction, returning rate, and market demand under centralized decision are optimal. When the cost coefficient of carbon emission reduction investment is less than a certain threshold (*i.e.*, $k < 0.5402$), the carbon emission reduction is the second-highest under the retailer-led, and the manufacturer-led carbon emission reduction is the lowest. With the

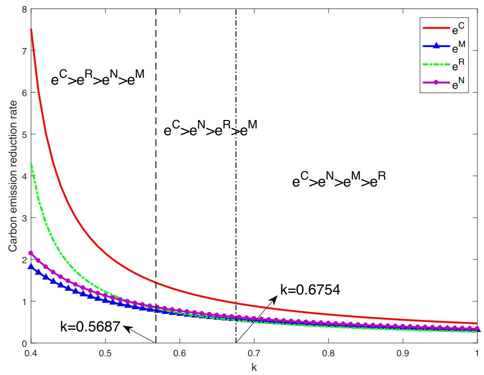

FIGURE 1. k *vs.* Carbon emission reduction.

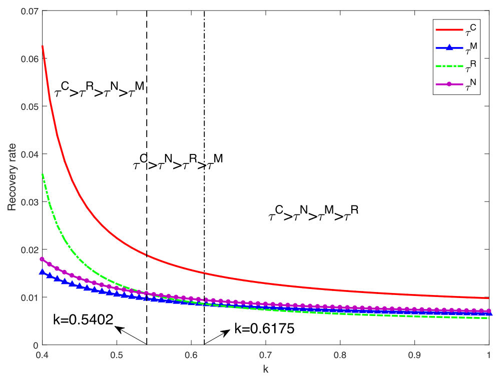

FIGURE 2. k *vs.* Recovery rate.

increase of the cost coefficient of carbon emission reduction investment, the carbon emission reduction increases under Nash Game. When the cost coefficient of carbon emission reduction investment is large enough(*i.e.*, $k > 0.6175$), the manufacturer-led carbon emission reduction is higher than retailer-led. The evolutionary trend of the used-product return rate is similar to carbon emission reduction. It will not repeat here. This verifies part of Conclusion 1 of this paper.

7.2. The effect of carbon emission reduction investment costs coefficient on profits

Figure 3 shows that, the manufacturer's profit decreases with the increase of carbon emission reduction investment cost coefficient. When the cost coefficient of carbon emission reduction investment is greater than a certain threshold (*i.e.*, $k > 0.4244$), the manufacturer's profit is the greatest under the manufacturer-led situation. When $0.4244 < k < 0.5400$, the manufacturer's profit is higher under retailer-led than Nash Game. Some conclusions of Proposition 5 and Conclusion 3 are verified. Figure 4 shows that, the retailer's profit is the lowest under the manufacturer-led situation, and the retailer's profit is the greatest under the retailer-led situation. Figure 5 shows that, as the cost coefficient of carbon emission reduction investment increases, the profit difference between manufacturer and retailer gradually decreases. When the manufacturer dominates, the profit difference between manufacturer and retailer is positive. However, the profit difference is negative in the state of retailer-led and channel power structures balance. Figure 6 shows that, the profit of low-carbon CLSC decreases with the increase of carbon emission reduction investment cost coefficient, and the profit of centralized low-carbon CLSC is always the highest. However, the profit is always the lowest in the manufacturer-

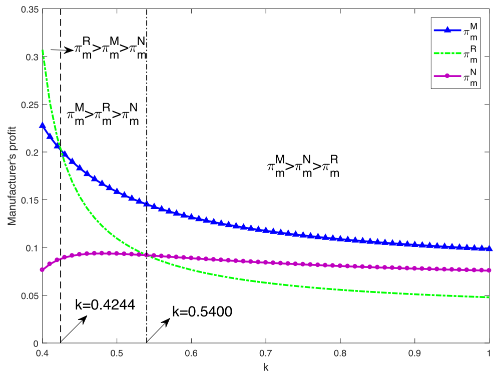

FIGURE 3. k *vs.* Manufacturer's profit.

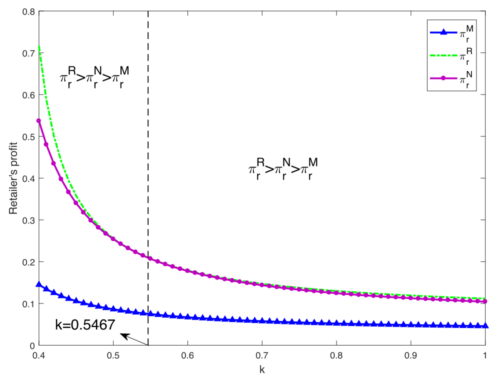

FIGURE 4. k *vs.* Retailer's profit.

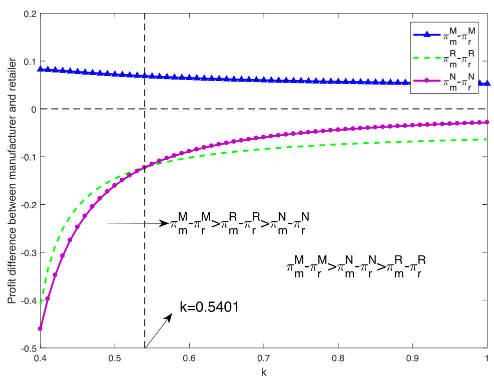

FIGURE 5. k *vs.* Profit difference.

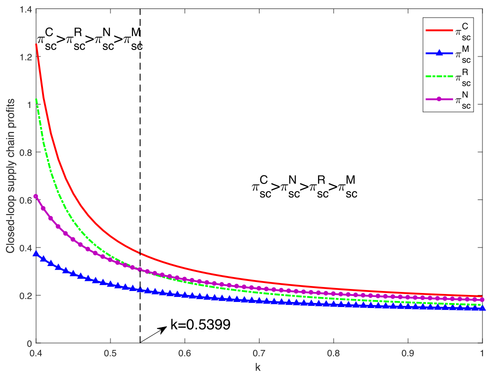

FIGURE 6. k *vs.* low-carbon CLSC profits.

TABLE 2. The influence of consumer carbon reduction sensitivity coefficient on enterprise decisions.

θ	Model C				Model M				Model R				Model N			
	τ	e	p	D	τ	e	p	D	τ	e	p	D	τ	e	p	D
0.05	0.006	0.031	1.501	0.251	0.004	0.023	1.715	0.143	0.003	0.017	1.715	0.143	0.005	0.025	1.601	0.201
0.35	0.007	0.258	1.590	0.295	0.005	0.188	1.808	0.161	0.004	0.147	1.765	0.168	0.006	0.199	1.683	0.227
0.65	0.013	0.861	2.059	0.530	0.008	0.518	2.195	0.239	0.007	0.492	2.034	0.303	0.008	0.563	2.038	0.346
0.85	0.064	5.501	6.174	2.588	0.015	1.298	3.291	0.458	0.036	3.143	4.385	1.479	0.018	1.532	3.163	0.721

led situation. Under certain conditions, the profit of low-carbon CLSC reaches the centralized level under Nash Game. This verifies part of Conclusion 6 of this paper.

7.3. The influence of consumer carbon reduction sensitivity coefficient on enterprise decisions

When $k = 0.85$, we assume $\theta < 0.89$. Table 2 analyzes the influence of consumer carbon reduction sensitivity coefficient on enterprise decisions.

Table 2 shows that, in four cases, with the increase of consumers' sensitivity to carbon emission reduction, the return rate, carbon emission reduction, and market demand of used products under the cross-shareholding strategy will increase. When the consumers' sensitivity to carbon emission reduction is higher than a certain threshold, the manufacturer and retailer benefit. The cross-shareholding of the manufacturer and the retailer realizes the sharing of risks and benefits. That gives manufacturer more incentive to cut emissions and retailer more incentive to sell. This verifies part of Proposition 4 in this article.

7.4. The influence of consumer carbon reduction sensitivity coefficient on profits

Table 3 shows that, (1) In four cases, profits of enterprises and low-carbon CLSC will increase with the improvement of consumers' environmental awareness. This verifies the content of Proposition 4. (2) When the manufacturer dominates, the manufacturer's profit is higher than the retailer. When the retailer dominates, the retailer's profit is all higher than the manufacturer. In Nash Game, the profits of the manufacturer and retailer are affected by the proportion of their shareholding ratios. The manufacturer's profit is the highest in the manufacturer-led situation. This verifies the content of Conclusion 3. (3) In Nash Game, the profit difference between the manufacturer and retailer is smaller than that between manufacturer-led and retailer-led, and the

TABLE 3. The influence of consumer carbon reduction sensitivity coefficient on profits.

θ	Model C	Model M			Model R			Model N		
	sc	m	r	sc	m	r	sc	m	r	sc
0.05	0.125	0.071	0.031	0.102	0.031	0.071	0.102	0.060	0.060	0.120
0.35	0.147	0.081	0.035	0.116	0.036	0.084	0.121	0.066	0.074	0.140
0.65	0.265	0.119	0.059	0.178	0.065	0.151	0.216	0.085	0.148	0.233
0.85	1.294	0.229	0.146	0.375	0.317	0.739	1.056	0.075	0.545	0.621

TABLE 4. The influence of shareholding ratio on enterprise decision-making.

θ		0.05			0.35			0.65			0.85		
α		0.05	0.25	0.35	0.05	0.25	0.35	0.05	0.25	0.35	0.05	0.25	0.35
β		0.05	0.25	0.35	0.05	0.25	0.35	0.05	0.25	0.35	0.05	0.25	0.35
Model C	τ		0.006			0.007			0.013			0.064	
	e		0.031			0.258			0.861			5.501	
	p		1.501			1.590			2.059			6.174	
	D		0.251			0.295			0.530			2.588	
Model M	τ	0.004	0.005	0.003	0.005	0.006	0.004	0.007	0.008	0.006	0.013	0.015	0.009
	e	0.022	0.023	0.019	0.181	0.188	0.154	0.485	0.518	0.391	1.151	1.298	0.800
	p	1.745	1.715	1.698	1.837	1.808	1.772	2.212	2.195	2.051	3.198	3.291	2.645
	D	0.128	0.143	0.151	0.144	0.161	0.168	0.209	2.195	0.228	0.379	0.458	0.357
Model R	τ	0.003	0.004	0.003	0.004	0.005	0.003	0.007	0.008	0.006	0.038	0.037	0.033
	e	0.018	0.017	0.016	0.152	0.147	0.132	0.506	0.492	0.442	3.236	3.143	2.821
	p	1.706	1.715	1.744	1.758	1.765	1.789	2.212	2.195	2.051	4.455	4.385	4.141
	D	0.147	0.143	0.128	0.173	0.168	0.151	0.311	0.303	0.272	1.522	1.479	1.327
Model N	τ	0.004	0.005	0.004	0.005	0.006	0.005	0.008	0.009	0.008	0.014	0.018	0.015
	e	0.023	0.025	0.024	0.186	0.199	0.191	0.510	0.563	0.526	1.260	1.533	1.339
	p	1.624	1.601	1.616	1.703	1.683	1.697	2.035	2.034	2.036	2.956	3.163	3.016
	D	0.189	0.201	0.193	0.213	0.227	0.218	0.313	0.346	0.324	0.593	0.721	1.010

profit of low-carbon CLSC reaches the centralized level under a certain condition. This verifies the content of Conclusion 6.

7.5. The influence of shareholding ratio on enterprise decision-makings

Table 4 shows that, (1) Under different channel power structures, the return rate of used products, carbon emission reduction of products, and market demand are positively proportional to the shareholding ratio of both parties. This verifies part of the conclusion of Propositions 1–3. (2) In Nash Game, the retail price is the lowest, which is best for consumers. However, the dominant Game between the manufacturer and retailer is driving up the retail price. (3) When the shareholding ratio of manufacturer and retailer is large enough, the manufacturer-led market demand is the highest. However, with the decrease of the shareholding ratio, the market demand in Nash Game is the highest. This verifies part of Conclusions 1–3. (4) In the manufacturer-led low-carbon CLSC, when $\alpha = 0.33$, $\beta = 0.4$, the carbon emission reduction and used-product return rate reach the centralized level. When $\alpha = 0.37$, $\beta = 0.36$, the retail price of the products reaches the centralized level. When $\alpha = 0.38$, $\beta = 0.44$, market demand reaches the centralized level. This verifies the content of Conclusion 4 in this paper.

TABLE 5. The influence of carbon emission reduction sensitivity coefficient and the shareholding ratio of consumers on corporate profits before and after coordination.

θ	α	β	Model C	Model M			Model R			Model TM			Model TR		
			sc	m	r	sc	m	r	sc	m	r	sc	m	r	sc
	0.05	0.30		0.064	0.031	0.095	0.030	0.073	0.104	0.075	0.049	0.125	0.047	0.077	0.125
0.05	0.25	0.25	0.125	0.071	0.030	0.102	0.030	0.071	0.102	0.081	0.044	0.125	0.044	0.081	0.125
	0.35	0.05		0.076	0.029	0.105	0.031	0.064	0.095	0.083	0.041	0.125	0.049	0.076	0.125
	0.05	0.30		0.072	0.035	0.107	0.035	0.086	0.122	0.089	0.057	0.147	0.053	0.094	0.147
0.35	0.25	0.25	0.147	0.081	0.035	0.116	0.036	0.084	0.121	0.093	0.054	0.147	0.060	0.086	0.147
	0.35	0.05		0.083	0.036	0.120	0.037	0.075	0.113	0.095	0.052	0.147	0.050	0.097	0.147
	0.05	0.30		0.105	0.054	0.159	0.064	0.155	0.220	0.121	0.144	0.265	0.095	0.169	0.265
0.65	0.25	0.25	0.265	0.119	0.058	0.178	0.065	0.151	0.216	0.137	0.127	0.265	0.098	0.166	0.265
	0.35	0.05		0.114	0.064	0.179	0.066	0.135	0.202	0.144	0.121	0.265	0.102	0.163	0.265
	0.05	0.30		0.189	0.114	0.303	0.313	0.761	1.075	0.692	0.602	1.294	0.456	0.838	1.294
0.85	0.25	0.25	1.294	0.229	0.146	0.375	0.317	0.739	1.056	0.567	0.726	1.294	0.471	0.823	1.294
	0.35	0.05		0.178	0.153	0.332	0.323	0.663	0.987	0.686	0.608	1.294	0.529	0.764	1.294

7.6. Contract coordination effect

Table 5 shows that, after the two-tariff contract is coordinated, (1) In the manufacturer-led and retailer-led low-carbon CLSC, the profits of both manufacturer and retailer are higher than those before coordination, and the profit difference between them is reduced. This will enhance the motivation for cross-shareholding and enhance the stability of the low-carbon CLSC. The profits of the low-carbon CLSC all reach the level of centralized decision-making and realize the perfect coordination of the low-carbon CLSC. This verifies the content of Conclusions 4 and 5 in this paper. (2) In the manufacturer-led low-carbon CLSC, the manufacturer's profit increment is proportional to the cross-shareholding ratio. In the retailer-led low-carbon CLSC, the manufacturer's profit increment decreases with the increase of the retailer's shareholding. While the retailer's profit increases with the increase of its shareholding. This verifies the Content of Conclusion 6.

8. Conclusion

Under the background of circular economy and low-carbon economy, this paper considers a low-carbon CLSC consisting of one manufacturer and one retailer, the influence of different channel power structures and cross-shareholding strategies on the optimal decision-making and profits are studied, and two-part-tariff contracts are proposed to coordinate the low-carbon CLSC. The results indicate that: (1) In four cases, the increased sensitivity of the consumer to carbon emission reduction is conducive to the manufacturer's remanufacturing and carbon emission reduction, expanding market demand and improving profits of enterprises and low-carbon CLSC. So, the manufacturer and retailer have an incentive to raise consumers' awareness of low-carbon environmental protection. (2) Under three different channel power structures, the improvement of the cross-shareholding ratio in the low-carbon CLSC is beneficial to improve the recovery rate, carbon emission reduction, and increase market demand. Under a certain condition, the sale price will increase, which is unfavorable to consumers. (3) Under a certain condition, the recovery rate and carbon emission reduction can reach the highest under retailer-led or Nash Game, which means resource and environmental benefits are the largest. However, Nash Game has the lowest retail price, which is the most beneficial to consumers. In the low-carbon CLSC, the profit of an enterprise is always maximization when it has dominant channel power, so the manufacturer and retailer will compete for the dominant power. The competition between the two sides intensifies the "Double Marginalization" which leads to the reduction of overall performance of the low-carbon CLSC. (4) Cross-shareholding led by retailer can't realize the coordination of low-carbon CLSC. When the shareholding ratio of the manufacturer and retailer is higher than a certain threshold, cross-shareholding led by the manufacturer can only realize partial coordination of low-carbon CLSC. However, cross-shareholding under Nash Game can achieve perfect coordination of low-carbon CLSC. (5) Under the two-part-tariff contracts, both manufacturer and retailer report higher profits, and the gap between them narrowed. At the same time, the profit of the low-carbon CLSC dominated by the manufacturer and retailer can reach the level of centralized decision-making. The two-part-tariff contract can realize the coordination of low-carbon CLSC.

This paper considers the low-carbon CLSC decision and coordination under information symmetry. But in practice, the information between enterprises in the low-carbon CLSC is usually asymmetric. Future research in this direction can be continued.

Appendix A.

Proof of Theorem 1

Proof. Substitute equation (1) into equation (2), Find the Hessian matrix

$$H_{3*3}^{C} = \begin{bmatrix} -2\delta & \theta(\Delta - A) & b(A-\Delta) \\ \theta(\Delta - A) & -k & \theta \\ b(A-\Delta) & \theta & -2b \end{bmatrix},$$

of Π_{sc}^{C}. When the scale parameter satisfies the condition $\delta > \delta_1$, $|H|_{1*1}^{C} = -2\delta < 0$, $|H|_{2*2}^{C} = 2\delta k - \theta^2(A-\Delta)^2 > 0$, $|H|_{3*3}^{C} = kb^2(A-\Delta)^2 - 4\delta kb + 2\delta\theta^2 < 0$, $|H|_{3*3}^{C}$ negative definite, Π_{sc}^{C} is a strictly concave function of τ, e, p. The optimal decisions and profits can be obtained from the first-order condition. □

The proof processes of Theorems 2–4 are similar to Theorem 1, so the detailed proof processes are omitted.

Proof of Proposition 1

Proof. (1)

$$\begin{aligned}
\frac{\partial e^{M*}}{\partial \alpha} &= \frac{8bk\theta\delta^2(a-bc_n)(1-\beta)}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^2} > 0,\\
\frac{\partial \tau^{M*}}{\partial \alpha} &= \frac{4b^2k^2\delta(a-bc_n)(\beta-1)(A-\Delta)}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^2} > 0,\\
\frac{\partial D^{M*}}{\partial \alpha} &= \frac{8b^2k^2\delta^2(a-bc_n)(\beta-1)^2}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^2} > 0,\\
\frac{\partial \Pi_m^{M*}}{\partial \alpha} &= \frac{4bk^2\delta^2(a-bc_n)^2(\beta-1)^2}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^2} > 0,\\
\frac{\partial p^{M*}}{\partial \alpha} &= \frac{8k\delta^2(1-\beta)(a-bc_n)(kb(\beta-1)+\theta^2)}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^2},\\
\frac{\partial \Pi_r^{M*}}{\partial \alpha} &= \frac{4bk^2\delta^2(1-\beta)((kb^2(A-\Delta)^2 + 2\delta\theta^2)(3\beta-1) + 4kb\delta\alpha(\beta-1)^2)}{(kb(b(A-\Delta)^2 - 4\delta(\alpha-2)(\beta-1)) + 2\delta\theta^2)^3},
\end{aligned}$$

when $\beta > \frac{kb-\theta^2}{kb}$, $\frac{\partial p^{M*}}{\partial \alpha} > 0$, On the other hand, $\frac{\partial p^{M*}}{\partial \alpha} < 0$, When $\alpha > \frac{(1-3\beta)(kb^2(A-\Delta)^2+2\delta\theta^2)}{4kb\delta(\beta-1)^2}$, $\frac{\partial \Pi_r^{M*}}{\partial \alpha} < 0$, on the other hand, $\frac{\partial \Pi_r^{M*}}{\partial \alpha} \geq 0$. □

The proof of (2) is similar to that of (1), so the detailed proof process is omitted.

The proof processes of Propositions 2–5 are similar to that of Proposition 1, so the detailed proof processes are omitted.

Proof of Conclusion 1

Proof.

$$\begin{aligned}e^{N*}-e^{M*} &= (4bk\theta\delta^2(a-bc_n)(\alpha(2\beta-1)+1-3\beta))/((kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))\\ &\quad +2\delta\theta^2)(kb(b(A-\Delta)^2+2\delta(\alpha+\beta-3))+2\delta\theta^2))>0,\\ e^{R*}-e^{M*} &= (4\theta\delta(a-bc_n)(kb(b(\beta-1)(A-\Delta)^2-4\delta\beta(\alpha-1)-\alpha)+2\delta\theta^2(\beta-1)))\\ &\quad /((\beta-2)(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)(kb(b(A-\Delta)^2-4bk\delta)+2\delta\theta^2)),\end{aligned}$$

when $\beta>\beta_1=\frac{kb((A-\Delta)^2-4\delta\alpha)+2\delta\theta^2}{kb((A-\Delta)^2-4\delta(\alpha-1))+2\delta\theta^2}$, $e^{R*}<e^{M*}$, on the other hand, $e^{R*}\geq e^{M*}$,

$$\begin{aligned}e^{R*}-e^{N*} &= (2\theta\delta(a-bc_n)(kb(b(\beta-1)(A-\Delta)^2-2\delta(\beta-\alpha-1))+2\delta\theta^2(\beta-1)))\\ &\quad /((\beta-2)(kb(b(A-\Delta)^2-4\delta(\alpha+\beta-3))+2\delta\theta^2)(kb(b(A-\Delta)^2-4bk\delta)+2\delta\theta^2)),\end{aligned}$$

when $\beta>\beta_2=\frac{kb((A-\Delta)^2-2\delta(\alpha+1))+2\delta\theta^2}{kb((A-\Delta)^2-2\delta)+2\delta\theta^2}$, $e^{R*}<e^{N*}$, on the other hand, $e^{R*}\geq e^{N*}$,

$$\begin{aligned}\tau^{N*}-\tau^{M*} &= (2b^2k^2\delta(a-bc_n)(\Delta-A)(\alpha(2\beta-1)+1-3\beta))/((kb(b(A-\Delta)^2\\ &\quad -4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)(kb(b(A-\Delta)^2+2\delta(\alpha+\beta-3))+2\delta\theta^2))>0,\\ \tau^{R*}-\tau^{M*} &= (bk(a-bc_n)(\Delta-A)(kb(b(\beta-1)(A-\Delta)^2-4\delta\beta(\alpha-1)-\alpha)+2\delta\theta^2(\beta-1)))\\ &\quad /((\beta-2)(kb(b(A-\Delta)^2-4\delta(\alpha-2)(\beta-1))+2\delta\theta^2)(kb(b(A-\Delta)^2-4bk\delta)+2\delta\theta^2)),\end{aligned}$$

when $\beta>\beta_1=\frac{kb((A-\Delta)^2-4\delta\alpha)+2\delta\theta^2}{kb((A-\Delta)^2-4\delta(\alpha-1))+2\delta\theta^2}$, $\tau^{R*}<\tau^{M*}$, on the other hand, $\tau^{R*}\geq\tau^{M*}$,

$$\begin{aligned}\tau^{R*}-\tau^{N*} &= (2bk(a-bc_n)(\Delta-A)(kb(b(\beta-1)(A-\Delta)^2-2\delta(\beta-\alpha-1))+2\delta\theta^2(\beta-1)))\\ &\quad /((\beta-2)(kb(b(A-\Delta)^2+2\delta(\alpha+\beta-3))+2\delta\theta^2)(kb(b(A-\Delta)^2-4bk\delta)+2\delta\theta^2)),\end{aligned}$$

when $\beta>\beta_2=\frac{kb((A-\Delta)^2-2\delta(\alpha+1))+2\delta\theta^2}{kb((A-\Delta)^2-2\delta)+2\delta\theta^2}$, $\tau^{R*}<\tau^{N*}$, on the other hand, $\tau^{R*}\geq\tau^{N*}$. □

The proof processes of Conclusions 2–6 are similar to that of Conclusion 1, so, the detailed proof processes are omitted.

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