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JOINT EFFECT OF SELLING PRICE AND PROMOTIONAL EFFORTS ON RETAILER'S INVENTORY CONTROL POLICY WITH TRADE CREDIT, TIME-DEPENDENT HOLDING COST, AND PARTIAL BACKLOGGING UNDER INFLATION

Sharad Kumar[1], S.R. Singh[2], Seema Agarwal[1] and Dharmendra Yadav[3,*]

Abstract. Technology improvements in the retail industry influence the buying behaviours of customers. In the retail industry, it has been observed that the selling price of goods and promotional efforts influence a customer's choice. In the retail sector, the popularity of financing schemes *i.e.*, trade credit offered by suppliers rather than financial institutions has also grown. Taking such a scenario into consideration and with reference to the retail sector, an inventory model has been developed for non-instantaneous deteriorating items. Effect of inflation also incorporated in model. Customers' demand is affected by the selling price of the product and the retailer's promotional efforts. During a shortage period, the backlogging rate of demand is considered a function of the waiting period. The retailer can also take advantage of a trade credit facility provided by the supplier. Furthermore, holding cost is time-dependent, and an investment is made to reduce ordering cost. Various theoretical results are obtained that maximize the retailer's total profit. To gain better managerial insights, sensitive analysis and numerical examples are provided. The results indicate that the retailer's profit increases as the trade credit period increases. Further, the profit of the retailer increases if the retailer deals in products with a longer non-deteriorating period. Time-dependent holding cost shows a significant impact on the profit of retail. In addition to this, different existing papers in literature show the special case of the current model.

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1. Introduction

Traditionally, researchers have addressed the issue of deterioration while designing the optimal policy for the retailer's inventory problem by considering that the deterioration process starts as soon as stock arrives in the stock [50]. Practically, this assumption gives vague results from the retailer's perspective. Many goods in the retail industry, such as vegetables, fruits, sweet potatoes, jimikand, electronic items, and so on, have a

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[1] Department of Mathematics, S.S.V. College, Hapur, Uttar Pradesh, India.

[2] Department of Mathematics, CCS University, Meerut, Uttar Pradesh, India.

[3] Department of Mathematics, Vardhaman College, Bijnor, Uttar Pradesh, India.

*Corresponding author: dharmendrayadav3580@gmail.com

shelf life. After that time, the quality of the goods began to deteriorate. Technically, this situation is termed non-instantaneous deterioration (Khan *et al.*, [30]). From a retailer's point of view, it is very important to think about deterioration that does not happen right away while making the optimal inventory control policy.

Traditionally, in inventory management, demand rate is considered independent of selling price, and promotional activities. However, in the market of developing countries like India, Pakistan, Bangladesh, etc., customer demand fluctuates because of the selling price [16,56] and the promotional activities run by the retailer [25,45,57]. Because of the promotional activities, customers become familiar with the features and descriptions of the products. Customers are attracted when they come across a product of the same quality at a lower price. This is common tendency that the customers are always willing to purchase a good product at a low price. So, in the market, there are many products that people are purchasing rapidly because of the fall in the selling price and the promotional efforts.

Traditionally, in inventory management, holding cost is considered a constant [17, 60]. But this assumption for deteriorating products is not relevant. This is because as time passes, holding cost increases because of deterioration. Holding cost increases with time for products such as medicines, fruits, vegetables, volatile liquids, etc. Thus, it is crucial to consider the time-dependent holding cost for such products while making inventory decisions.

Situation of inadequacy of certain products in the stock to satisfy the customer's demand is termed shortage. Different customers have different reactions to the stockout situation. In a monopolistic market situation, customers must wait for an upcoming lot to satisfy their demand. In a competitive market situation, some are willing to wait for an upcoming lot while others are left to purchase the products from another source. Therefore, partial backlogging is the practical tool to reflect the inventory situation in such a cutthroat competition.

Following important issues motivate us to come with the current work:

(1) To present the EOQ inventory model for finding the optimal profit for the retailer.
(2) In practice, the deterioration process does not begin as soon as it is received in the stock. So, non-instantaneous deterioration is considered.
(3) A more practical situation of shortages is considered in the model, *i.e.*, partial backlogging.
(4) Practically, promotional efforts have a positive impact on demand, whereas it is negatively affected by the selling price. In the current model, demand is selling price and promotional efforts dependent.
(5) A more frequent financing option opted for by the retailer in the market, *i.e.*, trade credit, is considered.

The rest of the paper is organized as follows: The literature review and the novelty of the work are contained in Section 2. Assumptions and notations are presented in Section 3. In Section 4, a mathematical formulation of the retailer's inventory model is presented. The solution procedure is presented in Section 5, and some theoretical results are incorporated in Section 6. Section 7 comprises numerical analysis, and Sections 7.1 and 7.2 consists of sensitivity analysis along with managerial insights. Concluding remarks and future extensions are provided in Section 8.

2. Literature review and novelty of research

In this section, literature review is carried out to identify the research gap.

2.1. Inventory model for non-instantaneous deterioration

Because of technological advancements, customers' expectations of product quality have shifted. They anticipated that the goods would be in good condition when they arrived. Non-instantaneous deterioration is the technical term for this situation. To decide the replenishment policy from the retailer's point of view, deterioration plays an important role. According to the literature, Wu *et al.* [86] were the first researchers to design the optimal policy for retailers by considering non-instantaneous deterioration where demand was stock-dependent. By considering permissible delays in payment, Ouyang *et al.* [46] further extended the model of Wu *et al.* [86]. Chang *et al.* [9] extended Wu *et al.* [86]'s model by taking stock-dependent demand into account. Dye

[19] explored the inventory model by considering non-instantaneous deteriorating items. They suggested that a higher service rate could be achieved because of high investment in preservation technology. Tiwari *et al.* [80] investigated the issue of non-instantaneous deterioration for a two-warehouse system when inflation is present, allowing for delay in payment and partial backlogging. For non-instantaneous deteriorating items, Jaggi *et al.* [27] shown that higher length of non-deteriorating item leads small replenishment cycle. Because of this, average profit of the system increases. For non-instantaneous deteriorating goods, Tayal *et al.* [79] studied a retailer's inventory problem by taking time-dependent holding cost and exponential demand. Li *et al.* [33] analyzed the effect of preservation technology on goods that deteriorate non-instantaneously. They observed that investment efficiency parameter of technology and the disposal cost have non-monotonic impacts on the optimal investment. In the model, they also considered the waiting-time-dependent backlogging rate, price-dependent demand, and time-dependent deterioration rate. For non-instantaneous deterioration, Liao *et al.* [34] developed an EOQ model with a partial backlogging and allowable payment delay. They developed four different theorems when the lot has some imperfect items. An integrated inventory model to control the deterioration rate with the help of preservation technology is proposed by Pervin *et al.* [54]. They suggested that vendors adopt the mechanism of preservation technology only if the rate of deterioration is high. By considering various business strategies and non-instantaneous deterioration process, Barman *et al.* [5] presented an integrated model with shortages. They observed that, from a financial point of view, an integrated model without shortage is better than an integrated model with shortage. Mashud *et al.* [39] presented an inventory model for non-instantaneous deterioration with backorder to represent the lifetime of product freshness. They suggested that on increasing the investment in green technology, profit of the system increases. Mashud *et al.* [40] developed an inventory model to control the deterioration rate, carbon emission, and ordering cost. In the case of a shortage, profits increased by 46.30%, while profits increased by 94.75% in the absence of a shortage. In the current work, non-instantaneous deterioration process is considered.

2.2. Inventory model with multi-variate demand

Promotional efforts undertaken by the company's sales team *via* electronic or print media are one of the most effective methodologies to make customers familiar with the properties and features of the product. Customer's demand is positively related to promotional efforts taken by the sales team. Further, it is also observed that an increase in selling price has a negative impact on demand especially in developing countries. Goyal and Gunasekaran [23] examined an inventory problem for the pharmaceutical and food industries by considering multi-stage production. They assumed that the selling price per unit of item and the frequency of advertisement determine demand. Under various pricing conditions, Dave *et al.* [15] developed an advertisement-inclusive production model. In this direction, a multi-item EOQ model was explored by Sana [62] for ameliorating and deteriorating items and investigated the effect of salesmen's effort on optimal solution. For non-instantaneous deteriorating items, Shah *et al.* [70] presented a retailer's inventory maximisation model where demand is determined by advertising and selling price. They observed that higher holding cost for non-instantaneous deteriorating items results lower selling price. A retailer inventory model is presented by Shastri *et al.* [73]. They considered that retailer accept trade credit period offered by supplier and assumed that customer demand is price dependent. Palanivel and Uthayakumar [49] investigated an important issue related to inventory problem *i.e.*, non-instantaneous deterioration process under the inflationary condition. They considered that demand depends on advertisement and assumed that partial backlogging rate is a function of waiting time. They suggested that more advertisement results in higher demand and hence, higher ordering cost. An EOQ model is investigated by Yadav *et al.* [88] by considering of selling price-dependent demand while accounting for inflation and learning. Manna *et al.* [37] presented an inventory model by considering advertisement-dependent demand. They assumed that advertisement rate increases over time with a decreasing rate due to the introduction of other brands of the same product. They demonstrated that as the cost of advertising increased, the average profit initially increased and then fell. They said that it happened because of market saturation. EOQ model was investigated by Goyal *et al.* [24]. They assumed that demand was affected by the frequency of advertisement. They conducted the analyses in an imprecise environment. Many researchers have worked in this area, including Mishra [42], Md

Mashud and Hasan [41], Shaikh *et al.* [72], Khan *et al.* [28], Aggarwal *et al.* [2], Pervin *et al.* [55], Rapolu and Kandpal [58], San-José *et al.* [63], Kumar [31], Mandal *et al.* [36], Choi *et al.* [10], Khan *et al.* (2022) and Dey *et al.* [16]. Researchers showed that advertisement is one of the most key factors in retail industries to enhance profit in a modern competitive marketing environment (Khan *et al.*, 2022). In this study, selling price and promotional efforts dependent demand is considered.

2.3. Inventory model with variable holding cost

Aside from that, many researchers use holding cost (HC) as a constant parameter. In many cases, this assumption is not practically correct. Ferguson *et al.* [20] relaxed this assumption and investigated an EOQ model by taking HC as a non-linear function of time. Numerically, they observed that there is significant cost reduction due to lower holding cost. [44] developed an EOQ model in this direction by considering HC as a linear function of time. EOQ model was explored by Dutta and Kumar [18] by considering partial backlogging, deterioration, time-dependent demand and holding cost. Pervin *et al.* [52] investigated an integrated model considering variable holding cost. They assumed that holding cost is time-dependent. In the interest of the customer, they considered partial backlogging. Under the imprecise environment and considering price dependent demand and time dependent HC, Garai *et al.* [21] presented an EOQ model. In this continuation, two different partially backlogged inventory models were presented by Khan *et al.* [28] for aggrandize products that considering selling and advertisement-dependent demand. They have taken HC as time-dependent. Further, EOQ model was explored by Swain *et al.* [78] for deteriorating items by taking HC as a function of time into account. They assumed that duration of waiting time affected the rate of backlogging. Paul *et al.* [51] formulated an inventory model considering variable holding cost and retail investments in green operations. Palanivel and Suganya [48] investigated an inventory model considering partial backlogging, storage period-dependent holding costs, and stock-and-selling price-dependent demand. They suggested that the total profit of the system increases because of partial backlogging. They showed that constant and variable coefficients of holding cost have a significant impact on profit. On observing the variable holding cost for deteriorating items, holding cost in the current work is assumed as the function of time.

2.4. Inventory model with inflation

Most optimal policies for solving inventory problems have been obtained by assuming that cost parameters remain constant throughout the planning period. In practice, this assumption is false because inflation is a critical factor in determining the optimal policy, particularly for developing countries such as Afghanistan, China, Jordan, and India, among others. As a result, it is critical to investigate how inflation affects inventory policy. First, Buzacott [6] investigated the impact of inflation on the EOQ model using various pricing policies. Sarker and Pan [68] investigated the effect of inflation on the EOQ model and determined the optimal order quantity and shortage quantity. Jaggi *et al.* [26] discovered an optimal inventory policy for deteriorating items under the influence of inflation. Sarkar and Moon [64] examined an imperfect production system with stochastic demand that follows a uniform rectangular distribution and a general distribution in an inflationary environment. Sarkar *et al.* [65] dealt with an economic manufacturing quantity model with imperfect manufacturing processes where demand was taken as a selling price and time-dependent. In an inflationary environment, they achieved optimal policies. Singh and Sharma [74] proposed a stochastic production model with and without shortages. They considered the effect of inflation when developing the inventory control policy. In addition, Vandana *et al.* [84], Saha and Sen [61], Singh and Sharma [75], Singh and Karuna [76], Chakraborty *et al.* [8], Handa *et al.* (2021), Sundararajan *et al.* [77], Barman *et al.* [4], and Padiyar *et al.* [47] have examined the inventory model in the context of inflation. To observe the real picture of financial situation in the model, effect of inflation is considered in the model.

2.5. Inventory model with trade credit

Market trends are changing dramatically these days. The retail industry is becoming more competitive day by day. Hence, trade credit policies are becoming more popular in the retail industry. Typically, suppliers offer trade credit to their retailer. During this time, the retailer took interest in the accumulated revenue. After that time, the supplier charges interest to his or her retailer. In recent years, researchers have conducted extensive research in this area. Some of the most cited papers in this area are Goyal [22], Aggarwal and Jaggi [1], Seifert *et al.* [69], Yadav *et al.* [88], Sarkar *et al.* [66,67], Tsao *et al.* [83], Tiwari *et al.* [81,82], Yang and Birge [90], Roy *et al.* [59,60], Wu *et al.* [87], Pervin *et al.* [53] and Das *et al.* [14]. In the current study, market-oriented financing scheme *i.e.*, trade credit is considered in the model (Tab. 1).

2.6. Novelty of this research

Researchers have done a lot of work on the EOQ model, which is based on several realistic assumptions. But a review of the research in this area shows that there is a clear research gap. Every retailer wishes to keep their system financially viable. According to the researchers' findings, inflation, time-dependent holding costs, investment to reduce ordering costs, trade credit, product selling prices, and promotional efforts by the retailer's sales team all play an important role in financial sustainability. Also, retailers who sell things like electronics, vegetables, fruits, bakery goods, and so on need to think about the idea of non-instantaneous deterioration when

TABLE 1. Summary of related work.

Name of contributors	Deterioration	Demand	Holding cost	Ordering cost	Shortages	Trade credit	Inflation
Chung and Liao [11]	Constant	Constant	Constant	Constant	Not Allowed	Yes	Yes
Chung and Liao [12]	No	Constant	Constant	Constant	Not allowed	Yes	Yes
Mishra [44]	Time-dependent	Time-dependent	Time-dependent	Constant	Partial backlogging	No	No
Lou and Wang [35]	No	Constant	Constant	Investment to reduce	No	Yes	No
Tiwari *et al.* [80]	Non-instantaneous	Constant	Constant	Constant	Partial backlogging	Yes	Yes
Alfares and Ghaithan [3]	No	Selling price dependent	Time-dependent	Constant	No	No	No
	Weibull deterioration rate	Time-dependent	Constant	Constant	Partial Backlogging	Yes	Yes
Mishra *et al.* [43]	Depends on investment on preservation	Selling price dependent	Constant	Constant	No	Yes	No
Li *et al.* [33]	Non-instantaneous	Selling price dependent	Constant	Constant	Partial backlogging		No
Shaikh *et al.* (2019)	Non-instantaneous	Stock-dependent	Constant	Constant	Partial Backlogging	No	Yes
Sundararajan *et al.* [77]	Non-instantaneous	Time and selling price dependent	Constant	Constant	Partial Backlogging	Yes	Yes
Current Study	Non-instantaneous	Price and promotional efforts dependent	Time dependent	Investment to reduce	Partial Backordering	Yes	Yes

making the optimal decisions about their financial situation. Taking cognizance of this, through this research, the financial sustainability of a retailer has been obtained under the effect of all this.

3. Problem definition, assumptions, and notations

3.1. Problem definition

Proposed model helps the retailer dealing in non-instantaneous deteriorating items to obtain the financial sustainability considering the effect of promotional efforts and selling price on the customers demand and investment to reduce ordering cost. Holding cost is considered as time dependent and shortages are partially backlogged. Whole of the analysis is carried out under the effect of inflation. Market oriented financing scheme *i.e.*, trade credit is considered in the model. Based on the value of trade credit, four different cases are considered. Some theoretical results are also obtained. Graphical representation of the current study can be represented as follows (Fig. 1).

3.2. Assumptions

(1) Single item is considered for modelling purpose in the current study.
(2) Lead time is negligible as well as planning horizon of the current study is taken infinite.
(3) In the current study, replenishment rate is considered as infinite.
(4) In the current study, deterioration process is taken non-instantaneous *i.e.*, deterioration of items occurs after the life time of the items [5,8].
(5) Holding cost ($h(t)$) consists two components: constant part (h_1) and the variable part ($= h_2t$) which increases as the holding period increases [3,18].
(6) In the current study, initial investment is made to reduce the ordering cost [35].
(7) Shortages consider in the present study which partially backlogged. Backlogging rate is $B(t) = e^{-\rho t}$ where "t" is the waiting period and $\rho(0 \leq \rho \leq 1)$ is backlogging parameter [8].
(8) Customer's demand is taken as the function of selling price and promotional effort. It is of the form:

$$D(s, \mu) = d_0 - d_1 s + d_2 \mu$$

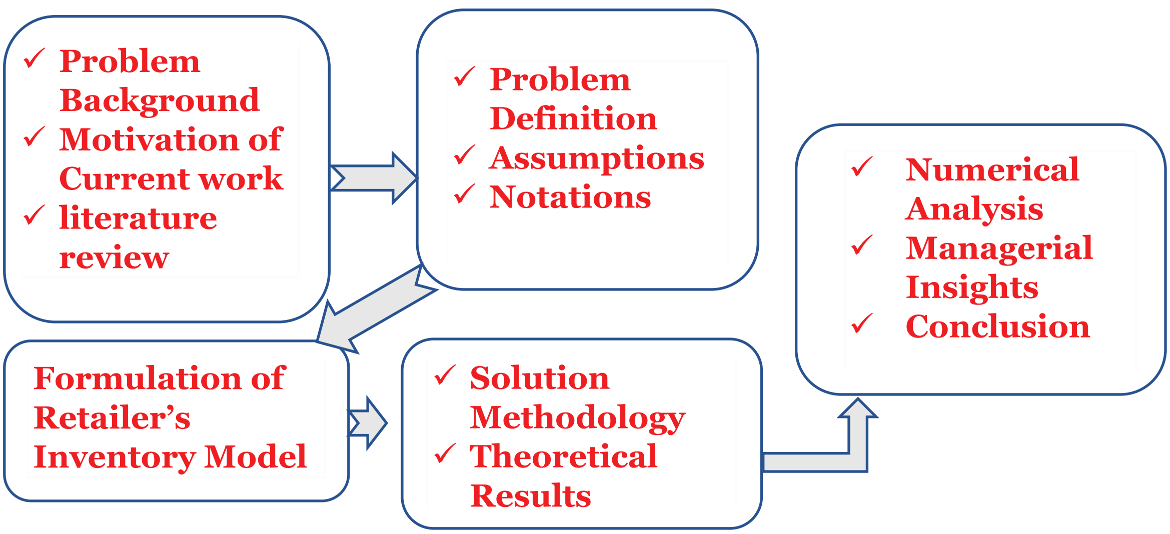

Figure 1. The flow of current study.

where d_0 is the initial demand, μ is the promotional effort, d_1 is scale parameter of demand change due to selling price, and d_2 is scale parameter of demand change which varies with the unit promotional effort μ [28,41].

(9) Credit facility is provided by the retailer to his/her customers [82,83].

(10) Promotional cost (PC) is the function of promotional effort (μ). Therefore, PC $= k\mu^n$ where $k(>0)$ and n are constant.

(11) Effect of inflation is incorporated in the present study [80,84].

3.3. Notations

Decision variables

T	Cycle time (unit time)
t_2	Duration of positive inventory (unit time)
s	Selling price (\$/unit)
μ	Unit promotional effort level of the retailer

Parameter

p	Purchasing cost (\$/unit)
K	Ordering cost (\$/order)
$h(t)$	$(h_1 + h_2 t)$ Holding cost (\$/unit)
C_d	Wastage cost due to deterioration (\$/unit)
L_s	Lost sale cost (\$/unit)
S_c	Shortage cost (\$/unit)
M	Trade credit (unit time)
I_p	Rate of interest paid (%)
I_e	Rate of interest earn (%)
θ	Deterioration rate (units/unit time)
r	Rate of inflation
Q	Order size (units/cycle)
Q_1	Maximum positive inventory level (units)
B	Maximum backorder quantity (units)

4. Formulation of model considering partial backlogging

In the current study, it is considered that in the beginning, Q units of items ordered by the retailer. Here, lifetime of items is considered as t_1 *i.e.*, after that deterioration process takes place. Thus, during the period $[0, t_1]$ inventory level decreases due to demand only whereas during the period $[t_1, t_2]$ inventory decline due to demand and deterioration and becomes zero at the end of the period. After that period, shortages occur during the period $[t_2, T]$ which partially backlogged. Inventory situation in each interval is illustrated in Figure 2.

During the period $[0, t_1]$, inventory level can be illustrated with the following differential equation:

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = -D(s,\mu), \qquad 0 \le t \le t_1 \tag{1}$$

with the boundary condition $I_1(0) = Q_1$.

Thus, inventory level during the interval $[0, t_1]$ is

$$I_1(t) = Q_1 - D(s,\mu)t. \tag{2}$$

During the period $[t_1, t_2]$, inventory level can be illustrated with the following differential equation:

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = -D(s,\mu) - \theta I_2(t), \qquad t_1 \le t \le t_2 \tag{3}$$

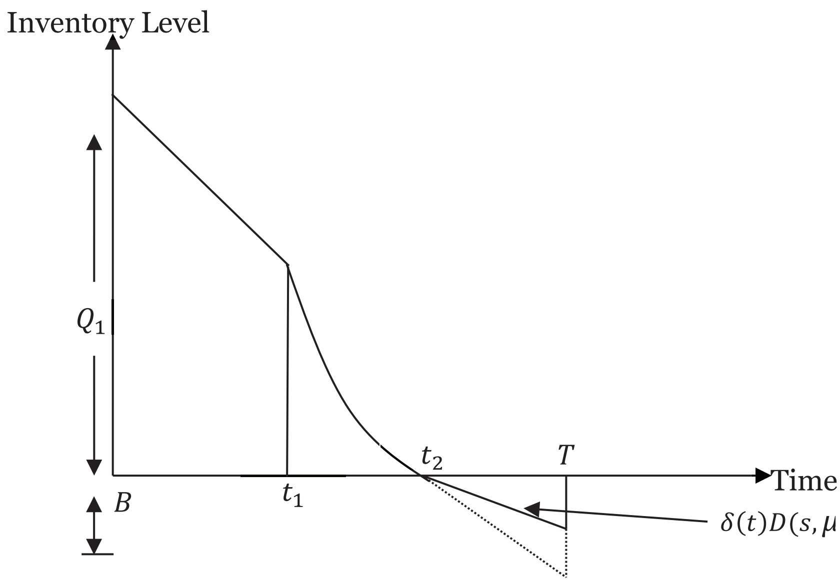

FIGURE 2. Graphical representation of sustainable inventory system.

with the boundary condition $I_1(t_2) = 0$.

Thus, inventory level during the interval $[t_1, t_2]$ is

$$I_2(t) = \frac{D(s,\mu)}{\theta}\left[e^{\theta(t_2-t)} - 1\right]. \tag{4}$$

During the period $[t_2, T]$, inventory level can be illustrated with the following differential equation:

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = -D(s,\mu)e^{-\rho(T-t)}, \qquad t_2 \le t \le T \tag{5}$$

with the boundary condition $I_3(t_2) = 0$.

Thus, inventory level during the interval $[t_2, T]$ is

$$I_3(t) = \frac{D(s,\mu)}{\rho}\left[e^{-\rho(T-t_2)} - e^{-\rho(T-t)}\right]. \tag{6}$$

From Figure 2, it is clear that $I_1(t_1) = I_2(t_1)$. This relation gives

$$Q_1 = D(s,\mu)t_1 + \frac{\boldsymbol{D(s,\mu)}}{\theta}\left[e^{\theta(t_2-t_1)} - 1\right]. \tag{7}$$

Using above relation in equation (2)

$$I_1(t) = D(s,\mu)(t_1 - t) + \frac{\boldsymbol{D(s,\mu)}}{\theta}\left[e^{\theta(t_2-t_1)} - 1\right]. \tag{8}$$

From Figure 2, it is clear that $I_3(T) = -B$. Thus

$$B = \frac{\boldsymbol{D(s,\mu)}}{\rho}\left[1 - e^{-\rho(T-t_2)}\right]. \tag{9}$$

Thus, order size placed by the retailer in the beginning of the inventory cycle is

$$Q = D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\Big[e^{\theta(t_2-t_1)} - 1\Big] + \frac{D(s,\mu)}{\rho}\Big[1 - e^{-\rho(T-t_2)}\Big]. \tag{10}$$

Now, different cost associated with system of retailer has been evaluated under the influence of inflation.

Retailer generate revenue by satisfying the customers demand during positive inventory and backlogged quantity during the negative inventory. Thus, revenue generated by the retailer under the effect of inflation is RG where

$$\mathrm{RG} = s\bigg\{\int_0^{t_2} D(s,\mu)e^{-rt}\,\mathrm{d}t + Be^{-rT}\bigg\} = sD(s,\mu)\bigg\{\frac{1}{r}\big(1-e^{-rt_2}\big) + \frac{e^{-rT}}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\bigg\}. \tag{11}$$

It is observed that ordering cost can be reduced by investing some additional amount. In the beginning of cycle, an order of quantity Q is placed by the retailer. So, ordering cost under the effect of inflation is OC where

$$\mathrm{OC} = Ke^{-\emptyset l}.$$

Here, supplier offers credit period to her/his retailer to settle the account. Thus, the purchasing cost under the effect of inflation is PC where

$$\mathrm{PC} = p\bigg\{D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\Big[e^{\theta(t_2-t_1)} - 1\Big] + \frac{D(s,\mu)}{\rho}\Big[1 - e^{-\rho(T-t_2)}\Big]\bigg\}e^{-rM}.$$

To carry the inventory in stock, retailer have to bear carrying cost for the items. Thus, holding cost under the influence of inflation is HC where

$$\begin{aligned}
\mathrm{HC} &= \int_0^{t_1} h(t)I_1(t)e^{-rt}\,\mathrm{d}t + \int_{t_1}^{t_2} h(t)I_2(t)e^{-rt}\,\mathrm{d}t \\
&= h_1\bigg[\frac{D(s,\mu)}{r^2}\big(e^{-rt_1} + rt_1 - 1\big) + \frac{D(s,\mu)}{\theta r}\Big(e^{\theta(t_2-t_1)} - 1\Big)\big(1 - e^{-rt_1}\big)\bigg] \\
&\quad + h_2\bigg[\frac{D(s,\mu)}{r^3}\big\{(t_1r+2)e^{-rt_1} + (t_1r-2)\big\} + \frac{D(s,\mu)}{\theta r^2}\Big(e^{\theta(t_2-t_1)} - 1\Big)\big(1 - e^{-rt_1} - t_1e^{-rt_1}\big)\bigg] + \frac{h_1D(s,\mu)}{\theta} \\
&\quad \times \bigg[\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\Big) + \frac{1}{r}\big(e^{-rt_2} - e^{-rt_1}\big)\bigg] + \frac{h_2D(s,\mu)}{\theta}\bigg[\frac{1}{\theta+r}\Big(t_1e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\Big) \\
&\quad + \frac{1}{(\theta+r)^2}\Big(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\Big) + \frac{1}{r}\big(t_2e^{-rt_2} - t_1e^{-rt_1}\big) + \frac{1}{r^2}\big(e^{-rt_2} - e^{-rt_1}\big)\bigg].
\end{aligned}$$

During the period $[0, t_1]$ deterioration not occur while during the period $[t_1, t_2]$ deterioration occurs. Therefore, wastage cost under the effect of inflation is DC where

$$\mathrm{WC} = C_d\int_{t_1}^{t_2} \theta I_2(t)e^{-rt}\,\mathrm{d}t = C_dD(s,\mu)\bigg[\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\Big) + \frac{1}{r}\big(e^{-rt_2} - e^{-rt_1}\big)\bigg].$$

During the period $[t_2, T]$ there is no inventory in the stock to satisfy the demand of customer. Therefore, under the effect of inflation, shortage cost and lost sale cost are SC and LS respectively where

$$\mathrm{SC} = S_c\int_{t_2}^{T} -I_3(t)e^{-rt}\,\mathrm{d}t = \frac{S_cD(s,\mu)}{\rho}\bigg[\frac{1}{r}\Big\{e^{-(\rho+r)T+\rho t_2} - e^{-\rho T+(\rho-r)t_2}\Big\} + \frac{1}{\rho-r}\Big\{e^{-rT} - e^{-\rho T+(\rho-r)t_2}\Big\}\bigg]$$

and

$$\mathrm{LS} = L_s\int_{t_2}^{T}\Big(1 - e^{-\rho(T-t)}\Big)D(s,\mu)e^{-rt}\,\mathrm{d}t = L_sD(s,\mu)\bigg[\frac{1}{r}\big(e^{-rt_2} - e^{-rT}\big) - \frac{1}{\rho-r}\Big(e^{-rT} - e^{-\rho T+(\rho-r)t_2}\Big)\bigg].$$

Here, retailer performs promotional activities with the help of print/online media. Thus, promotional cost is PC where

$$\mathrm{PC} = k\mu^n.$$

In the current study, it is taken that credit period (M) is offered by the supplier to her/his retailer. Thus, according to the credit period (M) following cases arise:

Case 1: $\mathbf{0 < M \leq t_1}$ *i.e.*, credit period is less than the lifetime of the items.

In this case, for the period $[0, M]$, interest is earned by the retailer and have to pay interest for the period $[M, t_2]$.

Under the effect of inflation, interest earned by the retailer is IE_1 where

$$\mathrm{IE}_1 = sI_e \int_0^M D(s,\mu) t e^{-rt}\,\mathrm{d}t = \frac{sI_e D(s,\mu)}{r^2}\left[1 - e^{-rM}(1 + rM)\right].$$

Interest paid by the retailer under the effect of inflation is IP_1 where

$$\begin{aligned}
\mathrm{IP}_1 &= pI_p\left(\int_M^{t_1} I_1(t) e^{-rt}\,\mathrm{d}t + \int_{t_1}^{t_2} I_2(t) e^{-rt}\,\mathrm{d}t\right)\\
&= pI_p\left[D(s,\mu)\left\{\frac{(t_1 - M)e^{-rM}}{r} + \frac{1}{r^2}\left(e^{-rt_1} - e^{-rM}\right)\right\} + \frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2 - t_1)} - 1\right)\left(e^{-rM} - e^{-rt_1}\right)\right.\\
&\quad \left. + \frac{D(s,\mu)}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2 - (r+\theta)t_1} - e^{-rt_1}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\}\right].
\end{aligned}$$

In this case, under the effect of inflation, total inventory cost per cycle is TC_1 where

$$\begin{aligned}
\mathrm{TC}_1 = \Bigg[& Ke^{\emptyset l} + p\left\{D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\left(e^{\theta(t_2 - t_1)} - 1\right) + \frac{D(s,\mu)}{\rho}\left(1 - e^{-\rho(T - t_2)}\right)\right\}e^{-rM}\\
& + h_1\left\{\frac{D(s,\mu)}{r^2}\left(e^{-rt_1} + rt_1 - 1\right) + \frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2 - t_1)} - 1\right)\left(1 - e^{-rt_1}\right)\right\} + h_2\left\{\frac{D(s,\mu)}{r^3}\right.\\
& \left.\times\left\{(t_1 r + 2)e^{-rt_1} + (t_1 r - 2)\right\} + \frac{D(s,\mu)}{\theta r^2}\left(e^{\theta(t_2 - t_1)} - 1\right)\left(1 - e^{-rt_1} - t_1 e^{-rt_1}\right)\right\}\\
& + \frac{h_1 D(s,\mu)}{\theta}\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2 - t_1) - rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + \frac{h_2 D(s,\mu)}{\theta}\\
& \times\left\{\frac{1}{\theta + r}\left(t_1 e^{\theta(t_2 - t_1) - rt_1} - e^{-rt_2}\right) + \frac{1}{(\theta + r)^2}\left(e^{\theta(t_2 - t_1) - rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(t_2 e^{-rt_2} - t_1 e^{-rt_1}\right)\right.\\
& \left. + \frac{1}{r^2}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + C_d D(s,\mu)\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2 - t_1) - rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\}\\
& + \frac{S_c D(s,\mu)}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho + r)T + \rho t_2} - e^{-\rho T + (\rho - r)t_2}\right) + \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T + (\rho - r)t_2}\right)\right\}\\
& + L_s D(s,\mu)\left\{\frac{1}{r}\left(e^{-rt_2} - e^{-rT}\right) - \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T + (\rho - r)t_2}\right)\right\} + k\mu^n pI_p\\
& \times\left\{D(s,\mu)\left(\frac{(t_1 - M)e^{-rM}}{r} + \frac{1}{r^2}\left(e^{-rt_1} - e^{-rM}\right)\right) + \frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2 - t_1)} - 1\right)\left(e^{-rM} - e^{-rt_1}\right) + \frac{D(s,\mu)}{\theta}\right.\\
& \left.\times\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2 - (r+\theta)t_1} - e^{-rt_1}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\}\right\} - \frac{sI_e D(s,\mu)}{r^2}\left\{1 - e^{-rM}(1 + rM)\right\}\Bigg].
\end{aligned} \tag{12}$$

Total profit of the retailer under the effect of inflation is TP_1 where

$$\mathrm{TP}_1 = \frac{1}{T}(\mathrm{RG} - \mathrm{TC}_1) \tag{13}$$

where value of RG and TC_1 are given in equations (11) and (12) respectively.

Case 2: $t_1 < M \leq t_2$ *i.e.*, credit period is more than the lifetime of the items.

In this case, for the period $[0, M]$ retailer earn the interest and for the period $[M, t_2]$ retailer have to pay interest.

Under the effect of inflation, interest earned by the retailer is IE_2 where

$$\mathrm{IE}_2 = sI_e \int_0^M D(s,\mu) t e^{-rt}\,\mathrm{d}t = \frac{sI_e D(s,\mu)}{r^2}\left[1 - e^{-rM}(1 + rM)\right].$$

Under the effect of inflation, interest paid by the retailer is IP_2 where

$$\mathrm{IP}_2 = pI_p \int_M^{t_2} I_2(t) e^{-rt}\,\mathrm{d}t = \frac{pI_p D(s,\mu)}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2 - (r+\theta)M} - e^{-rt_1}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rM}\right)\right\}.$$

In this case, under the effect of inflation, total inventory cost per cycle is TC_2 where

$$\begin{aligned}
\mathrm{TC}_2 = \Bigg[&Ke^{-\emptyset l} + p\left\{D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\left[e^{\theta(t_2-t_1)} - 1\right] + \frac{D(s,\mu)}{\rho}\left[1 - e^{-\rho(T-t_2)}\right]\right\}e^{-rM} \\
&+ h_1\left\{\frac{D(s,\mu)}{r^2}\left(e^{-rt_1} + rt_1 - 1\right) + \frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1}\right)\right\} + h_2\left\{\frac{D(s,\mu)}{r^3}\right. \\
&\times \left.\left\{(t_1 r + 2)e^{-rt_1} + (t_1 r - 2)\right\} + \frac{D(s,\mu)}{\theta r^2}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1} - t_1 e^{-rt_1}\right)\right\} \\
&+ \frac{h_1 D(s,\mu)}{\theta}\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + \frac{h_2 D(s,\mu)}{\theta} \\
&\times \left\{\frac{1}{\theta + r}\left(t_1 e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{(\theta + r)^2}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(t_2 e^{-rt_2} - t_1 e^{-rt_1}\right)\right. \\
&\left. + \frac{1}{r^2}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + C_d D(s,\mu)\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} \\
&+ \frac{S_c D(s,\mu)}{\rho}\left\{\frac{1}{r}\left\{e^{-(\rho+r)T + \rho t_2} - e^{-\rho T + (\rho - r)t_2}\right\} + \frac{1}{\rho - r}\left\{e^{-rT} - e^{-\rho T + (\rho - r)t_2}\right\}\right\} \\
&+ L_s D(s,\mu)\left\{\frac{1}{r}\left(e^{-rt_2} - e^{-rT}\right) - \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T + (\rho - r)t_2}\right)\right\} + k\mu^n \frac{pI_p D(s,\mu)}{\theta} \\
&\times \left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2 - (r+\theta)M} - e^{-rt_1}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rM}\right)\right\} - \frac{sI_e D(s,\mu)}{r^2}\left\{1 - e^{-rM}(1 + rM)\right\}\Bigg].
\end{aligned} \tag{14}$$

Total profit of the retailer under the effect of inflation is TP_2 where

$$\mathrm{TP}_2 = \frac{1}{T}(\mathrm{RG} - TC_2) \tag{15}$$

where value of RG and TC_1 are given in equations (11) and (14) respectively.

Case 3: $t_2 < M \leq T$ *i.e.*, credit period occurs when inventory level is not positive.

In this situation, interest is not paid by the retailer while retailer incur the interest on the accumulated revenue. Under the effect of inflation, interest earned by the retailer is IE_3 where

$$\begin{aligned}
\mathrm{IE}_3 &= sI_e\left\{\int_0^{t_1} D(s,\mu)te^{-rt}\,\mathrm{d}t + \int_{t_2}^{M}\left(\int_0^{t_1} D(s,\mu)\,\mathrm{d}t\right)e^{-rt}\,\mathrm{d}t\right\}\\
&= sI_e\left\{\frac{D(s,\mu)}{r^2}\left\{1-e^{-rt_1}(1+rt_1)\right\} + \frac{D(s,\mu)t_2\left(e^{-rt_2}-e^{-rM}\right)}{r}\right\}
\end{aligned}$$

$$\begin{aligned}
\mathrm{TC}_3 = \Bigg[&Ke^{-\emptyset l} + p\left\{D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\left[e^{\theta(t_2-t_1)}-1\right] + \frac{D(s,\mu)}{\rho}\left[1-e^{-\rho(T-t_2)}\right]\right\}e^{-rM}\\
&+ h_1\left\{\frac{D(s,\mu)}{r^2}\left(e^{-rt_1}+rt_1-1\right) + \frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}\right)\right\} + h_2\left\{\frac{D(s,\mu)}{r^3}\right.\\
&\left.\times\left\{(t_1r+2)e^{-rt_1}+(t_1r-2)\right\} + \frac{D(s,\mu)}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\right\}\\
&+ \frac{h_1D(s,\mu)}{\theta}\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\} + \frac{h_2D(s,\mu)}{\theta}\\
&\times\left\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right) + \frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right) + \frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)\right.\\
&\left.+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\right\} + C_dD(s,\mu)\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\\
&+ \frac{S_cD(s,\mu)}{\rho}\left\{\frac{1}{r}\left\{e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right\} + \frac{1}{\rho-r}\left\{e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right\}\right\}\\
&+ L_sD(s,\mu)\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right) - \frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\} + k\mu^n\\
&- sI_e\left\{\frac{D(s,\mu)}{r^2}\left\{1-e^{-rt_1}(1+rt_1)\right\} + \frac{D(s,\mu)t_2\left(e^{-rt_2}-e^{-rM}\right)}{r}\right\}\Bigg]. \qquad (16)
\end{aligned}$$

Total profit of the retailer under the effect of inflation is TP_3 where

$$TP_3 = \frac{1}{T}(RG - TC_3) \qquad (17)$$

where value of RG and TC_1 are given in equations (11) and (16) respectively.

Case 4: $T < M$

In this situation, interest is not earned by the retailer while retailer incur the interest on the accumulated revenue. Under the effect of inflation, interest earned by the retailer is IE_4 where

$$\begin{aligned}
\mathrm{IE}_4 &= sI_e\left\{\int_0^{t_1} D(s,\mu)te^{-rt}\,\mathrm{d}t + \int_{t_2}^{M}\left(\int_0^{t_1} D(s,\mu)\,\mathrm{d}t\right)e^{-rt}\,\mathrm{d}t\right\}\\
&= sI_e\left\{\frac{D(s,\mu)}{r^2}\left\{1-e^{-rt_1}(1+rt_1)\right\} + \frac{D(s,\mu)t_2\left(e^{-rt_2}-e^{-rM}\right)}{r}\right\}
\end{aligned}$$

$$
\begin{aligned}
\mathrm{TC}_4 = \Bigg[& Ke^{-\emptyset l} + p\Big\{D(s,\mu)t_1 + \frac{D(s,\mu)}{\theta}\Big[e^{\theta(t_2-t_1)}-1\Big] + \frac{D(s,\mu)}{\rho}\Big[1-e^{-\rho(T-t_2)}\Big]\Big\}e^{-rM} \\
& + h_1\Big\{\frac{D(s,\mu)}{r^2}\big(e^{-rt_1}+rt_1-1\big) + \frac{D(s,\mu)}{\theta r}\Big(e^{\theta(t_2-t_1)}-1\Big)\big(1-e^{-rt_1}\big)\Big\} \\
& + h_2\Big\{\frac{D(s,\mu)}{r^3}\{(t_1r+2)e^{-rt_1}+(t_1r-2)\} + \frac{D(s,\mu)}{\theta r^2}\Big(e^{\theta(t_2-t_1)}-1\Big)\big(1-e^{-rt_1}-t_1e^{-rt_1}\big)\Big\} \\
& + \frac{h_1D(s,\mu)}{\theta}\Big\{\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big) + \frac{1}{r}\big(e^{-rt_2}-e^{-rt_1}\big)\Big\} + \frac{h_2D(s,\mu)}{\theta} \\
& \times\Bigg\{\frac{1}{\theta+r}\Big(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big) + \frac{1}{(\theta+r)^2}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big) + \frac{1}{r}\big(t_2e^{-rt_2}-t_1e^{-rt_1}\big) \\
& + \frac{1}{r^2}\big(e^{-rt_2}-e^{-rt_1}\big)\Bigg\} + C_dD(s,\mu)\Big\{\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big) + \frac{1}{r}\big(e^{-rt_2}-e^{-rt_1}\big)\Big\} \\
& + \frac{S_cD(s,\mu)}{\rho}\Big\{\frac{1}{r}\Big\{e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\Big\} + \frac{1}{\rho-r}\Big\{e^{-rT}-e^{-\rho T+(\rho-r)t_2}\Big\}\Big\} \\
& + L_sD(s,\mu)\Big\{\frac{1}{r}\big(e^{-rt_2}-e^{-rT}\big) - \frac{1}{\rho-r}\Big(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\Big)\Big\} + k\mu^n \\
& - sI_e\Big\{\frac{D(s,\mu)}{r^2}\{1-e^{-rt_1}(1+rt_1)\} + \frac{D(s,\mu)t_2\big(e^{-rt_2}-e^{-rM}\big)}{r}\Big\}\Bigg]. \qquad (18)
\end{aligned}
$$

Total profit of the retailer under the effect of inflation is TP_4 where

$$\mathrm{TP}_4 = \frac{1}{T}(\mathrm{RG} - \mathrm{TC}_4) \qquad (19)$$

where value of RG and TC_1 are given in equations (11) and (18) respectively.

5. Solution procedure

Consider that s, μ, t_2, and T are non zero real numbers and such that there exist unique values of s, μ, t_2, and T such that it maximizes the profit of retailer and satisfies the first order derivative conditions which are as follows:

$$\frac{\partial \mathrm{TP}_i}{\partial s} = 0 \qquad (20)$$

$$\frac{\partial \mathrm{TP}_i}{\partial \mu} = 0 \qquad (21)$$

$$\frac{\partial \mathrm{TP}_i}{\partial t_2} = 0 \qquad (22)$$

$$\frac{\partial \mathrm{TP}_i}{\partial T} = 0. \qquad (23)$$

On solving the system of equations (20)–(23), we get the values of (s, μ, t_2, T). Check the nature of the Hessian matrix

$$H = \begin{bmatrix}
\frac{\partial^2 \mathrm{TP}_1}{\partial (s)^2} & \frac{\partial^2 \mathrm{TP}_1}{\partial s \partial \rho} & \frac{\partial^2 \mathrm{TP}_1}{\partial s \partial t_2} & \frac{\partial^2 \mathrm{TP}_1}{\partial s \partial T} \\
\frac{\partial^2 \mathrm{TP}_1}{\partial \rho \partial s} & \frac{\partial^2 \mathrm{TP}_1}{\partial (\rho)^2} & \frac{\partial^2 \mathrm{TP}_1}{\partial \rho \partial t_2} & \frac{\partial^2 \mathrm{TP}_1}{\partial \rho \partial T} \\
\frac{\partial^2 \mathrm{TP}_1}{\partial t_2 \partial s} & \frac{\partial^2 \mathrm{TP}_1}{\partial t_2 \partial \rho} & \frac{\partial^2 \mathrm{TP}_1}{\partial (t_2)^2} & \frac{\partial^2 \mathrm{TP}_1}{\partial t_2 \partial T} \\
\frac{\partial^2 \mathrm{TP}_1}{\partial T \partial s} & \frac{\partial^2 \mathrm{TP}_1}{\partial T \partial \rho} & \frac{\partial^2 \mathrm{TP}_1}{\partial T \partial t_2} & \frac{\partial^2 \mathrm{TP}_1}{\partial (T)^2}
\end{bmatrix}$$

at the obtained point.

If H is negative definite then $\mathrm{TP}_i(s^*, \mu^*, t_2^*, T^*)$ is the optimal profit for the system.

6. Theoretical results

It is clear from equations (13), (15), (17), and (19) that the profit function of retailer is highly non-linear. In practise, proving the concavity of the profit function with respect to the decision variables is a time-consuming task. Here, pseudo-concavity is obtained theoretically with the help of results from Cambini and Martien [7].

Theorem 1. *With respect to "T", TP_1 is a pseudo-concave and hence there exists an unique "T" that satisfy the following equation:*

$$\begin{aligned}
&\Bigg[sD(s,\mu)\bigg\{\frac{1}{r}(1-e^{-rt_2})+\frac{e^{-rT}}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\bigg\}-\Bigg[p\bigg\{D(s,\mu)t_1+\frac{D(s,\mu)}{\theta}\Big(e^{\theta(t_2-t_1)}-1\Big)\\
&\quad+\frac{D(s,\mu)}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\bigg\}e^{-rM}+h_1\bigg\{\frac{D(s,\mu)}{r^2}(e^{-rt_1}+rt_1-1)+\frac{D(s,\mu)}{\theta r}\Big(e^{\theta(t_2-t_1)}-1\Big)\Big(1-e^{-rt_1}\Big)\bigg\}\\
&\quad+h_2\bigg\{\frac{D(s,\mu)}{r^3}\{(t_1r+2)e^{-rt_1}+(t_1r-2)\}+\frac{D(s,\mu)}{\theta r^2}\Big(e^{\theta(t_2-t_1)}-1\Big)\Big(1-e^{-rt_1}-t_1e^{-rt_1}\Big)\bigg\}+\frac{h_1D(s,\mu)}{\theta}\\
&\quad\times\bigg\{\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big)+\frac{1}{r}\Big(e^{-rt_2}-e^{-rt_1}\Big)\bigg\}+\frac{h_2D(s,\mu)}{\theta}\bigg\{\frac{1}{\theta+r}\Big(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big)\\
&\quad+\frac{1}{(\theta+r)^2}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big)+\frac{1}{r}\Big(t_2e^{-rt_2}-t_1e^{-rt_1}\Big)+\frac{1}{r^2}\Big(e^{-rt_2}-e^{-rt_1}\Big)\bigg\}+C_dD(s,\mu)\\
&\quad\times\bigg\{\frac{1}{\theta+r}\Big(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\Big)+\frac{1}{r}\Big(e^{-rt_2}-e^{-rt_1}\Big)\bigg\}+\frac{S_cD(s,\mu)}{\rho}\bigg\{\frac{1}{r}\Big(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\Big)\\
&\quad+\frac{1}{\rho-r}\Big(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\Big)\bigg\}+L_sD(s,\mu)\bigg\{\frac{1}{r}\Big(e^{-rt_2}-e^{-rT}\Big)-\frac{1}{\rho-r}\Big(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\Big)\bigg\}+k\mu^n\\
&\quad\times pI_p\bigg\{D(s,\mu)\bigg(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\Big(e^{-rt_1}-e^{-rM}\Big)\bigg)+\frac{D(s,\mu)}{\theta r}\Big(e^{\theta(t_2-t_1)}-1\Big)\Big(e^{-rM}-e^{-rt_1}\Big)\\
&\quad+\frac{D(s,\mu)}{\theta}\bigg\{\frac{1}{(r+\theta)}\Big(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\Big)+\frac{1}{r}\Big(e^{-rt_2}-e^{-rt_1}\Big)\bigg\}\bigg\}-\frac{sI_eD(s,\mu)}{r^2}\{1-e^{-rM}(1+rM)\}\Bigg]\Bigg]\\
&\quad-T\bigg[sD(s,\mu)\bigg\{-e^{-rT}\Big((T-t_2)-\frac{\rho}{2}(T-t_2)^2\Big)+e^{-rT}(1-\rho(T-t_2))\bigg\}-pe^{-rM}D(s,\mu)\{1-\rho(T-t_2)\}\\
&\quad-\frac{S_cD(s,\mu)}{2\rho}\{2(2\rho+r)T-4\rho t_2\}+rL_sD(s,\mu)T\bigg]=0
\end{aligned}$$

and such that TP_1 is maximum.

Proof. Total profit of the retailer in case-1 is TP_1 where

$$\mathrm{TP}_1=\frac{1}{T}(\mathrm{RG}-\mathrm{TC}_1) \tag{24}$$

where value of RG and TC_1 are given in equations (11) and (12) respectively.

Necessary condition of the optimality is $\frac{\partial \mathrm{TP}_1}{\partial T}=0$ which gives

$$\begin{aligned}
&\Bigg[sD(s,\mu)\bigg\{\frac{1}{r}(1-e^{-rt_2})+\frac{e^{-rT}}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\bigg\}-\Bigg[p\bigg\{D(s,\mu)t_1+\frac{D(s,\mu)}{\theta}\Big(e^{\theta(t_2-t_1)}-1\Big)\\
&\quad+\frac{D(s,\mu)}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\bigg\}e^{-rM}+h_1\bigg\{\frac{D(s,\mu)}{r^2}(e^{-rt_1}+rt_1-1)+\frac{D(s,\mu)}{\theta r}\Big(e^{\theta(t_2-t_1)}-1\Big)\Big(1-e^{-rt_1}\Big)\bigg\}
\end{aligned}$$

$$
\begin{aligned}
&+h_2\left\{\frac{D(s,\mu)}{r^3}\left\{(t_1r+2)e^{-rt_1}+(t_1r-2)\right\}+\frac{D(s,\mu)}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\right\}+\frac{h_1D(s,\mu)}{\theta}\\
&\times\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{h_2D(s,\mu)}{\theta}\left\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)\right.\\
&\left.+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+C_dD(s,\mu)\\
&\times\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{S_cD(s,\mu)}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)\right.\\
&\left.+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+L_sD(s,\mu)\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+k\mu^n\\
&\times pI_p\left\{D(s,\mu)\left(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)+\frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(e^{-rM}-e^{-rt_1}\right)\right.\\
&\left.\left.+\frac{D(s,\mu)}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\right\}-\frac{sI_eD(s,\mu)}{r^2}\left\{1-e^{-rM}(1+rM)\right\}\right]\Bigg]\\
&-T\left[sD(s,\mu)\left\{-e^{-rT}\left((T-t_2)-\frac{\rho}{2}(T-t_2)^2\right)+e^{-rT}(1-\rho(T-t_2))\right\}-pe^{-rM}\right.\\
&\left.\times D(s,\mu)\{1-\rho(T-t_2)\}-\frac{S_cD(s,\mu)}{2\rho}\{2(2\rho+r)T-4\rho t_2\}+rL_sD(s,\mu)T\right]=0. \qquad (25)
\end{aligned}
$$

Further, consider $\mathrm{TP}_1=\frac{\omega(T)}{\psi(T)}$ where

$$
\begin{aligned}
\omega(T)=&\,sD(s,\mu)\left\{\frac{1}{r}\left(1-e^{-rt_2}\right)+\frac{e^{-rT}}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}-\left[p\left\{D(s,\mu)t_1+\frac{D(s,\mu)}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)\right.\right.\\
&\left.+\frac{D(s,\mu)}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}e^{-rM}+h_1\left\{\frac{D(s,\mu)}{r^2}\left(e^{-rt_1}+rt_1-1\right)+\frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}\right)\right\}\\
&+h_2\left\{\frac{D(s,\mu)}{r^3}\left\{(t_1r+2)e^{-rt_1}+(t_1r-2)\right\}+\frac{D(s,\mu)}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\right\}\\
&+\frac{h_1D(s,\mu)}{\theta}\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{h_2D(s,\mu)}{\theta}\left\{\frac{1}{\theta+r}\right.\\
&\times\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)\\
&\left.+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+C_dD(s,\mu)\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\\
&+\frac{S_cD(s,\mu)}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+L_sD(s,\mu)\\
&\times\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+k\mu^npI_p\left\{D(s,\mu)\left(\frac{(t_1-M)e^{-rM}}{r}\right.\right.\\
&\left.+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)+\frac{D(s,\mu)}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(e^{-rM}-e^{-rt_1}\right)+\frac{D(s,\mu)}{\theta}\\
&\left.\left.\times\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\right\}-\frac{sI_eD(s,\mu)}{r^2}\left\{1-e^{-rM}(1+rM)\right\}\right]
\end{aligned}
$$

and

$$\psi(T)=T.$$

For given value of s, μ, and t_2, first and second partial derivative of $\varphi(T)$ w.r.t. "T" are as follows:

$$\frac{\partial\omega(T)}{\partial T} = sD(s,\mu)\Big\{-e^{-rT}\Big((T-t_2)-\frac{\rho}{2}(T-t_2)^2\Big)+e^{-rT}(1-\rho(T-t_2))\Big\}$$
$$-pe^{-rM}D(s,\mu)(1-\rho(T-t_2))-\frac{S_cD(s,\mu)}{2\rho}\{2(2\rho+r)T-4\rho t_2\}+rL_sD(s,\mu)T$$
$$\frac{\partial^2\omega(T)}{\partial T^2} = -\Big[sD(s,\mu)e^{-rT}\Big\{(T-t_2)-\frac{\rho}{2}(T-t_2)^2-\rho\Big\}-p\rho D(s,\mu)e^{-rM}-L_srD(s,\mu)\Big].$$

It is clear that for value of $T>0$, $\frac{\partial^2\omega(T)}{\partial T^2}<0$. So, $\omega(T)$ is a differentiable and concave function. Also, $\psi(T)=T>0$ and affine function. Thus, for given s, μ, and t_2, TP_1 is a pseudo convex function w.r.t. "T". Hence, $\exists$ a unique value of T^* satisfying equation (25). □

Theorem 2. TP_1 *is a pseudo-concave function w.r.t. "t_2" and hence there exists an unique "t_2" which satisfy the following equation:*

$$sD(s,\mu)\{1-rt_2+e^{-rT}(-1+\rho(T-t_2))\}-p\{D(s,\mu)(1+\theta(t_2-t_1))+D(s,\mu)(-1+\rho(T-t_2))\}e^{-rM}$$
$$-h_1\Big\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))\big(1-e^{-rt_1}\big)\Big\}-h_2\Big\{\frac{D(s,\mu)}{r^2}(1+\theta(t_2-t_1))\big(1-e^{-rt_1}-t_1e^{-rt_1}\big)\Big\}$$
$$-\frac{h_1D(s,\mu)}{\theta}\Big\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)+(-1+rt_2)\Big\}-\frac{h_2D(s,\mu)}{\theta}$$
$$\times\Bigg\{\frac{1}{\theta+r}\Big(t_1e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)+\frac{1}{(\theta+r)^2}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)$$
$$+\frac{1}{r}\big(1-2rt_2+3r^2t_2^2\big)+\frac{1}{r^2}\big(-r+r^2t_2\big)\Bigg\}-C_dD(s,\mu)\Big\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)$$
$$+\frac{1}{r}\big(-r+r^2t_2\big)\Big\}+\frac{S_cD(s,\mu)}{2\rho}\{4\rho T-2(2\rho-r)t_2\}-L_sD(s,\mu)rt_2-pI_p\Big\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))$$
$$\times\big(e^{-rM}-e^{-rt_1}\big)+\frac{D(s,\mu)}{\theta}\Big\{\frac{1}{(r+\theta)}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)\Big\}\Big\}=0$$

and such that TP_1 *is maximum.*

Proof. Total profit of the retailer in case-1 is TP_1 where

$$\mathrm{TP}_1=\frac{1}{T}(\mathrm{RG}-\mathrm{TC}_1) \tag{26}$$

where value of RG and TC_1 are given in equations (11) and (12) respectively.

Necessary condition of the optimality is $\frac{\partial \mathrm{TP}_1}{\partial t_2}=0$ which gives

$$sD(s,\mu)\{1-rt_2+e^{-rT}(-1+\rho(T-t_2))\}-p\{D(s,\mu)(1+\theta(t_2-t_1))+D(s,\mu)(-1+\rho(T-t_2))\}e^{-rM}$$
$$-h_1\Big\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))\big(1-e^{-rt_1}\big)\Big\}-h_2\Big\{\frac{D(s,\mu)}{r^2}(1+\theta(t_2-t_1))\big(1-e^{-rt_1}-t_1e^{-rt_1}\big)\Big\}$$
$$-\frac{h_1D(s,\mu)}{\theta}\Big\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)+(-1+rt_2)\Big\}-\frac{h_2D(s,\mu)}{\theta}$$
$$\times\Bigg\{\frac{1}{\theta+r}\Big(t_1e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)+\frac{1}{(\theta+r)^2}\Big(e^{-(\theta+r)t_1}\big(\theta+\theta^2t_2\big)-\big(-r+r^2t_2\big)\Big)$$

$$+\frac{1}{r}\left(1-2rt_2+3r^2t_2^2\right)+\frac{1}{r^2}\left(-r+r^2t_2\right)\Bigg\}-C_dD(s,\mu)\Bigg\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)$$
$$+\frac{1}{r}\left(-r+r^2t_2\right)\Bigg\}+\frac{S_cD(s,\mu)}{2\rho}\{4\rho T-2(2\rho-r)t_2\}-L_sD(s,\mu)rt_2-pI_p\Bigg\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))$$
$$\times\left(e^{-rM}-e^{-rt_1}\right)+\frac{D(s,\mu)}{\theta}\Bigg\{\frac{1}{(r+\theta)}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)\Bigg\}\Bigg\}=0. \tag{27}$$

For given value of s, μ, and T, first and second partial derivative of TP_1 w.r.t. "t_2" are as follows:

$$\frac{\partial \mathrm{TP}_1}{\partial t_2}=sD(s,\mu)\Big\{1-rt_2+e^{-rT}(-1+\rho(T-t_2))\Big\}-p\{D(s,\mu)(1+\theta(t_2-t_1))+D(s,\mu)(-1+\rho(T-t_2))\}$$
$$\times e^{-rM}-h_1\Bigg\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))\left(1-e^{-rt_1}\right)\Bigg\}-h_2\Bigg\{\frac{D(s,\mu)}{r^2}(1+\theta(t_2-t_1))\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\Bigg\}$$
$$-\frac{h_1D(s,\mu)}{\theta}\Bigg\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)+(-1+rt_2)\Bigg\}-\frac{h_2D(s,\mu)}{\theta}$$
$$\times\Bigg\{\frac{1}{\theta+r}\Big(t_1e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)+\frac{1}{(\theta+r)^2}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)$$
$$+\frac{1}{r}\left(1-2rt_2+3r^2t_2^2\right)+\frac{1}{r^2}\left(-r+r^2t_2\right)\Bigg\}-C_dD(s,\mu)\Bigg\{\frac{1}{\theta+r}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)$$
$$+\frac{1}{r}\left(-r+r^2t_2\right)\Bigg\}+\frac{S_cD(s,\mu)}{2\rho}\{4\rho T-2(2\rho-r)t_2\}-L_sD(s,\mu)rt_2-pI_p$$
$$\times\Bigg\{\frac{D(s,\mu)}{r}(1+\theta(t_2-t_1))\Big(e^{-rM}-e^{-rt_1}\Big)+\frac{D(s,\mu)}{\theta}\Bigg\{\frac{1}{(r+\theta)}\Big(e^{-(\theta+r)t_1}\left(\theta+\theta^2t_2\right)-\left(-r+r^2t_2\right)\Big)\Bigg\}\Bigg\}$$
$$\frac{\partial^2 \mathrm{TP}_1}{\partial t_2^2}=-\Bigg[sD(s,\mu)\Big\{r+e^{-rT}\rho\Big\}+pD(s,\mu)(\theta-\rho)e^{-rM}+\frac{h_1\theta D(s,\mu)\left(1-e^{-rt_1}\right)}{r}+\frac{h_2\theta D(s,\mu)}{r^2}$$
$$\times\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)+\frac{h_1D(s,\mu)}{\theta}\Bigg\{\frac{(\theta^2-r^2)e^{-(\theta+r)t_1}}{\theta+r}+r\Bigg\}+\frac{h_2D(s,\mu)}{\theta}$$
$$\times\Bigg\{\frac{1}{\theta+r}\Big(\theta^2t_1e^{-(\theta+r)t_1}-r^2\Big)+\frac{1}{(\theta+r)^2}\Big(\theta^2e^{-(\theta+r)t_1}-r^2\Big)+(6rt_2-1)\Bigg\}$$
$$+C_dD(s,\mu)\Bigg\{\frac{(\theta^2-r^2)e^{-(\theta+r)t_1}}{\theta+r}+1\Bigg\}+\frac{S_cD(s,\mu)(2\rho-r)}{\rho}+L_sD(s,\mu)r$$
$$+pI_p\Bigg\{\frac{D(s,\mu)\theta}{r}\Big(e^{-rM}-e^{-rt_1}\Big)+\frac{D(s,\mu)}{\theta}\Bigg\{\frac{1}{(r+\theta)}\Big(\theta e^{-(\theta+r)t_1}+1\Big)\Bigg\}\Bigg\}\Bigg].$$

It is clear that for value of $t_2>0$, $\frac{\partial^2\mathrm{TP}_1}{\partial t_2^2}<0$. Thus, for given s,μ, and T, TP_1 is a pseudo convex w.r.t. "t_2". Hence, $\exists$ an unique value of t_2^* satisfying equation (27). □

Theorem 3. *With respect to "s",* TP_1 *is a pseudo-concave and hence there exists an unique "s" which satisfy the following equation:*

$$(d_0-2d_1s+d_2\mu)\Bigg\{\frac{1}{r}\left(1-e^{-rt_2}\right)+\frac{e^{-rT}}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\Bigg\}+\Bigg[d_1p\Bigg\{t_1+\frac{1}{\theta}\Big(e^{\theta(t_2-t_1)}-1\Big)$$
$$+\frac{1}{\rho}\Big(1-e^{-\rho(T-t_2)}\Big)\Bigg\}e^{-rM}+d_1h_1\Bigg\{\frac{1}{r^2}\left(e^{-rt_1}+rt_1-1\right)+\frac{1}{\theta r}\Big(e^{\theta(t_2-t_1)}-1\Big)\left(1-e^{-rt_1}\right)\Bigg\}$$
$$+d_1h_2\Bigg\{\frac{1}{r^3}\left\{(t_1r+2)e^{-rt_1}+(t_1r-2)\right\}+\frac{1}{\theta r^2}\Big(e^{\theta(t_2-t_1)}-1\Big)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\Bigg\}+\frac{d_1h_1}{\theta}$$

$$\times\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{d_1h_2}{\theta}\left\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)\right.$$
$$\left.+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+C_dd_1$$
$$\times\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{S_cd_1}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)\right.$$
$$\left.+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+L_sd_1\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+d_1pI_p$$
$$\times\left\{\left(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(e^{-rM}-e^{-rt_1}\right)+\frac{1}{\theta}\left\{\frac{1}{(r+\theta)}\right.\right.$$
$$\left.\left.\left.\times\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\right\}-\frac{(d_0-2d_1s+d_2\mu)I_e}{r^2}\left\{1-e^{-rM}(1+rM)\right\}\right]=0$$

and such that TP_1 *is maximum.*

Proof. Total profit of the retailer in case-1 is TP_1 where

$$\mathrm{TP}_1=\frac{1}{T}(\mathrm{RG}-\mathrm{TC}_1) \tag{28}$$

where value of RG and TC_1 are given in equations (11) and (12) respectively.

Necessary condition of the optimality is $\frac{\partial \mathrm{TP}_1}{\partial S}=0$ which gives

$$(d_0-2d_1s+d_2\mu)\left\{\frac{1}{r}\left(1-e^{-rt_2}\right)+\frac{e^{-rT}}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}+\left[d_1p\left\{t_1+\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)+\frac{1}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}\right.$$
$$\times e^{-rM}+d_1h_1\left\{\frac{1}{r^2}\left(e^{-rt_1}+rt_1-1\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}\right)\right\}+d_1h_2\left\{\frac{1}{r^3}\left\{(t_1r+2)e^{-rt_1}\right.\right.$$
$$\left.\left.+(t_1r-2)\right\}+\frac{1}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\right\}+\frac{d_1h_1}{\theta}\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)\right.$$
$$\left.+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{d_1h_2}{\theta}\left\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)\right.$$
$$\left.+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+C_dd_1\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}$$
$$+\frac{S_cd_1}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+L_sd_1\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)\right.$$
$$\left.-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+d_1pI_p\left\{\left(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\right.$$
$$\left.\times\left(e^{-rM}-e^{-rt_1}\right)+\frac{1}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\right\}-\frac{(d_0-2d_1s+d_2\mu)I_e}{r^2}$$
$$\left.\times\left\{1-e^{-rM}(1+rM)\right\}\right]=0. \tag{29}$$

For given value of s, μ, and T, first and second partial derivative of TP_1 w.r.t. "t_2" are as follows:

$$\frac{\partial \mathrm{TP}_1}{\partial S} = \frac{1}{T}[(d_0 - 2d_1 s + d_2\mu)\left\{\frac{1}{r}\left(1 - e^{-rt_2}\right) + \frac{e^{-rT}}{\rho}\left(1 - e^{-\rho(T-t_2)}\right)\right\} + \left[d_1 p\left\{t_1 + \frac{1}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)\right.\right.$$
$$+ \frac{1}{\rho}\left(1 - e^{-\rho(T-t_2)}\right)\Big\}e^{-rM} + d_1 h_1\left\{\frac{1}{r^2}\left(e^{-rt_1} + rt_1 - 1\right) + \frac{1}{\theta r}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1}\right)\right\}$$
$$+ d_1 h_2\left\{\frac{1}{r^3}\left\{(t_1 r + 2)e^{-rt_1} + (t_1 r - 2)\right\} + \frac{1}{\theta r^2}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1} - t_1 e^{-rt_1}\right)\right\} + \frac{d_1 h_1}{\theta}$$
$$\times\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + \frac{d_1 h_2}{\theta}\left\{\frac{1}{\theta + r}\left(t_1 e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right)\right.$$
$$+ \frac{1}{(\theta + r)^2}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(t_2 e^{-rt_2} - t_1 e^{-rt_1}\right) + \frac{1}{r^2}\left(e^{-rt_2} - e^{-rt_1}\right)\Big\} + C_d d_1$$
$$\times\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\} + \frac{S_c d_1}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2} - e^{-\rho T+(\rho-r)t_2}\right)\right.$$
$$+ \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T+(\rho-r)t_2}\right)\Big\} + L_s d_1\left\{\frac{1}{r}\left(e^{-rt_2} - e^{-rT}\right) - \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T+(\rho-r)t_2}\right)\right\}$$
$$+ d_1 p I_p\left\{\left(\frac{(t_1 - M)e^{-rM}}{r} + \frac{1}{r^2}\left(e^{-rt_1} - e^{-rM}\right)\right) + \frac{1}{\theta r}\left(e^{\theta(t_2-t_1)} - 1\right)\left(e^{-rM} - e^{-rt_1}\right) + \frac{1}{\theta}\left\{\frac{1}{(r + \theta)}\right.\right.$$
$$\times\left(e^{\theta t_2-(r+\theta)t_1} - e^{-rt_1}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\Big\}\Big\} - \frac{(d_0 - 2d_1 s + d_2\mu)I_e}{r^2}\left\{1 - e^{-rM}(1 + rM)\right\}\Big]\Big]$$
$$\frac{\partial^2 \mathrm{TP}_1}{\partial s^2} = -4d_1\left[\left\{\frac{1}{r}\left(1 - e^{-rt_2}\right) + \frac{e^{-rT}}{\rho}\left(1 - e^{-\rho(T-t_2)}\right)\right\} + \frac{I_e}{r^2}\left\{1 - e^{-rM}(1 + rM)\right\}\right].$$

It is clear that for value of $s > 0$, $\frac{\partial^2 \mathrm{TP}_1}{\partial s^2} < 0$. Thus, for given t_2, μ, and T, TP_1 is a pseudo convex w.r.t. "s". Hence, $\exists$ a unique value of s^* satisfying equation (29). □

Theorem 4. *With respect to "μ", TP_1 is a pseudo-concave and hence there exists an unique "μ" which satisfy the following equation:*

$$(d_0 - 2d_1 s + d_2\mu)\left\{\frac{1}{r}\left(1 - e^{-rt_2}\right) + \frac{e^{-rT}}{\rho}\left(1 - e^{-\rho(T-t_2)}\right)\right\} + \left[d_1 p\left\{t_1 + \frac{1}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) + \frac{1}{\rho}\left(1 - e^{-\rho(T-t_2)}\right)\right\}\right.$$
$$\times e^{-rM} + d_1 h_1\left\{\frac{1}{r^2}\left(e^{-rt_1} + rt_1 - 1\right) + \frac{1}{\theta r}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1}\right)\right\} + d_1 h_2\left\{\frac{1}{r^3}\left\{(t_1 r + 2)e^{-rt_1}\right.\right.$$
$$+ (t_1 r - 2)\Big\} + \frac{1}{\theta r^2}\left(e^{\theta(t_2-t_1)} - 1\right)\left(1 - e^{-rt_1} - t_1 e^{-rt_1}\right)\Big\} + \frac{d_1 h_1}{\theta}\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right)\right.$$
$$+ \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\Big\} + \frac{d_1 h_2}{\theta}\left\{\frac{1}{\theta + r}\left(t_1 e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{(\theta + r)^2}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right)\right.$$
$$+ \frac{1}{r}\left(t_2 e^{-rt_2} - t_1 e^{-rt_1}\right) + \frac{1}{r^2}\left(e^{-rt_2} - e^{-rt_1}\right)\Big\} + C_d d_1\left\{\frac{1}{\theta + r}\left(e^{\theta(t_2-t_1)-rt_1} - e^{-rt_2}\right) + \frac{1}{r}\left(e^{-rt_2} - e^{-rt_1}\right)\right\}$$
$$+ \frac{S_c d_1}{\rho}\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2} - e^{-\rho T+(\rho-r)t_2}\right) + \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T+(\rho-r)t_2}\right)\right\} + L_s d_1\left\{\frac{1}{r}\left(e^{-rt_2} - e^{-rT}\right)\right.$$
$$- \frac{1}{\rho - r}\left(e^{-rT} - e^{-\rho T+(\rho-r)t_2}\right)\Big\} + d_1 p I_p\left\{\left(\frac{(t_1 - M)e^{-rM}}{r} + \frac{1}{r^2}\left(e^{-rt_1} - e^{-rM}\right)\right) + \frac{1}{\theta r}\left(e^{\theta(t_2-t_1)} - 1\right)\right.$$

$$\times\left(e^{-rM}-e^{-rt_1}\right)+\frac{1}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\Bigg\}-\frac{(d_0-2d_1s+d_2\mu)I_e}{r^2}$$
$$\times\left\{1-e^{-rM}(1+rM)\right\}\Bigg]=0$$

and such that TP_1 *is maximum.*

Proof. Total profit of the retailer in case-1 is TP_1 where

$$\mathrm{TP}_1=\frac{1}{T}(\mathrm{RG}-\mathrm{TC}_1) \tag{30}$$

where value of RG and TC_1 are given in equations (11) and (12) respectively.

Necessary condition of the optimality is $\frac{\partial \mathrm{TP}_1}{\partial\mu}=0$ which gives

$$\begin{aligned}
&d_2\left\{\frac{1}{r}\left(1-e^{-rt_2}\right)+\frac{e^{-rT}}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}-\Bigg[d_2p\left\{t_1+\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)+\frac{1}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}e^{-rM}\\
&\quad+d_2h_1\left\{\frac{1}{r^2}\left(e^{-rt_1}+rt_1-1\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}\right)\right\}+h_2d_2\Bigg\{\frac{1}{r^3}\left\{(t_1r+2)e^{-rt_1}+(t_1r-2)\right\}\\
&\quad+\frac{1}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\Bigg\}+\frac{h_1d_2}{\theta}\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\\
&\quad+\frac{h_2d_2}{\theta}\Bigg\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)\\
&\quad+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\Bigg\}+C_dd_2\left\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}+\frac{S_cd_2}{\rho}\\
&\quad\times\left\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\right\}+L_sd_2\Bigg\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)\\
&\quad-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\Bigg\}+nk\mu^{n-1}d_2pI_p\Bigg\{\left(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)\\
&\quad+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(e^{-rM}-e^{-rt_1}\right)+\frac{1}{\theta}\left\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\right\}\Bigg\}\\
&\quad-\frac{sI_e1}{r^2}\left\{1-e^{-rM}(1+rM)\right\}\Bigg]=0.
\end{aligned} \tag{31}$$

For given value of s, t_2, and T, first and second partial derivative of TP_1 w.r.t. "μ" are as follows:

$$\begin{aligned}
\frac{\partial \mathrm{TP}_1}{\partial\mu}&=\frac{1}{T}\Bigg[d_2\left\{\frac{1}{r}\left(1-e^{-rt_2}\right)+\frac{e^{-rT}}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}-\Bigg[d_2p\left\{t_1+\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)+\frac{1}{\rho}\left(1-e^{-\rho(T-t_2)}\right)\right\}\\
&\quad\times e^{-rM}+d_2h_1\left\{\frac{1}{r^2}\left(e^{-rt_1}+rt_1-1\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}\right)\right\}+h_2d_2\Bigg\{\frac{1}{r^3}\big\{(t_1r+2)e^{-rt_1}\\
&\quad+(t_1r-2)\big\}+\frac{1}{\theta r^2}\left(e^{\theta(t_2-t_1)}-1\right)\left(1-e^{-rt_1}-t_1e^{-rt_1}\right)\Bigg\}+\frac{h_1d_2}{\theta}\Bigg\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)\\
&\quad+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\Bigg\}+\frac{h_2d_2}{\theta}\Bigg\{\frac{1}{\theta+r}\left(t_1e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)+\frac{1}{(\theta+r)^2}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)
\end{aligned}$$

$$+\frac{1}{r}\left(t_2e^{-rt_2}-t_1e^{-rt_1}\right)+\frac{1}{r^2}\left(e^{-rt_2}-e^{-rt_1}\right)\Bigg\}+C_dd_2\Bigg\{\frac{1}{\theta+r}\left(e^{\theta(t_2-t_1)-rt_1}-e^{-rt_2}\right)$$
$$+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\Bigg\}+\frac{S_cd_2}{\rho}\Bigg\{\frac{1}{r}\left(e^{-(\rho+r)T+\rho t_2}-e^{-\rho T+(\rho-r)t_2}\right)+\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\Bigg\}$$
$$+L_sd_2\Bigg\{\frac{1}{r}\left(e^{-rt_2}-e^{-rT}\right)-\frac{1}{\rho-r}\left(e^{-rT}-e^{-\rho T+(\rho-r)t_2}\right)\Bigg\}+nk\mu^{n-1}d_2pI_p$$
$$\times\Bigg\{\left(\frac{(t_1-M)e^{-rM}}{r}+\frac{1}{r^2}\left(e^{-rt_1}-e^{-rM}\right)\right)+\frac{1}{\theta r}\left(e^{\theta(t_2-t_1)}-1\right)\left(e^{-rM}-e^{-rt_1}\right)$$
$$+\frac{1}{\theta}\Bigg\{\frac{1}{(r+\theta)}\left(e^{\theta t_2-(r+\theta)t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(e^{-rt_2}-e^{-rt_1}\right)\Bigg\}\Bigg\}-\frac{sI_e1}{r^2}\left\{1-e^{-rM}(1+rM)\right\}\Bigg]\Bigg]$$
$$\frac{\partial^2\mathrm{TP}_1}{\partial\mu^2}=-kn(n-1)\mu^{(n-2)}.$$

It is clear that for value of $\mu>0$, $\frac{\partial^2\mathrm{TP}_1}{\partial\mu^2}<0$. Thus, for given t_2, s, and T, TP_1 is a pseudo convex function w.r.t. "μ". Hence, $\exists$ a unique value of μ^* satisfying equation (31). □

Theorem 5. TP_i, $i=2,3,4$ *is a pseudo-concave function w.r.t. "T".*

Proof. Proof same as Theorem 1. □

Theorem 6. TP_i, $i=2,3,4$ *is a pseudo-concave w.r.t. "t_2".*

Proof. Proof same as Theorem 2. □

Theorem 7. TP_i, $i=2,3,4$ *is a pseudo-concave w.r.t. "s".*

Proof. Proof same as Theorem 3. □

Theorem 8. TP_i, $i=2,3,4$ *is a pseudo-concave w.r.t. "μ".*

Proof. Proof same as Theorem 4. □

7. Numerical analysis

In this section, to validate the proposed inventory model numerical analysis has been carried out. For this, data has been taken from the Tiwari *et al.* [80] with appropriate modification. All the calculation is performed with the help of software MATHEMATICA (version 5.2).

Example 1. In order to illustrate the first case, following data has been considered:

$K=245\$$, $\emptyset=1.5$, $l=0.06$, $d_o=258$; $d_1=4.3$, $d_2=11.5$, $p=71\$$, $t_1=0.3$ year, $\theta=0.05$, $r=0.09$, $M=0.28$ year, $h_1=0.49\$$, $h_2=0.15\$$, $C_d=10\$$, $S_c=4\$$, $L_s=5\$$, $k=6$, $n=1.5$, $I_e=0.12\%$, $I_p=0.15\%$, $\delta=0.93$.

On applying the solution methodology, required optimal solutions are as follows:

s^* (\$)	μ^*	t_2^* (year)	T^* (year)	Q^* (units)	TP_1^* (\$)
172	74	0.74	0.84	310	24 420

Concavity of objective function: now, we check the nature of Hessian matrix H at $(172, 74, 0.74, 0.84)$.

$$H_{11} = -7.98268 < 0; \quad H_{22} = 108.974 > 0;$$
$$H_{33} = -1.1579 \times 10^8 < 0; \quad H_{44} = 1.5146 \times 10^{13} > 0.$$

Thus, the Hessian matrix is negative semi-definite and hence the profit is maximum at $(0.74, 0.84, 172, 74)$.

Example 2. To illustrate the second case, data remain same except the following: $M = 0.38(\$)$.
On applying the solution methodology, required optimal solutions are as follows:

s^* (\$)	μ^*	t_2^* (year)	T^* (year)	Q^* (units)	TP_1^* (\$)
149	76	0.64	0.79	310	28 569

Concavity of objective function: now, we check the nature of Hessian matrix H at $(149, 76, 0.64, 0.79)$

$$H_{11} = -7.9703 < 0; \quad H_{22} = 108.384 > 0;$$
$$H_{33} = -2.13744 \times 10^8 < 0; \quad H_{44} = 3.83868 \times 10^{12} > 0.$$

Thus, the Hessian matrix is negative semi-definite and hence the profit is maximum at $(149, 76, 0.64, 0.79)$.

Example 3. To illustrate the second case, data remain same except the following: $M = 0.88(\$)$.
On applying the solution methodology, required optimal solutions are as follows:

s^* (\$)	μ^*	t_2^* (year)	T^* (year)	Q^* (units)	TP_1^* (\$)
164	74	0.71	0.94	363	29 641

Concavity of objective function: now, we check the nature of Hessian matrix H at $(164, 74, 0.71, 0.94)$

$$H_{11} = -7.77898 < 0; \quad H_{22} = 103.875 > 0;$$
$$H_{33} = -2.16458 \times 10^8 < 0; \quad H_{44} = 1.12464 \times 10^{12} > 0.$$

Thus, the Hessian matrix is negative semi-definite and hence the profit is maximum at $(164, 74, 0.71, 0.94)$.

Example 4. To illustrate the second case, data remain same except the following: $M = 0.91(\$)$.
On applying the solution methodology, required optimal solutions are as follows:

s^* (\$)	μ^*	t_2^* (year)	T^* (year)	Q^* (units)	TP_1^* (\$)
165	75	0.74	0.89	361	31 460

Concavity of objective function: now, we check the nature of Hessian matrix H at $(165, 75, 0.74, 0.89)$

$$H_{11} = -8.01682 < 0; \quad H_{22} = 110.241 > 0;$$
$$H_{33} = -7.40706 \times 10^7 < 0; \quad H_{44} = 1.50409 \times 10^{13} > 0.$$

Thus, the Hessian matric is negative semi-definite and hence the profit is maximum at $(165, 75, 0.74, 0.89)$.

Remark. As far as our knowledge is concerned, no work is reported in the literature considering all burning issues as reported in the current model. So, it is not possible to compare our results quantitatively with the previous work. On ignoring some of the challenging issues facing by the organization as considered in current work, our developed model and the previously quoted work in literature become same.

(1) If we consider constant rate of deterioration in place of non-instantaneous deterioration, constant demand, constant holding cost, no investment to reduce the ordering cost, and with shortages then the current model reduces the inventory model of Chung and Liao [11].
(2) In the integrated environment, if we consider deterioration in place of non-instantaneous deterioration, skip the effect of inflation and ordering cost reduction investment, and consider demand as stock-dependent then the current model reduces the retailer model of Pervin *et al.* [52].
(3) In the production environment, if we consider deterioration in place of non-instantaneous deterioration, skip the effect of inflation, ignore the investment to reduce the ordering cost, and consider time dependent demand then the current model reduces the inventory model of Roy *et al.* [60].
(4) If we consider time and selling price dependent, constant holding cost, and no investment to reduce the ordering cost then the current model reduces the inventory model of Sundararajan *et al.* [77].

7.1. Sensitivity analysis

This section is devoted to sensitivity analysis in order to observe the effect of parameters on the optimal decision taken by the decision-maker.

7.1.1. Effect of different demand parameters

Figures 3a–3c shows the effect of various demand parameters on the system's optimal order quantity and profit. It has been discovered that increasing d_0 and d_1 leads to an increase in order quantity and profit for the system. It is observed that as the value of d_1 increases, the system's order quantity and profit decrease. Demand decreases by one-third as the value of d_1 increases from -25 to 25%. Due to this, order size dropped by decision-maker by one-third and hence the profit dropped by one-fourth. Similarly, as the value of d_2 increases from -25 to 25%, demand increases roughly four times. As a result, the system's profit shows a sharp inclination.

7.1.2. Effect of purchase cost

Figure 4 depicts the impact of variation in purchase cost on the total profit of the system. The results show that the decision-maker must compromise on profit due to an increase in purchase costs. The change in purchase cost causes a 46% decrease in profit.

7.1.3. Effect of partial backlogging parameter

Figure 5 depicts the effect of the partial backlog parameter on profit. As the partial backlog parameter value increases, more customers' demand is met during the shortage period. This accounts for a 0.7% (approx.) decrease in the system's profit.

7.1.4. Effect of deterioration rate

Figure 6 depicts the effect of deterioration rate on order quantity and wastage cost. It has a negative economic impact. Analysis indicates that as the deterioration rate increases, the optimal order quantity also increases to meet the demand. The findings also show that as the deterioration rate increases, so does the cost of waste.

7.1.5. Effect of interest earned rate and interest paid rate

The effect of a change in the interest earned rate and interest paid rate on the system's profit is depicted in Figure 7. The system's profit decreased due to an increase in the interest rate paid. As the interest rate paid rises from -25% to 25%, the system's profit falls by 7.2%. When the interest rate earned increases from -25% to 25%, the system's profit increases by 0.7%.

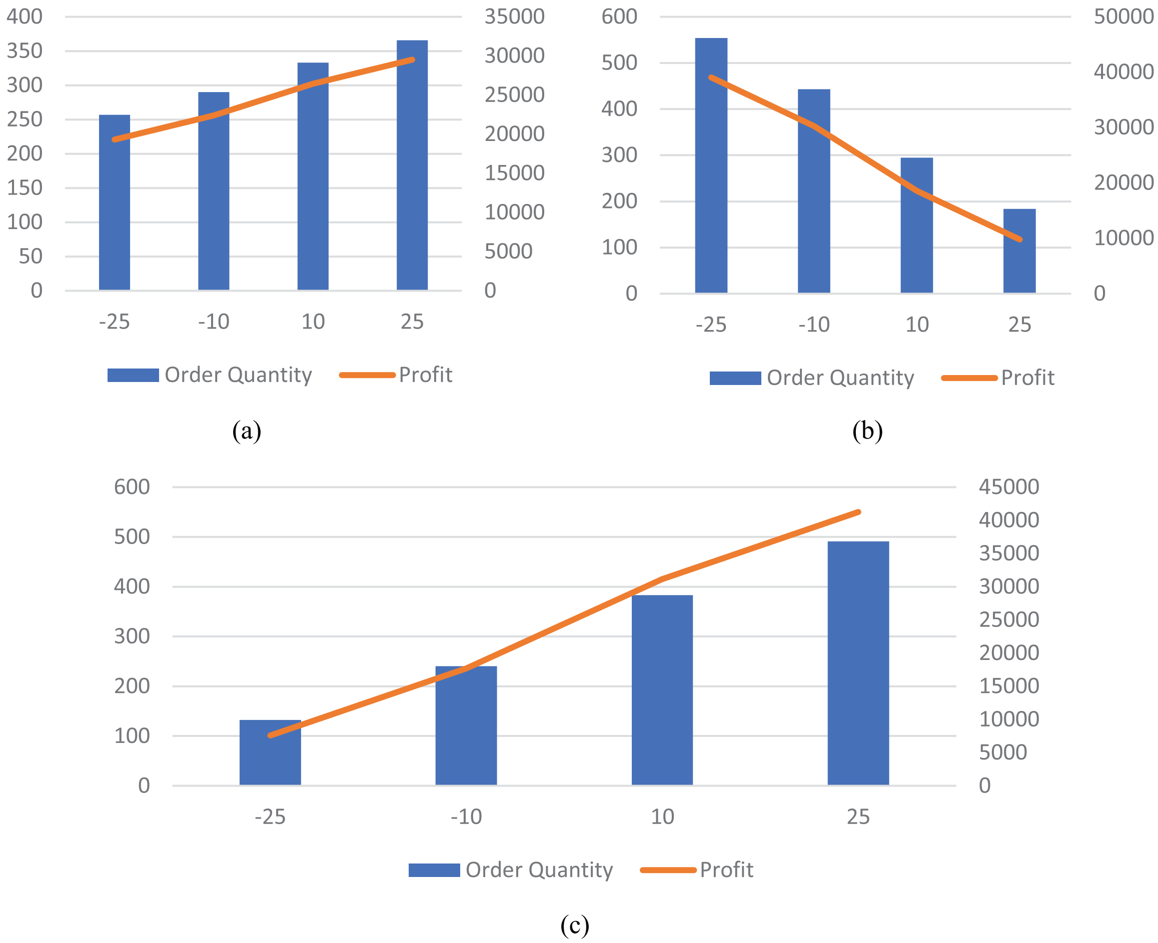

FIGURE 3. (a) Effect of base demand (d_0). (b) Effect of scalar parameter of selling price (d_1). (c) Effect of scalar parameter of promotional activities (d_2).

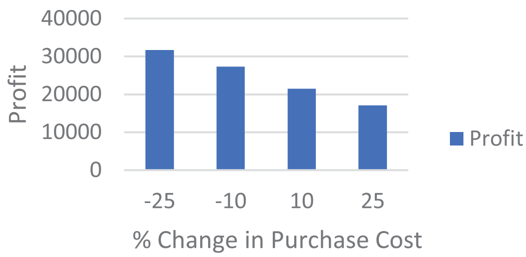

FIGURE 4. Effect of purchase cost on profit.

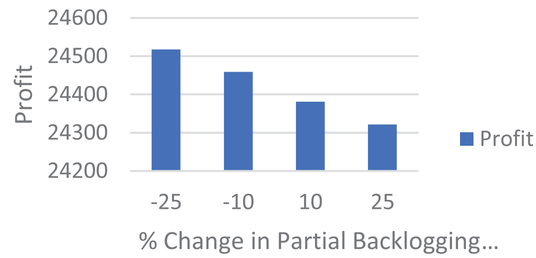

Figure 5. Effect of partial backlogging parameter.

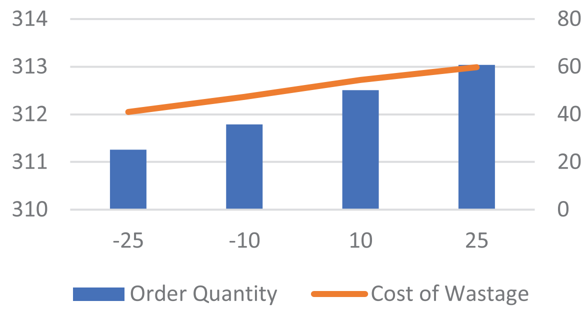

Figure 6. Effect of deterioration rate on profit.

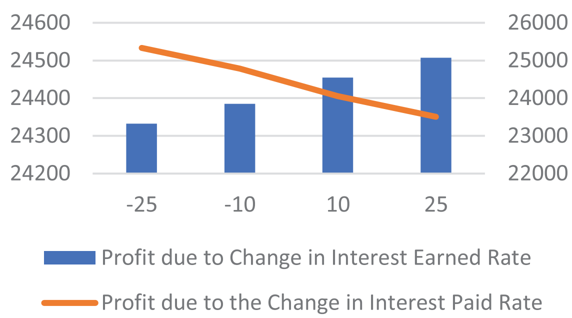

Figure 7. Effect of interest earned rate and interest paid rate on profit.

7.1.6. Effect of inflation on profit

Figure 8 depicts the effect of inflation on the system's profit. As a result, as the rate of inflation rises from −25% to 25%, profit falls from \$26 069 to \$22 669. As a result, profit suffers as a result of inflation.

7.1.7. Effect of shortage cost and lost sale cost

As illustrated in Figure 9, shortage costs and lost sale costs reduce profit. The result shows that as the shortage cost rises from −25% to 25%, the system's profit falls from \$24 424 to \$24 415. Profit decreases from \$24 425 to \$24 415 with the same variation in lost sale cost.

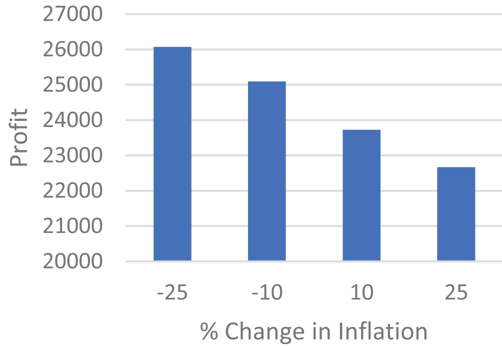

FIGURE 8. Effect of inflation on profit of system.

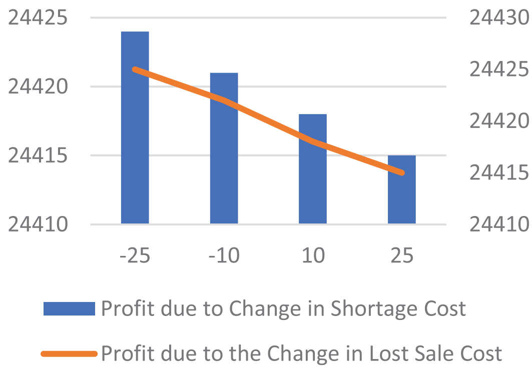

FIGURE 9. Effect of shortage cost and lost sale cost.

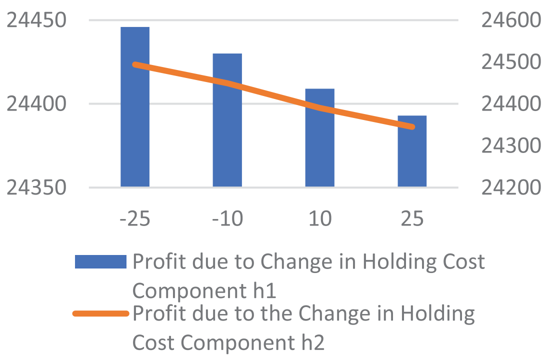

FIGURE 10. Effect of different components of holding cost.

7.1.8. Effect of different components of holding cost

Figure 10 depicts the effect of various components of holding cost on the system's profit. Profit is negatively related to both components. Profit decreases by 0.21% and 0.60% when h_1 and h_2 increase from -25% to 25%, respectively.

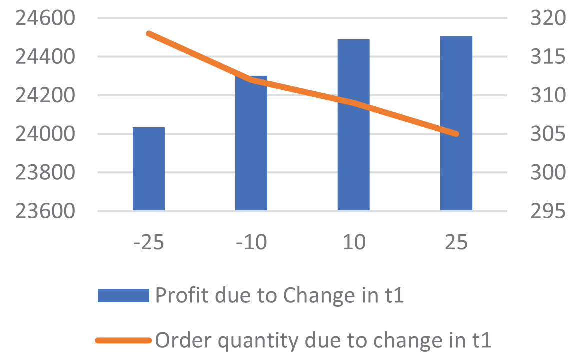

FIGURE 11. Effect of non-deteriorating period on profit and order quantity.

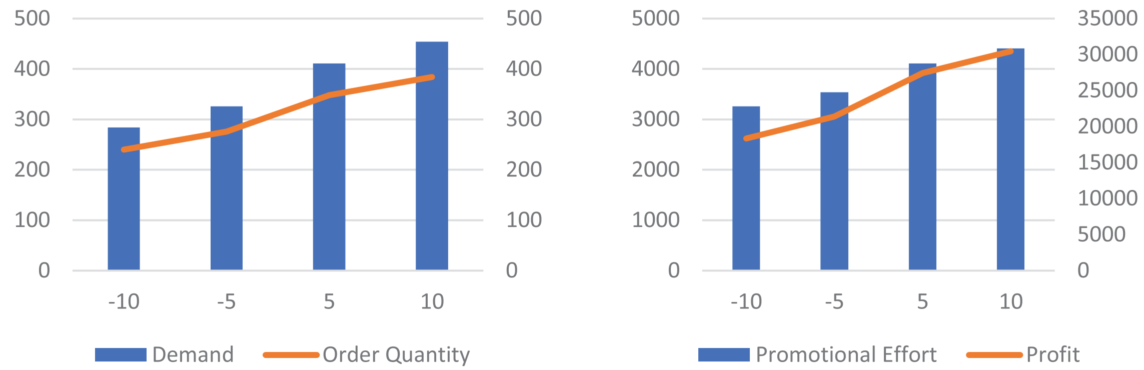

FIGURE 12. Effect of promotional effort.

7.1.9. Effect of non-deteriorating period

Figure 11 reflects the effect of the non-deteriorating period on the profit and order quantity of the system. As the non-deteriorating period increases from −25% to 25%, the profit of the system rises from \$24 035 to \$24 506 and the order quantity declines from 318 to 305.

7.1.10. Effect of promotional effort

Figure 12 depicts the impact of promotional effort on demand, order quantity, and profit. Each component is positively related to the promotional effort. As the promotional effort increase from −10% to 10%, there is a 39% increase in demand and order quantity, and a 42% increase in profit. To accomplish this, decision-makers must increase promotional spending by 24%.

7.2. Managerial insights

(1) Results suggest that scale parameters of selling price and promotional activities have an impact on the decision-making process of a decision-maker. So, while making an optimal decision, decision-makers take cognizance of how these parameters which are related to the selling price and cost associated with promotion activities.

(2) It is observed that on increasing the parameter of promotional activities or decreasing the scale parameter of selling price results decline in optimal cycle length and hence increases the total profit of the system. Therefore, it is suggested to the retailer that balance these two parameters in such a way the items can

be sale as much as possible before the start of the deterioration process. In addition to this, investment in promotional activities should be made keeping the available resources in mind so the stockout situation can be reduced as much as possible.

(3) Larger period of non-deterioration results small replenishment cycle and total profit of the system increases. Thus, present study suggests that decision-makers should pay special attention on different mechanisms such as preservation technology to increase the period of non-deterioration. This has positive impact on the environment as well as economy.
(4) Numerically it is observed that lower interest paid rate or higher interest earned rate results high profit for the retailer. Thus, present study suggests that decision-makers must select those suppliers where the interest paid rate is lower and the interest earned rate is higher as this is important for financial sustainability.
(5) Practically, because of high inflation rate inventory cost of the retailer's system increases and hence overall profit of the system declines. Thus, present study suggests that decision-makers must try to reduce the total inventory cost of the system and always consider inflation while designing the optimal polices for the inventory system.
(6) Result suggests the decision-maker must consider time dependent holding costs especially dealing in deteriorating products to get a correct picture of the financial position.
(7) The results give the direction to the decision-maker that they must pay special attention while choosing the supplier to meet their requirements as purchase cost plays a very crucial role.
(8) This study instructs the decision-maker on how much customer satisfaction costs him. As a result, the study directs decision-makers on how to balance profit and customer satisfaction.

8. Conclusion and future work

A retail inventory model has been developed that considers non-instantaneous deteriorating items, inflation, trade credit, shortages with partial backlog, and time dependent holding cost. Furthermore, the demand rate has been established as a function of the selling price and promotional efforts. The proposed model has a wide variety of applications in different industries, such as textile industries, fashionable products, electronic industries, industries related to volatile substances, food items, and others. Results indicate that trade credit periods as well as inflation affected the optimal decisions taken by the decision-maker for inventory control. The findings suggest that decision-makers pay special attention when determining the selling price when demand is highly dependent on the selling price, as changing the selling price parameters results in a 69% drop in total profit. Similarly, a positive increase in profit is observed as a result of the change in promotional efforts. According to the findings, increasing the deterioration rate from −25% to 25% increased the cost of waste by 31%. As a result, decision-makers implement some mechanism to control the rate of deterioration, which is beneficial from both an economic and an environmental standpoint. Because an increase in interest paid reduces profit by 7%, the decision-maker must pay attention while selecting a supplier. Analysis also shows that decision-makers pay special attention to the selection of promotional activities, as their efforts result in a 40% increase in profit. According to the findings of this study, a decision-maker can earn more profit if the supplier is chosen correctly, as a longer trade credit period means more profit for the system. Furthermore, analysis shows that as the non-deteriorating period lengthens, so does the system's profit. This result suggests that the decision-maker take some steps to extend the non-deteriorating period. There are numerous fertile areas where current work can be expanded. The current work can be expanded by considering different mechanisms to control the rate of deterioration, such as preservation technology ([89]; [38]). Carbon emissions are unavoidable due to various inventory-related activities such as stock holding, deterioration, ordering the product, and so on. Thus, different carbon regulation mechanisms ([85]; [32]) can be incorporated in the future in the current model. Impreciseness at various costs [13, 75] can be considered in the current study to increase its practical utility.

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