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EFFECT OF RELIABILITY AND MEMORY ON FRACTIONAL INVENTORY MODEL INCORPORATING PROMOTIONAL EFFORT ON DEMAND

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Abstract. This paper presents a study of inventory replenishment strategy for efficiently managing sales of a deteriorating item in a retail store. The study addresses pertinent effect on sales pattern due to promotional initiatives. The memory effect generated in the consumers' mind due to various factors like branding and the stock visibility to customers is incorporated in our model by formulating it as a Caputo–Fabrizio fractional differential equation. Even, in practice, consumers' purchase patterns are noticed to get influenced by the reliability of product, the same is modelled through demand rate formulation. Influence of both these factors is incorporated into the proposed formulation by representing them as model parameters. The study aims at determining the optimal replenishment quantity and its reordering time for the addressed item in terms of said factors estimated as parameters. Results of the study are analyzed through the data set obtained from a retail store. The analysis of model-parameters infers some managerial insights which match the reality of sales patterns. Our study provides a decision support framework for determining replenishment quantities along with an estimate of replenishment time in connection with promotional initiatives and reliability of the product for achieving minimal total cost incurred while keeping the selling price of the product as fixed.

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1. Introduction

Optimal inventory strategies in terms of the economic order quantity (EOQ) and optimal replenishment cycle have been studied in the literature in detail. Various models have been developed to address different requirements of decision-makers handling inventory sites in a connection with coordination of the demand and supply ecosystem [1, 2]. The inventory of perishable items is managed through various initiatives [3, 4] and planning strategies [5, 6]. Customers' purchase patterns of perishable products are seen to be deeply influenced by the likelihood of a product staying best-to-use for the time marked as its shelf-life. The probability of product's performance during the specified shelf-life time assessed under a specified environment is termed as

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reliability of the product [7,8], which is conceivably and appropriately adopted in context of perishable items. The conception of reliability is originally discussed in literature for technological products, as the probability that a product performs its functions up till the mean time to failure. The notion can be adopted in context of a perishable product by considering the shelf-life as mean-time-to-failure and usable condition as the functioning of the product. This reliability based customers' purchase behaviour is important to be modeled for an efficient planning of inventory system of especially perishable items.

Further, different marketing strategies focusing to raise the demand rate of perishable products are employed by retail stores and their production firms as well for minimizing the deterioration of products at their shelves. Consequently, retailers face the challenge of handling the inventory and its replenishment decisions according to the promoted rate of demand. Also, a memory effect is generated into the consumers mind as a consequence of these promotions, raised affinity to the product after its repetitive use, and even due to the reliability of the product. This memory effect eventually starts influencing the demand rates of such items at the retail stores. Accordingly, inventory strategies to determine optimal replenishment quantities, formally termed as EOQ, and optimal replenishment cycles require incorporating the effect of this memory effect along with reliability and efforts of promotional effects.

To the best of our knowledge, no study available in the literature has considers the persistent effect of promotions as well as the reliability of a product on customers' purchase behavior for identifying an optimal inventory replenishment strategy. The study presented in this paper is motivated to address this challenge faced by a retailer struggling to efficiently manage the procurement and sale of such a product in their retail outlet.

In this paper, we study the situation of replenishment decisions to be taken by retailers who sell a deteriorating item in their retail stores in a marketing environment influenced by the strategies marked above. The memory effect observed in the demand patterns is incorporated into the inventory model by using the fractional derivatives in light of the existing research [9–11] while propounding the order of fractional derivative as memory index. In this line, the most recent development in this context, Caputo–Fabrizio fractional derivative [12] is appropriately used in our model due to solution capabilities.

Whereas, the incremented demand rate is expressed in our model parameterized in coefficients of promotional effect and reliability of the product. The formulated inventory model is further analyzed on a data set obtained from a reputed retailer having a chain of retail store. A review of literature on developments on Caputo–Fabrizo fractional order derivatives and inventory models developed by far for the environment discussed in our study is focused in Section 2. Section 3 presents notations and assumptions for proposing a mathematical model. Section 4 puts forward the mathematical formulation of our inventory model as a Caputo–Fabrizio fractional order differential equation and then furnishes optimality conditions for EOQ. Section 5 demonstrates a case study from a retail store facing issues in preparing inventory replenishment plans in a sales environment same as that address in our model. Managerial implications of adopting the suggested model for determining optimal values of replenishment quantities and replenishment time are listed in Section 5. Section 6 propounds managerial implications of our study followed by a conclusion of our study presented in Section 7.

2. Literature review

2.1. Inventory models for deteriorating items

There is a plethora of inventory models in the literature discussing different types of demand rates. We count here some of the addressed types as price-dependent demand [13], time-dependent demand [14], time-and-price dependent demand [15], fuzzy random demand [16], generalized demand [17], ramp type demand [18], uncertainty demand [19], reliability depended demand [20–22], intermittent demand [23], and multivariate demand [24], stock-sensitive demand [13]. Inventory models available in the literature formulate the deterioration effect on the rate of inventory exhaustion through the following differential equation

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} + \phi I(t) = -D, \quad 0 \leq t \leq T, \tag{1}$$

where, $I(t)$ represents inventory level at any time t of the replenishment cycle $[0, T]$, D is the customers' demand rate of the item at any time t and ϕ is described as the rate of deterioration of the item. Inventory models not considering the deterioration factor do not involve the second term on the left hand side of equation (1). According to the situation being modeled in any particular study, the customers' demand D is considered as constant [25] or dependent on some variable or parameters [2, 5]. In this context, a typical development in literature which develops inventory planning model for handling multiple products of deteriorating nature with demand rate depending on price as well as the stock-levels, is worth citing [13, 26].

Customers' purchase pattern is observed in practice as significantly influenced by the reliability of product, along with different other factors depending upon the product or the situation considered. Reliability is a term with a wide perspective, which is used for assessment of various aspects including expenditures, systems, products. Cost reliability is discussed in the literature for planning of logistics [27]. System's reliability is referred as the probability that a system or a device performs its purpose sufficiently for specified time-duration under specific operating conditions. This concept of system's reliability and techniques to assess as well improve the same are explained in detail in a monograph by Billinton and Allan [28]. The reliability of a product conceptualized and assessed as discussed in literature cited hereby is used for the formulation of demand rate in connection with inventory planning. Such formulations in recent developments for incorporating the reliability influenced demand rate in inventory planning are worth citing here [21, 29].

2.2. Application of Fractional derivative to inventory modelling

Fractional calculus since its inception has been an area of its interest due to its multiple interpretations. Monographs by Podlubny [30], and Miller and Ross [10] can be referred for fundamental understanding of fractional calculus and differential equations based on fractional order derivatives. Further the monographs by Diethelm [11] and Kilbas *et al.* [31] provide a detailed analysis and application of fractional differential equations. Diethelm [11] demonstrates the use of this concept as a tool to model the memory-dependent human behavior for their reaction to external influences. Accordingly, fractional calculus appears as an appropriate technique to include memory effect in mathematical models of economical and biological problems. Thus one can speak that it is the appropriate mathematical approach to understand the real nature.

Some author have used fractional calculus in some of general inventory models for incorporating memory effects [32]–[41]. But no study by far is seen to address the memory effect of promotional efforts for studying the demand patterns of a deteriorating item and thereby develop an appropriate model for inventory replenishment.

One of the biggest challenges which impedes researchers from using the fractional order systems is the unavailability of solution methods for solving the system. A recent definition for fractional order derivative is proposed by Caputo and Fabrizio [12]. A merit of this approach is it authors have conceptualized further Laplace transform and inverse Laplace transform for their expressions of fractional order derivatives. Consequently, the approach has been used in various fields [41]–[44].

With this backdrop, we are ready to present our inventory model for determining optimal replenishment strategy of a deteriorating item with demand rate influenced by the noted factors.

3. Development of inventory system

3.1. Notations

For a better comprehension, nomenclature of notations used in the mathematical formulation of model is classified into five categories *viz*, decision variables, parameters, functions and variables dependent on parameters, constants and other notations for different cost heads.

Decision variables

T stock replenishment cycle time (optimal value of the variable to be denoted as T^*)

S order quantity at the time of replenishment (optimal value of the variable to be denoted as S^*, known with the prevalent term, EOQ)

Parameters

Inventory parameters

r coefficient of reliability (value lies between 0 and 1)
ρ coefficient of promotional effect (value lies between 0 and 1)

Fractional differential and integral parameters

α differential memory index
β integral memory index

Function

$I(t)$ inventory level at any time t (independent variable t varying over replenishment cycle $t \in [0, T]$)

Constants

h inventory holding cost per-unit of item per-unit time
A fixed cost of placing a purchase order

Dependent variables

$\mathrm{HC}_{\alpha,\beta}(T)$ inventory holding cost for the item in the replenishment cycle
$\mathrm{PC}_{\alpha}(T)$ purchase cost of quantity ordered
OC fixed cost of ordering incurred to the retailer at every replenishment
PEC promotional effort cost
$\mathrm{TC}_{\alpha,\beta}(T)$ total cost incurred to the retailer for dealing with the item in a replenishment cycle of duration $[0, T]$

3.2. Assumptions

- As the study focuses on developing an inventory strategy for retail stores selling deteriorating item, it is assumed that ample investment potential as well as shelf space is available for storage-cum-display of different items of such kind. This allows us to study the inventory model for each item separately. Moreover, different items can have different suppliers with different supply and inventory setups. Thereby, it suffices to study the inventory system for single item considered at a time.
- The demand rate D is considered to be incremented by the promotional effect achieved through creative and focused advertisements.
- As the product is of deteriorating nature, the customers purchase behavior is reasonably governed by consideration on actual life of the product compared to the specified on its label. Therefore, the demand rate is considered to get influenced further by the reliability of the product perceived by consumers after a long use or due to market reputation of the product. For measuring the reliability of a perishable product, an analogy is developed from the notion "reliability of technological products" [8]. For this purpose, the analogous of mean-time-to-failure for functionality of technological products is considered as the shelf-life of a perishable item. This parallel is drawn between technological products and perishable products, based on the fact that in both cases the product ceases to be used after a time. This time duration is empirically estimated by the manufacturer and even labeled on the packaging for customers' information as warranty for technological products and expiry time or best-before-time for perishable products. Accordingly, the reliability of a perishable item is measured as the probability that it does not perish before the best-before-time when kept under appropriate storage or preservation as the case be. This measure of reliability of a perishable item is termed in the present study as *coefficient of reliability* (r).

- The incremental effect of these factors on the demand rate has been modeled through their coefficients of both the factors. Through a statistical analysis of the data available for our study, the demand rate D is assessed to be dependent on these two factors according to the following function

$$D = d(1 + \rho r). \tag{2}$$

  where, ρ is the coefficient of promotional efforts $(0 < \rho < 1)$, r is the coefficient of reliability $(0 < r < 1)$, and d is the demand rate independent of any promotional influence or consideration of reliability factor. Thereby, a continuous demand rate is considered here in terms of parameters ρ and r
- The promotional efforts are considered as an initiative of the retailer for boosting the demand rate. Thereby, the cost of the same is incurred by the retailer.
- The cost of generating this promotional effect, which is predominantly termed as promotional effort cost (PEC), is estimated through the study of the data as an exponential function of the promotional effort as following.

$$\text{PEC} = k\rho^m, \tag{3}$$

  where, $k > 0$ and m are constants. Those values of k and m are selected which give a best fit of the promotional cost function.
- It has been observed that with an increase in the reliability of an item as a consumable product, players at each echelon of the supply chain of the product raise the price of selling to the player of the next echelon.
- An analysis of data received from records of the retailer firm has established an exponential influence of the reliability factor of product to its procurement price. Accordingly, the per-unit purchase cost of item is considered as a function of reliability coefficient of the item, *viz* through the expression

$$P(b, r) = bg^r, \tag{4}$$

  where, r is the coefficient of reliability of item $(r > 0)$ and b is the base price of the item (representing the price of the product without any account of its reliability).
- Persistent effect of promotional efforts on customers' purchase behavior can be broadly categorized into two factors. One of the factors being due to promotional initiatives which influence demand rate directly. Whereas, the other one being due to the memory effect generated in customers' mind through the visibility of inventory stocks in terms of their display at prime or tactical locations, for example putting shelves for such an item near the billing counters.
- As the first memory factor is associated with the demand rate, it is termed as the differential memory index (α). On the other hand, as the second factor is associated with available stock, it is appropriately termed as integral memory index (β). Both the memory factors are used appropriately at different places while formulating the inventory model for optimal replenishment policy.
- First one is termed as differential memory index and the other one as integral memory index.
- Shortages and backorders are not allowed.
- Replenishment rate is infinite and lead time is zero.
- Lowering the selling price through discounts is not considered as part of promotional efforts.
- Deterioration rate of the considered item is assumed to be constant throughout the replenishment cycle.
- Continuous review system is in place.
- EOQ lot-sizing inventory replenishment policy is considered in which the replenishment is done instantaneously. This amounts to conclude that lead time to the replenishment is negligible.
- Every replenishment cycle starts with replenishment of stock and finished at stock out. Accordingly, a replenishment order is placed as and when the previous stock gets sold out. Thereby it provides boundary conditions for inventory level $I(t)$ as, $I(0) = S$ (S being the order quantity at the time of replenishment) and $I(T) = 0$ (representing the stock-out and the end of replenishment cycle simultaneously.

4. Mathematical formulation and analysis of inventory model

4.1. Caputo–Fabrizio fractional order inventory model

The memory factor generated in customers' mind due to promotional efforts influences their purchase pattern to induce an incremental effect on the demand rate of the item. Therefore it is convincing to consider this factor through a differential memory index (α) representing the fractional order of Caputo–Fabrizio derivative in the differential equation representing the inventory exhaustion rate. It is proposed to incorporate the same as following.

$$\frac{d^{\alpha}I(t)}{\mathrm{d}t^{\alpha}}+\phi I(t)=-d(1+\rho r),\quad 0\le t\le T,\quad 0\le\alpha\le 1. \tag{5}$$

As the replenishment is considered to take place at beginning of each cycle with an order quantity S with no stock from the previous cycle considered in-hand, thereby it introduces a condition characterizing the initial inventory level as $I(0)=S$. Further, due to the time $t=T$ representing the complete exhaustion of the stock, the inventory system formulated in equation (5) gets another boundary condition namely $I(T)=0$.

This summarises the formulation of our inventory model as the fractional order differential equation (5) with boundary conditions $I(0)=S,\ I(T)=0$.

4.2. Solving the proposed model for inventory level function

For analyzing the proposed model to identify an optimal cost strategy a requisite is to solve the Caputo–Fabrizio fractional order differential equation (5) along with its boundary conditions to obtain an expression for the inventory level at any time t. First, we solve the fractional order differential equation given in equation (5) for a general solution by employing the Laplace transform for fractional order derivatives as suggested by Caputo and Fabrizio [12]. Equation (5) be expressed by applying the definition of Caputo–Fabrizio fractional derivative, as following

$$\frac{1}{1-\alpha}\int_0^t\exp\left[-\frac{\alpha(t-\xi)}{1-\alpha}\right]\frac{\mathrm{d}}{\mathrm{d}\xi}I(\xi)d\xi+\phi I(t)=-d(1+\rho r).$$

By the application of convolution theorem, the above equation becomes

$$\frac{1}{1-\alpha}\exp\left[-\frac{\alpha t}{1-\alpha}\right]\frac{d}{dt}I(t)=-\phi I(t)-d(1+\rho r).$$

The application of Laplace transform on both sides of the above equation gives

$$L\left\{\frac{1}{1-\alpha}\exp\left[-\frac{\alpha t}{1-\alpha}\right]\frac{\mathrm{d}}{\mathrm{d}t}I(t)\right\}=L\{-\phi I(t)-d(1+\rho r)\}$$

$$\Rightarrow\frac{1}{1-\alpha}L\left\{\exp\left[-\frac{\alpha t}{1-\alpha}\right]\right\}L\left\{\frac{\mathrm{d}}{\mathrm{d}t}I(t)\right\}=L\{-\phi I(t)-d(1+\rho r)\}$$

$$\Rightarrow\frac{1}{1-\alpha}\frac{1}{s+\frac{\alpha}{1-\alpha}}\left[s\bar{I}(s)-I(0)\right]=-\phi\bar{I}(s)-\frac{d(1+\rho r)}{s}$$

$$\Rightarrow\bar{I}(s)\left[\frac{s}{s+\frac{\alpha}{1-\alpha}}\frac{1}{1-\alpha}+\phi\right]=\frac{1}{1-\alpha}\frac{I(0)}{s+\frac{\alpha}{1-\alpha}}-\frac{d(1+\rho r)}{s}$$

$$\Rightarrow\bar{I}(s)\left[\frac{s\left(\frac{1}{1-\alpha}+\phi\right)+\frac{\alpha\phi}{1-\alpha}}{s+\frac{\alpha}{1-\alpha}}\right]=\frac{1}{1-\alpha}\frac{I(0)}{s+\frac{\alpha}{1-\alpha}}-\frac{d(1+\rho r)}{s}$$

$$\Rightarrow\bar{I}(s)=\frac{I(0)}{1+\phi(1-\alpha)}\left[\frac{1}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right]-\frac{d(1+\rho r)(1-\alpha)}{(1+\phi(1-\alpha))}\frac{1}{s}\left[\frac{s+\frac{\alpha}{1-\alpha}}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right]$$

$$\Rightarrow \bar{I}(s) = \frac{I(0)}{1+\phi(1-\alpha)}\left[\frac{1}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right] - \frac{d(1+\rho r)(1-\alpha)}{(1+\phi(1-\alpha))}\frac{1}{s}\left[\frac{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}\frac{\alpha}{1-\alpha}-\frac{\alpha\phi}{1+\phi(1-\alpha)}}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right]$$

$$\Rightarrow \bar{I}(s) = \frac{I(0)}{1+\phi(1-\alpha)}\left[\frac{1}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right] - \frac{d(1+\rho r)(1-\alpha)}{(1+\phi(1-\alpha))}\left[\frac{1}{s}+\frac{\frac{1}{s}\left(\frac{\alpha}{1-\alpha}-\frac{\alpha\phi}{1+\phi(1-\alpha)}\right)}{s+\frac{\alpha\phi}{1+\phi(1-\alpha)}}\right].$$

Using inverse Laplace transform on both sides of the above equation gives expression for $I(t)$ as following.

$$I(t) = \frac{I(0)}{1+\phi(1-\alpha)}\exp\left[\frac{-\alpha\phi t}{1+\phi(1-\alpha)}\right] - \frac{d(1+\rho r)(1-\alpha)}{1+\phi(1-\alpha)}\left[1+\left(\frac{\alpha}{1-\alpha}-\frac{\alpha\phi}{1+\phi(1-\alpha)}\right)\int_0^t \exp\left[\frac{-\alpha\phi(t-\xi)}{1+\phi(1-\alpha)}\right]\mathrm{d}\xi\right],$$

which simplifies to

$$I(t) = \frac{I(0)}{1+\phi(1-\alpha)}\exp\left[\frac{-\alpha\phi t}{1+\phi(1-\alpha)}\right] - \frac{d(1+\rho r)(1-\alpha)}{1+\phi(1-\alpha)}\left[1-\frac{1}{(1-\alpha)\phi}\left(\exp\left[\frac{-\alpha\phi t}{1+\phi(1-\alpha)}\right]-1\right)\right]. \tag{6}$$

Using the boundary condition $I(T) = 0$ in (6) and simplifying, we get

$$I(0) = d(1+\rho r)(1-\alpha)\left[1-\frac{1}{(1-\alpha)\phi}\left(\exp\left[\frac{-\alpha\phi T}{\phi(1-\alpha)+1}\right]-1\right)\right]\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]. \tag{7}$$

Using (7) back in (6), we get the expression for the inventory level at any time t as following.

$$I(t) = \frac{d(1+\rho r)}{\phi}\left(\exp\left[\frac{\alpha\phi(T-t)}{1+\phi(1-\alpha)}\right]-1\right). \tag{8}$$

Lemma 1. *For given values of fractional order of differentiation (α), promotional effort coefficient (ρ), reliability coefficient (r), deterioration rate (ϕ), the inventory level at any time t of the inventory cycle $[0, T]$ is given by the expression in equation (8).*

Lemma 2. *The initial inventory level representing the replenishment quantity at the commencement of each replenishment cycle is given by*

$$S = \frac{d(1+\rho r)}{\phi}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]-1\right). \tag{9}$$

Proof. Using boundary condition $I(0) = S$ in (8), gives the required expression. □

Equations (8) and (9) furnish expressions for inventory level $I(t)$ at any time t in a replenishment cycle $[0, T]$ and replenishment quantity S, respectively, in terms of the decision variable T. A replenishment quantity S ordered for any given value of replenishment time T can be termed as an economic order quantity (EOQ) based on whether this inventory decision is cost optimal for the retailer. Accordingly, the next section presents optimality conditions achieved through an analysis of all the costs involved while executing various inventory operations in a replenishment cycle.

4.3. Cost analysis

The objective of our study is to identify the optimal replenishment quantity S^* and optimal length of replenishment cycle (*viz*, time span T^* by which the replenished stock would be exhausted) for any given values of parameters $r, \rho, \alpha, \beta, \phi$. The cost optimal value S^* Here the optimality is considered in terms of minimal total cost incurred for carrying out various the inventory operations during each single replenishment cycle $[0, T]$. This calls for listing here each cost component of inventory operations.

Inventory holding cost

The total inventory holding cost is accounted depending on the duration for which a proportion of the stock is held. Accordingly, with the consideration of inventory level $I(t)$ as a continuous function of time variable t, taking integral of the inventory level function appropriately computes the total holding cost, with per-unit holding cost h being a constant. Over that, it is observed that the inventory exhaustion pattern is influenced by the memory effect on customers' purchase decisions with location of display shelves for stock visibility and stock remaining on shelves. Accordingly, the stock holding cost influenced by the effect of integral memory index (β) is given by the following expression.

$$\begin{aligned}\mathrm{HC}_{\alpha,\beta} &= hD^{-\beta}(I(t)) = h(1-\beta)I(T) + h\beta\int_0^T I(t)\mathrm{d}t \\ &= \frac{h\beta d(1+\rho r)}{\phi}\left[\frac{1+\phi(1-\alpha)}{\alpha\phi}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]-1\right)-T\right]. \qquad (10)\end{aligned}$$

Purchasing cost

The total cost of purchase of inventory stock in any replenishment cycle ($\mathrm{PC}_{\alpha,\beta}$), can be calculated by multiplying the expression for per-unit purchase cost given in (9) with the purchase quantity.

$$\mathrm{PC}_{\alpha,\beta} = P(b,r)S = \frac{bg^r d(1+\rho r)}{\phi}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]-1\right). \qquad (11)$$

Fixed ordering cost

The fixed cost of ordering is considered to be constant (A).

$$\mathrm{OC} = A. \qquad (12)$$

Promotional effort cost

The cost of promotional efforts is considered as described in (3).

$$\mathrm{PEC} = k\rho^m. \qquad (13)$$

Total cost of inventory operations

The total cost of various inventory operations listed above can now be aggregated as

$$\begin{aligned}\mathrm{TC}_{\alpha,\beta}(T) &= \frac{1}{T}(\mathrm{HC}_{\alpha,\beta} + \mathrm{PC}_{\alpha,\beta} + \mathrm{OC} + \mathrm{PEC}) \\ &= \frac{h\beta d(1+\rho r)}{T\phi}\left[\frac{1+\phi(1-\alpha)}{\alpha\phi}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]-1\right)-T\right] \\ &\quad + \frac{bg^r d(1+\rho r)}{T\phi}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]-1\right) + A + k\rho^m\end{aligned}$$

$$= U_1 T^{(-1)} + U_2 + U_3\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right)T^{(-1)}, \tag{14}$$

where,

$$U_1 = (A + k\rho^m), \tag{15}$$

$$U_2 = \frac{-h\beta d(1+\rho r)}{\phi}, \tag{16}$$

$$U_3 = \left(\frac{h\beta d(1+\rho r)}{\phi}\frac{1+\phi(1-\alpha)}{\alpha\phi} + \frac{bg^r d(1+\rho r)}{\phi}\right). \tag{17}$$

4.4. Optimality conditions

The objective of our study to identify cost optimal order quantity and replenishment time corresponding to it can now be expressed as following optimization problem.

$$\begin{aligned} &\text{Min} \qquad \mathrm{TC}_{\alpha,\beta}(T) = U_1 T^{(-1)} + U_2 + U_3\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right)T^{(-1)} \\ &\text{subject to } T \geqslant 0 \end{aligned}$$

where U_1, U_2 and U_3 are expressions given by (15), (16) and (17).

We suggest applying the second order derivative test for obtaining the conditions for minimum total cost. *i.e.*,

$\frac{\mathrm{d}(\mathrm{TC}_{\alpha,\beta}(T))}{\mathrm{d}T} = 0$ and $\frac{\mathrm{d}^2(\mathrm{TC}_{\alpha,\beta}(T))}{\mathrm{d}T^2} > 0.$

Now as,

$$\mathrm{TC}_{\alpha,\beta}(T) = U_1 T^{(-1)} + U_2 + U_3\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right)T^{(-1)}, \tag{18}$$

$$\frac{\mathrm{d}(\mathrm{TC}_{\alpha,\beta}(T))}{\mathrm{d}T} = -\frac{U_1}{T^2} - \frac{U_3}{T^2}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right) + \frac{U_3}{T}\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\frac{\alpha\phi}{1+\phi(1-\alpha)},$$

$$\begin{aligned}\frac{d^2(\mathrm{TC}_{\alpha,\beta}(T))}{\mathrm{d}T^2} &= \frac{2U_1}{T^3} + \frac{2U_3}{T^3}\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right) - 2\frac{U_3}{T^2}\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\frac{\alpha\phi}{1+\phi(1-\alpha)} \\ &\quad + \frac{U_3}{T}\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\left(\frac{\alpha\phi}{1+\phi(1-\alpha)}\right)^2.\end{aligned}$$

Therefore, equations gives the following conditions

$$-U_1 - U_3\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right) + U_3 T\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\frac{\alpha\phi}{1+\phi(1-\alpha)} = 0 \tag{19}$$

and

$$\begin{aligned}&2U_1 + 2U_3\left(\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right] - 1\right) - 2TU_3\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\frac{\alpha\phi}{1+\phi(1-\alpha)} \\ &\qquad + T^2 U_3\exp\left[\frac{\alpha\phi T}{1+\phi(1-\alpha)}\right]\left(\frac{\alpha\phi}{1+\phi(1-\alpha)}\right)^2 > 0.\end{aligned} \tag{20}$$

Thus, for any given values of parameters $\alpha,\ \beta,\ \phi,\ \rho$, the solution of (19) in terms of the decision variable T which satisfies the inequality (20) represents the cost-optimal replenishment time (denoted as $T^*_{\alpha,\beta}$) and substituting the same in condition (18) provides the minimum total cost incurred (denoted as $\mathrm{TC}^*_{\alpha,\beta}(T^*)$). Further, substituting $T^*_{\alpha,\beta}$ in (9) provides value of optimal replenishment quantity or EOQ (denoted as S^*).

TABLE 1. Values of inventory and other parameters.

A	k	ρ	d	ϕ	r	b	g	m	h
500	400	0.1	20	0.2	0.5	10	2	2	1

5. Case study from a retail store

In this section, we present our case study of a retail store that has grappled with efficient management of procurement and sales of a perishable item. Their procurement and sales records for this item do not tally with suggestions of relevant inventory models existing in literature. A majority of times, the replenishment quantities ordered by following the decision support of existing inventory models get exhausted quite earlier than the time estimated for the replenishment cycle. This leaves the retailer in a stock-out situation and unprepared for next replenishment order, thereby resulting in an opportunity loss for unfulfilled demands. It has been inferred through extensive interviews with sales managers of the retail store that promoted sales and appropriate placement of an item at more visible locations of the store influence the customers' purchase behavior incrementally.

All the challenges noted above which are faced by the retail store managers have been addressed in our proposed model for inventory replenishment decisions. We have employed our model on the available data of inventory parameters at this retail store for testing the appropriateness of our proposal of incorporating the memory effect. Optimal replenishment quantity and the length of replenishment cycle provided by our proposed model has matched the realization of the actual sales and inventory exhaustion rates during the subsequent sales experiences of the retail stores. For employing our model to the mentioned case, the inventory parameters have been estimated during this case study as given in Table 1.

Based on the validation of computational results obtained through our inventory model, we have further carried out an analysis of effect of variation of various parameters on addressed inventory decisions. This analysis is aimed at studying patterns of variation in optimal values of decision variables which would result as a consequence of variations in different parameters.

In the first part of the analysis, we demonstrate the effect of variation in memory factors represented by differential and integral memory indices. For this, first we vary the differential memory index (α) with 10 different values while fixing the integral memory index $\beta = 1$ and other inventory parameters as mentioned in Table 1. This represents different levels of memory factors influencing the consumers' purchase behavior through promotional initiatives while placing the shelf for the item at a best possible place in the retail store.

Table 2 exhibits increase in EOQ (S^*) with a precipitous shrinkage in optimal replenishment cycle ($T^*_{\alpha,\beta}$) with an increase in differential memory index (α). As the setup of our study considers the end of replenishment cycle at the time of inventory stock-out, results in Table 2 clearly indicate an assessment of the effect of differential memory index on the demand rate.

Now we demonstrate a similar effect in Table 3 by a variation in the integral memory index (β). Table 3 depicts that increase in value of β while fixing $\alpha = 1$ shortens the optimal replenishment cycle ($T^*_{\alpha,\beta}$), while the EOQ value (S^*) remains unchanged. This explicates an increase in demand rate due to better visibility of the product in the retail store. On the other hand, no effect of variation in β on EOQ is seen. The computational investigations compiled in Tables 2 and 3 establish that amplifying any of the two memory parameters shrinks replenishment cycle while increasing in optimal replenishment quantities. A surge is further visible in the optimal total cost $TC^*_{\alpha,\beta}$ across Tables 2 and 3 as a consequence of increase in optimal purchase quantities S^* along with intensification of memory parameters. Trends of variations discussed in Tables 2 and 3 are depicted in Figure 1, for a better insight of the computational outputs.

In a subsequent part of this analysis, we now examine the effect of variation in various inventory parameters while considering two combinations of memory parameters. One of the combinations considered as $\alpha = 0.2$ and

TABLE 2. Effect of variation in α on $T^*_{\alpha,\beta}$, $\mathrm{TC}^*_{\alpha,\beta}$ and S^* (for $\beta = 1$).

α	$T^*_{\alpha,\beta}$	$\mathrm{TC}^*_{\alpha,\beta}$	S^*
0.1	19.1548	75.0952	40.2735
0.2	11.9032	130.4790	53.2879
0.3	8.7460	185.2212	61.3803
0.4	6.9130	240.8326	67.0433
0.5	5.6984	297.8834	71.2664
0.6	4.8285	356.6907	74.5512
0.7	4.1724	417.4807	77.1867
0.8	3.6587	480.4423	79.3502
0.9	3.2449	545.7506	81.1569
1.0	2.9042	613.5780	82.6916

TABLE 3. Effect of variation in β on $T^*_{\alpha,\beta}$, $\mathrm{TC}^*_{\alpha,\beta}$ and S^* (for $\alpha = 1$).

β	$T^*_{\alpha,\beta}$	$\mathrm{TC}^*_{\alpha,\beta}$	S^*
0.1	3.2430	577.6759	82.6916
0.2	3.1989	581.9177	82.6916
0.3	3.1567	586.0892	82.6916
0.4	3.1162	590.1940	82.6916
0.5	3.0774	594.2350	82.6916
0.6	3.0401	598.2152	82.6916
0.7	3.0042	602.1372	82.6916
0.8	2.9697	606.0035	82.6916
0.9	2.9364	609.8164	82.6916
1.0	2.9042	613.5780	82.6916

$\beta = 1$, whereas the other one $\alpha = 0.8$ and $\beta = 1$. Both the cases indicate that the item is placed on a best visible shelf in the retail store for indicating the integral memory index $\beta = 1$. The differential memory index $\alpha = 0.2$ in one of the combinations represents a lower memory factor influencing the purchase behavior of customers, which is seen during initial stages of promotional campaigns. The other combination with differential memory index $\alpha = 0.8$ represents a higher memory factor governing the purchase behavior of customers seen eventually after promotional campaigns for long duration.

Variations in the optimal values of decision variables *viz*, replenishment cycle ($T^*_{\alpha,\beta}$), order quantity (S^*), and the optimal total cost ($T^*_{\alpha,\beta}$) as a consequence of variation in coefficient of promotional efforts (ρ) are presented in Table 4, for memory parameters $\alpha = 0.8$ and $\alpha = 0.2$, while fixing $\beta = 1$. Data listed in each of these tables shows an eventual surge in optimal values of both the decision variables and optimal total cost as a result of intensifying promotional effort indicated through increase in value of the parameter ρ. Figure 2 demonstrates graphically this pattern of variation. The J-shaped curve of replenishment time $T^*_{\alpha,\beta}$ against ρ depicted in Figure 2 observed in a connection with an increase in optimal order quantity S^* confirms to a general fact that initial stages of promotions gives an immediate boost to the sales. Thereby, placing bigger replenishment orders becomes more economical. Eventually, the time to stock-out for bigger volumes of replenishment also increases.

Owing to an escalation in demand due to elevation in reliability of an item the sales become faster. Even for such a situation, the proposed model suggests reducing the order quantities to have shorter replenishment cycle. Computational results in this state of affairs have been tabulated in Table 5 and exhibited through Figure 3. It can be inferred from Table 6 that our model suggests to reduce replenishment quantities for the orders in

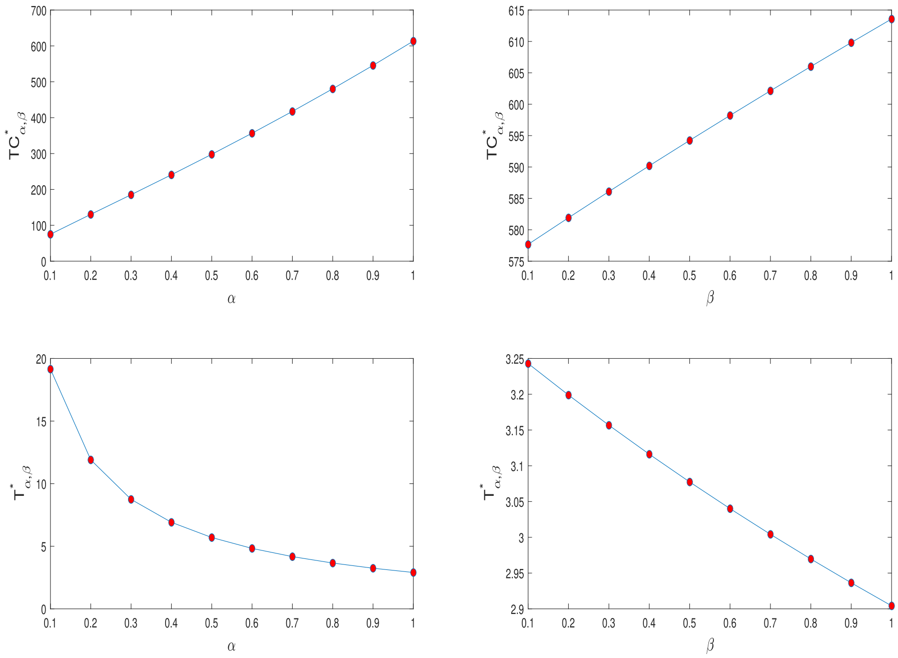

FIGURE 1. Variation pattern of $\text{TC}^*_{\alpha,\beta}$ and $T^*_{\alpha,\beta}$ against variation in α and β.

TABLE 4. Effect of variation in ρ on $T^*_{\alpha,\beta}$, $\text{TC}^*_{\alpha,\beta}$ and S^*.

ρ	$\alpha = 0.8$			$\alpha = 0.2$		
	$T^*_{\alpha,\beta}$	$\text{TC}^*_{\alpha,\beta}$	S^*	$T^*_{\alpha,\beta}$	$\text{TC}^*_{\alpha,\beta}$	S^*
0.1	3.6587	480.4423	79.3502	11.9032	130.4790	53.2879
0.2	3.6237	500.0249	82.0917	11.7839	135.6791	55.1446
0.3	3.6139	521.7987	85.5206	11.7507	141.5528	57.4537
0.4	3.6266	545.7826	89.6481	11.7939	148.1061	60.2199
0.5	3.6587	571.9551	94.4645	11.9032	155.3322	63.4380
0.6	3.7072	600.2637	99.9525	12.0688	163.2140	67.0978
0.7	3.7693	630.6350	106.0892	12.2810	171.7281	71.1816
0.8	3.8425	662.9835	112.8500	12.5314	180.8468	75.6722
0.9	3.9245	697.2187	120.2050	12.8124	190.5411	80.5497
1.0	4.0134	733.2500	128.1280	13.1174	200.7818	85.7942

the situation of higher deterioration rates of the sellable item. This replenishment time is also seen get shorter along this pattern for both the addressed situations of memory effect of promotions. This pattern of variation is evinced through Figure 4.

Variation in the optimal total cost ($\text{TC}^*_{\alpha,\beta}$) resulting due to a variation in the holding cost per-unit of item (h) against the change in differential memory index (α) is listed in Table 7. Whereas, the variation in optimal replenishment time ($T^*_{\alpha,\beta}$) due to variation in these parameters is listed in Table 8. The optimal total cost can be observed to increase along α for each value of h taken from 1 through 10, as well as along h also for each value of α from 0.1 through 1. Whereas, a shrinkage is seen through Table 8 in optimal replenishment time along increase in each of these parameters with value of other parameter fixed. Similarly, variations in the values of

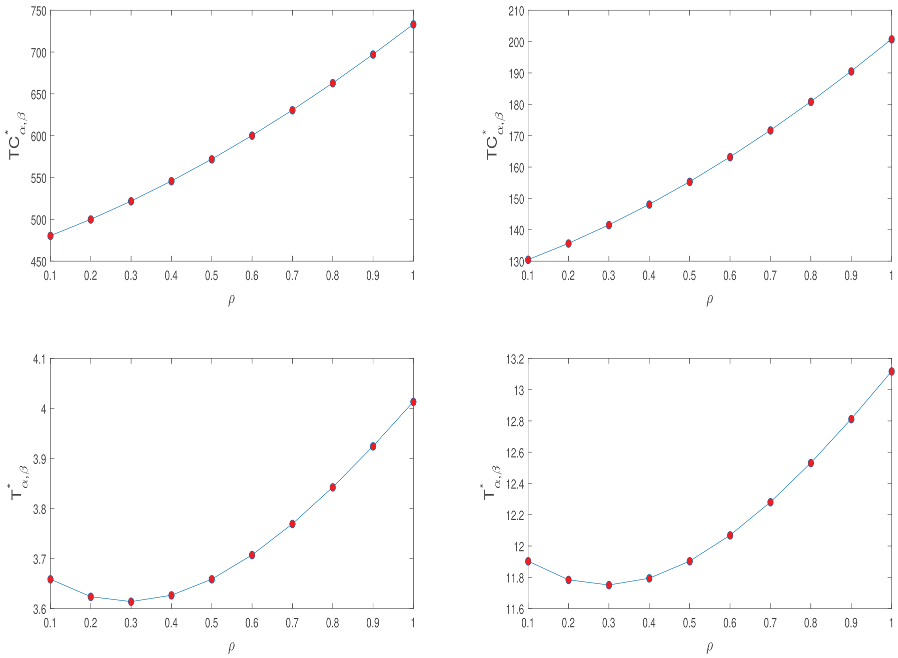

FIGURE 2. Variation pattern of $\mathrm{TC}^*_{\alpha,\beta}$ and $T^*_{\alpha,\beta}$ against variation in ρ ($\alpha = 0.2$ and 0.8, $\beta = 1$).

TABLE 5. Effect of variation in r on $T^*_{\alpha,\beta}$, $\mathrm{TC}^*_{\alpha,\beta}$ and S^*.

r	$\alpha = 0.8$			$\alpha = 0.2$		
	$T^*_{\alpha,\beta}$	$\mathrm{TC}^*_{\alpha,\beta}$	S^*	$T^*_{\alpha,\beta}$	$\mathrm{TC}^*_{\alpha,\beta}$	S^*
0.1	4.0086	394.6950	86.1346	12.5519	112.2461	54.7021
0.2	3.9210	413.9746	84.4575	12.3942	116.3381	54.3910
0.3	3.8335	434.6247	82.7679	12.2336	120.7255	54.0520
0.4	3.7460	456.7449	81.0634	12.0699	125.4310	53.6842
0.5	3.6587	480.4423	79.3502	11.9032	130.4790	53.2879
0.6	3.5716	505.8318	77.6287	11.7336	135.8956	52.8636
0.7	3.4850	533.0369	75.9079	11.5611	141.7091	52.4113
0.8	3.3988	562.1901	74.1852	11.3856	147.9494	51.9303
0.9	3.3132	593.4340	72.4665	11.2074	154.6492	51.4223
1.0	3.2282	626.9218	70.7521	11.0264	161.8434	50.8868

$\mathrm{TC}^*_{\alpha,\beta}$ and $T^*_{\alpha,\beta}$ due to variation and the per-unit base-price (b) for the purchase of the product by retailer and in differential memory index (α) are recorded in Tables 9 and 10, respectively.

6. MANAGERIAL IMPLICATIONS

The proposed inventory model provides an appropriate decision-support to an inventory manager for identifying cost-optimal quantities of replenishment orders for deteriorating items while considering important factors which influence demand rates through customers' purchase behavior. Variation analysis presented through a case study facilitates decision making for a tactical management to handle the changes in sales environment due to the addressed factors. Outcomes of our study can be appropriately applied to similar situations for inventory

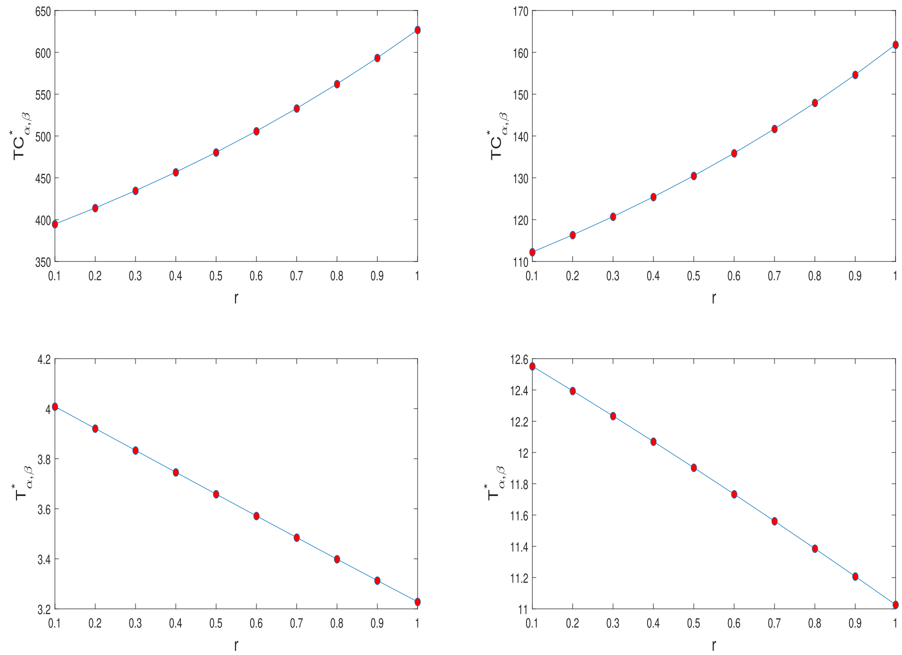

FIGURE 3. Variation pattern of $\text{TC}^*_{\alpha,\beta}$ and $T^*_{\alpha,\beta}$ against variation in r ($\alpha = 0.2$ and 0.8, $\beta = 1$).

TABLE 6. Effect of variation in ϕ on $T^*_{\alpha,\beta}$, $\text{TC}^*_{\alpha,\beta}$ and S^*.

r	$\alpha = 0.8$			$\alpha = 0.2$		
	$T^*_{\alpha,\beta}$	$\text{TC}^*_{\alpha,\beta}$	S^*	$T^*_{\alpha,\beta}$	$\text{TC}^*_{\alpha,\beta}$	S^*
0.1	4.7487	432.8169	94.7691	13.1997	128.3761	58.1500
0.2	3.6587	480.4423	79.3502	11.9032	130.4790	53.2879
0.3	3.0667	518.9836	70.1681	11.0749	131.4873	49.6269
0.4	2.6869	551.5937	63.8883	10.5058	131.7958	46.7387
0.5	2.4196	579.8969	59.2431	10.0956	131.6415	44.3824
0.6	2.2198	604.8727	55.6214	9.7900	131.1754	42.4112
0.7	2.0641	627.1716	52.6935	9.5568	130.4970	40.7292
0.8	1.9389	647.2549	50.2587	9.3759	129.6743	39.2721
0.9	1.8360	665.4656	48.1993	9.2337	128.7545	37.9925
1.0	1.7496	682.0679	46.4187	9.1210	127.7709	36.8566

management. For example, the inventory restocking decisions for the situations of a production firm manufacturing a perishable item for its sale in its own retail outlets can also addressed. Sale of milk based sweets is an appropriate illustration of the same. The model can be adopted by online retail stores also as e-marketing firms invest significant amount of efforts for creating a pertinent memory effect while promoting some typical items categorized as FMCG.

7. Conclusions, challenges and future scope

This work studies an inventory strategy for capturing effects of reliability, promotional efforts, and their persistent effect on customers' purchase behavior. The influence of reliability and promotional efforts on demand

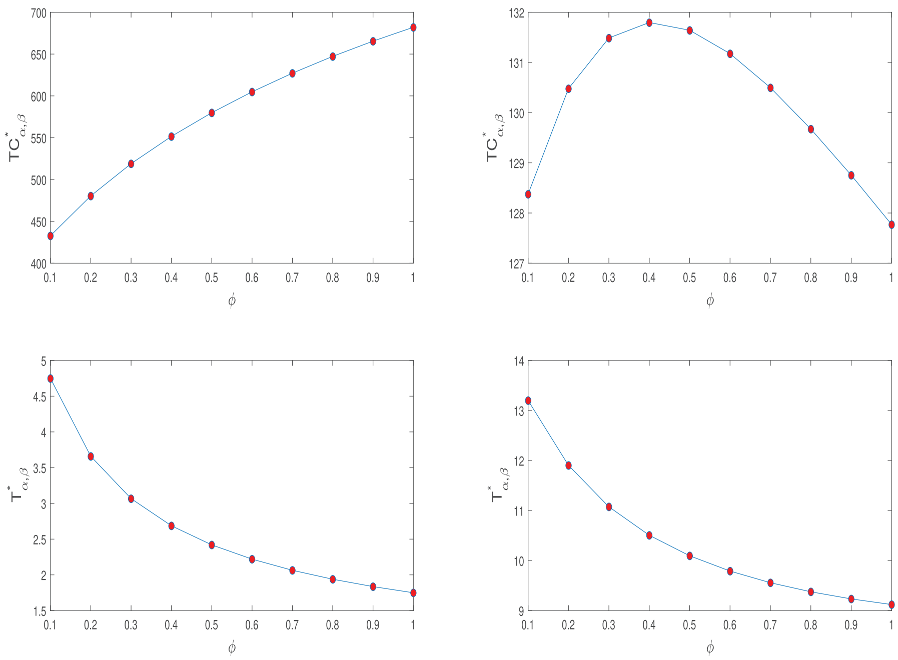

FIGURE 4. Variation pattern of $\mathrm{TC}^*_{\alpha,\beta}$ and $T^*_{\alpha,\beta}$ against variation in ϕ ($\alpha = 0.2$ and 0.8, $\beta = 1$).

TABLE 7. Variation in $\mathrm{TC}^*_{\alpha,\beta}$ against variation in α and h

$h \downarrow \alpha \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	75.0952	130.4790	185.2212	240.8326	297.8834	356.6907	417.4807	480.4423	545.7506	613.5781
2	91.3419	152.1208	210.3532	268.4942	327.4893	387.8489	449.9118	513.9389	580.1537	648.7627
3	104.1999	169.6815	231.1713	291.8094	352.8170	414.8496	478.3337	543.5878	610.8754	680.4321
4	115.1815	184.8531	249.3424	312.3479	375.3139	439.0134	503.9440	570.4714	638.8923	709.4668
5	124.9266	198.4065	265.6755	330.9144	395.7588	461.0816	527.4412	595.2435	664.8133	736.4318
6	133.7777	210.7701	280.6361	347.9870	414.6279	481.5201	549.2755	618.3349	689.0483	761.7150
7	141.9432	222.2111	294.5212	363.8769	432.2375	500.6441	569.7565	640.0473	711.8888	785.5965
8	149.5614	232.9097	307.5339	378.8005	448.8105	518.6785	589.1084	660.6017	733.5509	808.2864
9	156.7295	242.9939	319.8205	392.9151	464.5109	535.7908	607.4996	680.1656	754.2000	829.9469
10	163.5189	252.5588	331.4905	406.3395	479.4634	552.1094	625.0604	698.8697	773.9662	850.7066

TABLE 8. Variation in $T^*_{\alpha,\beta}$ against variation in α and h.

$h \downarrow \alpha \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	19.1548	11.9033	8.7460	6.9130	5.6984	4.8286	4.1724	3.6587	3.2450	2.9043
2	14.6287	9.4736	7.1744	5.8057	4.8772	4.1979	3.6756	3.2595	2.9192	2.6350
3	12.3255	8.1287	6.2459	5.1159	4.3426	3.7716	3.3287	2.9728	2.6793	2.4323
4	10.8651	7.2411	5.6123	4.6316	3.9576	3.4577	3.0680	2.7534	2.4927	2.2722
5	9.8315	6.5978	5.1436	4.2666	3.6627	3.2135	2.8624	2.5782	2.3418	2.1414
6	9.0497	6.1032	4.7781	3.9785	3.4270	3.0164	2.6948	2.4338	2.2165	2.0318
7	8.4312	5.7074	4.4826	3.7432	3.2330	2.8527	2.5545	2.3122	2.1101	1.9381
8	7.9259	5.3811	4.2371	3.5463	3.0695	2.7138	2.4347	2.2078	2.0183	1.8568
9	7.5029	5.1060	4.0288	3.3783	2.9292	2.5940	2.3309	2.1168	1.9379	1.7853
10	7.1418	4.8699	3.8490	3.2326	2.8070	2.4893	2.2397	2.0366	1.8668	1.7218

TABLE 9. Variation in $\text{TC}^*_{\alpha,\beta}$ against variation in α and b.

$b \downarrow \alpha \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2	51.1540	79.0913	104.0531	127.9061	151.3751	174.8419	198.5460	222.6574	247.3085	272.6111
4	57.1700	92.0930	124.7305	156.8600	189.1646	222.0061	255.6172	290.1701	325.8061	362.6508
6	63.1647	104.9836	145.1239	185.2699	226.0623	267.8439	310.8418	355.2314	401.1634	448.7779
8	69.1393	117.7755	165.2763	213.2365	262.2583	312.6698	364.6960	418.5178	474.2974	532.1907
10	75.0952	130.4790	185.2212	240.8326	297.8834	356.6907	417.4807	480.4423	545.7506	613.5781
12	81.0334	143.1026	204.9845	268.1134	333.0322	400.0515	469.4010	541.2792	615.8753	693.3792
14	86.9548	155.6537	224.5875	295.1213	367.7758	442.8581	520.6034	601.2214	684.9159	771.8943
16	92.8604	168.1385	244.0475	321.8901	402.1692	485.1910	571.1976	660.4108	753.0496	849.3396
18	98.7510	180.5624	263.3789	348.4472	436.2560	527.1126	621.2676	718.9553	820.4104	925.8767
20	104.6273	192.9301	282.5939	374.8150	470.0714	568.6730	670.8798	776.9393	887.1021	1.0016e+03

TABLE 10. Variation in $T^*_{\alpha,\beta}$ against variation in α and b .

$b \downarrow \alpha \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2	20.6533	13.5838	10.4472	8.5773	7.3024	6.3628	5.6344	5.0488	4.5654	4.1579
4	20.2446	13.0940	9.9243	8.0427	6.7678	5.8350	5.1174	4.5452	4.0767	3.6850
6	19.8601	12.6561	9.4769	7.6027	6.3424	5.4276	4.7291	4.1763	3.7267	3.3532
8	19.4975	12.2614	9.0881	7.2316	5.9928	5.0999	4.4227	3.8899	3.4589	3.1027
10	19.1548	11.9033	8.7460	6.9130	5.6984	4.8286	4.1724	3.6587	3.2450	2.9043
12	18.8303	11.5761	8.4417	6.6352	5.4458	4.5987	3.9626	3.4666	3.0685	2.7418
14	18.5223	11.2757	8.1686	6.3901	5.2258	4.4004	3.7833	3.3036	2.9197	2.6053
16	18.2294	10.9986	7.9216	6.1716	5.0318	4.2272	3.6275	3.1628	2.7917	2.4885
18	17.9506	10.7418	7.6968	5.9751	4.8590	4.0739	3.4905	3.0394	2.6800	2.3868
20	17.6845	10.5029	7.4909	5.7971	4.7036	3.9369	3.3686	2.9302	2.5814	2.2973

rate is formulated for developing an inventory strategy accordingly, whereas their pertinent effect on customers' behavior is formulated appropriately through Caputo–Fabrizio fractional order derivatives. A prominent situation of procurement and sale of a deteriorating item is considered for the study. On that account, an inventory model for identifying optimal values of replenishment order quantity and associated replenishment time is formulated by incorporating mentioned factors influencing the demand rate. A case study is conducted further on inventory operations of a retail store, which verifies acclaimed suggestions of the formulated model. An analytical investigation on the variation in values of crucial decision variables resulting through variation in various parameters confirms to the reality and reveals further even the pattern of the variation. Higher sales rates are observed in Tables 2 and 3 consequent to amplification in memory effect. The curve depicting optimal replenishment time $T^*_{\alpha,\beta}$ against ρ presented through Figure 2 confirms to the market dynamics that initial stages of promotions give immediate boost to the sales, and results in Table 4 further validate that placing bigger replenishment orders becomes more economical. Results of numerical computations and graphical representations have been obtained using MATLAB software.

Assessment of parameters of the study is a major challenge. The estimation of various costs and verification of corresponding function curves with the available data requires a statistical expertise for an effective use of this model. Similar model with inexactness of parameters is planned be studied in future under fuzzy environment for volatile market situations.

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