QUALITY AND PRICE COMPETITION IN A DUOPOLY UNDER PRODUCT LIABILITY AND TRACEABILITY

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Abstract. This study aims to explore how traceability affects quality and price competition in a duopoly. A theoretical model is developed in which both enterprises are liable to recall low-quality (defective) products identified by traceability and compensate consumers for the disutility caused by unidentified low-quality products. The equilibrium results are derived by determining the optimal product quality and sales price decisions. The key findings are highlighted below. (1) Traceability affects product quality primarily through the expected social cost (the sum of expected recall cost and expected consumer disutility), i.e., higher (lower) expected social cost induced by increased traceability of each enterprise leads to higher (lower) product quality for both enterprises, enhancing (reducing) quality competition. (2) If an enterprise’s improved (decreased) product quality is driven by its own increased traceability, it will result in lower (higher) consumer demand and profit, while it will result in higher (lower) consumer demand and profit if caused by its competitor’s increased traceability. (3) If an enterprise’s increased traceability results in higher expected social cost, the enterprise’s sales price falls while the competitor’s sales price rises; otherwise, both enterprises’ sales prices fall, leading to greater price competition. (4) Quality competition is unaffected by product liability but is improved by increased recall cost and consumer disutility, whereas price competition is intensified by increased consumer disutility but is reduced by increased product liability and recall cost. Managerial insights are also discussed.

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1. Introduction

In recent years, product quality problem has attracted growing attention due to severe consumer injuries and enormous financial losses for enterprises. For instance, Toyota recalled nearly 9 million vehicles in 2010 due to the unintended acceleration of the gas pedal [21]. In 2021, GM cost $1.2 billion for recalling and repairing over 7 million vehicles in connection with the faulty Takata airbags [34]. When confronted with product quality incidents, enterprises are generally motivated to improve product quality. Nevertheless, a more pressing issue for

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enterprises is whether and how they can identify and recall quality defective products in a timely and effective manner when detecting certain prior indications of a potential quality defective problem [30]. If enterprises are successful in this endeavor, their liability costs to compensate customers for damages will be reduced [3]. Traceability is typically regarded as an essential strategic tool for identifying the sources of quality problems, improving the effectiveness and efficiency of product recalls, and assuring the quality and safety of products [7].

With an effective traceability system, complete information about every aspect, stage, and step of a product can be recorded [61] and shared among all links in the chain [32], allowing every enterprise to track each unit of product from production to delivery to final consumers and trace the sources of quality defective problems from final consumers to the entire supply chain [65]. Enterprises can then quickly and effectively identify the sources of quality defective problems, including the responsible enterprise(s), process(es), and material(s) that may have caused the defects. This helps target recall batches and avoid recalling non-defective products [17,25]. Moreover, the more quality defective products that are recalled, the lower the risk of product harm that consumers experience as a result of a defect [30]. Accordingly, the (expected) liability cost will be limited and reduced given the empirical evidence that the unit recall cost per product is generally lower than the liability cost [30]. According to existing research, enterprises usually are inspired to improve product quality mainly because of higher allocated costs by a liability transfer (or cost-sharing) mechanism dependent on traceability [10,53]. Naturally, there is some doubt regarding whether an enterprise still has an incentive to improve product quality if traceability reduces the enterprise’s overall expenses for product recall and liability compensation.

Actually, traceability is widely applied by food, clothing, pharmaceutical, and electron enterprises and so on, aiming to ensure and improve the quality and safety of products [1, 2, 58]. For example, Walmart can track the movement of food in supply chains in real-time by utilizing a traceability system to improve food quality and safety [17]. However, consumers who are unfamiliar with traceability are unwilling to pay a higher price for traceable products [36,64]. This evidence appears to indicate a hazy relationship between traceability and product quality. Theoretical studies, such as Starbird and Amanor-Boadu [60], Pouliot and Sumner [53], Resende-Filho and Hurley [54], Dai et al. [18], Cui et al. [17], and Dong et al. [19], have shown that the impact of traceability on an enterprise’s product quality decision is influenced by a variety of exogenous conditions. However, all of them primarily focus on supply chain operations, neglecting to study the relation between traceability and product quality in a horizontal competitive setting. Our study tends to consider the impact of traceability on duopolistic enterprises’ competitive strategies regarding product quality and sales price.

Competition is a common commercial activity that can be seen in almost all industries [5], such as the competition between Walmart and Kroger in the retail industry, Tyson Foods and Golden State Foods in the food industry, Dole and Driscoll’s in the fruit industry, and Alibaba and JD in the e-commerce industry. All of the aforementioned competitors, in particular, have deployed traceability systems [17,23]. According to empirical evidence, traceability has become a key factor influencing quality and price competition. For example, by not maintaining the same mandatory traceability standards for cattle as competing export countries do in terms of demonstrated animal traceability, the United States has placed itself in a vulnerable position, with decreasing export share and increasing food safety occurrences [57]. The theoretical research, such as Cui et al. [17], has also highlighted that differences in the competitive structures of upstream suppliers (vertical and horizontal competitions) in the supply chain will directly determine whether traceability improves or decreases product quality. However, Cui et al. [17] conduct their research on the assumption of exogenous sales price and customer demand. Thus, it is worthwhile to investigate the impact of traceability on product quality and sales price decisions from a duopolistic standpoint.

Based on the aforementioned traceability observations and to fill a gap, our study aims to explore how traceability impacts the decisions of two competing enterprises regarding product quality and sales price. More specifically, we focus on the following questions.

1. What are the optimal pricing and quality decisions of two competing enterprises in a duopoly?
2. How does traceability impact the quality and price competition between the two enterprises, as well as consumer demand and the corresponding performance of the two enterprises?
(3) How do liability-related factors (e.g., product liability, recall cost, and consumer disutility) and transportation costs influence the competition decisions, consumer demand, and profits of the two enterprises?

To answer the above questions, we consider a duopoly market in which two horizontally competing enterprises manufacture and sell homogenous products to final consumers. The product may be of high (non-defective) or low quality (defective), and a product that is found to be of low quality may bring about some disutility or harm consumers. Suppose that both enterprises have established a traceability system that enables them to trace the source of the quality defective problem and guide them to conduct a recall for the low-quality product. We further assume that traceability is able to completely identify high-quality products, so there is no need for enterprises to be concerned that the recalling products are actually non-defective. However, due to the precision of a traceability system, a few quality defective products may not be accurately identified, driving enterprises to be liable to compensate consumers for the disutility or harm brought about by these unrecalled low-quality products. Even so, the identified low-quality products can be recalled and remedied to be of high quality once more and will never again make disutility or harm to consumers.

After solving the above game model, we obtain the following findings. First, if an enterprise’s increased traceability leads to a higher expected social cost (the expected total cost caused by a low-quality product, i.e., the sum of expected recall cost and expected consumer disutility), quality competition between two enterprises becomes more intense, while the enterprise’s sales price falls and the competitor’s sales price rises. However, if the expected social cost decreases, quality competition weakens while price competition intensifies. Second, an increase in product liability has no impact on quality competition but weakens price competition; similarly, an increase in recall cost weakens price competition but intensifies quality competition; nevertheless, an increase in consumer disutility intensifies both quality and price competition. Third, higher (lower) expected social cost induced by an enterprise’s increased traceability would reduce (increase) its consumer demand and profit, but increase (reduce) its competitor’s consumer demand and profit.

Our study makes four significant contributions that are summarized below.

(1) Existing studies that have explored the impact of traceability on product quality mainly focus on supply chain operations [17–19, 53, 54, 60]. Additionally, they usually neglect to distinguish between recall cost and product liability, instead lumping them together. We enrich existing studies by extending them to the scenario of horizontal competing enterprises and explore the impact of traceability on quality competition under the assumption that the expected liability cost decreases as traceability increases.

(2) Existing studies usually assume a quantity-fixed final consumer demand and lack research on the impact of traceability on price decisions [17, 19, 54, 60], while we complement by incorporating product quality, traceability, consumer disutility, and product liability into a final consumer demand function and allowing the sales price of the product to be endogenously determined by enterprises, which enables us to explore the impact of traceability on price competition between enterprises.

(3) Existing studies on the impact of liability-related costs on quality decisions [22,35,39,40,51,55] and on quality and price competition [5,11,20,27,47,50,66] have been investigated. However, the impact of traceability on quality and pricing decisions has received little attention in these studies.

(4) Our study reveals the mechanism by which traceability impacts the quality competition of two enterprises via the expected social cost rather than the expected liability cost. More specifically, while an enterprise’s expected liability cost decreases with its level of traceability, the expected social cost caused by this enterprise’s low-quality products increases (decreases) when the recall cost is greater (lesser) than the consumer disutility, motivating the enterprise and its competitor to simultaneously improve (decrease) product quality. This indicates that the increased (decreased) expected social cost as a result of traceability improves (decreases) the quality competition between the two enterprises.

The remainder of our study is organized as follows. In Section 2, we first summarized the relevant literature. Section 3 introduces the monopoly and duopoly models. Section 4 characterizes the optimal decisions made by the two enterprises regarding product quality and sales price, as well as consumer demand and corresponding
profits. In Section 5, the impact of traceability, liability-related factors, and transportation costs on the aforementioned equilibrium results is analyzed. Section 6 is the extension, and Section 7 is the managerial insights. The conclusion of our work is summarized in Section 8.

2. Literature review

Our study is mainly related to three streams of literature, which are reviewed and listed below.

2.1. Supply chain traceability

Our study contributes to the literature on supply chain traceability. This stream of literature focuses on how traceability impacts supply chain quality decisions. For example, by adopting a principal-agent model based on adverse selection, Starbird and Amanor-Boadu [60] examine the role of a cost-sharing contract with traceability in enabling supply chain enterprises to select safe products. Under the assumption that the demand function is dependent on both the market price and the expected consumer loss, Pouliot and Sumner [53] study the effectiveness of traceability for promoting the quality improvement decisions of an upstream farmer and a downstream marketer. Resende-Filho and Hurley [54] investigate the impact of traceability on an upstream supplier’s quality effort by considering that non-defective raw materials may be misidentified due to a downstream processor’s traceability precision. Cui et al. [17] study the different impacts of traceability on suppliers’ optimal quality decisions and supply chain profits under different supply chain structures. Assuming that traceability is used to enhance the effectiveness of product recalls, Dai et al. [18] explore optimal decisions for traceability and product reliability within a competitive supply chain. Dong et al. [19] investigate the adoption of traceability in reducing product contamination risks and improving supply chain performance in a three-tier supply chain.

However, our study differs from the aforementioned literature primarily in three aspects. First, in the studies by Starbird and Amanor-Boadu [60], Resende-Filho and Hurley [54], Cui et al. [17], and Dong et al. [19], it is commonly assumed that both sales price and consumer demand are exogenously given. However, to fill these gaps, we assume that they are endogenous in our study, which allows us to capture the impact of product quality, traceability, sales price, and liability-related factors on consumer demand, as well as the impact of traceability and liability-related factors on the sales price decisions. Second, while Pouliot and Sumner [53] and Dai et al. [18] modeled product liability and product recall, respectively, and explored the impact of traceability on consumer demand, none of the studies in this stream of literature take product liability, product recall, and consumer disutility (harm) into account simultaneously. We complement this stream of literature by incorporating them into our model and hence explicitly modeling their impacts on consumer demand, strategic decisions, and enterprise profits. Finally, while the existing studies focus primarily on supply chain operations, our study focuses on the strategic interactions in terms of product quality and sales price between two horizontal competitive enterprises, which captures the impact of traceability and liability-related factors on quality and price competition. To further understand the novelty of our study, Table 1 outlines the research gaps between our study and the most relevant studies on traceability and product quality.

2.2. Product liability and quality

The second stream of literature related to our study is the work on the impact of liability-related costs on product quality decisions. Leng et al. [40] explore a retailer’s gatekeeping activity in order to encourage a manufacturer to improve product quality by penalizing it for defective products. Plambeck and Taylor [51] investigate how a buyer can incentivize a supplier to improve product quality by increasing auditing, providing loans to the supplier, and penalizing the supplier for harm. Lee and Li [39] assume that a buyer is responsible for a supplier’s quality defects and propose three types of strategies (investment, incentives, and inspection) for motivating the supplier to improve product quality. Iyer and Singh [35] study how consumer moral hazard and liability-related cost sharing between the consumer and the firm affect the firm’s incentive to seek safety certification. Fan et al. [22] investigate the impact of liability cost sharing on quality improvement decisions of the upstream firm under different channel structures. Sarkar and Bhuniya [55] investigate green quality and
service decisions by considering the responsibility to collect and remanufacture used products. However, the aforementioned studies primarily concentrate on quality decisions from the supply chain’s perspective, and none of them have considered traceability and its impact on quality decisions. Differing from these studies, our study focuses on quality and price competition in a duopoly market and explores the impact of traceability and liability-related factors on competition decisions.

2.3. Quality and price competition

The third stream of literature related to our study contributes to the work on quality and price competition. For example, Banker et al. [5] investigate the impact of competitive intensity on quality and price decisions by developing three different models of oligopolistic competition. Chambers et al. [11] study how variable production costs influence the competitive decisions of two manufacturers in terms of product quality and price. Matsubayashi and Yamada [47] explore how asymmetry in consumers’ loyalty to two competing firms affects quality and price competition. Dubovik and Janssen [20] investigate the firms’ quality and price competition in an oligopolistic market, assuming that consumers have heterogeneous quality and price information. Zhang et al. [66] optimize product quality and pricing decisions of two horizontally competitive firms and investigate the impact of government subsidies on green investment decisions for reducing carbon emissions. Geng et al. [27] examine the effects of quality spillover on quality and price competition in a duopoly market under either committed pricing or dynamic pricing scheme. Choi et al. [15] examine online and offline price decisions under the assumption that service quality positively influences consumer demand. Pal et al. [50] investigate the price competition between retail and online channels, as well as the decisions on green quality and promotional effort. Other studies in this stream of literature include Bi et al. [6], Meng et al. [48], Cellini et al. [9], Klumpp and Su [38], Hauck et al. [31], Ling and Xu [42], Ling et al. [43], Hafezi et al. [29], Awasthy et al. [4], Geng et al. [26], Zhao et al. [68], etc. However, the aforementioned studies fail to model consumers’ disutility (harm) and competitive enterprises’ liability behaviors (e.g., product recall, liability compensation) caused by quality defective products. In our study, we explicitly model product liability and consumer disutility and incorporate them into the consumer demand utility function and enterprise profits. Our study, in particular, focuses on traceability and how traceability and liability-related factors affect quality and price competition in a duopoly.

3. MODEL ASSUMPTIONS

Consider a Hotelling linear city model in a market with a duopoly. There are two enterprises, enterprise 1 and enterprise 2, located at \( l = 0 \) and \( l = 1 \), respectively. Enterprises produce homogenous products and compete with each other in terms of quality and price. Suppose that consumers are distributed uniformly within the range of \([0, 1]\). A consumer living at \( l \in [0, 1] \) can buy a product from each of the two enterprises. To consume the product, suppose the consumer incurs a transportation cost due to the distance itself to enterprises, and the
unit transportation cost is \( t \), then the consumer must pay a transportation cost \( tl \) when purchasing a product from enterprise 1, or pay \( t(1 - l) \) when purchasing from enterprise 2.

The product is produced at the same constant marginal cost by both enterprises. Assume that the marginal cost is standardized to zero. This zero-cost hypothesis does not interfere with the analysis results from our study below [66]. The quality of products manufactured by enterprises may be defective. As a result, enterprises have an incentive to invest in improving the quality of their products. We assume that each enterprise incurs a quality improvement cost \( c(\theta_i) \) in order to achieve a quality level \( \theta_i (i = 1, 2) \), where \( \theta_i \in (0, 1) \) denotes the probability that the product is of high quality or non-defective. We will denote \( \theta_i \) as the level of quality below. Correspondingly, \( 1 - \theta_i \) denotes the probability of the product being of low quality or defective. Similar assumptions can be seen in the studies of Chen and Hua [12], Cui et al. [17], and Fan et al. [22]. For simplicity, we define the function of the quality improvement investment cost of enterprises as \( c(\theta_i) = k\theta_i^2/2 \), where \( k(> 0) \) is the enterprises’ quality improvement coefficient and a higher \( k \) corresponds to a lower level of quality improvement efficiency. In other words, enterprises need to invest more to improve the quality of their products per unit. This quadratic cost function captures the characteristic of declining return on investment in quality improvement and has also been widely used in the literature [14, 24, 33, 41, 44, 63, 67].

Each enterprise sells the product to a final consumer at a sales price \( p_i \) (\( i = 1, 2 \)). A consumer cannot precisely recognize whether a product is of high quality or low quality before purchasing, but can learn the true information about the product quality level \( (\theta_i) \) by capturing certain quality information through common channels such as the Consumer Product Safety Committee (CPSC) in German and the China Association for Quality Inspection (CAQI), and then figuring out a prior distribution of quality level [21]. If a consumer purchases a product that is deemed to be of low quality, the consumer may suffer from potential disutility. Both enterprises have set up a traceability system to identify the sources of quality defective problems. The traceability system is operational before a quality safety incident occurs and a low-quality product causes actual disutility to the consumer. That is, when prior indications of a potential quality defective problem are detected, if the source of the quality defective problem is identified by the traceability system, the enterprise can exactly know that the product from this source is of low quality and which batch of products comes from this source [17, 19, 67]. The enterprise can recall the identified low-quality product and repair or replace it with a high-quality product. In consequence, consumers will no longer be subjected to quality incidents. Both enterprises have a cost of recalling an identified low-quality product, denoted by \( c(> 0) \). Naturally, it is difficult to identify all products of low quality. We thus assume that the traceability is not fully effective, and the traceability level, expressed as the probability that the source of a low-quality product is accurately identified, is \( s_i \in (0, 1) (i = 1, 2) \). Assume that the traceability level \( s_i \) is exogenous [53, 60]. For the sake of argument, we also assume that no costs will be incurred by enterprises in setting up the traceability system. This assumption of zero-cost traceability is consistent with the majority of the existing literature, including Starbird and Amanor-Boadu [60], Pouliot and Sumner [53], Cui et al. [17], and Dong et al. [19], among others. It is also supported by practice. For example, the cost of implementing CattleTrace for feedlots ranged from $0.33 to $0.55/head [59]. In 2021, the FDA named a dozen winners in its “Low- or No-Cost Tech-Enabled Traceability Challenge” in which the winners develop traceability systems and platforms that are low-cost or nocost to the end user.

However, if the source of a low-quality product cannot be accurately identified, the product will not be recalled, resulting in actual disutility \( D(>0) \) for the consumer. In this regard, under product liability regulations, both enterprises must compensate consumers for utility loss. Suppose that the enterprise’s product liability is \( L \) and \( L > c \). This assumption is necessary because the establishment of the traceability system by two enterprises can reduce the expected liability cost and inspire the enterprises to build the traceability system and enhance the traceability efficiency. Moreover, the practical evidences also demonstrates that the unit recall cost is generally lower than product liability [30].

Assume that the enterprises may bear partial liability \( (L < D) \), full liability \( (L = D) \), or punitive damage liability \( (L > D) \) under the liability regulations. This assumption has been applied in studies by Pouliot and Sumner [53], Chen and Hua [12], and Fan et al. [22], among others. Specifically, Pouliot and Sumner [53] studied how enterprises determine quality effort decisions when they assume full liability. Chen and Hua [12]...
compared various quality investment strategies under different liability rules: partial, full, and punitive. Fan et al. [22] investigated the impact of liability cost sharing on quality decisions under the assumptions of partial and punitive damage liability. Additionally, different liability rules exist in many countries and regions. For example, as of 2005, 19 states in the United States had adopted liability legislation allowing punitive damage claims, while others had not [45]. According to Buzby and Frenzen [8], in 56% of 294 food poisoning cases, enterprise compensation cannot cover the consumers’ losses. Given that a product is of low quality, the expected liability cost borne by enterprises is $x_i = cs_i + L (1 - s_i)$ ($i = 1, 2$), while the expected net disutility borne by consumers is $y_i = (1 - s_i) (D - L)$. Naturally, $x_i + y_i = cs_i + D (1 - s_i)$ indicates the expected total cost caused by a low-quality product, which denotes as the expected social cost below.

We assume that each customer requires a unit or none of the product. $v(>0)$ represents the initial utility derived from consumption to the consumer. Suppose that $v$ is large enough that either of the two enterprises’ products can provide a nonnegative net utility, and hence the market is fully covered and all consumers are willing to purchase one unit of the product. Let $U_i$ ($i = 1, 2$) denote the net utility obtained by the consumer from each unit of purchased product. There is no doubt that the net utility $U_i$ equals the difference between the initial utility $v$ and the sum of the sales price $p_i$, the expected loss $(1 - \theta_i) y_i$ that the consumer suffers from per unit of product, and the transportation cost $tl$ or $t(1 - l)$.

Thus, the net utility of the consumer living at $l$ when purchasing from enterprise 1 and enterprise 2 is given:

$$U_i^l = \begin{cases} 
U_1^l = v - p_1 - y_1 (1 - \theta_1) - tl & \text{if purchasing from enterprise 1} \\
U_2^l = v - p_2 - y_2 (1 - \theta_2) - t (1 - l) & \text{if purchasing from enterprise 2}
\end{cases}$$

The consumer purchases the product from the enterprise that maximizes his/her net utility and remains indifferent between buying from enterprise 1 and enterprise 2 when $U_1 = U_2$. Accordingly, the location $l$ of a consumer with undifferentiated preference is given by:

$$l = \frac{p_2 - p_1 + t + y_2 (1 - \theta_2) - y_1 (1 - \theta_1)}{2t}.$$ 

So, the consumer’s demands $D_1$ and $D_2$ for enterprise 1 and enterprise 2 are given by:

$$D_1 = l = \frac{p_2 - p_1 + t + y_2 (1 - \theta_2) - y_1 (1 - \theta_1)}{2t},$$

$$D_2 = 1 - l = \frac{p_1 - p_2 + t + y_1 (1 - \theta_1) - y_2 (1 - \theta_2)}{2t}.$$ 

The profit functions of enterprise 1 and enterprise 2 can be expressed as follows:

$$\pi_1 (\theta_1, p_1) = [p_1 - (1 - \theta_1) x_1] D_1 - \frac{1}{2} k \theta_1^2,$$ 

$$\pi_2 (\theta_2, p_2) = [p_2 - (1 - \theta_2) x_2] D_2 - \frac{1}{2} k \theta_2^2.$$ 

The two enterprises choose product quality and sales price to maximize their own profits by engaging in a two-stage Nash game. The decision sequences in the game model are as follows: In the first stage of the game, the two enterprises simultaneously choose their optimal product quality decisions $\theta_i$ ($i = 1, 2$); and then in the second stage, both enterprises simultaneously determine their optimal sales prices $p_i$.

4. Equilibrium

4.1. The monopoly model

In this subsection, we first consider a monopoly model in which only enterprise 1 produces and sells products in the market, which serves as a benchmark case for the effect of competition in a duopoly. Without loss of
generality, assume that enterprise 1 still locates at \( l = 0 \). So, under any given product quality \( \theta_1 \) and sales price \( p_1 \), the net utility for a consumer purchasing a product from enterprise 1 is given by \( U_1 = v - p_1 - y_1 (1 - \theta_1) - t \). It is clear that if enterprise 1 charges a sales price \( p_1 \leq v - y_1 (1 - \theta_1) - t \), all consumers’ net utility is nonnegative, indicating that all consumers purchase the product from enterprise 1. As a result, the equilibrium consumer demand is \( D_1^{m*} = 1 \) (The superscript “\( m \)” represents the monopoly model.). If enterprise 1 determines its sales price \( p_1 > v - y_1 (1 - \theta_1) - t \), the consumer located at \( l = 1 \) gets net utility \( v - p_1 - y_1 (1 - \theta_1) - t < 0 \), that is, the market is partially covered by enterprise 1’s products and the consumer demand is given by \( D_1 = l = \frac{v - p_1 - y_1 (1 - \theta_1)}{y} \). Therefore, the profit function of enterprise 1 can be written as follows:

\[
\pi_1(\theta_1, p_1) = \begin{cases} 
 p_1 - (1 - \theta_1) x_1 - \frac{1}{2} k \theta_1^2 & \text{if } p_1 \leq v - y_1 (1 - \theta_1) - t \\
 [p_1 - (1 - \theta_1) x_1] D_1 - \frac{1}{2} k \theta_1^2 & \text{if } v - y_1 (1 - \theta_1) - t < p_1 \leq v - y_1 (1 - \theta_1)
\end{cases}
\]

Enterprise 1 determines the optimal product quality \( \theta_1 \) and sales price \( p_1 \) to maximize its profit \( \pi_1(\theta_1, p_1) \). Thus, in the monopoly model, if \( v > x_1 + y_1 \) and \( k > v(x_1 + y_1)/2t \), the optimal product quality, sales price, consumer demand, and profit for enterprise 1 are as follows:

\[
\begin{align*}
\theta_1^{m*} &= \frac{(v - x_1 - y_1)(x_1 + y_1)}{2kt - (x_1 + y_1)^2}, \\
p_1^{m*} &= \frac{kt(v + x_1 - y_1) - vx_1 (x_1 + y_1)}{2kt - (x_1 + y_1)^2}, \\
D_1^{m*} &= \frac{k(v - x_1 - y_1)}{2kt - (x_1 + y_1)^2}, \\
\pi_1^{m*} &= \frac{k(v - x_1 - y_1)^2}{2 \left[ 2kt - (x_1 + y_1)^2 \right]}. 
\end{align*}
\]

All proofs are in the Appendix A.

It should be noted that relevant parameters in this monopoly model should satisfy \( v > x_1 + y_1 \) and \( k > v(x_1 + y_1)/2t \). First, the technical condition \( v > x_1 + y_1 \) is required to ensure that all equilibrium results are nonnegative and that the consumer’s initial utility for the product is greater than the expected social cost caused by per unit of low-quality product. Otherwise, if the product’s value cannot cover the total cost caused by the product, the enterprise may have no incentive to manufacture the product, or the consumer may not purchase the product. Furthermore, the technical condition \( k > v(x_1 + y_1)/2t \) is sufficient to ensure that \( \theta_1^{m*} \in (0, 1) \) holds and that the profit function \( \pi_1(\theta_1, p_1) \) of enterprise 1 is strictly concave in its decisions on product quality \( \theta_1 \) and sales price \( p_1 \). Similar assumptions for a high enough quality improvement investment coefficient can be seen in the literature such as Fan et al. [22], Chen et al. [14], and Hong et al. [33]. Additionally, this assumption implies that improving product quality is a difficult task that requires a large investment of resources, such as capital and labor, which is consistent with the observation in practice [22].

4.2. The duopoly model

As described above for the duopoly model, the game can be solved by backward induction. In the second stage, enterprise 1 and enterprise 2 choose their decisions of sales price, i.e., \( p_1 \) and \( p_2 \), to maximize their profits, respectively. By substituting equations (1) and (2) into equations (3) and (4), one can see that \( \frac{\partial^2 \pi_1(\theta_1, p_1)}{\partial p_1^2} < 0 \) and \( \frac{\partial^2 \pi_2(\theta_2, p_2)}{\partial p_2^2} < 0 \). This implies that \( \pi_1(\theta_1, p_1) \) and \( \pi_2(\theta_2, p_2) \) are strictly concave in \( p_1 \) and \( p_2 \), respectively. By taking the first-order condition of enterprise 1’s profit \( \pi_1(\theta_1, p_1) \) with respect to \( p_1 \), we obtain:

\[
\frac{\partial \pi_1(\theta_1, p_1)}{\partial p_1} = p_2 - 2p_1 + t + y_2 (1 - \theta_2) - y_1 (1 - \theta_1) + x_1 (1 - \theta_1).
\]
Similarly, the first-order condition for enterprise 2 by maximizing $\pi_2(\theta_2, p_2)$ with respect to $p_2$ is given by:

$$\frac{\partial \pi_2(\theta_2, p_2)}{\partial p_2} = \frac{p_1 - 2p_2 + t + y_1(1 - \theta_1) - y_2(1 - \theta_2) + x_2(1 - \theta_2)}{2t}.$$

Solving $\frac{\partial \pi_1(\theta_1, p_1)}{\partial p_1} = 0$ and $\frac{\partial \pi_2(\theta_2, p_2)}{\partial p_2} = 0$ for $p_1$ and $p_2$, we derive:

$$p_1 = \frac{3t + (1 - \theta_2)(x_2 + y_2) + (1 - \theta_1)(2x_1 - y_1)}{3},$$
$$p_2 = \frac{3t + (1 - \theta_1)(x_1 + y_1) + (1 - \theta_2)(2x_2 - y_2)}{3}.$$  \hspace{1cm} (5)
$$\hspace{1cm} (6)$$

With equations (5) and (6), Corollary 1 is derived as follows.

**Corollary 1.** Given the value of product quality level, it holds that:

1. as both product liability and recall cost increase, both enterprises’ sales prices will increase, i.e., $\partial p_i/\partial L > 0$, $\partial p_i/\partial c > 0$; and if $(1 - \theta_j)(1 - s_j) \geq (<) (1 - \theta_i)(1 - s_i)$, then both enterprises’ sales prices will increase (decrease) in the consumer disutility, i.e., $\partial p_i/\partial D \geq (<)0$.

2. if $L \geq (>) (D + 2c)/3$, an enterprise’s sales price will decrease (increase) in its own traceability level, i.e., $\partial p_i/\partial s_i \leq (>0)$; if $c \geq (>) D$, an enterprise’s sales price will increase (decrease) in the competitor’s traceability level, i.e., $\partial p_i/\partial s_j \geq (<)0$ ($i = 1, 2; j = 1, 2; i \neq j$).

Corollary 1(1) shows that increases in product liability $L$ and recall cost $c$ both raise enterprises’ expected liability cost, providing an incentive for enterprises to raise their sales prices. Nevertheless, the impact of consumer disutility $D$ on enterprises’ sales price decisions is determined by a value comparison of the probabilities that two competing enterprises’ low-quality products are not accurately tracked. More specifically, if the probability $((1 - \theta_j)(1 - s_j))$ of a competitor’s low-quality product not being accurately identified by traceability is greater than that $((1 - \theta_i)(1 - s_i))$ of the enterprise itself, the expected disutility that the consumer experiences by purchasing the competitor’s product is comparatively higher, prompting the consumer to lower demand for the competitor’s product. Thus, the consumer will in general buy more from the enterprise. This naturally drives the enterprise to raise its sales price. Otherwise, if the enterprise’s low-quality product is more likely to be misidentified, the enterprise will decide to lower the sales price.

Furthermore, Corollary 1(2) demonstrates that if product liability $L$ is greater (lesser) than a certain threshold, namely, $L \geq (>) (D + 2c)/3$, an enterprise’s decision on sales price will decrease (increase) in its own traceability level. It is easy to see that when product liability is sufficiently high, the enterprise’s increased traceability level is more helpful in reducing its expected liability cost, thereby allowing the enterprise to charge a lower sales price for the product. On the contrary, it raises the sales price. However, the impact of the competitor’s traceability on the enterprise’s sales price decision is based solely on a comparison of the enterprise’s unit recall cost and consumer disutility, rather than product liability. When the enterprise’s unit recall cost is greater (lesser) than the consumer disutility, an increase in the competitor’s traceability level leads the enterprise to raise (lower) its product sales price. Obviously, the competitor’s increased traceability will result in a larger (lower) expected social cost if the recall cost is greater (lesser) than the consumer disutility, lowering (raising) consumer demand for the competitor’s products (Prop. 4 also demonstrates this). As a result, the enterprise has an incentive to raise (lower) its sales price.

In the first stage, the two enterprises simultaneously decide on the optimal product quality levels ($\theta_1$ and $\theta_2$). By substituting equations (5) and (6) into equations (3) and (4), we can show that if $k > \max\{(x_1 + y_1)^2/9t, (x_2 + y_2)^2/9t\}$, $\pi_1(\theta_1, p_1(\theta_1, \theta_2))$ and $\pi_2(\theta_2, p_2(\theta_1, \theta_2))$ are strictly concave in $\theta_1$ and $\theta_2$, respectively.
respectively. Hence, the optimal product quality, i.e., $\theta^*_1$ and $\theta^*_2$, can be derived as follows:

$$
\theta^*_1 = (x_1 + y_1) \frac{3k \left(3t - x_1 - y_1 + x_2 + y_2\right) - 2 \left(x_2 + y_2\right)^2}{3k \left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
$$

(7)

$$
\theta^*_2 = (x_2 + y_2) \frac{3k \left(3t - x_2 - y_2 + x_1 + y_1\right) - 2 \left(x_1 + y_1\right)^2}{3k \left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
$$

(8)

To ensure that $\theta^*_1 \in (0, 1)$ and $\theta^*_2 \in (0, 1)$, the following conditions should be satisfied: $3k \left(3t - x_i - y_i + x_j + y_j\right) - 2 \left(x_j + y_j\right)^2 > 0$ and $3k [9kt - (x_i + y_i)^2 - (x_j + y_j)^2] > (x_i + y_i) [3k \left(3t - x_i - y_i + x_j + y_j\right) - 2 \left(x_j + y_j\right)^2] (i = 1, 2; j = 1, 2; i \neq j)$.

Let $K_i = \frac{2(x_i+y_i)^2}{3[3t-2(x_i+y_i)+x_j+y_j]^4}$, $\overline{K}_i = \frac{2(x_i+y_i)}{3}$, $T_i = \frac{2(x_i+y_i)-x_j-y_j}{3}$, and $T_j = \frac{3k(x_j+y_j)x_j+y_j^2-2(x_j+y_j)^2-4(x_j+y_j)^2(x_j+y_j)}{9k[3k-2(x_j+y_j)]}$ ($i = 1, 2; j = 1, 2; i \neq j$). To simplify the mathematical derivations of the models and avoid some weird results below, we further assume that $k > \max\{K_i, \overline{K}_i\}$ and $t > \max\{T_i, T_j\}$. These two assumptions are sufficient to ensure that $\theta^*_1 \in (0, 1)$ and $\theta^*_2 \in (0, 1)$, as well as all equilibrium results in the duopoly are nonnegative. Furthermore, a high $k$ obviously implies a significant investment cost for quality improvement, implying that it is not so easy for enterprises to improve the quality level of the product. A great $t$ means that the consumer has to pay a high transportation cost for purchasing, which usually indicates that the two enterprises have significant product differentiation. Similar assumptions can be seen in studies by Chen et al. [14], Hong et al. [33], and Fan et al. [21].

Therefore, with equations (7) and (8), we can derive Corollary 2 as follows.

**Corollary 2.** Given the values of other parameters, it holds that:

1. as the expected social cost $x_i + y_i$ caused by an enterprise’s own low-quality product increases, the enterprise’s decision on product quality level will increase, i.e., $\partial \theta^*_i / \partial (x_i + y_i) > 0$.
2. as the expected social cost $x_j + y_j$ caused by a competitor’s low-quality product increases, the enterprise will improve its quality level of product, i.e., $\partial \theta^*_i / \partial (x_j + y_j) > 0$ ($i = 1, 2; j = 1, 2; i \neq j$).

Corollary 2 explores the impact of the expected social cost on enterprises’ product quality decisions. First, Corollary 2(1) illustrates that an enterprise will respond by improving its product quality level to a higher expected social cost caused by its own low-quality product. This indicates that when making a quality decision, an enterprise should consider the negative impact on society as a whole, particularly the consumers’ loss (disutility) as a result of his failure in quality control, rather than just his own liability-related costs. Given that the expected social cost is made up of the expected recall cost ($c_s$) incurred by enterprises and the expected disutility ($D(1-s_i)$) endured by consumers, a higher cost resulting from the former and a lower demand caused by the latter can both prompt an enterprise to spend more effort on quality improvement. In addition, Corollary 2(2) shows that an increase in the expected social cost brought about by a competitor’s low-quality product also induces the enterprise to invest more in improving the quality of the product. The thought behind this is that an enterprise’s quality decision is positively influenced by the external spillover effect of its competitor. As a result, the enterprise has an incentive to improve its product quality level in order to avoid a decline in consumer demand and enterprise performance caused by the negative impact of the competitor’s higher expected social cost.

Substituting equations (7) and (8) into equations (5) and (6), the equilibrium sales prices of the two enterprises are obtained by, respectively:

$$
p^*_1 = \frac{3t + (1 - \theta^*_2) (x_2 + y_2) + (1 - \theta^*_1) (2x_1 - y_1)}{3},
$$

(9)

$$
p^*_2 = \frac{3t + (1 - \theta^*_1) (x_1 + y_1) + (1 - \theta^*_2) (2x_2 - y_2)}{3}.
$$

(10)
Finally, by substituting equations (7), (8), (9), and (10) into equations (1), (2), (3), and (4), we can obtain the consumer demands for the two enterprises’ product and the two enterprises’ equilibrium profits:

\[
D_1^* = \frac{3k (3t - x_1 - y_1 + x_2 + y_2) - 2 (x_2 + y_2)^2}{2 (9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2)},
\]

(11)

\[
D_2^* = \frac{3k (3t - x_2 - y_2 + x_1 + y_1) - 2 (x_1 + y_1)^2}{2 (9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2)},
\]

(12)

\[
\pi_1^* = \frac{9kt - (x_1 + y_1)^2}{18k} \left[ \frac{3k (3t - x_1 - y_1 + x_2 + y_2) - 2 (x_2 + y_2)^2}{2 (9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2)} \right],
\]

(13)

\[
\pi_2^* = \frac{9kt - (x_2 + y_2)^2}{18k} \left[ \frac{3k (3t - x_2 - y_2 + x_1 + y_1) - 2 (x_1 + y_1)^2}{2 (9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2)} \right].
\]

(14)

The following Corollary 3 summarizes the findings by comparing the equilibrium results in the monopoly and duopoly models. To make the comparative results clearer and more straightforward, we assume that the two enterprises in the duopoly are identical, i.e., \(s_1 = s_2\). As a result, \(D_1^* = D_2^* = 1/2\). However, an implicit assumption in the duopoly is that \(v - p_1^* - y_1 (1 - \theta_1^*) - l/2 > 0\), implying that \(v > x_1 + y_1 + \frac{3t}{2} - \frac{(x_1 + y_1)^2}{2}\). Otherwise, the two enterprises cannot cover the market, indicating that there is no difference for enterprise 1’s decisions between the monopoly and duopoly models. Let \(v^\# = \frac{[2kt-(x_1+y_1)^2][3kt-x_1(x_1+y_1)]-3kt^2(x_1-y_1)}{3k[kt-x_1(x_1+y_1)]}\).

**Corollary 3.** By comparing the equilibrium results in the monopoly and duopoly models, it holds that: \(D_1^{1\ast} > D_1^*; \theta_1^{m\ast} > \theta_1^*; \pi_1^{m\ast} > \pi_1^*\); and if \(k > x_1(x_1+y_1)/t\) and \(v \geq (\ast)v\), \(p_1^{m\ast} \geq (\ast)p_1^*\).

The above corollary first shows that in a duopoly, both consumer demand and product quality are lower than in a monopoly. It is obvious that a new entrant (enterprise 2) will take a portion of the existing enterprise’s (enterprise 1) market share, resulting in a decrease in consumer demand for the existing enterprise’s products. A decline in product quality level, on the other hand, indicates that competition does not motivate enterprises to improve product quality to a higher level. This finding is consistent with Banker et al. [5]’s observation that as competition increases, the average industry quality level decreases. The observation in the airline industry that deregulation reduces service quality demonstrates it as well [5]. Furthermore, Corollary 3 demonstrates that competition does not necessarily result in a decrease in sales price. More specifically, if the quality improvement investment efficiency is low enough, enterprise 1 will set a lower sales price in the duopoly case than in the monopoly case if consumers’ initial utility for the product exceeds a certain threshold. This follows from common sense that increased competition and lower quality levels both motivate enterprises to lower their sales prices. If the initial utility is low enough, however, enterprise 1 chooses to raise the sales price. In general, a low consumer initial utility indicates that it is difficult for a new entrant to successfully enter the market and change consumers’ purchasing preferences; thus, the existing enterprise (enterprise 1) has the incentive to set a higher sales price in the context of a loss of market share due to the new entrant (enterprise 2). It is obvious that, regardless of sales price decisions, lower consumer demand leads to lower profitability in the duopoly.

5. Comparative statics

This section characterizes the impacts of traceability, liability-related factors (e.g., product liability, recall cost, and consumer disutility), and transportation cost on optimal product quality decisions, sales price decisions, consumer demand, and corresponding performance for the duopoly enterprises. As stated in the preceding section, due to the complexity of some equilibrium results, all analyses in this section are performed under...
the assumptions $k > \max \{K_i, \overline{K}_i\}$ and $t > \max \{T_i, \overline{T}_i\}$. Meanwhile, we conduct numerical studies to further validate the results of our analysis. Additionally, as some analytical results, such as the impact of traceability on equilibrium sales prices, are quite challenging to demonstrate and obtain, we will use the numerical studies directly to explore the impacts of relevant parameters on these equilibrium results.

5.1. The impact of traceability and liability-related factors on product quality decisions

**Proposition 1.** Given the values of other parameters, it holds that:

1. as the traceability level $s_i$ increases, the enterprise $i$’s product quality level increases (decreases) if $c \geq (<)D$, i.e., $\partial \theta_i^* / \partial s_i \geq (<)0$.
2. as the traceability level $s_j$ of an enterprise’s competitor increases, the enterprise $i$’s product quality level increases (decreases) if $c \geq (<)D$, i.e., $\partial \theta_i^* / \partial s_j \geq (<)0$ ($i = 1, 2; j = 1, 2; i \neq j$).

The above proposition investigates the impact of traceability on the quality decisions of the two enterprises. First, Proposition 1(1) makes an intriguing observation: raising an enterprise’s traceability level does not always motivate the enterprise to improve its level of quality. This depends primarily on a comparison between the enterprise’s recall cost and consumer disutility. In other words, as demonstrated in Corollary 2, the traceability of an enterprise influences its quality decision essentially through the expected social cost ($c s_i + D (1 - s_i)$). As an enterprise’s level of traceability increases, so does its expected recall cost ($c s_i$), creating an incentive for the enterprise to improve product quality. Nevertheless, the consumer’s expected disutility ($D (1 - s_i)$) decreases, which reduces the enterprise’s willingness to invest in quality improvement. At the point when the recall cost is greater (lesser) than the consumer disutility, the positive impact of the recall cost on the enterprise’s quality decision is greater (lesser) than the negative impact of the disutility, inducing the enterprise to increase (decrease) product quality.

This finding reveals that when deciding product quality in response to increased traceability level, an enterprise should balance the effects of increased recall cost and reduced consumer disutility. If the recall cost is sufficiently high, the increased recall cost caused by traceability motivates enterprises to improve product quality. This is consistent with Pouliot and Sumner’s [53] argument that traceability increases the incentives for a farmer or marketer to supply safer food by increasing the liability burden. However, the finding that traceability is negatively associated with product quality when consumer disutility is high enough is also partially supported by Starbird and Amanor-Boadu [60], Resende-Filho and Hurley [54], Cui et al. [17], and Dong et al. [19], where traceability is not an unambiguous indicator of higher product quality and increasing (decreasing) traceability can lead to lower (higher) product quality under certain conditions. Nonetheless, by assuming endogenous sales price and consumer demand in a duopoly situation, this finding supplements the aforementioned studies to some extent.

Further, the result of Proposition 1(2) indicates that an enterprise’s and its competitor’s traceability levels have the same effect on the enterprise’s quality decision. That is, if $c \geq (<)D$, an enterprise’s product quality is increasing (decreasing) in the traceability level of its competitor. Clearly, a greater (smaller) $c$ than $D$ means an increase (a decrease) in the expected social cost caused by a competitor’s low-quality product as its level of traceability increases, leading the enterprise to improve (decrease) the product quality. We can also see, practically speaking, that in a market with two competing enterprises selling homogeneous products to consumers, if the competitor of an enterprise decides to improve product quality due to the increased traceability, the enterprise will accordingly respond by improving the quality of its products in the hope of avoiding the loss of demand caused by greater product differentiation (due to the quality factor). If the competitor chooses to lower product quality due to the effective role that traceability plays in reducing consumers’ expected disutility, the enterprise will also make the same decision to decrease the product quality.

Next, we conduct a numerical analysis to illustrate the above results presented in Proposition 1. We first set $t = 4, k = 10, c = 4, L = 6,$ and $s_2 = 0.5$, and then examine the relationship between traceability and the quality decisions of the two enterprises in scenarios $D = 3$ and $D = 8$. We can see from Figure 1 that enterprise 1’s and enterprise 2’s product quality decisions ($\theta_1^*$ and $\theta_2^*$) are both increasing (decreasing) in enterprise 1’s
traceability level \( s_1 \) if \( c \geq (>)D \). Therefore, the results of Proposition 1 are verified. Furthermore, Figure 1 shows that if \( s_1 \leq (>)0.5 \), then \( \theta_1^* \leq (>)\theta_2^* \) in scenario \( c > D \). This is because the expected social cost and hence the enterprises’ quality levels are increasing in the traceability if \( c > D \). Therefore, if enterprise 1’s traceability is lower (higher) than that of enterprise 2 (\( s_1 \leq (>)s_2 = 0.5 \)), then enterprise 1’s quality level will also be lower (higher) in comparison to enterprise 2. However, we can also see from Figure 1 that in scenario \( c < D \), if \( s_1 \leq (>)0.5 \), then \( \theta_1^* \geq (>)\theta_2^* \). Clearly, the expected social cost, and thus the quality levels of the enterprises, are decreasing in traceability if \( c < D \). Thus, if \( s_1 \leq (>)s_2 = 0.5 \), a higher (lower) expected social cost for enterprise 1 relative to enterprise 2 will result in a higher product quality. In addition, it can be found from Figure 1 that relative to the scenario \( D = 3 \), the two enterprises will decide on a higher product quality in scenario \( D = 8 \), indicating that an increase in consumer disutility leads to an increase in the quality level of the product.

**Proposition 2.** Given the values of other parameters, it holds that:

1. as product liability \( L \) increases, the product quality of both enterprises remains unchanged, i.e., \( \partial \theta_i^*/\partial L = 0 \).
2. as the recall cost \( c \) increases, both enterprises’ product quality is increased, i.e., \( \partial \theta_i^*/\partial c > 0 \).
3. as the consumer disutility \( D \) increases, both enterprises’ product quality is increased, i.e., \( \partial \theta_i^*/\partial D > 0 \) (\( i = 1, 2 \)).

According to Proposition 2(1), changes in both enterprises’ product quality decisions are unrelated to product liability. Actually, equations (7), (8), and Corollary 2 imply that an enterprise’s product quality decision is primarily influenced by the expected social costs (as well as the quality improvement efficiency and the transportation cost), which are independent of product liability. Noting that, given the product quality level \( (\theta_i) \), as product liability rises, so does an enterprise’s expected liability cost, i.e., \( \partial (1 - \theta_i) x_i/\partial L = (1 - \theta_i) (1 - s_i) \). Thus, the enterprise can choose to improve product quality in order to reduce the expected liability cost or to raise the products’ sales price in order to transfer the increased cost. However, as demonstrated in Corollary 1, \( \partial p_i/\partial L = (1 - \theta_i) (1 - s_i) \), indicating that the enterprise raises its sales price as product liability increases, and the increase in sales price equals the increase in the expected liability cost. As a result, the enterprise could choose to increase the sales price to balance the effect of product liability while maintaining its product quality choice. This finding directly contradicts Loureiro’s [45] argument that product liability provides greater incentives to improve product quality. It is consistent, however, with realistic observations by Buzby and Frenzen [8] that US enterprises are weakly motivated to produce safer food and by Polinsky and Shavell [52] that product liability had no effect on product quality and safety in the industries of general aviation aircraft, automobiles, and childhood vaccines.
Furthermore, Propositions 2(2) and 2(3) show that an increase in both recall cost and consumer disutility contributes to product quality improvement. Obviously, increasing the recall cost raises the expected liability cost, while increasing the disutility reduces the consumer’s willingness to consume more products. As such result, enterprises prefer to respond by improving product quality. This finding is supported by Kalaignanam et al. [37], who conducted an empirical study involving 27 automobile manufacturers between 1995 and 2011 and found that product recalls improve product reliability and safety by lowering future injuries and recalls. In fact, the well-known Toyota recall in 2009 also confirms this finding: in response to the enormous economic loss in recall and repair expenses, as well as the tremendous loss of life in serious accidents, Toyota established a new global quality committee for defect analysis and new designs [21].

Figure 2 illustrates the impact of the recall cost $c$ on enterprise quality decisions in scenarios $D = 3$ and $D = 8$ with parameters $t = 4$, $k = 10$, $L = 6$, $s_1 = 0.3$, and $s_2 = 0.5$. We can first see from Figure 2 that both enterprises will choose to improve their product quality as the cost of recall increases. Furthermore, product quality is higher in scenario $D = 8$ than in scenario $D = 3$, indicating that increased consumer disutility motivates enterprises to improve product quality. Besides this, as seen in Figure 2, in scenario $D = 3$, $\theta_1^* \geq (\leq) \theta_2^*$ if $c \leq (> )3$. This is because, when $c \leq (> )3$, the expected social cost resulting from enterprise 1’s low-quality product is higher (lower) than that of enterprise 2. Similarly, in scenario $D = 8$, the expected social cost for enterprise 1 is always higher relative to enterprise 2, motivating enterprise 1 to choose a higher product quality level ($\theta_1^* > \theta_2^*$). Obviously, these results in degree corroborate the results in Proposition 1 (Fig. 1).

5.2. The impact of traceability and liability-related factors on sales price decisions

Due to the complex expressions of sales prices in equations (9) and (10), we will conduct a direct numerical analysis to investigate the impact of traceability on the sales price decisions of two competing enterprises. We begin by setting the parameters $t = 4$, $k = 10$, $c = 4$, $L = 6$, and $s_2 = 0.5$, and then we discuss scenarios $D = 3$ and $D = 8$.

As shown in Figure 3(a), if $c > D$, as enterprise 1’s traceability level $s_1$ increases, enterprise 1 tends to reduce its pricing decision $p_1^*$, whereas enterprise 2 chooses to raise its pricing decision $p_2^*$. The reason for this is that as enterprise 1’s traceability level $s_1$ increases, the expected liability cost incurred by enterprise 1 decreases, allowing it to sell the product at a lower sales price and hence capturing a larger market share in the competitive market even as its product quality increases. This means that enterprise 1’s pricing decision is incompatible with its product quality decision. It is not surprising, however, that enterprise 2’s decision to raise the sales price is motivated by improved product quality. In contrast, if $c < D$, enterprise 2 will also choose to lower the sales price of its product. In an intuitive sense, in a situation where the product quality levels of two competing
enterprises both decrease with the traceability level $s_1$, enterprise 2 can only maintain its market share through price competition by deciding on a lower sales price in line with enterprise 1. Clearly, this refers to a price war between the two competing enterprises.

Furthermore, as shown in Figure 3(a), if $s_1 \leq (>) s_2 = 0.5$ in scenario $c > D$, then $p_1^* \geq (<) p_2^*$, indicating that a lower (higher) traceability level for enterprise 1 relative to enterprise 2 means a higher (lower) sales price. Given that the expected liability cost decreases with traceability, if enterprise 1 has a lower (higher) traceability level than enterprise 2, enterprise 1 will determine a higher (lower) sales price than enterprise 2 due to a higher (lower) expected liability cost. However, when joined with the finding in Figure 1 that enterprise 1’s lower (higher) traceability level leads to a lower (higher) product quality level compared to enterprise 2, the implication is that higher product quality does not always suggest a higher sales price, and the expected liability cost relative to product quality plays a more important role in determining enterprises’ pricing decisions.

Moreover, Figure 3(a) shows that if $s_1 \leq (>) s_2 = 0.5$ in scenario $c < D$, then $p_1^* \leq (>)(>) p_2^*$, indicating that despite the fact that enterprise 1’s expected liability cost is higher (lower) than enterprise 2, enterprise 1 chooses a lower (higher) sales price. This obviously differs from the result in scenario $c > D$. The main reason for this is that when consumer disutility is high enough, enterprises tend to base their pricing decisions on the negative impact of the consumer’s expected disutility on demand rather than the expected liability cost. There is no
doubt that a higher (lower) consumer’s expected disutility as a result of a lower (higher) traceability level will lead enterprises to determine a lower (higher) sales price in order to avoid a decrease in consumer demand. In light of the findings in Figure 1 about enterprises’ quality decisions, we can conclude that even if an enterprise incurs a higher (lower) expected liability cost and decides on a higher (lower) product quality level, the enterprise will respond with a lower (higher) sales price if the consumer’s expected disutility caused by the enterprise is higher (lower) relative to its competitor.

To test the robustness of the comparative statics results provided in Figure 3(a), we assume full liability ($L = D$) of both enterprises in the duopoly market and the corresponding numerical study is carried out as follows. Nevertheless, it should be noted that Section 3 assumed that the recall cost is not greater than the liability cost, therefore the numerical study is performed only in scenario $c < D$. The corresponding results are shown in Figure 3(b). It shows, under the full liability assumption ($L = D$), that the comparative statics results of both enterprises’ pricing decisions with regard to traceability are qualitatively consistent with those under assumptions of partial liability ($L < D$) and punitive damage liability ($L > D$). However, one difference between Figures 3(b) and 3(a) is that if $s_1 \leq (>) s_2 = 0.5$ in scenario $c < D$, then $p'_i \geq (<) p'_2$, suggesting that enterprise 1 chooses a higher (lower) sales price than enterprise 2 if its traceability level is lower (higher) than that of enterprise 2. The reason for this is that the full liability mechanism makes the consumer’s expected net disutility zero, therefore both enterprises determine their pricing decisions based on the expected liability cost. Evidently, a lower (higher) traceability level for enterprise 1 vs. enterprise 2 results in a greater (lower) expected liability cost and, as a result, a higher (lower) sales price.

Next, we study the impact of product liability on the two enterprises’ pricing decisions and derive Proposition 3 as follows.

**Proposition 3.** Given the values of other parameters, as product liability $L$ increases, both enterprises’ sales prices will increase, i.e., $\partial p'_i / \partial L > 0$ ($i = 1, 2$).

The above proposition demonstrates that an increase in product liability increases the expected liability cost, which in turn enhances the enterprises’ incentive to raise their sales prices. This reflects the price-signaling benefit of product liability [52], which states that if enterprises are subject to product liability, the prices of their products will rise to reflect expected accident losses. However, Proposition 3 reveals that higher product liability is beneficial in alleviating the price competition between the two enterprises. When combined with the finding in Proposition 2 that the change in product liability does not affect the quality decision of either enterprise, hence, this indicates that product liability only plays an effective role in influencing the price competition instead of the quality competition. In addition, to illustrate Proposition 3, Figure 4 shows a numerical analysis with the parameters $t = 4$, $k = 10$, $c = 4$, $s_1 = 0.3$, $s_2 = 0.5$, and $D = 3$ or $D = 8$. According to Figure 4, we can observe that the sales prices of the two enterprises are both increasing in terms of product liability. Besides that, as shown in Figure 4, compared to scenario $D = 3$, when consumers suffer a relatively higher disutility ($D = 8$), the two enterprises will simultaneously lower their sales prices, indicating that the price competition between the two enterprises is becoming more intense as prices are reduced to prevent a decrease in consumer demand.

With the parameters $t = 4$, $k = 10$, $L = 6$, $s_1 = 0.3$, and $s_2 = 0.5$, we also examine the relationship between the recall cost and the two enterprises’ sales prices in scenarios $D = 3$ and $D = 8$. As shown in Figure 5, there is a positive relationship between recall cost and sales prices, that is, the higher the recall cost, the higher the sales price determined by enterprises ($p'_i$ and $p'_2$). However, in addition to this direct impact of the recall cost, an indirect impact resulting from the increased quality level of the product also strengthens each enterprise’s motivation to charge a higher sales price. Furthermore, it is straightforward to further observe from Figure 5 that, in contrast to scenario $D = 3$, both enterprises tend to lower their sales prices when the consumer disutility $D = 8$, which is consistent with the finding in Figure 4.

### 5.3. The impact of traceability and liability-related factors on consumer demand

**Proposition 4.** Given the values of other parameters, it holds that as an enterprise $i$’s traceability level $s_i$ increases, the consumer demand for this enterprise’s product is decreased (increased) if $c \geq (<) D$, i.e.,
\[ \partial D_i^* / \partial s_i \leq (\geq) 0. \] Accordingly, the consumer demand for the competitor j’s product of this enterprise will increase (decrease), i.e., \( \partial D_j^* / \partial s_i > (\leq) 0 \) \( (i = 1, 2; \ j = 1, 2; \ i \neq j) \).

Proposition 4 summarizes the impact of traceability on consumer demand for both enterprises’ products. The results suggest that as an enterprise’s traceability level increases, the expected social cost rises (falls) when the recall cost is higher (lower) than the consumer disutility \( (c \geq (<) D) \), bringing about lower (higher) consumer demand for the enterprise. Nevertheless, due to the trade-off in consumer product preferences between two competing enterprises, demand for the competitor’s product will increase (drop). Specifically, one can see from equations (1) and (2) that a consumer’s demand choice for an enterprise’s product is primarily influenced by two aspects. On one hand, a higher aggregate expenditure by the consumer for purchasing the enterprise’s product, including the sales price paid to the enterprise and the expected net disutility for each unit of product, i.e., \( p_i + y_i (1 - \theta_i) \), will reduce consumer demand for the enterprise (we denote it as the cost effect). On the other hand, a higher aggregate expenditure for purchasing the competitor’s product, i.e., \( p_j + y_j (1 - \theta_j) \), induces the consumer to shift demand away from the competitor and toward the enterprise (we denote it as the competition effect). Thus, whether consumer demand for the enterprise increases or decreases with the level of traceability depends on the comparison between the cost and competition effects. At the point when \( c \geq D \), the consumer demand choice for the enterprise is overwhelmed by the cost effect as opposed to the competition effect, then...
the demand for the enterprise rises and the demand for the competitor falls. Similarly, we can also explain the scenario \( c < D \). It is worth noting that the majority of existing studies, including Starbird and Amanor-Boadu [60], Resende-Filho and Hurley [54], Cui et al. [17], and Dong et al. [19], assume that consumer demand is exogenously given, leaving out the effect of traceability. The findings of Proposition 4, which partially fill the research gap left by previous studies, demonstrate how traceability influences customers’ purchasing decisions.

Figure 6 examines the results given in Proposition 4 through the numerical study based on the scenarios \( D = 3 \) and \( D = 8 \) by fixing the parameters \( t = 4 \), \( k = 10 \), \( c = 4 \), \( L = 6 \), and \( s_2 = 0.5 \). First, we can see from Figure 6 that when \( D = 3 \), which implies \( c \geq D \), consumer demand \( D^*_1 \) for enterprise 1’s product decreases in enterprise 1’s traceability level \( s_1 \), whereas consumer demand \( D^*_2 \) for enterprise 2’s product increases. However, Figure 6 also shows that when \( c < D \), the consumer’s demand decisions are the inverse of those in scenario \( c \geq D \). Thus, Proposition 4’s results are demonstrated. Furthermore, when \( c \geq D \), if \( s_1 \leq s_2 = 0.5 \), then \( D^*_1 \geq (<)D^*_2 \), implying that consumer demand for enterprise 1 is relatively higher (lower) than that for enterprise 2. Noting that enterprise 1 chooses a lower product quality and a higher sales price than enterprise 2 if \( s_1 \leq s_2 \), this indicates that lower product quality and higher sales prices do not result in lower consumer demand for enterprise 1. The reason for this is that when enterprise 1’s traceability level is relatively low, the consumer’s expected net disutility for purchasing enterprise 1’s product is lower, which leads to lower aggregate expenditure. Therefore, the consumer is motivated to purchase more products from enterprise 1 instead of enterprise 2. However, if \( s_1 > s_2 \), the higher expected net disutility caused by enterprise 1’s low-quality product to the consumer leads to lower consumer demand, even if enterprise 1 sells a higher quality product at a lower sales price. Moreover, we can see from Figure 6 that when \( c < D \), if \( s_1 \leq (>)= s_2 = 0.5 \), then \( D^*_1 \leq (>)= D^*_2 \), indicating that the consumer will purchase fewer (more) products from enterprise 1 if enterprise 1’s traceability level is relatively lower (higher) than enterprise 2. The reasons can be explained in the same way as in scenario \( c \geq D \).

Proposition 5. Given the values of other parameters, it holds that:

1. as product liability \( L \) increases, the consumer demand for the products of the two enterprises remains unchanged, i.e., \( \partial D^*_i / \partial L = 0 \).
2. as the recall cost \( c \) increases, the consumer demand for enterprise \( i \)’s product increases (decreases) if \( s_i \partial D^*_i / \partial (x_i + y_i) + s_j \partial D^*_j / \partial (x_j + y_j) \geq (<)0 \), i.e., \( \partial D^*_i / \partial c \geq (<)0 \). Accordingly, the demand for enterprise \( j \)’s product decreases (increases), \( \partial D^*_j / \partial c \leq (>)=0 \).
3. the consumer demand for enterprise \( i \)’s product is decreasing (increasing) in the consumer disutility \( D \) if \( (1 - s_i) \partial D^*_i / \partial (x_i + y_i) + (1 - s_j) \partial D^*_j / \partial (x_j + y_j) \geq (<)0 \), i.e., \( \partial D^*_j / \partial D \geq (<)0 \). Meanwhile, the demand for the competitor \( j \)’s product is increased (decreased), i.e., \( \partial D^*_j / \partial D \leq (>)=0 \) \( (i = 1, 2; j = 1, 2; i \neq j) \).
First, Proposition 5(1) states that product liability does not change consumer demand. It is clear from the previous analysis that the increase in the expected liability cost as a result of product liability is equivalent to the increase in the sales price. Consequently, the consumer’s net utility will no longer be affected by product liability, resulting in unchanged consumer demand. This finding is somewhat in line with the observation of Buzby and Frenzen [8] that the majority of foodborne illness cases in the United States do not result in legal action. Even when consumers file lawsuits, they are typically paid less than their losses, and they must pay more than 60% of product liability awards to attorneys as fees for services rendered. With this understanding, it is clear that product liability has little or no impact on the product preferences of consumers.

According to Proposition 5(2), in a monopoly situation, consumer demand declines as the recall cost increases. Evidently, an increase in the recall cost rises the expected social cost, which motivates the enterprise to improve product quality and, consequently, its sales price. This obviously reduces consumer demand for the enterprise’s products. This finding is directly supported by empirical studies that show a significant negative correlation between a product recall and market demand in the automobile [16] and meat industries [46]. From the perspective of the supply chain, Fan et al. [21]’s theoretical results also show that higher recall costs (and consumer disutility) reduce consumer demand. Unlike the monopoly situation, however, in a duopoly market, the demand choices of a consumer for two enterprises’ products are a tradeoff (both Fig. 6 above and Fig. 7 below demonstrate this). This is due to the fact that customer demand for an enterprise’s product is influenced by two sources. On the one hand, similar to the monopoly scenario, increased recall cost will have a direct impact on consumer demand, and \( s_i \partial D_i^r/\partial (x_i + y_i) \) indicates this direct and negative impact of the recall cost on consumer demand for enterprise \( i \)'s product. Higher recall cost, on the other hand, raises the competitor’s sales price, driving consumer demand for the enterprise’s product. \( s_j \partial D_j^r/\partial (x_j + y_j) \) refers to the positive and competitive effect of the recall cost on consumer demand for enterprise \( i \)'s product. Hence, if the direct effect of the recall cost on consumer demand exceeds the competitive effect, consumer demand for the enterprise \( i \)'s product falls, resulting in higher demand for the competitor \( j \)'s product. Instead, consumer demand for the enterprise’s product rises, reducing demand for the competitor \( j \)'s product. Similarly, how consumer disutility affects the consumer demand for the two enterprises can be explained in the same way that the recall cost can.

Figure 7 examines the impact of the recall cost on consumer demand using a numerical study in scenarios \( D = 3 \) and \( D = 8 \) with the parameters \( t = 4 \), \( k = 10 \), \( L = 6 \), \( s_1 = 0.3 \), and \( s_2 = 0.5 \). As shown in Figure 7, the parameters satisfy the condition \( s_1 \partial D_1^r/\partial (x_1 + y_1) + s_2 \partial D_2^r/\partial (x_2 + y_2) \geq 0 \), and thus consumer demand for enterprise 1’s product \( D_1^r \) rises (falls) as the recall cost \( c \) (the consumer disutility \( D \)) increases. Meanwhile, these parameters also satisfy the condition \((1 - s_i) \partial D_i^r/\partial (x_i + y_i) + (1 - s_j) \partial D_j^r/\partial (x_j + y_j) \leq 0 \), indicating that consumer demand for enterprise 2’s product \( D_2^r \) decreases (increases) as the recall cost (consumer disutility) rises.

### 5.4. The impact of traceability and liability-related factors on profits

**Proposition 6.** Given the values of other parameters, it holds that:

1. As an enterprise \( i \)'s traceability level \( s_i \) increases, the enterprise’s profit decreases (increases) if \( c \geq (<)D \), i.e., \( \partial \pi_i^r/\partial s_i \leq (>0) \).

2. As the competitor \( j \)'s traceability level \( s_j \) increases, the enterprise \( i \)'s profit will increase (decrease) if \( c \geq (<)D \), i.e., \( \partial \pi_i^r/\partial s_j \geq (<0) \) (\( i = 1, 2; j = 1, 2; i \neq j \)).

Proposition 6 explores the effect of traceability on the profits of the two enterprises. The result demonstrates that if the recall cost is greater (less) than the consumer disutility, an enterprise’s profit decreases (increases) with its own traceability level while increasing (decreasing) with that of its competitor. This illustrates how an increase in the expected social cost caused by an enterprise’s increased traceability harms the enterprise itself while benefiting its competitor. More specifically, the preceding results show that, while increasing an enterprise’s traceability level encourages the enterprise to improve product quality when \( c \geq D \), it still raises the enterprise’s quality investment cost and reduces both the sales price and consumer demand. As a consequence, the enterprise’s profit will be reduced. Nonetheless, as the competitor’s traceability level increases, the enterprise
can sell more products to consumers at a higher sales price due to an improvement in product quality and a corresponding reduction in consumer expected net disutility. Thus, by increasing the traceability of its competitors, the enterprise’s profit is increased. These findings suggest that when the cost of the recall is high but the consumer disutility is low, establishing a traceability system can stimulate both enterprises to sell higher quality products. However, the enterprise that establishes this traceability system will lose money, while its competitor’s profit increases. This means that both enterprises may choose not to set up a traceability system. In contrast to the scenario $c \geq D$, if $c < D$, increasing an enterprise’s traceability level is useful for increasing its own profit, but it reduces the profit of its competitor. This can also be explained in the same manner. These findings indicate that when the recall cost is low and the consumer disutility is high, an enterprise can benefit from its own traceability system and hence enhance its competitiveness. Therefore, each enterprise may strive to build the most effective traceability system possible.

Proposition 6 reveals that an enterprise’s increased traceability does not always imply higher profit for the enterprise and can provide a free-riding opportunity for its competitor under certain situations. These findings are roughly consistent with those of Dong et al. [19] and Cui et al. [17]. For example, Dong et al. [19] show that in some supply chain networks, suppliers’ traceability adoption might lead to increased contamination risk and lower profit while always benefitting the retailer. Cui et al. [17] demonstrate that in a parallel supply chain, traceability helps the buyer while diminishing suppliers’ efforts to enhance product quality, resulting in decreased profitability. However, in contrast to the previous studies, Proposition 6 also demonstrates that an enterprise’s quality decisions caused by increased traceability are in conflict with its profitability, i.e., higher product quality does not result in higher profitability, and lower product quality results in higher profitability. This finding explains why certain enterprises are so active or inactive in implementing the traceability system.

To clearly depict the results in Proposition 6, Figure 8 runs a numerical study with the parameters $t = 4$, $k = 10$, $c = 4$, $L = 6$, and $s_2 = 0.5$ in scenarios $D = 3$ and $D = 8$. As illustrated in Figure 8, as the traceability level of enterprise 1 $s_1$ rises, if $c \geq (\prec)D$, then the profit of enterprise 1 $\pi_1^*$ decreases (increases), while the profit of its competitor (enterprise 2) $\pi_2^*$ increases (decreases). Therefore, the aforementioned Proposition 6 results are certified. Furthermore, it can be seen that when $c \geq (\prec)D$, if $s_1 \leq 0.5$, the profit of enterprise 1 $\pi_1^*$ is larger (smaller) than the profit of its competitor (enterprise 2) $\pi_2^*$; and in contrast, the profit of enterprise 1 $\pi_1^*$ is smaller (larger) than the profit of enterprise 2 $\pi_2^*$ if $s_1 \leq 0.5$. In addition, Figure 9 depicts the effect of traceability on the total profit of two competing enterprises ($\pi_c^* = \pi_1^* + \pi_2^*$). It clearly shows that as the traceability level of enterprise 1 $s_1$ increases, the total profit decreases (increases) if $c \geq (\prec)D$. The reason for this is that if the recall cost is higher (lower) than the consumer disutility, increased traceability leads to an increased expected social cost, resulting in a lower (higher) total profit.
Proposition 7. Given the values of other parameters, it holds that:

1. as product liability $L$ increases, the profits of both enterprises remain unchanged, i.e., $\partial \pi^*_i / \partial L = 0$.
2. as the recall cost $c$ increases, the enterprise $i$’s profit increases (decreases) if
   $$s_i \partial D^*_i / \partial (x_i + y_i) + s_j \partial D^*_j / \partial (x_j + y_j) \geq (<) s_i (x_i + y_i) D^*_i / [9kt - (x_i + y_i)^2]$$
   i.e., $\partial \pi^*_i / \partial c \geq (<) 0$.
3. as the consumer disutility $D$ increases, the enterprise $i$’s profit decreases (increases) if
   $$(1 - s_i) \partial D^*_i / \partial (x_i + y_i) + (1 - s_j) \partial D^*_j / \partial (x_j + y_j) \geq (<) (1 - s_i) (x_i + y_i) D^*_i / [9kt - (x_i + y_i)^2]$$
   i.e., $\partial \pi^*_i / \partial D \geq (<) 0$ (i = 1, 2; j = 1, 2; i $\neq$ j).

Proposition 7 demonstrates the influence of liability-related factors on enterprise profits. First, Proposition 7(1) states that product liability does not change the profits of the two enterprises. As illustrated in Propositions 2, 3, and 5, if an enterprise can fully transfer increased product liability by raising its sales price while maintaining product quality and consumer demand constant, the enterprise’s profit will remain constant. As for enterprise profit, we can deduce from equations (13) to (14) that
$$\pi^*_i = 2t (D^*_i)^2 - k (\theta^*_i)^2 / 2,$$
implying that whether enterprise $i$’s profit rises or falls with the recall cost is essentially determined by the relative impact of the recall cost on consumer demand and product quality. As previously demonstrated in the analysis, higher recall costs may increase or reduce consumer demand, but they always result in higher quality...
investment costs. Based on this, Propositions 7(2) shows that, when \( s_i \partial D_i^* / \partial (x_i + y_i) + s_j \partial D_j^* / \partial (x_j + y_j) \geq s_i (x_i + y_i) D_i^* / [9kt - (x_i + y_i)^2] \), enterprise \( i \)'s profit increases as the recall cost increases. The reason for this is straightforward. According to Proposition 5(2), if the competitive effect of recall costs on consumer demand for an enterprise’s products is greater than the direct effect and exceeds a threshold, higher recall cost results in higher sales revenue for the enterprise through increasing consumer demand, and sales revenue growth exceeds the increase in quality investment costs, resulting in higher enterprise profit. But, when \( 0 < s_i \partial D_i^* / \partial (x_i + y_i) + s_j \partial D_j^* / \partial (x_j + y_j) < s_i (x_i + y_i) D_i^* / [9kt - (x_i + y_i)^2] \), enterprise \( i \)'s profit decreases as the recall cost increases even if consumer demand for enterprise \( i \)'s product increases. This is because, when the increase in consumer demand loses its dominance in determining the profit of an enterprise, the increase in quality investment cost will result in a drop in profit. When \( s_i \partial D_i^* / \partial (x_i + y_i) + s_j \partial D_j^* / \partial (x_j + y_j) < 0 \), the direct effect is stronger than the competitive effect, implying that higher recall cost reduces consumer demand, leading to a decrease in enterprise profit.

This finding shows how an enterprise’s profit varies as recall costs rise, and how it is primarily influenced by consumer demand, quality investment costs, and competition. In a monopoly situation, increasing recall costs directly reduces the enterprise’s profit. This negative impact is consistent with the findings of empirical studies. Chen et al. [13], for example, use CPSC recalls from 1996 to 2007 to demonstrate that product recall reduces firm value. Toyota’s recall is also associated with a 15% drop in share value and a 19% drop in cumulative abnormal returns [21]. In addition, Govindaraj et al. [28] illustrate that the major competitors of Bridgestone Corporation in the tire and auto industries experienced a significant increase in the market value of their stocks as a result of Bridgestone’s recall of Firestone tires. This naturally fits with the finding of Proposition 7(2) in the duopoly scenario. Finally, a similar explanation can be used to explain how customer disutility affects enterprise profits in Proposition 7(3).

Figure 10 depicts the impact of the recall cost on enterprise profits in further numerical research based on scenarios \( D = 3 \) and \( D = 8 \) using the parameters \( t = 4, k = 10, L = 6, s_1 = 0.3, \) and \( s_2 = 0.5 \). As shown in Figure 10, the parameters for enterprise 1 fulfill \( s_1 \partial D_1^* / \partial (x_1 + y_1) + s_2 \partial D_2^* / \partial (x_2 + y_2) \geq s_1 (x_1 + y_1) D_1^* / [9kt - (x_1 + y_1)^2] \) such that as the recall cost \( c \) (consumer disutility \( D \)) increases, enterprise 1’s profit \( \pi_1^* \) increases (decreases); while the parameters for enterprise 2 also satisfy \( (1 - s_i) \partial D_j^* / \partial (x_i + y_i) + (1 - s_j) \partial D_j^* / \partial (x_j + y_j) < (1 - s_i) (x_i + y_i) D_i^* / [9kt - (x_i + y_i)^2] \). Thus, as the recall cost \( c \) (consumer disutility \( D \)) increases, enterprise 2’s profit \( \pi_2^* \) decreases (increases). Moreover, Figure 11 shows the impact of the recall cost \( c \) (consumer disutility \( D \)) on the total profit. It shows that as the recall cost \( c \) (consumer disutility \( D \)) increases, so does the total profit due to the increased expected social cost.
5.5. The impact of transportation cost on equilibrium results

Proposition 8. Given the values of other parameters, as the transportation cost \( t \) increases, it holds that:

1. the enterprise \( i \)'s product quality level will increase (decrease) while its competitor \( j \)'s product quality level will decrease (increase) if \( x_i + y_i \geq (<)x_j + y_j \), i.e., \( \partial \theta^*_i / \partial t \geq (<)0 \) and \( \partial \theta^*_j / \partial t \leq (>0) \).
2. both enterprises’ sales prices will increase, i.e., \( \partial p^*_i / \partial t > 0 \).
3. the consumer demand for the enterprise \( i \)'s product will increase (decrease) while demand for the competitor \( j \)'s product will decrease (increase) if \( x_i + y_i \geq (<)x_j + y_j \), i.e., \( \partial D^*_i / \partial t \geq (<)0 \) and \( \partial D^*_j / \partial t \leq (>0) \).
4. both enterprises’ profits will increase, i.e., \( \partial \pi^*_i / \partial t > 0 \) \( (i = 1, 2; j = 1, 2; i \neq j) \).

Proposition 8 investigates the effect of the transportation cost on the duopoly’s equilibrium results. First, it demonstrates that as the transportation cost rises, an enterprise chooses to improve (decrease) product quality if the expected social cost caused by the enterprise is greater (lower) than that of its competitor, whereas the enterprise will always raise its sales price. In general, the greater the cost of transportation, the greater the product differentiation between the two enterprises, implying that each enterprise has stronger monopoly power over surrounding consumers. As also shown in equations (1) and (2), an increase in the transportation cost reduces the consumer’s sensitivity to the sales price and product quality; as a result, the intensity of price and quality competition between the two enterprises will be reduced, motivating enterprises to increase sales prices and decrease product quality levels. However, a higher expected social cost incurred by an enterprise relative to its competitor will change the enterprise’s motivation to improve product quality, as the enterprise must mitigate the negative impact of the low-quality product. Furthermore, Proposition 8 illustrates that when the enterprise faces a relatively lower expected social cost, its decisions to reduce product quality and increase sales price reduce consumer demand for the product. Otherwise, an increase in product quality induced by a higher expected social cost is carried out to increase consumer demand. Finally, Proposition 8 demonstrates a universal truth: increasing product differentiation benefits enterprise profitability.

6. Extensions

6.1. The interaction impact of traceability and liability-related factors on profits

In the previous Subsection 5.4, the impact of traceability and liability-related factors on duopoly profits, respectively, is examined. This subsection aims to investigate the interaction impact of traceability and liability-related factors on profits. First, by simultaneously taking the derivative of \( \pi^*_i \) \( (i = 1, 2) \) with respect to \( L \) and
We first set examine the interaction impact of traceability and recall cost (and customer disutility) on enterprise profits. With the result that implementing a more efficient traceability system will exacerbate the negative impact of the recall cost on its profit. Therefore, enterprise 1 should refrain from increasing the level of traceability. In contrast, if \( s_1 \) decreases in \( s_1 \), raising the traceability level of enterprise 1 of traceability will reduce this positive impact. In this situation, enterprise 1 would be wise to reduce the level of traceability.

Similarly, \( \frac{\partial^2 \pi^*_i}{\partial c \partial s_i} \) and \( \frac{\partial^2 \pi^*_i}{\partial D \partial s_i} \) are derived as follows.

\[
\frac{\partial^2 \pi^*_i}{\partial c \partial s_i} = \frac{4D^*_i}{9k} \left\{ \left[ 9kt - (x_i + y_i)^2 \right] \frac{\partial D^*_i}{\partial (x_i + y_i)} - (x_i + y_i) D^*_i \right\} + \frac{4(c - D) s_2}{9k} \left\{ \left[ 9kt - (x_i + y_i)^2 \right] \left( \frac{\partial D^*_i}{\partial (x_i + y_i)} \right)^2 \right. + \left. D_i \frac{\partial^2 D^*_i}{\partial (x_i + y_i)^2} \right\}
\]

\[
-4D^*_i (x_i + y_i) \frac{\partial D^*_i}{\partial (x_j + y_j)} - (D^*_i)^2 \right\}
\]

\[
+ \frac{4(c - D) \left( 1 - s_2 \right)}{9k} \left\{ \left[ 9kt - (x_i + y_i)^2 \right] \left( \frac{\partial D^*_i}{\partial (x_j + y_j)} \right)^2 \right. + \left. D_i \frac{\partial^2 D^*_i}{\partial (x_j + y_j)^2} \right\}
\]

\[
-2D^*_i (x_i + y_i) \frac{\partial D^*_i}{\partial (x_j + y_j)} \right\}
\]

Since obtaining analytic results for the above two equations is challenging, we opt for numerical studies to examine the interaction impact of traceability and recall cost (and customer disutility) on enterprise profits.

We first set \( t = 4, k = 10, L = 6, D = 3, c = 4, \) and \( s_2 = 0.5 \) in such a way that the values of these parameters satisfy the requirements \( k > \max \{ K_i, K_i \} \) and \( t > \max \{ T_i, T_i \} \), and then investigate the impact of enterprise 1’s traceability level on its profit (See Prop. 7 for details). As can be seen in Figure 12, as \( s_1 \) increases, \( \frac{\partial \pi^*_i}{\partial c} \) decreases while \( \frac{\partial \pi^*_i}{\partial D} \) increases. More specifically, it can be seen that \( \frac{\partial \pi^*_i}{\partial c} > 0 \) if \( s_1 \leq 0.435 \), which implies that higher recall cost benefits enterprise 1’s profit. In this situation, enterprise 1 would be wise to reduce the level of traceability. In contrast, if \( s_1 > 0.435 \), it shows that \( \frac{\partial \pi^*_i}{\partial D} < 0 \). The recall cost hurts enterprise 1’s profit, and a higher traceability level will accelerate this impact. That is, raising the traceability level of enterprise 1 will exacerbate the negative impact of the recall cost on its profit. Therefore, enterprise 1 should refrain from implementing a more efficient traceability system.

Figure 12 further shows that \( \frac{\partial \pi^*_i}{\partial c} < 0 \) if \( s_1 \leq 0.575 \), indicating that a higher recall cost results in a lower profitability for enterprise 2. With the result that \( \frac{\partial \pi^*_i}{\partial D} \) increases in \( s_1 \), increasing enterprise 1’s traceability level reduces the negative impact of the recall cost on enterprise 2’s profit. Therefore, a reasonable decision for enterprise 1 as a competitor is to reduce the traceability level. Figure 12 also shows that \( \frac{\partial \pi^*_i}{\partial D} > 0 \) if \( s_1 > 0.575 \), showing that the recall cost has a positive effect on the profit of enterprise 2. Hence, increasing enterprise 1’s traceability level benefits enterprise 2’s profit when the recall cost increases. Therefore, it is preferable for
Figure 12. Interaction impact of traceability and recall cost on profits.

Figure 13. Interaction impact of traceability and consumer disutility on profits.

enterprise 1 to decrease the traceability level in order to limit the increase in profit for enterprise 2. Figure 13 demonstrates the interaction impact of traceability and customer disutility on enterprise profits, which may also be explained similarly.

6.2. The impact of positive traceability costs

In this subsection, we relax the zero-cost assumption for the traceability of duopoly enterprises and assume that the traceability cost per unit of product is $c_s > 0$. Of course, the assumption that the traceability costs of both enterprises are the same stems from the fact that numerous competitive enterprises may use the same traceability level. For example, in the food industry, both Tyson Foods and Golden State Foods, in the fruit industry, both Dole and Driscoll's, and in the retailing industry, both Kroger and Walmart use IBM blockchain
technology for traceability. Accordingly, the equilibrium results are as follows.

\[
\theta_1^{**} = \frac{(x_1 + y_1)\left[3k \left(3t - x_1 - y_1 + x_2 + y_2\right) - 2(x_2 + y_2)^2\right]}{3k\left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
\]

\[
\theta_2^{**} = \frac{(x_2 + y_2)\left[3k \left(3t - x_2 - y_2 + x_1 + y_1\right) - 2(x_1 + y_1)^2\right]}{3k\left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
\]

\[
p_1^{**} = \frac{3t + (1 - \theta_2^{**}) (x_2 + y_2) + (1 - \theta_1^{**}) (2x_1 - y_1)}{3} + c_s,
\]

\[
p_2^{**} = \frac{3t + (1 - \theta_1^{**}) (x_1 + y_1) + (1 - \theta_2^{**}) (2x_2 - y_2)}{3} + c_s,
\]

\[
D_1^{**} = \frac{3k \left(3t - x_1 - y_1 + x_2 + y_2\right) - 2(x_2 + y_2)^2}{2\left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
\]

\[
D_2^{**} = \frac{3k \left(3t - x_2 - y_2 + x_1 + y_1\right) - 2(x_1 + y_1)^2}{2\left[9kt - (x_1 + y_1)^2 - (x_2 + y_2)^2\right]},
\]

\[
\pi_1^{**} = \frac{9kt - (x_1 + y_1)^2}{18k} \left[3k \left(3t - x_1 - y_1 + x_2 + y_2\right) - 2(x_2 + y_2)^2\right]^2 - \frac{9kt - (x_1 + y_1)^2}{18k} \left[3k \left(3t - x_2 - y_2 + x_1 + y_1\right) - 2(x_1 + y_1)^2\right]^2.
\]

\[
\pi_2^{**} = \frac{9kt - (x_2 + y_2)^2}{18k} \left[3k \left(3t - x_2 - y_2 + x_1 + y_1\right) - 2(x_1 + y_1)^2\right]^2 - \frac{9kt - (x_1 + y_1)^2}{18k} \left[3k \left(3t - x_1 - y_1 + x_2 + y_2\right) - 2(x_2 + y_2)^2\right]^2.
\]

Comparing the above equilibrium results with those with no traceability costs, it holds that \(\theta_i^{**} = \theta_i^*\), \(D_i^{**} = D_i^*\), and \(\pi_i^{**} = \pi_i^*\) \((i = 1, 2)\), indicating that the equilibrium product quality, equilibrium consumer demand, and equilibrium profits stay unchanged. The direct reason is that as the cost of traceability grows, both enterprises prefer to raise their sales prices, allowing them not to adjust their product quality decisions. At the same time, the consumer’s preferences for the two enterprises’ products are unaffected by the same positive traceability cost. As a result, under the positive traceability cost assumption, the enterprises’ profits stay constant.

6.3. The impact of traceability when \(k \leq \max\{K_i, \bar{K}_i\}\) or \(t \leq \max\{T_i, \bar{T}_i\}\)

Section 5 specified some more stringent requirements for quality improvement efficiency \((k > \max\{K_i, \bar{K}_i\})\) and transportation costs \((t > \max\{T_i, \bar{T}_i\})\). This subsection is to discuss whether the results from Section 5 about the impact of traceability on equilibrium results still hold when \(k \leq \max\{K_i, \bar{K}_i\}\) or \(t \leq \max\{T_i, \bar{T}_i\}\). The results of the comparative statics regarding traceability are shown below.

We first relax the transportation cost limitation and conduct numerical studies according to \(t \leq \max\{T_i, \bar{T}_i\}\) and \(k > \max\{K_i, \bar{K}_i\}\). Similar to Section 5, we investigate the effect of the level of enterprise 1’s traceability on the equilibrium results in scenarios \(D = 3\) and \(D = 8\), respectively, by setting \(t = 1.5\), \(k = 10\), \(L = 6\), \(c = 4\), and \(s_2 = 0.5\), where the parameters are consistent with Section 5 except for the transportation cost \(t\). Figures 14–17 report the numerical results. It can be seen from Figures 14 to 17 that in scenario \(c > D\), relaxing the limitation of transportation cost does not change the comparative static results with regard to traceability, except that the size comparison between the two enterprises’ quality levels is altered. It shows that \(\theta_1^* \geq (\geq)\theta_2^*\) if \(s_1 \leq (>)0.5\), which is completely opposite to that in Figure 1 where the transportation cost is high enough \((t = 4)\).
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**Figure 14.** Impact of traceability on quality ($t = 1.5$).

**Figure 15.** Impact of traceability on sales prices ($t = 1.5$).

**Figure 16.** Impact of traceability on demand ($t = 1.5$).

However, in scenario $c < D$, Figure 14 shows that enterprise 2’s product quality decision is decreasing in enterprise 1’s traceability, which is consistent with that in Figure 1, whereas enterprise 1’s product quality decision will increase first and then decrease in its traceability level, which differs from that in Figure 1. Additionally, if $s_1 \leq (>)0.5$, $\theta^*_1 \leq (>)\theta^*_2$, implying that enterprise 1’s quality level is lower (higher) than that of enterprise 2. Moreover, neither of them will always choose a higher product quality level in scenario $c < D$ than in scenario $c > D$. Evidently, these two results differ from those in Figure 1. Regarding sales prices, consumer demand, and enterprise profits, Figures 15–17 show that the comparative statics results with regard to traceability are also qualitatively consistent with those in Section 5 (specifically in Figs. 3, 6, and 8), except
that the comparative static result of enterprise 1’s sales price with regard to traceability is slightly changed. Specifically, enterprise 1’s sales price decision first decreases and then increases in its traceability level.

Figures 18–21 show the comparative statics results with regard to traceability when the limitation of quality improvement efficiency is relaxed. The numerical studies are carried out in scenarios \( D = 3 \) and \( D = 8 \) with \( t = 4, k = 4, L = 6, c = 4, \) and \( s_2 = 0.5 \). These parameters, of course, meet the requirements \( t > \max \{ T_i, \bar{T}_i \} \) and \( k \leq \max \{ K_i, \bar{K}_i \} \). Figures 18–21, when compared to the corresponding results in Section 5, show that in scenarios \( c > D \), the comparative statics results with regard to traceability are qualitatively consistent with those in Section 5, with the exception of the size comparison result between the two enterprises’ quality levels. This comparison result, however, is identical to that in Figure 14 where the limitation of transportation cost is relaxed.

Additionally, the comparative statics results with regard to traceability when relaxing the limitation of quality improvement efficiency in scenario \( c < D \) differ significantly from those in Section 5. More specifically, Figure 18 shows that enterprise 1’s product quality decision decreases in its traceability, which is consistent with that in Figure 1, but enterprise 2’s product quality decision is different from that in Figure 1 in that it increases in traceability first and subsequently decreases. According to Figure 19, when enterprise 1’s traceability level increases, so does its sales price, but enterprise 2’s sales price rises first and subsequently decreases. The sales price decisions of the two enterprises differ from those shown in Figure 3. More interestingly, in contrast to Figures 6 and 20 illustrates that the consumer demand for enterprise 1’s products decreases first and subsequently increases in enterprise 1’s traceability level. Instead, the consumer demand for enterprise 2’s products increases first, then decreases. Similarly, as seen in Figure 21, as enterprise 1’s traceability level increases, enterprise 1’s profit decreases first and then increases, but enterprise 2’s profit increases first and then decreases.
Figure 19. Impact of traceability on sales prices ($k = 14$).

Figure 20. Impact of traceability on demand ($k = 14$).

Figure 21. Impact of traceability on profits ($k = 14$).
7. Managerial insights

From the preceding comparative statics based on the equilibrium results in the duopoly, we derive the following managerial insights.

1. A higher level of traceability does not necessarily motivate enterprises to improve product quality. As illustrated in Corollary 2 and Proposition 1, whether the enterprises improve their product quality levels as traceability increases is primarily determined by the change in the expected social cost. This compels the managers to reconsider the relationship between traceability and product quality, as well as to systematically consider the significance of the expected social cost. Further, as long as one enterprise’s traceability level changes, both enterprises in the duopoly market make the same product quality decisions synchronously. This finding assists the government in managing the market and makes enterprises clearly aware of the impact of their traceability strategies on the overall competitive market.

2. A change in an enterprise’s traceability makes its quality performance to be at odds with its financial performance, while it causes its competitor’s quality performance to be in sync with its financial performance. Specifically, if an enterprise’s increased traceability motivates the enterprise to improve product quality by increasing the expected social cost, the enterprise’s quality improvement leads to a decrease in consumer demand and profitability, whereas the competitor’s product quality, consumer demand, and profitability all increase as the enterprise’s traceability increases. This strikes at the managers’ motivations for establishing a traceability system and suggests the government take actions to avoid free-riding in a competitive market or subsidize enterprises to improve traceability levels. Otherwise, an enterprise benefits from increased traceability even if it results in a lower quality level, while the competitor suffers profit loss. This case will inspire all competitive enterprises to increase their traceability levels.

3. Increasing an enterprise’s product liability to compensate consumers for the disutility caused by a low-quality product may not help to improve product quality and may have no effect on consumer demand and the enterprise’s profitability. This forces the government to carefully rethink the effectiveness of liability regulations or enact measures to preclude enterprises from transferring liability costs through their pricing strategies. However, increased recall costs as well as consumer disutility motivate enterprises to improve product quality, which is supported by Yadav et al. [62], who show that increasing the expense of pollution control leads to a decrease in the quantity of pollutants. Combining their effects on enterprise profitability, this suggests to managers that controlling recall cost and reducing consumer disutility subtly can provide enterprises with opportunities to improve operational performance while investing less in quality improvement.

8. Conclusion

Currently, the majority of existing studies involving traceability and quality generally assume that consumer demand is exogenously given, making it lack characterizing the impact of traceability and liability-related factors on consumers’ demand choices. Furthermore, most of them fail to differentiate between recall and liability (compensation) behaviors of the enterprise, thereby ignoring the impact of traceability on enterprise (expected) liability cost and, as a result, the impact of liability-related factors on quality decisions. Additionally, existing studies primarily focus on supply chain operations rather than competition in a duopoly market. To fill the gaps in existing studies, we consider a duopoly game model, where the products manufactured by enterprises and sold to consumers may be of low quality, leading enterprises to recall the low-quality product identified by traceability and compensate the consumer for the unidentified low-quality product. The main findings are outlined below.

1. When evaluating the impact of traceability on product quality decisions, the expected social cost is a crucial factor to consider. If an enterprise’s higher level of traceability raises (lowers) the expected social cost, the enterprise is more (less) motivated to improve product quality, and its duopolistic competitor makes quality decisions in accordance with it, suggesting that the duopoly’s quality competition is enhanced (weakened).
(2) If traceability positively affects the expected social cost, an enterprise’s sales price falls as its traceability increases but rises as that of its competitor, indicating that increased traceability, and thus increased product quality, does not always imply an increase in sales price. Otherwise, as its own and competitor’s traceability increases, the enterprise’s sales price falls, implying increased price competition between the two competing enterprises.

(3) Even if an enterprise can improve product quality while lowering the sales price as its traceability increases, this does not imply an increase in consumer demand and enterprise profit. In contrast, if the enterprise’s increased traceability reduces the expected social cost, a decrease in product quality and sales price may contribute to increased consumer demand for its products and the corresponding profit. However, its competitor’s increased (decreased) product quality as a result of this enterprise’s increased traceability leads to an increase (decrease) in both consumer demand and profitability.

(4) Higher product liability reduces price competition between the two enterprises but has no effect on quality competition, consumer demand, and profitability. Increased recall cost increases quality competition while decreasing price competition between the two enterprises. However, higher consumer disutility stimulates both quality and price competition.

(5) An increase in both recall cost and consumer disutility does not always imply a decrease in consumer demand and enterprise profits. If higher recall cost and consumer disutility have a greater negative impact on an enterprise’s competitor, the enterprise will be able to sell more products to consumers and achieve higher profitability. Under certain conditions, however, a reduction in both recall cost and consumer disutility can lead to higher profitability for enterprises with lower product quality levels.

Our study, to some extent, helps to enrich the research on the impact of traceability on product quality. However, it is also worth extending our study in the following several directions for future studies. First, in our model, we take the traceability level as exogenously given to characterize traceability’s impacts on quality and price competition. It would be interesting to take traceability as a decision variable [18] and then explore traceability competition between two enterprises as well as the interplay of quality decisions, price decisions, and traceability decisions. Second, in our model, we only consider competition between two enterprises and assume that they are symmetrical. These assumptions, however, can be relaxed to explore competition among multiple enterprises and the impact of enterprise asymmetries, such as enterprises with different production costs, traceability costs, and initial market sizes. Moreover, our study is based on the assumption of complete information between two competing enterprises and between enterprises and customers. Nonetheless, it is interesting to investigate such an incomplete information game in which quality and traceability information are private. Checking the robustness of our findings under the assumption of uncertain consumer demand is also worthwhile [55, 56]. Another extension of our study is to use actual survey data to validate the findings of our theoretical studies [49]. Finally, it is also a challenge to extend our study to supply chains [17–19] and explore the impact of traceability on product quality decisions in competitive supply chains.

**Appendix A. Proofs**

**Proof of the equilibrium of the monopoly model**

In the monopoly model, if enterprise 1 charges a sales price \( p_1 \leq v - y_1 \left( 1 - \theta_1 \right) - t \), then the optimal sales price decided by enterprise 1 is \( p_1 = v - y_1 \left( 1 - \theta_1 \right) - t \), thus, with the optimal consumer demand \( D_{11}^{m1*} = 1 \), the profit function can be written by

\[ \pi_1(\theta_1) = v - t - (x_1 + y_1)(1 - \theta_1) - \frac{1}{2}k\theta_1^2. \]

Enterprise 1 chooses the optimal product quality \( \theta_1 \) to maximize the profit. Since \( \pi_1(\theta_1) \) is strictly concave in \( \theta_1 \), differentiating \( \pi_1(\theta_1) \) with respect to \( \theta_1 \) and equating it to zero, we obtain

\[ \theta_1^{m1*} = \frac{x_1 + y_1}{k}. \]
Accordingly, the equilibrium sales price and profit of enterprise 1 are

\[ p_{1}^{m1*} = v - t - y_{1} + \frac{y_{1}(x_{1} + y_{1})}{k}, \]
\[ \pi_{1}^{m1*} = v - t - (x_{1} + y_{1}) + \frac{(x_{1} + y_{1})^{2}}{2k}. \]

Similarly, if enterprise 1 charges a sales price \( p_{1} > v - y_{1} (1 - \theta_{1}) - t \), in the condition \( 2kt - (x_{1} + y_{1})^{2} > 0 \) that ensures the concavity of \( \pi_{1} (\theta_{1}, p_{1}) \) in \( \theta_{1} \) and \( p_{1} \), differentiating \( \pi_{1} (\theta_{1}, p_{1}) \) with respect to \( \theta_{1} \) and \( p_{1} \) and equating them to zero, we obtain

\[ \theta_{1}^{m2*} = \frac{(v - x_{1} - y_{1})(x_{1} + y_{1})}{2kt - (x_{1} + y_{1})^{2}}, \]
\[ p_{1}^{m2*} = \frac{kt(v + x_{1} - y_{1}) - vx_{1}(x_{1} + y_{1})}{2kt - (x_{1} + y_{1})^{2}}. \]

Accordingly, the equilibrium consumer demand and profit of enterprise 1 are

\[ D_{1}^{m2*} = \frac{k(v - x_{1} - y_{1})}{2kt - (x_{1} + y_{1})^{2}}, \]
\[ \pi_{1}^{m2*} = \frac{k(v - x_{1} - y_{1})^{2}}{2[2kt - (x_{1} + y_{1})^{2}]} \]

By comparing \( \pi_{1}^{m1*} \) and \( \pi_{1}^{m2*} \), it holds that \( \pi_{1}^{m1*} < \pi_{1}^{m2*} \), indicating that enterprise 1 does not have the incentive to determine a low enough sales price to fully cover the market. That is, the market is partially covered and the optimal product quality and sales price are \( \theta_{1}^{m2*} \) and \( p_{1}^{m2*} \), respectively. However, to ensure that the condition \( \theta_{1}^{m2*} \in (0, 1) \) holds and all equilibrium results are nonnegative, the following conditions are required: \( v > x_{1} + y_{1} \) and \( k > v(x_{1} + y_{1})/2t \).

**Proof of Corollary 2.** With equations (7) and (8), by taking derivatives of \( \theta_{1}^{*} \) with respect to \( x_{i} + y_{i} \) and \( x_{j} + y_{j} \), respectively, we derive

\[ \frac{\partial \theta_{1}^{*}}{\partial (x_{i} + y_{i})} = \frac{6k(x_{i} + y_{i})[3k(3t - x_{i} - y_{i} + x_{j} + y_{j}) - 2(x_{j} + y_{j})^{2}]}{[9kt - (x_{i} + y_{i})^{2} - (x_{j} + y_{j})^{2}]^{2}} + \frac{9kt - (x_{i} + y_{i})^{2} - (x_{j} + y_{j})^{2}}{[9kt - (x_{i} + y_{i})^{2} - (x_{j} + y_{j})^{2}]^{2}} \]
\[ \frac{\partial \theta_{1}^{*}}{\partial (x_{j} + y_{j})} = \frac{(x_{i} + y_{i})\left\{ 9kt[3k - 2(x_{j} + y_{j})] + 4(x_{i} + y_{i})^{2}(x_{j} + y_{j}) - 3k[(x_{i} + y_{i} + x_{j} + y_{j})^{2} - 2(x_{j} + y_{j})^{2}] \right\}}{3k[9kt - (x_{i} + y_{i})^{2} - (x_{j} + y_{j})^{2}]} \]

Let \( T_{i} = \frac{2(x_{i} + y_{i}) - x_{i} - y_{i}}{3}, \overline{T}_{i} = \frac{3k[(x_{i} + y_{i} + x_{j} + y_{j})^{2} - 2(x_{i} + y_{i})^{2} - 4(x_{i} + y_{i})^{2}(x_{j} + y_{j})]}{9kt[3k - 2(x_{j} + y_{j})]}, \) \( K_{i} = \frac{2(x_{i} + y_{i})^{2}}{3k[3k - 2(x_{j} + y_{j}) + x_{i} + y_{i}]}, \) and \( \overline{K}_{i} = \frac{2(x_{i} + y_{i})^{2}}{3k[3k - 2(x_{j} + y_{j}) + x_{i} + y_{i}]} \).

Thus, if \( t > \max \{ T_{i}, \overline{T}_{i} \} \) and \( k > \max \{ K_{i}, \overline{K}_{i} \} \), we have \( \frac{\partial \theta_{1}^{*}}{\partial (x_{i} + y_{i})} > 0 \) and \( \frac{\partial \theta_{1}^{*}}{\partial (x_{j} + y_{j})} > 0 \). \( \square \)

**Proof of Corollary 3.** First, in the case that the two enterprises in the duopoly are identical, we have \( D_{1}^{*} = D_{2}^{*} = \frac{1}{2}, \theta_{1}^{*} = \theta_{2}^{*} = \frac{x_{1} + y_{1}}{3k}, p_{1}^{*} = p_{2}^{*} = t + x_{1} - \frac{x_{1}(x_{1} + y_{1})}{3k} \), and \( \pi_{1}^{*} = \pi_{2}^{*} = \frac{9kt - (x_{1} + y_{1})^{2}}{18k^{2}} \).
Thus, when $v > x_1 + y_1 + \frac{2t}{2} - \frac{(x_1 + y_1)^2}{2k}$, $k > v(x_1 + y_1)/2t$, $t > \max \{ T_i, T_i \}$, and $k \geq \max \{ K_i, K_i \}$, it first follows

$$D_i^m - D_i^s = \frac{2k(v - x_1 - y_1) - 2kt + (x_1 + y_1)^2}{2kt - (x_1 + y_1)^2} > 0,$$

$$\theta_i^m - \theta_i^s = \frac{(x_1 + y_1) [3kv - 3k (x_1 + y_1) - 2kt + (x_1 + y_1)^2]}{3k [2kt - (x_1 + y_1)^2]} > 0,$$

$$\pi_i^m - \pi_i^s = \frac{11kt (x_1 + y_1)^2 - (x_1 + y_1)^4 - 9k^2 [2t^2 - (v - x_1 - y_1)^2]}{18k [2kt - (x_1 + y_1)^2]} > 0.$$

Next, $p_i^m - p_i^s$ is calculated as

$$p_i^m - p_i^s = \frac{3kv [kt - (x_1 + y_1)] + 3k^2 t (x_1 - y_1) - [2kt - (x_1 + y_1)^2] [3k (t + x_1) - x_1 (x_1 + y_1)]}{3k [2kt - (x_1 + y_1)^2]}.$$

Thus, when $k > x_1 (x_1 + y_1)/t$, we have $p_i^m \geq (p_i^s)$ if $v \geq (\leq v^*)$, where $v^* = \frac{[2kt - (x_1 + y_1)^2][3k(t + x_1) - x_1(x_1 + y_1)] - 3k^2t(x_1 - y_1)}{3k[kt - x_1(x_1 + y_1)]}$.

Proof of Proposition 1. Following the result of Corollary 1, we have $\frac{\partial \theta_i^*}{\partial s_i} \geq (0)$ and $\frac{\partial \theta_i^*}{\partial s_i} \geq (0)$ if $c \geq (0)$.

Proof of Proposition 2. With equations (7) and (8), $\frac{\partial \theta_i^*}{\partial L}$ can be calculated as

$$\frac{\partial \theta_i^*}{\partial L} = \frac{\partial \theta_i^*}{\partial (x_i + y_i)} \frac{\partial (x_i + y_i)}{\partial L} + \frac{\partial \theta_i^*}{\partial (x_j + y_j)} \frac{\partial (x_j + y_j)}{\partial L}.$$

Thus, with $\frac{\partial (x_i + y_i)}{\partial L} = \frac{\partial (x_j + y_j)}{\partial L} = 0$, we have $\frac{\partial \theta_i^*}{\partial L} = 0$.

With equations (7) and (8), $\frac{\partial \theta_i^*}{\partial c}$ can be calculated as

$$\frac{\partial \theta_i^*}{\partial c} = \frac{\partial \theta_i^*}{\partial (x_i + y_i)} \frac{\partial (x_i + y_i)}{\partial c} + \frac{\partial \theta_i^*}{\partial (x_j + y_j)} \frac{\partial (x_j + y_j)}{\partial c} = s_i \frac{\partial \theta_i^*}{\partial (x_i + y_i)} + s_j \frac{\partial \theta_i^*}{\partial (x_j + y_j)}.$$

Thus, with $t > \max \{ T_i, T_i \}$ and $k \geq \max \{ K_i, K_i \}$, we have $\frac{\partial \theta_i^*}{\partial c} > 0$.

With equations (7) and (8), $\frac{\partial \theta_i^*}{\partial D}$ can be calculated as

$$\frac{\partial \theta_i^*}{\partial D} = \frac{\partial \theta_i^*}{\partial (x_i + y_i)} \frac{\partial (x_i + y_i)}{\partial D} + \frac{\partial \theta_i^*}{\partial (x_j + y_j)} \frac{\partial (x_j + y_j)}{\partial D} = (1 - s_i) \frac{\partial \theta_i^*}{\partial (x_i + y_i)} + (1 - s_j) \frac{\partial \theta_i^*}{\partial (x_j + y_j)}.$$

Thus, with $t > \max \{ T_i, T_i \}$ and $k \geq \max \{ K_i, K_i \}$, we have $\frac{\partial \theta_i^*}{\partial D} > 0$.

Proof of Proposition 4. With equations (11) and (12), differentiating $D_i^s$ with respect to $x_i + y_i$ and $x_j + y_j$, respectively, we have

$$\frac{\partial D_i^s}{\partial (x_i + y_i)} = \frac{3k [(x_i + y_i + x_j + y_j)^2 - 2(x_i + y_i)^2] - 4(x_j + y_j)^2 (x_i + y_i) - 9kt [3k - 2 (x_i + y_i)]}{2 [9kt - (x_i + y_i)^2 - (x_j + y_j)^2]^2},$$

$$\frac{\partial D_i^s}{\partial (x_j + y_j)} = \frac{9kt [3k - 2 (x_j + y_j)] - 3k [(x_i + y_i + x_j + y_j)^2 - 2(x_j + y_j)^2] + 4(x_i + y_i)^2 (x_j + y_j)}{2 [9kt - (x_i + y_i)^2 - (x_j + y_j)^2]^2}.$$
Thus, with $t > \max\{T_i, T_1\}$ and $k \geq \max\{K_i, K_1\}$, we have $\frac{\partial D_i^*}{\partial (x_i+y_i)} < 0$ and $\frac{\partial D_i^*}{\partial (x_j+y_j)} > 0$.

Then, $\partial D_i^*/\partial s_i$ and $\partial D_i^*/\partial s_j$ can be calculated as

$$\frac{\partial D_i^*}{\partial s_i} = \frac{\partial D_i^*}{\partial (x_i+y_i)} \frac{\partial (x_i+y_i)}{\partial s_i} = (c-D) \frac{\partial D_i^*}{\partial (x_i+y_i)},$$

$$\frac{\partial D_i^*}{\partial s_j} = \frac{\partial D_i^*}{\partial (x_j+y_j)} \frac{\partial (x_j+y_j)}{\partial s_j} = (c-D) \frac{\partial D_i^*}{\partial (x_j+y_j)}.$$ 

Thus, we have $\frac{\partial D_i^*}{\partial s_i} \leq (>)0$ and $\frac{\partial D_i^*}{\partial s_j} \geq (>)0$ if $c \geq (>)D$. \hfill $\square$

**Proof of Proposition 5.** With equations (11) and (12), $\partial D_i^*/\partial L$ can be calculated as

$$\frac{\partial D_i^*}{\partial L} = \frac{\partial D_i^*}{\partial (x_i+y_i)} \frac{\partial (x_i+y_i)}{\partial L} + \frac{\partial D_i^*}{\partial (x_j+y_j)} \frac{\partial (x_j+y_j)}{\partial L} = 0.$$

With equations (11) and (12), $\partial D_i^*/\partial c$ can be calculated as

$$\frac{\partial D_i^*}{\partial c} = \frac{\partial D_i^*}{\partial (x_i+y_i)} \frac{\partial (x_i+y_i)}{\partial c} + \frac{\partial D_i^*}{\partial (x_j+y_j)} \frac{\partial (x_j+y_j)}{\partial c} = s_i \frac{\partial D_i^*}{\partial (x_i+y_i)} + s_j \frac{\partial D_i^*}{\partial (x_j+y_j)}.$$ 

Thus, we have $\frac{\partial D_i^*}{\partial c} \geq (>)0$ and $\frac{\partial D_i^*}{\partial c} = -\frac{\partial D_i^*}{\partial c} \leq (>)0$ if $s_i \frac{\partial D_i^*}{\partial (x_i+y_i)} + s_j \frac{\partial D_i^*}{\partial (x_j+y_j)} \geq (>)0$.

With equations (11) and (12), $\partial D_i^*/\partial D$ can be calculated as

$$\frac{\partial D_i^*}{\partial D} = \frac{\partial D_i^*}{\partial (x_i+y_i)} \frac{\partial (x_i+y_i)}{\partial D} + \frac{\partial D_i^*}{\partial (x_j+y_j)} \frac{\partial (x_j+y_j)}{\partial D} = (1-s_i) \frac{\partial D_i^*}{\partial (x_i+y_i)} + (1-s_j) \frac{\partial D_i^*}{\partial (x_j+y_j)}.$$ 

Thus, we have $\frac{\partial D_i^*}{\partial D} \leq (>)0$ and $\frac{\partial D_i^*}{\partial D} = -\frac{\partial D_i^*}{\partial D} \geq (>)0$ if $(1-s_i) \frac{\partial D_i^*}{\partial (x_i+y_i)} + (1-s_j) \frac{\partial D_i^*}{\partial (x_j+y_j)} \geq (>)0$. \hfill $\square$

**Proof of Proposition 6.** With equations (13) and (14), by taking derivatives of $\pi_i^*$ with respect to $x_i+y$ and $x_j+y$, respectively, we have

$$\frac{\partial \pi_i^*}{\partial (x_i+y_i)} = \frac{4D_i^*}{9k} \left\{ \frac{[9k-(x_i+y_i)^2]}{\partial (x_i+y_i)} \frac{\partial D_i^*}{\partial (x_i+y_i)} - (x_i+y_i) D_i^* \right\},$$

$$\frac{\partial \pi_i^*}{\partial (x_j+y_j)} = \frac{4D_i^*}{9k} \left[ 9k-(x_i+y_i)^2 \right] \frac{\partial D_i^*}{\partial (x_j+y_j)}.$$

With the result of Proposition 4, if $t > \max\{T_i, T_1\}$ and $k \geq \max\{K_i, K_1\}$, we have $\frac{\partial D_i^*}{\partial (x_i+y_i)} < 0$ and $\frac{\partial D_i^*}{\partial (x_j+y_j)} > 0$, thus, we derive $\frac{\partial \pi_i^*}{\partial (x_i+y_i)} < 0$ and $\frac{\partial \pi_i^*}{\partial (x_j+y_j)} > 0$.

Thus, we can calculate $\partial \pi_i^*/\partial s_i$ and $\partial \pi_i^*/\partial s_j$ as

$$\frac{\partial \pi_i^*}{\partial s_i} = \frac{\partial \pi_i^*}{\partial (x_i+y_i)} \frac{\partial (x_i+y_i)}{\partial s_i} = (c-D) \frac{\partial \pi_i^*}{\partial (x_i+y_i)},$$

$$\frac{\partial \pi_i^*}{\partial s_j} = \frac{\partial \pi_i^*}{\partial (x_j+y_j)} \frac{\partial (x_j+y_j)}{\partial s_j} = (c-D) \frac{\partial \pi_i^*}{\partial (x_j+y_j)}.$$ 

Thus, we have $\frac{\partial \pi_i^*}{\partial s_i} \leq (>)0$ and $\frac{\partial \pi_i^*}{\partial s_j} \geq (>)0$ if $c \geq (>)D$. \hfill $\square$
Proof of Proposition 7. With equations (13) and (14), \( \frac{\partial \pi_i^*}{\partial L} \) can be calculated as
\[
\frac{\partial \pi_i^*}{\partial L} = \frac{\partial \pi_i^*}{\partial (x_i + y_i)} \frac{\partial (x_i + y_i)}{\partial L} + \frac{\partial \pi_i^*}{\partial (x_j + y_j)} \frac{\partial (x_j + y_j)}{\partial L} = 0.
\]

With equations (13) and (14), \( \frac{\partial \pi_i^*}{\partial c} \) can be calculated as
\[
\frac{\partial \pi_i^*}{\partial c} = \frac{\partial \pi_i^*}{\partial (x_i + y_i)} \frac{\partial (x_i + y_i)}{\partial c} + \frac{\partial \pi_i^*}{\partial (x_j + y_j)} \frac{\partial (x_j + y_j)}{\partial c} = s_i \frac{\partial \pi_i^*}{\partial (x_i + y_i)} + s_j \frac{\partial \pi_i^*}{\partial (x_j + y_j)}.
\]

Thus, we have \( \frac{\partial \pi_i^*}{\partial c} \geq (<)0 \) if \( s_i \frac{\partial D_i^*}{\partial (x_i + y_i)} + s_j \frac{\partial D_i^*}{\partial (x_j + y_j)} \geq (>) s_i (x_i + y_i) D_i^* - \frac{9}{9k} (x_i + y_i)^2 \).

Proof of Proposition 8. With equations (7) and (8), \( \frac{\partial \theta_i^*}{\partial t} \) can be calculated as
\[
\frac{\partial \theta_i^*}{\partial t} = \frac{3}{9k} \frac{(x_i + y_i)(x_i + y_i - x_j - y_j)(3k - x_i - y_i - x_j - y_j)}{(k + (x_i + y_i)^2 - (x_j + y_j)^2)^2}.
\]

Thus, we have \( \frac{\partial \theta_i^*}{\partial t} \geq (>0)0 \) if \( x_i + y_i \geq (>0)x_j + y_j \).

With equations (9) and (10), \( \frac{\partial \pi_i^*}{\partial t} \) can be calculated as
\[
\frac{\partial \pi_i^*}{\partial t} = 1 - \frac{x_j + y_j \frac{\partial \theta_i^*}{\partial t}}{3} - \frac{2x_i - y_j \frac{\partial \theta_i^*}{\partial t}}{3}.
\]

Thus, we have \( \frac{\partial \pi_i^*}{\partial t} > 0 \) and \( \frac{\partial \pi_i^*}{\partial t} > 0 \).

With equations (11) and (12), \( \frac{\partial D_i^*}{\partial t} \) can be calculated as
\[
\frac{\partial D_i^*}{\partial t} = \frac{9k}{2} \frac{(x_i + y_i - x_j - y_j)(3k - x_i - y_i - x_j - y_j)}{(k + (x_i + y_i)^2 - (x_j + y_j)^2)^2}.
\]

Thus, we have \( \frac{\partial D_i^*}{\partial t} \geq (>0)0 \) if \( x_i + y_i \geq (>0)x_j + y_j \).

With equations (13) and (14), \( \frac{\partial \pi_i^*}{\partial t} \) can be calculated as
\[
\frac{\partial \pi_i^*}{\partial t} = \frac{9k}{2} \frac{3k(3t - x_i - y_i + x_j + y_j)}{(k + (x_i + y_i)^2 - (x_j + y_j)^2)^2} \left[ \frac{9k - (x_i + y_i)^2}{(x_i + y_i)^2 - (x_j + y_j)^2} \right] + \frac{9k - (x_i + y_i)^2}{(x_i + y_i)^2 - (x_j + y_j)^2} \frac{9k - (x_i + y_i)^2}{(x_i + y_i)^2 - (x_j + y_j)^2}.
\]

Thus, we have \( \frac{\partial \pi_i^*}{\partial t} > 0 \) and \( \frac{\partial \pi_i^*}{\partial t} > 0 \).

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