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ANALYSIS OF SERIES–PARALLEL SYSTEM'S SENSITIVITY IN CONTEXT OF COMPONENTS FAILURES

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Abstract. This research examines the reliability characteristics of a broad series–parallel system. Complex system, having elements in series and parallel configuration is considered here to compute the reliability. Due to the failure of component, the system can either breakdown or can work with reduced frequency. To analyze the effect of component failure, sensitivity of the system with respect to reliability parameters is analyzed. In this study, generally distributed and exponentially distributed repair rates are taken into consideration from failed state to the working state of the system. The mathematical model of the designed structure is developed using Markov process and supplementary variable technique. The comparative study of the system availability and its sensitivity with respect to general distribution and general as well as exponential distribution has been examined. The Gumbel–Hougaard family of copula is used to analyze the effect of both the distributions together. For a better explanation of this work, a numerical example has been provided and shown the results graphically.

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1. Introduction

In today's modern environment, everyone depends on the continuous operation of complicated systems/equipment. When purchasing equipment or working on a system, the first thing a person wants to know is how reliable it is, or how well it works or acts in the way that person expects it to. Reliability is therefore most prevalent crucial attributes of components, goods, and complicated systems. Leitch [17] defined "reliability" as a transient but desirable characteristic of a product or service that is typically assessed quite subjectively in everyday life. Being able to quantify reliability is essential because for an engineer, it has important financial and potentially even safety implications. Regarding both qualitative and quantitative research, Roberts and Priests [26] have shown validity and reliability. Both the integrity of research methodologies and the dependability of research outcomes are demonstrated and communicated using the terms reliability and validity. According to Mohajan [22], the reliability of a measuring device relates to one's confidence in the data obtained using the device or the degree to which any measuring equipment corrects for random mistake. The validity of a measuring device is concerned with what it measures and how well it does so.

Keywords. Series-parallel system, sensitivity analysis, Markov process, supplementary variable technique.

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Concerns about issue characteristics and methods of solution continue to emphasize on the reliability of a series–parallel system. Also, in pursuance of improving these systems, several researches have been carried out. Juang *et al.* [13] explained a series–parallel system comprehensively. In their opinion, a series–parallel system includes subsystems that have many components connected in series first, then in parallel, or subsystems that have many components connected in series first, then in parallel. A system and its functions can be defined primarily by its components, the logical connections between those components (series, parallel, etc.), and its functions. According to Schorr [27], there are primarily two categories of systems: those that cannot be repaired and those that can. A system is considered unrepairable if it cannot be repaired to carry out its specialized duties after failing. This does not always indicate that it can never be fixed. It simply means that a system failure when it is in use has consequences that are essentially irreversible. In contrast, a system is deemed repairable if, after malfunctioning, it can be rectified by swapping out the damaged components for new ones or being restored in some other way. The definition of a repairable system, according to Tyagi *et al.* [29], is one that can be updated to keep performing its duties.

A reliability allocation challenge was given by Yalaoui *et al.* [33] in a series–parallel system taking into account the Tillman and Truelove functions and examined the performances using different cost functions. In order to examine a repairable parallel–series multi-state system, Bisht and Singh [1] introduced an interval universal generating function (IUGF) technique and employed the Markov process to assess the probability of various components. Tian *et al.* [28] and Levitin *et al.* [19] also examined reliability and its performance for multi-state series–parallel systems. An issue of a series–parallel system's bi-objective reliability and cost was identified by Garg [4] using a collaborative approach. He then used the PSO (Particle Swarm Optimization) technique to resolve the issue, and then the genetic approach was used. PSO is based on communication and interaction, where individuals (particles) within the population (swarm) share information with one another. This technique attempts to improve a solution iteratively with respect to a specified metric of quality in order to optimize the problem. Beginning with a swarm of particles whose positions are initial solutions and whose velocities are randomly given in the search area, PSO describes the process. Nourelfath and Nahas [23] have effectively addressed a problem of series–parallel structural redundancy optimization utilizing multi-state models. Kumar and Kumar [14] used the Laplace transformation to resolve the system's Markov model, and looked at reliability metrics of the two parallel-operating tripod turnstile machines. The supplementary variable approach and the Laplace transformation were used by Yuan and Meng [36], Manglik and Ram [21], and Ram *et al.* [25] to overcome challenges in their individual investigations on reliability characteristics and steady-state system indices utilizing the Markov process. A comparison of the Universal Generating Function (UGF) & the recursive methods, C-K theorem and differential equations was performed by Guilani *et al.* [8] for the evaluation of the reliability of non-repairable systems using Markov model and found that the beta distribution is the same as the distribution of time to failure. Tyagi *et al.* [30] taken into account sensitivity analysis and reliability modelling of an Internet of Things (IoT) based Flood Alerting System (FAS) and used Markov process to obtain its state transition probabilities which were solved using Laplace transformation. By merging the structural functions of Markov processes and s-coherent multi-state system, Xue and Yang [32] were able to determine the parameters for two-state reliability and examine the reliability of multi-state systems. By using two Markov models with standby, low coverage, reboots, and common cause failure, Jain and Kumar [11] projected the development of a repairable fault-tolerant system. Model II, which included certain realistic elements, was used to study a two-unit system, whereas Model I in this work was made up of a functioning unit and a backup unit. They finally arrived at the explicit words for the various indices after some time. Additionally, Yang and Tsao [34] took into account a matrix-analytic method to determine the availability and reliability of standby systems with working vacations. They conducted a sensitivity analysis using Laplace transformation to evaluate the MTTF and the reliability function, and the results show that increasing the number of spare parts and maintenance rates can increase system reliability. Gopalan and Venkatachalam [5] computed the reliability and availability of a two-unit, two-server system that has undergone preventative maintenance and repair. They also derived explicit Laplace transforms of mean downtime and mean time to failure of the system. Additionally, a precise equation for the steady-state availability is found. For a repairable system with several vacations and incomplete fault coverage, Jain and Gupta [10] devised the ideal replacement strategy and used the Laplace transformation and supplementary variable technique to calculate the reliability indices.

The desire to come up with fresh, effective solutions to the issues presented is what gave rise to the current work. The primary goal of this work is to demonstrate how to assess the reliability of complex systems. This is how the paper is organized: Section 2 briefly describes the model's specifics, including the system description, state description and notations. The formulation of mathematical model is covered in Section 3. Some significant measures, including availability, MTTF, reliability and cost analysis are discussed in Section 4. In Section 5, the sensitivity analysis of certain significant indices, such as availability, MTTF and reliability is presented. In Sections 6 and 7, respectively, the results discussion and conclusion are offered.

2. Model details

2.1. System description

An examination of a general series–parallel system's reliability is presented in this work. The system is made up of two subsystems, A and B, which are linked in series, sub-system A having two components A_1 and A_2 that arranged in parallel combination while sub-system B having two components, B_1 and B_2 connected in series. Further, they both are connected to one another component, B_3 in parallel. Home solar power systems and water treatment plants are real-world examples of this type of system. The system configuration is shown in the Figure 1. With the aid of the Markov process and the supplementary variable technique, the entire system is mathematically modelled. For complex systems, it is quite complicated to portray them in the right way. According to Ram [24], the working of any complex system is done in three states: good state, partially failed (degraded) state, and completely failed state. It is assumed that a system's failure can be either partial or complete. A partial failure can result from an internal component failing, which renders the system less effective, and a complete failure can cause the breakdown of the whole system [12,31].

2.2. Assumptions

(i) Initially we assumed that the system is in good working condition.
(ii) After repair, the system works like a new one.
(iii) It is assumed that components A_1 and A_2 have same failure rate (λ_A) and in the same way components B_1, B_2 and B_3 have same failure rate (λ_B).
(iv) It is considered that the failure rate follows exponential distribution.

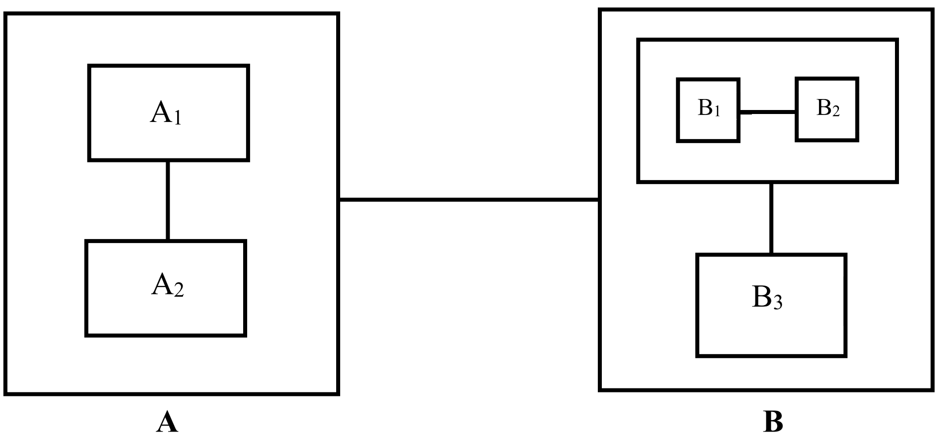

Figure 1. System configuration.

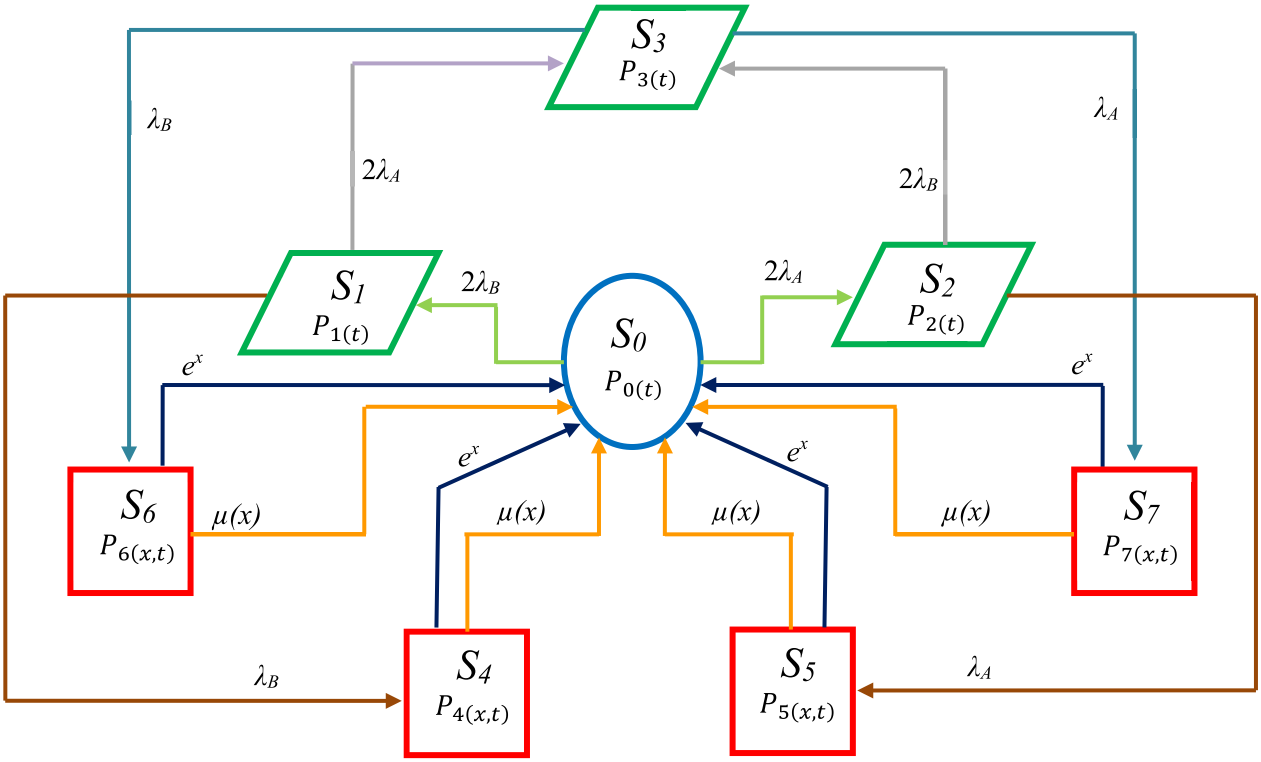

FIGURE 2. State transition diagram.

(v) The repair rate follows two types of distribution *i.e.* general distribution and exponential distribution.
(vi) 100% revenue is available.

2.3. State description

The various states as shown in Figure 2 are described in Table 1.

2.4. Notations

The notations concerned with the work are listed in Table 2.

TABLE 1. State description.

State	Description
S_0	All of the units are in good operational condition
S_1	Degraded state due to breakdown of B_3
S_2	Degraded state due to breakdown of A_1
S_3	Degraded state due to breakdown of A_2 and B_3
S_4	Completely failed state because B is completely broken while A is functional
S_5	Completely failed state because A has completely failed while B is operating
S_6	Completely failed state because A_1 and B has completely failed
S_7	Completely failed state because B_1, B and A has completely failed

Table 2. Notations.

Notation	Description
S	Laplace transformation variable
x	Supplementary variable for elapsed time
$P_i(t)$	The system's probability at state S_i, where $i = 0, 1, 2, 3$
$P_j(x,t)$	The system's at state S_j, where $j = 4, 5, 6, 7$
$\overline{P}(t)$	Laplace transformation of $P(t)$
λ_A	Rate of failure for subsystem A
λ_B	Rate of failure for subsystem B
$\mu(x)$	Repair rate for the completely failed states
e^x	Exponential distribution
$P_{\mathrm{up}}(t)$	Probability of the up states
$P_{\mathrm{down}}(t)$	Probability of the down states
k_2	Service cost

3. Mathematical model formulation

A variable whose significance is entirely determined by its current state, without reference to any past behavior, can have its value predicted using Markov analysis. According to Russian mathematician Andrei Andreyevich Markov, the state transitions are exponentially distributed, which means that the transition states are constant. The Markov process shatters and transforms into a non-Markovian process when either the failure time or the repair time are time dependent. The non-Markovian process develops into Markovian by adding a supplementary variable [2]. The hazard rate function, and essential boundary and initial conditions are required when the elapsed service time is used as a supplementary variable.

A copula known as the Gumbel–Hougaard family copula permits any particular degree of (upper) tail dependency between individual variables. It is an asymmetric Archimedean copula that exhibits more dependence in the positive tail than in the negative tail. It is given as,

$$C_\theta(u_1, u_2) = \exp\left(-\left((-\log u_1)^\theta + (-\log u_2)^\theta\right)^{1/\theta}\right), \qquad 1 \le \theta \le \infty.$$

The Gumbel–Hougaard copula models independence at $\theta = 1$, and it converges to comonotonicity at $\theta \to \infty$.

As a result of employing the Markov process and the mathematical model that is described, the following differential equations are produced:

$$\left(\frac{\partial}{\partial t} + 2\lambda_B + 2\lambda_A\right)P_0(t) = \int_0^\infty P_4(x,t)\mu\,\mathrm{d}x + \int_0^\infty P_5(x,t)\mu\,\mathrm{d}x + \int_0^\infty P_6(x,t)\mu\,\mathrm{d}x + \int_0^\infty P_7(x,t)\mu\,\mathrm{d}x. \tag{1}$$

$$\left(\frac{\partial}{\partial t} + 2\lambda_A + \lambda_B\right)P_1(t) = 2\lambda_B P_0(t) \tag{2}$$

$$\left(\frac{\partial}{\partial t} + 2\lambda_B + \lambda_A\right)P_2(t) = 2\lambda_A P_0(t) \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \lambda_A + \lambda_B\right)P_3(t) = 2\lambda_A P_1(t) + 2\lambda_B P_2(t) \tag{4}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu\right)P_4(x,t) = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu\right)P_5(x,t)=0 \tag{6}$$
$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu\right)P_6(x,t)=0 \tag{7}$$
$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu\right)P_7(x,t)=0. \tag{8}$$

The necessary boundary conditions are given as,

$$P_4(0,t)=\lambda_B P_1(t) \tag{9}$$
$$P_5(0,t)=\lambda_A P_2(t) \tag{10}$$
$$P_6(0,t)=\lambda_B P_3(t) \tag{11}$$
$$P_7(0,t)=\lambda_A P_3(t) \tag{12}$$

$P_0(0)=1$ is the beginning condition, and at time $t=0$, all other state probabilities are zero. (13)

Once we apply the Laplace transformation to equations (1) through (12) and use equation (13), we obtain,

$$(s+2\lambda_B+2\lambda_A)\overline{P_0}(s)=1+\int_0^\infty \overline{P_4}(x,s)\mu\,\mathrm{d}x+\int_0^\infty \overline{P_5}(x,s)\mu\,\mathrm{d}x+\int_0^\infty \overline{P_6}(x,s)\mu\,\mathrm{d}x+\int_0^\infty \overline{P_7}(x,s)\mu\,\mathrm{d}x \tag{14}$$
$$(s+2\lambda_A+\lambda_B)\overline{P_1}(s)=2\lambda_B\overline{P_0}(s) \tag{15}$$
$$(s+\lambda_A+2\lambda_B)\overline{P_2}(s)=2\lambda_A\overline{P_0}(s) \tag{16}$$
$$(s+\lambda_A+\lambda_B)\overline{P_3}(s)=2\lambda_A\overline{P_1}(s)+2\lambda_B\overline{P_2}(s)0 \tag{17}$$
$$\left(\frac{\partial}{\partial x}+s+\mu\right)\overline{P_4}(x,s)=0 \tag{18}$$
$$\left(\frac{\partial}{\partial x}+s+\mu\right)\overline{P_5}(x,s)=0 \tag{19}$$
$$\left(\frac{\partial}{\partial x}+s+\mu\right)\overline{P_6}(x,s)=0 \tag{20}$$
$$\left(\frac{\partial}{\partial x}+s+\mu\right)\overline{P_7}(x,s)=0 \tag{21}$$

and boundary conditions,

$$\overline{P_4}(0,s)=\lambda_B\overline{P_1}(s) \tag{22}$$
$$\overline{P_5}(0,s)=\lambda_A\overline{P_2}(s) \tag{23}$$
$$\overline{P_6}(0,s)=\lambda_B\overline{P_3}(s) \tag{24}$$
$$\overline{P_7}(0,s)=\lambda_A\overline{P_3}(s). \tag{25}$$

Now, solving equations (14)–(21) using equations (22)–(25), we get the transition state probabilities,

$$\overline{P_0}(s)=\frac{1}{\left[(s+C_1)+\overline{S_\mu}(s)\left\{\left(\frac{2\lambda_B^2}{s+C_2}\right)+\left(\frac{2\lambda_A^2}{s+C_3}\right)+\frac{C_4}{s+C_4}\left(\left(\frac{4\lambda_A\lambda_B}{s+C_2}\right)+\left(\frac{4\lambda_A\lambda_B}{s+C_3}\right)\right)\right\}\right]} \tag{26}$$
$$\overline{P_1}(s)=\left(\frac{2\lambda_B}{s+C_2}\right)\overline{P_0}(s) \tag{27}$$
$$\overline{P_2}(s)=\left(\frac{2\lambda_A}{s+C_3}\right)\overline{P_0}(s) \tag{28}$$

$$\overline{P_3}(s) = \frac{4\lambda_A\lambda_B}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\overline{P_0}(s) \tag{29}$$

$$\overline{P_4}(s) = \frac{2\lambda_B^2}{s+C_2}\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\overline{P_0}(s) \tag{30}$$

$$\overline{P_5}(s) = \frac{2\lambda_A^2}{s+C_3}\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\overline{P_0}(s) \tag{31}$$

$$\overline{P_6}(s) = \frac{4\lambda_A\lambda_B^2}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\overline{P_0}(s) \tag{32}$$

$$\overline{P_7}(s) = \frac{4\lambda_A^2\lambda_B}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\overline{P_0}(s). \tag{33}$$

Then, this is how the system's up-state probability underwent Laplace transformation:

$$\overline{P_{\text{up}}}(s) = \overline{P_0}(s)+\overline{P_1}(s)+\overline{P_2}(s)+\overline{P_3}(s)$$

$$\overline{P_{\text{up}}}(s) = \left(1+\left(\frac{2\lambda_B}{s+C_2}\right)+\left(\frac{2\lambda_A}{s+C_3}\right)+\frac{4\lambda_A\lambda_B}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\right)\overline{P_0}(s). \tag{34}$$

And, this is how the system's down-state probability is transformed using Laplace transformation:

$$\overline{P_{\text{down}}}(s) = \overline{P_4}(s)+\overline{P_5}(s)+\overline{P_6}(s)+\overline{P_7}(s)$$

$$\begin{aligned}\overline{P_{\text{down}}}(s) = &\left[\frac{2{\lambda_B}^2}{s+C_2}\left(\frac{1-\overline{S_\mu}(s)}{s}\right)+\frac{2{\lambda_A}^2}{s+C_3}\left(\frac{1-\overline{S_\mu}(s)}{s}\right)+\frac{4\lambda_A{\lambda_B}^2}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\right.\\ &\left.+\frac{4{\lambda_A}^2\lambda_B}{s+C_4}\left(\frac{1}{s+C_2}+\frac{1}{s+C_3}\right)\left(\frac{1-\overline{S_\mu}(s)}{s}\right)\right]\overline{P_0}(s).\end{aligned} \tag{35}$$

4. Some important measures of the system

4.1. Availability analysis

The availability of a system is determined by the number of failures that occur and how quickly they got rectified [15]. By setting the failure rates as $\lambda_A = 0.03$ and $\lambda_B = 0.04$ and repair rate as $\mu = 1$ in (34) and using the inverse Laplace, the system's availability is given as,

$$\begin{aligned}A = &-0.00568e^{(-0.99438t)}+0.042522e^{(-0.16057t)}\cos(0.048407)+0.24482e^{(-0.16057t)}\sin(0.04840)\\ &+0.00005e^{(-0.10446)}+0.96310.\end{aligned} \tag{36}$$

Also, by setting the failure rates as $\lambda_A = 0.03$ and $\lambda_B = 0.04$ and repair rate as $\mu = 2.71828$ in (34) and using the inverse Laplace, the system's availability with copula is given as,

$$\begin{aligned}A = &-0.00070e^{(-2.71636t)}+0.14582e^{(-0.15872t)}\cos(0.046291t)+0.00861e^{(-0.15872t)}\sin(0.04629t)\\ &+0.00002e^{(-0.10445t)}+0.98610.\end{aligned} \tag{36a}$$

Now, varying time $t = 0, 1, 2, \ldots, 30$ unit in (36) and (36a), we obtain numerous availability metrics for the system in numerical form, presented in Table 3 and its graphical illustration is shown in Figure 3.

4.2. Mean time to failure (MTTF) analysis

MTTF calculates the mean time expected until the first failure of a non-repairable system. So, putting the repair rate $\mu = 0$ and taking the limit tends to zero in (34), we obtain MTTF as,

$$\text{MTTF} = \lim_{s\to 0}\overline{P_{\text{up}}}(s).$$

TABLE 3. System's availability.

Time (t)	Availability (without copula)	Availability (with copula)
0	1.00000	1.00000
1	0.99823	0.99884
2	0.99478	0.99726
3	0.99103	0.99582
4	0.98746	0.99455
5	0.98422	0.99342
6	0.98133	0.99243
7	0.97880	0.99156
8	0.97658	0.99080
9	0.97464	0.99014
10	0.97296	0.98956
11	0.97150	0.98906
12	0.97025	0.98862
13	0.96916	0.98825
14	0.96823	0.98793
15	0.96743	0.98765
16	0.96675	0.98741
17	0.96617	0.98721
18	0.96567	0.98703
19	0.96525	0.98688
20	0.96489	0.98675
21	0.96459	0.98665
22	0.96433	0.98656
23	0.96412	0.98648
24	0.96394	0.98641
25	0.96379	0.98636
26	0.96366	0.98631
27	0.96356	0.98628
28	0.96347	0.98624
29	0.96340	0.98622
30	0.96334	0.98619

Hence,

$$\mathrm{MTTF} = \frac{1 + \frac{2\lambda_B}{2\lambda_A+\lambda_B} + \frac{2\lambda_A}{\lambda_A+2\lambda_B} + \frac{4\lambda_A\lambda_B}{(\lambda_A+\lambda_B)(2\lambda_A+\lambda_B)} + \frac{4\lambda_A\lambda_B}{(\lambda_A+\lambda_B)(\lambda_A+2\lambda_B)}}{2\lambda_A + 2\lambda_B}. \tag{37}$$

Putting $\lambda_A = 0.03$ and $\lambda_B = 0.04$ and then varying λ_A and λ_B one by one respectively as 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50 in equation (37), we determined the MTTF values in relation to both of the failure rates presented in Table 4, and it is graphically depicted in Figure 4.

4.3. Reliability analysis

The percentage of times for which a system doesn't fail in a given period of time is known as system reliability [9, 35]. For this particular system, the reliability may be described as,

$$R(t) = -2e^{(-(2\lambda_A+\lambda_B)t)} - 2e^{(-(\lambda_A+2\lambda_B)t)} + 4e^{(-(\lambda_A+\lambda_B)t)} + e^{(-(2\lambda_A+2\lambda_B)t)}. \tag{38}$$

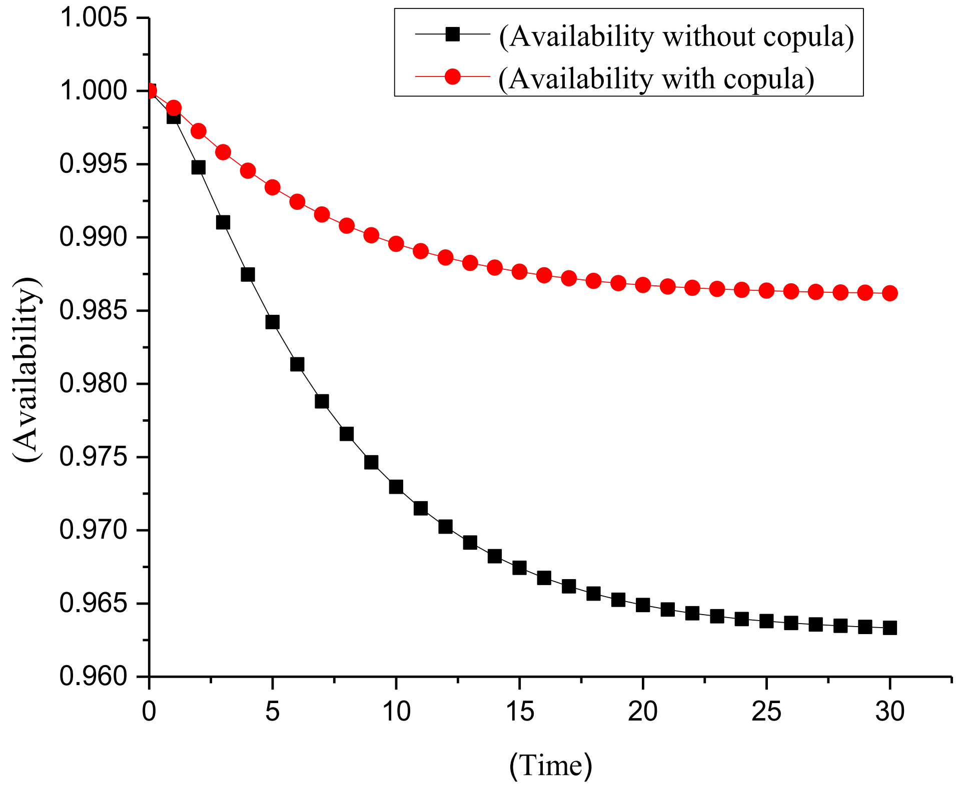

FIGURE 3. Time *vs.* availability.

TABLE 4. MTTF with failure rate.

Variation in λ_A and λ_B	MTTF	
	λ_A	λ_B
0.10	12.69841	13.41973
0.15	9.10620	9.41558
0.20	7.06168	7.22174
0.25	5.75293	5.84623
0.30	4.84713	4.90620
0.35	4.18459	4.224333
0.40	3.67965	3.70765
0.45	3.28242	3.30289
0.50	2.96198	2.97738

Putting the failure rates as $\lambda_A = 0.03$ and $\lambda_B = 0.04$ and repair rate as $\mu = 0$ in (34) and taking inverse Laplace, we get

$$R(t) = -2e^{(-0.10000t)} - 2e^{(-0.11000t)} + 4e^{(-0.07000t)} + e^{(-0.14000t)}. \tag{39}$$

Now, varying t from 0 to 30 in equation (38), we get the estimated reliability shown in Table 5 and their interrelation is demonstrated in Figure 5.

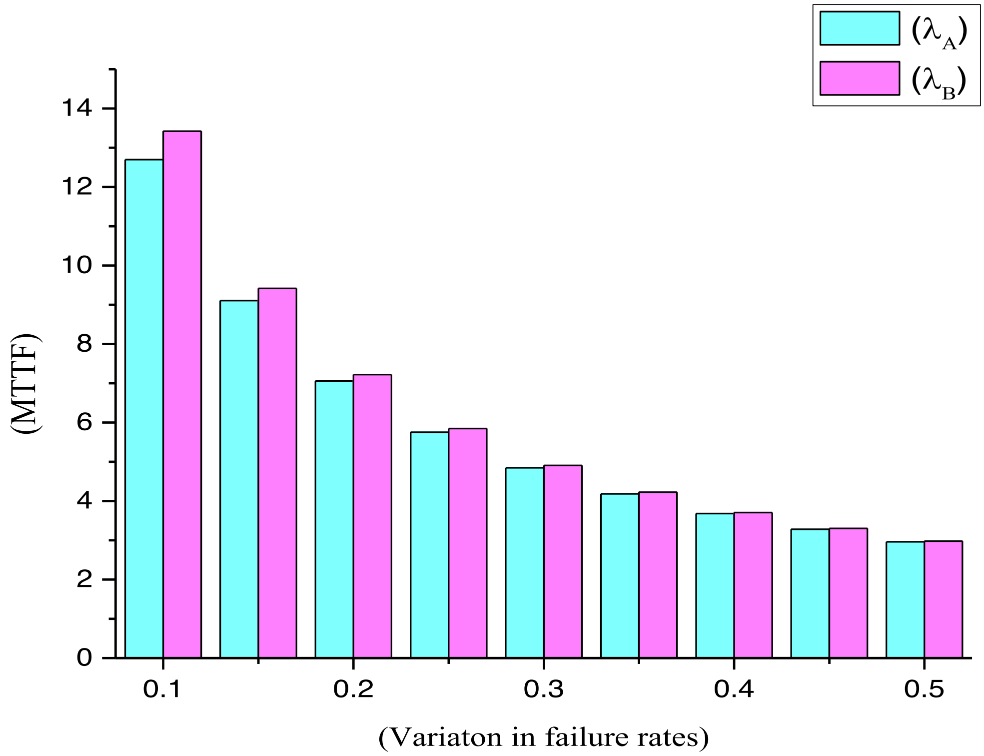

FIGURE 4. Failure rates *vs.* MTTF.

4.4. Cost analysis

The expected profit, taking into account service costs for the range $[0, t]$, is as follows if the service facility is considered to be accessible at all times,

$$E_p(t) = k_1 \int_0^t P_{\text{up}}(t)\,\mathrm{d}t - tk_2. \tag{40}$$

Using equation (34) in equation (40) and by putting $k_1 = 1$ and $k_2 = 0.1, 0.2, 0.3, 0.4, 0.5$ respectively we obtained Table 6 and Figure 6 in that order.

5. Sensitivity analysis

Sensitivity analysis, in general, is a technique for determining how various independent variable values influence a certain dependent variable under a specified set of assumptions [3,7]. According to Goyal and Ram [6], a function's partial derivative with respect to a certain factor gives an indication of how sensitive the function is to that component. Here we have determined the sensitivity of availability, sensitivity of MTTF and sensitivity of reliability in relation to the assumed failure rates.

5.1. Sensitivity of availability

The inverse Laplace equation (34) is differentiated to determine the sensitivity of availability (with and without copula), and then t is changed from 0 to 30 with respect to λ_A and λ_B one by one respectively, displayed in Table 7, and Figure 7 displays a graphical depiction of them.

TABLE 5. System's reliability.

Time (t)	Reliability
0	1.00000
1	0.99759
2	0.99071
3	0.97989
4	0.96563
5	0.94837
6	0.92857
7	0.90661
8	0.88289
9	0.85772
10	0.83143
11	0.80429
12	0.77655
13	0.74844
14	0.72014
15	0.69184
16	0.66369
17	0.63582
18	0.60833
19	0.58134
20	0.55492
21	0.52913
22	0.50403
23	0.47966
24	0.45607
25	0.43326
26	0.41127
27	0.39009
28	0.36973
29	0.35020
30	0.33148

5.2. Sensitivity of MTTF

For calculating the sensitivity of MTTF, differentiating (37) with respect to λ_A and λ_B one by one and varying them as 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, then we get the values for sensitivity of MTTF in relation to both the failure rates, shown in Table 8 and Figure 8 depicts it graphically.

5.3. Sensitivity of reliability

For calculating the sensitivity of reliability, differentiating (38) one by one with respect to λ_A and λ_B respectively and varying t from 0 to 30 we obtained numerical figures of sensitivity of reliability displayed in Table 9 and Figure 9 shows its graphical illustration.

6. Result discussion

In the present work, authors have examined the reliability indices and sensitivity for a series–parallel system comprising component failures. After performing our considered methodology on the designed system, some key outcomes are obtained those are discussed as

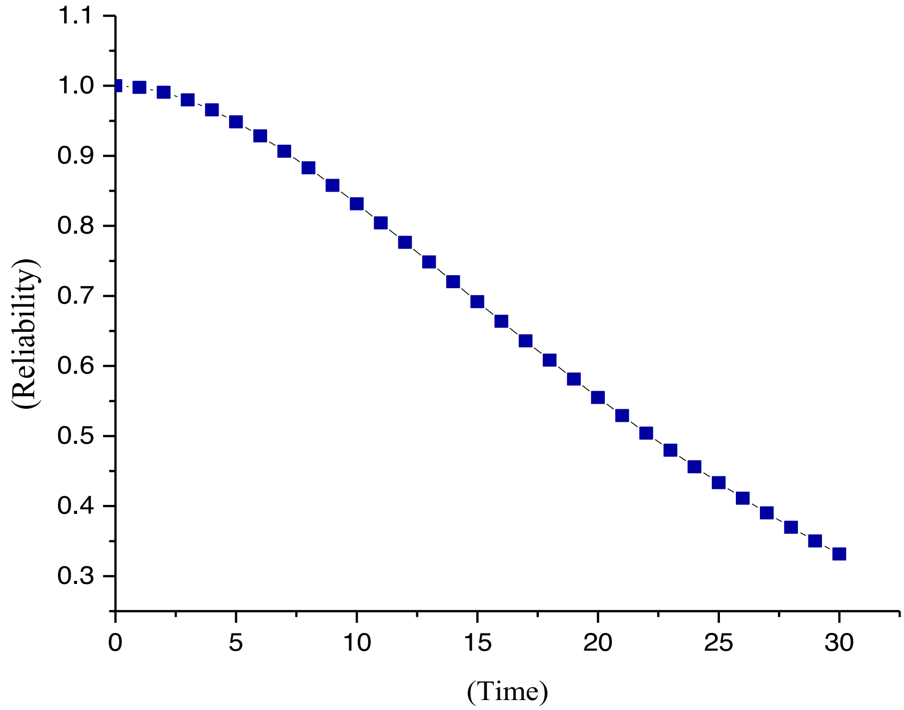

FIGURE 5. Time *vs.* reliability.

TABLE 6. Cost analysis.

Time (t)	$k_2 = 0.1$	$k_2 = 0.2$	$k_2 = 0.3$	$k_2 = 0.4$	$k_2 = 0.5$
0	0	0	0	0	0
1	0.89935	0.79935	0.69935	0.59935	0.49935
2	1.79592	1.59592	1.39592	1.19592	0.99592
3	2.68883	2.38883	2.08883	1.78883	1.48883
4	3.57806	3.17806	2.77806	2.37806	1.97806
5	4.46387	3.96387	3.46387	2.96387	2.46387
6	5.34662	4.74662	4.14662	3.54662	2.94662
7	6.22666	5.52666	4.82666	4.12666	3.42666
8	7.10433	6.30433	5.50433	4.70433	3.90433
9	7.97992	7.07992	6.17992	5.27992	4.37992

- With the aid of Table 3 and corresponding Figure 3, authors are able to demonstrate that if the failure rates are set at varying levels, the system's availability varies with the passage of time t. For the fixed values of failure rates as $\lambda_A = 0.03$ and $\lambda_B = 0.04$, the system's availability reduces with time and its probability of failure rises. After a long run it becomes constant. Figure 3 also evidently shows that the availability of the system exceeded highly after employing Gumbel–Hougaard family of copula approach for repairs.
- The MTTF of the system is determined by considering variations in λ_A and λ_B as described in Table 4. The graphs of MTTF shown in Figure 4 demonstrate that the system's MTTF decreases gradually in context of both the failure rates, λ_A and λ_B.

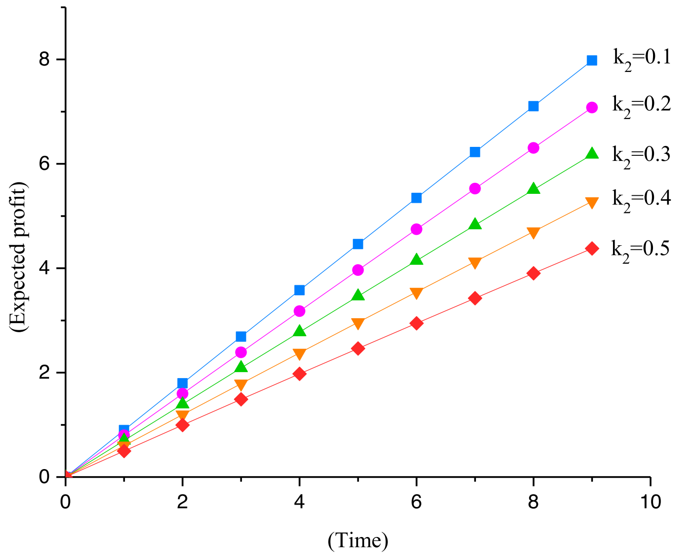

FIGURE 6. Time *vs.* expected profit.

TABLE 7. Sensitivity of availability.

Time (t)	Availability's sensitivity (without copula)		Availability's sensitivity (with copula)	
	λ_A	λ_B	λ_A	λ_B
0	0	0	0	0
1	−0.02821	−0.05513	−0.02743	−0.03596
2	−0.09873	−0.15789	−0.06384	−0.08219
3	−0.17662	−0.26276	−0.09564	−0.12102
4	−0.24939	−0.35499	−0.12237	−0.15240
5	−0.31303	−0.43113	−0.14448	−0.17732
6	−0.36677	−0.49169	−0.16247	−0.19677
7	−0.41107	−0.53845	−0.17687	−0.21164
8	−0.44684	−0.57350	−0.18817	−0.22269
9	−0.47512	−0.59882	−0.19681	−0.23060
10	−0.49695	−0.61620	−0.20323	−0.23597
11	−0.51331	−0.62719	−0.20778	−0.23930
12	−0.52510	−0.63313	−0.21081	−0.24100
13	−0.53312	−0.63514	−0.21261	−0.24146
14	−0.53808	−0.63416	−0.21341	−0.24096
15	−0.54060	−0.63096	−0.21345	−0.23975
16	−0.54119	−0.62618	−0.21289	−0.23803
17	−0.54030	−0.62032	−0.21189	−0.23597
18	−0.53832	−0.61380	−0.21058	−0.23371
19	−0.53555	−0.60693	−0.20906	−0.23134
20	−0.53224	−0.59996	−0.20741	−0.22894
21	−0.52860	−0.59309	−0.20569	−0.22657
22	−0.52480	−0.58644	−0.20397	−0.22428
23	−0.52094	−0.58012	−0.20227	−0.22211
24	−0.51715	−0.57419	−0.20063	−0.22006
25	−0.51348	−0.56869	−0.19907	−0.21816
26	−0.50998	−0.56364	−0.19760	−0.21641
27	−0.50670	−0.55904	−0.19624	−0.21481
28	−0.50365	−0.55488	−0.19497	−0.21337
29	−0.50085	−0.55115	−0.19382	−0.21206
30	−0.49829	−0.54783	−0.19277	−0.21090

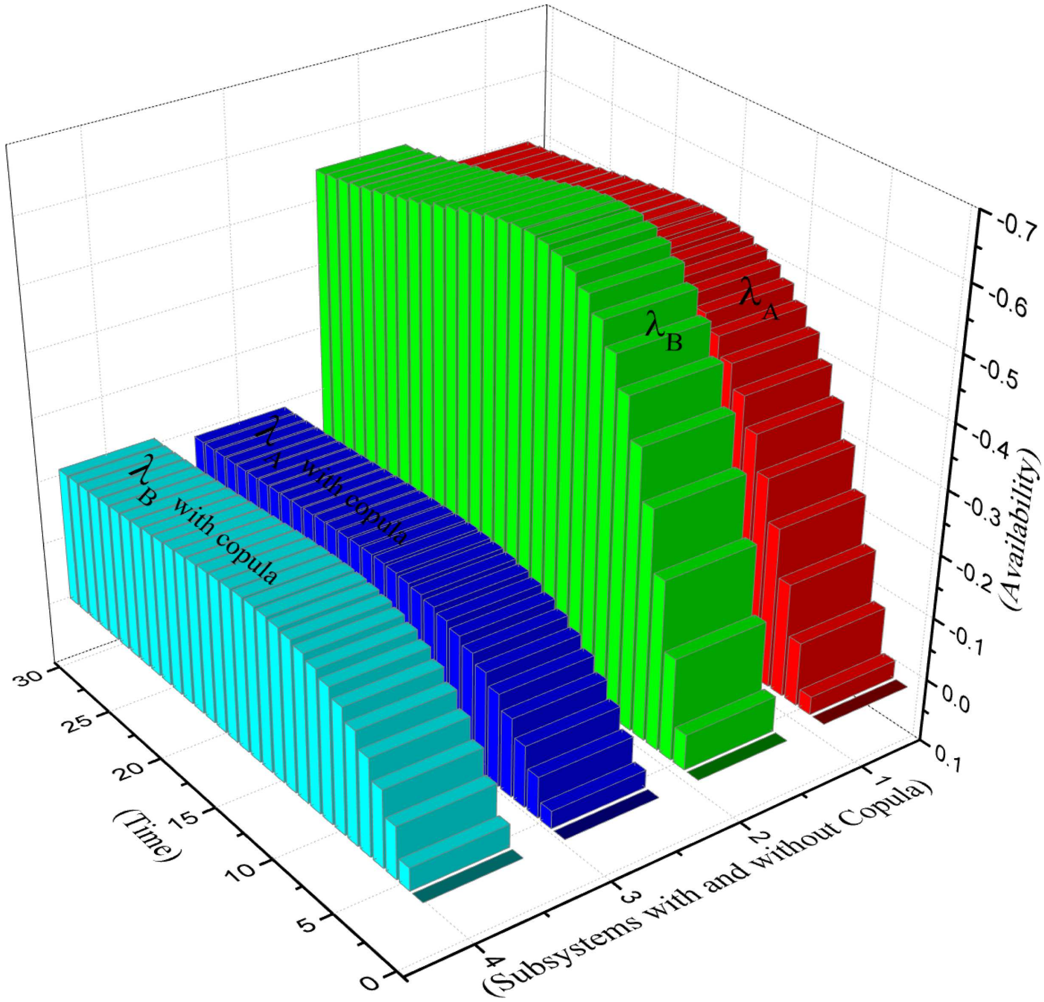

FIGURE 7. Sensitivity of availability.

- Table 5 reveals that the change in time causes the change in system's reliability, and its behavior is illustrated in Figure 5. From the graph of reliability, one can see that the system's reliability decreases uniformly as time grows and after a long run, it tends to zero.
- Figure 6 illustrates the analysis of expected profit with respect to service cost as well as time graphically. From the critical examination of graphs, one can see that the system gives the profit increasingly as time passes but if service cost increases then the profit attained by the system is decreases.
- Table 7 shows the sensitivity of availability of the system with and without implementing Gumbel–Hougaard family of copula approach with respect to both the failure rates *i.e.* λ_A and λ_B. From the critical analysis of graph one can observe that the sensitivity availability of the system with respect failure rate λ_A initially decreases till $t = 15$ units and after that it becomes constant. Similarly, sensitivity availability with respect to failure rate λ_B first decreases till $t = 10$ units. Then from $t = 10$ to 25 units it increases very slightly and after that it becomes constant. Again, after employing copula, the sensitivity availability of the system with respect to both the failure rates λ_A and λ_B it initially decreases slightly and after $t = 10$ it becomes constant.

TABLE 8. Sensitivity of MTTF.

Variation in λ_A and λ_B	MTTF's sensitivity λ_A	λ_B
0.10	−98.41899	−112.53282
0.15	−52.24448	−56.80646
0.20	−31.95363	−33.84704
0.25	−21.42483	−22.34636
0.30	−15.31129	−15.81210
0.35	−11.46453	−11.75965
0.40	−8.89431	−9.07932
0.45	−7.09528	−7.21708
0.50	−5.78857	−5.87200

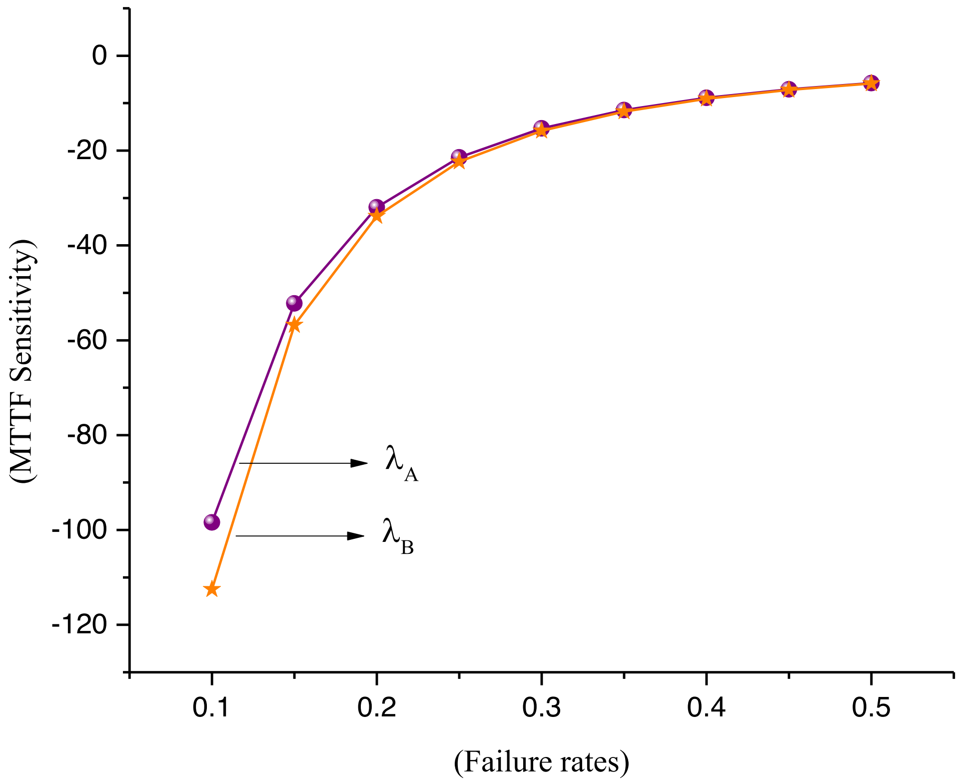

FIGURE 8. Sensitivity of MTTF.

– Critical analysis of the graphs demonstrated in Figure 8 gives that when failure rates vary from 0.1 to 0.2, the sensitivity of the MTTF with respect to λ_A and λ_B has some difference but after the value of failure rate 0.25, the sensitivity of MTTF is equal with respect to both. Also, as the failure rate increases, the sensitivity of MTTF first increases straightly and after that it increases smoothly.
– The graphs of system's sensitivity of reliability with regard to both the failure rates λ_A and λ_B is shown in Figure 9. It reveals that the sensitivity of reliability decreases very smoothly as time increases. Initially, the

TABLE 9. Sensitivity of reliability.

Time (t)	Reliability's sensitivity	
	λ_A	λ_B
0	0	0
1	−0.05727	−0.07528
2	−0.21807	−0.28292
3	−0.46593	−0.59729
4	−0.78480	−0.99506
5	−1.15950	−1.45531
6	−1.57595	−1.95947
7	−2.02132	−2.49133
8	−2.48411	−3.03685
9	−2.95418	−3.58406
10	−3.42275	−4.12291
11	−3.88229	−4.64511
12	−4.32651	−5.14392
13	−4.75024	−5.61403
14	−5.14935	−6.05140
15	−5.52065	−6.45305
16	−5.86176	−6.81701
17	−6.17108	−7.14211
18	−6.44762	−7.42789
19	−6.69097	−7.67447
20	−6.90121	−7.88249
21	−7.07878	−8.05298
22	−7.22451	−8.18730
23	−7.33946	−8.28705
24	−7.42492	−8.35405
25	−7.48237	−8.39022
26	−7.51337	−8.39762
27	−7.51960	−8.37832
28	−7.50279	−8.33444
29	−7.46470	−8.26807
30	−7.40706	−8.18130

reliability sensitivity is approximately same with respect to both the failure rates but after a short period it has a significant difference with respect to λ_A and λ_B.

7. Conclusion

This work carried out the analysis of reliability measures and their sensitivities of a series–parallel system consisting of two sub-systems with different configuration incorporating component failures. Laplace transformation, supplementary variable approach and Gumbel–Hougaard family of copula approach have been used in this study. A wide literature is available on series–parallel system ([16, 18, 20], etc.) but they did not consider two types of repair facilities. The results of this study enable us to draw the conclusion that the system's availability and reliability deteriorate over time and increased failure rates lead to a reduction in the system's MTTF. However, comparative study shows that system's availability increases eminently after utilizing copula approach.

Through the overall study, it is concluded that the system is highly sensitive with respect to failure of sub-system B. Hence, it stands to reason that lowering the system's failure rates makes it less sensitive and controlling

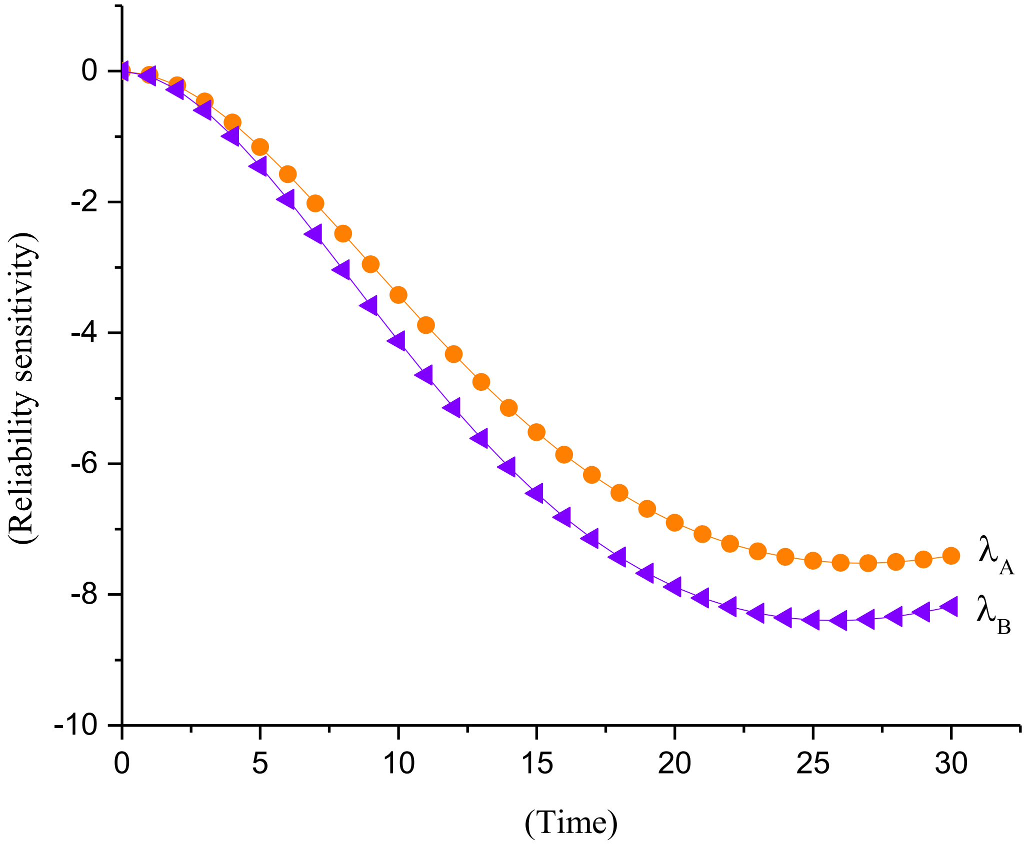

FIGURE 9. Sensitivity of reliability.

the service cost, makes the system more profitable. The results obtained in this paper after employing copula approach helps the designers and engineers to make a highly reliable and more profitable system. In future, authors can develop mathematical model to maximize reliability and minimize cost and sensitivity for cost effective systems. The model can be developed with the aid of reliability function to optimize reliability and other characteristics of the system by using meta-heuristics.

REFERENCES

[1] S. Bisht and S.B. Singh, Reliability evaluation of repairable parallel–series multi-state system implementing interval valued universal generating function. *J. Reliab. Stat. Stud.* **14** (2021) 81–120.

[2] D.R. Cox, The analysis of non-Markovian stochastic processes by the inclusion of supplementary variables, in Mathematical Proceedings of the Cambridge Philosophical Society. Vol. 51. Cambridge University Press (1955, July) 433–441.

[3] A.U.G.U.S.T.O. Gandini, Importance and sensitivity analysis in assessing system reliability. *IEEE Trans. Reliab.* **39** (1990) 61–70.

[4] H. Garg, Bi-objective reliability-cost interactive optimization model for series–parallel system. *Int. J. Math. Eng. Manage. Sci.* **6** (2021) 1331.

[5] M.N. Gopalan and P. Venkatachalam, Availability and reliability of a 2-unit 2-server system subject to preventive maintenance and repair. *RAIRO: Oper. Res.* **12** (1978) 85–92.

[6] N. Goyal and M. Ram, Software development life cycle testing analysis: a reliability approach. *Math. Eng. Sci. Aerosp.* **5** (2014) 313–329.

[7] N. Goyal, M. Ram, S. Amoli and A. Suyal, Sensitivity analysis of a three-unit series system under k-out-of-n redundancy. *Int. J. Qual. Reliab. Manage.* **34** (2017) 770–784.

[8] P.P. Guilani, M. Sharifi, S.T.A. Niaki and A. Zaretalab, Reliability evaluation of non-reparable three-state systems using Markov model and its comparison with the UGF and the recursive methods. *Reliab. Eng. Syst. Saf.* **129** (2014) 29–35.

[9] Z. Hu and X. Du, A sampling approach to extreme value distribution for time-dependent reliability analysis. *J. Mech. Design* **135** (2013) 071003.

[10] M. Jain and R. Gupta, Optimal replacement policy for a repairable system with multiple vacations and imperfect fault coverage. *Comput. Ind. Eng.* **66** (2013) 710–719.

[11] M. Jain and P. Kumar, Availability prediction of repairable fault-tolerant system with imperfect coverage, reboot, and common cause failure, in Performance Prediction and Analytics of Fuzzy, Reliability and Queuing Models. Springer, Singapore (2019) 93–103.

[12] P.A. Jensen and M. Bellmore, An algorithm to determine the reliability of a complex system. *IEEE Trans. Reliab.* **18** (1969) 169–174.

[13] Y.S. Juang, S.S. Lin and H.P. Kao, A knowledge management system for series–parallel availability optimization and design. *Expert Syst. App.* **34** (2008) 181–193.

[14] P. Kumar and A. Kumar, Tripod turnstile machines performance analysis for the system safety and security without considering simultaneous failures using reliability approach. *Int. J. Math. Eng. Manage. Sci.* **6** (2021) 383.

[15] G. Kumar, V. Jain and O.P. Gandhi, Availability analysis of repairable mechanical systems using analytical semi-Markov approach. *Qual. Eng.* **25** (2013) 97–107.

[16] A. Kumar, M. Garia, M. Ram and S.C. Dimri, Assessment of fuzzy reliability and signature of series–parallel multistate system, in Safety and Reliability Modeling and its Applications. Elsevier (2021) 397–409.

[17] R.D. Leitch, Reliability analysis for engineers: an introduction, in OUP Catalogue. Oxford University Press (1995).

[18] G. Levitin, Incorporating common-cause failures into nonrepairable multistate series–parallel system analysis. *IEEE Trans. Reliab.* **50** (2001) 380–388.

[19] G. Levitin, L. Xing, H. Ben-Haim and Y. Dai, Reliability of series–parallel systems with random failure propagation time. *IEEE Trans. Reliab.* **62** (2013) 637–647.

[20] S.C. Malik, S.K. Chauhan and N. Ahlawat, Reliability analysis of a non series–parallel system of seven components with Weibull failure laws. *Int. J. Syst. Assur. Eng. Manage.* **11** (2020) 577–582.

[21] M. Manglik and M. Ram, Reliability analysis of a two-unit cold standby system using Markov process. *J. Reliab. Stat. Stud.* **6** (2013) 65–80.

[22] H.K. Mohajan, Two criteria for good measurements in research: validity and reliability. *Ann. Spiru Haret Univ. Econ. Ser.* **17** (2017) 59–82.

[23] N. Nahas and M. Nourelfath, Non-linear threshold algorithm for the redundancy optimization of multi-state systems. *Int. J. Math. Eng. Manage. Sci.* **6** (2021) 416.

[24] M. Ram, Reliability measures of a three-state complex system: a copula approach. *App. Appl. Math. Int. J. (AAM)* **5** (2010) 10.

[25] M. Ram, S.B. Singh and V.V. Singh, Stochastic analysis of a standby system with waiting repair strategy. *IEEE Trans. Syst. Man Cybern. Syst.* **43** (2013) 698–707.

[26] P. Roberts and H. Priest, Reliability and validity in research. *Nursing Stand.* **20** (2006) 41–46.

[27] B. Schorr, *Introduction to reliability theory* (No. CERN–74-16). European Organization for Nuclear Research (1974).

[28] Z. Tian, M.J. Zuo and H. Huang, Reliability-redundancy allocation for multi-state series–parallel systems. *IEEE Trans. Reliab.* **57** (2008) 303–310.

[29] V. Tyagi, R. Arora, M. Ram and I.S. Triantafyllou, Copula based measures of repairable parallel system with fault coverage. *Int. J. Math. Eng. Manage. Sci.* **6** (2021) 322.

[30] V. Tyagi, N. Rawat and M. Ram, Reliability modelling and sensitivity analysis of IoT based flood alerting system. *J. Qual. Maintenance Eng.* **27** (2021) 292–307

[31] P. Weber and L. Jouffe, Complex system reliability modelling with dynamic object-oriented Bayesian networks (DOOBN). *Reliab. Eng. Syst. Saf.* **91** (2006) 149–162.

[32] J. Xue and K. Yang, Dynamic reliability analysis of coherent multistate systems. *IEEE Trans. Reliab.* **44** (1995) 683–688.

[33] A. Yalaoui, C. Chu and E. Chatelet, Reliability allocation problem in a series–parallel system. *Reliab. Eng. Syst. Saf.* **90** (2005) 55–61.

[34] D.Y. Yang and C.L. Tsao, Reliability and availability analysis of standby systems with working vacations and retrial of failed components. *Reliab. Eng. Syst. Saf.* **182** (2019) 46–55.

[35] B. Yang and M. Xie, A study of operational and testing reliability in software reliability analysis. *Reliab. Eng. Syst. Saf.* **70** (2000) 323–329.

[36] L. Yuan and X.Y. Meng, Reliability analysis of a warm standby repairable system with priority in use. *Appl. Math. Modell.* **35** (2011) 4295–4303.