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ANALYSING A LEAN MANUFACTURING INVENTORY SYSTEM WITH PRICE-SENSITIVE DEMAND AND CARBON CONTROL POLICIES

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Abstract. Production lot-sizing techniques used by lean practitioners to lower waste inventories and increase production efficiency in the manufacturing industry, are the subject of this paper's speculation. Lean manufacturing aims to incorporate innovative tools into the manufacturing process to improve productivity and reduce processing time. In view of this, the model anticipates a flexible production rate based on labor, energy, and tool/die costs, to meet the demand while minimizing wastage. Moreover, a discrete investment in set-up costs is considered to lower the initial set-up cost since it is a critical component of smooth manufacturing operations. Further, it is found that price plays a significant role in stimulating a product's demand; consequently, demand is presumed to be price-sensitive. Besides this, to reduce the carbon footprint in the production systems, two methods namely "Carbon tax" and "Cap-and-trade", have been employed. The purpose of the developed model is to maximize total profit by jointly optimizing the production rate, selling price, and set-up cost. Numerical experiments are performed to validate the model findings. Results suggest that manufacturers' production time decreases simultaneously with the introduction of advanced labor and technologies. With respect to carbon policies, the cap-and-trade policy performs better with an increase in total profit and a higher production rate as compared to a carbon tax. Also, sensitivity analysis is performed to support the manufacturer in the decision-making process for ancillary benefits of the optimal policy.

Mathematics Subject Classification. 90B05.

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1. Introduction

The growing global market competition has led to the rise of lean manufacturing techniques in various industries. Lean manufacturing finds its roots in the Toyota Production System (TPS), Japanese manufacturing, pioneered by Mr. Taiichi Ohno, an industrial engineer. His main focus was on productivity improvement, labor empowerment, and reducing the use of inventories and waste. The technique ameliorated the performance of Toyota and is still widely used today. Based on a global industry analysis, it has been identified that manufacturing companies are strongly committed to implementing lean concepts. As a result of lean manufacturing techniques, customers' needs can be met to the fullest with waste elimination. Some sources of waste are overproduction, underproduction, faulty inventory products or defects, transportation of irrelevant products, and

Keywords. Lean manufacturing, flexible production rate, price-sensitive demand, set-up cost reduction, carbon emissions.

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unnecessary waiting time. The implementation of lean manufacturing techniques helps to increase production efficiency while reducing processing time. Most of the existing research assumed the production rate to be fixed. But on the contrary, with lean management, a significant impact on a company's operations and overall productivity are observed. Therefore, a constant production rate is not valid. Hence, to achieve a flexible production rate, the cost of material, labor and energy cost and tool/die costs are utilized. Moreover, in addition to reducing waste, the lean manufacturing idea helps to develop a sustainable inventory structure. Today, it is observed almost every industry is adopting lean manufacturing methods, including textiles [23] and healthcare [40] industries, to avoid non-value-added activities and ensure a cleaner environment. Further, the consideration of green investment as a form of lean management was studied by Paul *et al.* [37] under the effects of prepayment in a production model.

To maintain the demand for any product, the price of an item has an immense effect on which it is sold. In most of the existing models, the demand is considered to be variable, for instance, You [67] assumed demand to be dependent on time and price, whereas [16] considered demand to be influenced by advertisement in a smart manufacturing system. Thus, in the actual world, the selling price has a significant impact on the product's demand. There is no escaping a product's selling price, with an inverse relationship between the price and demand for goods. Thus, to optimize the total profit, a demand function that depends on the selling price of the product is a practical assumption.

Traditional models often consider a fixed set-up cost for their model. However, for a smooth operation of the industry, investment in the set-up cost is a must. Using an initial investment for updating machinery and other modifications to improve the system helps to lower each set-up cost. Thus, a wise investment in set-up cost not only maximizes the profit but also serves as a quick response to the customer's demands. Finally, with regard to the manufacturing sector, the contribution of carbon emissions to the environment is a worldwide concern. These emissions originate from processes such as production, storing items in a warehouse, and transportation which acts as the major sources of greenhouse gases [7]. As the proposed model focuses on the ideology of lean manufacturing, carbon emissions generated by the industries serve as major waste produced. Reducing these emissions play an important role in addressing sustainability. Thus to attain sustainability, two distinct carbon policies namely, "carbon cap-and-trade and carbon tax" mechanisms are implemented.

1.1. Research gaps and our contributions

The main gap in the literature and the contribution of the study are discussed as followed:

(1) Several researchers have considered the concept of lean manufacturing by introducing it into an inventory model in various ways – as smart production [17], as waste management [50], as a form of rework [6]. However, the concept of lean manufacturing with a discrete investment in set-up cost to minimize the initial set-up cost, along with carbon emission control strategies (namely, carbon tax and carbon cap and trade) is yet to be explored.
(2) Many models have been designed under the consideration of selling price-dependent demand and flexible production rates [15,51], but the concept of adopting the selling price-dependent demand along with flexible production rate, and an investment in the set-up cost with carbon control policies is still not analyzed. Thus, this study fills this literature gap as a result of its approach.
(3) Several models have addressed the effects of carbon emissions on an inventory system [11,60]. However, the inclusion and comparison of the two carbon control policies – carbon cap-and-trade and a carbon tax in an inventory system with lean manufacturing and price-sensitive demand, along with an investment in set-up cost have not been proposed so far to the best of our knowledge. The present model attempts to provide a novel approach in this regard.

Considering the settings of the model, the proposed research attempts to answer the following questions:

– How can lean manufacturing be implemented in an inventory scenario?
– What would be the optimum production rate, selling price, and investment in set-up costs in such a system?

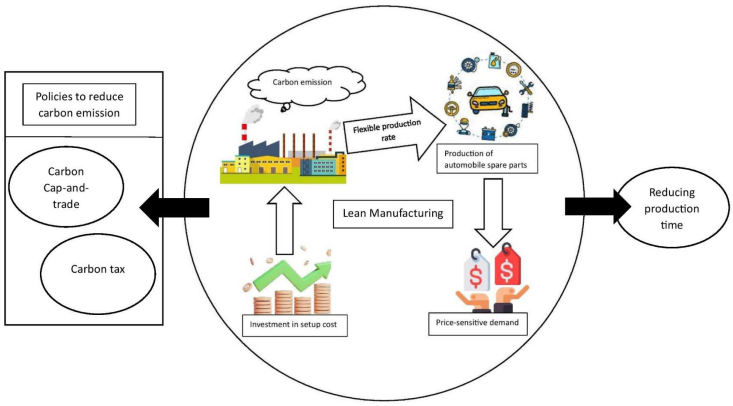

FIGURE 1. Graphical representation of the problem.

– What effect would flexible production rate and selling price-dependent demand have on the inventory policy?
– Which methodology yields the higher profit when comparing carbon emission policies?

1.2. Motivation and objective

In our existing linear economy, people have started considering the notion of sustainability for the welfare of the environment. In a production industry, waste is generated as a by-product at almost every step. Wastes produced by industries create an adverse effect on the environment. To dispose of these wastes, another cost is incurred, which increases the overall production cost. It is true, many researchers have conducted several types of studies on sustainable inventory models and waste management to reduce the overall total cost. But integrating the concept of lean manufacturing with the inventory model along with carbon control policies is an area with wide possibilities which can be explored. Lean manufacturing enhances efficiency, minimizes waste leading to reduced costs, and allows the company to provide more value to its customers. Motivated by this, the present study aims to develop a lean production lot-sizing inventory model, which will help in assisting the manufacturing industry to tilt toward a cleaner environment.

Moreover, the present study incorporates some practical aspects, *viz.*, a price-sensitive demand, a flexible production rate, and set-up cost reduction which are noteworthy contributions. When companies sell their products, the demand for the product depends on the customers' willingness to buy at a certain price point. Thus, a production rate that will meet the ever-changing demand is a must for an industry. In this model, a flexible production rate is considered that depends on the cost of material, labor and energy cost, and tool/die costs which help to improve production efficiency and reduce processing time. Further, an investment in the set-up cost is taken as a decision variable to reduce the initial set-up cost and for keeping the industry running smoothly. While talking about lean culture, the harmful carbon emissions produced as a repercussion of the activities involved in production cannot be ignored. Thus, two carbon emission control policies – "carbon cap-and-trade and carbon tax" are studied which will further ensure the sustainability of the model.

Based on the concepts discussed above, the proposed study examines a lean manufacturing model under the influence of carbon control policies which includes price-sensitive demand and investment in set-up costs. The model optimizes the selling price, production rate, and set-up cost investment to maximize the overall profit of the system, by developing the profit function for the two policies. Finally, numerical illustrations and the impact of different parameters have been summarized, with some valuable managerial insights. The proposed study finds its wide application in the automotive spare parts industry. The graphical representation of the problem under consideration has been presented in Figure 1.

1.3. Orientation

The following is how the current study is structured: a literature overview is presented in the following section. Sections 3 and 4 give the notation and assumptions on which the existing model is developed respectively. The mathematical model is presented in Section 5, followed by Section 6. In this section, the optimality of the total profit function of the two cases and the solution procedure has been discussed. Section 7 provides the numerical analysis, and Section 8 includes the sensitivity analysis of some of the model parameters with some significant managerial insights. Lastly, in Section 9, the paper concludes with directions for further research.

2. Literature overview

There are five streams of literature addressed in this work, including research on lean manufacturing, price-sensitive demand, flexible production rate, set-up cost reduction, and control policies for carbon emissions. The following subsections provide an in-depth overview of existing research and the research gap along with a clear motivation within this field along with the authors' contribution table.

2.1. Impact of price on demand

Almost in most retail and manufacturing industries, it is observed that the demand for a product is affected by a variety of factors. The selling price of an item is one such factor where it has a significant impact on its demand. When demand is variable and price-sensitive, it is likely to have an impact on both the manufacturer's and the consumer's costs. It is observed that the selling price of a product is an important determining element when defining the demand for that product. In the event of a price drop, the demand tends to rise. As a result, selling price-dependent demand is considered to determine the optimal price at which a product may be sold to maximize the demand. Polatoglu and Sahin [38] in their paper highlighted the optimal procurement policy under price-sensitive demand. A scenario under price-sensitive demand for deteriorating items was discussed by Bhunia and Shaikh [8]. Saha and Sen [43] extended the deteriorating item inventory model to incorporate inflationary effects on price and time-dependent demand and shortages. Agrawal and Yadav [1] studied price and profit structuring of an integrated inventory supply chain under price-sensitive demand conditions. Singer and Khmelnitsky [55] explored a production-inventory problem with price-sensitive demand. Later in the same year, Paul *et al.* [35] considered the effect of price-sensitive demand on a deteriorating inventory model. They discovered that when a price-sensitive demand was adopted, profit increased due to the rise in demand.

2.2. Importance of lean manufacturing on inventory system

The gold standard of modern operations is frequently considered to be lean production. As a potential solution, a growing number of organizations are turning to lean manufacturing. Lean manufacturing provides economic benefits such as shorter lead times and better throughput and a lower work-in-process. Lean manufacturing is a method that focuses on improving production efficiency with waste minimization. One of the main contributors to waste in the lean culture is overproduction. While there has been much research on sustainability and waste management alone, few researchers have considered lean manufacturing and sustainability on inventory models. A road map for how Value Stream Mapping (VSM) can be an important tool to define, analyze, and quantify waste was illustrated by Sullivan *et al.* [57]. They considered the defects of products by visualizing the source of the current state which can help managers more easily and pursue the adoption of lean manufacturing. Sundar *et al.* [58] reviewed the implementation of lean techniques for organizations. They further stated this technique is becoming a core competency for any type of organization to sustain. Tayyab and Sarkar [61] studied the impact of lean manufacturing in a textile sector for an imperfect production system. Further, Tayyab *et al.* [62] analyzed the imperfect manufacturing process that produces defective products at an uncertain rate. Sanabria Machuca *et al.* [44] discussed lean manufacturing techniques combined with inventory management in a small and medium-sized enterprise (SME) company dedicated to commercialization. Recently, Dey and Seok [13]

proposed a model for automated inspection which uses the concept of lean manufacturing and different service strategies to maximize profit.

2.3. Effect of flexible manufacturing on the inventory system

Most academicians have expressly assumed that the demand for a product is indubitably known. However, as discussed above, in the real world the demand for a particular product is not purely deterministic but varies depending on various factors such as price. Accordingly, the production process needs to be adjusted depending on the product's demand and availability. Therefore, as a result, when the demand for a particular product rises, the company must adapt by increasing the production rate to meet the rising customer demand. Thus, with the fluctuation in the demand for the product, the manufacturer has to alter the rate at which products are produced. Sethi and Sethi [54] introduced the theory of flexible production into the inventory model as an initial contribution to the field. Ayed *et al.* [4] jointly optimized the maintenance and production policies considering random demand and flexible production rates. AlDurgam *et al.* [2] investigated an integrated inventory model with stochastic demand and flexible production rate. Sarkar *et al.* [49] explored the effects of flexible production rates on the quality of products. In some cases, like launching a new product, it is very practical that the company may not be able to decide the appropriate production rate. In that case, they may go according to the demand rate [42]. Yadav *et al.* [66] discussed the impact of a sustainable production model with volume agility. Recently, as the world has grown more knowledgeable about sustainability, Sarkar *et al.* [52] proposed a flexible manufacturing-re-manufacturing model with green investment. The result shows major positive differences compared to the models which do not include flexible production. In the same year, Dey *et al.* [19] studied a flexible production rate in supply chain management to improve sustainability from economic, environmental, and social perspectives.

2.4. Set-up cost reduction

One of the cornerstones of the lean manufacturing concept is the idea that reducing the waste in the system saves precious resources and minimizes unnecessary spending. While talking about the manufacturing sector, it is crucial to talk about set-up costs for the smooth functioning of the company. It is a well-known fact that discrete capital investment is instrumental in controlling set-up costs. In other words, the introduction of capital investment can reduce the set-up costs from the original set-up cost level. One of the pioneers to introduce set-up cost in an inventory model [39], talked about the critical sales level such that investment is made in reducing set-ups. Using the Just-in-Time (JIT) manufacturing philosophy [12] studied inventory reductions which require investment in the reduction of set-up costs. Sarker and Coates [53] proposed an inventory model with set-up cost reduction under variable lead times and finite opportunities for investment. Optimal investment in set-up reduction in manufacturing systems with work-in-process (WIP) inventories was taken into account by Nye *et al.* [33]. The idea of investment in set-up cost reduction and quality improvement was proposed by Freimer *et al.* [20], who also considered the number of defects produced. Talking about defective products, Gautam and Khanna [21] explored an inventory model with set-up cost reduction in case of imperfect items. Sarkar and Moon [46] discussed an inventory model with set-up cost reduction, quality improvement, and variable backorder rate. A discrete set-up cost reduction under an inventory scenario formulated by Dey *et al.* [15] demonstrated that this model reduced the initial set-up cost and improved the initial quality. Taking large-scale industries, Karthick and Uthayakumar [29] assumed investment in the set-up cost to reduce the expense of the consignor during the huge production of items.

2.5. Carbon emission policies

As we know, with globalization and increasing environmental consciousness, carbon emissions have become one of the main concerns of industries worldwide. Both governmental and non-governmental groups are taking different measures to control harmful emissions. Beyond what individuals can do to reduce their carbon footprint and increase efficiency, companies that reduce carbon emissions play a critical role in improving our chances

of combating climate change. Reduced carbon emissions benefit the bottom line for many businesses because efficient practices reduce operating costs and help increase productivity. Carbon cap-and-trade and carbon tax are two prevalent ideas under carbon emission control policies. Under the former case, the government sets a limit on the amount of emissions, wherein companies are charged only if they exceed the limit. Whereas in the latter one, the emission-related expenditures include paying taxes when there are emission laws in place. He *et al.* [27] studied the lot size and emissions decisions in particular. To maximize the production of the product, Hua *et al.* [28] considered an inventory model with freshness-dependent demand under carbon emissions constraints. Ghosh *et al.* [24] developed a supply chain model under a strict carbon cap policy. Alkahtani *et al.* [3] discussed how to optimize resources and carbon emissions. Khanna and Yadav [30] under preservation technology, compared the effects of carbon cap-and-trade and carbon tax mechanisms on an inventory scenario. Hasan *et al.* [26] studied three models to optimize inventory levels and technology investments under carbon policies. Paul *et al.* [36] showed under the carbon tax system, reducing the level of emissions will result in greater benefits with a greening level. With sustainability and environmentally friendly technologies being developed daily, wastes are a major concern to the environment. They are produced in almost every step of the production process, and managing them is one of the main goals. Dey *et al.* [18] and Ghosh *et al.* [25] proposed a waste management model which helps reduce the production of carbon emissions. As a result, the model recommends a lean manufacturing model and carbon control strategies for reducing harmful emissions. A comparative framework of the proposed model with the existing literature has been presented in Table 1.

3. Notations

The notations considered are shown below to assist demonstrate the concept more clearly:

Decision variables

P	Production rate (unit/cycle)
s	Selling price ($/unit)
ξ	Investment in set-up cost ($)

Parameters

t_1	The time when the production stops (unit time)
T	Cycle length (unit time)
K	Set-up cost per order ($/setup)
K_0	Initial set-up cost ($/setup)
m	Known parameter
$I_{\max}$	Maximum inventory level in a cycle at t_1
μ	Cost of material per unit ($/unit)
g	Labor and energy cost ($)
δ	Tool/die cost ($)
e_1	Amount of emission per production run due to set-up (kg/time)
e_2	Average emission per unit of production during the production process (kg/time)
e_3	Average emission per unit of production run time generated due to machining operation (kg/time)
e_4	Average emission per unit item per unit time holding (kg/time)
h	Storage cost of the item ($/unit/unit time
e_p	Carbon emission cost in production ($/kg)
e_h	Carbon emission cost in inventory holding ($/kg)
$D(s)$	Rate of demand as a function of the unit selling price (unit/unit time)
a	Demand function intercept
b	Demand sensitivity parameter related to the price of the product
C_p	Quota price of carbon ($/unit (tonne))
z	Emissions quota of carbon (per unit time (tonne))

TABLE 1. Comparison of the current study with earlier research.

Author(s)	Lean manufacturing/ Flexible production	Demand rate	Set-up cost reduction	Carbon emission policies to reduce waste	
				Carbon cap and trade	Carbon tax
Dey *et al.* [15]	NA	Selling price dependent	Yes	NA	NA
Dey *et al.* [14]	Yes	Constant	Yes	NA	NA
Ruidas *et al.* [42]	Yes	Stock and selling price dependent	NA	NA	NA
Hasan *et al.* [26]	NA	Green technology and promotion investment dependent	NA	Yes	Yes
Ghosh *et al.* [25]	Yes	Constant	NA	Yes	Yes
Dey *et al.* [19]	Yes	Selling price and quality dependent	Yes	NA	NA
Sarkar *et al.* [50]	Yes	Selling price dependent	NA	NA	NA
Dey and Seok [13]	Yes	Service dependent	NA	NA	NA
Paul *et al.* [36]	NA	Selling price and green-sensitive dependent	NA	NA	Yes
Taleizadeh *et al.* [60]	NA	Selling price and carbon emission dependent	NA	Yes	Yes
Bachar *et al.* [6]	Yes	Selling price dependent	NA	NA	NA
Barman *et al.* [7]	NA	Selling price and green leveling dependent	NA	Yes	NA
This paper	Yes	Selling price dependent	Yes	Yes	Yes

Notes. NA – Not Applicable.

w Tax charged on carbon ($/unit (tonne))

Optimal values

P^* The optimal rate of production (unit/cycle)
s^* Optimal selling price ($/unit)
ξ^* The optimal amount of investment in set-up cost ($/production run)

4. Assumptions

The proposed paper presents a model for a production inventory system while managing waste produced. One of the key factors for the smooth operation of a production system is an initial investment. Furthermore, a price-dependent demand function and a flexible production rate are also incorporated. The following assumptions are considered to make this model:

(1) Demand is considered to be a function of price, and is given as:

$$D(s) = a - bs.$$

Here, $a < 0$ and $0 < b < 1$ is the scale and shape parameter respectively, and both are positive known constants [15, 32, 63].

(2) The production rate, P is flexible, where the production rate is greater than the demand, *i.e.*, $P > D(s)$ [34, 47].

(3) The unit production cost $(P) = \mu + \frac{g}{P} + \delta P$ where μ, g, δ are all positive constants. Here (μ) per unit item is fixed. As P rises, some costs, such as labor, are similarly distributed throughout a large number of units. Later, the cost of production per unit $(\frac{g}{P})$ declines as the rate of production P rises. The third term δ, linked with a tool/die costs is proportional to the rate of production [6, 52, 56].

(4) An investment ξ is considered in the production set-up which aids in minimizing the set-up cost. It is defined as $K(\xi) = K_0 e^{-m\xi}$, where m is the known parameter and K_0 the initial investment [15, 48].

(5) The present model assumes that carbon emissions are caused due to manufacturing and maintenance activities. During the production period, to produce the desired items, the manufacturing sector consumes a large number of raw materials and in turn, emits carbon gas immensely. Similarly, as the items are produced and held in holding, they require care and maintenance. As a result, numerous activities like ventilation, air-conditioning, and heating are required, all of which release carbon gas [59].

(6) This study compares two different carbon emission policies that are available for reducing carbon emissions, *viz.*, "Carbon cap-and-trade" and "Carbon tax" [9].

(7) Lead time is assumed to be negligible [31, 41].

(8) As the model assumes that the production rate is greater than the demand rate, so shortages do not occur [45, 64].

5. Mathematical model

Using the aforementioned assumptions and notations, consider an inventory production model. As demand depends upon the price of the product, it is considered to be price-sensitive. A flexible production rate is considered to meet the fluctuating demand and avoid shortages. In addition, at the start of each cycle, a discrete investment in the set-up cost is also taken into account. Figure 2 represents an inventory scenario that is divided into production time and non-production time. At the time $t = 0$, the cycle starts with no inventory and starts producing the inventory till the time $t = t_1$ at a rate of P. When the inventory reaches its maximum level, *i.e.*, $I_{\max}$ production ceases. During that time (t_1, T), the inventory declines due to demand alone. Finally, at the time $t = T$, the inventory gets depleted.

Let *I(t)* be inventory differential equations at time t in $[0, T]$:

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = P - (a - bs) \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = -(a - bs) \qquad t_1 \le t \le T. \tag{2}$$

Using condition

$$I_1(0) = 0 \quad \text{and} \quad I_1(t_1) = I_m$$

in equation (1), we get:

$$I_1(t) = Pt - at + bts \tag{3}$$

$$I_m = t_1(P - a + bs) \tag{4}$$

and using $I_1(T) = 0$ and $I_2(t_1) = I_m$ in equation (2), we get:

$$I_2(t) = (a - bs)(T - t) \tag{5}$$

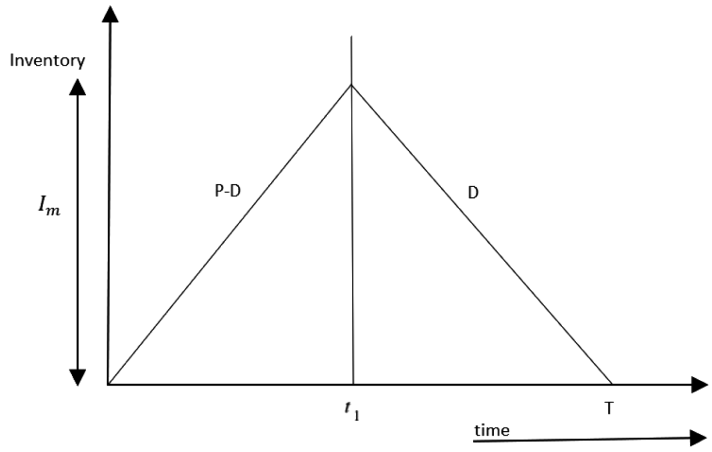

FIGURE 2. Graphical representation of Inventory over time T.

$$I_m = (a - bs)(T - t_1). \tag{6}$$

From equations (4) and (6), we get:

$$t_1 = \frac{T}{P}(a - bs). \tag{7}$$

Total average inventory:

$$\begin{aligned} I_{\text{avg}} &= \int_0^{t_1} I_1(t)\,\mathrm{d}t + \int_{t_1}^{T} I_2(t)\,\mathrm{d}t \left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right) \\ &= (P - (a - bs))\frac{t_1^2}{2} + (a - bs). \end{aligned} \tag{8}$$

The carbon emission in production and inventory holding:

$$\text{CE} = (Pt_1e_2 + e_1 + e_3t_1) + e_4\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}. \tag{9}$$

Now, the total profit of the inventory is given by: Total Profit = "Sales Revenue – Production cost – Set-up cost – Holding cost – Carbon emission due to production – Carbon emission due to holding".

The components of the total profit are calculated as follows:

Sales revenue

A company's sales revenue is the income it receives from the selling of goods or the supply of services. It is given by:

$$R = sTD(s). \tag{10}$$

Production cost

The cost incurred during the production process. It depends on the materials and machining cost, labor and energy cost, and tool/die cost. The production cost is given by:

$$\begin{aligned} \text{PC} &= \left(\mu + \frac{g}{P} + \delta P\right)\int_0^{t_1} P\,\mathrm{d}t \\ &= Pt_1\left(\mu + \frac{g}{P} + \delta P\right). \end{aligned} \tag{11}$$

Set-up cost

Set-up cost is one of the elementary costs for starting a business and continuously running a smooth business. Thus, an investment in the set-up cost is considered to minimize the cost of the initial set-up. The set-up cost of the model is given by:

$$K(\xi) = K_0 e^{-m\xi}. \tag{12}$$

Holding cost

One of the most significant costs associated with proper product storage is the holding cost. As a result, the manufacturer incurs inventory-holding costs to maintain the products. The total cost for holding the product is given by:

$$\mathrm{HC} = h\left\{(P-(a-bs))\frac{t_1^2}{2} + (a-bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}. \tag{13}$$

Carbon emission cost due to production

The manufacturing sector tends to emit carbon emissions into the environment during the production process. Thus, the cost of the carbon emissions due to production is given by:

$$C_1 = e_p(Pt_1e_2 + e_1 + e_3t_1). \tag{14}$$

Carbon emission cost due to holding

When the products are being held in the inventories by the manufacturing sectors, it tends to emit carbon emissions due to the relatively large share of electricity usage (*e.g.*, heating, ventilation, lighting). Hence, carbon emission cost due to the storage of items is given by:

$$C_2 = e_h e_4\left\{(P-(a-bs))\frac{t_1^2}{2} + (a-bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}. \tag{15}$$

Total carbon emission cost is given by

$$\mathrm{CE} = e_p(Pt_1e_2 + e_1 + e_3t_1) + e_he_4\left\{(P-(a-bs))\frac{t_1^2}{2} + (a-bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}. \tag{16}$$

Using equations (10)–(16). The total profit per unit of time is given by:

$$\begin{aligned}\mathrm{TP}(P,s,\xi) = \frac{1}{T}\Bigg[&(sTD(s)) - Pt_1(\mu + \frac{g}{P} + \delta P) - K_0e^{-m\xi} - \xi \\ &- h\left\{(P-(a-bs))\frac{t_1^2}{2} + (a-bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\} - e_p(Pt_1e_2 + e_1 + e_3t_1) \\ &- e_he_4\left\{(P-(a-bs))\frac{t_1^2}{2} + (a-bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}\Bigg].\end{aligned} \tag{17}$$

Lean is a concept that entails a systematic approach to identifying and eliminating waste and optimizing the production rate through continuous improvement. This is one of the most effective ways to improve environmental performance and make it more sustainable. Thus, to address the sustainable nature of the model, it contemplates the effect of carbon emissions on the inventory-production system and investigates two scenarios, *i.e.*, "Carbon cap-and-trade" and "Carbon tax" mechanisms.

Case 1: *Carbon cap-and-trade*

Carbon cap-and-trade is a system that uses a market-based strategy to limit emissions efficiently and cost-effectively by establishing a "cap" on maximum emissions to reduce the emissions.

Thus, the carbon emission cost under the carbon cap-and-trade cost is given as:

$$\text{Cap}^c = C_p(\text{CE} - z).$$

Using the value of CE from equation (16)

$$= C_p\left[e_p(Pt_1e_2 + e_1 + e_3t_1) + e_he_4\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\} - z\right]. \tag{18}$$

Total profit function due to cap-and-trade (TP_1) = "Total sales revenue – Production cost – Set-up cost – Holding cost – cap-and-trade cost".

$$\begin{aligned}\text{TP}_1(P, s, \xi) &= \frac{1}{T}(R - \text{PC} - \text{SC} - \text{HC} - \text{Cap}^c)\\ &= \frac{1}{T}\Big[(sTD(s)) - Pt_1(\mu + \frac{g}{P} + \delta P) - K_0e^{-m\xi} - \xi\\ &\quad - h\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}\\ &\quad - C_p\left[e_p(Pt_1e_2 + e_1 + e_3t_1) + e_he_4\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\} - z\right]\Big].\end{aligned} \tag{19}$$

Case 2: *Carbon tax*

A carbon tax is a retaliation imposed on companies that emit excessive carbon emissions. Typically, the polluters/emitters must pay for each tonne of emissions they generate.

Thus, carbon emission cost under the carbon tax cost is given as:

$$\text{Tax}^c = w(\text{CE}).$$

Using the value of CE from equation (16)

$$= w\left[e_p(Pt_1e_2 + e_1 + e_3t_1) + e_he_4\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}\right]. \tag{20}$$

Total profit function due to a carbon tax (TP_2) = "Total sales revenue – Production cost – Set-up cost – Holding cost – carbon tax cost"

$$\begin{aligned}\text{TP}_2(P, s, \xi) &= \frac{1}{T}(R - \text{PC} - \text{SC} - \text{HC} - \text{Cap}^c)\\ &= \frac{1}{T}\Big[(sTD(s)) - Pt_1\Big(\mu + \frac{g}{P} + \delta P\Big) - K_0e^{-m\xi} - \xi\\ &\quad - h\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}\\ &\quad - w\left[e_p(Pt_1e_2 + e_1 + e_3t_1) + e_he_4\left\{(P - (a - bs))\frac{t_1^2}{2} + (a - bs)\left(\frac{T^2}{2} - Tt_1 + \frac{t_1^2}{2}\right)\right\}\right]\Big].\end{aligned} \tag{21}$$

6. Optimality and solution procedure

The objective of the proposed study is to maximize the total profit by finding the optimal values of the selling price (s), rate of production (P), and investment in set-up cost (ξ). Since the two carbon policies – "Carbon cap-and-trade" and "Carbon tax" – are being compared, distinct cases for each are taken.

Case 1: ***Carbon cap-and-trade***

The necessary conditions to establish the optimality of the total profit function are given:

$$\frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta s}=0,\quad \frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta P}=0,\quad \text{and}\quad \frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta \xi}=0$$

$$\begin{aligned}\frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta s}&=\frac{1}{T}\left[aT-2bsT-\frac{hbt_1^2}{2}+hb\left(T^2-Tt_1+\frac{t_1^2}{2}\right)-\frac{C_pe_he_4bt_1^2}{2}+C_pb\left(T^2-Tt_1-\frac{t_1^2}{2}\right)\right]\\&=0.\end{aligned}\tag{22}$$

The optimal value of s is given as s^*:

$$s^*=\frac{1}{2b}\left[a-\frac{hbt_1^2}{2T}+\frac{hb}{T}\left(T^2-Tt_1+\frac{t_1^2}{2}\right)-\frac{C_pe_he_4bt_1^2}{2T}+\frac{C_pb}{T}\left(T^2-Tt_1-\frac{t_1^2}{2}\right)\right]\tag{23}$$

$$\begin{aligned}\frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta P}&=\frac{1}{T}\left[-\mu t_1-2P\delta t_1-\frac{ht_1^2}{2}-t_1e_pe_2-C_pe_pt_1e_2-\frac{C_pe_he_4t_1^2}{2}\right]\\&=0.\end{aligned}\tag{24}$$

The optimal value of P is given as P^*:

$$P^*=-\frac{1}{2\delta}\left[\mu+e_pe_2+C_pe_pe_2+t_1\left(\frac{h}{2}+\frac{C_pe_he_4}{2}\right)\right]\tag{25}$$

$$\begin{aligned}\frac{\delta \mathrm{TP}_1(s,P,\xi)}{\delta \xi}&=\frac{1}{T}\left[mK_0e^{-m\xi}-1\right]\\&=0.\end{aligned}\tag{26}$$

The optimal value of P is given as P^*:

$$\xi*=\frac{1}{m}\ln(mK_0).\tag{27}$$

Case 2: ***Carbon tax***

The necessary condition to establish the optimality of the total profit function is given:

$$\frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta s}=0,\quad \frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta P}=0,\quad \text{and}\quad \frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta \xi}=0$$

$$\begin{aligned}\frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta s}&=\frac{1}{T}\left[aT-2bsT-\frac{hbt_1^2}{2}+hb\left(T^2-Tt_1+\frac{t_1^2}{2}\right)-\frac{we_he_4bt_1^2}{2}+wb\left(T^2-Tt_1+\frac{t_1^2}{2}\right)\right]\\&=0.\end{aligned}\tag{28}$$

The optimal value of s is given as s^*:

$$s^*=\frac{1}{2b}\left[a-\frac{hbt_1^2}{2T}+\frac{hb}{T}\left(T^2-Tt_1+\frac{t_1^2}{2}\right)-\frac{we_he_4bt_1^2}{2T}+\frac{wb}{T}\left(T^2-Tt_1+\frac{t_1^2}{2}\right)\right]\tag{29}$$

$$\begin{aligned}\frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta P}&=\frac{1}{T}\left[-\mu t_1-2P\delta t_1-\frac{ht_1^2}{2}-t_1e_pe_2-we_pt_1e_2-\frac{we_he_4t_1^2}{2}\right]\\&=0.\end{aligned}\tag{30}$$

The optimal value of P is given as P^*:

$$P^*=-\frac{1}{2\delta}\left[\mu+e_pe_2+we_pe_2+t_1\left(\frac{h}{2}+\frac{we_he_4}{2}\right)\right]\tag{31}$$

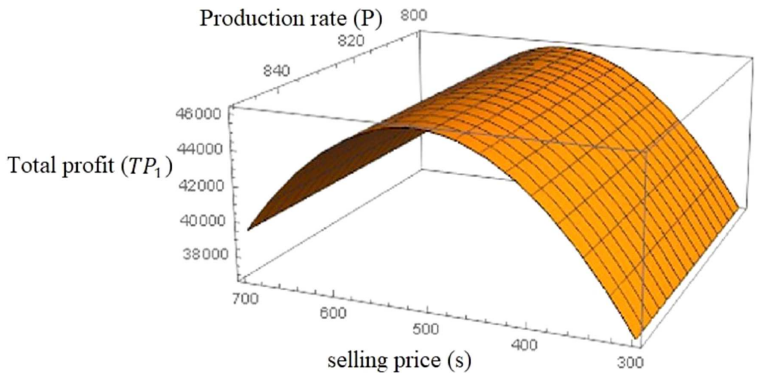

FIGURE 3. Concavity of (TP_1) w.r.t selling price (s) and production rate (P).

$$\begin{aligned}\frac{\delta \mathrm{TP}_2(s,P,\xi)}{\delta \xi} &= \frac{1}{T}\left[mK_0 e^{-m\xi} - 1\right] \\ &= 0. \end{aligned} \tag{32}$$

The optimal value of ξ is given as $\xi*$:

$$\xi^* = \frac{1}{m}\ln(mK_0). \tag{33}$$

The sufficient conditions for maximizing the profit functions (TP_1 and TP_2) are $D_1(s,P,\xi) < 0$, $D_2(s,P,\xi) > 0$, and $D_3(s,P,\xi) < 0$; where D_1, D_2 and D_3 are the minors of the Hessian matrix H. The matrix H and its minors are estimated as follows:

$$H = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi^2} \end{bmatrix},$$

$$\text{and} \quad D_1 = \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2},$$

$$D_2 = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} \end{bmatrix}$$

$$D_3 = \det H = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi^2} \end{bmatrix}.$$

The second-order derivatives for both cases are presented in Appendices A and B respectively. Also, the calculations of the Hessian matrix and sufficient conditions for maximizing the total profit function have been mentioned in Appendix C. Further, the concavity of the total profit function for the cap-and-trade scenario is also established graphically, using Mathematica 11.3, as shown below (Figs. 3–5):

Now, the solution procedure has been illustrated using the following flow chart (see Fig. 6).

7. Numerical analysis

In today's global industrialization, a lean culture is quite prevalent in many industries including Toyota, Intel, and Nike. For the present study, consider an inventory system concerned with the production of automobile

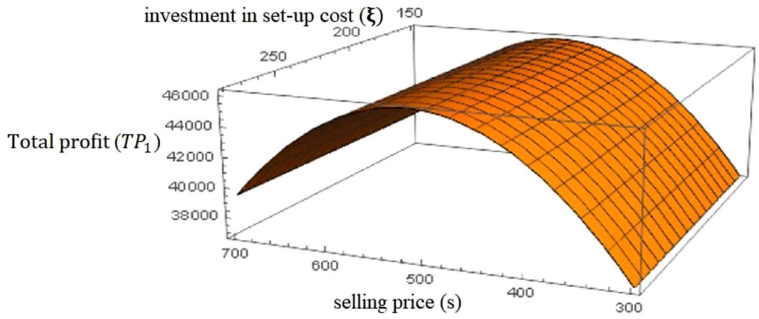

FIGURE 4. Concavity of (TP_1) w.r.t selling price (s) and investment in set-up cost (ξ).

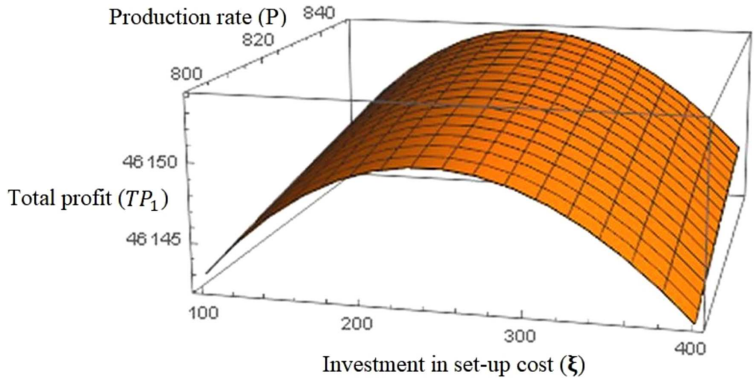

FIGURE 5. Concavity of (TP_1) w.r.t production rate (P) and investment in set-up cost (ξ).

spare parts; for example, car batteries, engines, airbags, fuel pumps, etc., where special attention is required during manufacturing and storage processes. Manufacturing of these units takes a toll on the environment as it emits large amounts of carbon dioxide. Controlling these emissions is one of the major concerns for the industry's practitioners. Some advanced tools and techniques are incorporated into the manufacturing system which enables a reduction of waste and focuses on maximizing productivity. Thus, a flexible production rate is considered, which will help manage overproduction and underproduction. Such technologies are required to help companies achieve and maintain competitive advantages in their industries. Thus, taking into account a hypothetical automobile industry, the following numerical parameters have been considered which are also influenced by the following research papers [21, 30, 66].

7.1. Numerical examples

In this section, two distinct examples for each of the two carbon emission strategies – carbon cap-and-trade and carbon tax – are taken for comparison.

Example 1 (Carbon cap-and-trade case)**.** This illustration takes into account the carbon cap-and-trade case, a system of government regulation created to encourage businesses to cut their carbon emissions. The following parameter values are taken in appropriate units for numerical illustration (Table 2).

Using equations (23), (25), and (27) the optimal policy w.r.t. production rate, selling price, and investment in set-up cost is obtained. Accordingly, the total profit for carbon cap-and-trade is calculated using the equation (19). The optimal results are given in Table 3.

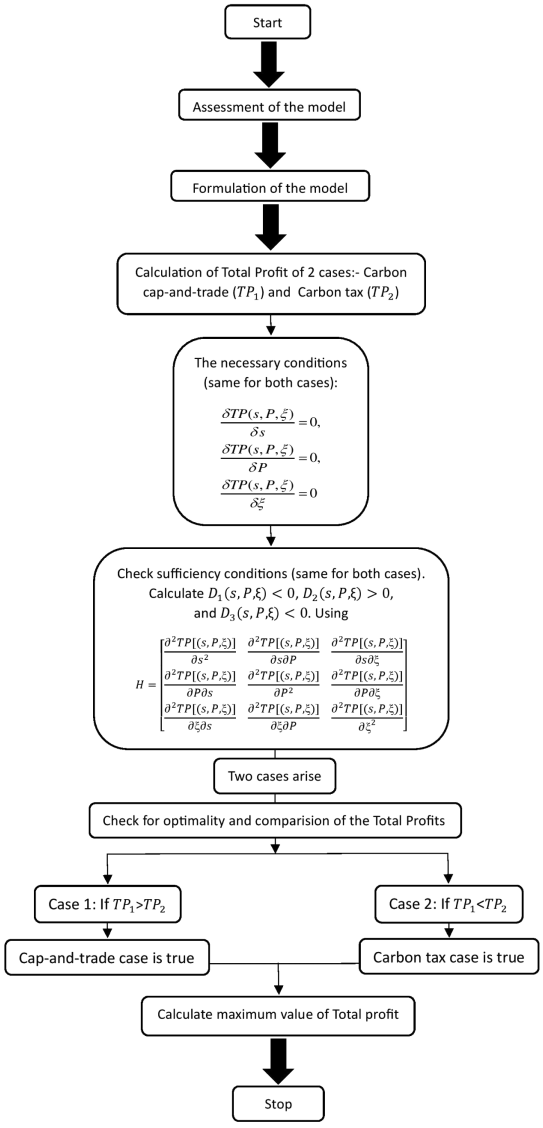

FIGURE 6. Flowchart of the solution procedure of the model.

TABLE 2. Parameters for Carbon cap-and-trade case.

$T = 2$ months	$\delta = \$0.0008$	$e_4 = \$2/\text{kg}$	$c_p = \$1/\text{kg}$
$a = 200$	$K_0 = \$1000/\text{order}$	$e_1 = 2\,\text{kg/time}$	$z = 60\,\text{kg}$
$b = 0.2$	$m = 0.0014$	$e_2 = 1\,\text{kg/time}$	
$\mu = \$25$	$h = \$2/\text{unit/unit time}$	$e_3 = 3\,\text{kg/time}$	
$g = \$1100$	$e_p = \$4/\text{kg}$	$e_4 = 2\,\text{kg/time}$	

TABLE 3. Optimal values for Example 1.

Parameters	Optimal values
Total profit function due to cap-and-trade (TP_1)	\$45 983.93
Production rate (P)	816.52 unit/cycle
Selling price (s)	\$517.80/unit
Investment in set-up cost (ξ)	\$240.34 per production run

TABLE 4. Parameters for Carbon tax case.

$T = 2$ months	$\delta = \$0.0008$	$e_4 = \$2/\text{kg}$	$w = \$1.5/\text{kg}$
$a = 200$	$K_0 = \$1000/\text{order}$	$e_1 = 2\,\text{kg/time}$	
$b = 0.2$	$m = 0.0014$	$e_2 = 1\,\text{kg/time}$	
$\mu = \$25$	$h = \$2/\text{unit/unit time}$	$e_3 = 3\,\text{kg/time}$	
$g = \$1100$	$e_p = \$4/\text{kg}$	$e_4 = 2\,\text{kg/time}$	

TABLE 5. Optimal values for Example 2.

Parameters	Optimal values
Total profit function due to cap-and-trade (TP_2)	\$45 591.15
Production rate (P)	660.20 unit/cycle
Selling price (s)	\$519.45/unit
Investment in set-up cost (ξ)	\$238.66 per production run

Example 2 (Carbon tax case)**.** This scenario shows the carbon tax case which is defined as a charge on carbon emissions that are generated during the production of goods and services. The following parameter values are taken in appropriate units for numerical illustration (Table 4).

From equations (29), (31), and (33) the optimal policy w.r.t. production rate, selling price, and investment in set-up cost is obtained. Accordingly, the total profit for a carbon tax is calculated using the equation (21). The optimal results are given in Table 5.

7.2. Comparative analysis of carbon policies

This model incorporates the concept of lean manufacturing, which focuses on two main concepts- maximizing productivity while minimizing waste. By combining these two concepts with an inventory model under carbon control policies, a sustainable model can be achieved. When a product's demand in the manufacturing sector is price-sensitive, it responds to changes in the selling price. This implies that as an item's selling price falls, its demand rises. As a result, the production rate will increase in tandem to meet the item's rising demand and prevent any potential shortages. Furthermore, it is abundantly clear that a reasonable investment in startup costs is required for a business to run smoothly which will help yield a better profit. Consequently, a comparison of the two policies – carbon cap-and-trade and carbon tax – has been carried out to assess their performance on different fronts. Table 6 offers the most accurate comparison between the two, and added to this, Figure 7 displays a graphical representation of the two case scenarios.

According to the results given in Table 6, the total profit is higher in the "Carbon cap-and-trade" case than in the "Carbon tax" case. Since in the former case, a "cap" has been put on emissions, it is intuitive that it will limit the emissions. Moreover, the company is charged only if it exceeds the prescribed limit, leading to a

TABLE 6. Comparative Analysis of Carbon cap-and-trade and Carbon tax policies.

	Carbon cap-and-trade	Carbon trade
Total profit	\$45 983.93	\$45 591.15
Production rate (P)	816.52 unit/cycle	660.20 unit/cycle
Selling price (s)	\$517.80/unit	\$519.45/unit
Investment in set-up cost (ξ)	\$240.34 per production run	\$238.66 per production run

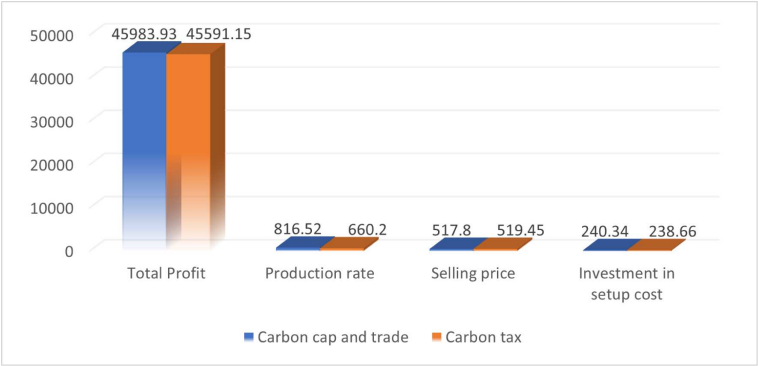

FIGURE 7. Graphical comparative analysis of the two scenarios.

decrease in carbon emissions costs. Thus, the "Carbon cap-and-trade" policy gives better results than that of the latter. It is also noted that this approach suggests a larger investment in set-up costs, resulting in a more efficient startup in each cycle. The firm is further recommended to reduce the selling price in this case, which would help in fetching more demand. It is therefore pragmatic that the production rate will increase in such a situation. Hence, the decision maker must implement a "Carbon cap-and-trade" policy to maximize his profit. The comparative analysis of the two policies – Carbon cap-and-trade and Carbon tax – is given in Table 6.

8. Sensitivity analysis and managerial insights

The sensitivity analysis of the proposed model is crucial for checking the limitations and variations of the system when the important factors and parameters are changed. It is essential to show the effects of varying data on the final objective of the model, *i.e.*, total profit. As the present paper focuses on sustainable lean manufacturing, the parameters considered for the sensitivity analysis are chosen to give centralized control to decision-makers to take care of various considerations.

8.1. Impact of demand and cost parameters on the optimal policy

For any sustainable production-inventory model, the demand and cost parameters (specifically carbon emission cost in production, carbon emission costs in inventory holding, and storage cost) are considered the performance indicators. Thus, in this section the sensitivity of the proposed model has been carried out w.r.t. demand parameters a, and b, carbon emission cost in production e_p, carbon emission costs in inventory holding e_h, and storage cost h. The objective is to highlight certain crucial features of the created model which is given in Table 7.

TABLE 7. Sensitivity analysis of the demand and cost parameters.

Parameter		s	P	ξ	t_1	D	CE cost	TP_1
a	100	268.180	1,020.900	240.272	0.091	46.364	734.068	10 284.112
	125	330.598	973.859	240.237	0.121	58.880	923.059	16 861.871
	150	393.008	924.398	240.309	0.154	71.398	1108.112	25 004.286
	175	455.409	872.212	240.336	0.192	83.918	1288.408	34 711.555
	200	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929
b	0.15	684.457	812.408	240.328	0.240	97.331	1474.891	62 634.763
	0.20	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929
	0.25	417.807	820.590	240.341	0.233	95.548	1450.562	35 999.776
	0.30	351.149	824.746	240.352	0.230	94.655	1438.325	29 348.971
	0.35	303.538	828.726	240.451	0.226	93.762	1426.038	24 602.943
e_p	2.0	516.791	810.980	240.334	0.238	96.642	1072.999	46 179.722
	2.5	517.043	812.376	240.335	0.238	96.591	1170.594	46 130.736
	3.0	517.295	813.750	240.336	0.237	96.541	1268.082	46 081.774
	3.5	517.547	815.136	240.338	0.237	96.491	1365.469	46 032.839
	4.0	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929
e_h	0.5	516.668	1013.656	241.006	0.191	96.666	958.516	46 243.021
	1.0	517.059	952.398	240.821	0.203	96.588	1130.310	46 155.892
	1.5	517.438	887.019	240.391	0.218	96.512	1298.780	46 069.477
	2.0	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929
	2.5	518.135	739.175	240.289	0.261	96.373	1620.192	45 899.469
h	1	517.438	887.363	240.391	0.218	96.512	1470.834	46 069.477
	1.5	517.621	852.510	240.341	0.226	96.476	1466.987	46 026.583
	2	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929
	2.5	517.971	778.912	240.324	0.248	96.406	458.007	45 941.545
	3	518.139	739.417	240.291	0.261	96.372	1452.598	45 899.469

Observations and managerial insights

The following analysis is executed as follows:

(a) As the demand parameter a increases, it is observed the selling price and the demand rate escalates. For the manufacturer to meet the growing demand, he has to increase the production rate and production time. Since the demand is increasing, the production time increases which leads to higher carbon emission costs. However, the overall profit of the system increases.

(b) The price-sensitive parameter b impacts the demand and total profit in an undesirable way. The production rate increases whereas the production time decrease, moreover the carbon emission cost also decreases. For such a scenario, it is suggested to decrease the selling price. Also, the manufacturer should produce bigger lots for a short period.

(c) As the carbon emission cost in production e_p and inventory holding e_h increases, the selling price increases whereas the demand and total profit decrease. Concurrently, an increase in production rate and production time is observed. Obviously, as production increases, carbon emission cost increases in both cases. To avoid emissions during the manufacture and storage of the goods in such a situation, it is vital to monitor the demand and produce only the appropriate amount.

(d) With the increase in the storage cost of the item h, a decline in the total profit is observed. The production rate decreases while the production time increases to manage the high holding costs. Moreover, it is observed that high holding cost has a positive impact on the carbon emissions cost as the total produced lot decreases. In this situation, it is beneficial for the producer to have less inventory on hand to reduce the produced lot's holding costs.

TABLE 8. Sensitivity analysis of the lean manufacturing parameters.

		s	P	ξ	t_1	D	CE cost	TP_1	$K(\xi)$
m	0.0012	517.799	816.511	151.938	0.236	96.440	1462.749	45 968.606	833.330
	0.0013	517.799	816.514	201.820	0.236	96.440	1462.751	45 975.715	769.227
	0.0014	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929	714.284
	0.0015	517.798	816.542	270.314	0.236	96.440	1462.763	45 992.752	666.666
	0.0016	517.798	816.602	293.754	0.236	96.440	1462.771	46 001.864	624.997
μ	23	516.797	815.606	240.336	0.237	96.641	1465.485	46 177.009	714.285
	24	517.298	816.055	240.337	0.237	96.540	1464.119	46 080.419	714.286
	25	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929	714.285
	26	518.300	816.963	240.338	0.236	96.340	1461.385	45 887.539	714.286
	27	518.801	817.429	240.339	0.235	96.240	1460.017	45 791.249	714.286
g	900	517.567	645.277	240.335	0.299	96.487	1439.955	46 010.324	714.286
	950	517.695	735.904	240.336	0.262	96.461	1453.371	45 996.355	714.286
	1000	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929	714.285
	1050	517.886	889.859	240.338	0.217	96.423	1469.776	45 972.626	714.285
	1100	517.964	957.585	240.339	0.201	96.407	1475.284	45 962.188	714.284
δ	0.0004	517.711	1154.604	240.337	0.167	96.458	1488.863	46 020.832	714.285
	0.0006	517.759	942.779	240.337	0.205	96.448	1474.693	46 000.809	714.285
	0.0008	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929	714.285
	0.0010	517.834	730.341	240.338	0.264	96.433	1452.236	45 969.058	714.286
	0.0012	517.866	666.735	240.338	0.289	96.427	1442.734	45 955.614	714.286
K_0	800	517.798	816.474	80.950	0.236	96.440	1462.751	46 063.623	714.287
	900	517.798	816.491	165.078	0.236	96.440	1462.754	46 021.557	714.288
	1000	517.799	816.516	240.338	0.236	96.440	1462.752	45 983.929	714.285
	1100	517.799	816.523	308.410	0.236	96.440	1462.748	45 949.889	714.282
	1200	517.798	816.548	370.566	0.236	96.440	1462.757	45 918.814	714.280

8.2. Impact of lean manufacturing on the production system

The ability to maximize production is one of the fundamental components of lean manufacturing, which is also one of the main goals of this study. In this section, the impact of lean production parameters *viz.*, set-up cost parameter (m), cost of material (μ), labor and energy cost (g), tool/die cost (δ), initial set-up cost (K_0) has been investigated on the optimal policy. The observations are recorded in Table 8.

Discussion: as we can see from Table 8, the production rate somewhat increases when a hike in the value of the parameter is taken into account. An increase in the parameter's value increases the investment in set-up cost, however, it has a positive influence on the total set-up cost. This demonstrates that when an investment in the set-up cost is taken into consideration, the entire set-up cost is reduced, which will improve the efficiency of the businesses. Furthermore, the total profit increases, which benefits industries; thus, investment in set-up costs is recommended.

The entire system costs are impacted by the cost of the production system. With an increase in the cost of the material, the selling price increases slightly. The production rate likewise intensifies which leads to a reduction in the production time. The total profit is noticed to decline in this case since the material cost increases.

When the labor and energy cost increases, an increase in the production rate is observed, with a decrease in the production time. This is quite intuitive since higher energy will amplify and improve the efficacy of the production process. As the intensity of production increases, carbon emission cost increases slightly, thereby impacting the total profit. However, as the tool/die cost increases, the production rate decreases with an increase noticed in the production time.

Lastly, a high initial set-up cost requires a major investment which can improvise the production process. Thus, with an increase in the initial set-up cost, a major change in the investment in the set-up cost is observed which leads to a slight decrease in the total profit.

9. Concluding remarks

9.1. Conclusion

Carbon emissions are being emitted every day in excess due to the expansion of the global economy and the rise in the needs of the manufacturing sector. To reduce carbon footprint, which is waste produced by the manufacturing industries, the current study developed a mathematical model using the idea of lean manufacturing and sustainable practices. The goal is to improve productivity and minimize the waste produced. In light of this, the paper examines a flexible production lot-sizing model with price-dependent demand and investment in set-up cost under the impact of carbon control policies. In the event of price-sensitive demand, determining the right price plays a crucial role in dictating yearly demand. Moreover, lean manufacturing techniques such as advanced labor and energy tools are constantly adopted by manufacturing facilities to increase production efficiency and reduce processing time. Additionally, the model addressed the sustainability concept by considering carbon emissions, and control strategies for the same. Consequently, emissions from production and inventory holding are examined. Further, to reduce harmful emissions, a comparative analysis of the firm's performance under the two regulations, *i.e.*, "carbon cap-and-trade" and "carbon tax" is also carried out. The profit-maximization models are illustrated to optimize production rate, selling price, and set-up cost. Numerical examples have been solved to validate the concept and demonstrate its resilience. According to the findings, companies should implement a "carbon cap-and-trade" policy to increase their profits while also limiting harmful emissions. The key environmental advantage of a "cap-and-trade" over a "carbon tax" is its ability to provide more certainty about how much emissions reductions will be achieved. The company is only liable to pay if it exceeds the prescribed limit, resulting in the reduction of carbon emissions cost. Further, the effect of various model parameters *viz.*, demand, carbon emission cost in production and inventory holding, and storage cost has been studied using sensitivity analysis, which offers crucial managerial insights. Demand parameters a and b have a contrasting impact on total profit: as a increases, the profit rises whereas with an increase in b, the total profit declines. It is observed that with a decrease in carbon emission costs and holding costs, the total profit of the system increases. However, more investment is required in labor and energy costs to improve the efficiency of the production system. Thus, the proposed framework can prove to be beneficial in many manufacturing industries, for instance, textiles and automobiles, where the business strives for maximum productivity with an increased focus on sustainability.

9.2. Limitations and future extensions of the study

The developed model has certain limitations. Demand is considered to be price sensitive. However, the demand can be influenced by various other factors such as advertisements, quality-dependent, environment, etc, [16, 19, 36]. Moreover, the model assumes a constant lead time and no shortage, which may not be the case always. Considering a variable lead time will be a remarkable extension of this study [10, 65]. The proposed study can also be extended for an imperfect production system along with the rework of the defectives [5]. Further, the model can be explored for multi-stage manufacturing [62]. Nowadays to implement sustainability, manufacturing firms are also exploring various product recovery techniques [22] which can also be a significant extension of this study.

Appendix A.

The second order partial derivatives for Case 1: Carbon cap-and-trade:

$$\frac{\delta^2 \mathrm{TP}_1(s, P, \xi)}{\delta s^2} = -2b$$

$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P^2} = -\frac{1}{T}\left(2t_1\left(\delta - \frac{g}{P^2}\right) - \frac{2gt_1}{P^2}\right)$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi^2} = -\frac{1}{T}\left(m^2 K_0 e^{-m\xi}\right).$$

Appendix B.

The second order partial derivatives for Case 2: Carbon tax:

$$\frac{\delta^2 \mathrm{TP}_2(s,P,\xi)}{\delta s^2} = -2b$$
$$\frac{\delta^2 \mathrm{TP}_2(s,P,\xi)}{\delta P^2} = -\frac{1}{T}\left(2t_1\left(\delta - \frac{g}{P^2}\right) - \frac{2gt_1}{P^2}\right)$$
$$\frac{\delta^2 \mathrm{TP}_2(s,P,\xi)}{\delta \xi^2} = -\frac{1}{T}\left(m^2 K_0 e^{-m\xi}\right).$$

Appendix C.

Case 1. Cap *and trade.*

The Hessian matrix H is given as:

$$H = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi^2} \end{bmatrix},$$

$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta s^2} = -2b$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P^2} = -\frac{1}{T}\left(2t_1\left(\delta - \frac{g}{P^2}\right) - \frac{2gt_1}{P^2}\right)$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi^2} = -\frac{1}{T}\left(m^2 K_0 e^{-m\xi}\right)$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P \delta s} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta s \delta P} = 0$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi \delta P} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P \delta \xi} = 0$$
$$\frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta s \delta \xi} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi \delta s} = 0.$$

For sufficient conditions of optimality D_1, D_2 and D_3 are evaluated as follows:

$$D_1 = \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2}$$
$$\mathrm{Det}(D_1) = -2b < 0$$
$$D_2 = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} \end{bmatrix}$$

$$X1 = \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} = -2b < 0$$

$$X2 = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P^2} = -\frac{1}{T}\left(2t_1\left(\delta - \frac{g}{P^2}\right) - \frac{2gt_1}{P^2}\right) < 0$$

$$X3 = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P \delta s} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta s \delta P}$$

$$= 0 \geq 0$$

$$\mathrm{Det}(D_2) = (X1xX2) - (X3)^2 > 0$$

$$D_3 = \det\ H = \begin{bmatrix} \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta s \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P^2} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta P \delta \xi} \\ \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta s} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi \delta P} & \frac{\delta^2 \mathrm{TP}_1[(s,P,\xi)]}{\delta \xi^2} \end{bmatrix}$$

$$Y1 = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi^2} = -\frac{1}{T}(m^2 K_0 e^{-m\xi}) < 0$$

$$Y2 = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi \delta P} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta P \delta \xi}$$

$$= 0 \geq 0$$

$$Y3 = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta s \delta \xi} = \frac{\delta^2 \mathrm{TP}_1(s,P,\xi)}{\delta \xi \delta s}$$

$$= 0 \leq 0$$

$$\mathrm{Det}(D_3) = X1((X2 \times Y1) - (Y1)^2) - X3((X3 \times Y1) - (Y2 \times Y3)) + Y3((X3 \times Y2) - (Y3 \times X2)) \leq 0.$$

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