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NEUTROSOPHIC PROGRAMMING APPROACH TO MULTILEVEL DECISION-MAKING MODEL FOR SUPPLIER SELECTION PROBLEM IN A FUZZY SITUATION

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Abstract. In supply chain management, the selection of suppliers is vital and plays a valuable role in the performance of organizations. A hierarchical structure, with different levels in the selection of suppliers, is employed, wherein sequential decisions are made from the highest to the lowest level. Decision variables, called controlling factors, are divided into several categories. In the decision-making process, often because of the lack of confidence or uncertainty, It becomes challenging for decision-makers to give explicit/crisp values to any parameter, resulting in uncertainty in the problem. In this paper, we address a multi-level supplier selection problem with fuzzy supply and demand. To avoid decision conflicts, superior or upper-level decision-makers give tolerances that could be used as a possible relaxation. Thus, the problem is employed with fuzzy constraints. Based on a neutrosophic decision set, the neutrosophic compromise programming approach (NCPA) is used as a solution technique with the idea of an indeterminacy degree as well as different objectives for membership and non-membership degrees. Membership functions (Linear-type) are used to develop satisfactory solutions by fuzzily describing objective functions and controlling factors. A numerical illustration is provided to demonstrate the validity and appropriateness of NCPA.

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1. Introduction

The supplier's role is crucial for an organization to excel in a competitive market. Selecting a reliable supplier who can be advantageous for the organization, considering various restrictions related to price, trait, continuance, demand, and supply is called a supplier selection problem (also called a sourcing decision problem). In a competitive milieu, customers' satisfaction is directly proportional to the selection of the best supplier, which consequently improves the efficiency of the supply chain. Moreover, the selection of felicitous suppliers enhances quality measures by reducing the levels of non-conformity of the products, enhancing resilience to fulfil the needs of end customers and lowering lead time during different phases. The supplier selection problem (SSP)

Keywords. Supply chain, Supplier selection, Multilevel programming, Indeterminacy membership function, Neutrosophic compromise programming approach.

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comprises several characteristics, including both qualitative and quantitative elements. SSP can be expressed as a linear programming problem. The multiobjective SSP is considered under the uncertain situation. It is assumed that an automobile company places an ordered quantity to the different suppliers for multiple parts to identify the quota allocation in a supply chain. For this purpose, the decision-makers are strictly against the shortage of parts, specific with the different capacities and budget allocation. Due to real-life complexity, vagueness and ambiguousness among the parameters are taken as type-2 triangular fuzzy numbers (T-2TF), which is more realistic. The decision-makers main aim is to handle the T-2TF parameters so that the minimum total cost associated with ordering the aggregate demand, the rejected items of the suppliers, and the late delivered items are obtained. The supplier selection problem is a well-known and integral component of the supply chain planning problems. The selection of best suppliers depends on various criteria such as overall performance ratings, less rejection of items, timely delivery, fulfilling aggregate demand, etc. The literature suggested that a large part is dedicated to SSP as a multi-criteria decision-making problem in interval type-2 fuzzy uncertainty.

1.1. Research gaps and motivation

The neutrosophic programming approach is a well-renowned and emerging optimization technique and can be applied to a vast range of real-life decision-making problems such as multilevel decision-making models for supplier selection problems. It has several work motivations that make it a promising area of research. Some of these work motivations include:

– Dealing with uncertainty and imprecision: the supplier selection problem in a fuzzy environment is characterized by a high degree of uncertainty and imprecision, making it challenging to make informed decisions. The neutrosophic programming approach provides a means of dealing with such uncertainties by allowing for the representation of partial truths and indeterminacy.
– Flexibility in modeling: the neutrosophic programming approach provides a flexible modeling framework that can accommodate different levels of decision-making, including strategic, tactical, and operational decision levels. This flexibility allows for a more comprehensive and integrated approach to supplier selection problems.
– Handling complex and conflicting criteria: the supplier selection problem involves multiple criteria, often with conflicting objectives. The neutrosophic programming approach can be used to handle such complex and conflicting criteria by incorporating multiple objectives and constraints in the decision-making process.
– Integration of domain knowledge: the neutrosophic programming approach can be integrated with domain knowledge to improve the decision-making process. This integration allows for the incorporation of expert knowledge and experience into the model, which can enhance the accuracy and relevance of the decision-making process.
– Potential for practical applications: the neutrosophic programming approach has the potential for practical applications in supplier selection problems in various industries, including manufacturing, logistics, and supply chain management. The development and application of neutrosophic programming models can lead to better decision-making and improved performance in these industries.
– Overall, the neutrosophic programming approach provides several work motivations that make it an exciting and promising area of research for the supplier selection problem in a fuzzy environment.
– Multi-level supplier selection problem with fuzzy supply and demand is considered.
– Neutrosophic compromise programming approach is used to solve above considered problem.
– Membership functions (Linear type) are also used to develop satisfactory solutions by fuzzily describing objective functions.

The paper is organised and structured in a way that makes sense in light of the proposed work. The first two sections are an introduction and literature review, while the third contains some preliminary material. After introducing the concept of multilevel decision-making in Section 4, we then formulate the model in Section 5 with an emphasis on supplier selection problems involving fuzzy parameters. The neutrosophic compromise programming approach to the above problem and its algorithm has been covered in detail in Sections 6 and 7. Section 8 provides a case study that was analysed computationally. The proposed work's managerial benefits are outlined in Section 9. The conclusion is presented in Section 10, along with some recommendations for the future.

2. Literature review

Several researchers have conducted various studies on SSP. In the early twenty-first century, different methods were employed by researchers for the solution of SSP [12]. Kumar *et al.* [32] developed a model using fuzzy sets; that reduced total purchasing costs, late deliveries, and rejected goods; and satisfied all objectives of a multi-objective integer programming problem concerning the selection of vendors. Analytic hierarchy process (AHP) and fuzzy linear programming were used by Kumar *et al.* [33] to find the most cost-effective suppliers. Lee [34] used Fuzzy AHP for supplier selection and ranked suppliers according to profit, marketing opportunity, cost, and risk. Xu *et al.* [50] worked on a deterministic multi-item aggressive lot size problem with collaborative business loads discount. Ghodsypour and O' Brain [26] examined the overall cost of logistics in supplier selection under multiple sources, multiple criteria, and capacity constraint conditions. Crama *et al.* [21] employed optimal procurement decisions in the presence of total quantity discounts and alternative product recipes. Xia and Wu [49] considered multi-criteria SSP in the volume discount problem. Mansini *et al.* [36] studied quantity discounts and truckload shipping in supplier selection problems. The inventory, supplier selection, and transportation programming problems were modelled by Choudhary and Shankar [20] using goal programming, and the solutions were compared to examine the benefits and drawbacks of each approach. Kar [30] utilized a mix of the Neural Network Algorithm, AHP, and fuzzy sets to choose suppliers. Cardenas-Barron *et al.* [18] created a strategy for handling SSP-based lot sizing for multi-product, multi-period inventories. Muneeb *et al.* [40] developed a bi-level decision planning model for municipal solid waste management with cost reliability under an uncertain environment. Muneeb *et al.* [48] developed a bi-level decision-making approach for the vendor selection problem with random supply and demand.

Multilevel organizations work as hierarchical structures composed of mutual decision-making units. Decisions are performed sequentially in a hierarchical system, beginning at the top and proceeding to the bottom. Each decision-maker attempts to maximize his earnings, yet the decisions of higher-level decision-makers influence his decision via after-effects. The upper-level decision maker initially defines the aim, and then subordinate levels are free to have their responses or optima, calculated in insulation. For the organization's overall benefit, the top/upper-level decision maker can modify the decisions at a lower level. In multilevel programming, the fundamental problem is that a single objective is considered at each level. In the work of Migdalas *et al.* [38], the approach involving multilevel single-objective problems could be found. However, considering multilevel, multiobjective problems is a more realistic approach. In this regard, Fuzzy Goal Programming was used by Baky [14] to carve a bi-level multiobjective problem model and bi-level multiobjective fractional programming problems. Based on the uncertainty, Ke *et al.* [31] contributed with an approach by amalgamating a genetic algorithm, simulations, and neural network into multilevel programming. The multilevel multiobjective problem was solved using Fuzzy Goal Programming by Pramanik *et al.* [44]. Adhami *et al.* [2] developed a multilevel decision-making model for the supplier selection problem in a fuzzy situation. They used membership functions for the controlling factors and goals for optimal solutions.

In realistic problems, specifically with the problems in supplier selection, it is only sometimes viable to ascertain the accurate values of the parameters involved in the problem. Only a limited amount of information may be feasible depending on prior knowledge and understanding. Then comes the element of uncertainty. As a result, the parameters can have various types of uncertainty, such as fuzzy numbers and random variables with known mean and variance. If the parameters are random variables, following some probability distribution, these problems might be dealt with a stochastic programming approach. Fuzzy techniques can be used if the uncertainties are due to vagueness or obscureness. These uncertainties in the parameters can be attributed to a lack of appropriate information, sudden shifts in the harmony of the environment, a need for newly cast products, and shortages of highly desired outcomes, and in the current scenario of the pandemic, intermittent lockdowns, among other things.

The fuzzy programming approach (FPA) came into existence after the invention of fuzzy sets (FS) by Zadeh [51]. Zimmermann [52] applied the concepts of FS theory with suitable membership functions to solve linear programming problems involving multiobjective functions, wherein multiple objectives are converted into a single objective, after which FPA maximizes the membership function (belongingness). Applications of FPA can be found in abundance in various problems of optimization [25, 35, 45]. The FPA considers the degree of belongingness; in some cases, dealing with a non-membership function may be critical (non-belongingness). To overcome this issue, the intuitionistic fuzzy set (IFS), a continuation of FS, was proposed by Atanassov [13]. Because it also addresses non-membership function (non-belongingness) or the element's failure in the collection, IFS is more cognition-based than FS. Based on IFS, the intuitionistic fuzzy programming approach (IFPA) has grown in prominence among real-time multiobjective optimization techniques. Several researchers have used IF and IFS, with varying degrees of modification, to solve optimization problems. Angelov [11] introduced the optimization technique to handle practical issues in an intuitionistic fuzzy environment. Bharti and Singh [17] solved multiobjective linear programming issues in an interval-valued intuitionistic fuzzy context. A new method for a multiobjective, multiple-choice, intuitionistic fuzzy transportation problem was proposed by Chakraborty *et al.* [19], which makes use of a chance operator. Sayed and Abo-Sinna [24] developed an approach for a fully intuitionistic fuzzy multiobjective fractional transportation problem. Adhami and Ahmad [1] created an interactive Pythagorean-hesitant fuzzy computational method for the multiobjective transportation issue under uncertainty.

Along with the previously mentioned advances of IF and IFS, a set known as a neutrosophic set (NS) has recently emerged. The word "neutrosophic," which distinguishes it from FS and IFS, literally means "knowledge of neutral thoughts" [47]. Smarandache introduced the idea of a neutrosophic set [47]. Future studies in this area will build on this idea of neutrality/indeterminacy in NS. The neutrosophic compromise programming technique (NCPA) has been developed based on NS to find the optimal answer to the multiobjective optimization problem. The NCPA considers three membership functions: maximizing truth (belongingness), minimizing falsity (non-belongingness), and indeterminacy (belongingness to some extent) [47]. Rizk-Allah *et al.* [46] built a neutrosophic compromise programming model to discover the best compromise solution, and they validated it by evaluating the ranking degree with the TOPSIS technique. Simultaneously, Pramanik [43] established the concept of neutrosophic linear goal programming for multiobjective optimization with uncertainty and indeterminacy. Pamucar *et al.* [23] proposed a model that is a recollection of a new weight aggregator, which uses pairwise comparison for fuzzy neutrosophic decision-making tactics for supplier assessment and selection. The NCPA optimizes the indeterminacy/neutral degree of satisfaction, maximizing and minimizing the decision makers' satisfaction and dissatisfaction.

Pervin *et al.* [41] proposed investigating the ideal retailer's replenishment options for deteriorating items, including time-dependent demand, to demonstrate more practical situations within economic-order quantity frameworks. To combine the facility placement problem and the transportation problem inside a multi-objective context, Das and Roy [22] created the multi-objective transportation-p-facility location model. Mondal *et al.* [39] and Ahmad [3,4] created a multi-objective multi-product multi-period two-stage sustainable opened- and closed-loop supply chain planning to ensure supply across production centers and hospitals. Barman *et al.* [15] offer a multi-objective sustainable economic production quantity model with partial back ordering shortages, in which the consequences of sustainability are examined and resolved using the Fuzzy Goal Programming approach. Pervin *et al.* [42], Ahmad and John [5] and Ahmad *et al.* [8,9] studies develop an integrated vendor-buyer model for decaying items. Shortages are permitted for both the vendor and the buyer to regulate the degradation rate. Ghosh *et al.* [27] initiate a multi-objective solid transportation problem with a preservation technology connection in a Pythagorean fuzzy environment. Mardanya *et al.* [37] established a transportation problem strategy that considers the multi-modal transport framework to optimize overall transportation cost under the rough interval approximation. Multi-objective decision-making was created by Ghosh *et al.* [28] and used in the design of real-world transportation problems. Using neutrosophic linear programming, fuzzy programming, and the global criteria technique, a compromise solution to the multi-objective transportation problem is developed. Giri and Roy [29] and Ahmad *et al.* [6,7] introduced neutrosophic programming (NP) and Pythagorean hesitant fuzzy programming (PHFP) to extract a better compromise solution for a multi-objective, green, four-dimensional, fixed-charge transportation problem.

This paper considers a multilevel supplier selection problem with fuzzy demand and supply. Decision impasses are alleviated by allowing for possible exemptions in the form of tolerances offered by higher-level decision-makers. Fuzzy restrictions are used to solve the problem. The NCPA, based on a neutrosophic decision set, is used as a solution technique with the concept of indeterminacy degree and membership and non-membership degree of various objectives.

3. Preliminaries

This section discusses some definitions related to FS, IFS, and NS.

3.1. Fuzzy set (FS)

Definition 3.1 ([16])**.** A fuzzy set A on a universe of discourse X is defined by a membership function $\mu_{\tilde{A}}(x)$, which maps each element x of X to a value between 0 and 1, denoting the degree of membership of x in A. The membership function $\mu_{\tilde{A}}(x)$ is a real valued function defined on X

$$\mu_{\tilde{A}}(x) : X \rightarrow [0,1].$$

Definition 3.2 ([16])**.** The triplet $\tilde{X}(p,q,r)$, indicating the lower, middle, and upper values of a membership function are called parabolic fuzzy number if its membership function expressed as

$$\mu_{\tilde{X}}(y) = \begin{cases} \left(\frac{y-p}{q-p}\right)^2, & \text{if } \ p \leq y \leq q; \\ 1, & \text{if } \ y = q; \\ \left(\frac{r-y}{r-q}\right)^2, & \text{if } \ q \leq y \leq r; \\ 0, & \text{otherwise.} \end{cases} \tag{3.1}$$

Defuzzification of parabolic number [25]: Defuzzification is the process of finding a crisp or deterministic value of the fuzzy number. The defuzzified value function d of the parabolic fuzzy number $\tilde{X}(p,q,r)$ is given as

$$d\left(\tilde{X}\right) = \frac{(p+2q+r)}{4}. \tag{3.2}$$

3.2. Intuitionistic fuzzy set (IFS)

Definition 3.3 ([13])**.** Let there be a universal set Y; then, an IFS W in Y, is given by the ordered triplets as follows:

$$W = \left\{y,\, \mu_W(y),\, v_W(y) \middle| y \in Y\right\},$$

where $\mu_W(y) : Y \rightarrow [0,1]$; $v_W(y) : Y \rightarrow [0,1]$.

With conditions $0 \leq \mu_W(y) + v_W(y) \leq 1$. Where $\mu_W(y)$ and $v_W(y)$, denote the membership and non-membership functions of the elements $y \in Y$ into the set W.

3.3. Neutrosophic set (NS)

Definition 3.4 ([47])**.** Let there be universal set Y, such that $y \in Y$, then a neutrosophic set A in Y is expressed by three membership functions, viz., truth $T_A(y)$, indeterminacy $I_A(y)$ and a falsity $F_A(y)$ and is denoted by the following form:

$$A = \left\{y,\, T_A(y),\, I_A(y),\, F_A(y) \middle| y \in Y\right\}, \tag{3.3}$$

where $T_A(y),\ I_A(y),$ and $F_A(y)$ are real standard or non-standard subsets belonging to $]0^-,1^+]$, also given as $T_A(y) : Y \to]0^-,1^+],\ I_A(y) : Y \to]0^-,1^+]$, and $F_A(y) : Y \to]0^-,1^+]$. Also, there is no restriction on the sum of $T_A(y),\ I_A(y),$ and $F_A(y)$, so we have,

$$0^- \leq \sup T_A(y) + I_A(y) + \sup F_A(y) \leq 3^+. \tag{3.4}$$

Definition 3.5 ([47])**.** A single valued neutrosophic set (SVNS) A over a universal set Y, is defined as

$$A = \{y,\ T_A(y),\ I_A(y),\ F_A(y) \big| y \in Y\}, \tag{3.5}$$

where $T_A(y)$, $I_A(y)$, and $F_A(y) \in [0,1]$ and $0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3$, for each $y \in Y$.

Definition 3.6 ([47])**.** Let A and B be the two SVNS'^{s}, then the union and intersection of A and B is defined by SVNS $C$and D that is, $C = (A \cup B)$ and $D = (A \cap B)$ respectively, whose truth $T_C(y)$, indeterminacy $I_C(y)$ and falsity $F_C(y)$ membership functions are given by

$$\begin{aligned} T_C(y) &= \max(T_A(y),\ T_B(y)),\\ I_C(y) &= \max(I_A(y),\ I_B(y)),\\ F_C(y) &= \min(F_A(y), F_B(y)) \text{ for each } y \in Y. \end{aligned}$$

4. Model for multilevel decision making

An n-level, the multilevel programming problem is considered, in which the objective is to be minimized at each level. An n-level multilevel problem, in addition to the set of constraints, may be represented as follows:

$$\left.\begin{array}{c} \min\limits_{x_1} Z_1 = z_1(x_1, x_2, \ldots, x_n)\\ \min\limits_{x_2} Z_2 = z_2(x_1, x_2, \ldots, x_n)\\ \vdots\\ \min\limits_{x_n} Z_n = z_n(x_1, x_2, \ldots, x_n)\\ \text{subject to the constraints :}\\ \text{g}(x_1, x_2, \ldots, x_n) \quad (\leq / \geq / =)b\\ x_i \geq 0, \quad i = 1, 2, \ldots n \end{array}\right\} \tag{4.0}$$

In the formulation given in equation (4.0), the decision variables are partitioned into different levels. This means that the first level controls the first decision variable, the second level controls the second decision variable, and so on. The hierarchy is from the first level to the last level. The first-level decision-maker is the first to execute his policies and develop the solution to set the goal. He asks others at the lower levels for their optima, which are obtained separately. The first-level decision-maker then amends these lower-level decisions to boost the organization's overall performance. This method is repeated until a satisfactory solution at all levels is reached.

Multilevel decision-making for the supplier selection problem

The general formulation for the multilevel supplier selection problem is provided in this section. The following assumptions are being considered:

- Only one supplier is to be assigned to purchase a particular item.
- For any item, shortages from suppliers are not permissible.
- No discounts of any type are considered.
- Demand and supply of an item are considered to be fuzzy.
- All the objectives, viz., minimizing the whole cost, the aggregate number of rejected items, and the total number of late deliveries, are fuzzy.

4.1. Denotations

Z_1	Cost incurred for ordering the total demand
Z_2	Total number of the items that are rejected
Z_3	Number of the items that are delivered late
w_i	Quantity to be purchased from the supplier i
c_i	Cost incurred for ordering the aggregate demand
q_i	Proportion of the rejected items delivered by the supplier i.
l_i	Proportion of the late deliveries by the supplier i.
D	Total demand for the product over a definite planning period
P	Minimum tolerable rating of a supplier
F	Least resilience value in a supplier's supply quota.
B_i	Budget availability for the supplier i.
U_i	Capacity of the supplier i.
r_i	Rating of the supplier i.
s_i	Quota resilience for the supplier i.

The formulation of the problem involving three objectives with a set of system and policy constraints for a supplier selection problem can be as follows:

I level

$$\text{Min} \;\; Z_1 = \sum_{i=1}^{n} c_i w_i, \tag{4.1}$$

where some of w_i satisfies.

II level

$$\text{Min} \;\; Z_2 = \sum_{i=1}^{n} q_i w_i, \tag{4.2}$$

where some of w_i satisfies.

III level

$$\text{Min} \;\; Z_3 = \sum_{i=1}^{n} l_i w_i \tag{4.3}$$

$$\text{subject to} \;\; \sum_{i=1}^{n} w_i \geq D \tag{4.4}$$

$$w_i \leq U_i, \qquad i = 1, 2, \ldots, n \tag{4.5}$$

$$\sum_{i=1}^{n} s_i w_i \geq F \tag{4.6}$$

$$\sum_{i=1}^{n} r_i w_i \geq P \tag{4.7}$$

$$c_i w_i \leq B_i, \qquad i = 1, 2, \ldots, n \tag{4.8}$$

$$w_i \geq 0, \text{ and integer}, \qquad i = 1, 2, \ldots, n. \tag{4.9}$$

The objective function (4.1) is to minimize the entire cost associated with the problem. The objective function (4.2) seeks to minimize the total number of goods rejected after delivery. The objective function (4.3) aims to minimize the supplier's overall number of late deliveries. The constraint (4.4) applies to the demand. The constraint (4.5) is for the providers' maximum capacity. Based on the various quotas ordered from other vendors, constraint (4.6) has the least adaptability. Supplier ratings constitute the constraint (4.7). The limits represented by constraint (4.8) ensure that the purchase price does not exceed the budget allotted to particular providers. The constraint (4.9) assures that the decision variable is non-negative and has an integer value.

5. Multilevel decision-making for the supplier selection problem with fuzzy parameters

As discussed in Section 1, the exact value of the parameters is only sometimes possible to attain. Vagueness in the parameters is due to non-exact estimates of the parameters provided by the decision maker. In such circumstances, this uncertainty must be considered while solving the problem. This vagueness can be dealt with fuzzy techniques. In this multilevel decision-making model for the supplier selection problem (MLSSP), ambiguous problem uncertainties are converted into deterministic types using ranking function techniques.

The multilevel decision-making problem for the supplier selection with fuzzy parameters can be given as follows:

I level

$$\text{Min} \;\; Z_1 = \sum_{i=1}^{n} \tilde{c}_i w_i, \tag{5.1}$$

where some of w_i satisfies.

II level

$$\text{Min} \;\; Z_2 = \sum_{i=1}^{n} \tilde{q}_i w_i, \tag{5.2}$$

where some of w_i satisfies.

III level

$$\text{Min } Z_3 = \sum_{i=1}^{n} \tilde{l}_i w_i \tag{5.3}$$

$$\text{subject to } \sum_{i=1}^{n} w_i \geq \tilde{D} \tag{5.4}$$

$$w_i \leq \tilde{U}_i, \qquad i = 1, 2, \ldots, n \tag{5.5}$$

$$\sum_{i=1}^{n} s_i w_i \geq F \tag{5.6}$$

$$\sum_{i=1}^{n} r_i w_i \geq P \tag{5.7}$$

$$c_i w_i \leq B_i, \qquad i = 1, 2, \ldots, n \tag{5.8}$$

$$w_i \geq 0, \;\; \text{and integer}, \qquad i = 1, 2, \ldots, n. \tag{5.9}$$

where $\tilde{c}_l$,$\tilde{q}_l$, $\tilde{l}_l$, $\tilde{U}_l$ are the fuzzy parameters of the total cost, the total number of rejected items, the total number of late deliveries and total capacity. In addition, supply and demand are also assumed to be fuzzy. These can be defuzzfied by using equation (3.2). The problem can then be represented as follows:

I level

$$\text{Min} \;\; Z_1 = \sum_{i=1}^{n} d(\tilde{c}_i) w_i, \tag{5.10}$$

where some of w_i satisfies.

II level

$$\text{Min}\ \ Z_2 = \sum_{i=1}^{n} d(\tilde{q}_i) w_i, \tag{5.11}$$

where some of w_i satisfies.

III level

$$\text{Min}\ Z_3 = \sum_{i=1}^{n} d\left(\tilde{l}_i\right) w_i \tag{5.12}$$

$$\text{subject to}\ \sum_{i=1}^{n} w_i \geq d\left(\tilde{D}\right) \tag{5.13}$$

$$w_i \leq d\left(\tilde{U}_i\right), \qquad i = 1, 2, \ldots, n \tag{5.14}$$

$$\sum_{i=1}^{n} s_i w_i \geq F \tag{5.15}$$

$$\sum_{i=1}^{n} r_i w_i \geq P \tag{5.16}$$

$$c_i w_i \leq B_i, \qquad i = 1, 2, \ldots, n \tag{5.17}$$

$$w_i \geq 0,\ \text{ and integer}, \qquad i = 1, 2, \ldots, n \tag{5.18}$$

where $d(\tilde{c}_i)$, $d(\tilde{q}_i)$, and $d\left(\tilde{U}_i\right)$ are the defuzzified values of $\tilde{c}_i$, $\tilde{q}_i$, and $\tilde{U}_i$ respectively.

6. Neutrosophic compromise programming approach (NCPA) for multilevel decision-making for the supplier selection problem with fuzzy parameters

An approach, conceptualized on neutrosophic set theory has been promulgated to solve multilevel decision-making for the supplier selection problem with fuzzy parameters. In the neutrosophic compromise programming approach (NCPA), three membership functions are considered: maximization of the degree of truth, maximization of indeterminacy, and minimization of falsity membership function.

If fuzzy decision is denoted by D, fuzzy goal by G, and fuzzy constraints by C, then the neutrosophic decision set, denoted by D_N, can be defined as:

$$D_N = \left(\bigcap_{k=1}^{K} G_k\right)\left(\bigcap_{l=1}^{L} C_l\right) = (w,\, T_D(w),\, I_D(w),\, F_D(w)), \tag{6.1}$$

$$\text{where } T_D(w) = \max\left\{\begin{matrix} T_{G_1}(w),\, T_{G_2}(w),\, T_{G_3}(w) \\ T_{C_1}(w),\, T_{C_2}(w) \end{matrix}\right\}, \qquad \forall\, w \in W \tag{6.2}$$

$$I_D(w) = \max\left\{\begin{matrix} I_{G_1}(w),\, I_{G_2}(w),\, I_{G_3}(w) \\ I_{C_1}(w),\, I_{C_2}(w) \end{matrix}\right\}, \qquad \forall\, w \in W \tag{6.3}$$

$$F_D(w) = \min\left\{\begin{matrix} F_{G_1}(w),\, F_{G_2}(w),\, F_{G_3}(w) \\ F_{C_1}(w),\, F_{C_2}(w) \end{matrix}\right\}, \qquad \forall\, w \in W \tag{6.4}$$

where $T_D(w)$, $I_D(w)$, and $F_D(w)$ are the truth membership function, indeterminacy membership function and a falsity membership function, respectively, of neutrosophic decision set D_N.

At different levels, we define each objective function's lower and upper bounds. These bounds can be obtained as follows:

First, the objective function at an individual level is solved under the given constraints. After that, K solution sets are obtained. Let these solution sets be denoted by $W^1, W^2, \ldots, W^K$ the obtained solutions are then substituted for each objective function, yielding the lower and upper bounds for each objective function as follows:

$$U_k = \max\left[Z_k\left(W^k\right)\right] \quad \text{and} \quad L_k = \min\left[Z_k\left(W^k\right)\right], \quad k = 1, 2, \ldots, K. \tag{6.5}$$

The lower and upper bounds can now be obtained as follows [26]:

$$\text{For truth membership:} \quad U_k^T = U_k, \; L_k^T = L_k \tag{6.6}$$

$$\text{For indeterminacy membership:} \quad U_k^I = L_k^T + s_k, \; L_k^I = L_k^T \tag{6.7}$$

$$\text{For falsity membership:} \quad U_k^F = U_k^T, \; L_k^F = L_k^T + t_k \tag{6.8}$$

where s_k and $t_k \in (0, 1)$ are predetermined real numbers assigned by decision-makers.

Under a neutrosophic environment, the linear membership functions are defined using lower and upper bounds.

$$T_k(Z_k(w)) = \begin{cases} 1 & \text{if} \quad Z_k(w) < L_k^T \\ \dfrac{U_k^T - Z_k(w)}{U_k^T - L_k^T} & \text{if} \quad L_k^T \le Z_k(w) \le U_k^T \\ 0 & \text{if} \quad Z_k(w) > U_k^T \end{cases} \tag{6.9}$$

$$I_k(Z_k(w)) = \begin{cases} 1 & \text{if} \quad Z_k(w) < L_k^I \\ \dfrac{U_k^I - Z_k(w)}{U_k^I - L_k^I} & \text{if} \quad L_k^I \le Z_k(w) \le U_k^I \\ 0 & \text{if} \quad Z_k(w) > U_k^I \end{cases} \tag{6.10}$$

$$F_k(Z_k(w)) = \begin{cases} 1 & \text{if} \quad Z_k(w) > U_k^F \\ \dfrac{Z_k(w) - L_k^F}{U_k^F - L_k^F} & \text{if} \quad L_k^F \le Z_k(w) \le U_k^F \\ 0 & \text{if} \quad Z_k(w) < L_k^F \end{cases} \tag{6.11}$$

For all objective functions $L_k^{(.)} \neq U_k^{(.)}$. If $L_k^{(.)} = U_k^{(.)}$, then the value of the membership will be equal to 1.

Using the approach in [16], the MLSSP can be expressed as follows:

$$\begin{aligned} &\text{Max} \min_{k=1,2,\ldots,K} T_k(Z_k(w)) \\ &\text{Max} \min_{k=1,2,\ldots,K} I_k(Z_k(w)) \\ &\text{Min} \max_{k=1,2,\ldots,K} F_k(Z_k(w)) \\ &\text{subject to} \sum_{i=1}^{n} w_i >= d(D) \\ &\qquad w_i \le d\left(\tilde{U}_i\right), \qquad\qquad i = 1, 2, \ldots, n \\ &\qquad \sum_{i=1}^{n} s_i w_i \ge F \\ &\qquad \sum_{i=1}^{n} r_i w_i \ge P \end{aligned}$$

$$
\begin{aligned}
& c_i w_i \leq B_i, && i = 1, 2, \ldots, n \\
& w_i \geq 0, \ \text{ and integer}, && i = 1, 2, \ldots, n
\end{aligned}
$$

Using auxiliary parameters, the above problem can be transformed into the following form:

$$
\begin{aligned}
& \text{Max } \alpha && && (6.12) \\
& \text{Max } \beta && && (6.13) \\
& \text{Min } \gamma && && (6.14) \\
& \text{subject to } T_K(Z_k(w)) \geq \alpha && && (6.15) \\
& \qquad I_k(Z_k(w)) \geq \beta && && (6.16) \\
& \qquad F_k(Z_k(w)) \leq \gamma && && (6.17) \\
& \qquad \sum_{i=1}^{n} w_i \geq d\left(\tilde{D}\right) && && (6.18) \\
& \qquad w_i \leq d\left(\tilde{U_i}\right), && i = 1, 2, \ldots, n && (6.19) \\
& \qquad \sum_{i=1}^{n} s_i w_i \geq F && && (6.20) \\
& \qquad \sum_{i=1}^{n} r_i w_i \geq P && && (6.21) \\
& \qquad c_i w_i \leq B_i, && i = 1, 2, \ldots, n && (6.22) \\
& \qquad w_i \geq 0, \ \text{ and integer}, && i = 1, 2, \ldots, n && (6.23)
\end{aligned}
$$

The same problem can be further expressed using a linear membership function as follows:

$$
\begin{aligned}
& \text{Max } \alpha + \beta - \gamma && && (6.24) \\
& \text{subject to } \sum_{i=1}^{n} w_i \geq d\left(\tilde{D}\right) && && (6.25) \\
& \qquad w_i \leq d\left(\tilde{U_i}\right), && i = 1, 2, \ldots, n && (6.26) \\
& \qquad \sum_{i=1}^{n} s_i w_i \geq F && && (6.27) \\
& \qquad \sum_{i=1}^{n} r_i w_i \geq P && && (6.28) \\
& \qquad c_i w_i \leq B_i, && i = 1, 2, \ldots, n && (6.29) \\
& \qquad w_i \geq 0, \ \text{ and integer}, && i = 1, 2, \ldots, n && (6.30) \\
& \qquad Z_k(w) + \left(U_k^T - L_k^T\right)\alpha \leq U_k^T && && (6.31) \\
& \qquad Z_k(w) + \left(U_k^I - L_k^I\right)\beta \leq U_k^I && && (6.32) \\
& \qquad Z_k(w) - \left(U_k^F - L_k^F\right)\gamma \leq L_k^F && && (6.33) \\
& \qquad \alpha \geq \beta, \ \alpha \geq \gamma, \ \alpha + \beta + \gamma \leq 3 && && (6.34) \\
& \qquad \alpha, \ \beta, \ \gamma \in (0, 1) && && (6.35)
\end{aligned}
$$

Ultimately, the above model gives the compromise solution for MLSSP.

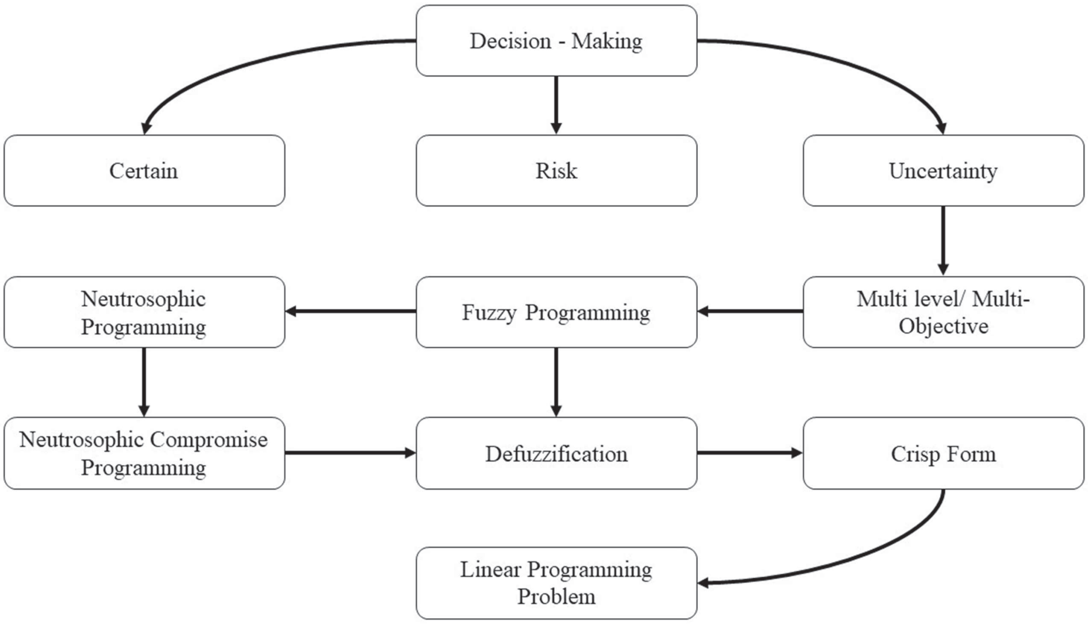

Figure 1. Flow chart of proposed model.

7. Algorithm for neutrosophic compromise programming approach

A step-wise summary of the proposed method can be presented as follows:

Step 1. Formulate the multilevel supplier selection problem with fuzzy parameters, as discussed in Section 5.
Step 2. Using the defuzzification method, given in equation (3.2), transform the problem into a crisp form.
Step 3. Solve objective functions at each level individually with the set of constraints and contrive the pay-off matrix, as shown in Table 2.
Step 4. For each level, determine each objective function's upper and lower bounds.
Step 5. For truth, indeterminacy, and falsity membership functions, define the upper and lower bounds as in equations (6.6)–(6.8).
Step 6. Under a neutrosophic environment, define the liner membership function as in equations (6.9)–(6.11).
Step 7. Formulate the neutrosophic problem defined by equations (6.12)–(6.23) and convert it to the neutrosophic compromise programming problem specified by equations (6.24)–(6.35).
Step 8. Solving the converted multilevel decision-making for the supplier selection problem parameters using an optimizing software package.

8. Computational study

To demonstrate the procedure, an illustrative example is considered [10]. An automobile company which orders auto parts from different suppliers is considered. The resources are limited, and few parameters are fuzzy in nature. As a result, specific professionals are assigned to design the selection criteria and select providers based on the specified quality and its restrictions. The proposed NCPA approach is utilised to solve MLSSP with three unique objectives: minimize overall ordering cost, rejection rate, and item delivery time within a given set of resources at the level I, level II and level III respectively. The data as shown in Table 1 is considered. The solution results are obtained using LINGO 16.

Table 1. Input parameters.

Parameters	Suppliers (i)					
	1	2	3	4	5	6
C_i(\$)	$(2, 3,\ 4)$	$(1, 2, 3)$	$(3, 4, 5)$	$(1, 1, 1)$	$(4, 5, 6)$	$(4, 6, 8)$
q_i	$(0.04, 0.05, 0.06)$	$(0.01, 0.02, 0.07)$	$(0, 0, 0)$	$(0.03, 0.04, 0.05)$	$(0.01, 0.02, \mathrm{y}0.03)$	$(0, 0.02, 0.04)$
$U_i('00)$	$(40, 60, 100)$	$(100, 150, 240)$	$(30, 40, 70)$	$(20, 30, 40)$	$(25, 35, 65)$	$(30, 40, 50)$
l_i	$(0.01, 0.02, 0.03)$	$(0, 0.01, 0.02)$	$(0.07, 0.08, 0.09)$	$(0.01, 0.02, 0.03)$	$(0, 0.01, 0.02)$	$(0.01, 0.02, 0.03)$
r_i	0.85	0.80	0.97	0.81	0.82	0.90
s_i	0.01	0.02	0.06	0.04	0.02	0.03
B_i	14 000	27 000	12 000	1900	18 000	5000
	7	8	9	10	11	12
C_i (\$)	$(5, 7, 9)$	$(3, 6, 9)$	$(1, 2, 3)$	$(2, 6, 6)$	$(1, 1, 1)$	$(2, 7, 8)$
q_i	$(0.01, 0.02, 0.03)$	$(0, 0.01, 0.02)$	$(0.04, 0.06, 0.08)$	$(0, 0, 0)$	$(0.02, 0.03, 0.04)$	$(0.01, 0.02, 0.07)$
$U_i('00)$	$(15, 25, 35)$	$(15, 20, 25)$	$(40, 60, 80)$	$(16, 25, 34)$	$(20, 30, 40)$	$(15, 20, 25)$
l_i	$(0, 0.02, 0.04)$	$(0.03, 0.04, 0.05)$	$(0.02, 0.03, 0.04)$	$(0.01, 0.02, 0.03)$	$(0, 0.01, 0.02)$	$(0.01, 0.02, 0.03)$
r_i	0.92	0.87	0.86	0.97	0.80	0.84
s_i	0.05	0.02	0.02	0.04	0.03	0.06
B_i	2000	9000	10 000	12 000	3000	9000

D	F	P
$(22\,000, 30\,000, 54\,000)$	1020	31 280

The formulation of the problem using the data in Table 1 is given as follows:

I level

$$\begin{aligned}\text{Min}\ \ Z_1 = {} & (2,3,4)w_1 + (1,2,3)w_2 + (3,4,5)w_3 + (1,1,1)w_4 + (4,5,6)w_5 \\ & + (4,6,8)w_6 + (5,7,9)w_7 + (3,6,9)w_8 + (1,2,3)w_9 \\ & + (2,6,6)w_{10} + (1,1,1)w_{11} + (2,7,8)w_{12}\end{aligned}$$

where $w_1,\ w_2,\ w_3,\ w_4$ satisfy

II level

$$\begin{aligned}\text{Min}\ \ Z_2 = {} & (0.04, 0.05, 0.06)w_1 + (0.01, 0.02, 0.07)w_2 + (0,0,0)w_3 + (0.03, 0.04, 0.05)w_4 + (0.01, 0.02, 0.03)w_5 \\ & + (0, 0.02, 0.04)w_6 + (0.01, 0.02, 0.03)w_7 + (0, 0.01, 0.02)w_8 + (0.04, 0.06, 0.08)w_9 \\ & + (0,0,0)w_{10} + (0.02, 0.03, 0.04)w_{11} + (0.01, 0.02, 0.07)w_{12}\end{aligned}$$

where $w_5,\ w_6,\ w_7,\ w_8$ satisfy

III level

$$\begin{aligned}\text{Min}\ Z_3 = {} & (0.01, 0.02, 0.03)w_1 + (0, 0.01, 0.02)w_2 + (0.07, 0.08, 0.09)w_3 + (0.01, 0.02, 0.03)w_4 + (0, 0.01, 0.02)w_5 \\ & + (0.01, 0.02, 0.03)w_6 + (0, 0.02, 0.04)w_7 + (0.03, 0.04, 0.05)w_8 + (0.02, 0.03, 0.04)w_9 \\ & + (0.01, 0.02, 0.03)w_{10} + (0, 0.01, 0.02)w_{11} + (0.01, 0.02, 0.03)w_{12}\end{aligned}$$

$$\begin{aligned}\text{subject to}\ & w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 \\ & + w_8 + w_9 + w_{10} + w_{11} + w_{12} \geq (22\,000, 30\,000, 54\,000) \\ & w_1 \leq (40, 60, 100)\ 100 \\ & w_2 \leq (100, 150, 240)\ 100\end{aligned}$$

$$
\begin{aligned}
&w_3 \leq (30, 40, 70)\ 100\\
&w_4 \leq (20, 30, 40)\ 100\\
&w_5 \leq (25, 35, 65)\ 100\\
&w_6 \leq (30, 40, 50)\ 100\\
&w_7 \leq (15, 25, 35)\ 100\\
&w_8 \leq (15, 20, 25)\ 100\\
&w_9 \leq (40, 60, 80)\ 100\\
&w_{10} \leq (16, 25, 34)\ 100\\
&w_{11} \leq (20, 30, 40)\ 100\\
&w_{12} \leq (15, 20, 25)\ 100\\
&0.01w_1 + 0.02w_2 + 0.06w_3 + 0.04w_4 + 0.02w_5 + 0.03w_6\\
&\quad + 0.05w_7 + 0.02w_8 + 0.02w_9 + 0.04w_{10} + 0.03w_{11} + 0.06w_{12} \geq 1020\\
&0.85w_1 + 0.80w_2 + 0.97w_3 + 0.81w_4 + 0.82w_5 + 0.90w_6\\
&\quad + 0.92w_7 + 0.87w_8 + 0.86w_9 + 0.97w_{10} + 0.80w_{11} + 0.84w_{12} \geq 31\,280\\
&(2, 3, 4)w_1 \leq 14\,000\\
&(1, 2, 3)w_2 \leq 27\,000\\
&(3, 4, 5)w_3 \leq 12\,000\\
&(1, 1, 1)w_4 \leq 1900\\
&(4, 5, 6)w_5 \leq 18\,000\\
&(4, 6, 8)w_6 \leq 5000\\
&(5, 7, 9)w_7 \leq 2000\\
&(3, 6, 9)w_8 \leq 9000\\
&(1, 2, 3)w_9 \leq 10\,000\\
&(2, 6, 6)w_{10} \leq 12\,000\\
&(1, 1, 1)w_{11} \leq 3000\\
&(2, 7, 8)w_{12} \leq 9000\\
&w_i \geq 0, \text{ and integer}, \qquad\qquad i = 1, 2, \ldots, 12
\end{aligned}
$$

The crisp model using equation (3.2) will be as follows:

I level

$$
\begin{aligned}
\text{Min } Z_1 = {} & 3w_1 + 2w_2 + 4w_3 + w_4 + 5w_5 + 6w_6 + 7w_7\\
& + 6w_8 + 2w_9 + 5w_{10} + w_{11} + 6w_{12}
\end{aligned}
$$

where $w_1,\ w_2,\ w_3,\ w_4$ satisfy.

II level

$$
\begin{aligned}
\text{Min } Z_2 = {} & 0.05w_1 + 0.03w_2 + 0w_3 + 0.04w_4 + 0.02w_5\\
& + 0.02w_6 + 0.02w_7 + 0.01w_8 + 0.06w_9\\
& + 0w_{10} + 0.03w_{11} + 0.03w_{12}
\end{aligned}
$$

where $w_5,\ w_6,\ w_7,\ w_8$ satisfy.

III level

$$\begin{aligned}
\text{Min } Z_3 = {} & 0.02w_1 + 0.01w_2 + 0.08w_3 + 0.02w_4 + 0.01w_5 \\
& + 0.02w_6 + 0.02w_7 + 0.04w_8 + 0.03w_9 \\
& + 0.02w_{10} + 0.01w_{11} + 0.02w_{12} \\
\text{subject to } & w_1 + w_2 + w_3 + w_4 + w_5 \\
& \quad + w_6 + w_7 + w_8 + w_9 \\
& \quad + w_{10} + w_{11} + w_{12} \geq 34\,000 \\
& 0.01w_1 + 0.02w_2 + 0.06w_3 + 0.04w_4 + 0.02w_5 \\
& \quad + 0.03w_6 + 0.05w_7 + 0.02w_8 + 0.02w_9 \\
& \quad + 0.04w_{10} + 0.03w_{11} + 0.06w_{12} \geq 1020 \\
& 0.85w_1 + 0.80w_2 + 0.97w_3 + 0.81w_4 + 0.82w_5 \\
& \quad + 0.90w_6 + 0.92w_7 + 0.87w_8 + 0.86w_9 \\
& \quad + 0.97w_{10} + 0.80w_{11} + 0.84w_{12} \geq 31\,280 \\
& w_1 \leq 6500 \\
& w_2 \leq 16\,000 \\
& w_3 \leq 4500 \\
& w_4 \leq 3000 \\
& w_5 \leq 4000 \\
& w_6 \leq 4000 \\
& w_7 \leq 2500 \\
& w_8 \leq 2000 \\
& w_9 \leq 6000 \\
& w_{10} \leq 2500 \\
& w_{11} \leq 3000 \\
& w_{12} \leq 2000 \\
& 3w_1 \leq 14\,000 \\
& 2w_2 \leq 27\,000 \\
& 4w_3 \leq 12\,000 \\
& w_4 \leq 1900 \\
& 5w_5 \leq 18\,000 \\
& 6w_6 \leq 5000 \\
& 7w_7 \leq 2000 \\
& 6w_8 \leq 9000 \\
& 2w_9 \leq 10\,000 \\
& 5w_{10} \leq 12\,000 \\
& w_{11} \leq 3000 \\
& 6w_{12} \leq 9000 \\
& w_i \geq 0, \text{and integer}, \qquad\qquad i = 1, 2, \ldots, 12
\end{aligned}$$

The payoff matrix is created by considering each objective separately, as shown in Table 2.

TABLE 2. Individual best and worst solution.

Decision variables	Individual objective values		
	Z_1	Z_2	Z_3
(3415.6, 13500, 3000, 1900, 2227.9, 833.3, 285.7, 0, 5000, 2400, 3000, 1500)	**103286.4**	1153.719	783.9719
(3510.5, 13500, 3000, 1900, 3600, 833.3, 285.7, 1500, 2080.4, 2400, 3000, 1500)	113592.5	**1025.735**	772.0051
(2425.6, 13500, 2707.6, 1900, 3600, 833.3, 285.7, 0, 5000, 2400, 3000, 1500)	106007.3	1131.659	**754.5037**

Notes. The bold values represents the "Individual best and worst solutions."

The upper and lower bound for each objective function are as follows:

$$U_1 = 113592.5; \quad L_1 = 103286.4$$
$$U_2 = 1153.719; \quad L_2 = 1025.735$$
$$U_3 = 783.9719; \quad L_3 = 754.5037$$

Using equations (6.6)–(6.8), the upper and lower bound for the truth, indeterminacy, and falsity membership functions are as follows:

$$U_1^T = U_1 = 113592.5; \qquad L_1^T = L_1 = 103286.4$$
$$U_1^I = L_1^T + s_1 = 103286.4 + s_1; \qquad L_1^I = L_1^T = 103286.4$$
$$U_1^F = U_1^T = 113592.5; \qquad L_1^F = L_1^T + t_1 = 103286.4 + t_1$$

$$T_1(Z_1(w)) = \begin{cases} 1 & \text{if} \quad Z_1(w) < 103286.4 \\ 11.02 - \frac{Z_1(w)}{10306.1} & \text{if} \quad 103286.4 \le Z_1(w) \le 113592.5 \\ 0 & \text{if} \quad Z_1(w) > 113592.5 \end{cases}$$

$$I_1(Z_1(w)) = \begin{cases} 1 & \text{if} \quad Z_1(w) < 103286.4 \\ 1 + \frac{103286.4 - Z_1(w)}{s_1} & \text{if} \quad 103286.4 \le Z_1(w) \le 103286.4 + s_1 \\ 0 & \text{if} \quad Z_1(w) > 103286.4 + s_1 \end{cases}$$

$$F_1(Z_1(w)) = \begin{cases} 1 & \text{if} \quad Z_1(w) > 113592.5 \\ \frac{Z_1(w) - 103286.4 - t_1}{10306.1 - t_1} & \text{if} \quad 103286.4 + t_1 \le Z_1(w) \le 113592.5 \\ 0 & \text{if} \quad Z_1(w) < 103286.4 + t_1 \end{cases}$$

Similarly for the second objective;

$$U_2^T = U_2 = 1153.719; \qquad L_2^T = L_2 = 1025.735$$
$$U_2^I = L_2^T + s_2 = 1025.735 + s_2; \qquad L_2^I = L_2^T = 1025.735$$
$$U_2^F = U_2^T = 1153.719; \qquad L_2^F = L_2^T + t_2 = 1025.735 + t_2$$

$$T_2(Z_2(w)) = \begin{cases} 1 & \text{if} \quad Z_2(w) < 1025.735 \\ 9.01 - \frac{Z_2(w)}{127.984} & \text{if} \quad 1025.735 \le Z_2(w) \le 1153.719 \\ 0 & \text{if} \quad Z_2(w) > 1153.719 \end{cases}$$

$$I_2(Z_2(w)) = \begin{cases} 1 & \text{if} \quad Z_2(w) < 1025.735 \\ 1 + \frac{1025.735 - Z_2(w)}{s_2} & \text{if} \quad 1025.735 \le Z_2(w) \le 1025.735 + s_2 \\ 0 & \text{if} \quad Z_2(w) > 1025.735 + s_2 \end{cases}$$

$$F_3(Z_2(w)) = \begin{cases} 1 & \text{if} \quad Z_2(w) > 1153.719 \\ \frac{Z_2(w) - 1025.735 - t_2}{127.984 - t_2} & \text{if} \quad 1025.735 + t_2 \le Z_2(w) \le 1153.719 \\ 0 & \text{if} \quad Z_2(w) < 1025.735 + t_2 \end{cases}$$

For the third objective:

$$U_3^T = U_3 = 783.9719; \qquad L_3^T = L_3 = 754.5037$$
$$U_3^I = L_3^T + s_3 = 754.5037 + s_3; \qquad L_3^I = L_3^T = 754.5037$$
$$U_3^F = U_3^T = 783.9719; \qquad L_3^F = L_3^T + t_3 = 754.5037 + t_3$$

$$T_3(Z_3(w)) = \begin{cases} 1 & \text{if} \quad Z_3(w) < 754.5037 \\ 26.61 - \frac{Z_3(w)}{29.47} & \text{if} \quad 754.5037 \le Z_3(w) \le 783.9719 \\ 0 & \text{if} \quad Z_3(w) > 783.9719 \end{cases}$$

$$I_3(Z_3(w)) = \begin{cases} 1 & \text{if} \quad Z_3(w) < 754.5037 \\ 1 + \frac{754.5037 - Z_3(w)}{s_3} & \text{if} \quad 754.5037 \le Z_3(w) \le 754.5037 + s_3 \\ 0 & \text{if} \quad Z_3(w) > 754.5037 + s_3 \end{cases}$$

$$F_3(Z_3(w)) = \begin{cases} 1 & \text{if} \quad Z_3(w) > 783.9719 \\ \frac{Z_3(w) - 754.5037 - t_3}{29.4682 - t_3} & \text{if} \quad 754.5037 + t_3 \le Z_3(w) \le 783.9719 \\ 0 & \text{if} \quad Z_3(w) < 754.5037 + t_3 \end{cases}$$

Now, the simplified neutrosophic model for MLPP is given as follows:

$$\begin{aligned}
&\text{Max } \alpha + \beta - \gamma \\
&\text{subject to } \; w_1 + w_2 + w_3 + w_4 + w_5 \\
&\qquad + w_6 + w_7 + w_8 + w_9 + w_{10} + w_{11} + w_{12} \ge 34\,000 \\
&\qquad 0.01w_1 + 0.02w_2 + 0.06w_3 + 0.04w_4 + 0.02w_5 \\
&\qquad\quad + 0.03w_6 + 0.05w_7 + 0.02w_8 + 0.02w_9 \\
&\qquad\quad + 0.04w_{10} + 0.03w_{11} + 0.06w_{12} \ge 1020 \\
&\qquad 0.85w_1 + 0.80w_2 + 0.97w_3 + 0.81w_4 + 0.82w_5 \\
&\qquad\quad + 0.90w_6 + 0.92w_7 + 0.87w_8 + 0.86w_9 \\
&\qquad\quad + 0.97w_{10} + 0.80w_{11} + 0.84w_{12} \ge 31\,280 \\
&\qquad w_1 \le 6500 \\
&\qquad w_2 \le 16\,000 \\
&\qquad w_3 \le 4500 \\
&\qquad w_4 \le 3000 \\
&\qquad w_5 \le 4000 \\
&\qquad w_6 \le 4000 \\
&\qquad w_7 \le 2500 \\
&\qquad w_8 \le 2000 \\
&\qquad w_9 \le 6000 \\
&\qquad w_{10} \le 2500 \\
&\qquad w_{11} \le 3000 \\
&\qquad w_{12} \le 2000 \\
&\qquad 3w_1 \le 14\,000 \\
&\qquad 2w_2 \le 27\,000 \\
&\qquad 4w_3 \le 12\,000 \\
&\qquad w_4 \le 1900 \\
&\qquad 5w_5 \le 18\,000
\end{aligned}$$

$$
\begin{aligned}
&6w_6 \leq 5000\\
&7w_7 \leq 2000\\
&6w_8 \leq 9000\\
&2w_9 \leq 10\,000\\
&5w_{10} \leq 12\,000\\
&w_{11} \leq 3000\\
&6w_{12} \leq 9000\\
&3w_1 + 2w_2 + 4w_3 + w_4 + 5w_5\\
&\quad + 6w_6 + 7w_7 + 6w_8 + 2w_9\\
&\quad + 5w_{10} + w_{11} + 6w_{12} + (113592.5 - 103286.4)\alpha \leq 113592.5\\
&0.05w_1 + 0.03w_2 + 0w_3 + 0.04w_4 + 0.02w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.01w_8 + 0.06w_9\\
&\quad + 0w_{10} + 0.03w_{11} + 0.03w_{12} + (1153.719 - 1025.735)\alpha \leq 1153.719\\
&0.02w_1 + 0.02w_2 + 0.08w_3 + 0.02w_4 + 0.01w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.04w_8 + 0.03w_9 + 0.02w_{10}\\
&\quad + 0.01w_{11} + 0.01w_{12} + (783.9719 - 754.5037)\alpha \leq 783.9719\\
&3w_1 + 2w_2 + 4w_3 + w_4 + 5w_5\\
&\quad + 6w_6 + 7w_7 + 6w_8 + 2w_9\\
&\quad + 5w_{10} + w_{11} + 6w_{12} + (103286.4 + s1 - 103286.4)\beta \leq 103286.4 + s1\\
&0.05w_1 + 0.03w_2 + 0w_3 + 0.04w_4 + 0.02w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.01w_8 + 0.06w_9\\
&\quad + 0w_{10} + 0.03w_{11} + 0.03w_{12} + (1025.735 + s2 - 1025.735)\beta \leq 1025.735 + s2\\
&0.02w_1 + 0.02w_2 + 0.08w_3 + 0.02w_4 + 0.01w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.04w_8 + 0.03w_9\\
&\quad + 0.02w_{10} + 0.01w_{11} + 0.01w_{12} + (754.5037 + s3 - 754.5037)\beta \leq 754.5037 + s3\\
&3w_1 + 2w_2 + 4w_3 + w_4 + 5w_5\\
&\quad + 6w_6 + 7w_7 + 6w_8 + 2w_9\\
&\quad + 5w_{10} + w_{11} + 6w_{12} - (113592.5 - 103286.4 - t1)\gamma \leq 103286.4 + t1\\
&0.05w_1 + 0.03w_2 + 0w_3 + 0.04w_4 + 0.02w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.01w_8 + 0.06w_9\\
&\quad + 0w_{10} + 0.03w_{11} + 0.03w_{12} - (1153.719 - 1025.735 - t2)\gamma \leq 1025.735 + t2\\
&0.02w_1 + 0.02w_2 + 0.08w_3 + 0.02w_4 + 0.01w_5\\
&\quad + 0.02w_6 + 0.02w_7 + 0.04w_8 + 0.03w_9\\
&\quad + 0.02w_{10} + 0.01w_{11} + 0.01w_{12} - (783.9719 - 754.5037 - t3)\gamma \leq 754.5037 + t3\\
&w_i \geq 0, \text{ and integer}, \qquad i = 1, 2, \ldots, 12\\
&\alpha \geq \beta,\ \alpha \geq \gamma,\ \alpha + \beta + \gamma \leq 3\\
&\alpha,\ \beta,\ \gamma \in (0,1)
\end{aligned}
$$

The above problem is solved using LINGO 16.0, on an Intel(R) core i5-4210U CPU @1.7 GHz and 8 GB of RAM. The result is shown in Table 3.

TABLE 3. Compromise solution results.

Decision variables												Objective values		
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	Min Z_1	Min Z_2	Min Z_3
2940	13 500	3000	1900	3600	833	286	460	3837	2400	3000	1356	108 291	1088	968

9. Managerial benefits

Using Neutrosophic Programming (NP) in multi-level decision making for supplier selection problems with fuzzy parameters can provide a number of managerial benefits, such as:

- Improved handling of uncertainty: Neutrosophic Programming enables the explicit representation of uncertainty and indeterminacy in decision-making. When dealing with fuzzy parameters like cost, quality, and delivery times, this can help managers make more informed decisions.
- Enhanced risk management: by incorporating various levels of uncertainty and indeterminacy, Neutrosphic Programming can assist managers in a more effective risk management approach to supplier selection. Managers are able to identify potential risks associated with each supplier and make more informed decisions as a result.
- Better problem-solving: Neutrosophic Programming provides a structured approach to problem-solving that can assist managers in analysing and resolving complex supplier selection problems involving fuzzy parameters.
- Increased efficiency: managers can spend less time and money choosing suppliers by using Neutrosophic Programming to help them make more informed and effective decisions.
- Improved supplier performance: the performance of suppliers can be enhanced over time by using a more structured and informed approach to selection, which will result in better quality, faster delivery, and lower costs.
- Sensitivity analysis: managers can determine which factors have the biggest influence on the final solution and make wise decisions by using the sensitivity analysis of Neutrosophic Programming in multi-level decision making for supplier selection problems with fuzzy parameters. By taking into consideration the potential trade-offs between various variables and objectives, this can assist managers in making decisions that are more effective and efficient. Sensitivity analysis in Neutrosophic Programming involves analyzing the objective function values as decision variables change. This can help determine which variables are most important and which can be changed without affecting the solution.

10. Conclusions and future scopes of the study

The supplier selection problem is vital in overall project management in the present scenario. Herein, we have considered uncertainties that can be involved in the parameters of the mathematical model for SSP. These uncertainties are fuzzy in nature and match many real-life situations. A hierarchical model was developed for SSP and was solved by NCPA to obtain the best compromise solution. The contributions of this paper are summarized below:

- The parameters in the concerned problem are considered fuzzy, which may be the case in many real-life situations.
- To the best of the author's knowledge, neutrality/indeterminacy is an area that has hitherto not been well explored for SSP.
- The NCPA can effectively and efficiently be applied to the developed multilevel SSP model.
- The present work can efficiently apply to advertizing, portfolio selection, and other problems.

Finally, in supplier selection issues with fuzzy parameters, neurosophic programming offers a powerful tool for multi-level decision making. The explicit representation and manipulation of uncertainty and indeterminacy made possible by Neutrosophic Programming can aid managers in making more informed, effective, and efficient supplier selection decisions. This could lead to better risk management, increased transparency, and increased decision-making efficiency from suppliers. Managers can achieve their objectives and improve outcomes for their organisations by using NP in supplier selection. There are still some gaps in the research that require filling in at some point. The following are some of the research gaps that currently exist in this area:

– Lack of practical applications: although NP has been investigated on a theoretical level, there is a severe shortage of practical applications and real-world case studies that demonstrate the usefulness of NP in multi-level decision making for fuzzy-parameter supplier selection problems.
– Limited research in specific industries: NP has been applied to a variety of decision-making problems, but research on its use in specific industries, such as the manufacturing and retail industries, is limited.
– Integration with other decision-making techniques: integration of NP with other decision-making techniques is possible, but research on the combination of NP with other optimization methods and decision-making techniques is limited.
– Comparison with other methods: NP has been compared to other decision-making techniques, but more extensive comparisons with other optimization and decision-making techniques are required to demonstrate its effectiveness and advantages.

Addressing these gaps can facilitate in expanding the practical applications of NP and enhancing its efficiency in real-world decision-making scenarios.

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