

OPTIMAL PRICING STRATEGY FOR NEW PRODUCTS CONSIDERING REFERENCE PRICE EFFECT IN ADVANCE SELLING

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Abstract. Previous research has shown that retailers' operations and consumers' purchase choices are significantly influenced by reference prices. This study explores a retailer selling some new products to strategic consumers in advance selling, and addresses the impact of reference price effect on the advance selling strategy and corresponding pricing decisions. Consumers' choices are determined by their purchasing utilities which are dependent on the selling price in current period and the reference price. We first drive optimal selling prices and corresponding profits of the retailer under the scenarios of no advance selling, advance selling without considering the reference price effect, and advance selling with considering the reference price effect, respectively. We find an advance selling strategy is not always beneficial for the retailer. Besides, results present that the retailer benefits from considering reference price effects only if the positive reference effect is relatively high. Finally, numerical studies show that dynamic pricing is dominated by price commitment, which reaches the maximum when positive and negative reference effects parameters are both the highest.

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1. INTRODUCTION

With the development of online retailing, advance selling has been widely used by retailers selling products, such as, fashion products or consumer electronics, to increase sales. Prior researchers defined advance selling as a marketing strategy, which implies that before the time of consumption, consumers have opportunities offered by retailers to make purchases [32, 40]. This strategy can benefit the retailer by reducing demand uncertainty and updating the demand forecast [24]. In China, “Double 11”, characterized with advance selling, is increasingly becoming popularity as a selling format. In 2022, Alibaba disclosed that in just four hours from 20:00 to 24:00 on October 24, Taobao Live produced 130 livestreaming rooms whose sales with advance selling are more than 10 million yuan. The number of viewers in Li Jiaqi's livestreaming room reached 460 million¹.

However, from the point view of consumers, consumers cannot realize the precise valuation of the product in the first period (*i.e.*, advance selling period) due to the first launch or limited information. Thus they may choose to wait until the second period (*i.e.*, regular selling period) during which their valuation uncertainty

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¹<https://baijiahao.baidu.com/s?id=1747735703270078328&wfr=spider&for=pc>

disappears. As a response, a retailer may offer an advance purchase discount to encourage them to purchase in advance. Simultaneously, two kinds of pricing strategies, namely, price commitment and dynamic pricing, respectively, are often used to discourage the consumer's strategic waiting [22]. Under such circumstances, the price difference between the two periods will lead the gain-loss utility to consumers, which can be understood as the reference price effect. This effect will greatly influence the consumers' purchase behavior and then the retailer's strategic decisions [8, 19, 25].

The retailer's decisions and consumers' behavior are different under different pricing strategies. Specifically, with dynamic pricing strategy, the retailer sequentially announces two selling prices at the start of each period. In the first period, consumers are not impacted by the reference effect due to no historical price. In the second period, after knowing the advance price, consumers will form a reference price. But in the price commitment strategy, the retailer simultaneously offers two prices before the advance selling period. Thus consumers will form reference prices in two periods. With an advance price discount, consumers purchasing from the first period perceive a gain and obtain positive transaction utility in their mind, but consumers purchasing later feel a loss and obtain negative transaction utility.

However, it is unclear that which of the pricing strategy should be chosen by the retailer to obtain the maximum profit. As mentioned above, retailers usually offer price discounts to attract consumers to buy products early. Thus in the dynamic pricing strategy, although the retailer has the flexibility to change the price in the second period, consumers' purchasing decisions are negatively effected by the reference price, leading to less regular demand. In the price commitment strategy, the retailer faces a tradeoff between the expected profits from both periods, also resulting from the positive and negative effects of reference prices.

By incorporating aspects of consumer's reference effect behavior into the retailer's advance selling strategies, three research questions are presented. Firstly, we explore the optimal selling prices and ordering quantities with different pricing strategies under the scenarios with or without considering consumers' reference price effect. Next, should the retailer make decisions taking the reference effect into consideration? Finally, how do the factors such as the procurement cost and reference price effect parameters affect the retailer's strategy choice and corresponding optimal decisions?

Based on these questions, we present a two-period model in which consumers who are informed of pre-order options compare expected utilities from two periods and make the best choices. Remaining uninformed consumers with knowing their precise product valuations decide whether to purchase in the second period. Besides, we focus on the following three scenarios: the scenario of no advance selling, that is, as a benchmark; and scenarios of advance selling with and without considering consumers' reference price effect.

The main contributions of this study are as follows. Firstly, our research is constructed on the background of advance selling, in which consumers face product valuation uncertainty and need to decide which period to purchase (*i.e.*, advance or regular selling period). Furthermore, we explore the impact of reference price effect, which results from the price difference between the two periods, on the retailer's decisions and consumers' purchasing behavior. Through analysis, the results show that taking consumers' reference price effect into consideration is not always beneficial for the retailer. Next, the previous literature on reference price effect mainly focuses on dynamic pricing strategy. However, price commitment is a common used pricing strategy for retailers operating advance selling. Under these two pricing strategies, the impact of reference price effect on consumers' choices, the retailer's decisions including pricing and ordering quantity, and the corresponding expected profit are different. By comparison, we find that the retailer gets more profit from choosing the price commitment strategy no matter whether the retailer takes the consumer's reference effect into decisions or not. In summary, the combination of advance selling and reference price is not well explored in the previous literature. Finally, with respect to methodological approach, we formulate a game between a retailer and consumers, and derive optimal results using the utility theory. Furthermore, we characterize consumers' total expected utilities that are the combination of economic surplus and transaction utility resulted from the reference price effect.

In the following, some related literature and problem settings are presented, respectively. Next, we investigate the retailer's decisions under no advance selling, advance selling without considering the reference price effect,

and advance selling with considering the reference price effect, respectively. We then discuss the effect of the reference effect parameters and obtain the retailer's optimal strategy choice. The final section is conclusions.

2. LITERATURE REVIEW

There are two streams of literature, namely, advance selling and reference price effect, which are closely related to this paper.

A substantial body of literature has focused on the advance selling strategy [16, 36, 40, 41]. The main benefit of advance selling is price discrimination. Xie and Shugan (2000) pointed out that valuation uncertainty exists when consumers buy new products in the advance selling period, which increases the profit of the firm. On the contrary, Zhao *et al.* [42] focused on a decentralized supply chain including a supplier and a retailer and discussed the impact of the retailer's advance selling function. They showed that the profits of both the retailer and the total supply chain decrease in advance selling. Cachon and Feldman (2017) showed that advance selling under the context of competition is less beneficial and may be harmful. Besides, several pricing strategies are chosen by retailers who provide advance selling options, including price commitment, dynamic pricing, and price guarantee. Taking both consumer demand and valuation uncertainties into consideration, Pang *et al.* [22] compared above mentioned three strategies. They found that the optimal pricing strategy is depending upon consumer valuation and demand uncertainties. When either uncertainty is relatively high in the advance selling period, a pre-order price guarantee should be adopted by the retailer. Besides, Pang *et al.* [22] showed that the retailer prefers dynamic pricing rather than price commitment when the retailer is able to decide whether or not to offer a price guarantee under dynamic pricing optimally. Xiao *et al.* [31] considered that advance selling has several new features (*e.g.*, consumer awareness and product quality). By comparing dynamic pricing with price commitment strategies, they found that price commitment dominates dynamic pricing when the consumer's awareness is high but the consumer fitness differentiation is low in advance selling. With respect to price guarantee, Peng *et al.* [23] studied the benefit of this pricing policy for a seller who sells products in advance and takes the preorder-dependent social learning into consideration. In contrast, our study considers the impact of reference prices on consumers' purchasing behavior and then analyzes and compared two pricing strategies under the framework of advance selling.

The second stream of related studies is reference price effect, which has been extensively studied in both empirical and analytical modeling literature [5, 6]. The concept of reference price originates from the adaptation level theory [7], which stated that the consumer's response to the current selling price of a product comes from comparing it with the reference point in the consumer's mind, which depends on the consumer's past experience with the product price. Here, we only review analytical modeling literature, which mainly focused on dynamic pricing strategy and simultaneously considered the effect of reference price [2, 20]. Assuming that the demand has relations with reference price and real price, Kopalle *et al.* [13] developed optimal policies of dynamic pricing considering a monopoly retailer and oligopoly retailers, respectively. Hu *et al.* [10] studied under what conditions a firm should choose the skimming pricing or high-low pricing strategy. Besides, some studies also focused on strategic consumers. When these consumers form reference prices *via* past experiences, Wu *et al.* [30] characterized the properties of the optimal ordering and markdown decisions in a multi-season setting. Based on consumers' purchasing behavior, Zhao *et al.* [43] explored the optimal pricing decisions and corresponding revenue of a retailer by taking the effects of reference price and price-matching into consideration. Their results showed that the retailer's profit always decrease even if using a price matching strategy when considering the reference price effect. Chen *et al.* [3] considered that a firm sells products to strategic consumers over two stages by adopting two dynamic pricing policies. They found that the consumer strategic behavior harms the firm's profit, and the reference price effect benefits (harms) the firm when consumer strategic behavior is low (high). Other papers include inventory strategy [1, 29], assortment strategy [28], advertising strategy [17], preservation technology investment [4], positioning [19], green-product problem [9], durable goods [40], and supply chain [15, 37–39]. Our study is different from the above literature in the following sides. Firstly, besides uninformed consumers, we focus on a particular group of consumers who are informed pre-order options and thus referred

TABLE 1. Notation.

V	Consumer valuation
X	The product price in the first period
N_1	Number of consumers who are informed of pre-order options
D_1	The first period's demand
U_A	Expected utility from advance purchases
p	The product price in the second period
N_2	Number of consumers who are not informed of pre-order options
D_2	The second period's demand
U_W	Expected utility from purchasing later
Q	Ordering quantity
Π	The total expected profit

to as informed consumers. They will time their purchases between both periods. Secondly, different from the case that consumers can always perceive the precise valuation of the product, we consider that consumers face uncertainty in their valuations during the advance selling period. Thirdly, under each pricing strategy: price commitment and dynamic pricing, the impacts of reference prices on consumers' purchasing decisions are discussed. Finally, we simultaneously analyze and compare two different pricing strategies to find the optimal operational strategy of the retailer.

In summary, this paper firstly incorporates the aspect of consumer's reference effect behavior into the retailer's advance selling strategy, and then explores which of the pricing strategy (*i.e.*, two common used strategies: dynamic pricing and price commitment) should be chosen by the retailer. The results show that retailers' operations are significantly influenced by reference price effect. On the other hand, in the background of advance selling, this study constructs a two-period model in which consumers cannot realize the precise valuation of the product in the first period (*i.e.*, advance selling period). They may choose to wait until the second period (*i.e.*, regular selling period) during which their valuation uncertainty disappears. Considering the reference price effect, consumers' behaviors are different under two pricing strategies, which leads to the research gap about studying the retailer's decisions.

3. MODEL

3.1. Retailer settings

A retailer (called, "he") sells new products to consumers (called, "she") in advance selling during two periods, which are also called as advance selling period and regular selling period, respectively. For the retailer, his unit procurement cost is c and the selling prices of two periods are X and p , respectively. To increase the number of pre-orders from consumers, many retailers usually provide a pre-order bonus, that is, a price discount (*i.e.*, $X < p$). When the regular selling period starts, these advance-orders are fulfilled. In this paper, we assume that the stock-out risk does not exist in the period of regular selling because of enough capacity of products. All parameters are summarized in Table 1.

Two decisions should be made by the retailer. Firstly, whether or not he should choose an advance selling option? If so, he needs to choose a pricing strategy between dynamic pricing (referred to as "DP") and price commitment (referred to as "PC"). More specifically, when the retailer adopts DP, then he needs to announce the selling price sequentially at the start of each period. when he chooses PC, then he should announce selling prices X and p simultaneously before the first period starts. Besides, the retailer decides the ordering quantity (*i.e.*, Q) including both the advance sales and the regular demand.

3.2. Consumer settings

Depending on whether consumers are informed of information about the product in advance selling or not, consumers are divided into two types, which are referred to as the informed and uninformed consumers, respectively. Number of consumers of each type is N_1 and N_2 , respectively. In order to focus on the effect of reference price, we assume that N_1 and N_2 are deterministic and $N_1 = N_2 = N$, where N is an exogenous parameter. This assumption is consistent with the previous literature *e.g.*, [31, 32, 42]. For both types, each consumer who has bought one will leave the market.

With respect to consumer valuation, we assume that product valuations for consumers are uncertain during the first period. The reason is as follows. When the retailer launches a new product, consumers lack enough information to grasp all the features of the product, including product reviews or description. For analytical tractability, we assume that consumers' valuations V follow a uniform distribution from 0 to 1 [12, 14, 26]. Informed consumers' actual valuations are not realized until they receive products during the regular selling period, implying that consumers are *ex-ante* homogeneous but *ex-post* heterogeneous (*i.e.*, different realized valuations). This assumption is also consistent with the previous literature *e.g.*, [11, 33, 42].

From a psychological perspective, when consumers observe different prices in each period, they will compare them to make choices, which is called the reference price effect. Based on this, for an advance discount strategy, all consumers who make purchases in the second period usually take the selling price in the first period as the reference price. When the regular price is larger than the reference price, they feel that they suffer a loss and obtain negative transaction utility in their mind. Similarly, informed consumers usually take the second period's selling price as the reference price, and obtain positive transaction utility. For each type of consumer, there exists a same reference dependence coefficient r ($0 < r < 1$), which means the sensitivity coefficient of the difference between the actual selling price and the reference price. Specifically, we define r_1 as the gain reference dependence coefficient and r_2 as the loss reference dependence coefficient. Under uncertain conditions, all consumers are more sensitive to loss compared to gain (*i.e.*, $r_2 > r_1$) [21]. Besides, we assume that the transaction utility of consumers is linearly dependent of the difference of selling prices in each period.

With respect to consumer surplus, if a consumer has an opportunity to purchase in advance, her surplus is $V - X$. But if this consumer decides to wait and purchase in the following period, her surplus is $V - p$; otherwise her surplus is 0 if she doesn't buy anything. Therefore, if a consumer purchases in the second period, the surplus is $\max\{V - p, 0\}$. For uninformed consumers, they only make purchases and the surplus $v \geq p$ under the condition of realized valuations in the second period; otherwise, they don't buy anything. With respect to expected utility, when facing uncertainty, consumers obtain the total utility which is the combination of economic surplus and transaction utility resulted from the reference price effect, which is also depending on the pricing strategy the retailer chooses. In specific, we let the subscript A represent that a consumer places a pre-order, and the subscript W represent that a consumer purchases in the latter period. Therefore, when the retailer chooses DP, a consumer's expected utility is

$$U_A^{DP} = E_V(V - X) = \frac{1}{2} - X, \quad (1)$$

if purchasing in advance. Otherwise, if waiting for the next period and then realizing valuations, consumers make purchases and the corresponding expected utility is

$$U_W^{DP} = \int_{p+r_2(p-X)^+-r_1(X-p)^+}^1 [v - p - r_2(p - X)^+ + r_1(X - p)^+] dv. \quad (2)$$

Next, when the retailer chooses PC, an informed consumer buys a product in the first period or second period and obtains the expected utility

$$U_A^{PC} = E_V(V - X) + r_1(p - X) = \frac{1}{2} - X + r_1(p - X), \quad (3)$$

or

$$U_W^{PC} = \int_{p+r_2(p-X)}^1 [v - p - r_2(p - X)] dv = \frac{[1 - p - r_2(p - X)]^2}{2}, \quad (4)$$

respectively.

When the retailer announces the pricing strategy, informed consumers strategically decide when to purchase and they prefer the first period if $U_A^j \geq U_W^j$, where $j \in \{DP, PC\}$ [27]. In the following period, all remaining consumers make choices according to the relative size between the realized valuation and the selling price p .

4. MODEL ANALYSIS

This section first examines the scenario in which the retailer does not adopt the advance selling strategy, that is, as a benchmark. Then equilibrium results with and without considering consumers' reference price effect under the strategy of advance selling are derived and compared to show the impacts of the reference price effect and the advance selling strategy on the retailer's decisions and profit. For presentation convenience, we let the superscript "NAS" represent the scenario of no advance selling option, and superscripts "ASDP" and "ASPC" represent the scenarios when the retailer sells with dynamic pricing and price commitment strategies in advance selling, respectively. Let superscripts "WR" and "NR" represent the scenarios with or without considering reference price effect, respectively.

4.1. Benchmark: no advance selling

We first consider the scenario in which all consumers have realized their product valuations and decide whether to buy new products or not during the regular season. In general, those consumers will choose to purchase if and only if their valuations are not less than the selling price p . Thus total demand is $D_2 = 2N \cdot (1 - p)$ and the profit of the retailer is given by $\Pi_{NAS} = 2(p - c)(1 - p)N$. Evidently, the optimal decision is $p_{NAS}^* = \frac{1+c}{2}$; the ordering quantity is $Q_{NAS}^* = (1 - c)N$, and consequently $\Pi_{NAS}^* = \frac{(1-c)^2 N}{2}$.

4.2. Advance selling without considering reference price effect

4.2.1. Dynamic pricing strategy

If the retailer adopts the advance selling strategy, informed consumers can purchase early, and then uninformed consumers arrive in the regular selling period. Under the DP strategy, the retailer decides the prices of new products at each period separately. Using the approach of backward induction, we first study the optimal regular selling price, followed by the analysis of AS price.

In the regular period, consumers whose realized valuations is not less than the selling price will choose to purchase. Thus this period's demand is $D_2 = N \cdot (1 - p)$. Similar to the benchmark, the optimal regular selling price is $p_{ASDP}^{NR*} = \frac{1+c}{2}$ and the regular period's profit is $\frac{(1-c)^2 N}{4}$.

During the advance selling period, each informed consumer anticipates the optimal regular price p_{ASDP}^{NR*} and compare expected utilities from purchasing in advance and from waiting. If purchasing in advance, the expected utility is $U_A^{DP} = \frac{1}{2} - X$. If waiting, the expected utility is $U_W^{DP} = \frac{(1-p_{ASDP}^{NR*})^2}{2}$. Thus to induce more pre-orders, the advance selling price should satisfy $X \leq \frac{1}{2} - \frac{(1-c)^2}{8}$. The total expected profit in advance selling is written as $\Pi_{ASDP}^{NR} = (X - c) \cdot N + \frac{(1-c)^2 N}{4}$. The optimal advance selling price should maximize Π_{ASDP}^{NR} while motivating more informed consumers to purchase with pre-orders; otherwise, two types of consumers choose to wait, leading to the same results in the benchmark. Then Lemma 1 follows.

Lemma 1. *Without considering consumers' reference price effect, when the retailer adopts DP, his optimal prices, total ordering quantity, and expected profit are $X_{ASDP}^{NR*} = \frac{1}{2} - \frac{(1-c)^2}{8}$, $p_{ASDP}^{NR*} = \frac{1+c}{2}$, $Q_{ASDP}^{NR*} = \frac{(3-c)N}{2}$, and $\Pi_{ASDP}^{NR*} = \frac{(c^2 - 10c + 5)N}{8}$.*

Lemma 1 presents that prices in different selling periods are only contingent on the procurement cost c and increasing in it, indicating a negative effect on the retailer's expected profit. Besides, we find that the advance selling price is lower than the regular price, indicating that the retailer should not charge a low regular price even if he has the flexibility to change in the second period.

4.2.2. Price commitment strategy

Under the PC strategy, the retailer should announce the price X in the first period and the price p in the second period simultaneously before the starting of the advance selling. For informed consumers, they consider the selling price p as the reference price and decide whether to purchase early or to wait, depending on comparing their corresponding expected utilities. If purchasing in advance, the expected utility is $U_A^{PC} = \frac{1}{2} - X$. If waiting, the expected utility is $U_W^{PC} = \frac{(1-p)^2}{2}$. Regarding the uninformed consumers, they make purchases only if their surpluses are nonnegative, that is, $v \geq p$. To encourage more consumers to purchase early, the retailer maximizes the total expected profit Π_{ASPC}^{NR} while the advance selling price should satisfy $X \leq \frac{1}{2} - \frac{(1-p)^2}{2}$, where $\Pi_{ASPC}^{NR} = (X - c)N + (p - c)(1 - p)N$. Therefore, the following lemma presents the pricing decisions under the PC strategy.

Lemma 2. *Without considering consumers' reference price effect, when the retailer adopts PC, his optimal prices, total ordering quantity, and expected profit are $X_{ASPC}^{NR*} = \frac{-c^2}{18} + \frac{c}{9} + \frac{4}{9}$, $p_{ASPC}^{NR*} = \frac{2+c}{3}$, $Q_{ASPC}^{NR*} = \frac{(4-c)N}{3}$, and $\Pi_{ASPC}^{NR*} = \frac{(c^2-8c+4)N}{6}$.*

Lemma 2 shows that, similar to Lemma 1, the higher the procurement cost of the retailer, the higher the prices of the two periods and the lower the expected profit of him.

Next, we compare the retailer's pricing decisions and expected profits without considering reference price effect under no advance selling, dynamic pricing strategy in advance selling, and price commitment strategy in advance selling. Then Proposition 1 follows.

Proposition 1. *In equilibrium,*

- (i) *the advance selling price satisfies $X_{ASDP}^{NR*} < X_{ASPC}^{NR*}$;*
- (ii) *the regular selling price satisfies $p_{NAS}^* = p_{ASDP}^{NR*} < p_{ASPC}^{NR*}$;*
- (iii) *the total ordering quantity satisfies $Q_{NAS}^* < Q_{ASPC}^{NR*} < Q_{ASDP}^{NR*}$;*
- (iv) *the retailer's expected profit satisfies $\Pi_{ASDP}^{NR*} < \Pi_{ASPC}^{NR*}$; if $c \in (0, \frac{\sqrt{3}-1}{2})$, $\Pi_{ASPC}^{NR*} > \Pi_{NAS}^*$; otherwise, $\Pi_{ASPC}^{NR*} \leq \Pi_{NAS}^*$.*

When the retailer uses the dynamic pricing strategy to sell products in advance selling, informed consumers will face the future price risk, compare to that under the price commitment strategy. It may induce them to hesitate to purchase early. To retrieve this situation, the retailer can charge a relatively low advance selling price to attract informed consumers to purchase in advance. Simultaneously, when entering into the regular selling period, the retailer still has the opportunity to adjust the price and then charge a relatively low regular selling price, which is the same as that in the scenario of no advance selling, to maximize the regular selling profit. Therefore, relatively lower selling prices in both periods lead to higher total demand of products. In addition, the retailer has to offer a price discount to counteract consumer waiting behavior and thus all informed consumers prefer to purchase in advance under the price commitment strategy. According to Proposition 1 (i) and (ii), a relatively higher regular price makes the retailer obtain less profit from the regular season. However, the regular loss can be compensated by more advance profit (*i.e.*, extracting more surplus from informed consumers) by charging a relatively higher advance selling price. Thus the retailer obtains more profit when adopting price commitment, compared to dynamic pricing strategy. Finally, comparing advance selling with price commitment with no advance selling, we find there exists a threshold about the procurement cost which determines the retailer's decision. Intuitively, a lower procurement cost favors advance selling strategy for retailers.

4.3. Advance selling with considering reference price effect

4.3.1. Dynamic pricing strategy

In this scenario, informed consumers make purchase choices depending on the announced advance price and anticipated selling price in the second period. They are not influenced by the reference effect. However, uninformed consumers view the advance price as the reference price. Thus, if an uninformed consumer purchases in the second period, her obtained expected utility is: $U_W^{DP} = \int_{p+r_2(p-X)+-r_1(X-p)}^1 [v - p - r_2(p-X)^+ + r_1(X-p)^+] dv$, which is as follows:

$$U_W^{DP} = \begin{cases} \frac{[1-p-r_2(p-X)]^2}{2} & \text{if } p > X \\ \frac{[1-p-r_1(p-X)]^2}{2} & \text{if } p \leq X. \end{cases} \tag{5}$$

Similar to Subsection 4.2, consumers prefers to buy new products in regular period if she gets nonnegative utility from purchase, *i.e.*, $U_W^{DP} \geq 0$. Then the demand from this selling period is as follows:

$$N_2 = \begin{cases} [1 - p - r_2(p - X)]N & \text{if } p > X \\ [1 - p + r_1(X - p)]N & \text{if } p \leq X. \end{cases} \tag{6}$$

Denote Π_{ASDP2}^{WR} as the expected profit from sales in this period. Thus the retailer solves the following problem:

$$\Pi_{ASDP2}^{WR} = \begin{cases} (p - c) \cdot [1 - p - r_2(p - X)]N & \text{if } p > X \\ (p - c) \cdot [1 - p + r_1(X - p)]N & \text{if } p \leq X. \end{cases} \tag{7}$$

By solving this problem, the optimal regular selling price and the regular profit of the retailer are as follows:

$$p_{ASDP}^{WR*} = \begin{cases} \frac{(X+c)r_2+1+c}{2(1+r_2)} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ X & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{(X+c)r_1+1+c}{2(1+r_1)} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1} \end{cases} \tag{8}$$

and

$$\Pi_{ASDP2}^{WR*} = \begin{cases} \frac{[(X-c)r_2+1-c]^2N}{4(1+r_2)} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ (X - c)(1 - X)N & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{[(X-c)r_1+1-c]^2N}{4(1+r_1)} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1}. \end{cases} \tag{9}$$

When products are sold with pre-orders, each informed consumer will compare expected utilities of purchasing between two periods and place a pre-order if and only if $U_A^{DP} \geq U_W^{DP'}$, where

$$U_A^{DP} - U_W^{DP'} = \begin{cases} \frac{1}{2} - X - \frac{[1-p^*-r_2(p^*-X)]^2}{2} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ \frac{1}{2} - X - \frac{(1-X)^2}{2} & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{1}{2} - X - \frac{[1-p^*-r_1(p^*-X)]^2}{2} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1}. \end{cases} \tag{10}$$

Based on above analysis, the following lemma illustrates the retailer’s pricing decisions and the corresponding total expected profit.

Lemma 3. *With considering consumers’ reference price effect, when the retailer adopts DP, his optimal prices, total ordering quantity, and expected profit are as follows:*

$$X_{ASDP}^{WR*} = \frac{cr_2^2 + cr_2 + 2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - r_2 - 4}{r_2^2}, \tag{11}$$

$$p_{ASDP}^{WR*} = \frac{cr_2^2 + cr_2 + 2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - 2}{r_2(1+r_2)}, \tag{12}$$

$$Q_{ASDP}^{WR*} = \frac{r_2 + \sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - 2}{r_2} \cdot N, \tag{13}$$

and

$$\Pi_{ASDP}^{WR*} = \frac{[4 - 2(1 - r_2)\sqrt{(1 - 2c)r_2^2 + (2 - 2c)r_2 + 4} - cr_2^2 - (c + 3)r_2] \cdot N}{r_2^2(1 + r_2)}. \quad (14)$$

It can be concluded from Lemma 3 that the retailer's price and ordering decisions, and the corresponding expected profit are affected by both the procurement cost (c) and loss reference dependence coefficient (r_2). Next, we turn to compare the optimal advance selling price and regular price in the dynamic pricing strategy, as shown in Theorem 1.

Theorem 1. *Through comparing the optimal advance selling price and regular price, we have $X_{ASDP}^{WR*} < p_{ASDP}^{WR*}$.*

Theorem 1 suggests that the retailer should offer an advance price discount to attract more informed demands in advance selling. Besides, a relatively low advance price will prevent informed consumers behaving strategic behavior, under which informed consumers may choose to wait until their uncertain valuations realize, that is, the regular selling period. Our findings differ from the literature on dynamic pricing facing strategic consumers, for example, [3], who showed that the seller always uses the pricing strategy of high-low path, that is, the selling price announced by the retailer in the first period is higher than that in the second period. The reason behind this is that pre-ordering consumers have to face fitting risk. They cannot experience and check products and thus may realize a negative surplus. Therefore, the retailer has to offer an advance discount to obtain profits from informed consumers.

4.3.2. Price commitment strategy

Under the PC strategy, the retailer will simultaneously announce the selling prices of each period when the product launches into the market. We assume that the pre-announced prices are credible. For each informed consume, she prefers to place a pre-order if and only if $U_A^{PC} = \frac{1}{2} - X + r_1(p - X) \geq U_W^{PC} = \frac{[1 - p - r_2(p - X)]^2}{2}$. Similarly, the retailer make a decision about advance price to get the maximum total expected profit; simultaneously, this price can result in more pre-orders, that is, the highest advance selling price is

$$X_{ASPC}^{WR*} = \frac{\sqrt{(1 + r_1 + r_2)[(1 + r_1 + r_2) - 2pr_2]} + r_2^2p - r_2(1 - p) - r_1 - 1}{r_2^2}. \quad (15)$$

When entering into the regular selling period, the uninformed consumers who obtain nonnegative utility will purchase. Thus, the total expected profit Π_{ASPC}^{WR*} for the retailer under the price commitment strategy is as follows:

$$\Pi_{ASPC}^{WR*} = (X_{ASPC}^{WR*} - c) \cdot N + (p - c)[1 - p - r_2(p - X_{ASPC}^{WR*})] \cdot N, \quad (16)$$

here, two terms denote the profit of the retailer in each period, respectively.

With respect to the regular price p , by solving the retailer's objective function, we have

$$p_{ASPC}^{WR*} = \frac{c(r_2^2 + r_1r_2 + r_2) + (r_1 + r_2)^2 + r_1 + r_2 + (r_2 - r_1)J}{3r_2(1 + r_1 + r_2)}, \quad (17)$$

where $J = \frac{r_1 - r_2 + \sqrt{4(r_1^2 + r_1r_2 + r_2^2) - 6c(r_2^2 + r_1r_2 + r_2) + 12(r_1 + r_2) + 9}}{3}$.

Finally, we substitute equation (17) into equations (15) and (16) and get the optimal advance selling X_{ASPC}^{WR*} , total ordering quantity (*i.e.*, $N + [1 - p - r_2(p - X_{ASPC}^{WR*})] \cdot N$), and total expected profit Π_{ASPC}^{WR*} . Accordingly, we conclude that both prices are affected by gain and loss reference dependence coefficients (r_1 and r_2). Furthermore, it can be found that the optimal decisions under the PC strategy are complex such that we will discuss the impacts of key parameters and compare different strategies in the following numerically.

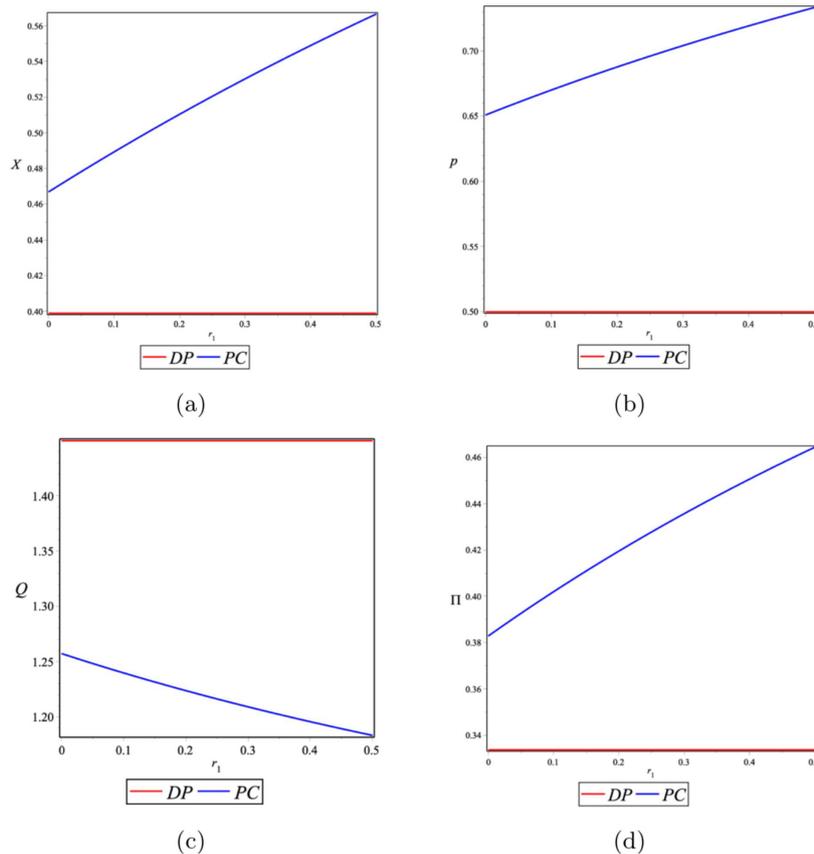


FIGURE 1. The effect of r_1 . Note: $c = 0.2$, $r_2 = 0.5$, $N = 1$.

5. NUMERICAL ANALYSIS

In what follows, we first compare dynamic pricing with price commitment, and then study the impacts of parameters (r_1 and r_2) from the reference price effect on the selling prices in each period, total ordering quantity, and the retailer's expected profit, respectively. Then, we use numerical examples to conduct strategy comparisons with respect to key model parameters. It's should be noted that the change of parameter c will not affect the main results.

5.1. Sensitivity analysis

5.1.1. Impact of r_1 on prices, ordering quantity, and expected profit (DP vs. PC)

We now explore the impact of the gain reference dependence coefficient (r_1) on the optimal selling prices in each period, total ordering quantity, and the retailer's total expected profit, as shown in Figure 1 (*i.e.*, red line represents dynamic pricing and blue line represents price commitment).

From Figure 1, we find that the selling prices of the product and the profit are always increasing in the gain reference dependence coefficient (r_1) under the PC strategy, but remain unchanged under the DP strategy. The total ordering quantity also remains unchanged in the DP strategy but increases with r_1 in the PC strategy. The reason is that the positive reference price effect only exists in the first period when the retailer chooses the price commitment strategy with an advance discount. Thus, when informed consumers are more sensitive to

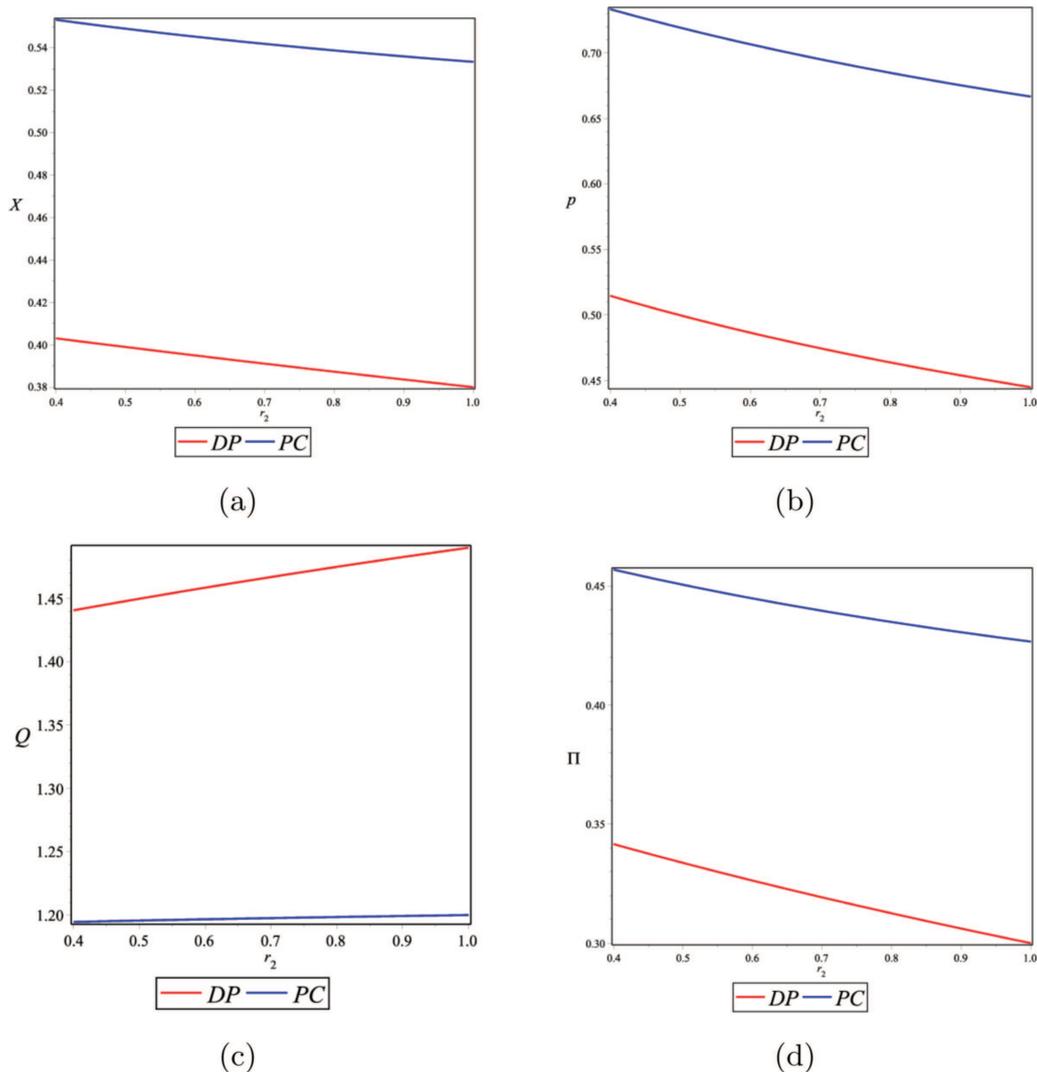


FIGURE 2. The effect of r_2 . Note: $c = 0.2$, $r_1 = 0.4$, $N = 1$.

gain (*i.e.*, the gain reference dependence coefficient becomes larger), they feel more utility from purchasing in advance. Simultaneously, the retailer can raise the regular selling price to extract more surplus from uninformed consumers. Relatively high prices lead to the decreasing of ordering quantity. In summary, the positive reference price effect is beneficial to the retailer.

5.1.2. Impact of r_2 on prices, ordering quantity, and expected profit (DP vs. PC)

Next, the impact of the loss reference dependence coefficient (r_2) on the optimal selling prices in each period, total ordering quantity, and the retailer's expected profit is shown in Figure 2 (*i.e.*, red line represents dynamic pricing and blue line represents price commitment). We find that optimal prices and the retailer's profit are decreasing in r_2 no matter what kind of strategy is adopted, dynamic pricing or price commitment. Regarding the total ordering quantity, it increases with r_2 in the DP or PC strategy. The reason behind this is that the negative reference price effect exists in the second period where uninformed consumers regard the product price

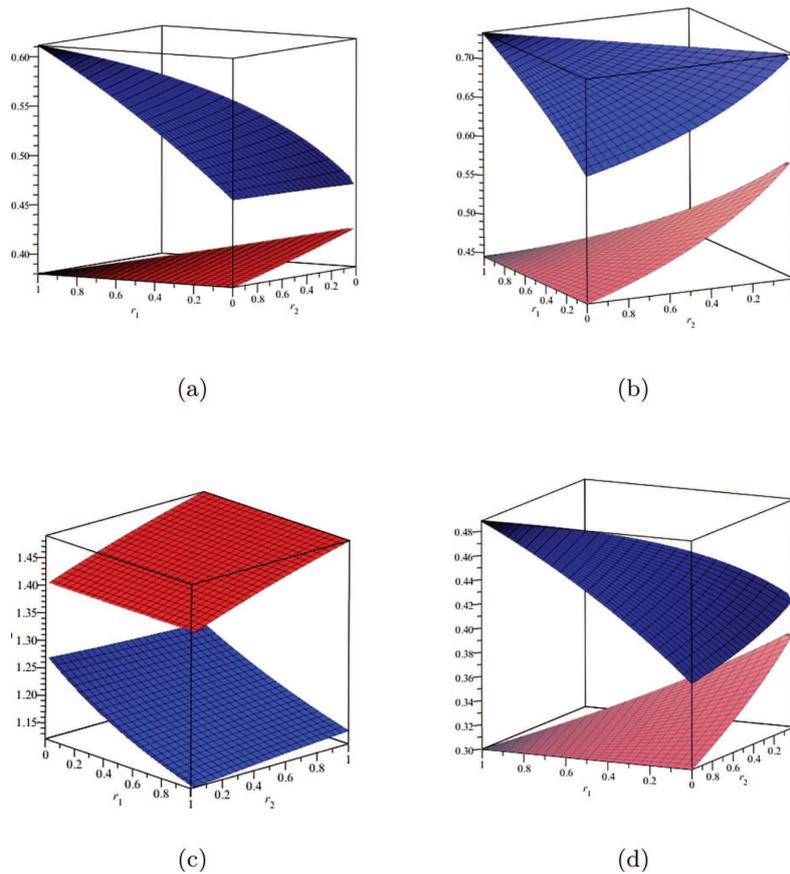


FIGURE 3. The impacts of r_1 and r_2 on prices and profits. Note: $c = 0.2$, $N = 1$.

of the first period as a reference price. Thus uninformed consumers feel less utility when they are more sensitive to loss (*i.e.*, the loss reference dependence coefficient becomes larger). As a response, the retailer will decrease the selling price in the second period to attract them to buy products.

Next, the explanation of the effect of negative reference effect on the advance selling price is different under different strategies. When the retailer chooses the PC strategy, he has to offer a higher advance discount to attract more pre-orders due to the decreasing of the regular selling price. When the retailer turns to adopt the DP strategy, the decreasing of the advance selling price is unintuitive because the purchasing decisions of informed consumers are not influenced by the reference price in the first period. If informed consumers are more sensitive to loss, the expected utility of waiting becomes smaller. Thus the retailer can charge a relatively higher price to make informed consumers purchase early. However, actually, the expected utility of waiting may not be low even if the loss reference dependence coefficient (r_2) is relatively high. As for the reason, under such circumstances, the regular price and the difference between the advance price and regular price are both becoming small. Therefore, the retailer still offers a lower advance price to mitigate the negative reference price effect. Relatively low prices lead to the increasing of ordering quantity. However, the impacts of the parameter r_2 on selling prices are larger than that of the ordering quantity. Finally, it leads to that the negative reference price effect is detrimental to the retailer.

5.1.3. Impacts of r_1 and r_2 on prices and expected profit (DP vs. PC)

Here, we explore the joint impacts of reference price effects parameters r_1 and r_2 on the optimal selling prices in each period, total ordering quantity, and the retailer's expected profits, as shown in Figure 3 (*i.e.*, red curve represents dynamic pricing and blue curve represents price commitment). In Figures 3(a), (b), and (c), the selling prices of two periods under the DP strategy are only influenced by the parameter r_2 and decrease with it. But the total ordering quantity increases with r_2 . However, under the PC strategy, both the reference effect parameters will impact the advance selling and regular selling prices. Specifically, the advance selling price is the largest when the gain and loss reference dependence coefficients are both relatively higher. The regular selling price is the largest when the gain and loss reference dependence coefficients are both relatively high or low. A special case should be mentioned that the selling price in the first period is increasing with the loss reference dependence coefficient r_2 when the gain reference dependence coefficient r_1 is zero. Recall that with a positive value of r_1 , the advance selling price decreases with r_2 under the PC strategy in Figure 2(a). The reason is that the positive reference price effect will disappear if the value of r_1 is zero. The retailer can increase the selling price of the product in the first period if the negative reference effect is becoming large because informed consumers are more sensitive to loss and thus discouraged to wait until the next period. Besides, the total ordering quantity is the largest when the gain and loss reference dependence coefficients are relatively low.

Figure 3(d) illustrates that under the DP strategy, the expected profit of the retailer is only decreasing in the negative reference effect parameter r_2 , but his profit under the PC strategy is both affected by the parameters r_1 and r_2 and increases with r_1 and decreases with r_2 . Besides, the retailer's expected profit is the highest when the positive and negative reference effects are relatively large under the PC strategy but the highest when the positive and negative reference effects are relatively small under the DP strategy. In summary, the impacts of reference price effect parameters on the retailer's expected profit is dependent of the strategy he chooses when implementing advance selling. In specific, in the case of considering consumers' reference price effect, the retailer benefits less if consumers are more sensitive to loss under the dynamic pricing strategy. However, the retailer can benefit more from the price commitment strategy if consumers are more sensitive to gain and loss. Two reasons can explain this difference. First, it's no doubt that the retailer can benefit from the increasing of the positive reference effect because informed consumers obtain more utility from purchasing in advance. Second, under the positive impact of the reference price effect, the selling price of the product in the first period is higher but the selling price in the second period stays the same if the negative reference effect is extremely strong compared to those if the negative reference effect is extremely weak. The increased advance profit results from the increasing of the negative reference effect is higher than the decreased regular profit. Thus the retailer's expected profit reaches the maximum value when the gain and loss reference dependence coefficients are highest. According to the result, the retailer should reduce consumer awareness of price differentiation between the advance selling and regular selling periods when he adopts the dynamic pricing strategy, and improve their awareness of price differentiation when announcing both prices before the advance selling period, that is, price commitment strategy. A common measure of a firm that releases a new product is remaining the product price for a certain period before a post selling period starts in practice. For example, Huawei smartphones, as a kind of electronic products, the release prices remain unchanged for months.

The influences of reference prices on the retailer's prices and profit are dramatically different from previous literature when simultaneously considering consumer valuation uncertainty and strategic behavior. With the assumption that consumers are myopic, Zhang *et al.* [38] considered a supply chain comprised of a manufacturer and a retailer, and found that the reference price effect will increase the selling prices in two periods and the supply chain performance. Ma *et al.* [18] concluded that the consumers' reference price effect benefits the seller. When the parameter on reference price is relatively small, the seller's profits increase quickly. With an assumption that consumers are strategic, Zhao *et al.* [43] found that the reference price effect always harms the retailer and Chen *et al.* [3] also presented this trend when consumer strategic behavior is high (*i.e.*, consumers can obtain relatively high utility from the second period). By contrast, in our work, consumers face valuation uncertainty. The retailer needs to offer an advance discount to make pre-purchases. Thus the positive reference

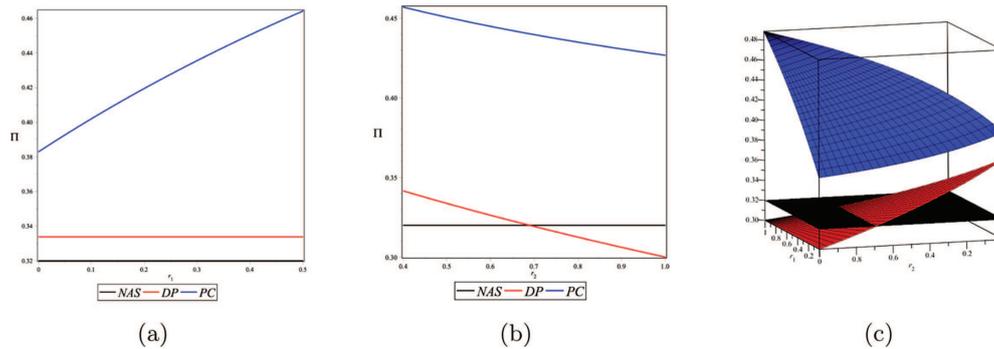


FIGURE 4. The comparisons of the expected profit. Note: $c = 0.2$, $r_1 = 0.4$, $r_2 = 0.5$, $N = 1$.

price effect is gone during the regular selling period in a dynamic environment. The negative reference price effect is detrimental to the retailer due to the decreasing of the selling prices in each period. However, the reference price effects will always increase the revenue of the retailer when he chooses to adopt the price commitment strategy.

5.2. Strategy comparison

This subsection first analyzes the optimal profits of the retailer under no advance selling, dynamic pricing strategy in advance selling, and price commitment strategy in advance selling considering the consumer's reference price effect. Next, we answer the question that "Should the retailer make decisions taking consumers' reference price effect into consideration?" by comparing the expected profit between the scenarios with or without considering reference price effect.

Remark 1. From Figure 4², we find that the retailer will benefit the most from choosing the price commitment among the above mentioned strategies. Comparing no advance selling with the dynamic pricing strategy, the retailer prefers the latter only if the loss reference dependence coefficient r_2 is relatively small.

With respect to the price commitment strategy, the retailer faces a tradeoff between the expected profits from both periods. On one hand, under the price commitment strategy, the retailer can make more profit from advance demand by setting a relatively high selling price in this period (also called the positive effect of reference price). On the other hand, due to the negative effect of reference price, which results from that the selling price in the second period is larger than the advance price, less uninformed consumers choose to buy products. Thus the retailer obtains less profit from the regular season. Our result demonstrates that this negative effect of reference price is dominated by the positive effect, leading to more expected profit for the retailer when adopting price commitment rather than dynamic pricing strategy.

Informed consumers will face price risk under the dynamic pricing strategy. In this paper, the negative reference price effect can be regarded as the degree of risk of loss. When consumers are less sensitive to loss (*i.e.*, less risk-averse), informed consumers may be encouraged to purchase in advance and uninformed consumers are also preferring to buy products in the second period. Thus this situation allows the retailer to charge different prices for consumers with different valuations, also called achieving price discrimination. In practice, the retailer prefers to adopt the dynamic pricing strategy when consumers want to be the first to access new products, but not focusing on price differentiation in advance selling. That is to say, the negligible effect of reference price on consumers' purchasing behavior improves dynamic pricing performance in the setting of advance selling.

²Red represents dynamic pricing, blue represents price commitment, and black represents no advance selling.

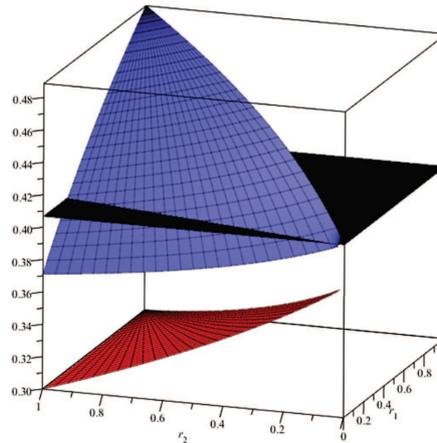


FIGURE 5. The comparisons of the expected profit between the scenarios with or without considering reference price effect.

Remark 2. From Figure 5³, we find that the retailer may not always benefit from making optimal decisions considering the reference price effect; it is beneficial to the retailer only if the gain reference dependence coefficient r_1 is relatively high.

Remark 2 tells us that the retailer should take consumers' reference price effect into consideration only when consumers focus on the difference of the selling prices in each period and are more sensitive to gain. The reason is that the positive effect of reference price will increase the product's valuation of consumers. In practice, during the Double 11 festival, a large number of merchants (*e.g.*, apparel and cosmetics industries) on different platforms (*e.g.*, Tmall, JD) offer deep discounts to consumers during the pre-sale period. During Double 11, consumers can receive the highest discount from purchases; simultaneously, most of merchants also obtain a considerable part of the annual revenue from the sales of this shopping festival⁴. As an important e-commerce platform in China, Tmall's sales reach 54.03 billion CNY in 2021⁵.

6. CONCLUSION AND MANAGERIAL IMPLICATIONS

This paper investigates that a retailer sells new products with advance selling to consumers in the market who arrive sequentially in two selling periods. The consumers' total utilities of purchasing include two parts: economic surplus and transaction utility resulted from the reference price effect. We specifically consider two pricing strategies. The first is the dynamic pricing strategy, under which the retailer offers two prices sequentially at the beginning of each period. The second is the price commitment strategy, under which he announces the product prices in each period simultaneously before the advance selling starts. Taking the consumer's reference price effect into consideration, we have found several interesting results about the retailer's policy.

Firstly, we show that the retailer cannot benefit from selling products in advance selling if the procurement cost is relatively high. Regarding to consumers' reference price effects, our results present that the retailer may not always obtain more profit from making optimal decisions taking these effects into consideration; it is beneficial to the retailer only if the positive reference effect is relatively high. Secondly, we examine how the selling prices in each period and the retailer's expected profit are affected by the reference price effect. We find that under the

³Red represents dynamic pricing with considering reference price effect, blue represents price commitment with considering reference price effect, and black represents price commitment without considering reference price effect.

⁴<https://www.chinainternetwatch.com/tag/double-11/>

⁵<https://new.qq.com/omn/20211113/20211113A00AKJ00.html>

dynamic pricing strategy, the product prices in each period and the expected profit are only decreasing the loss reference dependence coefficient. That is to say, when consumers are more sensitive to loss, the retailer obtains less profit. However, the selling prices in each period and the retailer's expected profit are increasing the gain reference dependence coefficient but decreasing the loss reference dependence coefficient under the strategy of price commitment. When considering both positive and negative effects of reference price, relatively higher gain and loss reference dependence coefficients will induce the retailer to charge relatively higher selling prices in each period, leading to the increase of the expected profit. Besides, a relatively higher regular selling price will also be announced when the gain and loss reference dependence coefficients are both lower. Thirdly, regarding the question of which strategy the retailer should choose between dynamic pricing and price commitment strategies, we suggest that the retailer offer an advance selling discount when facing strategic consumers with reference price effects. Furthermore, through numerical studies, we find that the retailer can always get the most profit from choosing the price commitment among the above mentioned strategies.

Based on the above findings, we believe our study has important implications for academics, practitioners, and policy makers. With respect to theoretical research, how reference price affects the retailer's operations in the context of advance selling has not been fully explored. This paper firstly incorporates the aspect of consumer's reference effect behavior into the retailer's advance selling strategy, and then studies which of the pricing strategy (*i.e.*, dynamic pricing and price commitment) is optimal for the retailer. The result shows that the retailer considering an advance price discount should take consumers' reference price behavior into consideration if and only if the positive reference effect is relatively high. This suggests that when consumers are more sensitive to a price discount from purchasing in advance, retailers should implement an advance selling strategy based on consumers' psychology. Furthermore, the retailer should not emphasize a deep price discount, but increase the advance selling price to extract consumer surplus. This result provides important managerial insights to retailers that engage in advance selling activities during the Double 11 festival. During this period, because consumers mainly focus on price discounts, retailers provide them with a great quantity of discounted products. Besides, when some products are firstly released by retailers, such as smartphones or video games, consumers usually prefer to enjoy and experience them firstly rather than focus on price. Under such circumstances, retailers should adopt a dynamic pricing strategy to obtain profits from these consumers. However, the reference price behavior of consumers will hurt the profit of retailers. As a response, retailers should slow down the speed and range of price discounts even if receiving a soft market response when the advance selling period finishes.

For future research, during the two periods, we assume that the supply of new products is sufficient. However, if consumers face stock-out risk, what are the optimal pricing decisions and the total expected profit? Secondly, we can further consider the case that the number of each type of consumers are stochastic and correlated. Lastly, because competition exists in the market, the scenario that two firms sell products in advance selling can be studied.

APPENDIX A. THE EFFECT OF UNIT PROCUREMENT COST IS c

As shown in these figures, the change of parameter c will not affect the main decisions. We conclude that the main conclusions still hold.

APPENDIX B. PROOFS

B.1. Proof of Lemma 1

Without considering consumers' reference price effect, when the retailer adopts the dynamic pricing strategy, the expression of his expected profit is $\Pi_{ASDP}^{NR} = (X - c) \cdot N + \frac{(1-c)^2 N}{4}$. Obviously, it is monotonously increasing the advance selling price X .

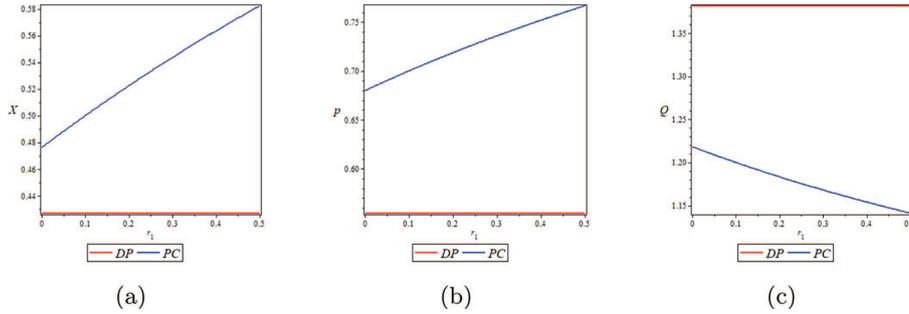


FIGURE A.1. The effect of r_1 . Note: $c = 0.3, r_2 = 0.5, N = 1$.

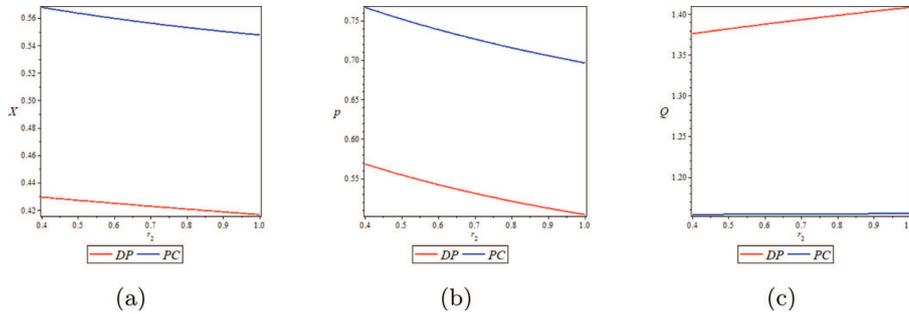


FIGURE A.2. The effect of r_2 . Note: $c = 0.3, r_1 = 0.4, N = 1$.

In order to obtain more advance sales, the selling price in the first period should meet the requirements of $X \leq \frac{1}{2} - \frac{(1-c)^2}{8}$. Thus the optimal advance price and regular selling price are $X_{ASDP}^{NR*} = \frac{1}{2} - \frac{(1-c)^2}{8}$ and $p_{ASDP}^{NR*} = \frac{1+c}{2}$, respectively.

Finally, substituting optimal prices into the demand functions and the profit function of the retailer, we have $Q_{ASDP}^{NR*} = \frac{(3-c)N}{2}$ and $\Pi_{ASDP}^{NR*} = \frac{(c^2-10c+5)N}{8}$.

B.2. Proof of Lemma 2

Without considering consumers' reference price effect, when the retailer adopts the price commitment strategy, the expression of his expected profit is $\Pi_{ASPC}^{NR} = (X - c)N + (p - c)(1 - p)N$. Obviously, it is monotonously increasing the advance selling price X .

In order to obtain more advance sales, the selling price in the first period should meet the requirements of $X \leq \frac{1}{2} - \frac{(1-p)^2}{2}$. Thus the profit function of the retailer is rewritten as $\Pi_{ASPC}^{NR} = [\frac{1}{2} - \frac{(1-p)^2}{2} - c] \cdot N + (p - c)(1 - p)N$.

Then with respect to p , differentiating the retailer's profit expression, we have

$$\frac{\partial \Pi_{ASPC}^{NR}}{\partial p} = (2 + c - 3p) \cdot N, \tag{B.1}$$

and

$$\frac{\partial^2 \Pi_{ASPC}^{NR}}{\partial p^2} = -3N. \tag{B.2}$$

Because the second partial derivative is less than zero, by letting $\frac{\partial \Pi_{ASPC}^{NR}}{\partial p} = 0$, we obtain the optimal regular selling price p_{ASPC}^{NR*} , i.e., $p_{ASPC}^{NR*} = \frac{2+c}{3}$.

Finally, by substitution, we obtain the selling price in advance is

$$X_{ASPC}^{NR*} = \frac{-c^2}{18} + \frac{c}{9} + \frac{4}{9}, \tag{B.3}$$

the optimal total ordering quantity is

$$Q_{ASDP}^{NR*} = \frac{(3-c)N}{2}, \tag{B.4}$$

and the profit of the retailer is

$$\Pi_{ASPC}^{NR*} = \frac{(c^2 - 8c + 4)N}{6}. \tag{B.5}$$

B.3. Proof of Proposition 1

Firstly,

$$\begin{aligned} X_{ASDP}^{NR*} - X_{ASPC}^{NR*} &= \frac{1}{2} - \frac{(1-c)^2}{8} - \left(\frac{-c^2}{18} + \frac{c}{9} + \frac{4}{9}\right) \\ &= \frac{-5(1-c)^2}{72} < 0. \end{aligned} \tag{B.6}$$

Then,

$$\begin{aligned} p_{ASDP}^{NR*} - p_{ASPC}^{NR*} &= \frac{1+c}{2} - \frac{2+c}{3} \\ &= \frac{c-1}{6} < 0. \end{aligned} \tag{B.7}$$

Next,

$$\begin{aligned} Q_{ASDP}^{NR*} - Q_{ASPC}^{NR*} &= \frac{(3-c)N}{2} - \frac{(4-c)N}{3} \\ &= \frac{(1-c)N}{6} > 0, \end{aligned} \tag{B.8}$$

and

$$\begin{aligned} Q_{ASPC}^{NR*} - Q_{NAS}^* &= \frac{(4-c)N}{3} - (1-c)N \\ &= \frac{(1+2c)N}{3} > 0. \end{aligned} \tag{B.9}$$

Lastly,

$$\begin{aligned} \Pi_{ASDP}^{NR*} - \Pi_{ASPC}^{NR*} &= \frac{(c^2 - 10c + 5)N}{8} - \frac{(c^2 - 8c + 4)N}{6} \\ &= \frac{-(1-c)^2 \cdot N}{24} < 0, \end{aligned} \tag{B.10}$$

and

$$\begin{aligned} \Pi_{ASPC}^{NR*} - \Pi_{NAS}^* &= \frac{(c^2 - 8c + 4)N}{6} - \frac{(1-c)^2 N}{2} \\ &= \frac{(1 - 2c - 2c^2) \cdot N}{6}, \end{aligned} \tag{B.11}$$

thus when $c \in (0, \frac{\sqrt{3}-1}{2})$, $\Pi_{ASPC}^{NR*} > \Pi_{NAS}^*$; otherwise, $\Pi_{ASPC}^{NR*} \leq \Pi_{NAS}^*$.

B.4. Proof of Lemma 3

In the strategy of dynamic pricing, if purchasing early, the expected utility is $U_A^{DP} = \frac{1}{2} - X$ for each informed consumer; if purchasing later, the expected utility is $U_W^{DP} = \int_{p+r_2(p-X)^+ - r_1(X-p)^+}^1 [v - p - r_2(p-X)^+ + r_1(X-p)^+] dv$, which can be changed to

$$U_W^{DP} = \begin{cases} \frac{[1-p-r_2(p-X)]^2}{2} & \text{if } p > X \\ \frac{[1-p-r_1(p-X)]^2}{2} & \text{if } p \leq X. \end{cases} \tag{B.12}$$

For uninformed consumers, if they obtain nonnegative surplus, that is, $v - p - r_2(p-X)^+ + r_1(X-p)^+ \geq 0$, they will choose to buy products. Therefore, the demand function of regular selling period is

$$N_2 = \begin{cases} [1 - p - r_2(p - X)]N & \text{if } p > X \\ [1 - p + r_1(X - p)]N & \text{if } p \leq X. \end{cases} \tag{B.13}$$

The profit function in this period is

$$\Pi_{ASDP2}^{WR} = \begin{cases} (p-c) \cdot [1-p-r_2(p-X)]N & \text{if } p > X \\ (p-c) \cdot [1-p+r_1(X-p)]N & \text{if } p \leq X. \end{cases} \quad (\text{B.14})$$

Then with respect to p , differentiating the retailer's profit expression, we have

$$\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} = \begin{cases} [1-2p+c+r_2(X+c-2p)]N & \text{if } p > X \\ [1-2p+c+r_1(X+c-2p)]N & \text{if } p \leq X \end{cases} \quad (\text{B.15})$$

and

$$\frac{\partial^2 \Pi_{ASDP2}^{WR}}{\partial p^2} = \begin{cases} -2(1+r_2)N & \text{if } p > X \\ -2(1+r_1)N & \text{if } p \leq X. \end{cases} \quad (\text{B.16})$$

As $\frac{\partial^2 \Pi_{ASDP2}^{WR}}{\partial p^2} < 0$, the profit expression is a concave function of price p .

Because

$$\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=0} = [1+c+r_1(X+c)]N > 0, \quad (\text{B.17})$$

$$\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=1} = [c-1+r_2(X+c-2)]N < 0, \quad (\text{B.18})$$

$$\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=X^-} = [1+c-2X+r_1(c-X)]N, \quad (\text{B.19})$$

$$\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=X^+} = [1+c-2X+r_2(c-X)]N, \quad (\text{B.20})$$

and $r_1 < r_2$, $\frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=X^+} < \frac{\partial \Pi_{ASDP2}^{WR}}{\partial p} \Big|_{p=X^-}$.

Therefore, the optimal regular selling price and the regular profit are as follows:

$$p_{ASDP}^{WR*} = \begin{cases} \frac{(X+c)r_2+1+c}{2(1+r_2)} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ X & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{(X+c)r_1+1+c}{2(1+r_1)} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1} \end{cases} \quad (\text{B.21})$$

and

$$\Pi_{ASDP2}^{WR*} = \begin{cases} \frac{[(X-c)r_2+1-c]^2N}{4(1+r_2)} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ (X-c)(1-X)N & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{[(X-c)r_1+1-c]^2N}{4(1+r_1)} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1}. \end{cases} \quad (\text{B.22})$$

For informed consumers, they will compare expected utilities of purchasing between two periods, namely

$$U_A^{DP} - U_W^{DP} = \begin{cases} \frac{1}{2} - X - \frac{[1-p^*-r_2(p^*-X)]^2}{2} & \text{if } X < \frac{1+c+r_2c}{2+r_2} \\ \frac{1}{2} - X - \frac{(1-X)^2}{2} & \text{if } \frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1} \\ \frac{1}{2} - X - \frac{[1-p^*-r_1(p^*-X)]^2}{2} & \text{if } X \geq \frac{1+c+r_1c}{2+r_1}. \end{cases} \quad (\text{B.23})$$

In the following, we discuss it in three cases:

i) if $X < \frac{1+c+r_2c}{2+r_2}$, we have

$$U_A^{DP} - U_W^{DP} = \frac{-r_2^2 X^2}{8} + \left[\frac{cr_2^2}{4} + \frac{(c-1)r_2}{4} - 1 \right] X + \frac{3}{8} + \frac{c}{4} - \frac{c^2}{8} - \frac{(c-1)cr_2}{4} - \frac{c^2 r_2^2}{8}. \quad (\text{B.24})$$

We observe that the expression of $U_A^{DP} - U_W^{DP}$ is decreasing in X and $U_A^{DP} - U_W^{DP}|_{X=\frac{1+c+r_2c}{2+r_2}} = -\frac{(1+c+r_2c)^2}{2(2+r_2)^2} < 0$. By letting $U_A^{DP} - U_W^{DP}$ be equal to zero, the optimal advance selling price is obtained, that is,

$$X_{ASDP}^{WR*} = \frac{cr_2^2 + cr_2 + 2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - r_2 - 4}{r_2^2}. \quad (\text{B.25})$$

By substituting X_{ASDP}^{WR*} into the expression $p_{ASDP}^{WR*} = \frac{1+c+r_2(X+c)}{2(1+r_2)}$, we then have

$$p_{ASDP}^{WR*} = \frac{cr_2^2 + cr_2 + 2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - 2}{r_2(1+r_2)}. \quad (\text{B.26})$$

Noted that with considering consumers' reference price effect, when the retailer adopts the dynamic pricing strategy, the expected profit of the retailer is $\Pi_{ASDP}^{WR} = (X-c) \cdot N + \frac{[1-c+r_2(X-c)]^2 N}{4(1+r_2)}$. Thus substituting optimal product prices into the demand and profit functions, we have

$$Q_{ASDP}^{WR*} = \frac{r_2 + \sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - 2}{r_2} \cdot N, \quad (\text{B.27})$$

and

$$\Pi_{ASDP}^{WR*} = \frac{[4 - 2(1-r_2)\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - cr_2^2 - (c+3)r_2] \cdot N}{r_2^2(1+r_2)}. \quad (\text{B.28})$$

ii) if $\frac{1+c+r_2c}{2+r_2} \leq X < \frac{1+c+r_1c}{2+r_1}$, leading to $U_A^{DP} - U_W^{DP} < 0$, informed consumers will not buy products in advance. Thus the retailer's decision is same the scenario of no advance selling.

iii) if $X \geq \frac{1+c+r_1c}{2+r_1}$, we observe that the expression of $U_A^{DP} - U_W^{DP}$ is decreasing in X and $U_A^{DP} - U_W^{DP}|_{X=\frac{1+c+r_1c}{2+r_1}} = -\frac{(1+c+r_1c)^2}{2(2+r_1)^2} < 0$, implying that no informed consumers purchase in the first period. Thus no equilibrium results exist in this case.

B.5. Proof of Theorem 1

Because

$$X_{ASDP}^{WR*} - p_{ASDP}^{WR*} = \frac{2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4} - 4 - (1-c)r_2^2 - (3-c)r_2}{r_2^2(1+r_2)}, \quad (\text{B.29})$$

due to $[2\sqrt{(1-2c)r_2^2 + (2-2c)r_2 + 4}]^2 - [4 + (1-c)r_2^2 + (3-c)r_2]^2 = -r_2[16 + (1-c)^2r_2^3 + (2c^2 - 8c + 6)r_2^2 + (c^2 - 6c + 13)r_2] < 0$, we have $X_{ASDP}^{WR*} < p_{ASDP}^{WR*}$.

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