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SEARCH ENGINE MARKETING FOR DIFFERENT COMPETITION MODES – INTERFIRM AND INTRAFIRM

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Abstract. Brands seeking for prominent positions in online consumer search may invest in two types of search engine marketing: paid search marketing (PSM) and search engine optimization (SEO). This paper investigates how firms should allocate marketing resources between PSM and SEO under different competition modes (interfirm and intrafirm competition). We develop an analytical model where within an interfirm competition, two brands compete for every step, while for the intrafirm competition, multi-brand firm coordinates the search engine marketing decisions for its brands, but delegates the price decisions to the brand managers. We find that the resource allocation decision may depend on the firm's brand strength in organic search. Our analysis shows an increasing curve relation between resources allocated to PSM and the brand strength for interfirm competition and inverted-U shape for intrafirm competition. More specifically, as the brand strength goes up, for the interfirm competition, the resources allocated to PSM increase till 100%, while it should increase only when brand strength is sufficiently low, but decrease when brand strength is high for the intrafirm competition. This nontrivial result underscores the challenge facing a multi-brand firm in balancing between maximizing the search prominence for the category and minimizing the internal competition.

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1. Introduction

Online search has become an important stage of purchase process for most consumers [26]. To increase the likelihood of consideration and purchase, it becomes critical for brands to have prominent positions on search pages [2]. Brands can achieve online search prominence through paid search marketing (PSM) and/or search engine optimization (SEO). In PSM, a brand bids for the keywords relevant to its product and market characteristics. When a consumer's search terms match the keywords that the brand has bid, the brand's website link will likely appear in the sponsored (or paid) region. Another approach, SEO, intends to improve the prominence of the brand in the organic (or natural) links [32,37]. A brand may spend resources in optimizing the website contents and design according to the search engine's algorithm and ranking mechanics. Both PSM

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and SEO are important in achieving search prominence. According to industry estimates, in the U.S. market alone, in 2016 the total spending on SEO was about $65 billion, spending on PSM was about $35 billion, and majority of firms engage in both SEO and PSM.[1] Given the increasing importance of search engine marketing, it is important to understand how a firm should allocate resources between SEO and PSM.

A key distinction between PSM and SEO is the extent to which a brand's marketing effort can influence the search prominence. The amount of resources allocated to PSM is the sole determinant for the brand's expected prominence in the sponsored region. In contrast, the brand's prominence in the organic region depends on both *brand strength* and resources devoted to SEO. Here we define brand strength by the chance that a brand will be prominent in the organic region when the brand does not engage in SEO. Factors that affect a brand's strength may include the brand's popularity and quality reputation. Given this distinction between SEO and PSM, how would the search engine marketing decisions depend on brand strength? Should a firm with stronger brand strength allocate more resources to PSM?

This paper addresses the above research question mainly in the context of interfirm competition and intrafirm competition. Interfirm is the competition between different firms and intrafirm is the competition within different brands in a multi-brand firm. Interfirm competition is common to us, but in this paper we also want to discuss the intrafirm competition is because there are many multibrand firms in the market and we are really curious about their decisions: is there anything different compared with the interfirm competition?

A multi-brand firm offers multiple brands catering to heterogeneous consumer preferences in the same category. Well-known examples of multi-brand firms include Proctor & Gamble in the package good industry and General Motors in durable goods industry. For example, Proctor & Gamble offers multiple brands in laundry detergent category (Tide, Cheer, Dreft, and Gain), and dishwashing category (Dawn, Joy, and Ivory). Although multi-brand companies try their best to make their own brand differentiation and avoid competition within brands, but in case of different brands of the same product, it is impossible for brands to be completely separated without overlapping [11]. These brands create internal competition for those consumers who care more about the core functions and less about the differentiations [12,38]. Multi-brand firms are important to study because these companies are large and impactful, and their decisions are more complex due to the need to coordinate marketing activities for high category-level profits.

Multi-brand companies have different architectural models, one of which is that each brand, as a department, is mainly responsible for its own brand profits and completes the profit target distributed by the head office. In this case, the brand manager can decide the product price to a certain extent according to the profit requirements of his department. Ziss [51] mainly discusses the problem of multi-brand integration under different models, one of which is the impact of brand integration on brands when there is price competition among multiple brands.

At the same time, the competition between brands also involves the competition of internal resources, such as advertising. There are two kinds of marketing input for multi-brand companies. One is that each brand has its own part of marketing expense, and the other is that the company determines the publicity intensity of each brand in terms of its overall strategy. The company decided on the brand advertising input from the overall strategic perspective. All decisions made at the overall level of the company are analyzed from the perspective of the overall profit of the company. The company can make more publicity for a certain brand in a certain period of time, or reduce the competition between its brands as much as possible from the perspective of advertising. Many articles [8,9,17,18,33] have studied the problem of multi-brand advertising. The sub-brands apply to the headquarter for advertising funds according to the department profits and product demands of the previous year and this year's profit target, and the general department decides to allocate advertising resources for each brand according to the overall strategy of the enterprise. Search engine Market is also a means of advertising publicity. Both PSM and SEO are advertising inputs, which are also directed by the head office.

In this paper, we develop an analytical model where firms considers both price and category-level search engine marketing. Two brands offer two horizontally differentiated products. Consumers are new to the category and rely on search engines to look for relevant products to meet their needs. For the competition between firms, two

[1]Sources: https://searchengineland.com (accessed on January 9, 2018) and a Search Engine Land survey reported by Sullivan [44].

firms are competitive in terms of both price and search engine marketing. However, for internal competition, only price level is competitive, while search engine marketing level is cooperative. For the multibrand firm, a category manager coordinates these two brands' search engine marketing decisions to maximize the total category profits. Each brand has a brand manager who makes the brand's price decision and is responsible for the brand's profit. Such a category-brand organization structure with advertising coordination at the category level is consistent with industry practices. As for the above discussion, we asked China Dongfeng motor company for details. Dongfeng has different business divisions, including Dongfeng Nissan, Dongfeng Qichen, Dongfeng Infiniti, and Zhengzhou Nissan. Dongfeng Fengshen under Dongfeng Nissan is similar to Dongfeng Demeanor brand launched by Zhengzhou Nissan about the performance, and there will be competition between the two. Although they both have the "double flying swallow" logo, they are independent between the brands. Each brand is responsible for its own products. The brand manager works hard to meet his own profit requirements. Each branch will decide its own keyword bidding budget and the keywords to be auctioned, but the branch will submit the plan book to the headquarters for approval, and the headquarters will consider whether to pass it or make adjustments and modifications according to the overall interests of the enterprise. We also asked optimization companies to confirm this claim.

Managing search engine prominence is not a straightforward task especially for multi-brand firms. First, for all the competitions, a brand's prominence in the search engine is not deterministic. In organic region, the algorithms behind the search engines are unknown to the brands. In the sponsored region, although the mechanism is more transparent, a brand's prominence is also subject to other brands' bidding amount. Second, for the multibrand firm, the category manager has to anticipate the effect of search outcomes on internal brand competition. Research shows that the brand links in the organic region are more prominent to consumers than the links in the sponsored region [20]. While for the competition between different firms, they will hope to defeat their rivals at any step and maximize their profits.

We talked earlier about intrafirm competition and maybe there would be a different strategy. For intrafirm competition, if both brands allocate all resources to SEO, both brands will be prominent in organic region but neither in sponsored region. Such *symmetrically prominence* will lead to intense price competition and reduced category-level profit. To mitigate the internal price competition, the firm needs to induce *asymmetric prominence* – two brands having differentiated prominence in search outcome. Importantly, the asymmetric prominence in online search enhances brand differentiation. Finally, the firm has to consider its brand strength. When the brand strength is very strong, the brand will naturally enjoy high prominence in organic region. One might conjecture that the firm should devote all of the resources to PSM. However, if both two brands are prominent in organic and sponsored links, symmetric prominence and hence undesirable price competition arise again.

Our analysis shows that if it is interfirm competition, firms will give priority to investing in SEO mainly based on the brand strength. With the improvement of brand strength, funds will gradually flow to PSM. When the brand strength reaches to a certain level, it will put all funds into PSM. The main reason is that it is certainly best for firms to appear in organic region and sponsored region at the same time, but it is not easy to do so due to budget constraints. Between SEO and PSM, they prioritize SEO because users trust the organic links more. But firms will not put all their resources to SEO as that if one dose not have any budget on PSM, the other one investments just a little bit of PSM will make his link appear in the sponsored region, which makes the firm have much opportunities to be shown in both organic region and sponsored region, so as to get more user clicks.

If it is the intrafirm competition, the optimal proportion of search engine marketing resources allocated to PSM exhibits an inverted U-shape relation with the brand strength – first increasing and then decreasing with the brand strength. To understand this result, note that the firm faces the tension between maximizing category-level revenue and minimizing internal competition. To minimize the internal competition, the firm seeks to increase the chance of asymmetric prominence. In coordinating the brands' search-engine marketing decisions, it is ideal for the brands to have a moderate chance (50% given our model specification) of prominence in the organic region and a moderate chance of prominence in the sponsored region. When the brand strength is

sufficiently low, the priority for the firm is to invest in SEO to increase the chance of prominence in the organic search region. As the brand strength increases, the firm allocates less resource to SEO. However, when the brand strength is sufficiently high that the brand's natural prominence in organic region exceeds the moderate level, the firm would have to rely on the prominence in sponsored region to differentiate these two brands. When the brand strength is very high, it becomes optimal to reduce the allocation to PSM to create asymmetric prominence.

Our research falls into the broad stream of work on search engine advertising. Previous research has studied several strategic issues related to search advertising. Some of them focused on firms' decisions on PSM. This literature has examined keyword bidding mechanisms [5, 27, 28], optimal number of sponsored links [29, 41, 45], the click rate of sponsored links and how to improve the click rate of sponsored links [6, 35, 40, 48], content of sponsored links [23], and the effect of organic links on bidding prices for PSM [28]. Some of them focused on the relation between PSM and firms' other decisions and consumer behavior. The literature has investigated the impact of PSM on retailers' pricing and advertising [1, 49] and the value of consumer clicks in paid search links [19]. The above research typically assumes the outcomes in organic search region as exogenous.

Like this paper, Sen [42] and Berman and Katona [10] study the decisions on both PSM and SEO, but these papers are sufficiently different from ours in the context, model, and results. First, both papers study competition in search engine marketing in duopoly models while we examine a model of coordination within a multi-brand firm. Second, the equilibrium results are very different. For instance, Sen [42] concludes that a brand should invest in either SEO or PSM, but not in both. In contrast, we show that a brand should invest in both SEO and PSM. Finally, models in these two papers and our paper are different in the formation of search prominence and consumer considerations. As an example, in Sen [42], a brand achieves search prominence as long as it appears in the organic region. In our paper, following the empirical findings in recent studies (*e.g.* Yang and Ghose [47]), a brand appearing in both organic and sponsored regions is considered as more prominent than a brand appearing in only the organic search region.

In addition, this paper studies how to allocate SEO and PSM in the case of limited capital budget [30, 34, 43]. With respect to the way we account for the budget-limited advertisers in search engine advertising, our work is closest to Zia and Rao [50]. Zia and Rao [50] investigate advertisers' budgeting and bidding strategies across multiple search platforms. They found that when advertisers are symmetric in their total budgets, they pursue asymmetric allocation strategies: one advertiser allocates a higher share of its budget to one of the search engines, and the other allocates the same higher share of its budget to the other search engine. Thus budget allocation is a balancing act between getting more clicks and keeping costs low. This paper studies the investment strategies of different search engine marketing modes of multi-brand enterprises on the same platform. The content of the two researches is different, while the intuition behind the strategy in budget allocation is same: reduce competition and maximize profits.

Since the intrafirm competition model of our research focuses on multi-brand firm cooperate in search engine marketing spending and compete in price setting, it is also related to the stream of literature studying co-opetition. Previous research has studied several strategic issues related to co-opetition between upstream and downstream enterprises in supply chain such as information-sharing and decision-making [21, 22, 31], return policies between supply chain enterprises [7], contract research on revenue distribution [13, 14, 39], competition for leadership positions [36]. Some scholars have studied the effect of vertical competition on the supply chain in which enterprises that have been cooperating with each other in different parts of the supply chain become involved in their partners' business areas and thus become competitors [3, 4].

This paper mainly analyzes the co-opetition strategies in search engine marketing. The problem of cooperative search advertising has not been addressed yet so far as we know. Cao and Ke [15] study manufacturers and retailers cooperate in search ad spendings, while at the same time, compete in search ad auctions. They focused on a manufacturer and multiple retailers' intra-brand competition and coordination in search advertising, but also count for the inter-brand competition with advertisers of other brands.

Next we introduce the models of search engine marketing within different kinds of firms.

2. The Model

Consider two brands offer two horizontally differentiated products, denoted by $i \in \{A, B\}$. We use the Hotelling model to capture the horizontal differentiation [25]. Specifically, the brands A and B are located at the two ends of unit interval [0, 1], brand A at point 0 and brand B at point 1. Each brand offers a single product or service. The products offered by the two brands have the identical value of vertical quality (*e.g.* core function of cleaning in laundry detergent) denoted by V.

Without loss of generality, we normalize the market size to one unit. Each consumer has the need to buy at most one unit of the product. We assume that the consumers are uniformly distributed along the unit interval. A consumer's location represents the relative preference for these two brands. As standard in the Hotelling model formulation, for a consumer located at x, the consumer's surplus from consuming product A and B are $U_A = V - p_A - tx$, and $U_B = V - p_B - t(1-x)$, respectively. Here p_i is the price charged by brand i and tx is the disutility of consuming a product positioned at a distance of x away from a consumer's location. Following the convention, we call t the unit transportation cost, which measures the weight of horizontal preferences in consumer choices.

Consumer search and prominence

We assume that all the consumers are new to the category and are unaware of either brand A or B. After conducting online search activities, a consumer sees search outcomes in two regions – organic links and sponsored links. For a brand (i), since it may or may not appear in each of these two regions, there are four possible scenarios of search outcome: brand i is not featured in either region, featured in organic region only, featured in sponsored region only, or featured in both organic and sponsored regions. Since each brand can be in one of the above four scenarios, there are $4 \times 4 = 16$ scenarios of search outcomes for the two brands. Among them, a trivial scenario has neither brand featured in either region; in this scenario, both brands have zero sales.

We divide the other fifteen scenarios into two categories based on search outcomes: *symmetric prominence* and *asymmetric prominence*. We have symmetric prominence when (1) both brands are prominent in the organic and sponsored search regions, (2) both brands are prominent in the organic region only, and (3) both brands are prominent in the sponsored region only. In the above scenarios of symmetric prominence, consumers will evaluate both brands and then choose one to purchase. Given price p_i for brand i, the market shares are determined by the location of the marginal consumers who are indifferent between these two brands. Specifically, brand i's market sales is $\frac{1}{2} + \frac{p_j - p_i}{2t}$, with $i \neq j,\ i, j = A, B$.

The two brands achieve asymmetric prominence in twelve possible scenarios of search outcomes. For the benefit of tractability and the convenience of exposition, we make the following assumption to determine the relative prominence.

Assumption on relative prominence: Featured in both search regions > featured only in organic region > featured only in sponsored region > not featured in either region.

Next we elaborate the above assumption in specific scenarios of search outcomes.

(i) If brand i is prominent in either organic links and/or sponsored links and brand j is not prominent in either type of links, then brand i enjoys sole prominence. This covers six scenarios of search outcomes, including three scenarios of prominence for each brand.

(ii) If brand i is prominent only in the organic region AND brand j is featured only in the sponsored region, then brand i enjoys sole prominence. Consumers perceive the organic links more relevant to their needs because the organic rankings are created by consumer preferences. This is consistent with the empirical evidences in the reports of Group MUK and Nielsen that the click through rate of the prominent organic links was much higher than the sponsored links [20].

(iii) If brand i is prominent in both the organic and the sponsored regions AND brand j is prominent in either one of these two regions, then brand i has sole prominence. This assumption is similar with the exposure effect assumed in Desai *et al.* [16] and is consistent with empirical findings in Ghose and Yang [19].

To summarize the above discussions, we introduce the notations for the probabilities associated with different types of search prominence. We let Φ_{AB} denote the probability that brands A and B achieve *symmetric prominence*, and let Φ_A and Φ_B denote the probability that brand A and B achieve the *asymmetric prominence* respectively. We let $\lambda_{i,SO}$ denote the probability that the search outcome for brand $i \in A, B$ is SO, where $S = 1$ if brand i is prominent in sponsored region and $S = 0$ otherwise, $O = 1$ if brand i is prominent in organic region and $O = 0$ otherwise. Symmetric prominence can be achieved in three scenarios of search outcomes,

$$\Phi_{AB} = \lambda_{A,11}\lambda_{B,11} + \lambda_{A,01}\lambda_{B,01} + \lambda_{A,10}\lambda_{B,10}. \tag{1}$$

Similarly, we can express the probabilities of asymmetric prominence by the scenarios of search outcomes. As described earlier, there are six scenarios of search outcomes leading to asymmetric prominence with brand A:

$$\Phi_A = \lambda_{A,11}(1 - \lambda_{B,11}) + \lambda_{A,01}\lambda_{B,00} + \lambda_{A,10}(\lambda_{B,01} + \lambda_{B,00}). \tag{2}$$

We can write a similar expression for Φ_B.[2]

Search engine marketing and search outcomes

Both brands engage in search engine marketing to attract new customers. Each brand has an exogenously determined and identical search engine marketing budget normalized to one. Brand i allocates η_i proportion of search engine marketing budget to PSM, and the rest $1 - \eta_i$ proportion to SEO, $i = A, B$. Given the budget allocation, brand i expects a probability $P(\eta_i)$ that its link will be prominent in the sponsored region, and a probability $Q(\eta_i)$ that its link will be prominent in the organic region. Since marketing budget allocation is a decision covering a longer period of time and search outcomes can vary over time during the period, these probabilities $P(\eta_i)$ and $Q(\eta_i)$ can be interpreted as the proportion of times the link is featured.

We assume the following functional specifications for $P(\eta_i)$ and $Q(\eta_i)$:

$$P(\eta_i) = \eta_i, \tag{3}$$

$$Q(\eta_i) = 1 - \eta_i + \alpha\eta_i = \alpha + (1 - \eta_i)(1 - \alpha), \quad i = A, B. \tag{4}$$

α is the brand strength parameter. Here we define brand strength by the chance that a brand will be prominent in the organic region when the brand does not engage in SEO. This is an exogenous variable, a fixed value. However, the probability that its link can be prominent in the sponsored region is not an exogenous variable. Firms can increase this probability by increasing investment in SEO. It can take any value between 0 and 1. α equals 0 means that if the company does not invest in SEO, the brand will not automatically appear in the organic region. While α equals 1 means when users search for a product, the brand will appear in the organic region without any investment in SEO.

The functional specifications in (3) and (1) have the following properties. First, when η_i is zero, there is zero probability that brand i will be prominent in the sponsored region. Second, as η_i increases, brand i has a higher probability to win bidding, hence higher chances of prominence [43]. We assume linearity for simplicity. Third, when brand i increases the resources for PSM, its investment in SEO would decrease accordingly. Note that when η_i is one, $Q(\eta_i)$, the probability of prominence in organic region is equal to α. We will call α the *brand strength* parameter. When $\alpha = 1$, the brand will always be prominent in organic region, independent of the amount of budget for SEO.

Given brand i's search engine marketing decision η_i, there are four possible scenarios: brand i is prominent in both organic and sponsored regions, prominent in the organic region only, prominent in sponsored region only, and not prominent in either region. The following probabilities denoted by λ depict the outcomes in both search regions.

$$\lambda_{i,11} = P(\eta_i)Q(\eta_i), \tag{5a}$$

[2] Here we assume two brands in symmetric prominence as equal. Hotchkiss *et al.* [24] tracked consumers' eye movements when they searched products and found no difference among the top three links in both the organic and sponsored regions. Our model of asymmetric prominence can be viewed as a special case of multi-stage sequential search with high search cost [46].

$$\lambda_{i,01} = (1 - P(\eta_i))Q(\eta_i), \tag{5b}$$
$$\lambda_{i,10} = P(\eta_i)(1 - Q(\eta_i)), \tag{5c}$$
$$\lambda_{i,00} = (1 - P(\eta_i))(1 - Q(\eta_i)). \tag{5d}$$

The above expressions (5a)–(5d) are necessary for equations (1) and (2).

Price competition between brands and expected market shares (second stage)

Given the outcomes of search prominence, the competing brands determine their prices simultaneously, and then the consumers make their purchase decisions. The firm delegates the price decision to the brand managers because prices need to be adopted to the changes in the market. Such price delegation and customization becomes necessary with the increasing availability of customer information and direct access to the customers.

Price decisions depend on the search prominence. Under asymmetric prominence, consumers will evaluate and consider only the prominent brand. The prominent brand's expected sales is determined by the marginal consumers who are indifferent between buying and not buying. In this case, sales of the prominent brand (i) is

$$x_i^i = \min\left\{1, \frac{V - p_i^i}{t}\right\}, \quad i = A, B, \tag{6}$$

where the superscripts of sales x_i^i and price p_i^i stand for brand i's prominence in consumer search. Each brand makes price decision to maximize its profit, $p_i^{i*} = \text{argmax}\Pi_i^i = x_i^i \times p_i^i$.

Under symmetric prominence, two brands set the prices simultaneously. Given the brands' prices, each brand's sales is determined by the marginal consumers who are indifferent between buying either brand. In this case, sales of the prominent brand (i) is

$$x_i^{AB} = \frac{1}{2} + \frac{p_j^{AB} - p_i^{AB}}{2t}, \quad i, j = A, B, \quad i \neq j, \tag{7}$$

where the superscripts in both market share and price variables represent symmetric prominence. Each brand makes price decision to maximize own profit. Specifically $p_i^{AB*} = \text{argmax}\Pi_i^{AB} = x_i^{AB} \times p_i^{AB}$.

The above discussions imply that a brand's expected sales and profit vary by the nature of search prominence. In the first stage, a brand (i) expects a probability Φ_{AB} of achieving symmetric prominence and profit Π_i^{AB}, and a probability Φ_i of achieving asymmetric prominence and profit Π_i^i. When the other brand (j) enjoys asymmetric prominence, this brand (i) expects zero profit. Therefore, a brand (i)'s expected profit is $\Pi_i = \Pi_i^{AB} \times \Phi_{AB} + \Pi_i^i \times \Phi_i, \quad i = A, B$.

(1) Interfirm competition
Each brand determines the investment of SEO and PSM based on their profit maximization decisions.

$$\eta_i^* = \underset{\eta}{\text{argmax}}\ \Pi_i \quad i = A, B. \tag{8}$$

(2) Intrabrand competition
In the multi-brand firm, the category manager coordinates the search engine marketing decisions for both brands within the same category. The resource allocation decision between SEO and PSM is set to maximize the total profits from two brands. Thus,

$$\eta^* = \underset{\eta}{\text{argmax}}\ \Pi_A + \Pi_B, \tag{9}$$

where the profit functions are $\Pi_i = \Pi_i^{AB} \times \Phi_{AB} + \Pi_i^i \times \Phi_i, \quad i = A, B$ as explained earlier. The brand manager is forward looking. That is, when making the search engine marketing decisions, the manager anticipates the impact of the decisions on the probabilities of symmetric and asymmetric prominences as well as the impact on subsequent price competition in the second stage.

Next we analyze the model and discuss the equilibrium results. In the main text of the paper, we will focus on describing the results and intuitions, leaving all mathematical proofs in the Appendix A.

3. Model analysis

We follow the backward induction approach to seek for subgame perfect equilibrium. Specifically, we start with the analysis of subgame price equilibrium, and then move backwards to the search engine marketing decisions. Our analysis focuses on the case of sufficiently large values of V ($V > 2.5t$) that the entire market is covered in the equilibrium.

3.1. Subgame pricing equilibrium in Stage 2

At the beginning of Stage 2, the firm has already made search engine marketing decisions. In this stage, each brand (i) makes price decision (p_i) to maximize own expected profit conditional on realized search outcomes. As explained earlier, we assume an online pricing model where the brands know the realized search prominence when offering the prices. Thus, the subgame equilibrium prices depend on search outcomes.

In case of *symmetric prominence*, both brands A and B achieve prominent positions in the results of online search, and consumers consider both brands. This is essentially the baseline Hotelling model with symmetric duopoly. In the subgame equilibrium, the equilibrium prices and profits are symmetric, as follows:

$$p_i^{AB*} = t, \quad \Pi_i^{AB*} = \frac{t}{2}, \quad i = A, B. \tag{10}$$

In case of *asymmetric prominence*, the prominent brand has the option to set a high price and serve a proportion of the market or to set a low price and serve the entire market. As shown in the Appendix A, since V/t is sufficiently large as assumed, it is optimal for the prominent brand to pursue the low margin and high volume strategy. The equilibrium prices and profits are

$$p_i^{i*} = V - t,\ \Pi_i^{i*} = V - t,\ \Pi_j^{i*} = 0; \quad i, j = A, B,\ i \neq j \tag{11}$$

The prominent brand gains more profit under the case of asymmetric prominence than under the case of symmetric prominence. For any brand that fails in achieving the prominence in search outcome, the profit will be zero. Also, the category profit is larger under the case of asymmetric prominence when V/t is sufficiently large as assumed.

3.2. Equilibrium search engine marketing decision in Stage 1

In stage one the brands decide η's (η_i for brand i) – the proportion of search engine marketing budget allocated to PSM. We characterize the equilibrium results in Proposition 1.

Consider the case with $V/t \geq 2.5$.

Proposition 1. *When the competition is between different firms:*

(i) *The optimal allocation of search engine marketing to PSM, $\eta_A^* = \eta_B^* = \eta^*$. When it is interfirm competition, η_D^* is defined by the first order condition: (12)*

$$\begin{aligned} &-(1-\alpha)^2\left(4\frac{V}{t} - 7\right)\eta_D^{*3} + \frac{1}{2}(-1+\alpha)\left((8\alpha - 14)\frac{V}{t} - 11\alpha + 23\right)\eta_D^{*2} \\ &+ \left(\left(-\alpha^2 + 5\alpha - 5\right)\frac{V}{t} + \frac{3}{2}\alpha^2 - 8\alpha + \frac{17}{2}\right)\eta_D^* + \frac{V}{t} + \frac{\alpha}{2} - 2 = 0, \\ &0 \leq \eta_D^* \leq 1. \end{aligned} \tag{12}$$

(ii) *There exists α^* such that $\frac{d\eta_D^*}{d\alpha} > 0$ when $\alpha < \alpha^*$ and $\eta_D^* = 1$ when $\alpha > \alpha^*$.*

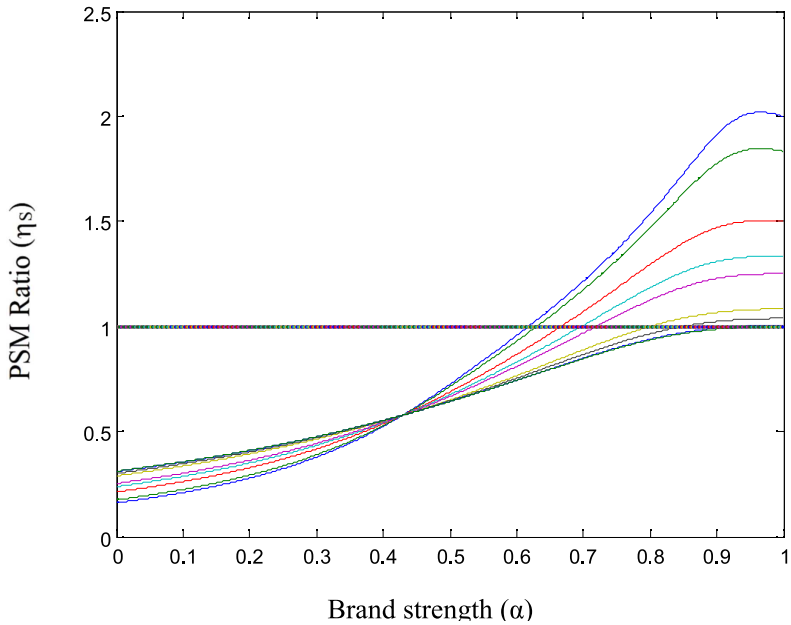

FIGURE 1. Relationship between η_D^* and α.

The first part of Proposition 1 indicates that two brands will make the same allocation decisions. This property is due to the symmetry – both in brand strength and vertical quality – assumed between two brands. Despite this symmetry, the first order condition is very complex. Equation (12) are implicit functions that involves high power of the decision variable. We show the concavity in the Appendix A to ensure that the first-order condition defines the optimal solution.

The second part of Proposition 1 is our key research question regarding the relation between the search engine marketing decision η_D^* and brand strength (α). We examine their relationship in the Appendix A and summarize the result in the next proposition.

We find that the firms should invest in both SEO and PSM when the brand strength is low. With the improvement of brand strength, firms will invest more money from SEO to PSM. When the brand strength improves to a certain level, firms will put all the fund into PSM. Since users trust organic results more, firms prioritize SEO. But any firm does not put all his fund into the SEO, because if one firm, let us say firm A, puts all the fund into SEO, only needs to put a small amount of money into PSM, firm B will be easy to occupy the sponsored region. And there is still a lot of fund left to invested in SEO, so firm B's organic rank even if a little bit lower, but his probability of appearing in both organic region and sponsored region is still very large.

Therefore, when the brand strength is low, the firm will put more fund in SEO, and a small amount in PSM. With the improvement of brand strength, the firm can appear in the organic region with a higher probability all by himself, he will reduce the investment in SEO and increase the investment in PSM. When the brand strength improves to a certain level, the firm can rest assured to put all the funds into the PSM. Since in this situation, if the enterprise takes out a part of funds from the PSM to invest in SEO, the natural search ranking will not be significantly improved, and the promotion link ranking will be easily defeated by the opponent, so the firm will not do that (Fig. 1).

Proposition 2. *When the competition is between brands in the same firm:*

(i) *The optimal allocation of search engine marketing to PSM, $\eta_A^* = \eta_B^* = \eta^*$. When it is interfirm competition, η_S^* is defined by the first order condition: (13)*

$$\begin{aligned} &-2(1-\alpha)^2\left(4\frac{V}{t}-7\right)\eta_S^{*3} - 3(1-\alpha)\left((2\alpha-4)\frac{V}{t}+7-3\alpha\right)\eta_S^{*2} \\ &+\left(-2(2-\alpha)^2\frac{V}{t}+3\alpha^2-14\alpha+15\right)\eta_S^* + (2-\alpha)\left(\frac{V}{t}-2\right) = 0, \\ &\text{where} \quad 0.5 < \eta_S^* < 1. \end{aligned} \tag{13}$$

(ii) *There exists α^* such that $\frac{d\eta_S^*}{d\alpha} > 0$ when $\alpha < \alpha^*$ and $\frac{d\eta_S^*}{d\alpha} < 0$ when $\alpha > \alpha^*$.*

The first part of Proposition 2 indicates that the two brands will make the same allocation decisions, which is the same with Proposition 1, and the reason is also the same. We show the concavity in the Appendix A to ensure that the first-order condition defines the optimal solution. $0.5 < \eta_S^* < 1$ means that a larger proportion of search engine marketing budget should be allocated to PSM than to SEO within multibrand firm. To understand this result, note that in the second stage, the multibrand firm's total profit is higher under asymmetric prominence (monopoly profit) than under symmetric prominence (duopoly profit). In other words, internal brand competition leads to lower category profit. Thus, the category manager of the multibrand firm prefers to have a lower Φ_{AB} and higher $\Phi_i (i = A, B)$.

The firm will not allocate all marketing resources to either SEO or PSM. If $\eta_S^* = 0$, both brands will be prominent in the organic region and none of the brands will be prominent in the sponsored region. In this case, the probability of symmetric prominence Φ_{AB} reaches one, the maximum chance for internal brand competition. This is clearly not an appealing outcome to the firm. The other extreme, $\eta_S^* = 1$, is not optimal for the similar reason. By allocating all of the resources to PSM, both brands will be certain to be prominent in the sponsored region. When $\alpha = 0$ or $\alpha = 1$, the search outcomes will always be symmetrically prominent, leading to maximum internal price competition again.

For the equilibrium property that $0.5 < \eta_S^* < 1$, note that the probability of asymmetric prominence (Φ_i) reaches maximum when each brand has 50% chance of prominence in both the organic and in the sponsored regions. Since brand strength α is positive, relatively less resource should be allocated to SEO and more resources allocated to PSM. Thus, $\eta_S^* > 0.5$.

The second part is our key research question regarding the relation between the search engine marketing decision η_S^* and brand strength (α). We find that the relationship of intrafirm competition is quite different from that of the interfirm competition. Proposition 2 indicates an inverted-U shaped relation between the optimal allocation to PSM and the brand strength. Specifically, the investment in PSM should be increasing with the brand strength when the brand strength is sufficiently low, but decreasing with the brand strength when the brand strength is sufficient high. Figure 2 shows the relationship between η_S^* and brand strength α, each curve corresponding to one unique value of V/t. The pattern is consistent with Proposition 2 and is robust with the value of V/t; the result is also consistent with Proposition 1.

Proposition 2 shows that, a firm should decrease the spending on SEO as the brand strength increases only when the brand strength is sufficiently low. In the low range of brand strength, as the brand strength increases and approaches to 0.5, a firm should indeed allocate more market budget to PSM. However, when the brand strength is sufficiently high and further increases to 1, the firm should shift more search engine marketing budget to SEO. This result appears surprising because the return from further spending on SEO is low when the brand strength is the natural likelihood of prominence in organic search region is already high.

Our analysis indicates the importance to consider the impact of search prominence on internal brand competition. With a high brand strength, it becomes optimal for the firm to further increase the spending on SEO, leaving PSM allocation closer to 0.5. The lower spending on PSM leads to higher probability of asymmetric prominence. Clearly, in achieving the balance between high market reach and low internal competition, a firm's strategic decision on the allocation for PSM and SEO can vary by the brand strength.

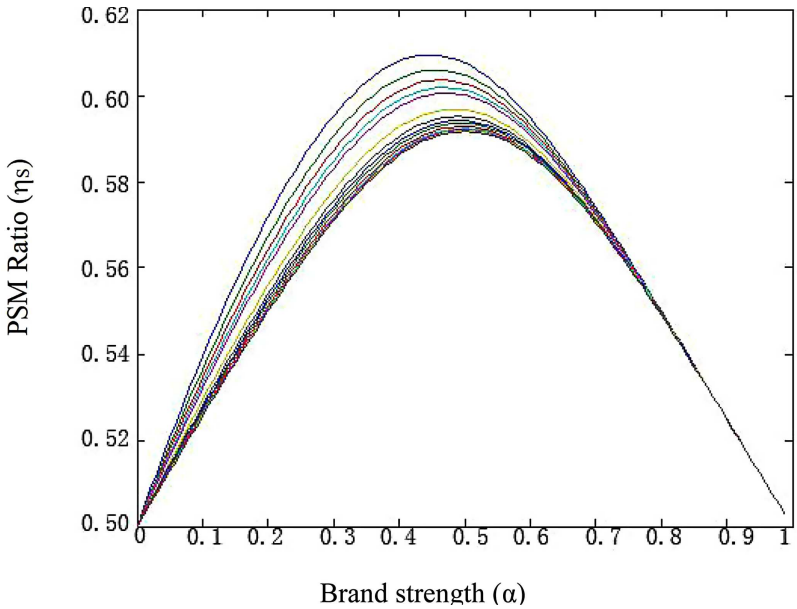

FIGURE 2. Relationship between η_S^* and α.

4. Conclusion and discussion

This paper employs an analytical model to study the search engine marketing decisions of different kinds of competitions. For the interfirm competition, according to the influence of organic region and sponsor region on users, firms give priority to SEO. With the improvement of brand strength, funds are gradually transferred to PSM. For the intrafirm competition, the firm allocates the resources between PSM and SEO, considering the strength of brands in natural search and anticipating the subsequent impact on the intensity of internal brand competition. We find an inverted-U shaped relation between the proportion of resources allocated to PSM and the brand strength. This result underscores the challenges that multi-brand firms face in managing the tension between maximizing brand prominence to attract consumers and minimizing internal price competition through differentiation.

Our main result suggests that all firms should determine their investment in search marketing based on their brand strength and competitors' brand strength. If it is the competition between different firms, firms need to seize the organic region. When the brand strength is low, invest more funds in SEO to improve the organic ranking, while when the brand strength is high, invest more funds in PSM to improve the sponsored ranking.

If it is the competition between brands in a multi-brand firm, the category manager should differentiate two brands through asymmetric prominence depends on the brand strength. If the brand strength is low, then the firm should use both organic and sponsored search outcomes to differentiate two brands' search prominence. In allocating the resource, the firm should balance the allocations between PSM and SEO, and shift more resources to PSM when the brand strength is stronger. However, if the brand strength is sufficiently high, then the firm should rely on only the sponsored search outcome to differentiate search prominence. In this case, the stronger the brand strength, the more resources the firm should allocate to SEO and use the prominence in organic search area to secure consumer exposure to the firm's product. Meantime, the firm may reduce the resources to PSM and differentiate two brands through the different prominent positions in the sponsored region. In this case, the stronger a brand's natural strength, the more prominent we shall observe the brand in the organic search region, but less prominent in the sponsored region.

This paper is analytical in nature and thus the quantitative results are subject to the assumptions and specifications. For instance, Equation (3) assumes that the chance of a brand's prominence in sponsored region goes up from zero to one when the allocated resource to PSM increases from zero to 100%. While such functional property is empirically valid, the specific first-order condition in (12), (13) and the result that $\eta^* > 0.5$ are subject to these assumptions. However, the qualitative result associated with the different line shape relations and the insights should be robust to the assumed specifications. We can make similar claims regarding the assumptions on the symmetry between brands and the monopoly nature of the model.

Appendix A. Coordinating search engine marketing for a multi-brand firm

I. Proofs of Section 2

(1) Analysis of symmetric prominence and asymmetric prominence

The two brands achieve asymmetric prominence in twelve possible scenarios of search outcomes. For the benefit of tractability and the convenience of exposition, we make the following assumption to determine the relative prominence.

Assumption on relative prominence: Featured in both search regions > featured only in organic region > featured only in sponsored region > not featured in either region.

Next we elaborate the above assumption in specific scenarios of search outcomes.

(i) If brand i is prominent in either organic links and/or sponsored links and brand j is not prominent in either type of links, then brand i enjoys sole prominence. This covers six scenarios of search outcomes, including three scenarios of prominence for each brand.

(ii) If brand i is prominent only in the organic region AND brand j is featured only in the sponsored region, then brand i enjoys sole prominence. Consumers perceive the organic links more relevant to their needs because the organic rankings are created by consumer preferences. This is consistent with the empirical evidences in the reports of Group MUK and Nielsen that the click through rate of the prominent organic links was much higher than the sponsored links [20].

(iii) If brand i is prominent in both the organic and the sponsored regions AND brand j is prominent in either one of these two regions, then brand i has sole prominence. This assumption is similar with the exposure effect assumed in Desai *et al.* [16] and is consistent with empirical findings in Ghose and Yang [19].

To summarize the above discussions, we introduce the notations for the probabilities associated with different types of search prominence. We let Φ_{AB} denote the probability that brands A and B achieve *symmetric prominence*, and let Φ_A and Φ_B denote the probability that brand A and B achieve the *asymmetric prominence* respectively. We let $\lambda_{i,SO}$ denote the probability that the search outcome for brand $i \in A, B$ is SO, where $S = 1$ if brand i is prominent in sponsored region and $S = 0$ otherwise, $O = 1$ if brand i is prominent in organic region and $O = 0$ otherwise. Symmetric prominence can be achieved in three scenarios of search outcomes,

$$\Phi_{AB} = \lambda_{A,11}\lambda_{B,11} + \lambda_{A,01}\lambda_{B,01} + \lambda_{A,10}\lambda_{B,10}. \tag{A.1}$$

Similarly, we can express the probabilities of asymmetric prominence by the scenarios of search outcomes. As described earlier, there are six scenarios of search outcomes leading to asymmetric prominence with brand A:

$$\Phi_A = \lambda_{A,11}(1-\lambda_{B,11}) + \lambda_{A,01}\lambda_{B,00} + \lambda_{A,10}(\lambda_{B,01} + \lambda_{B,00}). \tag{A.2}$$

We can write a similar expression for Φ_B.[3]

[3] Here we assume two brands in symmetric prominence as equal. Hotchkiss *et al.* [24] tracked consumers' eye movements when they searched products and found no difference among the top three links in both the organic and sponsored regions. Our model of asymmetric prominence can be viewed as a special case of multi-stage sequential search with high search cost [46].

(2) Search engine marketing and search outcomes

Both brands engage in search engine marketing to attract new customers. Each brand has an exogenously determined and identical search engine marketing budget normalized to one. Brand i allocates η_i proportion of search engine marketing budget to PSM, and the rest $1-\eta_i$ proportion to SEO, $i = A, B$. Given the budget allocation, brand i expects a probability $P(\eta_i)$ that its link will be prominent in the sponsored region, and a probability $Q(\eta_i)$ that its link will be prominent in the organic region. Since marketing budget allocation is a decision covering a longer period of time and search outcomes can vary over time during the period, these probabilities $P(\eta_i)$ and $Q(\eta_i)$ can be interpreted as the proportion of times the link is featured.

We assume the following functional specifications for $P(\eta_i)$ and $Q(\eta_i)$:

$$P(\eta_i) = \eta_i, \tag{A.3}$$

$$Q(\eta_i) = 1 - \eta_i + \alpha\eta_i = \alpha + (1-\eta_i)(1-\alpha), \quad i = A, B. \tag{A.4}$$

The functional specifications in (A.3) and (A.4) have the following properties. First, when η_i is zero, there is zero probability that brand i will be prominent in the sponsored region. Second, as η_i increases, brand i has a higher probability to win bidding, hence higher chances of prominence [43]. We assume linearity for simplicity. Third, when brand i increases the resources for PSM, its investment in SEO would decrease accordingly. Note that when η_i is one, $Q(\eta_i)$, the probability of prominence in organic region is equal to α. We will call α the *brand strength* parameter. When $\alpha = 1$, the brand will always be prominent in organic region, independent of the amount of budget for SEO.

Given brand i's search engine marketing decision η_i, there are four possible scenarios: brand i is prominent in both organic and sponsored regions, prominent in the organic region only, prominent in sponsored region only, and not prominent in either region. The following probabilities denoted by λ depict the outcomes in both search regions.

$$\lambda_{i,11} = P(\eta_i)Q(\eta_i), \tag{A.5a}$$

$$\lambda_{i,01} = (1-P(\eta_i))Q(\eta_i), \tag{A.5b}$$

$$\lambda_{i,10} = P(\eta_i)(1-Q(\eta_i)), \tag{A.5c}$$

$$\lambda_{i,00} = (1-P(\eta_i))(1-Q(\eta_i)). \tag{A.5d}$$

The above expressions (A.5a)–(A.5d) are necessary for equations (1), (2) and (3).

II. Proofs of Section 3: Model solution

We follow the backward induction approach, starting with analysis of subgame price equilibrium in stage 2, and then move backwards to study the search engine marketing decisions in stage 1. We assume that V/t is sufficiently large ($V > 2.5t$) such that the entire market is covered in the equilibrium. (Analysis and results of other cases with smaller V/t are available from the authors.)

Stage 2: Subgame price equilibrium

At the beginning of stage 2, there are two search outcomes – symmetric prominence or asymmetric prominence.

Symmetric prominence: In this case, consumers choose from brand A and B. The market shares of the brands are decided by the location of the marginal consumers indifferent between brand A and B.

$$V - p_A^{AB} - t \times x^{AB} = V - p_B^{AB} - t \times \left(1 - x^{AB}\right), \tag{A.6}$$

where $x^{AB} \in [0,1]$ is the distance from brand A, superscript "AB" indicating prominent brands. The profit functions for A and B are

$$\Pi_A^{AB} = x^{AB} \times p_A^{AB}, \qquad \Pi_B^{AB} = \left(1 - x^{AB}\right) \times p_B^{AB}. \tag{A.7}$$

Using the first-order conditions, we solve the optimal prices and obtain the corresponding profit.

$$p_i^{AB*} = t, \quad \Pi_i^{AB*} = \frac{t}{2}, \quad i = A, B. \tag{A.8}$$

Asymmetric prominence: In this case, consumers consider one brand only; effectively, the prominent brand serves as a monopoly. For the convenience of notation, let us take A as the prominent brand. The consumers who are close to brand A would choose to buy, and those who are located far away would choose not to buy. Sales of brand A is determined by the marginal consumer located at x who is indifferent between buying and not buying:

$$V - p_A^A - t \times x^A = 0. \tag{A.9}$$

From the equation we can solve brand A's market share:

$$x^A = \min\left\{1, \frac{V - p_A^A}{t}\right\}. \tag{A.10}$$

The upper bound at 1 in (A.10) is a natural constraint. The brand chooses the price level such that

$$\text{Max } \Pi_A^A = x^A \times p_A^A. \tag{A.11}$$

Since $V/t > 2.5$, we need to compare the results from a corner solution and the interior solution.

(1) *Interior solution.* Take the first-order condition, $\frac{\partial \Pi_A^A}{\partial p_A^A} = 0$, we have $p_A^A = 0.5V$. Since $V/t > 2.5$, sales of brand A is $x^A = \min\left\{1, \frac{V - p_A^A}{t}\right\} = 1$ and the corresponding profit for brand A is $0.5V$.

(2) *Corner solution.* We set the maximum price for the brand such that the brand's sales is equal to one. Thus, $x = 1$, and price $p_A^A = V - t$, leading to a profit equal to $V - t$.

Since $V/t > 2.5$, profit at corner solution $(V - t) > V/2$. Thus, the optimal price and profit in this case is

$$p_A^A = V - t \qquad \Pi_A^A = V - t. \tag{A.12}$$

Stage 1: Equilibrium decision on search engine marketing

Two brands A and B have the same chance (Φ_{AB}) of reaching symmetric prominence, and each brand i has the probability of Φ_i ($i = A, B$) achieving sole prominence. Summarizing the results from subgame equilibrium, the expected profits for brand i is as follows:

$$\Pi_i = \frac{t}{2} \times \Phi_{AB} + (V - t) \times \Phi_i \quad i = A, B. \tag{A.13}$$

As we can see Φ_{AB}, Φ_i from the above context, we can get the following formulas

$$\begin{aligned} \Phi_{AB} = &\Big(3(1-\alpha)^2\eta_B^2 - \big(\alpha^2 - 4\alpha + 3\big)\eta_B + (1-\alpha)\Big)\eta_A^2 \\ &+ \big(\big(-\alpha^2 + 4\alpha - 3\big)\eta_B^2 + \big(\alpha^2 - 4\alpha + 5\big)\eta_B - (2-\alpha)\big)\eta_A \\ &+ (1-\alpha)\eta_B^2 - (2-\alpha)\eta_B + 1 \end{aligned} \tag{A.14}$$

$$\begin{aligned}\Phi_i = &\Big(-2(1-\alpha)^2\eta_j^2 + \big(2\alpha^2 - 5\alpha + 3\big)\eta_j - (1-\alpha)\Big)\eta_i^2 \\ &+ \big((1-\alpha)\eta_j^2 + \big(-\alpha^2 + 3\alpha - 3\big)\eta_j + 1\big)\eta_i + (1-\alpha)\eta_j, \quad i = A, B\end{aligned} \tag{A.15}$$

As the two brands are symmetric, the allocation decisions for two brands are exogenously determined to be the same, *i.e.*, $\eta_A = \eta_B = \eta$.

(1) *Interfirm competition*

Each brand manager maximize his profit.

$$\Pi_i = t\left\{\begin{array}{l}\left[\begin{array}{l}-(1-\alpha)(1-\eta_j)(2\alpha\eta_j - 2\eta_j + 1)\eta_i^2 + ((1-\alpha)\eta_j^2 \\ +(-\alpha^2 + 3\alpha - 3)\eta_j + 1)\eta_i + (1-\alpha)\eta_j\end{array}\right]\frac{V}{t} \\ +\frac{1}{2}\left[\begin{array}{l}-(1-\alpha)(7(1-\alpha)\eta_j^2 + (9-5\alpha)\eta_j - 3)\eta_i^2 + ((1-\alpha)(\alpha-5)\eta_j^2 \\ +(3\alpha^2 - 10\alpha + 11)\eta_j + (\alpha-4))\eta_i + (1-\alpha)\eta_j^2 + (3\alpha-4)\eta_j + 1\end{array}\right]\end{array}\right\}. \tag{A.16}$$

The equilibrium level of location η is determined by $\eta_{iD}^* = \underset{\eta}{\operatorname{argmax}}\,\Pi_i \quad i = A, B$.

We first prove that (i) η_D^* *is an increasing relationship with* α, and then prove that (ii) *the first-order condition defines the equilibrium outcome*. Since to prove (ii) we need the result of (i).

Analysis: Relation between equilibrium ratio (η_D^*) and baseline value α

Each firm maximize his profit with respect to each η_i separately.

$$\frac{\partial \Pi_A}{\partial \eta_A} = 0, \quad \frac{\partial \Pi_B}{\partial \eta_B} = 0. \tag{A.17}$$

Since the equilibrium is symmetric, the optimal decision of each firm is the same, $\eta_A = \eta_B$. It is sufficient to study the property with one brand. We get the following condition with η_D^*.

$$\begin{aligned}&-(1-\alpha)^2\left(4\frac{V}{t} - 7\right)\eta_D^{*3} + \frac{1}{2}(-1+\alpha)\left((8\alpha - 14)\frac{V}{t} - 11\alpha + 23\right)\eta_D^{*2} \\ &+\left(\big(-\alpha^2 + 5\alpha - 5\big)\frac{V}{t} + \frac{3}{2}\alpha^2 - 8\alpha + \frac{17}{2}\right)\eta_D^* + \frac{V}{t} + \frac{\alpha}{2} - 2 = 0.\end{aligned} \tag{A.18}$$

Let

$$\begin{aligned}F_1 = &-(1-\alpha)^2\left(4\frac{V}{t} - 7\right)\eta_D^{*3} + \frac{1}{2}(-1+\alpha)\left((8\alpha - 14)\frac{V}{t} - 11\alpha + 23\right)\eta_D^{*2} \\ &+\left(\big(-\alpha^2 + 5\alpha - 5\big)\frac{V}{t} + \frac{3}{2}\alpha^2 - 8\alpha + \frac{17}{2}\right)\eta_D^* + \frac{V}{t} + \frac{\alpha}{2} - 2.\end{aligned} \tag{A.19}$$

Differentiating with respect to α, we can get the following expression.

$$\frac{\mathrm{d}\eta_D^*}{\mathrm{d}\alpha} = -\frac{\partial F_1/\partial\alpha}{\partial F_1/\partial\eta_D^*}, \tag{A.20}$$

$$\begin{aligned}\frac{\partial F_1}{\partial \eta_D^*} = &3(1-\alpha)^2\left(7 - 4\frac{V}{t}\right)\eta_D^{*2} - (1-\alpha)\left((8\alpha - 14)\frac{V}{t} + 23 - 11\alpha\right)\eta_D^* \\ &+\big(-\alpha^2 + 5\alpha - 5\big)\frac{V}{t} + \frac{3}{2}\alpha^2 - 8\alpha + 8.5.\end{aligned} \tag{A.21}$$

We find $\frac{\partial F_1}{\partial \eta_D^*} < 0$ because (a) the quadratic term coefficient of (A.21), which we denote by a_1, is negative ($a_1 < 0$), and (b) the discriminant of (A.21), denoted by Δ_1, is negative ($\Delta_1 < 0$).

To show (a), since $\frac{V}{t} > 2.5$, then $7 - 4\frac{V}{t} < 0$. So

$$\alpha_1 = 3(1-\alpha)^2\left(7 - 4\frac{V}{t}\right) < 0. \tag{A.22}$$

To show (b), note that

$$\Delta_1 = (\alpha-1)^2\left(\left(16\frac{V^2}{t^2} - 20\frac{V}{t} - 5\right)\alpha^2 + \left(16\frac{V^2}{t^2} - 128\frac{V}{t} + 166\right)\alpha - 44\frac{V^2}{t^2} + 184\frac{V}{t}\right). \tag{A.23}$$

We set $\Delta_1' = \left(16\frac{V^2}{t^2} - 20\frac{V}{t} - 5\right)\alpha^2 + \left(16\frac{V^2}{t^2} - 128\frac{V}{t} + 166\right)\alpha - 44\frac{V^2}{t^2} + 184\frac{V}{t}$. The quadratic term coefficient of Δ_1', which we denote by a_1', is positive ($a_1' > 0$) when $V/t > 2.5$.

$$a_1' = 16\frac{V^2}{t^2} - 20\frac{V}{t} - 5 > 0. \tag{A.24}$$

As the quadratic term coefficient is positive, we can get the biggest Δ_1' when α =0 or $\alpha = 1$

$$\alpha = 0 \quad \Delta_1' = -44\frac{V^2}{t^2} + 184\frac{V}{t} - 185 < 0 \quad \Delta_1' < 0, \tag{A.25}$$

$$\alpha = 1 \quad \Delta_1' = -12\frac{V^2}{t^2} + 36\frac{V}{t} - 24 < 0 \quad \Delta_1' < 0. \tag{A.26}$$

Then we know that Δ_1' is always negative. So as to Δ_1. As the quadratic term coefficient a_1 and the discriminant Δ_1 are all negative, we confirm $\frac{\partial F_1}{\partial \eta_D^*} < 0$.

$$\frac{\partial F_1}{\partial \alpha} = 2(1-\alpha)^2\left(4\frac{V}{t} - 7\right)\eta_D^{*3} + \left((8\alpha - 11)\frac{V}{t} + 17 - 11\alpha\right)\eta_D^{*2} + \left((5-2\alpha)\frac{V}{t} + 3\alpha - 8\right)\eta_D^* + \frac{1}{2}\cdot \tag{A.27}$$

Substituting the η_D^{*3} from (A.18) to (A.27), we can get

$$\frac{\partial F_1}{\partial \alpha} = -\frac{1}{2(1-\alpha)}\left(6\left(\frac{V}{t} - 2\right)(-1+\alpha)\eta_D^{*2} + \left((10-6\alpha)\frac{V}{t} - 18 + 10\alpha\right)\eta_D^* + 7 - \alpha - 4\frac{V}{t}\right). \tag{A.28}$$

We set

$$F_\alpha = 6\left(\frac{V}{t} - 2\right)(-1+\alpha)\eta_D^{*2} + \left((10-6\alpha)\frac{V}{t} - 18 + 10\alpha\right)\eta_D^* + 7 - \alpha - 4\frac{V}{t}\cdot \tag{A.29}$$

The quadratic term coefficient of the above formula, which we denote by a_2, is negative ($a_2 < 0$) when $V/t > 2.5$

$$\alpha_2 = 6\left(\frac{V}{t} - 2\right)(-1+\alpha) < 0. \tag{A.30}$$

We set it's symmetry axis as l_2.

$$l_2 = \frac{(3\alpha - 5)\frac{V}{t} + 9 - 5\alpha}{\left(6\frac{V}{t} - 12\right)(\alpha - 1)}\cdot \tag{A.31}$$

We find that $\frac{\partial l_2}{\partial \alpha} = \frac{(1-\alpha)^2}{3} > 0$. So l_2 goes up as α goes up, α goes up to a point where l_2 has to be greater than 1, but η_D^* can only go up to 1 at most. We find that: when $V/t < 3$, l_2 will always bigger than 1.

When $V/t \geq 3$, when $\alpha > (V/t - 3)/(3V/t - 7)$, $l_2 > 1$, the maximum value of $(V/t - 3)/(3V/t - 7)$ is $\frac{1}{3}\cdot$ So if $\alpha > \frac{1}{3}$, $l_2 > 1$.

In this case, F_α get the maximum value when $\eta_D^* = 1$, which is

$$F_\alpha = 1 - 3\alpha. \tag{A.32}$$

What we find from the graph is that η_D^* increases with α. We suppose that η_D^* increases with α, and then we prove that η_D^*does increases with α.

Setting $\eta_D^* = 1$, we get the smallest $\alpha = 0.6180$ ($V/t = 2.5$). So when $\eta_D^* = 1$, $\alpha \geq 0.6180$. Then $F_\alpha = 1 - 3\alpha > 0$. We show the result as follows.

When $\eta = 1$, we get

$$\left(3 - \frac{V}{t}\right)\alpha^2 + \left(2\frac{V}{t} - \frac{9}{2}\right)\alpha - \frac{V}{t} + 2 = 0. \tag{A.33}$$

We set

$$f = \left(3 - \frac{V}{t}\right)\alpha^2 + \left(2\frac{V}{t} - \frac{9}{2}\right)\alpha - \frac{V}{t} + 2, \tag{A.34}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}(V/t)} = -\frac{\partial f/\partial\alpha}{\partial f/\partial(V/t)}, \tag{A.35}$$

$$\frac{\partial f}{\partial\alpha} = 2(1-\alpha)\frac{V}{t} + 6\alpha - 4.5 > 2(1-\alpha) \times 2.5 + 6\alpha - 4.5 = \alpha + 0.5 > 0, \tag{A.36}$$

$$\frac{\partial f}{\partial(V/t)} = -(1-\alpha)^2 < 0. \tag{A.37}$$

So $\frac{\mathrm{d}\alpha}{\mathrm{d}(V/t)} > 0$. Then we get the smallest $\alpha = 0.6180$ when $V/t = 2.5$.

When $V/t > 3$ and $\alpha \leq (V/t - 3)/(3V/t - 7)$, we know that l_2 decreases with α, then we get the smallest l_2 which is $l_2 = 5V/t - 9V/6t - 12$, here l_2 is decreasing with the increasing of V/t, so we get the smallest $l_2 = 5/6$ when $\alpha = 0$ and $V/t = 3$.

We just assumed that η_D^* increases with the increasing of α. So when $\alpha = 1/3$, we get the biggest $\eta_D^* = 0.5$. While l_2 is $5/6$ at least. So when $\eta_D^* = 0.5$, we get the biggest $F_\alpha = \left(1 - 1.5\frac{V}{t}\right)\alpha + 1 - 0.5\frac{V}{t} < 0$.

To sum up, F_α is always negative. And $\frac{\partial F_1}{\partial\alpha}$ is always positive. Considering what we confirmed before $\frac{\partial F_1}{\partial\eta_D^*} < 0$, we can get $\frac{\mathrm{d}\eta_D^*}{\mathrm{d}\alpha} = -\frac{\partial F_1/\partial\alpha}{\partial F_1/\partial\eta_D^*} > 0$, which also verifies our hypothesis.

Proof. Solution for the claim that the first-order condition defines the equilibrium outcome.

We prove by showing that the second-order derivative with respect to η is negative.

$$\frac{\mathrm{d}^2\Pi_i}{\mathrm{d}\eta_i^2} = t(1-\alpha)\left(\left(4\frac{V}{t} - 7\right)(-1+\alpha)\eta_j^2 + \left((6-4\alpha)\frac{V}{t} - 9 + 5\alpha\right)\eta_j - 2\frac{V}{t} + 3\right) \quad i \neq j = A, B. \tag{A.38}$$

Let

$$L = \left(4\frac{V}{t} - 7\right)(-1+\alpha)\eta_j^2 + \left((6-4\alpha)\frac{V}{t} - 9 + 5\alpha\right)\eta_j - 2\frac{V}{t} + 3 \quad i = A, B. \tag{A.39}$$

As $t^*(1-\alpha) > 0$, all we have to prove is L is negative.

The quadratic term coefficient of (A.39), which we denote by a_3, is negative ($a_3 < 0$)

$$\alpha_3 = \left(4\frac{V}{t} - 7\right)(-1+\alpha) < 0. \tag{A.40}$$

We set it's symmetry axis as l_3.

$$l_3 = \frac{(6-4\alpha)\frac{V}{t} - 9 + 5\alpha}{2\left(4\frac{V}{t} - 7\right)(1-\alpha)} \qquad \frac{\mathrm{d}l_3}{\mathrm{d}\alpha} = \frac{\left(\frac{V}{t} - 2\right)\left(4\frac{V}{t} - 2\right)}{(1-\alpha)^2} > 0 \tag{A.41}$$

And we just showed you that η_D^* increases with α, and we know from the formula (A.41) that l_3 increases with α, so α increases to a point where l_3 would be greater than 1, and at this point η_D^* is going to have to be at most 1. We get the following formulas from the calculation.

When $\alpha > (2V/t - 5)/(4V/t - 9)$, $l_3 > 1$, and $(2V/t - 5)/(4V/t - 9) < 0.5$. So if $\alpha > 0.5$, $l_3 > 1$.

So this is where the maximum value of L, which is $L = 1 - 2\alpha$ when η_D^* is equal to 1, and we analyse that when η_D^* is equal to 1, $\alpha > 0.6180$, so $L < 0$.

$\alpha \leq 0.5$, we get the smallest $l_3 = \frac{3V/t - 4.5}{4V/t - 7}$ when $\alpha = 0$. And the smallest $l_3 = 0.75$ when $V/t = 2.5$. But we get the biggest $\eta_D^* = 0.7244$ ($V/t = 2.5$), which is smaller than l_3. We get the biggest $L = -0.2517$ when $\alpha = 0.5$, $\eta_D^* = 0.7244$ and $V/t = 2.5$, which is negative.

Since $\frac{\mathrm{d}L}{\mathrm{d}\alpha} = \eta_D^*\left((4\eta_D^* - 4)\frac{V}{t} + 5 - 7\eta_D^*\right) < 0$, we find that $\eta_D^* = 0.7244$ and $V/t = 2.5$, when $\alpha \geq 0.3771$, $L < 0$. While $\alpha < 0.3771$, $L > 0$. But now η_D^* is not 0.7244 any more. We calculate the actual η_D^* when $\alpha = 0.3771$, and the biggest $\eta_D^* = 0.4759$ ($V/t = 2.5$), now we get $L = -0.4549 < 0$.

When we set $\eta_D^* = 0.4759$, we find that only when $\alpha \geq 0.1035$, $L < 0$. While $\alpha < 0.1035$, $L > 0$. But now η_D^* is not 0.4759 any more. We calculate the actual η_D^* when $\alpha = 0.1035$, and the biggest $\eta_D^* = 0.2121$ ($V/t = 2.5$), now we get $L = -0.9582 < 0$. And we find for all the α, $L < 0$.

Based on analysis of that above, we get L is always negative. And $\frac{\mathrm{d}^2\Pi_i}{\mathrm{d}\eta_i^2}$ is always negative. □

(2) Intrafirm competition

The category manager maximizes the total category profit.

$$\Pi_A + \Pi_B = t\begin{pmatrix} -(\alpha-1)^2\left(4\frac{V}{t} - 7\right)\eta^4 + 2(\alpha-1)\left(2(\alpha-2)\frac{V}{t} - 3\alpha + 7\right)\eta^3 \\ +2\left(-(\alpha-2)^2\frac{V}{t} + \frac{3}{2}\alpha^2 - 7\alpha + \frac{15}{2}\right)\eta^2 + 2(2-\alpha)\left(\frac{V}{t} - 2\right)\eta + 1 \end{pmatrix}. \tag{A.42}$$

The equilibrium level of location η is determined by

$$\eta_{S,i}^* = \underset{\eta}{\operatorname{argmax}}(\Pi = \Pi_A + \Pi_B). \tag{A.43}$$

Proof. Solution for the claim that the first-order condition defines the equilibrium outcome.

We prove by showing that the second-order derivative with respect to η is negative.

$$\begin{aligned} \frac{\mathrm{d}^2\Pi}{\mathrm{d}\eta^2} &= -6(\alpha-1)^2\left(4\frac{V}{t} - 7\right)\eta^2 \\ &\quad + 6(\alpha-1)\left(2(\alpha-2)\frac{V}{t} - 3\alpha + 7\right)\eta + \left(-2(\alpha-2)^2\frac{V}{t} + 3\alpha^2 - 14\alpha + 15\right). \end{aligned} \tag{A.44}$$

The second-order derivative is negative because (1) the quadratic term coefficient of (A.44), which we denote by a, is negative ($a < 0$), and (2) the discriminant of (A.44), denoted by Δ, is negative ($\Delta < 0$).

To show (1), since $V/t > 2.5$, then $4V/t - 7 > 0$. So

$$\alpha = -6(\alpha-1)^2\left(4\frac{V}{t}-7\right) < 0. \tag{A.45}$$

To show (2), note that

$$\Delta = -12(\alpha-1)^2\Big(4(\alpha-2)^2\Big)\left(\frac{V}{t}\right)^2 + \big(-16\alpha^2+68\alpha-64\big)\frac{V}{t} + 15\alpha^2 - 70\alpha + 63). \tag{A.46}$$

We set

$$\Delta' = 4(\alpha-2)^2)\left(\frac{V}{t}\right)^2 + \big(-16\alpha^2+68\alpha-64\big)\frac{V}{t} + 15\alpha^2 - 70\alpha + 63. \tag{A.47}$$

Expression Δ' is a quadratic function of V/t that points upwards. We set it's symmetry axis as L.

$$L = \frac{16\alpha^2-68\alpha+64}{8(1-\alpha)^2} = 2 - \frac{4\alpha}{8(1-\alpha)^2} < 2. \tag{A.48}$$

Since the function of Δ' is upwards and it's symmetry axis is less than 2, we can get the smallest Δ' equals to 3 (setting $V/t = 2.5$). Since the smallest Δ' is positive, the Δ^* must be positive when $V/t > 2.5$. As $\Delta = -12(\alpha-1)^2\Delta'$, then we can conclude that Δ is always negative.

Analysis: Relation between equilibrium ratio (η^*) and baseline value α

Since the equilibrium is symmetric, it is sufficient to study the property with one brand. η^* must satisfy the following condition.

$$\frac{\partial \Pi_i}{\partial \eta} = \frac{t}{2}\frac{\partial \Phi_{AB}}{\partial \eta} + (V-t)\frac{\partial \Phi_i}{\partial \eta} = 0. \tag{A.49}$$

Substituting the derivatives from (A.14) and (A.15),

$$\begin{aligned}&\Big[14(1-\alpha)^2\eta^{*3} + 3\big(-3\alpha^2+10\alpha-7\big)\eta^{*2} + \big(3\alpha^2-14\alpha+15\big)\eta^* + 2\alpha - 4\Big]\frac{t}{V}\\ &\quad - 8(1-\alpha)^2\eta^{*3} + 3\big(2\alpha^2-6\alpha+4\big)\eta^{*2} + 2\big(-\alpha^2+4\alpha-4\big)\eta^* + 2 - \alpha = 0, \quad \text{where } 0 \le \eta^* \le 1.\end{aligned} \tag{A.50}$$

Then we confirm that the equilibrium result η_i^* first increases and then decreases with the quality.

Let

$$\begin{aligned}F = &\Big[14(1-\alpha)^2\eta^{*3} + 3\big(-3\alpha^2+10\alpha-7\big)\eta^{*2} + \big(3\alpha^2-14\alpha+15\big)\eta^* + 2\alpha - 4\Big]\frac{t}{V}\\ &- 8(1-\alpha)^2\eta^{*3} + 3\big(2\alpha^2-6\alpha+4\big)\eta^{*2} + 2\big(-\alpha^2+4\alpha-4\big)\eta^* + 2 - \alpha, \quad \text{where } 0 \le \eta^* \le 1.\end{aligned} \tag{A.51}$$

Differentiating with respect to α, we can get the following expression.

$$\begin{aligned}\frac{\mathrm{d}\eta^*}{\mathrm{d}\alpha} &= -\frac{\partial F/\partial \alpha}{\partial F/\partial \eta^*}\\ &= \frac{4(\alpha-1)\big(4\frac{V}{t}-7\big)\eta^{*3} - \big(-18\alpha+30+\frac{V}{t}(12\alpha-18)\big)\eta^{*2} - \big(6\alpha-14+\frac{V}{t}(8-4\alpha)\big)\eta^* + \frac{V}{t} - 2}{-6(1-\alpha)^2\big(4\frac{V}{t}-7\big)\eta^{*2} + 6(\alpha-1)\big(2(\alpha-2)\frac{V}{t} - 3\alpha + 7\big)\eta^* - 2(\alpha-2)^2\frac{V}{t} + 3\alpha^2 - 14\alpha + 15}.\end{aligned} \tag{A.52}$$

(1) Denominator: we can see that the denominator part is the same with (A.44), we confirm that (A.44) is negative.
(2) Numerator: we denote the numerator of function (A.52) as P.

$$P = 4(\alpha-1)\left(4\frac{V}{t}-7\right)\eta^{*3} - \left(-18\alpha+30+\frac{V}{t}(12\alpha-18)\right)\eta^{*2} - \left(6\alpha-14+\frac{V}{t}(8-4\alpha)\right)\eta^{*} + \frac{V}{t} - 2. \tag{A.53}$$

Substituting the η^{*3} from (A.50) to (A.53), we can get

$$P = -\frac{\left(\left(6\eta^{*2}-8\eta^{*}+3\right)-\alpha\left(6\eta^{*2}-4\eta^{*}+1\right)\right)\left(\frac{V}{t}-2\right)}{1-\alpha}. \tag{A.54}$$

We let $\eta^* > 0.5$, which we can confirm later. Then we can get

$6\eta^{*2}-8\eta^{*}+3 < 6\eta^{*2}-4\eta^{*}+1$ and $6\eta^{*2}-8\eta^{*}+3 > 0 \quad 6\eta^{*2}-4\eta^{*}+1 > 0$.

So when α is very small, like the extreme value 0, $6\eta^{*2}-8\eta^{*}+3 > \alpha\left(6\eta^{*2}-4\eta^{*}+1\right)$,we get $P < 0$; When α increases to a certain value, like the extreme 1, $6\eta^{*2}-8\eta^{*}+3 < \alpha\left(6\eta^{*2}-4\eta^{*}+1\right)$, we get $P > 0$.

According to the above discussion, we conclude that there exists α^* such that $\frac{\mathrm{d}\eta^*}{\mathrm{d}\alpha} > 0$ when $\alpha < \alpha^*$ and $\frac{\mathrm{d}\eta^*}{\mathrm{d}\alpha} < 0$ when $\alpha > \alpha^*$.

Given the above discussions, we have the smallest η^* when α equals 0 or 1. We set $\alpha = 0/1$ into equation (A.50), we get $\eta^* = 0.5$. Then every η^* is bigger than 0.5, which is consistent with the assumption $\eta^* > 0.5$. End of proofs. □

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