STRATEGIC INTRODUCTION OF PRIVATE BRAND IN THE ONLINE RETAIL PLATFORM

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Abstract. Private brands are frequently introduced by online platforms. Motivated by this phenomenon, this study aims to examine the strategic introduction of private brand involving the selling format choice in the online retail platform circumstance. The retail platform determines the private label introduction as well as the selling formats, i.e., reselling or agency selling. We develop a game-theoretic model to examine the private brand introduction problem for the online retail platform. The results indicate that when the potential market for private brand is moderate, the introduction of a private brand will interact with the choice of selling format. Clearly, it makes sense for the platform to exclude or include the private brand strategy based on the size of the potential market when it is extremely small versus relatively large. However, in the case of a medium-sized market for private brands, the platform’s private brand introduction and selling format will be determined by both the cost of order fulfillment and the intensity of competition between national brands and private brands. This study contributes to uncovering under what conditions an online retail platform can be beneficial by introducing a private brand.

Mathematics Subject Classification. 91B06, 91B38.

Received February 2, 2023. Accepted May 30, 2023.

1. Introduction

The contemporary market for private brands, also known as store brands, has proliferated markedly in the past two decades [7]. In 2021, private brand products reached a record $199 billion in sales, accounting for 17.7% of total retail sales [35]. Both traditional brick-and-mortar retailers and emerging online platform retailers are keen on developing their private brands. For instance, the e-commerce giant Amazon has launched its private brands since 2009. China’s online retail giant JD.com has developed private brands in a variety of industries, including fashion, digital, and skincare. The introduction of private brands can contribute to enhanced channel power over manufacturers [30], which generates additional revenue for the platform [2].

However, online private brand introductions might vex national brand manufacturers selling on the online platform. Amazon, for instance, owns 158,000 private brand products across 45 brands by 2020 [14]. China’s online retail giants Alibaba and JD.com are following Amazon’s footsteps by offering private brand products [12]. Such manufacturers worry about the competition between the private brands and their national brands.

Keywords. Private brand, retail platform, selling format, reselling format, agency selling format.

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Moreover, these manufacturers are even threatened by the possibility that the national brand may be replaced by the online platform’s private brand. It has been observed that an abundance of manufacturers results in greater diversity and appeal for the platform, as a result, a decreasing number of manufacturers may lower the platform’s appeal to consumers. This conjuncture begs the following question: When should the online platform introduce a private brand?

Moreover, the selling format of the platform may also be involved with the introduction of private brand. This issue, though, has been neglected in extant work. Being different from the traditional brick-and-mortar retailers, online platform can reach their customers either via the reselling format or via the agency selling format. In the reselling format, the online platform purchases products from manufacturers at a wholesale price and resells them to customers at a retail price. In contrast, in the agency selling format, the online platform acts as a marketplace provider for manufacturers, and manufacturers sell their products directly to customers through the platform (Note that the terms “agency selling” and “marketplace” are interchangeably used in this paper). The agency selling avenue is widely utilized in the Internet era. For example, the agency selling format accounted for 45% and 43% of total sales on Amazon and JD.com, respectively [43]. The selling format selection may interact with the decision of the private brand introduction. Most existing researches on private brand management have focused mainly on a reselling context [10, 20, 22]. Whether the presence of an agency selling format would bring any new insights pertaining to the introduction of a platform retailer’s private brands is an intriguing, but unexplored, area.

There are three crucial differences between the reselling and agency selling format. First, the selling option incurs a series of costs (e.g., inventory, transshipment) for the online platform – which we refer to as “order-fulfillment cost”. Alternatively, the agency selling format entails manufacturer’s selling their products directly through the platform and thus assuming the order fulfillment costs, which can reach up to as high as 25% of total sales revenue [21]. We examine the influence of order fulfillment cost on the private brand introduction by including the order fulfillment cost as a fixed cost according to Tian et al. [43]. Second, the decision maker who determines the national brand’s retail price differs between the two models. In the agency model, the manufacturer determines the price of the national brand, while the platform determines the price of the private brand. With the reselling format, the platform determines the price for both brands, which affords the platform the opportunity to mitigate competition between the national brand and private brand. As a result, the agency selling may result in a higher degree of competition between the two brands than the reselling format.

Third, the allocation of profit among channel members under the agency selling format differs significantly from that under the reselling format. Under the reselling option, channel members distribute the profit of the national brand as a wholesale contract. Under the agency option, though, the channel members utilize a revenue-sharing scheme, whereby the manufacturer gives a proportion $\beta$ of the total sales revenue of the national brand for the use of the platform and keeps the remaining $1 - \beta$ of the revenue. The differences in selling across channel members might generate new insights for the introduction of private brand for the platform.

Previous studies have widely investigated factors that might affect the choice of selling format under the context of platform. Such factors, for example, includes manufacturer competition and asymmetric information [16], cross-channel spillovers and retail competition [1], upstream competition and order fulfillment cost [43], multistage retailing and retail competition [6], and information sharing [50]. However, how the platform makes the decision of private brand introduction under the two foregoing selling formats is a critical question.

Motivated by these issues, we concentrate on the private brand introduction problem in the context of platform selling. We consider a distribution channel that consists of a national brand manufacturer and a platform seller. The manufacturer (he) sells his national brand products through the platform, and the platform (she) can utilize the reselling format and the agency model for the national brand. In the reselling format, the manufacturer wholesales national brand products to the platform, then the platform determines the retail price of the national brand and generates a fixed order-fulfillment cost. In the agency selling format, the manufacturer determines the retail price of the national brand and bears the order-fulfillment cost. The manufacturer shares $\beta$ proportion of the total revenue to the platform. Meanwhile, the platform contemplates launching a private brand that closely mirrors the national brand. To be specific, this paper aims to solve the following questions:
When should the platform choose to introduce the private brand incorporating the online retail selling format decision?

(2) Whether and how would the platform’s private brand introduction be influenced by the online retail platform’s selling format?

(3) What are the implications for the manufacturer in terms of the impact brought about by the introduction of the private brand?

To answer the above questions, we develop a game-theoretic model consisting a national brand manufacturer and an online retail platform. The retail platform determines whether a private brand should be introduced, as well as which selling formats, i.e., reselling or agency selling, should be employed. Then the platform and manufacturer make pricing decisions according to the platform’s private brand and selling format choice.

Our results show that the potential market for private brand plays a critical role of the private brand introduction. The private brand and selling format choice of the platform depends on the private brand’s potential market, order-fulfillment cost, and competition intensity between the national and private brand. Intuitively, the platform’s private brand strategy mainly depends on the potential market of the private brand. If the potential market of the private brand, i.e., the number of loyal customers for the platform or the private brand, is extremely small, the platform’s optimal choice is to exclude the private brand strategy whatever the selling format is. If the platform can develop a relatively large potential market for the private brand, the platform can be profitable by introducing the private brand no matter what the selling format is. If the potential market is moderate, the introduction of a private brand will depend on the order-fulfillment cost, i.e., only when it falls below a particular threshold can the platform introduce the private brand profitably.

Moreover, our results also indicate that when the potential market of the private brand is moderate, the private brand introduction will interact with the selling format choice. It is optimal for the platform to implement the agency selling format with private brand if there is little competition between the national brand and private brand, while implementing the reselling format with private brand if there is a high level of competition intensity.

The remainder of this paper proceeds accordingly. In Section 2, we review relevant literature. Section 3 enumerates research hypotheses and presents the model. Section 4 investigates the pricing decisions under different scenarios and their impacts on price and demand. Section 5 examines the platform’s private brand introduction incorporating the selling format choice. Section 6 concludes and presents the managerial implications of this paper. All proofs are provided in the Appendix A.

2. Literature review

We review existing literature in the following two streams that are relevant to our paper: platform selling format selection and private brand introduction. We focus our review on theoretical studies which use game theory approach to examine the platform selling selection as well as private brand issues.

2.1. Platform selling format selection

In recent years, online retail giants (e.g., Amazon and JD.com) have achieved abundant success and have thus drawn extensive attention from scholars. The prominent feature of the online retail platform is that it can either employ the agency selling or traditional reselling format to distribute products to customers. Existing studies have mainly focused on two aspects: whether to introduce the online platform [28,38,45–47] and whether to opt for the reselling or agency selling format [1,6,16,41,48–50]. Most studies in this area employ a theoretical approach to examine issues related to platform selling format selection in various supply chain environments.

Lots of studies are devoted to examining the issue that whether retailers should employ an online channel for selling besides the extant traditional reselling channel, such as Ryan et al. [38], Mantin et al. [28] and Yan et al. [46,47]. These studies focus on when retailers or manufacturers should introduce an online selling channel and the impact of an online channel on channel members’ reselling channel and revenue.

Another stream of literature has centered on the strategic choice between opting for the reselling or agency selling formats under various circumstances, such as asymmetric information and seller investment incentives.
[16,19], spillover effect [1], retailer competition [6], supplier competition [49], information sharing [50], and cross-sales structure [26]. Chen et al. [6] and Zennyo [49] investigated the strategic choice problem under competitive circumstances: the former vis-a-vis retailer competition; the latter in relation to supplier competition. Chen et al. [6] found that the agency selling format might lead to a higher price than the reselling format; this phenomenon contradicts the double marginalization, and Pareto improvement can be generated in a hybrid model of competitive e-tailers. Zennyo [49] suggested that the platform could design the agency and wholesale contract according to the product substitutability. Zhang and Zhang [50] showed that the e-tailer would make different information sharing decision under agency selling and reselling format depending on the supplier's offline entry cost.

2.2. Private brand introduction

A large literature has used either game theory or empirical methods to investigate issues associated with retailers’ private brands. We focus on review the studies using game-theoretical method related to private brand introduction issues.

In relation to the potential impact of private brands on retailers, researchers have discovered that private brands contribute to the amplification of their channel power in relation to upstream manufacturers [30], as well as the enhancement of their product mix [8,37].

Numerous studies have attended to the private brand introduction decision in various contexts. Raju et al. [36] employed a game-theoretical model to investigate the introduction of private brands in the presence of competing national brands. Sayman et al. [40] focused on retailers’ decisions regarding the introduction of multiple private brands. Groznik and Heese [15] and Jin et al. [20] explored this issue in the context of retailer competition. The former proposed that retailers prefer that competitors introduce the private brand rather than their launching it. The latter additionally took channel strategy into consideration and ascertained different equilibria under flexible and uniform wholesale price schemes. Cui et al. [13] incorporated the retailer’s risk attitude when making decisions about private brand introduction. Hara and Matsubayashi [17] examined the potential for collaboration between retailers and manufacturers in the context of private brand introduction.

To explore the potential for successful private brand introduction, it is crucial for private brands to adequately address the issue of differentiation from national brands. Numerous scholars have devoted their attention to addressing the positioning problem of private brands. Morton and Meyer [31] investigated the positioning problem of private brands within the context of retailer-manufacturer negotiations. Choi and Coughlan [9] examined the positioning problem from the perspectives of both quality and feature differentiation. Chung and Lee [11] delved into the problem of private brand quality and product line design. Hara and Matsubayashi [17] explored the positioning problem with a focus on collaboration between national brand manufacturers and retailers. Nalca et al. [32] considered the positioning problem in light of consumer taste information. Baumer et al. [5] investigated couponing strategies in conjunction with the positioning problem.

How the introduction of private brand would impact national brand has also attracted significant attentions. Some scholars have indicated that private brands can compel manufacturers to lower their national brand prices and potentially cannibalize the market share of national brands [23, 29, 34, 39]. Huang and Feng [18] discovered that retailers can strategically utilize money-back guarantees as a tool for developing private brands. Considering the potential adverse effects of private brands on national brands, scholars have also explored strategies manufacturers can employ in response to private brand introduction by retailers, such as online channel integration [3], product line strategies [33], and strategic timing of pricing and advertising decisions [22]. Scholars have also devoted attention to other aspects of private brand management, including store differentiation [10], information acquisition and disclosure [32], etc.

The existing studies have usually focused on the private brand introduction decision regarding traditional retailers rather than on an online platform. Kurata et al. [25] examined the pricing decisions when the national brand can distribute through both e-channel and physical channel and private brand only distribute through physical channel. Arce-Urriza and Cebollada [4] conducted an Chung rical research to investigate the competitive position of the private brand against the national brand with category-level relative analysis. However, the
online platform differs from traditional resellers, as the agency selling format can be implemented in the online platform. In a recent study, Li et al. [27] investigated the private brand introduction problem in the platform retailing circumstance. Their study focused mainly on the marketing investment effect by the manufacturer on the demand. Our study contributes to the literature in several aspects.

First, we examine the platform retailer’s private brand introduction problem by highlighting the agency selling format, which has been ignored in previous literature. While the agency selling format may result in greater brand competition between the manufacturer and platform, the reselling format can mitigate the competition between the national brand and private brand since the platform will provide pricing for both brands under the reselling format. We highlight the differences in selling formats and find that an agency platform would introduce a private brand only when the competition is low.

Second, we examine the interaction between selling format choice and private brand introduction, while previous literature tends to focus on either selling format choice or private brand introduction. Earlier studies have explored online selling format choice from the perspective of channel competition, spillover, and asymmetric information. However, the presence of private brand would result in direct competition between the national brand and private brand, and thus would influence the selling format preferences for the platform and manufacturer. We highlight this interaction and demonstrate that as competition between the two brands intensifies, the preference will shift from the agency model to the reselling format.

3. Model setup

Consider a supply chain consisting of a national brand manufacturer and an online retail platform. The manufacturer produces a national brand product and distributes it via the platform, who can either adopt a reselling or an agency selling format to distribute the national brand product. Also, the platform is considering launching a private brand that is to be a substitute for the manufacturer’s national brand.

Assume that the private brand product is produced by an alternative OEM (Original Equipment Manufacturer) at a fixed unit price, which could be regarded as the marginal production cost of the private brand [20,36]. To concentrate on the interaction between private brand and selling format, we assume that the unit production costs of both the national and private brand are equal to zero. This means that the difference between the private brand and national brand is therefore driven by demand rather than cost. It is straightforward, as the production technologies are more likely to be at the same level across different firms in contemporary industry settings [10,20].

As noted earlier, subsequently we use the pronoun “he” (“she”) to represent the manufacturer (the platform). The platform chooses the selling format and makes the private brand introduction decision. We use KK to signify the distribution model. Based on the platform’s selling format choice and private brand introduction decision, there are four possible distribution models: KK ∈ {NR, NA, FR, FA}, where N and F represent not to introduce the private brand and to introduce the private brand; R and A designate the reselling format and agency selling format, respectively.

The possible existence of each distribution model is driven by the platform’s selling format choice and private brand introduction decision. Assuming the existence of each selling format, the sequence of events for each model is stated below.

Model NR. In this model, the platform chooses to act as a reseller for the manufacturer’s national brand and not to introduce the private brand. The manufacturer first announces the wholesale price \( w_{m}^{\text{NR}} \) of the national brand, and then the platform determines the retail price \( p_{m}^{\text{NR}} \), accordingly.

Model NA. In this model, the platform chooses to act as a marketplace provider for the manufacturer by specifying a proportional fee \( \beta \) and not to introduce a private brand. If he accepts the marketplace offer, then the manufacturer determines the retail price \( p_{m}^{\text{NA}} \) for the national brand product.

Model FR. In this model, the platform chooses to act as a reseller for the manufacturer’s national brand and to introduce the private brand. The manufacturer first announces the wholesale price \( w_{m}^{\text{FR}} \) for the national
brand, and then the platform simultaneously determines the retail price for the national brand $p^{FR}_m$ and private brand $p^{FR}_f$.

**Model FA.** In this model, the platform chooses to act as a marketplace provider for the manufacturer by specifying a proportional fee $\beta$ and to introduce the private brand. Assume that the manufacturer accepts the online marketplace offer; then the manufacturer and the platform set the retail prices of the national brand $p^{FA}_m$ and private brand $p^{FA}_f$, respectively.

Empirical data show that the proportional fee $\beta$ values commonly range from 5% to 40%. Online travel agencies, such as Expedia and Booking.com, usually charge between 15% and 25% to book hotels online [24]. Digital publishing industries set a 30% commission fee for e-books [44]. Amazon sets the proportional fee between 6% to 25% of the sales revenue according to the product category. Therefore, we constric our analysis to $\beta < 0.5$, which reflects the proportion of sales revenue under the agency selling format that industry prefers the platform retains.

We follow established norms in the private brand introduction literature to describe how the private brand affects the national brand’s demand. We assume that total category demand is the same and normalized to one unit under different models. This assumption is consistent with extant private brand studies (e.g., [10, 20]). Use subscript $m/f$ to represent the national brand and private brand. The model notations are summarized in Table 1.

When the platform does not introduce the private brand, the demand for national brand is then:

$$q^{KK}_m = 1 - p^{KK}_m .$$

When the platform introduces the private brand, the demand functions for the national and private brand are formulated as:

$$q^{KK}_m = \frac{1}{1 + a} \left[ 1 - p^{KK}_m + \theta (p^{KK}_f - p^{KK}_m) \right],$$

$$q^{KK}_f = \frac{1}{1 + a} \left[ a - p^{KK}_f + \theta (p^{KK}_m - p^{KK}_f) \right],$$

where $a \in (0, 1)$ represents the private brand’s potential market, which captures the cannibalization effect that the private brand product derives from the national brand product. $a < 1$ indicates that the brand preference for the private brand is lower than for the national brand. A higher $a$ reflects that the preference for the private brand is getting close to the national brand and indicates a strong cannibalization effect. $\theta \in (0, 1)$ measures cross-price competition between the national and private brand.

The channel member who distributes products will incur an order fulfillment cost, which is denoted by $F$. Order fulfillment cost refers to the expenditures incurred for completing an order, which includes package handling and transshipment. As in Tian et al. [43], we view the order-fulfillment cost as a fixed cost. For simplicity, we set the order-fulfillment cost for the national brand undertaken by the manufacturer equals that by the platform, which is consistent with Tian et al. [43]. Note that, when the platform introduces the private brand, she will also incur an order-fulfillment cost. We assume that the order-fulfillment cost for the private brand is the same as for the national brand.

4. **Pricing decisions under four models**

This section will derive the equilibrium decisions under different models. And we also conduct a sensitivity analysis for the optimal prices and demands.

4.1. **Model NR**

We start with the scenario in which the platform acts as a traditional reseller, and she chooses not to introduce the private brand. With this model, the platform distributes the national brand product by setting the retail
Table 1. Model notations.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>KK</td>
<td>The distribution model, where KK ∈ {NR, NA, FR, FA}</td>
</tr>
<tr>
<td>θ</td>
<td>The competition intensity between the national brand and private brand</td>
</tr>
<tr>
<td>a</td>
<td>The potential market of the private brand</td>
</tr>
<tr>
<td>β</td>
<td>The proportion fee charged by the platform under the agency selling format</td>
</tr>
<tr>
<td>F</td>
<td>The order-fulfillment cost</td>
</tr>
<tr>
<td>(w_{m}^{KK})</td>
<td>The wholesale price of the national brand under model KK</td>
</tr>
<tr>
<td>(p_{m}^{KK}, p_{f}^{KK})</td>
<td>Retail price of the national brand and private brand under model KK, respectively</td>
</tr>
<tr>
<td>(q_{m}^{KK}, q_{f}^{KK})</td>
<td>Demand for the national brand and private brand under model KK, respectively</td>
</tr>
<tr>
<td>(\Pi_{m}^{KK}, \Pi_{f}^{KK})</td>
<td>Profit of the manufacturer and platform under model KK, respectively</td>
</tr>
</tbody>
</table>

price \(p_{m}^{NR}\) and bears the order-fulfillment cost \(F\). The demand is then \(q_{m}^{NR} = 1 - p_{m}^{NR}\). The profit functions of the manufacturer and platform are given as follows:

\[
\Pi_{m}^{NR} = w_{m}^{NR} q_{m}^{NR} - F, \\
\Pi_{f}^{NR} = (p_{m}^{NR} - w_{m}^{NR}) q_{m}^{NR} - F. 
\]

The optimal pricing decision under NR are given in Lemma 4.1.

**Lemma 4.1.** Under model NR, the equilibrium wholesale price and retail price of the national brand are \(w_{m}^{NR*} = \frac{1}{2}\) and \(p_{m}^{NR*} = \frac{3}{4}\), respectively. The optimal profit of the manufacturer and platform are \(\Pi_{m}^{NR*} = \frac{1}{8}\) and \(\Pi_{f}^{NR*} = \frac{1}{16} - F\), respectively.

Clearly, the platform might receive negative profit and thus exit the wholesale contract with the manufacturer if the order-fulfillment cost is too large. To focus on the case where the platform can always make a positive profit through reselling the national brand product and to enhance the implications from our investigation, however, we confine the order-fulfillment cost to \(F < \frac{1}{16}\) throughout this study.

4.2. Model NA

In this subsection, we investigate the scenario in which the platform chooses the agency selling format and not to introduce the private brand. The agency model reflects that the platform serves as an online marketplace provider through which the manufacturer distributes his national brand product directly by himself. The platform receives a proportion \(β\) of the sales revenue of the national brand, and the manufacturer retains \(1 - β\) of the sales revenue. The manufacturer selects the retail price of the national brand \(p_{m}^{NA}\) and assumes the order-fulfillment cost \(F\); then the demand realized as \(q_{m}^{NR} = 1 - p_{m}^{NR}\). Therefore, supply chain members’ objective functions are given as follows:

\[
\Pi_{m}^{NA} = (1 - β)p_{m}^{NA} q_{m}^{NA} - F, \\
\Pi_{f}^{NA} = β p_{m}^{NA} q_{m}^{NA}. 
\]

**Lemma 4.2.** Under model NA, the national brand’s retail price is \(p_{m}^{NA*} = \frac{1}{2}\). The channel members’ profits are calculated by \(\Pi_{m}^{NA*} = \frac{1}{4}(1 - β) - F\) and \(\Pi_{f}^{NA*} = \frac{β}{2}\), respectively.
The agency selling format generates a much lower retail price than the reselling format. The reason for this is that the agency selling format eliminates the double marginalization effect of the reselling format. As a result, the agency selling format will benefit customers more. The manufacturer’s and platform’s revenues are allocated according to the platform percentage fee. Further, the manufacturer will incur an order fulfillment cost for handling online orders under the agency selling format.

4.3. Model FR

Under model FR, the platform chooses to act as a traditional reseller to distribute the national brand product of the manufacturer and introduces her private brand to customers. The platform outsources the production of private brand to a third-party manufacturer with a fixed unit production cost. We assume that the unit production cost of the private brand is 0, the same as the national brand. Based on this assumption, we can eliminate the impact of cost differences and focus on the demand differences created by the national brand premium.

The demand functions of two brands are formulated as follows in the presence of brand competition:

\[ q_{m}^{FR} = \frac{1}{1 + a} \left[ 1 - p_{m}^{FR} + \theta(p_{f}^{FR} - p_{m}^{FR}) \right], \]
\[ q_{f}^{FR} = \frac{1}{1 + a} \left[ a - p_{f}^{FR} + \theta(p_{m}^{FR} - p_{f}^{FR}) \right], \]

where \( \theta \) represents the intensity of brand competition between the two brands. Because the platform handles the distribution of the two brands, she might be able to weaken competition by setting proper retail prices for the two brands. The platform will incur cost \( 2F \) to deal with the orders of the two brands. The manufacturer first announces the wholesale price \( w_{m}^{FR} \), and then the platform determines the retail prices \( p_{m}^{FR} \) and \( p_{f}^{FR} \), simultaneously. The profit functions of the supply chain members are formulated by:

\[ \Pi_{m}^{FR} = w_{m}^{FR} q_{m}^{FR}, \]
\[ \Pi_{f}^{FR} = (p_{m}^{FR} - w_{m}^{FR}) q_{m}^{FR} + p_{f}^{FR} q_{f}^{FR} - 2F. \]

Lemma 4.3. Under model FR, the equilibrium wholesale and retail price of the national brand are \( w_{m}^{FR*} = \frac{1}{2(1 + \theta)} \) and \( p_{m}^{FR*} = \frac{3 + 6\theta + 2a\theta + 26\theta^2 + 2a^2\theta^2}{4(1 + \theta)(1 + 2\theta)} \), respectively. The equilibrium retail price of the private brand is \( p_{f}^{FR*} = \frac{a + \theta + a\theta + 4a^2\theta^2}{2(1 + \theta)} \). The optimal profits of the channel members are calculated as:

\[ \Pi_{m}^{FR*} = \frac{1}{8(1 + a)(1 + \theta)}, \]
\[ \Pi_{f}^{FR*} = \frac{1 + 4a^2 + 2\theta + 8a\theta + 8a^2\theta + 4\theta^2 + 8a\theta^2 + 4a^2\theta^2}{16(1 + a)(1 + \theta)(1 + 2\theta)} - 2F. \]

Inferred from Lemma 4.3 is that \( w_{m}^{FR} < w_{m}^{NR} \) and \( \partial w_{m}^{FR}/\partial \theta < 0 \). It indicates that once the platform introduces her private brand, the manufacturer tries to strengthen the competitiveness of the national brand by lowering the wholesale price, which would lead to a reduction in the retail price of the national brand. The manufacturer would further reduce the wholesale price of the national brand to respond to the intensified competition.

4.4. Model FA

Under model FA, the platform chooses to act as a marketplace provider and introduce her private brand. The manufacturer distributes the national brand product on the platform and faces competition from the private brand. The manufacturer and platform determine the retail prices \( p_{m}^{FA} \) and \( p_{f}^{FA} \), independently and
simultaneously. Different from scenario FR, the manufacturer can attend to competition with the private brand by setting the retail price directly. The demands for the two brands are realized as follows:

\[ q_{FA}^m = \frac{1}{1 + a} \left[ 1 - p_{FA}^m + \theta (p_{FA}^f - p_{FA}^m) \right], \]
\[ q_{FA}^f = \frac{1}{1 + a} \left[ a - p_{FA}^f + \theta (p_{FA}^m - p_{FA}^f) \right]. \]

The manufacturer shares \( \beta \) proportion of the sales revenue of the national brand with the platform. The manufacturer and the platform handle the orders and bear the order-fulfillment cost, respectively. The profit functions of the supply chain members are given as follows:

\[ \Pi_{FA}^m = (1 - \beta) p_{FA}^m q_{FA}^m - F, \]
\[ \Pi_{FA}^f = \beta p_{FA}^m q_{FA}^m + p_{FA}^f q_{FA}^f - F. \]

**Lemma 4.4.** Under model FA, the equilibrium retail price for the national brand and the private brand are

\[ p_{FA}^m = \frac{2 + (2 + a) \theta}{4 + 8 \theta + (3 - \beta) \theta^2}, \]
\[ p_{FA}^f = \frac{(1 + \beta) \theta + 2a(1 + \theta)}{4 + 8 \theta + (3 - \beta) \theta^2}. \]

The profits of the manufacturer and platform are

\[ \Pi_{FA}^m = \frac{(1 - \beta)(1 + \theta)(2 + 2 \theta + a \theta)^2}{(1 + a)(4 + 8 \theta + 3 \theta^2 - \beta \theta^2)^2} - F, \]
\[ \Pi_{FA}^f = \frac{a^2(1 + \theta)[4 + 8 \theta + (4 - \beta) \theta^2] - (1 + \theta)M + a \theta N}{(1 + a)[4 + 8 \theta + (3 - \beta) \theta^2]^2} - F \]

where \( M = \theta^2 - \beta^2 \theta^2 + 4 \beta (1 + \theta)^2, N = \beta^2 \theta^2 - 4 (1 + \theta)^2 - \beta(4 + 8 \theta + 3 \theta^2). \) In the FA model, the manufacturer handles the national brand and the platform introduces the private brand. Therefore, the level of competition between the two brands may be higher than in the FR model. This may result in lower retail prices and thus benefit customers. Consequently, the agency model is more advantageous for consumers.

### 4.5. Sensitivity analysis

In this subsection, we examine the effect of private brand potential market \( a \) and competition intensity \( \theta \) on the retail pricing strategies and demand. Corollary 4.5 presents the main results of the sensitivity analysis under models FR and FA where the national brand competes with the private brand.

**Corollary 4.5.** The sensitivity analysis for the equilibrium prices and demand under the distribution models with private brand are summarized in Table 2.

First, the results show that an increase in private brand potential market will lead to higher retail prices for both brands. The augmented private brand potential market will enhance the market strength of the private brand and hence lead to a demand upturn for the private brand and a demand decrease for the national brand. The effects of private brand potential market on the agency and reselling format are the same; this indicates that the selling alternative will not create a difference for the influence of private brand potential market.

Second, the increase in competition intensity will lead to a reduction in the retail price of the national brand under both the agency and reselling format. Raju et al. [36] found that increased competition between national and private brands resulted in a decrease in the retail price of the national brand but would increase the retail price of the private brand. Our results are consistent with Raju et al. [36] in that the increased competition will lead to the reduction in the national brand’s product price in the reselling format. Moreover, we find that it is also true under the agency selling format. However, our results differ from Raju et al. [36] in that the platform
Table 2. Sensitivity analysis for equilibrium prices and demand.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model FR</th>
<th>Model FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{FR}^m$</td>
<td>$p_{FA}^m$</td>
<td>$q_{FR}^m$</td>
</tr>
<tr>
<td>$a$</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>$\theta$</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>

Notes. ↑: increasing; ↓: decreasing; -: unchanged; C: conditional on $a$ (if $a$ is small, $\partial p_{FA}^f/\partial \theta > 0$; otherwise, there exists a threshold value $\theta^*: \partial p_{FA}^f/\partial \theta \geq 0$ when $\theta \leq \theta^*$; $\partial p_{FA}^f/\partial \theta < 0$ when $\theta > \theta^*$).

will not necessarily raise the private brand’s retail price under the FA model. When the agency selling format is adopted, how the competition intensity influences the platform’s private brand price depends on the private brand’s potential market. If the private brand’s potential market is low, the platform’s private brand pricing is similar to that under the reselling format, i.e., raising private brand’s selling price over the competition intensity. If the private brand’s potential market is high, the platform will first improve and then reduce the private brand’s selling price over the competition intensity.

4.6. Platform’s proportional fee

Note that we have assumed that the manufacturer would accept the platform’s marketplace offer in models NA and FA. However, the manufacturer might not accept the offer if the platform charges a relatively high proportional fee. Therefore, the platform would need to design her proportional fee policy carefully. In practice, platforms likely pre-announce their proportional fee policy [43]. In this subsection, we examine the policy where the manufacturer agrees to guarantee the existence of models NA and FA.

It is straightforward to consider that the manufacturer would be induced to accept the marketplace offer unless he can make a profit under the agency model that is no less than under the reselling format. Consequently, the manufacturer will accede to the marketplace proposal given the platform does not introduce a private brand when $\Pi_{m}^{NA^*} \geq \Pi_{m}^{NR^*}$, and will concede to the deal given the platform introduces a private brand when $\Pi_{m}^{FA^*} \geq \Pi_{m}^{FR^*}$. The proof and expression of $\beta_1$ are given in the Appendix A.

**Proposition 4.6.** The manufacturer will accept the agency selling format provided $\beta \leq \tilde{\beta} = \min[\beta_1, \beta_2]$, where $\beta_1$ is the unique solution of $\Pi_{m}^{FA^*} - \Pi_{m}^{FR^*} = 0$ and $\beta_2 = \frac{1}{2}(1 - 8F)$ is the unique solution of $\Pi_{m}^{NA^*} - \Pi_{m}^{NR^*} = 0$.

**Corollary 4.7.** $\tilde{\beta}$ decreases with $F$.

Proposition 4.6 characterizes the sufficient condition under which the manufacturer will accept the marketplace offer from the platform. The profit of the manufacturer under the agency model monotonically decreases with the proportional fee; this indicates that the platform should set the proportional fee not too high to enhance the manufacturer’s motivation to engage in the agency model. Therefore, Proposition 4.6 provides guidance for the platform in establishing her proportional fee policy. Due to the fact that the proportional fee is pre-announced and remains steady over a significant period of time, we consider that the platform should adopt a policy that is not linked to the platform’s decision to introduce a private brand. In addition, this enables all models to be realized. In general, it has been observed that online platforms set a relatively low proportional fee, for instance, Amazon usually charges a proportional fee between 8% and 25%, and JD.com charges a proportional fee between 2% and 8%.

Corollary 4.7 shows that the platform should reduce the proportional fee when the order-fulfillment cost increases. A higher order-fulfillment cost would lead to a lower incentive for the manufacturer to engage in the marketplace offer. Therefore, the platform should improve the sales revenue for the manufacturer by lowering the proportional fee to ensure that the manufacturer will accede to the marketplace proposal.
Subsequently, we substitute $\tilde{\beta}$ into the equilibrium profit to represent the supply chain members’ optimal profits.

5. Analysis

5.1. Comparative analysis of prices and demands

In this subsection, we conduct comparative analyses to examine the influences of selling formats and private brand introduction on prices and demands. Corollaries 5.1 and 5.2 compare the prices and demand under the different models.

**Corollary 5.1.** If $a > \frac{29^2 - 1}{29(1+\theta)}$, $p_{m}^{\text{NR}} > p_{m}^{\text{FR}} > p_{m}^{\text{NA}} > p_{m}^{\text{FA}}$; otherwise, $p_{m}^{\text{NR}} > p_{m}^{\text{NA}} > p_{m}^{\text{FR}} > p_{m}^{\text{FA}}$.

First, Corollary 5.1 demonstrates that the reselling format will result in a higher retail price for the national brand ($p_{m}^{\text{FR}} > p_{m}^{\text{FA}}$, $p_{m}^{\text{NR}} > p_{m}^{\text{NA}}$), which is generated by the double-marginalization effect. Second, the competition level induced by the presence of the private brand will lead to a reduced retail price of the national brand ($p_{m}^{\text{NR}} > p_{m}^{\text{FR}}$, $p_{m}^{\text{NA}} > p_{m}^{\text{FA}}$); this is reflective of the competition effect. Therefore, both the agency selling format and the introduction of the private brand are conducive to consumers’ welfare. Additionally, model FA results in the lowest retail price for the national brand; this is derived from the direct pricing by the manufacturer as well as the competition effect. And model NR leads to the highest retail price of the national brand owing to the double-marginalization and monopoly effect.

**Corollary 5.2.** If $a > \frac{\sqrt{1+\beta}^6 - 1}{4+6\sqrt{1+\beta}^2}$, $q_{m}^{\text{NA}} > q_{m}^{\text{FA}} > q_{m}^{\text{NR}} > q_{m}^{\text{FR}}$; otherwise, $q_{m}^{\text{FA}} > q_{m}^{\text{NA}} > q_{m}^{\text{NR}} > q_{m}^{\text{FR}}$.

Regarding the demand, first, the agency model will always generate higher demand for the national brand when the private brand choice is the same, i.e., $q_{m}^{\text{NA}} > q_{m}^{\text{FA}}$ and $q_{m}^{\text{NR}} > q_{m}^{\text{FR}}$; this is a function of the direct pricing advantage with the agency model. Second, the demand for the national brand will be reduced by the introduction of the private brand under the reselling alternative ($q_{m}^{\text{NR}} > q_{m}^{\text{FR}}$). Furthermore, the demand for the national brand will be reduced under the agency mode when the private brand’s potential market is larger than a threshold. This can be referred to as the “erosion effect”, which represents the encroachment that the private brand will inevitably have on the market share of the national brand. Although the retail prices have been reduced to react to the encroachment of the private brand, the demand for the national brand is still reduced. Apparently, the competition effect will lead to a reduction in retail prices; it will not necessarily improve the demand for the national brand, however, which means the competition effect cannot outweigh the erosion effect. Interestingly, when the private brand’s potential market is smaller than a threshold, the demand of the national brand versus the private brand under the agency model will be higher than that without the private brand, i.e., $q_{m}^{\text{FA}} > q_{m}^{\text{NA}}$. This means that the competition effect dominates over the erosion effect in such circumstances.

5.2. Private brand effect

In this subsection, we examine the effect of private brand introduction and identify the conditions under which introducing the private brand is advantageous to the platform.

**Corollary 5.3.** $\Pi_m^{\text{NA}} > \Pi_m^{\text{FA}}$ and $\Pi_m^{\text{NR}} > \Pi_m^{\text{FR}}$.

Corollary 5.3 shows that the manufacturer’s profit will be eroded once the platform chooses to introduce the private brand. The introduction of the private brand always brings competition between the two brands; this in turn reduces the national brand’s price for the manufacturer (the competition effect). The presence of the private brand is more likely to encroach on the market share of the national brand; this leads to a loss in demand for the manufacturer (the erosion effect). Therefore, the competition and the erosion effect jointly cause a profit loss for the manufacturer under both the agency and the reselling format.
Proposition 5.4. Under the reselling format, when \( a \leq \frac{1}{4} \) and \( \theta \geq \frac{1-4a}{2(1+2a)} \), or, \( a > \frac{1}{4} \), there exists a threshold value \( F_1 \) such that \( \Pi_{f}^{FR} \geq \Pi_{f}^{NR} \) when \( F < \min\left(F_1, \frac{1}{16}\right) \).

Corollary 5.5. \( F_1 \) increases in \( \theta \) and \( a \).

Proposition 5.4 identifies the private brand introduction condition of the platform under the reselling format. The threshold value \( F_1 \) is the upper limit condition of the order-fulfillment cost, beyond which introducing the private brand would not be beneficial for the platform. A higher \( F_1 \) allows the platform to make a profit from introducing the private brand even when handling the order is costly. If the private brand’s potential market is small and the competition intensity between the national brand and private brand is low, this situation will never be favorable for the platform to introduce the private brand.

The potential market parameter \( a \) can be explained as customers’ perceived degree of value for the private brand. A higher potential market may arise from a strong platform image or high brand quality positioning. The private brand introduction decision thus is closely related to the potential market. The platform is likely to be profitable by introducing a private brand when the potential market is high and the order-fulfillment cost is below a threshold. However, when the platform has a low potential market, introducing a private brand can only be beneficial for the platform when intense competition between the two brands exists. This situation implies that the platform is beneficial to introduce a private brand which is close to the national brand. The rationale is that the platform’s profit would be encroached by the presence of private brand; as long as the profit from the private brand can outweigh the loss of the national brand, introducing a private brand will be favorable for her. To improve the implementation of the private brand, the platform should utilize advertising or a quality guarantee to augment private brand potential market to avoid intense competition with the national brand when the potential market is small.

Corollary 5.5 demonstrates that the platform is increasingly motivated to introduce the private brand under the reselling format if either the competition between the two brands augments or the private brand’s potential market rises. This is because a larger potential market or rising competition intensity raises the threshold values of \( F_1 \), thus fostering the platform’s decision to introduce the private brand. When the private brand’s potential market is relatively large, i.e., \( a > 0.809 \), the threshold value \( F_1 \) will surpass 1/16. The condition on order-fulfillment cost will always be met, as we have assumed that the order-fulfillment cost will not exceed 1/16. This means that the platform’s private brand introduction decision is independent of the order-fulfillment cost when the potential market is large enough and will always be favorable for introducing the private brand under such circumstances.

Proposition 5.6. Under agency selling format, when \( F_2(\beta) > 0 \) there exists a threshold value \( F_2(\beta) \) such that \( \Pi_{f}^{FA} \geq \Pi_{f}^{NA} \) when \( F < F_2(\beta) \).

When the agency selling format is adopted, the condition for the platform’s introducing a private brand is similar to that under the reselling alternative. This infers that introducing the private brand is beneficial to the platform only when the order-fulfillment cost is below a threshold.

Under the agency selling format, the platform makes a profit from the national brand by setting the proportional fee \( \beta \). As such, higher proportional fees benefit the platform under both FA and NA scenarios. Interestingly, \( \Pi_{f}^{FA} - \Pi_{f}^{NA} \) decreases with the proportional fee \( \beta \); this implies that, when the platform receives a large share of revenue from the national brand, she will consider the introduction of private brand to be less attractive. We have discussed the optimal proportional fee design for the platform in the previous subsection. Therefore, the optimal proportional fee \( \beta \) should be chosen not too high to ensure that \( F_2 \) is positive; otherwise, the platform will never introduce the private brand under the agency selling format.

The positive effect of the private brand introduction is the enhancement of the product mix in the platform. Such enhancement is favorable for the platform to attract more consumers, i.e., \( q_{m}^{FA} + q_{f}^{FA} > q_{m}^{NA} \). The negative effect of the private brand introduction is that it would reduce the revenue from national brand sales owing to increased sales of the private brand, i.e., \( \Pi_{f}^{NA} > \beta q_{m}^{FA} q_{m}^{FA} \). Moreover, if the private brand is introduced,
the platform has to bear the order-fulfillment cost. Therefore, the private brand benefits the platform when the order-fulfillment cost is below the threshold. The threshold value exists only when the proportional fee is not too high. This means that the platform would never introduce the private brand when she can share a sufficiently high proportion of revenue from the national brand.

5.3. Private brand strategy involving selling format

This subsection conducts a comparison analysis to provide insights into the selling format choice.

**Corollary 5.7.** The model NR will not be the optimal choice due to \( \Pi_{j}^{\text{NA}} > \Pi_{j}^{NR} \).

Corollary 5.7 indicates that the NR model would not be the optimal choice for the platform, as it always yields to the NA model. This result means that the agency selling format improves the platform’s profitability because of the elimination of the double-marginalization effect under the reselling format. The agency selling alternative is widely adopted in the Internet era; after all, the platform allows for easy tracking of the manufacturer’s sales volume, thus eliminating information asymmetry and facilitating the agency model’s implementation.

**Observation 1** summarizes the optimal private brand strategy choice for the platform involving the online selling format.

**Observation 1.** (i) When the private brand’s potential market is small, the NA model is the best choice; (ii) when the potential market is moderate, the NA model is the best choice if the order-fulfillment cost is high; otherwise, the FA (FR) model is the best choice if competition intensity is low (high); (iii) when the private brand’s potential market is large, the FA model is the best choice if competition intensity is low; the FR model is the best choice if competition intensity is high.

Observation 1 suggests that the platform’s selling model is related to the private brand’s potential market, order-fulfillment cost, and competition intensity between the two brands. Intuitively, when the private brand’s potential market is low, the platform’s best option is to exclude the private brand and adopt the agency selling format. That is, when the private brand’s potential market is lower than a threshold, there is no profit for the platform by introducing the private brand. Conversely, if the private brand’s potential market is relatively large, introducing the private brand would always be beneficial for the platform, regardless of the selling format. This implies that the platform should ameliorate the brand image of the private brand, which facilitates the private brand’s introduction to customers.

When the private brand’s potential market is moderate, the private brand can be beneficial to the platform only when the order-fulfillment cost is below a threshold. Its best choice is to adopt the agency selling format when the level of competition is low and adopt the reselling format when it is high. These implications are derived from the fact that the platform can mitigate the competition between the two brands when the competition intensity is high, which might have been generated by closely positioning the private brand to the manufacturer’s national brand. Moreover, when the potential market of the private brand becomes relatively large, the region having no private brand introduction will disappear; this implies that under such circumstances, the platform can always be profitable by introducing the private brand.

**Observation 1** (ii) and (iii) is slightly consistent with Hagiu [16] and Tian et al. [43], in which the reselling format is preferred when upstream competition intensity is high, and the agency selling alternative is favored when upstream competition intensity is low. However, our study highlights that the level of competition hinges on the substitutability between the store and national brand rather than upstream competition (Fig. 1).

6. Discussion and conclusions

This paper studies the private brand introduction decision, along with the selling format choice, in an online platform circumstance. The platform can serve as a traditional retailer or an online marketplace provider for the manufacturer. The platform is considering the introduction of a private brand that will substitute for the
manufacturer’s national brand. We first examine the equilibrium strategies under four selling formats vis-à-vis the selling format and private brand introduction. We then investigate the impact of selling format and private brand introduction on the prices and demands of the two brands. We further identify when the platform should introduce the private brand both under the reselling and agency models, as well as the choice of selling format from the four models.

First, if the private brand’s potential market is extremely small or relatively large, the platform’s private brand introduction would not be interacted with the selling format choice, i.e., the platform would introduce the private brand if the private brand’s potential market is relatively large, and not introduce it if the potential market is extremely small, regardless of the selling format. This implies that the potential market of the private brand exerts a significant influence on the platform’s decision to introduce the private brand. In intuition, online retail platforms who plan to introduce a private brand should examine carefully the private brand’s potential market or resort to cultivating their customers’ preference for their private brand. Since the private brand will not easily succeed if the private brand’s potential market, i.e., potential market, is extremely small. In 2017, the Chinese e-commerce giant JD.com launched its own brand, Jing Zao, which offers functional goods, such as kitchenware and health products. It is generally easier for consumers to accept new brands that are functionally focused. As a result, these functionally focused products have a greater chance of gaining a large potential market.

Second, if the private brand’s potential market is moderate, the platform’s private brand introduction will be interacted with the selling format choice, i.e., the decision of private brand introduction should be determined depending on the order-fulfillment cost and competition intensity. The platform can be profitable by introducing the private brand only when the order-fulfillment cost is below a threshold. Under such circumstances, the platform can utilize the selling format to earn a greater profit, that is, by adopting the agency model if the competition intensity is low and the reselling format if the competition intensity is high. Accordingly, the decision to introduce a private brand may be influenced by the selling format implemented by the manufacturer and platform. Reselling can be a useful method to mitigate competition between private brand and national brand when the level of competition is high. When the competition level of the two brands is not that keen, the agency selling format may be implemented to reduce the double-marginalization effect and to generate more profit.

Our study suggests several managerial implications for the online platform. Firstly, platform managers should make private brand decision according to the size of potential market, i.e., excluding or including it accordingly when the potential market is extremely small or relatively large. Platform managers should recognize the criticality of assessing the potential market when contemplating the introduction of a new private brand. Understanding the size and growth potential of a market segment is essential for determining the feasibility and success of a private brand launch. Platform firms should prioritize private brands with a large potential market, as it indicates a higher demand and increased likelihood of capturing market share. Additionally, investing in
developing the potential market for a private brand before it reaches a significant size can be a strategic step. By actively promoting and cultivating consumer interest in a specific category, platform firms can create a favorable market environment that supports the successful introduction of a private brand. Notably, companies like Amazon and JD.com tend to introduce functional categories as private brands, which aligns with the potential market requirements.

Second, the selling format plays a significant role in determining the platform firm’s private brand selection when the private brand’s potential market is moderate. Agency-oriented platforms should adopt a differentiated private brand strategy to circumvent competition from national brands. Conversely, reselling-oriented platforms can introduce private brands that closely resemble national brands, leveraging the platform’s ability to mitigate competition between the two. For instance, Alibaba’s agency-oriented platform Taobao Xinxuan specializes in premium quality products across diverse categories, aiming to differentiate its private brand from national brands. Reselling-oriented platforms like JD.com prioritize designing functionally-oriented private brand products, despite the potential for increased competition. However, these platforms can employ pricing mechanisms to alleviate competition between the private brand and national brand. In summary, platform firms can align their private brand preferences with their selling formats to increase the success possibility of a private brand launch.

There are several limitations in this study. Firstly, the study does not account for competition among peer manufacturers or platforms. In practice, multiple manufacturers often compete on the same platform, while similar platforms may also compete with each other. Future researches may examine how such competition among manufacturers or platforms could potentially impact the introduction of private brands on a platform. Secondly, we neglected to consider the channel choice of manufacturers. It is plausible that manufacturers may establish their own online channels or opt for alternative distribution platforms. Therefore, an interesting issue for further research would be to explore how manufacturers respond when a platform contemplates introducing a private brand. Thirdly, we did not incorporate the customer preferences, attitudes, or behavioral features in our theoretical model. Consequently, it would be valuable to investigate how customer preferences for private brands versus national brands may influence the platform’s decision to introduce private brands. Additionally, future researchers can conduct in-depth empirical studies utilizing platform data to scrutinize how platforms should strategically make decisions and introduce private brands amid national competition and diverse product attributes.

**APPENDIX A.**

See Table A.1.

**Proof of Corollary 4.5.** Take the first-order derivative of $p^F_A$ with respect to $\theta$,

$$\frac{\partial p^F_A}{\partial \theta} = \frac{4 + 4\beta - 3\theta^2 - 2\beta\theta^2 + \beta^2 \theta^2 + a(-8 - 12\theta + 4\beta\theta - 6\theta^2 + 2\beta\theta^2)}{(4 + 8\theta + 3\theta^2 - \beta\theta^2)^2}. $$

The denominator is always positive, so we only need to investigate the sign of the numerator. The numerator decreases with $a$, the maximum value can be achieved when $a = 0$, and the minimum value can be achieved when $a = 1$. Max $[4 + 4\beta - 3\theta^2 - 2\beta\theta^2 + \beta^2 \theta^2 + a(-8 - 12\theta + 4\beta\theta - 6\theta^2 + 2\beta\theta^2)]_{a=0} = 4 - 3\theta^2 + \beta^2 \theta^2 + \beta(4 - 2\theta^2) > 0$, Min $[4 + 4\beta - 3\theta^2 - 2\beta\theta^2 + \beta^2 \theta^2 + a(-8 - 12\theta + 4\beta\theta - 6\theta^2 + 2\beta\theta^2)]_{a=1} = -4 + 4\beta + (-12 + 4\beta)\theta + (-9 + \beta^2)\theta^2 < 0$. Therefore, there exists a root $a = \frac{(1 + \beta)[4 - 3\beta \theta^2]}{8 + 4(3 - \beta)\theta + 2(3 - \beta)\theta^2}$ when we set $4 + 4\beta - 3\theta^2 - 2\beta\theta^2 + \beta^2 \theta^2 + a(-8 - 12\theta + 4\beta\theta - 6\theta^2 + 2\beta\theta^2) = 0$. When $a > \frac{(1 + \beta)[4 - (3 - \beta)\theta^2]}{8 + 4(3 - \beta)\theta + 2(3 - \beta)\theta^2}$, $\frac{\partial p^F_A}{\partial \theta} < 0$, therefore, $p^F_A$ decreases with $\theta$. □

The rest of the sensitivity analysis could be conducted similarly and we omit the details here.
Proof of Corollary 4.7. The comparisons of the equilibrium retail prices for national brand are as follows:

\[
p_{m}^{\text{NR}} - p_{m}^{\text{NA}} = \frac{\theta}{4} > 0, \quad p_{m}^{\text{NR}} - p_{m}^{\text{FR}} = \frac{\theta[3 + 4\theta - 2a(1 + \theta)]}{4(1 + \theta)(1 + 2\theta)} > 0, \quad p_{m}^{\text{NR}} - p_{m}^{\text{FA}} = \frac{4 + 16\theta - 4a\theta + 9\theta^2 - 3\beta\theta^2}{4[4 + 8\theta + (3 - \beta)\theta^2]}.
\]

The denominator is always positive, so we only need to investigate the sign of the numerator. The numerator decreases with \(a\), and the numerator achieves its minimum value when \(a = 1\), which can be expressed as
therefore $p_a^m - p_a^F = 0$.

$\frac{p_m^F - p_m^A}{p_m^{NA} - p_m^A} = \frac{4+16\theta+256\theta^2-33\theta^2-a(4+12\theta+9\theta^2-3\theta^2)}{4(1+\theta)}$, the denominator is always positive, so we only need to investigate the sign of the numerator. The numerator increases with $\theta$, and the numerator achieves its minimum value when $a = 0$, which can be expressed as $\text{Min}[4+16\theta+256\theta^2-33\theta^2-a(4+12\theta+9\theta^2-3\theta^2)]_{a=0} = 4+16\theta+256\theta^2-33\theta^2-a(4+12\theta+9\theta^2-3\theta^2) > 0$, therefore $p_m^F - p_m^A > 0$. 

$\frac{p_m^F - p_m^{NA}}{p_m^{FR} - p_m^{NA}} = \frac{4+16\theta+256\theta^2+33\theta^2-a(4+12\theta+9\theta^2-3\theta^2)}{4(1+\theta)}$, the denominator is always positive, so we only need to investigate the sign of the numerator. The numerator increases with $\theta$, and the numerator achieves its minimum value when $a = 0$, which can be expressed as $\text{Min}[4+16\theta+256\theta^2+33\theta^2-a(4+12\theta+9\theta^2-3\theta^2)]_{a=0} = 4+16\theta+256\theta^2+33\theta^2-a(4+12\theta+9\theta^2-3\theta^2) > 0$. Therefore $p_m^F - p_m^{NA} > 0$. So $p_m^{NA} - p_m^A = 2(4+9\theta+9\theta^2) > 0$, which can be expressed as $\text{Min}[4+16\theta+256\theta^2-33\theta^2-a(4+12\theta+9\theta^2-3\theta^2)]_{a=0} = 4+16\theta+256\theta^2-33\theta^2-a(4+12\theta+9\theta^2-3\theta^2) > 0$.

Proof of Corollary 5.2. $\frac{q_m^{FA} - q_m^{NR}}{q_m^{FR} - q_m^{NR}} = \frac{4+8\theta+5\theta^2+\theta^2-a(4+4\theta-\theta^2-\theta^2)}{4(1+a)}$, the denominator is always positive, so we only need to investigate the sign of the numerator. The numerator decreases with $a$, and the numerator achieves its minimum value when $a = 0$, which can be expressed as $\text{Min}[4+8\theta+5\theta^2+\theta^2-a(4+4\theta-\theta^2-\theta^2)]_{a=0} = 4+8\theta+5\theta^2+\theta^2 > 0$. The minimum value is more than zero so that the numerator is positive, therefore $q_m^{FA} - q_m^{NR} > 0$.

$\frac{q_m^{FR} - q_m^{FR}}{q_m^{FR} - q_m^{FR}} = \frac{4+8\theta+5\theta^2+\theta^2-a(4+4\theta-\theta^2-\theta^2)}{4(1+a)}$, the denominator is always positive, so we only need to investigate the sign of the numerator. The numerator decreases with $a$, and the numerator achieves its minimum value when $a = 0$, which can be expressed as $\text{Min}[4+8\theta+5\theta^2+\theta^2-a(4+4\theta-\theta^2-\theta^2)]_{a=0} = 4+8\theta+5\theta^2+\theta^2 > 0$. The minimum value is more than zero so that the numerator is positive, therefore $q_m^{FA} - q_m^{NR} > 0$.

Proof of Corollary 5.3. The comparisons of the equilibrium profits for national brand are as follows:

$\Pi_m^{NR} - \Pi_m^{FR} = \frac{a + \theta + a\theta}{8(1+a)}(1+\theta) > 0$,

therefore $\Pi_m^{NR} > \Pi_m^{FR}$. And

$\Pi_m^{NA} - \Pi_m^{FA} = \frac{(1-\beta)H}{4(1+a)(4+8\theta+3\theta^2-\theta^2)}$, where $H = 16\theta+40\theta^2-8\theta^2+32\theta^3-16\theta^3+9\theta^4-6\theta^4+3\theta^3\theta^4-a^2[4\theta^2(1+\theta)] + a(16+48\theta+56\theta^2-8\beta\theta^2+32\theta^3+16\theta^3+9\theta^4-6\theta^4+3\theta^3\theta^4)$. The denominator is always positive and $(1-\beta) > 0$, we need to investigate the sign of $H$. $H$ is a concave function in terms of $a$, so the minimum value can be achieved when $a = 0$ and $a = 1$. The minimum value is either $H_{a=0} = 0$ or $H_{a=1} = 2$.

Therefore, $\Pi_m^{NA} - \Pi_m^{FA} > 0$. So $\Pi_m^{NA} > \Pi_m^{FA}$ and $\Pi_m^{NR} > \Pi_m^{FR}$ can be proved.
Proof of Proposition 5.4.

\[ \Pi^{\text{PR}}_J - \Pi^{\text{NR}}_J = \frac{-a + 4a^2 - \theta + 5a\theta + 8a^2\theta + 2\theta^2 + 6a\theta^2 + 4a^2\theta^2}{16(1 + a)(1 + \theta)(1 + 2\theta)} - F. \]

We derive \( F_1 = \frac{-a + 4a^2 - \theta + 5a\theta + 8a^2\theta + 2\theta^2 + 6a\theta^2 + 4a^2\theta^2}{16(1 + a)(1 + \theta)(1 + 2\theta)} \), the denominator is always positive, so we only need to investigate the sign of the numerator if we expect \( F_1 \) is positive. As we can see, \(-a + 4a^2 + (-1 + 5a + 8a^2)\theta + (2 + 6a + 4a^2)\theta^2 \) is a convex quadratic function of \( \theta \). We drive two roots from \(-a + 4a^2 + (-1 + 5a + 8a^2)\theta + (2 + 6a + 4a^2)\theta^2 = 0 \), one is \(-\frac{a}{1 + 4a} \), the other is \( \frac{1 - 4a}{2(1 + 2\theta)} \). Apparently, \(-\frac{a}{1 + 4a} \leq 0 \) if \( a \leq \frac{1}{4} \), then \( \frac{1 - 4a}{2(1 + 2\theta)} \geq 0 \). Therefore, when \( a \leq \frac{1}{4} \) and \( \theta \geq \frac{1 - 4a}{2(1 + 2\theta)} \), we have \( F_1 > 0 \); and when \( a > \frac{1}{4} \), \( \frac{1 - 4a}{2(1 + 2\theta)} \) is always negative, in this case we have \( F_1 > 0 \) regardless of \( \theta \). □

Proof of Corollary 5.5.

\[ \frac{\partial F_1}{\partial \theta} = \frac{L}{16(1 + a)(1 + \theta)^2(1 + 2\theta)^2}, \quad \text{where} \quad L = \left[-1 + 8a - 4a^2 + (4 + 16a - 8a^2)\theta + (8 + 8a - 4a^2)\theta^2\right]. \]

The denominator is always positive, so we only need to investigate the sign of the numerator. The numerator is convex quadratic function in \( \theta \).

1. When \( a \leq \frac{1}{4} \) and \( \theta \geq \frac{1 - 4a}{2(1 + 2\theta)} \):

\begin{align*}
L_{\theta=\frac{1 - 4a}{2(1 + 2\theta)}} &= \frac{3(1 - a)^2}{(1 + 2a)^2} > 0, \quad \text{and} \quad L_{\theta=1} = 11 + 32a - 16a^2 > 0.
\end{align*}

And \( \theta = \frac{-4a + 2a^2}{4 + 4a} \) is the function for axis of symmetry of numerator and it’s negative. \( \frac{-4a + 2a^2}{4 + 4a} \) \( < \frac{1 - 4a}{2(1 + 2\theta)} \) \( < 1 \), therefore, \( L \) is always positive when \( a \leq \frac{1}{4} \) and \( \theta \geq \frac{1 - 4a}{2(1 + 2\theta)} \).

2. And when \( a > \frac{1}{4} \):

\( L_{\theta=0} = -1 + 8a - 4a^2 \), as we can see \( -1 + 8a - 4a^2 \) is a concave quadratic function in \( a \) and \( a = 1 \) is the function for axis of symmetry of \( L_{\theta=0} \), therefore \( L_{\theta=0} \) increases in \( a \) when \( a \in (\frac{1}{4}, 1) \). The minimum value can be achieved when \( a = \frac{1}{4} \).\( \min(1 + 8a - 4a^2)_{a=\frac{1}{4}} = \frac{3}{8} > 0 \). So \( L_{\theta=0} \) is always positive when \( a > \frac{1}{4} \).

Combining with \( L_{\theta=1} > 0 \), we can conclude \( L \) is always positive when \( a > \frac{1}{4} \).

Therefore, \( F_1 \) increases with \( \theta \). Similarly, we could prove that \( F_1 \) increases with \( a \). □

Proof of Proposition 5.6. From \( \Pi^{\text{FA}}_J = \Pi^{\text{NA}}_J \), we derive

\[ F_2 = -\frac{\beta}{4} + \frac{-a^2(1 + \theta)(-4 - 8\theta + (-4 + \beta)\theta^2 - (1 + \theta)(-\theta^2 + \beta^2\theta^2 - 4\beta(1 + \theta)^2) + \beta\theta[4 + 8\theta + 3\theta(1 + \theta)^2]}{(1 + a)[4 + 8\theta - (3 + 3\beta)\theta^2]^2}. \]

To ensure that \( F_2 \) exists, we should confine that \( F_2(\beta) > 0 \), which is too tedious for us to obtain an explicit condition. □

Proof of Corollary 5.7. Since \( \Pi^{\text{NA}}_J[\beta = \beta_2] - \Pi^{\text{NR}}_J = \frac{1}{16} > 0 \), we can derive \( \Pi^{\text{NA}}_J[\beta = \beta_2] > \Pi^{\text{NR}}_J \).

By the use of Mathematica, we input \( \text{Resolve}[\exists{(a, \theta, F)}, \Pi^{\text{NA}}_J[\beta = \beta_1] - \Pi^{\text{NR}}_J < 0 \&\& \theta < 1 \&\& a < 1 \&\& 0 < F < \frac{1}{16}, \{a, \theta, F\}] \), and the output is “False”, which means there doesn’t exist \( \theta \in (0, 1) \), \( a \in (0, 1) \) and \( F \in (0, \frac{1}{16}) \) such that \( \Pi^{\text{NA}}_J[\beta = \beta_1] - \Pi^{\text{NR}}_J < 0 \) holds. And when we input \( \text{Resolve}[\exists{(a, \theta, F)}, \Pi^{\text{NA}}_J[\beta = \beta_1] - \Pi^{\text{NR}}_J > 0 \&\& 0 < \theta < 1 \&\& 0 < a < 1 \&\& 0 < F < \frac{1}{16}, \{a, \theta, F\}] \), and the output is “true”, which means when \( 0 \leq \theta \leq 0.1 \), \( a \in (0, 1) \) and \( F \in (0, \frac{1}{16}) \) such that \( \Pi^{\text{NA}}_J[\beta = \beta_1] > \Pi^{\text{NR}}_J \) can always hold. Therefore, \( \Pi^{\text{NA}}_J > \Pi^{\text{NR}}_J \). □
Acknowledgements. On behalf of all authors, the corresponding author states that there is no conflict of interest. This research was partly supported by National Natural Science Foundation of China (71971134, 72072111), Shanghai Soft Science Project (22692110900), Shanghai Pujiang Program (2020PJ066).

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