

BILATERAL “PRICING” FOR CONSULTATION COMPANIES’ COMPETITION CONSIDERING CUSTOMERS’ SWITCHING BEHAVIOR

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Abstract. Customers are either eager or obliged to switch from a physical consultation firm to an online consultation platform. Considering customers’ switching behavior and competition effect on bilateral users, the paper developed a three-echelon Stackelberg game framework where two consultation companies decide bilateral “pricing” (the service price and the wage), and the servicer decides service investment. In four scenarios combining two salary strategies (Employment or Sharing Strategy) and two game sequences (firm as leader or follower), stakeholders’ optimal decision-making and revenue were proved. We then theoretically analyzed the influence of some parameters on the equilibrium status and the gap between them and numerically simulated the joint effect of switching rate and parameters on the equilibrium revenue of stakeholders. It is deduced that in the consultation service market: (a) there is equilibrium status in all four scenarios when the switching rate, the servicer’s sensitivity to current revenue, and the service cost coefficient on the firm satisfy certain conditions; (b) the servicer’s potential market and the service investment cost coefficients negatively affect the optimal fixed wage set by the two companies and the servicer’s revenue; (c) two consultation companies can achieve greater revenue when both adopt Sharing Strategy; (d) servicer with higher perceived service cost are incited to devote more to servicing when both employ Employment strategy.

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1. INTRODUCTION

As a tertiary sector, the service sector has attracted increased funding and attention because it is a significant contributor to most countries’ economic and social development. The broad adoption of Internet information technology, particularly mobile technology, has recently led to substantial changes in the traditional consultation services industry as well as new channels and customer touchpoints. Because of the advantages of information technology, many platforms in the healthcare (Good Doctor Online), legal (Lawyer 365), accounting (hrblock.com), housing (Lianjia), and other consulting (support.com) industries play an essential role in daily life. They are widely adopted by the general public [1]. The consultation platforms provide a virtual place for convenient communication between servicers and customers at anytime and location. Due to the limited resources of bilateral users, the physical consultation firm is experiencing intense competition in the market for consultation services, where various consultation platforms feed into.

Keywords. Consultation platform, Service pricing, Salary strategy, Customer switching, Service investment.

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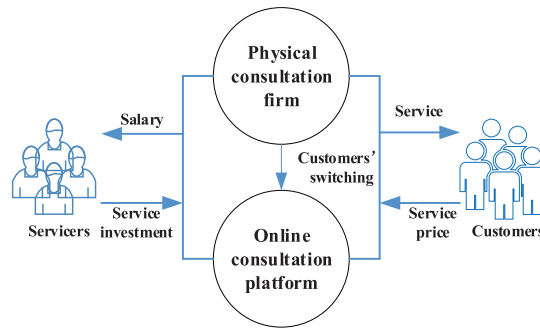


FIGURE 1. The bilateral “pricing” structure of consultation companies under customers switching.

Real-time customer switching across service systems can increase service effectiveness and reduce service capacity costs [2]. In fact, it’s interesting that some customers have switched from a physical consultation firm to an online one due to the pricing differential between the two types of consultation companies, their exploratory behavior, and other environmental considerations [1, 3]. For instance, consumers are drawn to the lower prices of online services and tend to terminate using services from a physical firm and switch to an online platform. Under COVID-19, there were a number of unforeseen issues in physical hospitals, including a lack of medical resources and cross-infection, which encouraged the use of healthcare platforms. As a result, physical hospitals had to close temporarily. Patients must switch to the healthcare platform to get the necessary medical services, such as consultation, diagnosis, re-examination, etc. [4, 5]. The demand market for physical consultation firm is obviously further diminished by customer behavior that switches from the firm to the platform.

Bilateral “pricing” is typically adopted by consultation companies (both the physical consultation firm and the online consultation platform) to control service supply and demand to survive intense competition effectively. Specifically, consultation companies compete for the servicer and consumer resources by paying salary and charging service price, referred to as bilateral “pricing.” To attract servicer resources, consultation companies often employ one of two salary strategies: the Employment or Sharing Strategy. With the Employment Strategy, consultation companies pay a fixed wage for recruiting servicers for a set period (a month or a year) to guarantee more reliable serviceability [6]. However, with the Sharing Strategy, consultation companies divide a portion of the revenue from each service transaction among the servicers engaged [5], which can draw in a lot of idle servicers. Additionally, motivated by wages, servicers devote time and experience to helping customers who need guidance and assistance [7]. For example, physicians invest their time and expertise in the medical field to provide medical services to patients who are impacted by contacting price, and they receive wages from medical facilities; attorneys invest their time and expertise in the legal field to provide legal services to clients who are impacted by charging attorney fee, and they receive a wage from law firms; accountants invest their time and expertise in the financial sector to provide financial services to investors who are impacted by contacting price, and they receive a wage from investment institutions; experts devote their own time and expertise to delivering consultation services for callers who are affected by negotiating consultation fees, and they receive a wage from consultation firms [1]. Figure 1 depicts two consultation companies’ bilateral “pricing” structure under customers’ switching. Consultation companies should jointly decide on salary and service price to maximize their revenue, and then servicers need to optimize service investment in consultation companies to pursue revenue.

Under the competitive consultation service market and customers’ switching behavior, bilateral “pricing” of consultation companies has become a challenging issue. And questions arise: what is the optimal decision for the two consultation companies and servicer in different market scenarios combining salary strategies and decision sequences? How do crucial factors affect stakeholders’ optimal decisions, revenues, and differences? How does customers’ switching behavior impact the ultimate revenue of stakeholders? To address these questions, we develop a three-echelon Stackelberg game model for competitive bilateral “pricing” of the two consultation

companies where customers switch from the physical consultation firm to the online consultation platform in period 2. Considering two salary strategies (the Employment Strategy or the Sharing Strategy) and two decision sequences (firm as the leader, platform as the follower, or platform as leader, firm as follower), the bilateral “pricing” model for consultation companies’ competition is proposed in the study. We proved the optimal decision-making and equilibrium revenue of stakeholders, compared the differences between them, and analyzed the influence of crucial parameters, particularly the combined effect of customer switching behavior and relevant factors.

The following are innovations in this paper: (a) we have complemented the relevant literature on service pricing by studying bilateral “pricing” in the competitive consultation service market. (b) This study’s primary concern is the impact of consumers’ switching behavior and salary strategy on the bilateral “pricing” of two consultation companies. The study’s findings can be utilized as guidance for the operation optimization of two consultation companies when deciding salary strategies and making bilateral price decisions. (c) The optimal decision-making and revenue of stakeholders (firm, platform, and servicer) and crucial factors’ impact were studied in four market scenarios, which can provide stakeholders with a theoretical foundation and decision assistance.

The remainder of the paper is laid out as follows. Section 2 reviews three streams of related literature. Section 3 presents assumptions and defines the demand of bilateral users on two consultation companies with different salary strategies. Section 4 theoretically analyzes the condition of the equilibrium status, the optimal decisions, and the revenue of stakeholders in four scenarios. Section 5 carries out a comparative analysis from the salary strategy and the game sequence. Section 6 numerically analyzes the combined effect of the switching rate and some crucial parameters on stakeholders’ revenue. Finally, Section 7 highlights some of the findings and discusses future research.

2. LITERATURE REVIEW

This section sheds light upon three streams of related literature: service pricing, bilateral “pricing” for service companies, and customers’ switching between service companies.

2.1. Service pricing

There is extensive literature on service pricing that studies the optimal service price and equilibrium revenue. Zhang *et al.* developed the price and quality equilibrium methods for data communication network services when two rivals offer multiple classes of priority-based services [8]. Chun and Choi analyzed the optimal pricing for cloud computing services from the standpoint of cloud computing service providers’ strategies: membership and pay-per-use [9]. Based on the two-stage non-cooperative game model, Do *et al.* investigated the competitive price between two heterogeneous cloud computing service providers. They determined the sufficient conditions for the existence and uniqueness of the Nash equilibrium of the optimal price [10]. Considering drivers’ choice of working hours, a bi-level optimization model of peak-hour pricing was constructed, and Zha *et al.* calculated the optimal online riding service price [7]. Liu *et al.* presented the optimal prices of the basic service and the additional service for the O2O platform, revealing key factors’ role in pricing decisions [11]. Tanwar *et al.* studied the revenue-risk trade-off for a healthcare service provider under a fixed-price contract and showed analytically that the optimal risk-adjusted model package prices reside between the efficient pricing interval [12]. Considering the customers’ aversion to waiting and the server’s vacation(s), Tian and Wang studied the equilibrium participation strategy of customers and the optimal service price of the single-server system [13].

Previous works on service pricing mainly concentrate on cloud computing and riding services; however, they rarely mention consultation service and ignore price competition for customers’ resources. In addition, little literature involves customers’ switching behavior and integrates the bilateral side competition. Our work contributes to the stream’s literature by studying the bilateral “pricing” for consultation companies considering customers’ switching.

2.2. Bilateral “pricing” for service companies

An important stream of the service company operation literature is concerned with bilateral “pricing”: the service price and the wage. Taylor researched how on-demand service platforms should set service price and wage [14]. To provide the optimal service price and wage rate, Bai *et al.* developed a monopoly service platform bilateral “pricing” model from the perspective of users’ utility [15]. Benjaafar *et al.* provided insights into how a monopoly service platform makes service price and wage rate decisions when independent servicers decide how much time to serve for the platform [16]. Under the registration fee and the transaction fee model, Li *et al.* jointly optimized the fee for doctors entering the monopoly healthcare platform and the drug price for patients and revealed that the registration fee model is superior to the transaction fee model in terms of the platform’s revenue [5].

Considering the bilateral congestion effect, Bernstein *et al.* compared the equilibrium price, the wage rate, and the revenue of ride-sharing platforms under drivers’ single-homing behavior and multi-homing behavior [17]. Siddiq and Taylor considered the impact of autonomous vehicles’ ownership on the service supply, built a two-stage bilateral “pricing” model of duopoly ride-hailing platforms competition, and suggested favorable conditions for the platform benefits [6]. Given the duopoly service platforms competition, Bai and Tang adopted a two-stage Stackelberg game model to select the equilibrium service price and wage rate and found that only one platform sustains profitability when the two platforms are symmetrical [18].

The literature mentioned above has been carried out from the perspective of user utility and assumes that service companies only adopt Sharing Strategy for the servicer side, but most ignore the servicer’s investment decisions. As far as we know, there is rare research on bilateral “pricing” of consultation companies’ competition taking servicers’ investment decisions into account.

2.3. Customers’ switching between service companies

Numerous studies about customers’ switching have examined the factors of customer switching in the power market [19], telecommunication industry [20], retail bank [21], restaurant [22], and online takeout service [23] by empirically analyzing general survey data. Taking banking services in China as an example, Lu *et al.* claimed that elements such as motivators and the inhabit that influence consumers’ propensity to switch their usage [24]. Meanwhile, there is a growing literature that studies referrals in the healthcare system [25, 26] or demands referrals in the online market [27, 28].

However, scholars have paid little attention to customers switching between service companies. Li *et al.* used a two-stage game to calculate the steady-state performance of a service system composed of two service queues and investigate their service and pricing competition, in which customers can choose a service queue and switch between queues based on the spot utility [2]. Cheng *et al.* studied the optimal bilateral “pricing” decision of a monopoly video platform, further considering the multi-homing behavior of advertisers, and derived the optimal pricing decision [29]. Regarding multi-homing behavior as switching behavior, Athey *et al.* analyzed the impact of consumers’ switching behavior between publishers on publisher’s advertising price, profit, and content strategy. They pointed out that consumers’ switching behavior reduces advertiser’s expectation of advertising value [30]. Regarding the issue of expert service priced by a servicer, Akçura and Ozdemir, and Yang *et al.* took into account the behavior that customers switching to offline service after experiencing the online service and studied the optimal online and offline service price and servicer’s revenue [1, 31].

To our knowledge, the aforementioned research on customers switching between service companies is rarely concerned with bilateral “pricing” issues. In our work, customers’ switching behavior is emphasized. Meanwhile, we conduct our study with the competition between the online consultation platform and the physical consultation firm. We try to figure out how the customers’ switching behavior affects the two consultation companies’ bilateral “pricing” decision-making and revenue.

3. ASSUMPTIONS AND MODELLING

This section presents basic assumptions and constructs the bilateral pricing model.

3.1. Assumptions

- (a) To simplify our model, two consultation companies (the physical consultation firm and the online consultation platform) only operate the same single type of paid consultation service in the market, referring to [1].
- (b) Both consultation companies who are homogenous ex-ante and ex-post adopt the same salary strategy to carry out bilateral “pricing” competition at the same period. And the bilateral “prices” remains unchanged in two-period operations.
- (c) Customers and servicers in the competition consultation service market are homogeneous. And all servicers pursue revenue maximization by devoting themselves to consultation companies.
- (d) The unit invested by servicers can provide unit consultation service on the online platform or physical firm, which is indifferent to customers.
- (e) All stakeholders, including the online platform, the physical firm, and the servicers, are risk-neutral and have complete consultation service market information in the decision-making process.

3.2. Modeling

3.2.1. Consultation service demand

Consultation service demand for customers is divided into two periods considering customers’ switching from the physical firm to the online platform. In this context, let f and m denote the physical consultation firm and the online consultation platform, respectively.

In period 1, customers’ service demand on $i, i \in \{f, m\}$ consultation company is a linear function of potential customer market $Q (Q > 0)$, service price, and prices competition [6, 32], noted as q_{i1} .

$$\begin{aligned} q_{f1} &= \eta Q - \alpha p_f + \beta p_m, \\ q_{m1} &= (1 - \eta)Q - \alpha p_m + \beta p_f, \end{aligned}$$

where η ($0 < \eta < 1$) is the consumer’s preference for the physical firm [33, 34]. α, β ($0 < \alpha, \beta$) represent the elasticity coefficients of the consultation service price and price competition, respectively.

In period 2, partial b ($0 < b < 1$) customers’ service demand on the physical firm flows into the online platform. Consumers’ service demand on i consultation company is noted as q_{i2} .

$$\begin{aligned} q_{f2} &= (1 - a_f)q_{f1} - bq_{f1}, \\ q_{m2} &= (1 - a_m)q_{m1} + bq_{f1}, \end{aligned}$$

where a_f, a_m ($0 < a_f, a_m < 1$) represent the customer churn from the physical firm and the online platform, respectively.

3.2.2. Consultation service provision

To attract servicer resources, both consultation companies provide a fixed wage for the servicers when adopting the Employment Strategy while sharing the revenue of each service transaction with the servicers when adopting the Sharing Strategy.

(1) Employment Strategy

Under the Employment Strategy, the provision of servicers devoting to $i, i \in \{f, m\}$ consultation company is a linear function of potential servicers’ market M ($M > 0$), service investment ($T_f, T_m > 0$), fixed wage

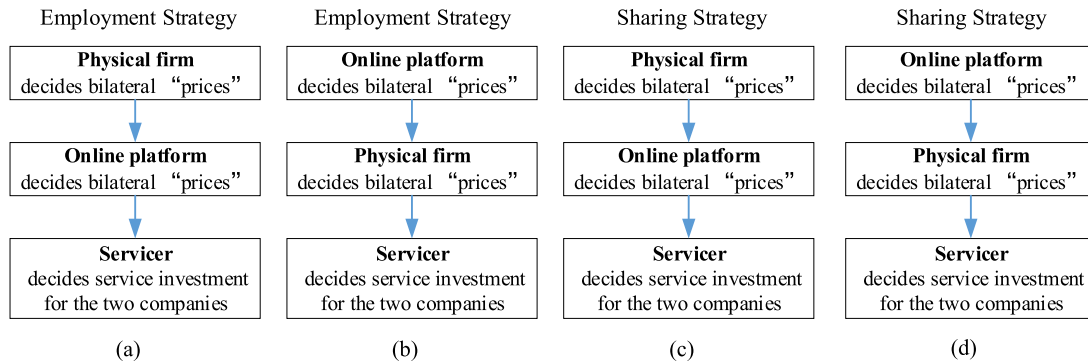


FIGURE 2. The frameworks of four Stackelberg game scenarios. (a) Scenario 1: the physical firm as the game leader, following the online platform, and the servicer as the final gamer under the Employment Strategy; (b) Scenario 2: the online platform as the game leader, following the physical firm, and the servicer as the final gamer under the Employment Strategy; (c) Scenario 3: the physical firm as the game leader, following the online platform, and the servicer as the final gamer under the Sharing Strategy; (d) Scenario 4: the online platform as the game leader, following the physical firm, and the servicer as the final gamer under the Sharing Strategy.

($w_f, w_m > 0$), and wage competition. When both consultation companies adopt the Employment Strategy, the servicers' provision on i consultation company is noted d_f^{E-E}, d_m^{E-E} , respectively.

$$d_f^{E-E} = vM + \gamma w_f - \rho T_f - \sigma w_m,$$

$$d_m^{E-E} = (1 - v)M + \gamma w_m - \rho T_m - \sigma w_f,$$

where $v(0 < v < 1)$ is the servicer's preference for the physical firm. $\gamma, \rho, \sigma(0 < \gamma, \rho, \sigma)$ represent the elasticity coefficients of fixed wage, service investment, and wage competition, respectively.

(2) Sharing Strategy

Under the Sharing Strategy, the provision of servicers devoting to $i, i \in \{f, m\}$ consultation company is a linear function of potential servicers' market $M (M > 0)$, service investment ($T_f, T_m > 0$), sharing revenue ($\theta_f p_f, \theta_m p_m > 0$) [7], and sharing revenue competition. When both consultation companies adopt the Sharing Strategy, the servicers' provision on i consultation company is noted d_f^{S-S}, d_m^{S-S} , respectively.

$$d_f^{S-S} = vM + \lambda \theta_f p_f - \rho T_f - \sigma \theta_m p_m,$$

$$d_m^{S-S} = (1 - v)M + \lambda \theta_m p_m - \rho T_m - \sigma \theta_f p_f,$$

where θ_f, θ_m represent the sharing rate of each consultation service transaction declared by the firm and the platform, respectively. $\lambda (\lambda > 0)$ is the elasticity coefficient of sharing revenue.

In reality, both consultation companies disclose wages, sharing rates, and other information before recruitment servicers and represent service prices for customers in advance. Therefore, the physical firm or the online platform first sets bilateral "prices" to maximize their total revenue, and servicers finally make service investment decisions on the two companies. There are four game scenarios by combing two decision sequences (the physical firm as game leader, the online platform and servicers as game followers, the online platform as game leader, the physical firm and servicers as game followers) and two salary strategies (Employment Strategy and Sharing Strategy), as Figure 2 shows.

4. THEORETICAL ANALYSIS

We theoretically proved the optimal decisions and the revenue of stakeholders in 4 Stackelberg game scenarios. Meanwhile, the effect of some parameters on equilibrium status is analyzed.

4.1. Employment strategy

4.1.1. Scenario 1

In scenario 1, both consultation service companies adopt the Employment Strategy for attracting servicers. The physical firm initially set the consultation service price p_f and fixed wage w_f for bilateral users. Then, the online platform, as the follower charges bilateral “prices” p_m, w_m to compete with the firm. Servicer finally decides how much to invest T_f, T_m in the two consultation companies.

The total service investment cost of the servicer on consultation company i is a quadratic function of service investment [16,32]. In scenario 1, the servicer’s revenue is denoted as

$$\pi_S^{E-E} = w_m + w_f - \frac{c_f T_f^2}{2} - \frac{c_m T_m^2}{2}, \tag{1}$$

where c_i ($c_i > 0$) represents the service investment cost coefficient on consultation company i , which is impacted by the company’s operational environment. The total service investment cost of the servicer on consultation company i is $\frac{c_i T_i^2}{2}$.

Customer demand can bring benefits, while hiring servicers is the operating cost for two consultation companies. Without loss of generality, the other costs of the two companies are assumed to be 0 [5, 6, 30]. In scenario 1, the revenue of both consultation companies are

$$\pi_f^{E-E} = (q_{f1} + q_{f2})p_f - w_f d_f^{E-E}, \tag{2}$$

$$\pi_m^{E-E} = (q_{m1} + q_{m2})p_m - w_m d_m^{E-E}. \tag{3}$$

Proposition 1. *If $0 < b < \min\{b_1, 1\}$ and $2\gamma^2 > \sigma^2$ (condition 1), there is an equilibrium in scenario 1. The optimal bilateral “pricing” decisions of the physical firm are*

$$p_f^1 = \frac{Q(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1 - \eta) + (4 + b)\eta - 4))}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)},$$

$$w_f^1 = \frac{c_m\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2\sigma A_{m2} - c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2))}{16c_m c_f \gamma^2(\sigma^2 - 2\gamma^2)^2},$$

where $b_1 = \frac{A_{m1}\beta}{\alpha} - \frac{2A_{m1}\alpha}{\beta}$, $A_{m1} = -2 + a_m < 0$, $A_{m2} = 2\gamma + \sigma > 0$.

The optimal bilateral “pricing” decisions of the online platform are

$$p_m^1 = \frac{Q(2A_{m1}\alpha + (4 - 2a_m - b)\alpha\eta + A_{m1}\beta\eta)}{4(A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2)},$$

$$w_m^1 = \frac{c_m\rho^2\sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2 A_{m2} + c_m M(2(v - 1)\gamma - v\sigma)(2\gamma^2 - \sigma^2))}{8c_m c_f \gamma(\sigma^2 - 2\gamma^2)^2}.$$

The optimal service investment of the servicer on two consultation companies are

$$T_f^1 = \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_f\gamma^3 - 4c_f\gamma\sigma^2}, \quad T_m^1 = \frac{\rho(2\gamma + \sigma)}{4c_m\gamma^2 - 2c_m\sigma^2}.$$

In scenario 1, by substituting the optimal decisions of stakeholders into equations (1)–(3), the optimal revenue of the physical firm, the online platform, and the servicer are obtained as follows.

$$\begin{aligned} \pi_f^1 &= \frac{1}{256} \left(\frac{16(2 - a_f - b)(2A_{m1}Q\alpha\beta + Q(4A_{m1}\alpha^2 + (4 - 2a_m + b)\alpha\beta - (A_{m1}\beta^2)\eta))^2}{\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} \right. \\ &\quad \left. + \frac{(c_m\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2\sigma A_{m2} - c_mM(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2)))^2}{c_m^2c_f^2\gamma^3(-2\gamma^2 + \sigma^2)^4} \right), \\ \pi_m^1 &= \frac{1}{2} \left(\frac{(c_m\rho^2\sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2A_{m2} + c_mM(2(v - 1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))^2}{64c_m^2c_f^2(2\gamma^3 - \gamma\sigma^2)^3} \right. \\ &\quad \left. - \frac{Q^2(2A_{m1}\alpha - (-4 + 2a_m + b)\alpha\eta + A_{m1}\beta\eta)^2}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} \right), \\ \pi_s^{11} &= \frac{c_m\rho^2(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2 + 4c_f\gamma(\gamma\rho^2A_{m2}^2 - 2c_mM(2\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - v\sigma^2))}{32c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2}. \end{aligned}$$

Please see Appendix A.1 for the proof of Proposition 1.

Corollary 1. *In scenario 1, if $\max\{b_2, b_3, b_4\} < b < \min\{1, b_5\}$, $v_1 < v < \min\{1, v_2\}$, $0 < M < M_1$, $2\gamma > \sigma$ (condition 2), then it can characterize the impact of key parameters on the optimal bilateral “pricing” where*

$$\begin{aligned} b_2 &= \frac{A_{m1}(2\alpha(1 - \eta) + \beta\eta)}{\alpha\eta}, & b_3 &= \frac{2A_{m1}(2\eta - 1)}{\eta}, \\ b_4 &= \frac{A_{m1}(2\alpha\beta(\eta - 1) + \beta^2\eta - 4\alpha^2\eta)}{\alpha\beta\eta}, & b_5 &= A_{m1} \left(\frac{\beta}{\alpha} - 2 \right), \\ v_1 &= \frac{\gamma(c_mM(4\gamma^2 - 2\sigma^2) - \rho^2A_{m2})}{c_mM(2\gamma - \sigma)(2\gamma^2 - \sigma^2)}, & v_2 &= \frac{2\gamma(2\gamma - \sigma)}{8\gamma^2 - 4\gamma\sigma - \sigma^2}, \\ M_1 &= \frac{\gamma\rho^2\sigma A_{m2}}{c_m(2\gamma^2 - \sigma^2)(4v\gamma^2 - 2(v - 1)\gamma\sigma - v\sigma^2)}. \end{aligned}$$

- (I) For consultation service pricing, the customers’ potential market (Q), the service price competition coefficient (β), the churn rate of the platform (a_m), and the customers’ switching rate (b) have a positive effect on the optimal service price of two consultation companies. Customer’s sensitivity to service price (α) and preference for the firm (η), however, negatively affect the optimal service price set by the platform.
- (II) For fixed wage, the servicer’s sensitivity to service investment (ρ) positively affects the optimal fixed wage provided by two consultation companies, while the servicers’ potential market (M) and the service investment cost coefficients (c_f, c_m) have a negative effect. The servicer’s preference for the physical firm (v) has the opposite effect on the optimal fixed wage provided by the firm and the platform, respectively. The former has a negative relationship, whereas the latter has a positive one.

(III) The customers’ potential market (Q) plays a positive role in the equilibrium revenue of two consultation companies. The churn rate of two consultation companies (a_f, a_m) and the service price competition (β) negatively affect the firm’s equilibrium revenue. The switching rate (b) and the preference of customers (η) for the firm negatively influence the platform’s equilibrium revenue. Moreover, the servicer’s sensitivity to service investment (ρ), the service investment cost coefficients (c_f, c_m), and the servicers’ potential market (M) have the same impact on the stakeholder’s equilibrium revenue as it has on the optimal wage set by the platform.

Please see Appendix A.2 for the proof of Corollary 1.

Corollary 1 shows that, the bigger service cost coefficients mean the servicer invests more in service companies when there is an equilibrium status in scenario 1, where the customers’ switching rate and the scale of potential servicers who prefer the firm are moderate. To attract servicers, the two consultation companies need to compensate servicers by increasing the fixed wage. Due to the increased potential of the servicers, the service supply is essentially sufficient, and the two companies can hire the servicers at a lower wage. Faced with great potential demand, consultation companies tend to charge higher service prices to achieve higher operational revenue. At the same time, the influence of these factors on bilateral “pricing” will be transmitted to the revenue of two companies through bilateral market operations. It is interesting noting that although factors such as the service price competition, the churn rate in the platform, and the customers’ switching rate have a favorable impact on the optimal service price set by two consultation companies, they have the exact opposite effect on the equilibrium revenue of the two firms. This indicates that raising service price arbitrarily by adjusting these factors cannot boost operational revenue.

Corollary 2. *Based on scenario 1 and Corollary 1, with the increase of service investment cost coefficients (c_f, c_m), the optimal service investment gap between two consultation companies will widen. The price gap is positively widened by the churn rate of the platform (a_m), switching rate (b), and customers’ potential market (Q). The servicers’ potential market (M) and preference for the firm (v) positively widen the fixed wage gap, while the service investment cost coefficient on the platform (c_m) plays a negative role. Moreover, the churn rate of the firm (a_f) has a positive impact on the equilibrium revenue gap.*

Proof. Based on scenario 1 and Corollary 1, with $T_m^1 - T_f^1 = \frac{4c_f\gamma^2\rho - 4c_m\gamma^2\rho + 2c_f\gamma\rho\sigma - 2c_m\gamma\rho\sigma + c_m\rho\sigma^2}{8c_m c_f \gamma^3 - 4c_m c_f \gamma \sigma^2}$, we derive

$$\frac{\partial T_m^1 - T_f^1}{\partial c_m} = \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{4c_f^2(2\gamma^3 - \gamma\sigma^2)} > 0, \quad \frac{\partial T_m^1 - T_f^1}{\partial c_f} = \frac{\rho A_{m2}}{2c_m^2(\sigma^2 - 2\gamma^2)} > 0.$$

With $p_m^1 - p_f^1 = \frac{Q(-A_{m1}\beta^2\eta + \alpha\beta(-4 + a_m(2 - 4\eta) + (8 + b)\eta) + 2\alpha^2(4 + (b - 8)\eta + a_m(4\eta - 2)))}{4\alpha(A_{m1}\beta^2 - 2A_{m1}\alpha^2 - b\alpha\beta)} > 0$, we obtain

$$\begin{aligned} \frac{\partial p_m^1 - p_f^1}{\partial a_m} &= \frac{bQ\alpha(2\alpha - \beta)(\beta + \alpha\eta - \beta\eta)}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, & \frac{\partial p_m^1 - p_f^1}{\partial b} &= -\frac{A_{m1}Q\alpha(2\alpha - \beta)(\beta + \alpha\eta - \beta\eta)}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \\ \frac{\partial p_m^1 - p_f^1}{\partial Q} &= \frac{A_{m1}\beta^2\eta - 2\alpha^2(4 + (b - 8)\eta + a_m(4\eta - 2)) + \alpha\beta(4 - (8 + b)\eta + a_m(4\eta - 2))}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0. \end{aligned}$$

With $w_m^1 - w_f^1 = \frac{c_m\rho^2(12\gamma^2\sigma^2 - \sigma^4 - 16\gamma^4) + 4c_f\gamma(\gamma\rho^2(4\gamma^2 - \sigma^2) + c_mM(2\gamma^2 - \sigma^2)((8v - 4)\gamma^2 + (2 - 4v)\gamma\sigma - v\sigma^2))}{16c_m c_f (\gamma\sigma^2 - 2\gamma^3)^2}$, we determine

$$\begin{aligned} \frac{\partial w_m^1 - w_f^1}{\partial c_m} &= \frac{\rho^2(\sigma^2 - 4\gamma^2)}{4c_m^2(\sigma^2 - 2\gamma^2)^2} < 0, & \frac{\partial w_m^1 - w_f^1}{\partial M} &= \frac{(8v - 4)\gamma^2 + (2 - 4v)\gamma\sigma - v\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} > 0, \\ \frac{\partial w_m^1 - w_f^1}{\partial v} &= \frac{M(8\gamma^2 - 4\gamma\sigma - \sigma^2)}{8\gamma^3 - 4\gamma\sigma^2} > 0. \end{aligned}$$

With $\pi_m^1 - \pi_f^1 = -\frac{Q^2(2A_{m1}\alpha - (-4+2a_m+b)\alpha\eta + A_{m1}\beta\eta)^2}{8\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} + \frac{1}{128c_f^2c_m^2(2\gamma^3 - \gamma\sigma^2)^3}(c_m^2\rho^4\sigma^2(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2 + 16c_f^2\gamma^2(\gamma^2\rho^4A_{m2}^2 + c_m\gamma\rho^2A_{m2}((4M(\rho - 1) - 2)\gamma - (1 + 2M\rho)\sigma)(2\rho^2 - \sigma^2) + c_m^2M(\sigma^2 - 2\gamma^2)^2(4(2 + M(v - 1)^2)\gamma^2 + 4(1 - M(v - 1)v)\gamma\sigma + v(Mv - 2)\sigma^2)) - 4c_m c_m \gamma \rho^2(4\gamma^2 + 2\gamma\sigma - \sigma^2)(-2\gamma\rho^2\sigma A_{m2} + c_m(2\gamma^2 - \sigma^2)(4\gamma^2 + 2(1 + 2M - 2Mv)\gamma\sigma + (2Mv - 1)\sigma^2)))$, we calculate

$$\frac{\partial\pi_m^1 - \pi_f^1}{\partial a_f} = \frac{(2A_{m1}Q\alpha\beta + Q(4A_{m1}\alpha^2 + (b - 2A_{m1})\alpha\beta - A_{m1}\beta^2)\eta)^2}{16\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0.$$

□

Corollary 2 reveals that, when there is an equilibrium status in scenario 1, there are significant differences in the optimal bilateral “pricing” and equilibrium revenue between two consultation companies. The service investment gap between the two companies widens as the platform service environment improves, while the wage gap provided by the two companies narrows. Thus, it suggests that the consultation platform should fully leverage its performance advantages to improve competitiveness.

4.1.2. Scenario 2

Like scenario 1, two consultation companies adopt the Employment Strategy to recruit servicers. Nevertheless, the online platform initially set the consultation service price p_m and the fixed wage w_m for bilateral users. Then, the physical firm as the follower, charges bilateral “prices” p_f, w_f to compete with the platform. Servicer ultimately decides how much to invest T_f, T_m in the two consultation companies.

Proposition 2. *If $\max\{b_1, 0\} < b < \min\{-\frac{A_{m1}\alpha}{\beta}, 1\}$ and $2\gamma^2 > \sigma^2$ (condition 3), there is an equilibrium status in scenario 2. The optimal bilateral “pricing” decisions of the online platform are*

$$p_m^2 = \frac{Q(-2A_{m1}\alpha^2(4 + a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(1 - \eta) + 2A_{m1}^2\eta - b^2\eta) + A_{m1}\beta^2(2 - a_m + (a_m + 3b - 2)\eta))}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)},$$

$$w_m^2 = \frac{4c_m\gamma^2\rho^2\sigma A_{m2} + c_f\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_m c_f M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma + (1 - v)\sigma^2)}{16c_m c_f \gamma^2(\sigma^2 - 2\gamma^2)^2}.$$

The optimal bilateral “pricing” decisions of the physical firm are

$$p_f^2 = \frac{Q(2A_{m1}\alpha\eta + \beta(a_m - a_m\eta + (2 + b)\eta - 2))}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2},$$

$$w_f^2 = \frac{\rho^2(8c_m\gamma^3 + 4(c_f + c_m)\gamma^2\sigma + 2c_f\gamma\sigma^2 - c_f\sigma^3)}{4c_m c_f \gamma(2\gamma^2 - \sigma^2)(4\gamma^2 - 2\sigma^2)} + \frac{M(v - 1)\sigma - 2Mv\gamma}{4c_m c_f \gamma(2\gamma^2 - \sigma^2)(4\gamma^2 - 2\sigma^2)}.$$

The optimal service investment of the servicer on two consultation companies are

$$T_f^2 = \frac{\rho A_{m2}}{4c_f\gamma^2 - 2c_f\sigma^2}, \quad T_m^2 = \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_m\gamma^3 - 4c_m\gamma\sigma^2}.$$

In scenario 2, by substituting the optimal decisions of stakeholders into equations (1)–(3), the optimal revenue of the physical firm, the online platform, and the servicer are obtained as follows.

$$\pi_f^2 = \frac{(-2 - a_f - b)Q^2(2A_{m1}\alpha\eta + \beta(-2 + a_m - a_m\eta + (2 + b)\eta))^2}{8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}$$

$$\begin{aligned}
 & + \frac{(-4c_m\gamma^2\rho^2A_{m2} + 4c_m c_f M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f\rho^2\sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma))^2}{128c_f^2c_m^2(2\gamma^3 - \gamma\sigma^2)^3}, \\
 \pi_m^2 = & \frac{1}{2} \left(- \frac{Q^2(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))^2}{8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} \right. \\
 & \left. + \frac{(4c_m\gamma^2\rho^2\sigma A_{m2} + c_f\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_m c_f M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2))^2}{128c_m^2c_f^2\gamma^3(\sigma^2 - 2\gamma^2)^4} \right), \\
 \pi_f^2 = & \frac{4c_m\gamma^2\rho^2A_{m2}^2 + c_f\rho^2(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2 - 8c_m c_f M\gamma(2\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma + (v - 1)\sigma^2)}{32c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2}.
 \end{aligned}$$

Please see Appendix A.3 for the proof of Proposition 2.

Corollary 3. *In scenario 2, if $\max\{b_6, b_7, -\frac{A_{m1}\alpha}{4\beta}\} < b < \min\{b_8, b_9, 1\}$, $\alpha > \beta$, $2\gamma > \sigma$, and $M_2 < M < M_3$ (condition 4), then it can reveal the impact of key parameters on the optimal bilateral “pricing” where*

$$\begin{aligned}
 b_6 &= \frac{A_{m1}(\beta(\eta - 1) - 2\alpha\eta)}{\beta\eta}, & b_7 &= A_{m1} - \frac{2A_{m1}\alpha}{\beta}, \\
 b_8 &= \frac{A_{m1}((2\alpha^2 + \beta^2)(\eta - 1) - 4\alpha\beta\eta)}{\beta^2\eta}, & b_9 &= \frac{A_{m1}(2\beta - 3\alpha)}{\beta}, \\
 M_2 &= \frac{\rho^2(8\gamma^3c_m + 4(c_m + c_f)\gamma^2\sigma + 2c_f\gamma\sigma^2 - c_f\sigma^3)}{4c_m c_f \gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2)}, \\
 M_3 &= \frac{\rho^2(2\gamma + \sigma)(4c_m\gamma^2\sigma + c_f(8\gamma^3 - 4\gamma\sigma^2 + \sigma^3))}{4c_m c_f \gamma(\sigma^2 - 2\gamma^2)((4\gamma^2 - \sigma^2)(v - 1) - 2v\gamma\sigma)}.
 \end{aligned}$$

- (I) Similar to scenario 1, the customers’ potential market (Q), the churn rate of the platform (a_m), and the service price competition (α) have a favorable impact on the optimal service price of the firm. The optimal service prices of two consultation companies are positively impacted by the customer’s preference for the firm (η) since the demand for consultation services of two companies rises due to the customers’ switching behavior.
- (II) The effect of the servicer’s sensitivity to service invested (ρ), the potential market (M), preference for the physical firm (v), and the service investment cost coefficients (c_f, c_m) on the optimal fixed wage in scenario 1 is the same as that in scenario 2.
- (III) The bilateral users’ potential market (Q, M) and the service investment cost coefficients (c_f, c_m) have the opposite impacts on the equilibrium revenue of the two companies, respectively. The former has a positive relationship, while the latter has a negative one. The churn rate of the platform (a_m) and the bilateral users’ preference for the firm (η, v) play a positive role in the firm’s equilibrium revenue. The firm’s equilibrium revenue will decrease as its churn rate (a_f) rises. The servicer’s preference for the firm (v) harms the servicer’s equilibrium revenue. The effect of other parameters is transmitted through the wage provided to the servicer’s revenue.

Please see Appendix A.4 for the proof of Corollary 3.

Corollary 3 shows that, when there is an equilibrium status in scenario 2, the customers’ switching rate and the servicer’s potential market are moderate, and the bilateral “price” competition is less intense, two consultation companies’ revenue exhibits different evolutionary trends with the increase of bilateral users’ potential market

and service cost coefficients, and the effect of critical parameters on the optimal wage is the same as in scenario 2. It indicates that although the change in game sequence significantly affects stakeholders' decision-making, it does not affect the sensitivity of the optimal wage to crucial parameters.

Corollary 4. *Based on scenario 2 and Corollary 3, the service investment gap between the two consultation companies will narrow when the service investment cost coefficients (c_f, c_m) increase. Customers strongly prefer the firm (η), which widens the optimal service price gap. The servicer's preference for the firm (v) and the service invested cost coefficient on the firm (c_f) positively expand the optimal fixed wage gap. Moreover, the customers' potential market (Q), the sensitivity to service investment (ρ), and the churn rate of the firm (a_f) play a positive role in the equilibrium revenue gap.*

Proof. Based on scenario 2 and Corollary 3, with $T_m^2 - T_f^2 = \frac{4c_f\gamma^2\rho - 4c_m\gamma^2\rho + 2c_f\gamma\rho\sigma - 2c_m\gamma\rho\sigma - c_f\rho\sigma^2}{8c_fc_m\gamma^3 - 4c_fc_m\gamma\sigma^2}$, we derive

$$\frac{\partial T_m^2 - T_f^2}{\partial c_m} = \frac{\rho(2\gamma + \sigma)}{2c_f^2(2\gamma^2 - \sigma^2)} < 0, \quad \frac{\partial T_m^2 - T_f^2}{\partial c_f} = \frac{\rho(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)}{4c_m^2(2\gamma^3 - \gamma\sigma^2)} < 0.$$

With $p_m^2 - p_f^2 = \frac{A_{m1}Q(4A_{m1}\alpha^2 + (3b - 2A_{m1})\alpha\beta - (A_{m1} + 2b)\beta^2) + Q(-2A_{m1}(4A_{m1} + b)\alpha^2 + (4A_{m1}^2 - 9A_{m1}b - b^2)\alpha\beta + (4 + a_m^2 - 2b(5 + b) + a_m(5b - 4))\beta^2)\eta}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}$, we obtain

$$\frac{\partial p_m^2 - p_f^2}{\partial \eta} = \frac{-(A_{m1} + b)Q}{4A_{m1}\alpha + 4b\beta} + \frac{(2A_{m1} - b)Q\beta - 3A_{m1}Q\alpha}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} > 0.$$

With $w_m^2 - w_f^2 = \frac{4c_m\gamma^2\rho^2(\sigma^2 - 4\gamma^2) + 4c_m c_f M\gamma(2\gamma^2 - \sigma^2)((8v - 4)\gamma^2 + (2 - 4v)\gamma\sigma - (v - 1)\sigma^2) + c_m\rho^2(16\gamma^4 - 12\gamma^2\sigma^2 + \sigma^4)}{16c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2}$, we determine

$$\frac{\partial w_m^2 - w_f^2}{\partial c_f} = \frac{\rho^2(4\gamma^2 - \sigma^2)}{4c_f^2(\sigma^2 - 2\gamma^2)^2} > 0, \quad \frac{\partial w_m^2 - w_f^2}{\partial v} = \frac{M(8\gamma^2 - 4\gamma\sigma - \sigma^2)}{8\gamma^3 - 4\gamma\sigma^2} > 0.$$

With $\pi_m^2 - \pi_f^2 = \frac{1}{2} \left(\frac{Q^2(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))}{8(A_{m1}\alpha + b\beta)(A_{m1}\beta^2 - 2A_{m1}\alpha^2 - b\alpha\beta)^2} + \frac{c_f\rho^2 A_{m2}^2}{(4c_f\gamma^2 - 2c_f\sigma^2)^2} + \frac{\rho^2(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2}{c_m(8\gamma^3 - 4\gamma\sigma^2)^2} + \frac{-4c_m\gamma^2\rho^2 A_{m2} + 4c_f c_m M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f\rho^2\sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)}{4c_f c_m \gamma(\sigma^2 - 2\gamma^2)^2} \right) + \frac{4c_m\gamma^2\rho^2\sigma A_{m2} + c_f\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2)}{8c_f c_m(\gamma\sigma^2 - 2\gamma^3)^2} + \frac{(4c_m\gamma^2\rho^2\sigma A_{m2} + c_f\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2))^2}{128c_f^2 c_m^2 \gamma^3(\sigma^2 - 2\gamma^2)^4}$, we calculate

$$\frac{\partial \pi_m^2 - \pi_f^2}{\partial Q} = \frac{Q(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))^2}{-8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0,$$

$$\frac{\partial \pi_m^2 - \pi_f^2}{\partial \rho} = \frac{1}{128c_f^2 c_m^2} \left(\frac{4\rho(8\gamma^3 c_m + 4(c_f + c_m)\gamma^2\sigma + 2\gamma\sigma^2 c_f - \sigma^3 c_f)}{(-4\gamma^2\rho^2(2\gamma + \sigma)c_m + 4c_m c_f M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f\rho^2\sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma))} \frac{(2\gamma^3 - \gamma\sigma^2)^3}{(2\gamma^3 - \gamma\sigma^2)^3} + \frac{2\rho(2\gamma + \sigma)(4c_m\gamma^2\rho^2\sigma(2\gamma + \sigma) + c_f\rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_m c_f M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma)) (4c_m\gamma^2\sigma + c_f(8\gamma^3 - 4\gamma\sigma^2 + \sigma^3))}{\gamma^3(\sigma^2 - 2\gamma^2)^4} \right)$$

> 0,

$$\frac{\partial \pi_m^2 - \pi_f^2}{\partial a_f} = \frac{Q^2(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2 + b)\eta))^2}{8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0.$$

□

Corollary 4 reveals that, when there is an equilibrium status in scenario 2, there are significant differences in the optimal bilateral “pricing” and revenue between two consultation companies. Moreover, the gap between two companies’ revenue will widen with the increase in bilateral potential markets and the churn rate in the firm.

4.2. Sharing Strategy

4.2.1. Scenario 3

The decision sequence in scenario 3 is the same as in scenario 1. However, the physical consultation firm and the online consultation platform adopt the Sharing Strategy for attracting servicers in scenario 3. Two consultation companies need to jointly make the service price and the sharing ratio decisions to maximize their revenue.

In scenario 3, the servicer’s revenue is denoted as

$$\pi_s^{S-S} = \theta_m p_m T_m + \theta_f p_f T_f - \frac{c_f T_f^2}{2} - \frac{c_m T_m^2}{2}. \quad (4)$$

The revenue of the two consultation companies are [15]

$$\pi_f^{S-S} = (q_{f1} + q_{f2})p_f - \theta_f p_f T_f d_f^{E-E}, \quad (5)$$

$$\pi_m^{S-S} = (q_{m1} + q_{m2})p_m - \theta_m p_m T_m d_m^{E-E}. \quad (6)$$

Proposition 3. *If $0 < b < \min\{b_{10}, b_{11}, b_{12}, 1\}$, $c_f > \max\left\{\frac{\rho}{2\lambda} + \frac{\lambda\rho}{2\lambda^2 - \sigma^2}, \frac{\rho}{2\lambda} + \frac{\rho(\rho - c_m\lambda)}{2\lambda\rho + c_m(\sigma^2 - 2\lambda^2)}\right\}$, and $2\lambda^2 > \sigma^2$ (condition 5), there is an equilibrium status in the scenario 3. The optimal bilateral “pricing” decisions of the physical firm are*

$$p_f^3 = \frac{Q(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1-\eta) + (4+b)\eta - 4))}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}, \quad \theta_f^3 = -\frac{F_f^{21}}{M_f^{21}},$$

where $b_{10} = 2 - a_f - \frac{T_f(\rho T_f + \sigma p_m \theta_m - Mv - 6\lambda p_f \theta_f)(+\rho T_f + \sigma p_m \theta_m - Mv - 2\lambda p_f \theta_f)}{4\alpha\lambda p_f^2}$, $b_{11} = \frac{A_{m1}(\beta^2 - 2\alpha^2)}{\alpha\beta} + \frac{\theta_m^2 T_m(2\lambda^2 - \sigma^2)}{\lambda\beta}$, $b_{12} = -\frac{1}{4p_m^2\alpha\beta\lambda(2\lambda^2 - \sigma^2)}(M^2 T_m \alpha(2(1-v)\lambda + v\sigma)^2 + T_m \alpha \rho^2(2\lambda T_m + \sigma T_f)^2 - 8\alpha\rho T_m \theta_m p_m(2T_m \lambda + \sigma T_f)(2\lambda^2 - \sigma^2) + 4p_m^2(2\lambda^2 - \sigma^2)(\lambda(A_{m1}(2\alpha^2 - \beta^2) + 6\alpha\lambda\theta_m^2 T_m) - 3\alpha\sigma^2\theta_m^2 T_m) + 2MT_m\alpha(2(v-1)\lambda - v\sigma)(\rho(2T_m\lambda + \sigma T_f) + 4\theta_m p_m(\sigma^2 - 2\lambda^2)))$, $F_f^{21} = M\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(\rho(4v\lambda^2 + 2(1-v)\lambda\sigma - v\sigma^2)(4\lambda\rho + c_m(\sigma^2 - 4\lambda^2)) + 4\lambda c_f(c_m(2\lambda^2 - \sigma^2)(4v\lambda^2 + 2(1-v)\lambda\sigma - v\sigma^2) + \lambda\rho(2(v-1)\lambda\sigma + 3v\sigma^2 - 8v\lambda^2)))$, $M_f^{21} = 2Q(4A_{m1}\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1-\eta) + (4+b)\eta - 4))\lambda(2\lambda^2 - \sigma^2)(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)$.

The optimal bilateral “pricing” decisions of the online platform are

$$p_m^3 = \frac{Q(2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2}, \quad \theta_m^3 = -\frac{F_m^{21}}{M_m^{21}},$$

where $F_m^{21} = M(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(4\lambda c_f(2(v-1)\lambda - v\sigma)(2\lambda^2 c_m - \lambda\rho - \sigma^2 c_m) + \rho(8(v-1)\lambda^2(\rho - 2\lambda c_m) + 4v\lambda(\lambda c_m - \rho)\sigma + 6c_m(v-1)\lambda\sigma^2 - v\sigma^3 c_m))$, $M_m^{21} = 2Q(\alpha(4 + 2a_m(\eta - 1) + (-4 + b)\eta) - A_{m1}\beta\eta)(2\lambda^2 - \sigma^2)(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)$.

The optimal service investment of the servicer on two consultation companies are

$$T_f^3 = \frac{M(4v\lambda(\rho - \lambda c_m) + 2c_m(v-1)\lambda\sigma + v\sigma^2 c_m)}{8\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2},$$

$$T_m^3 = \frac{M\lambda(2(v-1)(\lambda c_f - \rho) - v\sigma c_f)}{4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2}.$$

In scenario 3, by substituting the optimal decisions of stakeholders into equations (4)–(6), the optimal revenue of the physical firm, the online platform, and the servicer are obtained.

Please see Appendix A.5 for the proof of Proposition 3.

Corollary 5. *In scenario 3, if $\max\{b_1, b_4, b_{10}, b_{11}\} < b < \min\{1, b_5\}$, $\max\{\rho_1, \rho_2, \rho_3\} < \rho < \lambda c_f$ (condition 6), then it can characterize the impact of key parameters on the optimal bilateral “pricing” where*

$$b_{10} = \frac{A_{m1}(2\alpha^2 - \beta^2)(2\alpha(\eta - 1) - \beta\eta)}{\alpha(4\alpha\beta(1 - \eta) + 2\alpha^2\eta + \beta^2\eta)}, \quad b_{11} = \frac{A_{m1}((4\alpha^3 + 2\alpha\beta^2)(\eta - 1) - 6\alpha^2\beta\eta + \beta^3\eta)}{\alpha(\beta^2 - 2\alpha^2)\eta},$$

$$\rho_1 = c_f \left(\lambda + \frac{v\sigma}{2 - 2v} \right), \quad \rho_2 = \frac{1}{4}c_m \left(4\lambda + 2 \left(\frac{1}{v} - 1 \right) \sigma - \frac{\sigma^2}{\lambda} \right), \quad \rho_3 = c_m \left(2\lambda - \frac{\sigma^2}{\lambda} \right).$$

- (I) The service investment cost coefficients (c_f, c_m) respectively have a positive and negative impact on the optimal sharing rate of the firm. In contrast, the impact on the platform is just the opposite.
- (II) The servicers’ potential market (M) and the churn rate of the firm (a_f) negatively influence the equilibrium revenue of the firm. The servicers’ potential market (M) positively affects the equilibrium revenue of the platform. The firm’s and the platform’s equilibrium revenue are impacted in opposite ways by the customers’ potential market (Q) and the churn rate of the online platform (a_m) , respectively. The former has a positive relationship, while the latter has a negative one. The service investment cost coefficients (c_f, c_m) hurt the equilibrium revenue of the servicer.

Please see Appendix A.6 for the proof of Corollary 5.

Corollary 5 suggests that, when there is an equilibrium status in scenario 3, the customers’ switching rate and the servicer’s sensitivity to service investment are moderate, and the service investment cost coefficients on the two consultation companies have the opposite effect on their optimal sharing rate. Still, both are detrimental to the revenue of the servicer. Meanwhile, potential bilateral markets have opposite effects on the two companies’ revenue. It indicates that, the competitive impact between the two companies is more pronounced under the Sharing Strategy.

Corollary 6. *Based on scenario 3 and Corollary 5, and if $0 < \rho < \frac{c_f(4\lambda^3 - 2\lambda\sigma^2)}{4\lambda^2 + 2\lambda\sigma - \sigma^2}$ is satisfied, the service investment cost coefficients (c_f, c_m) negatively affect the optimal service investment gap between two consultation companies. The servicers’ potential market (M) and the service investment cost coefficient on the platform (c_m) positively expand the optimal sharing rate gap between the two companies, while the customers’ potential market (Q) and the service investment cost coefficient on the firm (c_f) mitigate the sharing rate gap. Moreover, the customers’ potential market (Q) , the churn rate of the platform (a_m) , the sensitivity of customers to service price (α) , and their preference for the firm (η) negatively affect the equilibrium revenue gap.*

Proof. Based on scenario 3 and Corollary 5, and if $0 < \rho < \frac{c_f(4\lambda^3 - 2\lambda\sigma^2)}{4\lambda^2 + 2\lambda\sigma - \sigma^2}$ is satisfied, with $T_m^3 - T_f^3 = \frac{M(4\lambda(c_f(v-1)\lambda + c_mv\lambda + \rho - 2v\rho) - 2(c_m(v-1) + c_fv)\lambda\sigma - c_mv\sigma^2)}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2}$, we obtain

$$\frac{\partial T_m^3 - T_f^3}{\partial c_f} = - \frac{M\lambda(4v\lambda(c_m\lambda - \rho) + 2c_m(1 - v)\lambda\sigma - c_mv\sigma^2)(c_m(2\lambda^2 - \sigma^2) - \rho(2\lambda + \sigma))}{(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_m(\rho - 2c_f\lambda)\sigma^2)^2} < 0,$$

$$\frac{\partial T_m^3 - T_f^3}{\partial c_m} = \frac{M\lambda(2(v - 1)(c_f\lambda - \rho) - c_fv\sigma)(4\lambda^2(c_f\lambda - \rho) - 2\lambda\rho\sigma + (\rho - 2c_f\lambda)\sigma^2)}{-(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_m(\rho - 2c_f\lambda)\sigma^2)^2} < 0.$$

Based on the optimal sharing rate and revenue of the two consultation companies, we derive

$$\begin{aligned} \theta_m^3 - \theta_f^3 > 0, \quad \frac{\partial \theta_m^3 - \theta_f^3}{\partial M} > 0, \quad \frac{\partial \theta_m^3 - \theta_f^3}{\partial c_m} > 0, \quad \frac{\partial \theta_m^3 - \theta_f^3}{\partial Q} < 0, \quad \frac{\partial \theta_m^3 - \theta_f^3}{\partial c_f} < 0, \\ \frac{\partial \pi_m^3 - \pi_f^3}{\partial Q} &= \frac{Q((2A_{m1} + b)\alpha\eta - 2A_{m1}\alpha - A_{m1}\beta\eta)^2}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0 \\ \frac{\partial \pi_m^3 - \pi_f^3}{\partial a_m} &= \frac{Q^2(A_{m1}\beta\eta - \alpha(4 + 2a_m(\eta - 1) + (b - 4)\eta))(A_{m1}\beta^3\eta + 2\alpha^2\beta(2b(\eta - 1) - A_{m1}\eta) - \alpha\beta^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha^3(8 + 4a_m(\eta - 1) - 2(4 + b)\eta))}{8\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial \pi_m^3 - \pi_f^3}{\partial \alpha} &= \frac{A_{m1}Q^2(-A_{m1}\beta\eta + \alpha(4 + 2a_m(\eta - 1) + (b - 4)\eta))(-6A_{m1}\alpha^2\beta\eta + A_{m1}\beta^3\eta + 2\alpha^3(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta^2(4 + a_m(\eta - 1) - (4 + b)\eta))}{8\alpha^2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial \pi_m^3 - \pi_f^3}{\partial \eta} &= \frac{Q^2((2A_{m1} + b)\alpha - A_{m1}\beta)(2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta)}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0. \end{aligned}$$

□

Corollary 6 implies that, when there is an equilibrium status in scenario 3 and the servicer’s sensitivity to service investment is less than a certain threshold, there are significant differences in the optimal bilateral “pricing” and equilibrium revenue between two companies. Meanwhile, the gap between two consultation companies’ revenue narrows as potential demand in the firm increases.

4.2.2. Scenario 4

Similar to scenario 3, both consultation companies adopt the Sharing Strategy for attracting servicers in scenario 4. However, the decision sequences in two scenarios are different. The online platform sets the consultation service price p_m and the sharing rate θ_m for bilateral users at first. Then, the physical firm as the follower, charges bilateral “prices” p_f, θ_f to compete with the platform. Servicer finally decides how much to invest T_f, T_m in the two consultation companies.

Proposition 4. *If $0 < b < \min\{b_1, b_{13}, b_{14}, \frac{-A_{m1}\alpha}{\beta}, 1\}$, $\max\{a_{f1}, a_{f2}\} < a_f$, $c_f > \max\{\frac{2\lambda\rho}{2\lambda^2 - \sigma^2}, \frac{4\lambda(c_m\lambda - \rho)\rho}{4c_m\lambda^3 - 4\lambda^2\rho - 2c_m\lambda\sigma^2 + \rho\sigma^2}\}$, and $2\lambda^2 > \sigma^2$ (condition 7), there is an equilibrium status in the scenario 4, where*

$$\begin{aligned} b_{13} &= \frac{\lambda\theta_m^2 T_m - \alpha A_{m1}}{\beta}, \quad b_{14} = \frac{\alpha A_{m1}}{\beta} + \frac{T_m(M(v - 1) - 6\lambda\theta_m p_m + \rho T_m + \sigma\theta_f p_f)}{(M(v - 1) - 2\lambda\theta_m p_m + \rho T_m + \sigma\theta_f p_f)4\beta\lambda p_m^2}, \\ a_{f1} &= \frac{\lambda((2 - b)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2) + 2T_f(A_{m1}\alpha + b\beta)\theta_f^2\lambda) - T_f(A_{m1}\alpha + b\beta)\theta_f^2\sigma^2}{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)\lambda}, \\ a_{f2} &= -\frac{G_1}{G_2}, \\ G_1 &= (A_{m1}\alpha + b\beta)(M^2 T_f(2v\lambda + \sigma - v\sigma)^2 + \rho^2 T_f(2\lambda T_f + \sigma T_m)^2 - 8\rho\theta_f T_f p_f(2\lambda T_f + \sigma T_m)(2\lambda^2 - \sigma^2) \\ &\quad + \frac{4p_f^2(2\lambda^2 - \sigma^2)(\lambda((-2 + b)(2A_{m1} + b\alpha\beta - A_{m1}\beta^2) + 6T_f(A_{m1}\alpha + b\beta)\theta_f^2\lambda) - 3T_f(A_{m1}\alpha + b\beta)\theta_f^2\sigma^2)}{A_{m1}\alpha + b\beta} \\ &\quad - 2MT_f(2v\lambda + \sigma - v\sigma)(\rho(2\lambda T_f + \sigma T_m) + 4\theta_f T_f f(-2\lambda^2 + \sigma^2))), \\ G_2 &= 4p_f^2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)\lambda(2\lambda^2 - \sigma^2). \end{aligned}$$

The optimal bilateral “pricing” decisions of the online platform are

$$p_m^4 = \frac{Q(-2A_{m1}\alpha^2(4 + a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(1 - \eta) + 2A_{m1}^2\eta - b^2\eta) + A_{m1}\beta^2((A_{m2} + 3b)\eta - A_{m1}))}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)},$$

$$\theta_m^4 = -\frac{F_m^4}{M_m^4},$$

$$F_m^4 = M(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(4\lambda\rho(4(1 - v)\lambda^2(2\lambda c_m - \rho) + 2v\lambda(\lambda c_m - \rho)\sigma + (v - 1)(3\lambda c_m - \rho)\sigma^2) + c_f(4(v - 1)\lambda^2 - 2v\lambda\sigma - (v - 1)\sigma^2)(8\lambda^3 c_m - 4\lambda^2\rho - 4\lambda\sigma^2 c_m + \rho\sigma^2)),$$

$$M_m^4 = 2Q(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (-4 + b)\eta) + \alpha\beta(3A_{m1}b(-1 + \eta) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))\lambda(2\lambda^2 - \sigma^2)(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_f(-2\lambda c_m + \rho)\sigma^2).$$

The optimal bilateral “pricing” decisions of the physical firm are

$$p_f^4 = \frac{Q(2A_{m1}\alpha\eta + \beta(a_m - a_m\eta - 2 + (2 + b)\eta))}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2},$$

$$w_f^4 = \frac{F_f^4}{M_f^4},$$

$$F_f^4 = M(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(8v\lambda^2(2\lambda c_f - \rho)(\lambda c_m - \rho) + 4(1 - v)\lambda(2\lambda^2 c_f c_m - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_f v\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v - 1)(4\lambda c_m - \rho)\sigma^3),$$

$$M_f^4 = 2Q(-2A_{m1}\alpha\eta + \beta(2 + a_m(\eta - 1) - (2 + b)\eta))(2\lambda^2 - \sigma^2)(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_f(\rho - 2\lambda c_m)\sigma^2).$$

The optimal service investment of the servicer on two consultation companies are

$$T_f^3 = \frac{M(4v\lambda(\rho - \lambda c_m) + 2c_m(v - 1)\lambda\sigma + v\sigma^2 c_m)}{8\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2}, \quad T_m^3 = \frac{M\lambda(2(v - 1)(\lambda c_f - \rho) - v\sigma c_f)}{4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2}.$$

In scenario 4, by substituting the optimal decisions of stakeholders into equations (4)–(6), the optimal revenue of the physical firm, the online platform, and the servicer are obtained.

Please see Appendix A.7 for the proof of Proposition 4.

Corollary 7. *If $0 < b < \min\{b_6, 1\}$, $c_m > \max\{c_{m1}, c_{m2}, c_{m3}\}$, and $\rho > \frac{1}{4}c_f\left(4\lambda - \frac{2v\sigma}{-1+v} - \frac{\sigma^2}{\lambda}\right)$ (condition 8), then it can characterize the impact of key parameters on the optimal bilateral “pricing” where*

$$c_{m1} = \frac{\rho(8v\lambda^2(2c_f\lambda - \rho) + 4(1 - v)\lambda(c_f\lambda - \rho)\sigma - 6c_f v\lambda\sigma^2 + c_f(v - 1)\sigma^3)}{4\lambda(2v\lambda + \sigma - v\sigma)(2c_f\lambda^2 - \lambda\rho - c_f\sigma^2)},$$

$$c_{m2} = \frac{\rho(4\lambda\rho(\sigma - 2\lambda) + c_f(16\lambda^3 - 4\lambda^2\sigma - 6\lambda\sigma^2 + \sigma^3))}{4\lambda(2\lambda - \sigma)(2c_f\lambda^2 - \lambda\rho - c_f\sigma^2)}, \quad c_{m3} = \frac{2v\rho}{2v\lambda + \sigma - v\sigma}.$$

- (I) The customers’ potential market (Q), the preference for firm (η), and the churn rate of the platform (a_m) positively affect the optimal sharing rate set by the firm. However, the servicer’s potential market (M) and the preference for the firm (v) have a negative impact.
- (II) There is a positive relationship between the equilibrium revenue of two consultation companies and the bilateral users’ market size (M , Q). The servicers’ potential market (M) contributes positively to the equilibrium revenue of the servicer, whereas the service investment cost coefficients (c_f, c_m) contribute negatively.

Please see Appendix A.8 for the proof of Corollary 7.

Corollary 7 shows that, when there is an equilibrium status in scenario 4, the customers’ switching rate is below a certain threshold, the service cost coefficient on the platform and the servicer’s sensitivity to service investment are larger, the supply and demand of consultation service in firm has the opposite effect on its optimal sharing rate, but both are beneficial for two consultation companies’ revenue.

Corollary 8. *Based on scenario 4 and Corollary 7, and if $\max\left\{\frac{c_m(4\lambda^3-2\lambda\sigma^2)}{4\lambda^2+2\lambda\sigma-\sigma^2}, c_m\left(\lambda + \frac{(1-v)\sigma}{2v}\right)\right\} < \rho < \frac{c_f(2\lambda^2-\sigma^2)}{2\lambda+\sigma}$, the optimal service investment gap between two consultation companies are impacted negatively by the service investment cost coefficients (c_f, c_m). The equilibrium revenue difference between two consultation companies decreases with the increase of customers’ potential market (Q).*

Proof. Based on scenario 4 and Corollary 7, and if $\max\left\{\frac{c_m(4\lambda^3-2\lambda\sigma^2)}{4\lambda^2+2\lambda\sigma-\sigma^2}, c_m\left(\lambda + \frac{(1-v)\sigma}{2v}\right)\right\} < \rho < \frac{c_f(2\lambda^2-\sigma^2)}{2\lambda+\sigma}$, with $T_m^4 - T_f^4 = \frac{M(4\lambda(c_f(v-1)\lambda+c_mv\lambda+\rho-2v\rho)-2(c_m(v-1)+c_fv)\lambda\sigma+c_f(1-v)\sigma^2)}{8\lambda(c_f\lambda-\rho)(c_m\lambda-\rho)+2c_f(\rho-2c_m\lambda)\sigma^2}$, we derive

$$\begin{aligned}\frac{\partial T_m^4 - T_f^4}{\partial c_f} &= -\frac{M\lambda(4\lambda^2(c_m\lambda - \rho) - 2\lambda\rho\sigma + (\rho - 2c_m\lambda)\sigma^2)(c_m(2v\lambda + \sigma - v\sigma) - 2v\rho)}{(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^2} < 0, \\ \frac{\partial T_m^4 - T_f^4}{\partial c_m} &= -\frac{M\lambda(4(v-1)\lambda(c_f\lambda - \rho) - 2c_fv\lambda\sigma + c_f(1-v)\sigma^2)(c_f(2\lambda^2 - \sigma^2) - \rho(2\lambda + \sigma))}{(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^2} < 0.\end{aligned}$$

Based on the revenue of two consultation companies, we determine

$$\frac{\partial \pi_m^4 - \pi_f^4}{\partial Q} = \frac{Q(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))^2}{-8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0.$$

□

Corollary 8 indicates that, when there is an equilibrium status in scenario 4 and the sensitivity of servicer to service investment is moderate, there are significant differences in the optimal bilateral “pricing” and equilibrium revenue between two companies. Meanwhile, the revenue gap between two consultation companies narrows as customers’ potential market increases.

5. COMPARATIVE ANALYSIS

In this section, we compare the differences in the optimal decisions of stakeholders under two game sequences or two salary strategies.

Proposition 5. *The bilateral “price” differences of two consultation companies under the two game sequences are compared when the salary strategy is the same. If $0 < b < \min\left\{1, \frac{-A_{m1}\beta}{\alpha}, \frac{-A_{m1}\alpha}{\beta}, \frac{-2A_{m1}\alpha}{\beta} + \frac{-A_{m1}\beta}{\alpha}\right\}$, then we can conclude that*

Compared to scenario 1 with 2, the customers’ potential market (Q) and the service price competition coefficient (β) favorably impact the optimal service price gap charged by the platform but harm that by the firm. The customer’s preference for the firm (η) negatively affects the optimal service price gap for two consultation companies. The servicer’s preference for the firm (v) and the service investment cost coefficient on the firm (c_f) play a positive role in the optimal fixed wage gaps for both two consultation companies, whereas the service investment cost coefficient on the platform (c_m) plays a negative role. As the customers’ potential market (M) increases, the optimal fixed wage gap set by the online platform decreases; nevertheless, the optimal fixed wage gap also widens for the physical firm.

Proof. With $p_m^1 - p_m^2 = \frac{Q\beta(b\alpha + A_{m1}\beta)(A_{m1} - (A_{m1} + b)\eta)}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0$, we derive that

$$\begin{aligned} \frac{\partial p_m^1 - p_m^2}{\partial \eta} &= \frac{-(A_{m1} + b)Q\beta(b\alpha + A_{m1}\beta)}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0, \\ \frac{\partial p_m^1 - p_m^2}{\partial \beta} &= \frac{Q(b^3\alpha^2\beta^2 - 2A_{m1}b^2\alpha\beta^3 - A_{m1}^2b(2\alpha^4 + 4\alpha^2\beta^2 + \beta^4) - 4A_{m1}^3\alpha^3\beta)((A_{m1} + b)\eta - A_{m1})}{4(A_{m1}\alpha + b\beta)^2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \\ \frac{\partial p_m^1 - p_m^2}{\partial Q} &= \frac{-\beta(b\alpha + \beta A_{m1})((A_{m1} + b)\eta - A_{m1})}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0. \end{aligned}$$

With $w_m^1 - w_m^2 = \frac{\sigma^2(c_f\rho^2(4\gamma - \sigma)A_{m2} + 4c_m c_f M(v-1)\gamma(2\gamma^2 - \sigma^2) - 2c_m\gamma\rho^2\sigma)}{16c_m c_f \gamma^2(\sigma^2 - 2\gamma^2)^2}$, we derive

$$\begin{aligned} \frac{\partial w_m^1 - w_m^2}{\partial M} &= \frac{(v-1)\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} < 0, \quad \frac{\partial w_m^1 - w_m^2}{\partial v} = \frac{M\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} > 0, \quad \frac{\partial w_m^1 - w_m^2}{\partial c_f} = \frac{\rho^2\sigma^3}{8c_f^2\gamma(\sigma^2 - 2\gamma^2)^2} > 0, \\ \frac{\partial w_m^1 - w_m^2}{\partial c_m} &= \frac{\rho^2\sigma^2(\sigma^2 - 8\gamma^2 - 2\gamma\sigma)}{16c_m^2\gamma^2(\sigma^2 - 2\gamma^2)^2} < 0. \end{aligned}$$

With $p_f^1 - p_f^2 = \frac{-Q\beta(b\alpha + \beta A_{m1})\eta}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0$, we derive

$$\begin{aligned} \frac{\partial p_f^1 - p_f^2}{\partial Q} &= \frac{-\beta(b\alpha + A_{m1}\beta)\eta}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0, \quad \frac{\partial p_f^1 - p_f^2}{\partial \beta} = \frac{-A_{m1}Q(2\alpha\beta A_{m1} + b(\alpha^2 + \beta^2))\eta}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial p_f^1 - p_f^2}{\partial \eta} &= \frac{-Q\beta(b\alpha + A_{m1}\beta)}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0. \end{aligned}$$

With $w_f^1 - w_f^2 = \frac{\sigma^2(c_m\rho^2(\sigma^2 - 8\gamma^2 - 2\gamma\sigma) + 2c_f\gamma(\rho^2\sigma + 2c_m Mv(2\gamma^2 - \sigma^2)))}{16c_m c_f (\gamma\sigma^2 - 2\gamma^3)^2}$, we derive

$$\begin{aligned} \frac{\partial w_f^1 - w_f^2}{\partial M} &= \frac{v\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} > 0, \quad \frac{\partial w_f^1 - w_f^2}{\partial v} = \frac{M\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} > 0, \quad \frac{\partial w_f^1 - w_f^2}{\partial c_f} = \frac{\rho^2(4\gamma - \sigma)\sigma^2 A_{m2}}{16c_f^2\gamma^2(\sigma^2 - 2\gamma^2)^2} > 0, \\ \frac{\partial w_f^1 - w_f^2}{\partial c_m} &= \frac{-\rho^2\sigma^3}{8c_m^2\gamma(\sigma^2 - 2\gamma^2)^2} < 0. \end{aligned}$$

□

Proposition 5 reveals that, when the customers' switching rate is less than a certain threshold, the same salary strategy is adopted by the two consultation companies, and there is an equilibrium status in the game system, there are significant differences in the two companies' optimal bilateral pricing under different decision-making sequences. If the consultation company decides as the follower, it tends to charge a higher service price because of the advantage of market information. Additionally, the wage gap under different decision-making sequences may widen as firms' service circumstances improve.

Proposition 6. *When the game sequence is the same, the optimal service investment gap by the servicer under two salary strategies is compared. The elasticity coefficient of wage (γ) negatively affects the optimal service investment gap in the platform or the firm under two salary strategies. The optimal service investment under the Employment Strategy is higher than that under the Sharing Strategy. The potential servicers with a preference for the firm (M, v) play a positive role in the optimal service investment gap when the service investment cost coefficients (c_f, c_m) exceed a certain threshold or the servicer's sensitivity to service investment (ρ) is moderate.*

Proof. Based on the optimal service invested by servicers in four scenarios, we derive

$$\begin{aligned}
 T_f^1 - T_f^3 &= \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_f\gamma^3 - 4c_f\gamma\sigma^2} - \frac{M(4v\lambda(\rho - c_m\lambda) + 2c_m(v - 1)\lambda\sigma + c_f v\sigma^2)}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2}, \\
 T_m^1 - T_m^3 &= \frac{\rho(2\gamma + \sigma)}{4c_m\gamma^2 - 2c_m\sigma^2} + \frac{M\lambda(2(1 - v)(c_f\lambda - \rho) + c_f v\sigma)}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_m(\rho - 2c_f\lambda)\sigma^2}, \\
 T_f^2 - T_f^4 &= \frac{\rho(2\gamma + \sigma)}{4c_f\gamma^2 - 2c_f\sigma^2} + \frac{M\lambda(c_m(2v\lambda + \sigma - v\sigma) - 2v\rho)}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2}, \\
 T_m^2 - T_m^4 &= \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_m\gamma^3 - 4c_m\gamma\sigma^2} + \frac{M(4(1 - v)\lambda(c_f\lambda - \rho) + 2c_f v\lambda\sigma + c_f(v - 1)\sigma^2)}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_f(\rho - 2c_m\lambda)\sigma^2}.
 \end{aligned}$$

We calculate

$$\begin{aligned}
 \frac{\partial T_f^1 - T_f^3}{\partial \gamma} &= \frac{-\rho(8\gamma^4 + 8\gamma^3\sigma - 2\gamma^2\sigma^2 + \sigma^4)}{4c_f(\gamma\sigma^2 - 2\gamma^3)^2} < 0, & \frac{\partial T_m^1 - T_m^3}{\partial \gamma} &= \frac{-\rho(2\gamma^2 + 2\gamma\sigma + \sigma^2)}{c_m(\sigma^2 - 2\gamma^2)^2} < 0, \\
 \frac{\partial T_f^2 - T_f^4}{\partial \gamma} &= \frac{-\rho(2\gamma^2 + 2\gamma\sigma + \sigma^2)}{c_f(\sigma^2 - 2\gamma^2)^2} < 0, & \frac{\partial T_m^2 - T_m^4}{\partial \gamma} &= \frac{-\rho(8\gamma^4 + 8\gamma^3\sigma - 2\gamma^2\sigma^2 + \sigma^4)}{4c_m(\gamma\sigma^2 - 2\gamma^3)^2} < 0.
 \end{aligned}$$

If $c_f > \max\left\{\frac{2(v-1)\rho}{2(v-1)\lambda - v\sigma}, \frac{2\rho}{2\lambda - \sigma}\right\}$, then $\frac{\partial T_m^1 - T_m^3}{\partial v} = \frac{M\lambda(2\rho + c_f(\sigma - 2\lambda))}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_m(\rho - 2c_f\lambda)\sigma^2} > 0$.

If $c_m > \max\left\{c_{m3}, \frac{2\rho}{2\lambda - \sigma}\right\}$, then $\frac{\partial T_f^2 - T_f^4}{\partial v} = \frac{M\lambda(2c_m\lambda - 2\rho - c_m\sigma)}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2} > 0$, $\frac{\partial T_f^2 - T_f^4}{\partial M} = \frac{\lambda(c_m(2v\lambda + \sigma - v\sigma) - 2v\rho)}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2} > 0$.

If $\max\left\{\frac{1}{4}c_f\left(4\lambda - 2\sigma - \frac{\sigma^2}{\lambda}\right), 0\right\} < \rho < \min\left\{\frac{1}{4}c_m\left(4\lambda - 2\sigma - \frac{\sigma^2}{\lambda}\right), \frac{1}{4}c_f\left(4\lambda - \frac{2v\sigma}{v-1} - \frac{\sigma^2}{\lambda}\right)\right\}$, then

$$\begin{aligned}
 \frac{\partial T_f^1 - T_f^3}{\partial v} &= \frac{-M(4\lambda\rho + c_m(2\lambda\sigma + \sigma^2 - 4\lambda^2))}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2} > 0, \\
 \frac{\partial T_f^1 - T_f^3}{\partial M} &= \frac{4v\lambda(c_m\lambda - \rho) + 2c_m(v - 1)\lambda\sigma + c_m v\sigma^2}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_m(\rho - 2c_f\lambda)\sigma^2} > 0, \\
 \frac{\partial T_m^2 - T_m^4}{\partial v} &= \frac{M(4\lambda\rho + c_f(2\lambda\sigma + \sigma^2 - 4\lambda^2))}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_f(\rho - 2c_m\lambda)\sigma^2} > 0, \\
 \frac{\partial T_m^2 - T_m^4}{\partial M} &= \frac{4(1 - v)\lambda(c_f\lambda - \rho) + 2c_f v\lambda\sigma + c_f(v - 1)\sigma^2}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_f(\rho - 2c_m\lambda)\sigma^2} > 0.
 \end{aligned}$$

□

Proposition 6 shows that, when the decision-making sequence is the same, different salary strategies are adopted by the two consultation companies, and the game system has an equilibrium status, the service investment gap between the two consultation companies will be reduced accordingly if the servicer pays more attention to the salary provided by the current company. In addition, when the service circumstance of the two companies is harsh, and the servicer is sensitive to investment cost, the increase in potential service supply in the firm will exacerbate the service investment gap, which implies that the two companies should improve service circumstance to maintain service consistency for customers.

6. NUMERICAL ANALYSIS

The numerical calculation method is utilized for simulation analysis to analyze the joint effect of crucial parameters and consumers’ switching rate (b) on stakeholders’ revenue. Combined with the relevant studies

TABLE 1. The values of critical parameters.

Parameter	Value	Parameter	Value	Parameter	Value
η	0.7	Q	100	α	0.6
β	0.55	a_f	0.1	a_m	0.2
b	0.15	v	0.4	M	50
γ	0.9	ρ	0.12	σ	0.5
λ	0.8	c_f	0.15	c_m	0.12

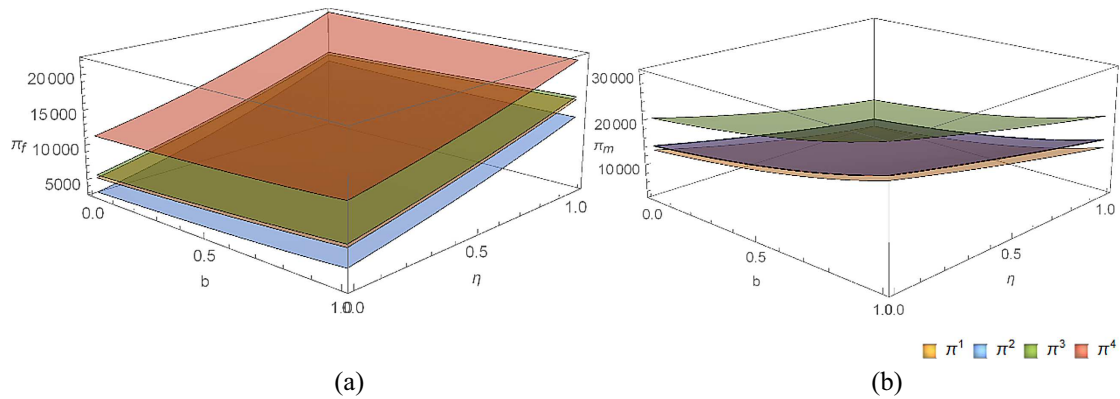


FIGURE 3. The joint effect of η and b . (a) The joint effect of η and b on the firm’s revenue. (b) The joint effect of η and b on the platform’s revenue.

[1, 35] and the model assumptions, the key parameters’ values are shown in Table 1. The value range of b is [0,1].

(1) The joint effect of η and b

The value range of η is [0,1]. The joint effect of η and b has no direct impact on the equilibrium revenue of servicer, and the combined effect on the equilibrium revenue of two consultation companies are shown in Figures 3(a) and 3(b), respectively. It can be seen from the figure that the physical firm obtains the most considerable equilibrium revenue in scenario 4, and the online platform achieves the most considerable equilibrium revenue in scenario 3, regardless of consumers’ preferences for the firm and the switching rate change. The consumers’ switching rate (b) has a positive effect on the equilibrium revenue of two companies in four scenarios. Consumers’ preference for the firm (η) has a positive impact on the firm’s revenue but a negative one on the platform’s revenue. The reason is that more consumers prefer the consultation firm, and more consumers flow into the firm, which can bring more benefits to the firm in the consultation market with a certain scale of potential consumers.

(2) The joint effect of Q and b

The value range of η is [0,200]. The joint effect of Q and b has no direct impact on the equilibrium revenue of servicer, and the combined effect on the equilibrium revenue of two consultation companies are shown in Figures 4(a) and 4(b), respectively. Figure shows that the equilibrium revenue of both consultation companies is more sensitive to the customers’ potential market in four scenarios. The equilibrium revenue of two companies is positively impacted by the joint effect of Q and b , which is strengthened with the increase of the values of Q and b . It indicates that two consultation companies can benefit from the booming demand market.

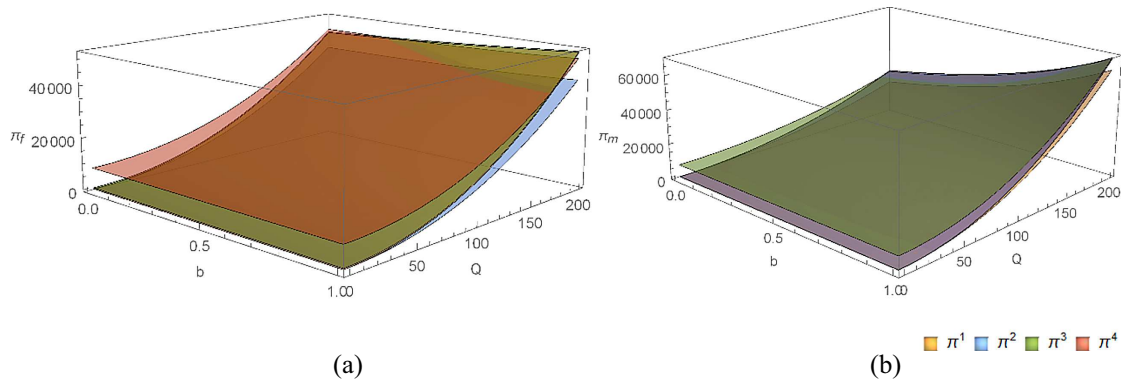


FIGURE 4. The joint effect of Q and b . (a) The joint effect of Q and b on the firm’s revenue and (b) the joint effect of Q and b on the platform’s revenue.

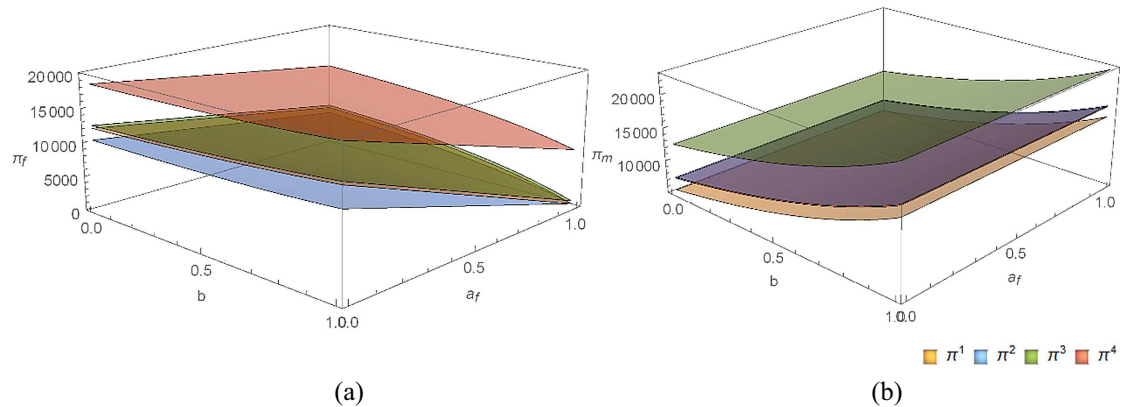


FIGURE 5. The joint effect of a_f and b . (a) The joint effect of a_f and b on the firm’s revenue and (b) the joint effect of a_f and b on the platform’s revenue.

(3) The joint effect of a_f and b

The value range of a_f is $[0,1]$. The joint effect of a_f and b has no direct impact on the equilibrium revenue of servicers, and the combined effect on the equilibrium revenue of two consultation companies are shown in Figures 5(a) and 5(b), respectively. It is for this reason that in markets where the churn rate from the firm is higher, some drawbacks lead to a decrease in the firm’s revenue. The platform’s competitiveness can be indirectly enhanced by the increase in the churn rate of the firm.

(4) The joint effect of v and b

The value range of v is $[0,1]$. The joint effect of v and b on the equilibrium revenue of the physical firm, the online platform, and the servicers are shown in Figures 6(a)–6(c), respectively. Seen figure, the equilibrium revenue of the platform, the firm, and the servicer is less sensitive to the servicer’s preference for the firm in scenarios 1 and 3. The joint effect of v and b has a U -shaped impact on the equilibrium revenue of servicers in scenarios 2 and 4. The combined effect negatively affects the firm’s equilibrium revenue, and the intensity of the effect decreases at first and then increases with the servicer’s preference for the firm. In contrast, the joint effect positively affects the platform’s equilibrium revenue. The intensity of the impact increases at first. Then it decreases with the increase of the servicer’s preference for the firm.

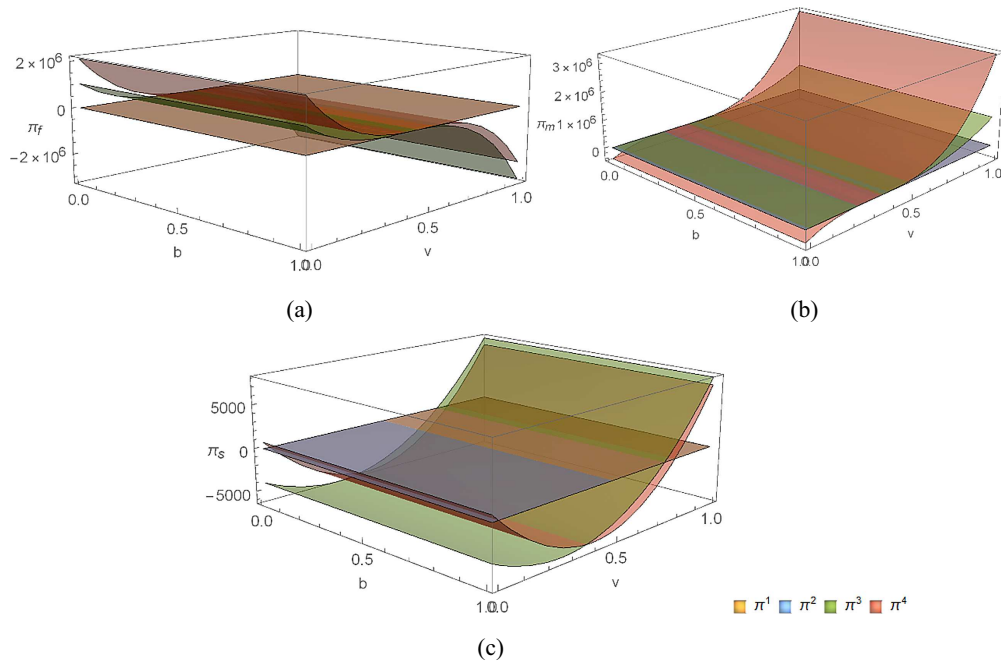


FIGURE 6. The joint effect of v and b . (a) The joint effect of v and b on the firm's revenue, (b) the joint effect of v and b on the platform's revenue and (c) the joint effect of v and b on the service's revenue.

At the same time, some theoretical conclusions in the study are verified by the numerical analysis.

7. CONCLUSION AND IMPLICATIONS

This study develops a three-echelon Stackelberg game model to investigate the bilateral optimal “pricing” for consultation companies' competition, taking into account customers switching from the physical firm to the online platform. Two consultation companies charge the consulting service price for customers and hire the servicers by adopting Employment Strategy or Sharing Strategy. The servicer attracted by revenue makes service investment decisions in two consultation companies. Under combinations of two consultation companies' game sequences and two salary strategies, we prove stakeholders' optimal decision-making and equilibrium revenue and theoretically analyze the effect of some parameters on the optimal decisions and the gap between them in four game scenarios. Moreover, the numerical analysis simulates the joint impact of customer switching behavior and other parameters on stakeholders' equilibrium revenue. Some notable and practical managerial insights have been summarized as follows.

- (1) When the customers' switching rate is moderate, no matter who makes the decision first, the customers' potential market (Q) and the churn rate of the online platform (a_m) have a positive effect on the firm's optimal service price, whereas customers preference for the firm (η) hurts the optimal service price of the platform. Moreover, the customer's preference for the firm (η) negatively affects the optimal service price gap for two consultation companies compared to scenario 1 with scenario 2. The service price can be indirectly affected by adjusting the customers' potential demand in the consultation service competition market.
- (2) Under the Employment Strategy, the servicer's sensitivity to service investment (ρ) positively influences the optimal fixed wage provided by two consultation companies when the servicer's potential market is

moderate, and the servicers’ sensitivity to the current revenue is more than half of the wage competition. Nevertheless, the servicer’s potential market (M) and the service investment cost coefficients (c_f, c_m) harm the optimal fixed wage set by the two companies and the servicer’s revenue. The servicer’s preference for the physical firm (v) respectively plays a negative and positive role in the optimal fixed wage set by the two companies. Compared to scenario 1 with 2, the servicer’s preference for the firm (v) and the service investment cost coefficient on the firm (c_f) play a positive role in the optimal fixed wage gaps for two consultation companies. In contrast, the service investment cost coefficient on the platform (c_m) plays a negative role.

- (3) Expanding customers’ potential market (Q) can promote the physical consultation firm’s revenue in four scenarios and benefit the online consultation platform’s revenue in scenarios 1, 2 and 4. In most scenarios, the churn rate of the firm (am) harms its revenue. Interestingly, the servicers’ potential market (M) plays a positive role in the equilibrium revenue of the platform when two companies adopt Sharing Strategy simultaneously. The churn rate of the firm (a_f) has a positive impact on the equilibrium revenue gap between the physical firm and the online platform when both companies utilize Employment Strategy; however, the customers’ potential market (Q) negatively affects the equilibrium revenue gap when both companies utilizing Sharing Strategy.
- (4) The service investment cost coefficients (c_f, c_m) have a positive impact on the optimal service investment gap between two consultation companies in scenario 1, but scenario 2, 3, and 4 show a negative one. Comparing the optimal service investment under two salary strategies indicates that Employment Strategy can motivate servicers to raise service investment when the servicer’s perceived service cost is higher. Meanwhile, expanding potential servicers in the firm can strengthen this incentive effect. It also partly explains the phenomenon that the servicers typically experience greater pressure when working in the physical firm, usually adopting Employment Strategy.
- (5) In the competition circumstance, there is an equilibrium status in all scenarios when the consumers’ switching rate is moderate, the service investment cost coefficient in the firm is greater than a certain threshold, and the servicers are more sensitive to the current revenue provided by the firm or the platform. The two companies’ revenue is positively impacted by the joint effect of customers’ potential market (Q) and the switching rate (b) but negatively by the combined impact of bilateral users’ preference for the firm (the churn rate of the firm) and the switching rate play an opposite role. It also suggests that two consultation companies can achieve more significant revenue when both adopt Sharing Strategy.

It’s deserved to study the bilateral “pricing” issue of service companies by relaxing or eliminating some model assumptions in the paper. There are directions for further research in the future. Firstly, for bilateral “pricing” strategies, service companies may adopt different salary strategies simultaneously, even utilizing mixed or service price strategies. Secondly, for the matching ability, some novel and practicable insights may be derived considering the effective service transaction and the network externality in service companies. Thirdly, for the data validation, it is better to verify the model’s validity and conclusions using the actual operation data of service companies.

APPENDIX A.

A.1. Equilibrium status in scenario 1

In scenario 1, the Jacobian matrix of the physical firm’s revenue function is

$$H_f^{E-E} = \begin{pmatrix} 2(a_f + b - 2)\alpha & 0 \\ 0 & -2\gamma \end{pmatrix}.$$

The corresponding values of the principal minor sequence are $H_{f1}^{E-E} = 2(a_f + b - 2)\alpha < 0$, $H_{f2}^{E-E} = 4(2 - a_f - b)\alpha\gamma > 0$, respectively. Thus, there is an optimal solution p_f^*, w_f^* to maximize the revenue of the physical firm.

Solving $\begin{cases} \frac{\partial \pi_f^1}{\partial p_f} = 0 \\ \frac{\partial \pi_f^1}{\partial w_f} = 0 \end{cases}$, then $\begin{cases} p_f^* = \frac{\beta p_m + Q\eta}{2\alpha} \\ w_f^* = \frac{\rho T_f + \sigma w_m - Mv}{2\gamma} \end{cases}$ is derived.

The Jacobian matrix of the online platform's revenue function is

$$H_m^{E-E} = \begin{pmatrix} 2A_{m1}\alpha + b\beta - \frac{A_{m1}\beta^2}{\alpha} & 0 \\ 0 & \frac{\sigma^2}{\gamma} - 2\gamma \end{pmatrix}.$$

If condition 1 was held, the corresponding values of the principal minor sequence are

$$H_{m1}^{E-E} = 2A_{m1}\alpha + b\beta - \frac{A_{m1}\beta^2}{\alpha} < 0,$$

$$H_{m2}^{E-E} = -\frac{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(2\gamma^2 - \sigma^2)}{\alpha\gamma} > 0.$$

Thus, there is an optimal solution p_m^* , w_m^* to maximize the revenue of the online platform.

Solving $\begin{cases} \frac{\partial \pi_m^1}{\partial p_m} = 0 \\ \frac{\partial \pi_m^1}{\partial w_m} = 0 \end{cases}$, then $\begin{cases} p_m^* = \frac{Q(2A_{m1}\alpha + (4-2a_m-b)\alpha\eta + A_{m1})\beta\eta}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} \\ w_m^* = \frac{2M(v-1)\gamma - Mv\sigma + \rho(2T_m\gamma + T_f\sigma)}{4\gamma^2 - 2\sigma^2} \end{cases}$

The Jacobian matrix of the servicer's revenue function is

$$H_S^{E-E} = \begin{bmatrix} -c_f & 0 \\ 0 & -c_m \end{bmatrix}.$$

The corresponding values of the principal minor sequence are

$$H_{s1}^{E-E} = -c_f < 0, \quad H_{s2}^{E-E} = c_f c_m > 0.$$

Thus, there is an optimal solution T_f^* , T_m^* to maximize the revenue of the servicer.

Solving $\begin{cases} \frac{\partial \pi_s^{11}}{\partial T_f} = 0 \\ \frac{\partial \pi_s^{11}}{\partial T_m} = 0 \end{cases}$, then $\begin{cases} T_f^* = \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_f\gamma^3 - 4c_f\gamma\sigma^2} \\ T_m^* = \frac{\rho(2\gamma + \sigma)}{4c_m\gamma^2 - 2c_m\sigma^2} \end{cases}$

By substituting T_f^* , T_m^* into p_m^* , w_m^* , p_f^* , w_f^* , Proposition 1 is proven.

A.2. Sensitivity analysis in scenario 1

Based on Proposition 1, we derive

$$\frac{\partial p_f^1}{\partial a_m} = \frac{bQ\alpha\beta(\beta + \alpha\eta - \beta\eta)}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \quad \frac{\partial p_f^1}{\partial b} = \frac{A_{m1}Q\alpha\beta(\beta\eta - \beta - \alpha\eta)}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0,$$

$$\frac{\partial p_f^1}{\partial Q} = \frac{\eta}{2\alpha} + \frac{\beta(2A_{m1} - (2A_{m1} + b)\alpha\eta + (A_{m1}\beta\eta))}{2\alpha(4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2)} > 0,$$

$$\frac{\partial p_f^1}{\partial \beta} = \frac{A_{m1}Q(A_{m1}\beta^2(\eta - 1) - 2A_{m1}\alpha\beta\eta + \alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta))}{-2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0,$$

$$\frac{\partial p_m^1}{\partial a_m} = \frac{bQ\alpha^2(\beta + \alpha\eta - \beta\eta)}{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \quad \frac{\partial p_m^1}{\partial b} = -\frac{A_{m1}Q\alpha^2(\beta + \alpha\eta - \beta\eta)}{(2A_{m1} + b\alpha\beta - A_{m1}\beta^2)^2} > 0,$$

$$\frac{\partial p_m^1}{\partial Q} = \frac{2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} > 0,$$

$$\frac{\partial p_m^1}{\partial \alpha} = \frac{A_{m1}Q(A_{m1}\beta^2(\eta - 1) - 2A_{m1}\alpha\beta\eta + \alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta))}{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0,$$

$$\frac{\partial p_m^1}{\partial \beta} = \frac{Q(b^2\alpha^2\eta + 2A_{m1}b\alpha(\alpha(\eta - 1) - \beta\eta) + A_{m1}^2(4\alpha\beta(1 - \eta) + 2\alpha^2\eta + \beta^2\eta))}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0,$$

$$\begin{aligned} \frac{\partial p_m^1}{\partial \eta} &= \frac{Q(A_{m1}\beta - (2A_{m1} + b)\alpha)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} < 0, \\ \frac{\partial w_f^1}{\partial c_m} &= \frac{-\rho^2\sigma A_{m2}}{4c_m^2(\sigma^2 - 2\gamma^2)^2} < 0, \quad \frac{\partial w_f^1}{\partial c_f} = \frac{\rho^2(\sigma - 2\gamma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{16c_f^2\gamma^2(\sigma^2 - 2\gamma^2)^2} < 0, \\ \frac{\partial w_f^1}{\partial M} &= \frac{2(v-1)\gamma\sigma + v\sigma^2 - 4v\gamma^2}{8\gamma^3 - 4\gamma\sigma^2} < 0, \quad \frac{\partial w_f^1}{\partial v} = \frac{M(2\gamma\sigma + \sigma^2 - 4\gamma^2)}{8\gamma^3 - 4\gamma\sigma^2} < 0, \\ \frac{\partial w_f^{11}}{\partial \rho} &= \frac{\rho A_{m2}(4c_m\gamma^2\sigma + c_m(8\gamma^3 - 4\gamma\sigma^2 + \sigma^3))}{8c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2} > 0, \\ \frac{\partial w_m^1}{\partial c_m} &= \frac{-\gamma\rho^2 A_{m2}}{2c_m^2(\sigma^2 - 2\gamma^2)^2} < 0, \quad \frac{\partial w_m^1}{\partial c_f} = \frac{\rho^2\sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)}{8c_m^2\gamma(\sigma^2 - 2\gamma^2)^2} < 0, \\ \frac{\partial w_m^1}{\partial M} &= \frac{2(v-1)\gamma - v\sigma}{4\gamma^2 - 2\sigma^2} < 0, \quad \frac{\partial w_m^1}{\partial v} = \frac{M(2\gamma - \sigma)}{4\gamma^2 - 2\sigma^2} > 0, \\ \frac{\partial w_m^1}{\partial \rho} &= \frac{\rho(8c_f\gamma^3 + 4(c_f + c_m)\gamma^2\sigma + 2c_m\gamma\sigma^2 - c_m\sigma^3)}{4c_m c_f\gamma(\sigma^2 - 2\gamma^2)^2} > 0, \\ \frac{\partial \pi_f^1}{\partial \alpha_f} &= \frac{(2A_{m1}Q\alpha\beta + Q(4A_{m1}\alpha^2 + (b - 2A_{m1})\alpha\beta - A_{m1}\beta^2)\eta)^2}{-16\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial \pi_f^1}{\partial a_m} &= \frac{b(2 - a_f - b)Q^2\alpha\beta(\beta + \alpha\eta - \beta\eta)(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1 - \eta) + (4 + b)\eta - 4))}{4(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^3} < 0, \\ \frac{\partial \pi_f^1}{\partial Q} &= \frac{(2 - a_f - b)Q(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1 - \eta) + (4 + b)\eta - 4))^2}{8\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \\ \frac{\partial \pi_f^1}{\partial \beta} &= \frac{A_{m1}(a_f + b - 2)Q^2(A_{m1}\beta^2(\eta - 1) - 2A_{m1})\alpha\beta\eta + \alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta)}{(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1 - \eta) + (4 + b)\eta - 4))}{4(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^3} < 0, \\ \frac{\partial \pi_f^1}{\partial c_m} &= \frac{\rho^2\sigma(2\gamma + \sigma)(c_m\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2\sigma A_{m2} - c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2)))}{-32c_f c_m^3\gamma(-2\gamma^2 + \sigma^2)^4} < 0, \\ \frac{\partial \pi_f^1}{\partial c_f} &= -\frac{1}{128c_f^3 c_m\gamma^3(\sigma^2 - 2\gamma^2)^4}\rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2)(c_m\rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(\gamma\rho^2\sigma A_{m2} - c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2))) < 0, \\ \frac{\partial \pi_f^1}{\partial v} &= \frac{M(4\gamma^2 - 2\gamma\sigma - \sigma^2)(c_m\rho^2(\sigma - 2\gamma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma(-\gamma\rho^2\sigma A_{m2} + c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2)))}{32c_m c_f\gamma^2(2\gamma^2 - \sigma^2)^3}, \\ \frac{\partial \pi_f^1}{\partial \rho} &= \frac{1}{64c_m^2 c_f^2\gamma^3(\sigma^2 - 2\gamma^2)^4}\rho A_{m2}(4c_f\gamma^2\sigma + c_m(8\gamma^3 - 4\gamma\sigma^2 + \sigma^3))(c_m\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma((\gamma\rho^2\sigma A_{m2} - c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2))) > 0, \\ \frac{\partial \pi_f^1}{\partial M} &= \frac{1}{32c_m c_f\gamma^2(2\gamma^2 - \sigma^2)^3}(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2)(c_m\rho^2(2\gamma - \sigma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f\gamma((\gamma\rho^2\sigma A_{m2} - c_m M(2\gamma^2 - \sigma^2)(4v\gamma^2 + 2(1 - v)\gamma\sigma - v\sigma^2))) < 0, \\ \frac{\partial \pi_m^1}{\partial b} &= \frac{Q^2(\alpha(4 + 2a_m(\eta - 1) + (b - 4)\eta) - A_{m1}\beta\eta)}{(4A_{m1}\alpha^2\eta - A_{m1}\beta^2\eta + \alpha\beta(2a_m(1 - \eta) + (4 + b)\eta - 4))}{-8(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial \pi_m^1}{\partial Q} &= \frac{Q(2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta)^2}{4\alpha(A_{m1}\beta^2 - 2A_{m1}\alpha^2 - b\alpha\beta)} > 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_m^1}{\partial \eta} &= \frac{Q^2((2A_{m1} + b)\alpha - A_{m1}\beta)(2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta)}{4\alpha(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} < 0, \\ \frac{\partial \pi_m^1}{\partial c_m} &= \frac{\rho^2 A_{m2}(c_m \rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f \gamma(\gamma \rho^2 A_{m2} + c_m M(2(v-1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))}{16c_f c_m^3 \gamma(\sigma^2 - 2\gamma^2)^3} < 0, \\ \frac{\partial \pi_m^1}{\partial c_f} &= \frac{\rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2)(c_m \rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f \gamma(\gamma \rho^2 A_{m2} + c_m M(2(v-1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))}{64c_m c_f^3 \gamma^3(\sigma^2 - 2\gamma^2)^3} < 0, \\ \frac{\partial \pi_m^1}{\partial v} &= \frac{M(2\gamma - \sigma)(c_m \rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f \gamma(\gamma \rho^2 A_{m2} + c_m M(2(v-1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))}{16c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2} > 0, \\ \frac{\partial \pi_m^1}{\partial \rho} &= \frac{\rho(8c_f \gamma^3 + 4(c_m + c_f)\gamma^2 \sigma + 2c_m \gamma \sigma^2 - c_m \sigma^3)(c_m \rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f \gamma(\gamma \rho^2 A_{m2} + c_m M(2(v-1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))}{32c_m^2 c_f^2(2\gamma^3 - \gamma\sigma^2)^3} > 0, \\ \frac{\partial \pi_m^1}{\partial M} &= \frac{(2(v-1)\gamma - v\sigma)(c_m \rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f \gamma(\gamma \rho^2 A_{m2} + c_m M(2(v-1)\gamma - v\sigma)(2\gamma^2 - \sigma^2)))}{16c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2} < 0, \\ \frac{\partial \pi_s^1}{\partial c_m} &= \frac{-\rho^2 A_{m2}^2}{8c_m^2(\sigma^2 - 2\gamma^2)^2} < 0, \quad \frac{\partial \pi_s^1}{\partial c_f} = -\frac{\rho^2(-4\gamma^2 - 2\gamma\sigma + \sigma^2)^2}{32c_f^2 \gamma^2(\sigma^2 - 2\gamma^2)^2} < 0, \\ \frac{\partial \pi_s^1}{\partial M} &= \frac{v\sigma^2 - 4\gamma^2 - 2\gamma\sigma}{8\gamma^3 - 4\gamma\sigma^2} < 0, \quad \frac{\partial \pi_s^1}{\partial v} = \frac{M\sigma^2}{8\gamma^3 - 4\gamma\sigma^2} > 0, \quad \frac{\partial \pi_s^1}{\partial \rho} = \frac{\rho(4c_f \gamma^2 A_{m2}^2 + c_m(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2)}{16c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2} > 0. \end{aligned}$$

Thus, Corollary 1 is proven.

A.3. Equilibrium status in scenario 2

In scenario 2, the Jacobian matrix of the online platform’s revenue function is

$$H_m^{E-E} = \begin{pmatrix} 2(A_{m1}\alpha + b\beta) & 0 \\ 0 & -2\gamma \end{pmatrix}.$$

If $0 < b < \left\{ -\frac{A_{m1}\alpha}{\beta}, 1 \right\}$, the corresponding values of the principal minor sequence are

$$H_{m1}^{E-E} = 2(A_{m1}\alpha + b\beta) < 0, \quad H_{m2}^{E-E} = -4(A_{m1}\alpha + b\beta)\gamma > 0.$$

Thus, there is an optimal solution p_m^*, w_m^* to maximize the revenue of the online platform.

Solving $\begin{cases} \frac{\partial \pi_m^2}{\partial p_m^*} = 0 \\ \frac{\partial \pi_m^2}{\partial w_m^*} = 0 \end{cases}$, then $\begin{cases} p_m^* = \frac{bp_f \alpha + A_{m1} p_f \beta + Q(A_{m1} - (A_{m1} + b)\eta)}{2A_{m1}\alpha + 2b\beta} \\ w_m^* = \frac{M(v-1) + \rho T_m + \sigma w_f}{2\gamma} \end{cases}$ is derived.

The Jacobian matrix of the physical firm’s revenue function is

$$H_f^{E-E} = \begin{pmatrix} \frac{(a_f + b - 2)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}{A_{m1}\alpha + b\beta} & 0 \\ 0 & \frac{\sigma^2}{\gamma} - 2\gamma \end{pmatrix}.$$

If $0 < b < \min\left\{ \frac{A_{m1}\beta}{\alpha} - \frac{2A_{m1}\alpha}{\beta}, 1 \right\}$, and $2\gamma^2 > \sigma^2$, the corresponding values of the principal minor sequence are

$$H_{f1}^{E-E} = \frac{(a_f + b - 2)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}{A_{m1}\alpha + b\beta} < 0,$$

$$H_{f2}^{E-E} = \frac{(2 - a_f - b)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(2\gamma^2 - \sigma^2)}{(A_{m1}\alpha + b\beta)\gamma} > 0.$$

Thus, there is an optimal solution p_f^*, w_f^* to maximize the revenue of the physical firm.

Solving $\begin{cases} \frac{\partial \pi_f^2}{\partial p_f} = 0 \\ \frac{\partial \pi_f^2}{\partial w_f} = 0 \end{cases}$, then $\begin{cases} p_f^* = \frac{Q(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2+b)\eta))}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} \\ w_f^* = \frac{M(v-1)\sigma + \rho(2\gamma T_f + \sigma T_m) - 2Mv\gamma}{4\gamma^2 - 2\sigma^2} \end{cases}$ is derived.

The Jacobian matrix of the servicer’s revenue function is

$$H_S^{E-E} = \begin{pmatrix} -c_f & 0 \\ 0 & -c_m \end{pmatrix}.$$

The corresponding values of the principal minor sequence are

$$H_{s1}^{E-E} = -c_f < 0, \quad H_{s2}^{E-E} = c_f c_m > 0.$$

Thus, there is an optimal solution T_f^*, T_m^* to maximize the revenue of the servicer.

Solving $\begin{cases} \frac{\partial \pi_s^2}{\partial T_f} = 0 \\ \frac{\partial \pi_s^2}{\partial T_m} = 0 \end{cases}$, then $\begin{cases} T_f^* = \frac{\rho A_{m2}}{4c_f\gamma^2 - 2c_f\sigma^2} \\ T_m^* = \frac{\rho(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{8c_m\gamma^3 - 4c_m\gamma\sigma^2} \end{cases}$ is derived.

By substituting T_f^*, T_m^* into $p_m^*, w_m^*, p_f^*, w_f^*$, Proposition 2 is proven.

A.4. Sensitivity analysis in scenario 2

Based on Proposition 2, we derive

$$\begin{aligned} \frac{\partial p_f^2}{\partial a_m} &= \frac{bQ\beta^2(\alpha - \alpha\eta + \beta\eta)}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, & \frac{\partial p_f^2}{\partial Q} &= \frac{2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2+b)\eta)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} > 0, \\ \frac{\partial p_f^2}{\partial \eta} &= \frac{Q(2A_{m1}\alpha + (b - A_{m1})\beta)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} > 0, \\ \frac{\partial p_f^2}{\partial \beta} &= \frac{A_{m1}Q(2A_{m1}\alpha^2(1 - \eta) + 4A_{m1}\alpha\beta\eta + \beta^2(A_{m1} - a_m\eta + (2+b)\eta))}{2(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} > 0, \\ \frac{\partial p_m^2}{\partial \eta} &= -\frac{(A_{m1} + b)Q}{4A_{m1}\alpha + 4b\beta} + \frac{A_{m1}Q(\beta - \alpha)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} > 0, \\ \frac{\partial w_f^2}{\partial c_m} &= \frac{\rho^2\sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)}{8c_m^2\gamma(\sigma^2 - 2\gamma^2)^2} < 0, & \frac{\partial w_f^2}{\partial c_f} &= \frac{-\gamma\rho^2 A_{m2}}{2c_f^2(-2\gamma^2 + \sigma^2)^2} < 0, & \frac{\partial w_f^2}{\partial M} &= \frac{(v-1)\sigma - 2v\gamma}{4\gamma^2 - 2\sigma^2} < 0, \\ \frac{\partial w_f^2}{\partial v} &= \frac{M(\sigma - 2\gamma)}{4\gamma^2 - 2\sigma^2} < 0, & \frac{\partial w_f^2}{\partial \rho} &= \frac{\rho(8c_m\gamma^3 + 4(c_f + c_m)\gamma^2\sigma + 2c_f\gamma\sigma^2 - c_f\sigma^3)}{4c_m c_f \gamma(\sigma^2 - 2\gamma^2)^2} > 0, \\ \frac{\partial w_m^2}{\partial c_m} &= \frac{\rho^2(\sigma - 2\gamma)A_{m2}(4\gamma^2 + 2\gamma\sigma - \sigma^2)}{16c_m^2\gamma^2(\sigma^2 - 2\gamma^2)^2} < 0, & \frac{\partial w_m^2}{\partial c_f} &= \frac{-\rho^2\sigma A_{m2}}{4c_f^2(-2\gamma^2 + \sigma^2)^2} < 0, \\ \frac{\partial w_m^2}{\partial M} &= \frac{1}{4} \left(\frac{-1 + v}{\gamma} + \frac{2(\gamma - v\gamma + v\sigma)}{\sigma^2 - 2\gamma^2} \right) < 0, & \frac{\partial w_m^2}{\partial v} &= \frac{M(4\gamma^2 - 2\gamma\sigma - \sigma^2)}{8\gamma^3 - 4\gamma\sigma^2} > 0, \\ \frac{\partial w_m^2}{\partial \rho} &= \frac{\rho A_{m2}(4c_m\gamma^2\sigma + c_f(8\gamma^3 - 4\gamma\sigma^2 + \sigma^3))}{8c_m c_f (\gamma\sigma^2 - 2\gamma^3)^2} > 0, \\ \frac{\partial \pi_f^2}{\partial a_f} &= \frac{Q^2(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2+b)\eta))^2}{8(A_{m1}\alpha + b\beta)(A_{m1}\beta^2 - 2A_{m1}\alpha^2 - b\alpha\beta)} < 0, \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_f^2}{\partial a_m} &= \frac{b(2 - a_f - b)Q^2\beta\left(\beta(3A_{m1}\alpha^2 + 2b\alpha\beta - A_{m1}\beta^2) + (\alpha - \beta)^2(2A_{m1}\alpha + (A_{m1} + b)\beta)\eta\right)}{(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2 + b)\eta))} > 0, \\ \frac{\partial \pi_f^2}{\partial M} &= \frac{(2v\gamma + \sigma - v\sigma)(+4c_f c_m M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f \rho^2 \sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma) - 4c_m \gamma^2 \rho^2(2\gamma + \sigma))}{16c_f c_m (\gamma\sigma^2 - 2\gamma^3)^2} > 0, \\ \frac{\partial \pi_f^2}{\partial v} &= \frac{M(2\gamma - \sigma)(4c_f c_m M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f \rho^2 \sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma) - 4c_m \gamma^2 \rho^2(2\gamma + \sigma))}{16c_f c_m (\gamma\sigma^2 - 2\gamma^3)^2} > 0, \\ \frac{\partial \pi_f^2}{\partial Q} &= \frac{(2 - a_f - b)Q(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2 + b)\eta))^2}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0, \\ \frac{\partial \pi_f^2}{\partial c_f} &= \frac{\rho^2(2\gamma + \sigma)(4c_f c_m M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f \rho^2 \sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma) - 4c_m \gamma^2 \rho^2(2\gamma + \sigma))}{16c_f^3 c_m \gamma(2\gamma^2 - \sigma^2)^3} > 0, \\ \frac{\partial \pi_f^2}{\partial c_m} &= \frac{\rho^2 \sigma(4\gamma^2 + 2\gamma\sigma - \sigma^2)(4c_f c_m M\gamma(2v\gamma + \sigma - v\sigma)(2\gamma^2 - \sigma^2) + c_f \rho^2 \sigma(\sigma^2 - 4\gamma^2 - 2\gamma\sigma) - 4c_m \gamma^2 \rho^2(2\gamma + \sigma))}{64c_f c_m^3 \gamma^3(2\gamma^2 - \sigma^2)^3} > 0, \\ \frac{\partial \pi_f^2}{\partial \eta} &= \frac{(2 - a_f - b)Q^2(2A_{m1}\alpha + (b - A_{m1})\beta)(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2 + b)\eta))}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0, \\ \frac{\partial \pi_m^2}{\partial Q} &= -\frac{Q(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))^2}{8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} < 0, \\ \frac{\partial \pi_m^2}{\partial M} &= \frac{(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2)(4c_m \gamma^2 \rho^2 \sigma(2\gamma + \sigma) + c_f \rho^2(4 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2))}{32c_f c_m \gamma^2(2\gamma^2 - \sigma^2)^3} < 0, \\ \frac{\partial \pi_m^2}{\partial v} &= \frac{M(4\gamma^2 - 2\gamma\sigma - \sigma^2)(c_m \gamma^2 \rho^2 \sigma(2\gamma + \sigma) + c_f \rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma - (v - 1)\sigma^2))}{32c_f c_m \gamma^2(2\gamma^2 - \sigma^2)^3} > 0, \\ \frac{\partial \pi_m^2}{\partial \rho} &= \frac{\rho(2\gamma + \sigma)(4c_m \gamma^2 \rho^2 \sigma(2\gamma + \sigma) + c_f \rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma + (v - 1)\sigma^2))}{64c_f^2 c_m^2 \gamma^3(\sigma^2 - 2\gamma^2)^4} > 0, \\ \frac{\partial \pi_m^2}{\partial c_f} &= -\frac{\rho^2 \sigma(2\gamma + \sigma)(4c_m \gamma^2 \rho^2 \sigma(2\gamma + \sigma) + c_f \rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma + (v - 1)\sigma^2))}{32c_f^3 c_m \gamma(\sigma^2 - 2\gamma^2)^4} < 0, \\ \frac{\partial \pi_m^2}{\partial c_m} &= -\frac{\rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2)(4c_m \gamma^2 \rho^2 \sigma(2\gamma + \sigma) + c_f \rho^2(4\gamma^2 - \sigma^2)(4\gamma^2 + 2\gamma\sigma - \sigma^2) + 4c_f c_m M\gamma(2\gamma^2 - \sigma^2)(4(v - 1)\gamma^2 - 2v\gamma\sigma + (v - 1)\sigma^2))}{128c_f c_m^3 \gamma^3(\sigma^2 - 2\gamma^2)^4} < 0, \\ \frac{\partial \pi_s^2}{\partial c_m} &= \frac{-\rho^2(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2}{32c_m^2 \gamma^2(-2\gamma^2 + \sigma^2)^2} < 0, \quad \frac{\partial \pi_s^2}{\partial c_f} = \frac{-\rho^2(2\gamma + \sigma)^2}{8c_f^2(\sigma^2 - 2\gamma^2)^2} < 0, \quad \frac{\partial \pi_s^2}{\partial M} = \frac{1}{4}\left(\frac{-1 + v}{\gamma} + \frac{2(\gamma + v\gamma + \sigma)}{-2\gamma^2 + \sigma^2}\right), \end{aligned}$$

$$\frac{\partial \pi_s^2}{\partial v} = \frac{M\sigma^2}{4\gamma\sigma^2 - 8\gamma^3} < 0, \quad \frac{\partial \pi_s^2}{\partial \rho} = \frac{\rho(4c_m\gamma^2(2\gamma + \sigma)^2 + c_f(\sigma^2 - 4\gamma^2 - 2\gamma\sigma)^2)}{16c_m c_f(\gamma\sigma^2 - 2\gamma^3)^2} > 0.$$

Thus, Corollary 3 is proven.

A.5. Equilibrium status in scenario 3

In scenario 3, the Jacobian matrix of the physical firm’s revenue function is

$$H_f^{S-S} = \begin{pmatrix} 2(a_f + b - 2)\alpha - 2\lambda\theta_f^2 T_f & T_f(T_f\rho + \sigma\theta_m p_m - Mv - 4\lambda\theta_f T_f) \\ T_f(T_f\rho + \sigma\theta_m p_m - Mv - 4\lambda\theta_f T_f) & -2\lambda T_f p_f^2 \end{pmatrix}.$$

If $0 < b < \min\{b_{10}, 1\}$, the corresponding values of the principal minor sequence are

$$2(a_f + b - 2)\alpha - 2\lambda\theta_f^2 T_f < 0,$$

$$H_{f2}^{S-S} = T_f(4(2 - a_f - b)\alpha\lambda p_f^2 + T_f(\rho T_f + \sigma\theta_m p_m - Mv - 6\lambda\theta_f p_f)(T_f\rho + \sigma\theta_m p_m - Mv - 2\lambda\theta_f p_f)) > 0.$$

Thus, there is an optimal solution p_f^*, θ_f^* to maximize the revenue of the physical firm.

Solving $\begin{cases} \frac{\partial \pi_f^3}{\partial p_f} = 0 \\ \frac{\partial \pi_f^3}{\partial \theta_f} = 0 \end{cases}$, then $\begin{cases} p_f^* = \frac{\beta p_m + Q\eta}{2\alpha} \\ \theta_f^* = \frac{\alpha(\rho T_f + \sigma\theta_m p_m - Mv)}{(\beta p_m + Q\eta)\lambda} \end{cases}$ is derived.

The Jacobian matrix of the online platform’s revenue function is

$$H_m^{S-S} = \begin{pmatrix} 2A_{m1}\alpha + b\beta - \frac{A_{m1}\beta^2}{\alpha} + \frac{\theta_m^2 T_m(\sigma^2 - 2\lambda^2)}{\lambda} & \frac{T_m(2M(v-1)\lambda - Mv\sigma + \rho(2\lambda T_m + \sigma T_f) + 4\theta_m p_m(\sigma^2 - 2\lambda^2))}{\lambda} \\ \frac{T_m(2M(v-1)\lambda - Mv\sigma + \rho(2\lambda T_m + \sigma T_f) + 4\theta_m p_m(\sigma^2 - 2\lambda^2))}{2\lambda} & \frac{T_m p_m^2(\sigma^2 - 2\lambda^2)}{\lambda} \end{pmatrix}.$$

If $0 < b < \min\{b_{11}, b_{12}, 1\}$, the corresponding values of the principal minor sequence are

$$H_{m1}^{S-S} = 2A_{m1}\alpha + b\beta - \frac{A_{m1}\beta^2}{\alpha} + \frac{\theta_m^2 T_m(\sigma^2 - 2\lambda^2)}{\lambda} < 0,$$

$$H_{m2}^{S-S} = -\frac{1}{4\alpha\lambda^2} T_m(M^2\alpha T_m(2(1-v)\lambda + v\sigma)^2 + \alpha\rho^2 T_m(2\lambda T_m + \sigma T_f)^2 - 8\alpha\rho p_m\theta_m T_m(2\lambda T_m + \sigma T_f)(2\lambda^2 - \sigma^2) + 4p_m^2(2\lambda^2 - \sigma^2)(2A_{m1}\alpha^2\lambda + b\alpha\beta\lambda - A_{m1}\beta^2\lambda + 3\alpha\theta_m^2 T_m(2\lambda^2 - \sigma^2)) + 2M\alpha T_m(2(v-1)\lambda - v\sigma)(\rho(2\lambda T_m + \sigma T_f) + 4\theta_m p_m(\sigma^2 - 2\lambda^2))) > 0.$$

Thus, there is an optimal solution p_m^*, θ_m^* to maximize the revenue of the online platform.

Solving $\begin{cases} \frac{\partial \pi_m^3}{\partial p_m} = 0 \\ \frac{\partial \pi_m^3}{\partial \theta_m} = 0 \end{cases}$, then $\begin{cases} p_m^* = \frac{Q(2A_{m1}\alpha - (2A_{m1} + b)\alpha\eta + A_{m1}\beta\eta)}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} \\ \theta_m^* = -\frac{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(2M(v-1)\lambda - Mv\sigma + \rho(2\lambda T_m + \sigma T_f))}{Q(-A_{m1}\beta\eta + \alpha(4 + 2a_m(-1 + \eta) + (-4 + b)\eta))(2\lambda^2 - \sigma^2)} \end{cases}$ is derived.

The Jacobian matrix of the servicer’s revenue function is

$$H_S^{S-S} = \begin{pmatrix} \frac{\rho}{2\lambda} + \frac{\lambda\rho}{2\lambda^2 - \sigma^2} - c_f & \frac{\rho\sigma}{2\lambda^2 - \sigma^2} \\ \frac{\rho\sigma}{2\lambda^2 - \sigma^2} & \frac{\rho\sigma}{\sigma^2 - 2\lambda^2} - c_m \end{pmatrix}.$$

If $c_f > \max\left\{\frac{\rho}{2\lambda} + \frac{\lambda\rho}{2\lambda^2 - \sigma^2}, \frac{\rho}{2\lambda} + \frac{\rho(\rho - \lambda c_m)}{2\lambda\rho + c_m(\sigma^2 - 2\lambda^2)}\right\}$, and $2\lambda^2 > \sigma^2$, the corresponding values of the principal minor sequence are

$$H_{s1}^{S-S} = \frac{\rho}{2\lambda} + \frac{\lambda\rho}{2\lambda^2 - \sigma^2} - c_f > 0, \quad H_{s2}^{S-S} = \frac{4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2}{4\lambda^3 - 2\lambda\sigma^2} < 0.$$

Thus, there is an optimal solution T_f^*, T_m^* to maximize the revenue of the servicer.

Solving $\begin{cases} \frac{\partial \pi_s^3}{\partial T_f} = 0 \\ \frac{\partial \pi_s^3}{\partial T_m} = 0 \end{cases}$, then $\begin{cases} T_f^* = \frac{M(4v\lambda(\rho - \lambda c_m) + 2c_m(v-1)\lambda\sigma + c_m v\sigma^2)}{8\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + 2c_m(\rho - 2\lambda c_f)\sigma^2} \\ T_m^* = \frac{M\lambda(2(v-1)(\lambda c_f - \rho) - v\sigma c_f)}{4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2} \end{cases}$ is derived.

By substituting T_f^*, T_m^* into $p_m^*, w_m^*, p_f^*, w_f^*$, Proposition 3 is proven.

A.6. Sensitivity analysis in scenario 3

Based on Proposition 3, we derive

$$\begin{aligned} \frac{\partial \theta_f^3}{\partial c_m} &= \frac{-4c_f M \alpha (2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2) \lambda^2 \rho \sigma (2(v-1)(\lambda c_f - \rho) - v\sigma c_f)}{Q(4A_{m1} \alpha^2 \eta - A_{m1} \beta^2 \eta + \alpha\beta(2a_m(1-\eta) + (4+b)\eta - 4))(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} < 0, \\ \frac{\partial \theta_f^3}{\partial c_f} &= \frac{M \alpha (2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2) \rho (4v\lambda(\rho - \lambda c_m) + 2c_m(v-1)\lambda\sigma + v\sigma^2 c_m) (4\lambda\rho + c_m(\sigma^2 - 4\lambda^2))}{Q(4A_{m1} \alpha^2 \eta - A_{m1} \beta^2 \eta + \alpha\beta(2a_m(1-\eta) + (4+b)\eta - 4))(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} > 0, \\ \frac{\partial \theta_m^3}{\partial c_m} &= \frac{4M(2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2) \lambda^2 (\lambda c_f - \rho) \rho (2(v-1)(\lambda c_f - \rho) - v\sigma c_f)}{Q(\alpha(4 + 2a_m(\eta - 1) + (b-4)\eta) - A_{m1} \beta \eta) (4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} > 0, \\ \frac{\partial \theta_m^3}{\partial c_f} &= \frac{-c_m M (2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2) \lambda \rho \sigma (4v\lambda(\lambda c_m - \rho) + 2c_m(1-v)\lambda\sigma - c_m v \sigma^2)}{Q(\alpha(4 + 2a_m(\eta - 1) + (b-4)\eta) - A_{m1} \beta \eta) (4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} < 0, \\ \frac{\partial \pi_f^3}{\partial M} &= \frac{K_3}{K_4} < 0, \\ K_3 &= -(3M^2(4v\lambda(\lambda c_m - \rho) + 2c_m(1-v)\lambda\sigma - c_m v \sigma^2)(16v\lambda^3(\rho - 2\lambda c_f)(\rho - \lambda c_m) \\ &\quad + 8(1-v)\lambda^2(2c_m c_f \lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma \\ &\quad - 4v\lambda(\rho - 3c_f \lambda)(\rho - 2c_m \lambda)\sigma^2 + 2c_m(1-v)\lambda(\rho - 4c_f \lambda)\sigma^3 + c_m v(4c_f \lambda - \rho)\sigma^4)^2, \\ K_4 &= 128\lambda(\sigma^2 - 2\lambda^2)^2(4\lambda(c_f \lambda - \rho)(c_m \lambda - \rho) + c_m(\rho - 2c_f \lambda)\sigma^2)^3, \\ \frac{\partial \pi_f^3}{\partial Q} &= \frac{(2 - a_f - b)Q(4A_{m1} \alpha^2 \eta - A_{m1} \beta^2 \eta + \alpha\beta(2a_m(1-\eta) + (4+b)\eta - 4))^2}{8\alpha(2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2)^2} > 0, \\ \frac{\partial \pi_f^3}{\partial a_m} &= \frac{b(2 - a_f - b)Q^2 \alpha \beta (\beta + \alpha \eta - \beta \eta) (4A_{m1} \alpha^2 \eta - A_{m1} \beta^2 \eta + \alpha \beta (2a_m(1-\eta) + (4+b)\eta - 4))}{4(2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2)^3} > 0, \\ \frac{\partial \pi_f^3}{\partial a_f} &= -\frac{(2A_{m1} Q \alpha \beta + Q(4A_{m1} \alpha^2 + (b - 2A_{m1}) \alpha \beta - A_{m1} \beta^2) \eta)^2}{16\alpha(2A_{m1} \alpha^2 + b\alpha\beta - A_{m1} \beta^2)^2} < 0, \\ \frac{\partial \pi_m^3}{\partial M} &= \frac{3M^2(2(v-1)(\lambda c_f - \rho) - v\sigma c_f)(4\lambda c_f(2(v-1)\lambda - v\sigma)(2\lambda^2 c_m - \lambda\rho - \sigma^2 c_m) \\ &\quad + \rho(8(v-1)\lambda^2(\rho - 2\lambda c_m) + 4v\lambda(\lambda c_m - \rho)\sigma + 6c_m(v-1)\lambda\sigma^2 - c_m v \sigma^3))^2}{32(2\lambda^2 - \sigma^2)(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^3} > 0, \\ \frac{\partial \pi_m^3}{\partial Q} &= \frac{Q(2A_{m1} \alpha - (2A_{m1} + b)\alpha\eta + A_{m1} \beta \eta)^2}{4\alpha(A_{m1} \beta^2 - 2A_{m1} \alpha^2 - b\alpha\beta)} < 0, \\ \frac{\partial \pi_m^3}{\partial a_m} &= \frac{Q^2(A_{m1} \beta \eta - \alpha(4 + 2a_m(\eta - 1) + (b-4)\eta))(A_{m1} \beta^3 \eta + 2\alpha^2 \beta(2b(\eta - 1) - A_{m1} \eta) \\ &\quad - \alpha \beta^2(4 + 2a_m(\eta - 1) + (b-4)\eta) + \alpha^3(8 + 4a_m(\eta - 1) - 2(4+b)\eta))}{8\alpha(2A_{m1} + b\alpha\beta - A_{m1} \beta^2)^2} < 0, \\ \frac{\partial \pi_s^3}{\partial c_m} &= \frac{-M^2 \lambda^2 (2(1-v)(\lambda c_f - \rho) + v\sigma c_f)^2}{2(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} < 0, \\ \frac{\partial \pi_s^3}{\partial c_f} &= \frac{M^2(4v\lambda(\rho - \lambda c_m) + 2c_m(v-1)\lambda\sigma + c_m v \sigma^2)^2}{8(4\lambda(\lambda c_f - \rho)(\lambda c_m - \rho) + c_m(\rho - 2\lambda c_f)\sigma^2)^2} < 0. \end{aligned}$$

Thus, Corollary 5 is proven.

A.7. Equilibrium status in scenario 4

In scenario 4, the Jacobian matrix of the online platform's revenue function is

$$H_m^{S-S} = \begin{pmatrix} 2(A_{m1}\alpha + b\beta - \lambda\theta_m^2 T_m) & T_m(M(v-1) - 4\lambda\theta_m p_m + \rho T_m + \sigma\theta_f p_f) \\ T_m(M(v-1) - 4\lambda\theta_m p_m + \rho T_m + \sigma\theta_f p_f) & -2\lambda T_m p_m^2 \end{pmatrix}.$$

If $0 < b < \min\{b_{13}, b_{14}, 1\}$, the corresponding values of the principal minor sequence are

$$\begin{aligned} H_{m1}^{S-S} &= 2(A_{m1}\alpha + b\beta - \lambda\theta_m^2 T_m) < 0, \\ H_{m2}^{S-S} &= -T_m(4p_m^2(A_{m1}\alpha + b\beta)\lambda + T_m(M(v-1) - 6\lambda\theta_m p_m + \rho T_m + \sigma\theta_f p_f)(M(v-1) - 2\lambda \\ &\quad \theta_m p_m + \rho T_m + \sigma\theta_f p_f)) > 0. \end{aligned}$$

Thus, there is an optimal solution p_m^*, θ_m^* to maximize the revenue of the online platform.

$$\begin{cases} \frac{\partial \pi_m^4}{\partial p_m} = 0 \\ \frac{\partial \pi_m^4}{\partial \theta_m} = 0 \end{cases}, \begin{cases} p_m^* = \frac{b\alpha p_f + \beta A_{m1} p_f + Q(A_{m1} - (A_{m1} + b)\eta)}{2A_{m1}\alpha + 2b\beta} \\ \theta_m^* = \frac{(A_{m1}\alpha + b\beta)(M(v-1) + \rho T_m + \sigma\theta_f p_f)}{(p_f(b\alpha + A_{m1}\beta) - Q((A_{m1} + b)\eta - A_{m1}))\lambda} \end{cases}$$

The Jacobian matrix of the physical firm's revenue function is

$$H_f^{E-E} = \begin{pmatrix} \frac{(a_f + b - 2)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}{A_{m1}\alpha + b\beta} + \frac{\theta_f^2 T_f (\sigma^2 - 2\lambda^2)}{\lambda} & \frac{T_f(2\lambda(\rho T_f - Mv - 4\lambda\theta_f p_f) + M(v-1)\sigma + \rho\sigma T_m + 4\sigma^2\theta_f p_f)}{2\lambda} \\ \frac{T_f(2\lambda(\rho T_f - Mv - 4\lambda\theta_f p_f) + M(v-1)\sigma + \rho\sigma T_m + 4\sigma^2\theta_f p_f)}{2\lambda} & \frac{T_f p_f^2 (\sigma^2 - 2\lambda^2)}{\lambda} \end{pmatrix}.$$

If $0 < b < \min\{b_{13}, \frac{-A_{m1}\alpha}{\beta}, 1\}$, $2\lambda^2 > \sigma^2$, and $\max\{a_{f1}, a_{f2}\} < a_f$, the corresponding values of the principal minor sequence are

$$\begin{aligned} H_{f1}^{S-S} &= \frac{(a_f + b - 2)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)}{A_{m1}\alpha + b\beta} + \frac{\theta_f^2 T_f (\sigma^2 - 2\lambda^2)}{\lambda} < 0, \\ H_{f2}^{S-S} &= \frac{1}{4\lambda^2} T_f (-M^2 T_f (2v\lambda + \sigma - v\sigma)^2 - T_f \rho^2 (2T_f \lambda + \sigma T_m)^2 + 8\theta_f p_f T_f \rho (2\lambda T_f + \sigma T_m)(2\lambda^2 - \sigma^2) \\ &\quad - \frac{4p_f^2 (2\lambda^2 - \sigma^2) (\lambda((a_f + b - 2)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2) + 6T_f (A_{m1}\alpha + b\beta)\theta_f^2 \lambda) - 3T_f (A_{m1}\alpha + b\beta)\theta_f^2 \sigma^2}{A_{m1}\alpha + b\beta} \\ &\quad + 2MT_f (2v\lambda + \sigma - v\sigma)(\rho(2\lambda T_f + \sigma T_m) + 4\theta_f p_f (\sigma^2 - 2\lambda^2))) > 0. \end{aligned}$$

Thus, there is an optimal solution p_f^*, θ_f^* to maximize the revenue of the physical firm.

$$\begin{cases} \frac{\partial \pi_f^4}{\partial p_f} = 0 \\ \frac{\partial \pi_f^4}{\partial \theta_f} = 0 \end{cases}, \begin{cases} p_f^* = \frac{Q(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + (2+b)\eta))}{4A_{m1}\alpha^2 + 2b\alpha\beta - 2A_{m1}\beta^2} \\ \theta_f^* = \frac{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(M(2v\lambda + \sigma - v\sigma) - \rho(2\lambda T_f + \sigma T_m))}{Q(+\beta(2+a_m(\eta-1) - (2+b)\eta) - 2A_{m1}\alpha\eta)(2\lambda^2 - \sigma^2)} \end{cases}$$

The Jacobian matrix of the servicer's revenue function is

$$H_S^{S-S} = \begin{pmatrix} \frac{2\lambda\rho}{2\lambda^2 - \sigma^2} - c_f & \frac{\rho\sigma}{2\lambda^2 - \sigma^2} \\ \frac{\rho\sigma}{2\lambda^2 - \sigma^2} & \frac{\rho}{2\lambda} + \frac{\lambda\rho}{2\lambda^2 - \sigma^2} - c_m \end{pmatrix}.$$

If $c_f > \max\left\{\frac{2\lambda\rho}{2\lambda^2 - \sigma^2}, \frac{4\lambda(c_m\lambda - \rho)\rho}{4c_m\lambda^3 - 4\lambda^2\rho - 2c_m\lambda\sigma^2 + \rho\sigma^2}\right\}$, the corresponding values of the principal minor sequence are

$$H_{s1}^{S-S} = \frac{2\lambda\rho}{2\lambda^2 - \sigma^2} - c_f < 0, \quad H_{s2}^{S-S} = \frac{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2}{4\lambda^3 - 2\lambda\sigma^2} > 0.$$

Thus, there is an optimal solution T_f^*, T_m^* to maximize the revenue of the servicer.

$$\text{Solving } \begin{cases} \frac{\partial \pi_s^{22}}{\partial T_f} = 0 \\ \frac{\partial \pi_s^{22}}{\partial T_m} = 0 \end{cases}, \text{ then } \begin{cases} T_f^* = \frac{M\lambda(2v\rho + c_m(v-1)\sigma - 2c_m v\lambda)}{4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2} \\ T_m^* = \frac{M(4(v-1)\lambda(c_f\lambda - \rho) - 2c_f v\lambda\sigma + c_f(1-v)\sigma^2)}{8\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + 2c_f(\rho - 2c_m\lambda)\sigma^2} \end{cases} \text{ is derived.}$$

By substituting T_f^*, T_m^* into $p_m^*, w_m^*, p_f^*, w_f^*$, Proposition 4 is proven.

A.8. Sensitivity analysis in scenario 4

Based on Proposition 4, we derive

$$\begin{aligned} \frac{\partial \theta_f^4}{\partial M} &= \frac{(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(8v\lambda^2(2c_f\lambda - \rho)(c_m\lambda - \rho) + 4(1-v)\lambda(2c_fc_m\lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_fv\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v-1)(4c_m\lambda - \rho)\sigma^3)}{2Q(\beta(2 + a_m(\eta - 1) - (2 + b)\eta) - 2A_{m1}\alpha\eta)(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} \\ &< 0, \\ \frac{\partial \theta_f^4}{\partial v} &= \frac{M(2A_{m1} + b\alpha\beta - A_{m1}\beta^2)(4\lambda(c_m\lambda - \rho)\rho(\sigma - 2\lambda) + 4c_fc_m\lambda(2\lambda - \sigma)(2\lambda^2 - \sigma^2) - c_f\rho(16\lambda^3 - 4\lambda^2\sigma - 6\lambda\sigma^2 + \sigma^3))}{2Q(+\beta(2 + a_m(\eta - 1) - (2 + b)\eta) - 2A_{m1}\alpha\eta)(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} \\ &< 0, \\ \frac{\partial \theta_f^4}{\partial Q} &= -\frac{M(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(8v\lambda^2(2c_f\lambda - \rho)(c_m\lambda - \rho) + 4(1-v)\lambda(2c_fc_m\lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_fv\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v-1)(4c_m\lambda - \rho)\sigma^3)}{2Q^2(\beta(2 + a_m(\eta - 1) - (2 + b)\eta) - 2A_{m1}\alpha\eta)(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} \\ &> 0, \\ \frac{\partial \theta_f^4}{\partial a_m} &= \frac{bM\beta^2(\alpha - \alpha\eta + \beta\eta)(8v\lambda^2(2c_f\lambda - \rho)(c_m\lambda - \rho) + 4(1-v)\lambda(2c_fc_m\lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_fv\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v-1)(4c_m\lambda - \rho)\sigma^3)}{2Q(\beta(2 + a_m(\eta - 1) - (2 + b)\eta) - 2A_{m1}\alpha\eta)^2(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} \\ &> 0, \\ \frac{\partial \theta_f^4}{\partial \eta} &= \frac{M(2A_{m1}\alpha + (b - A_{m1})\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)(8v\lambda^2(2c_f\lambda - \rho)(c_f\lambda - \rho) + 4(1-v)\lambda(2c_fc_m\lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_fv\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v-1)(4c_m\lambda - \rho)\sigma^3)}{2Q(\beta(2 + a_m(\eta - 1) - (2 + b)\eta) - 2A_{m1}\alpha\eta)^2(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} \\ &> 0, \\ \frac{\partial \pi_f^4}{\partial M} &= \frac{3M^2(8v\lambda^2(2c_f\lambda - \rho)(c_m\lambda - \rho) + 4(1-v)\lambda(2c_fc_m\lambda^2 - (c_f + c_m)\lambda\rho + \rho^2)\sigma + 2c_fv\lambda(3\rho - 4c_m\lambda)\sigma^2 + c_f(v-1)(4c_m\lambda - \rho)\sigma^3)^2(c_m(2v\lambda + \sigma - v\sigma) - 2v\rho)}{32(\sigma^2 - 2\lambda^2)(4\lambda(c_f\lambda - \rho)(c_f\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^3} \\ &> 0, \\ \frac{\partial \pi_f^{22}}{\partial Q} &= \frac{(2 - a_f - b)Q(2A_{m1}\alpha\eta + \beta(A_{m1} - a_m\eta + A_{m1}\eta))^2}{4(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)} > 0, \\ \frac{\partial \pi_m^{22}}{\partial M} &= \frac{K_1}{K_2} > 0, \\ K_1 &= 3M^2(4(v-1)\lambda(c_f\lambda - \rho) - 2c_fv\lambda\sigma + c_f(1-v)\sigma^2) \\ &\quad (4\lambda\rho(4(1-v)\lambda^2(2c_m\lambda - \rho) + 2v\lambda(c_m\lambda - \rho)\sigma + (v-1)(3c_m\lambda - \rho)\sigma^2) \\ &\quad + c_f(4(v-1)\lambda^2 - 2v\lambda\sigma + (1-v)\sigma^2)(8c_m\lambda^3 - 4\lambda^2\rho - 4c_m\lambda\sigma^2 + \rho\sigma^2))^2, \\ K_2 &= 128\lambda(\sigma^2 - 2\lambda^2)^2(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^3, \\ \frac{\partial \pi_m^{22}}{\partial Q} &= \frac{Q(2A_{m1}\alpha^2(4 + 2a_m(\eta - 1) + (b - 4)\eta) + \alpha\beta(3A_{m1}b(\eta - 1) - 2A_{m1}^2\eta + b^2\eta) + A_{m1}\beta^2(A_{m1} - (A_{m1} + 3b)\eta))^2}{-8(A_{m1}\alpha + b\beta)(2A_{m1}\alpha^2 + b\alpha\beta - A_{m1}\beta^2)^2} \\ &> 0, \end{aligned}$$

$$\frac{\partial \pi_s^{22}}{\partial M} = \frac{M(c_f(4(1-v)\lambda^2 + 2v\lambda\sigma + (v-1)\sigma^2)^2 + 4\lambda(c_m\lambda(2v\lambda + \sigma - v\sigma)^2 + \rho\lambda^2((8(1-v)v - 4) + 4(v-1)v\lambda\sigma + (v-1)^2\sigma^2)))}{8\lambda(2\lambda^2 - \sigma^2)(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)} > 0,$$

$$\frac{\partial \pi_s^{22}}{\partial c_m} = \frac{M^2(4(1-v)\lambda(c_f\lambda - \rho) + 2c_f v\lambda\sigma + c_f(v-1)\sigma^2)^2}{-8(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^2} < 0,$$

$$\frac{\partial \pi_s^{22}}{\partial c_f} = \frac{-M^2\lambda^2(c_m(2v\lambda + \sigma - v\sigma) - 2v\rho)^2}{2(4\lambda(c_f\lambda - \rho)(c_m\lambda - \rho) + c_f(\rho - 2c_m\lambda)\sigma^2)^2} < 0.$$

Thus, Corollary 7 is proven.

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Code availability. Code will be available from the authors on request.

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