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MARGINAL-UTILITY-ORIENTED OPTIMIZATION MODEL FOR COLLABORATIVE MEDICAL SUPPLY REBALANCING AND ALLOCATING IN RESPONSE TO EPIDEMICS

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Abstract. Large-scale epidemics impose significant burdens globally and cause an imbalance of medical supplies among different regions owing to the dissimilarly and unevenly distributed prevalence of the infection. Along with rebalancing the limited medical supplies to meet the demand and supply requirements, ensuring that the supplies are allocated to support the affected regions is also important. Hence, this study focuses on the collaborative medical supply rebalancing and allocating process to balance the demand and supply. The law of diminishing marginal utility is incorporated in this study to quantify the principle of fairness in rebalancing and allocating medical supplies. Accordingly, under uncertainty, a marginal-utility-oriented optimization model is proposed to formulate the rebalancing and allocation of collaborative medical supplies. Because the proposed model is nonlinear and computationally intractable, a linearization approach is adopted to obtain the global optimum that supports decision-making in response to epidemics. Furthermore, a real case study of the United States is implemented, where the sensitivity analysis of critical parameters is conducted on the coronavirus disease 2019. Computational results indicate that additional medical supplies, stock levels, and scenario constructions significantly influence the supply/demand point identification and outgoing/incoming shipments. Moreover, this study not only validates the effectiveness and feasibility of the method but also highlights the importance of incorporating the law of diminishing marginal utility into the collaborative medical supply rebalancing and allocating problem.

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1. Introduction

Large-scale disasters are associated with human health and mortality. Infectious diseases in densely populated areas continue to impose a significant health concern, particularly in resource-constrained settings. Coronavirus Disease 2019 (COVID-19) is one of the most majorly impacting infectious diseases in recent years. Based

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on the situation reports from the World Health Organization (WHO), by July 2020, more than 10 million COVID-19 cases were reported worldwide. COVID-19 affected more than 200 countries and was considered a global pandemic due to its extremely high transmission rate. In the early stages, the spread of the virus was accelerated owing to mutual exposure, shortage of detection reagents, and partial understanding of the virus, resulting in massive demand for daily household items and medical supplies [9–11, 24]. Continuous provision of medical supplies is critical in recovering from severely infectious diseases. The shortage of medical supplies due to reduced supply or production capacity reduces the success rate of treatment while increasing the risk of infection among medical workers, directly slowing the progress of epidemic prevention and control. Therefore, allocating the limited medical supplies efficiently to meet the enormous treatment, protection, and control demands is important [50].

The prominent characteristic of COVID-19 is the multiregional outbreak. Moreover, disease cases are unevenly distributed among different regions owing to the dissimilar prevalence of the infection. Therefore, some regions face strict medical supply shortages than others. The growing mismatch between supply and demand can increase mortality rates. A medical supply rebalancing strategy such as that suggested by Gao [18] provides a significant solution to relieve the detrimental situations caused by disasters as it allocates the right medical supplies to the right place in the right quantities. In addition, allocating medical supplies from the suppliers to the affected regions is also integrated into the rebalancing process, further improving medical service, reducing waste and costs, and increasing operational efficiency. To guild these relief operations, the law of diminishing marginal utility, a basic theory in modern economics, can be applied to consumption and other human behaviors [35, 37]. Consequently, the law of diminishing marginal utility is highly correlated with the satisfaction of infected patients and medical staff in the context of epidemics. However, applying the law of diminishing marginal utility to measure fairness involved in COVID-19 medical supply rebalancing activities is still in its infancy in humanitarian logistics.

In addition, the inherent uncertainty in data is a prominent characteristic of COVID-19. Particularly, the characteristics of the virus, such as transmissibility, severity, and mode of transmission, are uncertain at the beginning of a pandemic. Besides, the effectiveness of public health measures, such as social distancing, mask-wearing, and vaccination, can be uncertain, particularly if the measures are new. Moreover, the behavior of individuals and communities is unpredictable. By handling uncertain information using scenario planning and modeling, decision-makers can anticipate potential outcomes and develop contingency plans that can be adapted as new information becomes available for preventing and controlling pandemics. With the uncertain information, *it is an indispensable and urgent task for governments to coordinate closely with hospitals, companies, donation centers, and non-profit institutions to rebalance and allocate medical supplies such that the supply meets the demand at each location.*

Given the above challenges, this study focuses on rebalancing and allocating medical supplies among different regions with the help of companies, donation centers, and non-profit institutions. Given known stock levels of prepared medical supplies in hospitals, hospitals with slight shortages can share their supplies with hospitals facing serious shortages. Further, medical supplies from companies, donation centers, and non-profit institutions can also be strategically allocated in a rebalancing process to support hospitals with heavy shortages. This process, called the "collaborative medical supply rebalancing and allocating problem," is a pressing task in humanitarian logistics. Thus, we consider the following critical questions that were overlooked in previous studies:

(1) How can fairness be quantified in the collaborative medical supply rebalancing and allocating problem?
(2) How can we rebalance medical supplies among different regions through companies, donation centers, and non-profit institutions?
(3) How can we formulate and solve the collaborative medical supply rebalancing and allocating problem?

In light of the above questions, this study makes three main contributions. First, differing from the previous studies [18, 19, 21], to the best of our knowledge, this is the first study to focus on the collaborative medical supply rebalancing and allocating problem by maximizing total marginal utility (*i.e.*, customer satisfaction) with the help of companies, donation centers, and non-profit institutions. Second, extending the previous multi-commodity rebalancing problems defined by Gao [18], we propose a marginal-utility-oriented optimization (MUOO) model to formulate the collaborative medical supply rebalancing and allocating problems under uncertainty. In addition, a linearization method to reformulate the MUOO model helped determine the global optimum, overcoming the nonlinearity of the MUOO model. Finally, a numerical analysis using a real dataset from the United States illustrated the effectiveness of the proposed method. The study provides new managerial and actionable insights from theory and practice. For example, the study results suggest that the partners should collaborate to fight against the pandemic.

The remainder of this paper is organized as follows. Section 2 reviews previous relevant studies focused on this issue and highlights the novelty of this study. Section 3 describes the collaborative medical supply rebalancing and allocating problem under uncertainty. Based on the law of diminishing marginal utility, a MUOO model is proposed, and a linearization method is developed in Section 4. Then, a real case study of the United States is provided, and the results are obtained and discussed in Section 5. Finally, Section 6 concludes this study with contributions and managerial insights from theory and practice, outlining possible directions for further work.

2. Literature review

Based on the main concerns in the collaborative rebalancing and allocating of medical supplies, we position this study in the following three streams of the literature.

2.1. Collaborative medical supply rebalancing and allocating

Recently, many studies have been dedicated to designing the pre-positioning network and inventory strategy before disasters [1,17,19,36] and the distribution of relief commodities after disasters [2,6,14]. Although many research efforts have been dedicated to humanitarian logistics, issues related to virus outbreaks, such as COVID-19, are receiving increased attention [32,45,46]. Owing to the high unpredictability of COVID-19, the mismatch between the medical supply and demand has increased, increasing the mortality rate. Contrary to previous studies, this work aims to rebalance and allocate medical supplies to solve the mismatch between supply and demand in response to the epidemic. Two main tasks can be performed to relieve supply shortages. The first involves rebalancing the limited medical supplies among hospitals to meet the most urgent needs in the context of limited production capacity from suppliers. The second involves allocating the limited resources of medical supplies from companies, donation centers, and non-profit institutions.

2.2. Uncertain elements in humanitarian logistics

Previous studies on humanitarian logistics have considered numerous uncertain elements to reflect disaster environments accurately. Among these, demand and supply are inevitably uncertain elements. Mete and Zabinsky [33] proposed a mixed-integer programming model for distributing medical supplies under demand uncertainty. Erbeyoğlu and Bilge [17] also presented a mixed-integer programming model to address a humanitarian network design problem under demand uncertainty. Gao [18] proposed a mixed-integer nonlinear programming model for a commodity rebalancing problem considering uncertain demand and road availability. Davis *et al.* [13] proposed a mixed-integer programming model to solve the commodity distribution problem under three uncertain elements (*i.e.*, demand, supply, and transportation network). Safaei *et al.* [41] also considered uncertainty in demand and supply to propose a mixed-integer nonlinear programming model when designing a supply distribution relief network. Liu *et al.* [30] and Balcik *et al.* [4] considered the uncertain demand and supply in different problems in humanitarian logistics. Gao *et al.* [21] considered uncertain demand, supply, and road availability in a multi-commodity rebalancing problem with combined distances over a multimodal transportation system. Undoubtedly, these uncertainties make decision-making challenging in humanitarian logistics. Tirkolaee *et al.* [44] proposed a novel mixed-integer linear programming model to formulate the sustainable multi-trip location-routing problem with time windows for medical waste management during COVID-19 under uncertainty. Goli *et al.* [24] proposed a mathematical formulation and solution method to optimize organ transplant supply chains under shipment time uncertainty.

2.3. Measurements of fairness in humanitarian logistics

In contrast to the monetary objectives in business logistics, humanitarian logistics consider fairness as the first goal in guiding relief operations [7,20]. Several methods have been proposed to measure fairness in humanitarian logistics. Fairness is quantified as the penalty cost of unsatisfied demand at demand points [3, 29, 38] owing to the uneven distribution of commodities. Rivera-Royero *et al.* [39] measured fairness and human suffering level by considering the proportion of unmet demand with priority scores at each demand point. Gao [18] used the weighted unsatisfied demand while rebalancing commodities among relief centers to quantify fairness. Gao and Cao [20] measured fairness by minimizing the total weighted unmet demand at relief centers.

In addition to penalty costs, the law of diminishing marginal utility is applied to quantify customer satisfaction. According to the law of diminishing marginal utility, marginal utilities diminish across the ranges relevant to decision-making [25], where the first consumed unit of a good or service yields more utility than the second and subsequent units. As the basic theory in modern economics, the law of diminishing marginal utility can be applied to consumption and other human behaviors [35,37]. Consequently, the law of diminishing marginal utility is highly correlated with the satisfaction of infected patients and medical staff in the context of epidemics. However, its application to measure fairness in humanitarian logistics is still in its infancy. Therefore, we consider the law of diminishing marginal utility to measure fairness in rebalancing and allocating medical supplies in response to epidemics.

2.4. Contributions of the study

The differences between the current and previous studies are outlined in Table 1. Most researchers have focused on relief operations after non-pandemic disasters, but relief efforts in response to pandemics are rare. Existing literature concentrates on facility locations and relief distribution-related issues, whereas our study mainly focuses on solving the mismatch between supply and demand by rebalancing and allocating medical supplies among different regions under uncertainty in the context of epidemics. Moreover, previous studies applied unmet demand-based fairness measurements such that each unit of medical supplies yields the same utility. However, this method is unreasonable because initial units of medical supplies have more utility than latter units. Hence, this study emphasized the marginal utility caused by different units of medical supplies and proposed the MUOO model. The proposed mathematical optimization model incorporates increased parameters, making it different from methods presented in earlier studies. Furthermore, various heuristics approaches have been developed to handle the nonlinear optimization model; however, obtaining the global optimum is challenging. We overcome the nonlinearity of the proposed MUOO model by developing a linearization approach to determine the global optimum within a limited computation time.

We emphasize the novelties of this study as follows. First, to the best of our knowledge, our study is the first to optimize the collaborative medical supply rebalancing and allocating problem with uncertain information for successfully containing a pandemic. Second, this study defines customer satisfaction to measure fairness based on the law of diminishing marginal utility. Third, to determine the optimal medical supply rebalancing and allocation scheme, the collaborative medical supply rebalancing and allocating problem is formulated as the MUOO model and then reformulated as a linear optimization model to determine the global optimum. Finally, a numerical analysis based on a real dataset from the United States is implemented to illustrate the effectiveness of the proposed method. The analyses of the case in rebalancing and allocating medical supplies have resulted in some new managerial and actionable theoretical and strategic insights.

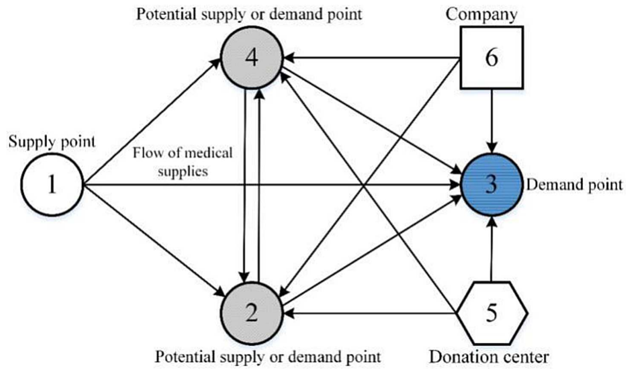


FIGURE 1. Illustration of the collaborative medical supply rebalancing and allocating problem.

3. Problem description

The impacts caused by epidemics are different from other disasters and are characterized by multiregional outbreaks, dynamic and uncertain information, and uneven distribution of infected patients). Moreover, the uneven prevalence of infection contributes to the rapidly growing imbalance between supply and demand for medical supplies in many countries [16]. Some of the worst-hit regions of COVID-19 (*e.g.*, New York in the United States, Lombardia in Italy, and Wuhan in China) had faced serious shortages of medical supplies (such as masks, protection suits, gloves, goggles, ventilators, and testing kits). A lack of medical supplies undoubtedly increased the potential risk of infection among medical staff. For example, by May 2020, the number of infected medical staff members in Italy was more than twice that in China. More seriously, the shortage of medical supplies may also lead to the collapse of the hospital system. For example, South Korea faced a bed shortage, resulting in some COVID-19 patients staying home while awaiting admission in early 2020. By May 2020, New York, with more infected patients, confronted severe shortages of facilities, such as beds, masks, ventilators, and testing kits, resulting in a higher number of deaths compared to other states.

Consequently, allocating medical supplies from the non-affected to the worst-hit areas is necessary to meet the most urgent needs. Hospitals in the unaffected regions can overcome adverse conditions and transfer essential medical supplies to support hospitals in the worst-hit and moderately affected regions. The hospitals in moderately affected regions can also share medical supplies with hospitals in the worst-hit regions. Moreover, involved organizations, including companies, donation centers, and non-profit institutions, can allocate medical supplies to hospitals in the affected regions.

The collaborative medical supply rebalancing and allocating problem, comprising four categories of regions and other organizations such as companies, donation centers, and non-profit institutions, is shown in Figure 1. The non-profit institutions can be considered donation centers. In addition to rebalancing medical supplies among hospitals, companies, and donation centers provide medical supplies to hospitals.

As shown in Figure 1, there are three regions of interest: (i) supply regions (point 1), (ii) demand regions (point 3), and (iii) potential demand or supply regions (points 2 and 4). The medical supplies at points 1, 2, and 4 can be transferred to support point 3. Further, potential demand or supply points 2 and 4 can support each other. Moreover, companies and donation centers can share medical supplies with points 2, 3, and 4. Thus, the main interdependent activity is determining the incoming and outgoing shipments of medical supplies so that the medical supplies can best match the demand in each region for a fair measurement.

As mentioned in Section 1, the demand for medical supplies during an epidemic is uncertain because predicting the number of infected cases is challenging. Undoubtedly, uncertain demand increases the complexity of collaborative medical supply rebalancing and allocating problem. Moreover, uncertain demand increases the uncertainty regarding supply in the potential demand or supply points, where supply is the difference value of

TABLE 1. Summary of the literature on humanitarian operations.

Article	Main problem	Fairness	Law of diminishing marginal utility	Virus outbreak	Approach
Elci and Noyan [15]	Humanitarian relief network design	No	No	No	Exact
Loree and Aros-Vera [31]	Facility location and inventory allocation	No	No	No	Heuristic
Stauffer *et al.* [42]	Asset supply network design	No	No	No	Exact
Liu *et al.* [30]	Relief distribution	No	No	No	Exact
Balcik *et al.* [4]	Pre-positioning network design	Yes	No	No	Exact
Arnette and Zobel [1]	Relief asset pre-positioning	Yes	No	No	Exact
Gao [18]	Commodity rebalancing	Yes	No	No	Exact
Gao and Cao [20].	Commodity rebalancing and transportation	Yes	No	No	Exact
Wei *et al.* [49]	Location-routing problem with relief distribution	No	No	No	Heuristic
Erbeyoğlu and Bilge [17]	Humanitarian network design	No	No	No	Exact
Briskorn *et al.* [5]	Road clearance and relief distribution	No	No	No	Exact
Haeri *et al.* [26]	Pre-positioning and distribution network design	No	No	No	Exact
Mohammadi *et al.* [34]	Humanitarian relief chain	Yes	No	No	Exact
Gao *et al.* [21]	Commodity rebalancing	No	No	No	Exact
Jamali *et al.* [28]	Sustainable humanitarian logistics	No	No	No	Exact
Cao *et al.* [8]	Relief distribution	No	No	No	Exact
Tirkolaee *et al.* [44]	Multi-trip location-routing problem	No	No	Yes	Exact
Sun *et al.* [43]	Location-transportation-allocation problem	No	No	No	Exact
Cao *et al.* [10]	Medical waste location-transport	No	No	Yes	Exact
Lotfi *et al.* [32]	Confirmed infection prediction	No	No	Yes	Exact
Gao *et al.* [24]	Medical staff rebalancing	Yes	Yes	Yes	Exact
Cao *et al.* [11]	Medical waste transportation	No	No	Yes	Exact
This study	Collaborative medical supply rebalancing and allocating	Yes	Yes	Yes	Exact

the stock level and uncertain demand. Generally, uncertain elements are represented as continuous or discrete events. Considering discrete numbers of infected patients (*i.e.*, discrete events), this study used a scenario-based approach to represent uncertain elements. The demand for medical supplies is classified into several independent scenarios, where a specific realization of the possible demand quantity is considered a scenario.

Before presenting the method for rebalancing and allocating collaborative medical supplies, assumptions used to derive the model are stated. (1) Each region is a separate unit. (2) Centralized decisions can be made for the regions and entities. (3) Utility functions are known in each region. (4) The number of infected cases can be estimated for the following two weeks.

4. Mathematical optimization model

4.1. Notations

The notations and decision variables used in the proposed mathematical model are as follows.

Sets

$\mathscr{S}$ Set of supply regions, indexed by $s \in \mathscr{S}$
$\mathscr{D}$ Set of demand regions, indexed by $d \in \mathscr{D}$
$\mathscr{H}$ Set of potential demand or supply regions, indexed by $h \in \mathscr{H}$
$\mathscr{C}$ Set of suppliers (*i.e.*, companies and donation centers), indexed by $c \in \mathscr{C}$
$\mathscr{Q}$ Set of medical supplies or items, indexed by $q \in \mathscr{Q}$
Ξ Set of scenarios, indexed by $\xi \in \Xi$

Parameters

L_{qs} Stock level of medical item q at supply region s
L_{qd} Stock level of medical item q at demand region d
L_{qh} Stock level of medical item q at potential demand or supply region h
O_{qc} Supply of medical item q at supplier c
$M_{qd\xi}$ Demand for medical item q at demand region d in scenario ξ
$M_{qh\xi}$ Demand for medical item q at potential demand or supply region h in scenario ξ
P_{ξ} Probability of occurrence in scenario ξ

Decision variables

a_{qs} Quantity of delivered medical item q from supply region s
b_{qd} Quantity of received medical item q at demand region d
k^{-}_{qh} Quantity of delivered medical item q from potential demand or supply region h
k^{+}_{qh} Quantity of received medical item q at potential demand or supply region h
e_{qc} Quantity of delivered medical item q from supplier c

4.2. Objective function

To maximize fairness in rebalancing and allocating medical supplies, the law of diminishing marginal utility is considered to measure fairness in this study. As a basic theory in modern economics, the law of diminishing marginal utility can be applied not only to consumption behavior but also to other behaviors of human beings [35,37]. In this sense, the law of diminishing marginal utility is highly correlated with the satisfaction of infected cases.

With the quantity of received medical supplies x, the marginal and total utilities are functions of x and are denoted by $\mathbb{M}(x)$ and $\mathbb{T}(x)$, respectively. The marginal utility value decreases from positive to negative when x increases. Nevertheless, the total utility increases first and then decreases when the marginal utility value

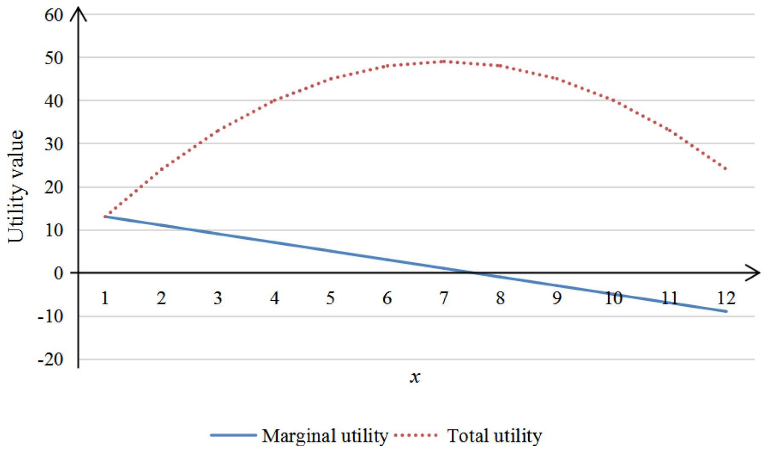


FIGURE 2. An illustrative example of marginal and total utility functions (*i.e.*, $\mathbb{M}(x)$ and $\mathbb{T}(x)$).

changes from positive to negative. In this sense, the change in the total utility (*i.e.*, $\mathbb{T}$) can be quantified by increasing the quantity of medical supplies from state x_0 to state x_1 ($x_1 > x_0$), which is given by

$$\Delta\mathbb{T} = \mathbb{T}(x_1) - \mathbb{T}(x_0) = \int_{x_0}^{x_1} \mathbb{M}(x)\mathrm{d}x \tag{1}$$

In addition to the above formulation in (1), this study also provides an illustrative example to present the general form of marginal and total utility functions (Fig. 2). As shown in Figure 2, given a marginal utility function $\mathbb{M}(x)$, the total utility function $\mathbb{T}(x)$ value increases first and then decreases when the marginal utility function $\mathbb{M}(x)$ value changes from positive to negative.

Because this study considers three categories of regions, three cases (*i.e.*, I, II, and III) are used to present their possible changes in the total utilities, where the existing quantity of medical supplies is denoted by x_0. The quantities of received and sent medical supplies are denoted by a and b, respectively.

(1) Case I

Hospitals in the worst-hit regions need to receive medical supplies so that the total utility can be increased. Thus, the change in the total utility with increasing the number of medical supplies from x_0 to $(x_0 + a)$ is given by

$$\Delta\mathbb{T} = \mathbb{T}(x_0 + a) - \mathbb{T}(x_0) = \int_{x_0}^{x_0+a} \mathbb{M}(x)\ \mathrm{d}x > 0. \tag{2}$$

(2) Case II

However, in the unaffected regions, medical supplies can be sent to the worst-hit and moderately affected regions. Accordingly, the total utility decreases when the quantity of medical supplies decreases from x_0 to $(x_0 - b)$, which is given by

$$\Delta\mathbb{T} = \mathbb{T}(x_0 - b) - \mathbb{T}(x_0) = \int_{x_0}^{x_0-b} \mathbb{M}(x)\ \mathrm{d}x < 0. \tag{3}$$

(3) Case III

Different from Cases I and II, the hospitals in the moderately affected regions have two possible strategies (*i.e.*, sending or receiving medical supplies), which are given by

$$\begin{aligned}
&\Delta\mathbb{T} = \mathbb{T}(x_0 + a) - \mathbb{T}(x_0) = \int_{x_0}^{x_0+a} \mathbb{M}(x)\,\mathrm{d}x > 0 \\
&\text{or} \\
&\Delta\mathbb{T} = \mathbb{T}(x_0 - b) - \mathbb{T}(x_0) = \int_{x_0}^{x_0-b} \mathbb{M}(x)\,\mathrm{d}x < 0
\end{aligned} \tag{4}$$

Note that different marginal and total utility functions are considered in different regions. And these marginal and total utility functions reflect their urgent demand for medical supplies, which makes sure the available medical supplies going to the right places. Besides, this study proposes a stochastic optimization model to formulate the collaborative medical supply rebalancing and allocating problem with uncertain information. Moreover, the proposed model can be continuously applied so that the quantities of sending and receiving medical supplies can be updated in different periods.

In terms of the above fairness measurement, each region should decide whether to send or receive medical supplies for the objective function.

(i) Group of supply regions

In the supply regions, medical supplies can be shared with other regions because there are no infected cases. Let $\mathbb{M}_{qs}(x)$ be the marginal utility function of the medical item q in region s. Thus, with the outgoing quantity of medical item q, denoted by a_{qs}, the lost utility in supply region s, denoted by V_{qs}, is given by

$$V_{qs} = \int_{L_{qs}}^{L_{qs}-a_{qs}} \mathbb{M}_{qs}(x)\,\mathrm{d}(x). \tag{5}$$

(ii) Group of demand regions

In demand regions, the key to estimating the required medical supplies is based on the uncertain number of undiagnosed cases. A large number of infected cases make the medical supplies in region d seriously insufficient, which leads to this region receiving medical supplies. To guarantee fairness, we consider satisfaction based on the marginal utility function. Let $\mathbb{M}_{qd\xi}(x)$ be the marginal utility function of medical item q in region d in scenario ξ, where x denotes the quantity of medical supplies. Thus, with the estimated incoming quantity of medical item q in region d in scenario ξ, denoted by b_{qd}, the increased utility, denoted by $V_{qd\xi}$, is given by

$$V_{qd\xi} = \int_{L_{qd}}^{L_{qd}+b_{qd}} \mathbb{M}_{qd\xi}(x)\,\mathrm{d}(x). \tag{6}$$

Then, the expected increased utility caused by receiving a quantity of medical item q in region d over all scenarios is given by

$$\mathrm{E}(V_{qd\xi}, \xi) = \sum_{\xi\in\Xi} P_\xi \int_{L_{qd}}^{L_{qd}+b_{qd}} \mathbb{M}_{qd\xi}(x)\,\mathrm{d}(x). \tag{7}$$

(iii) Potential demand or supply regions.

In the moderately affected regions, the number of infected cases is not large compared with that in the worst-hit regions. Region h can be chosen to share medical supplies with the worst-hit regions or receive medical supplies from the non-affected regions. Let $\mathbb{M}_{qh\xi}(x)$ be the marginal utility function of medical item q in region h in scenario ξ, where x denotes the quantity of medical supplies. Thus, with the outgoing and incoming quantities of medical item q in region h in scenario ξ, denoted by k^-_{qh} and k^+_{qh}, the increased utility, denoted by $V_{qh\xi}$, is given by

$$V_{qh\xi} = \int_{L_{qh}}^{L_{qh}+k^+_{qh}-k^-_{qh}} \mathbb{M}_{qh\xi}(x)\,\mathrm{d}(x). \tag{8}$$

Then, the expected lost or gained utility caused by sharing or receiving medical item q in region h over all scenarios is given by

$$\mathrm{E}(V_{qh\xi},\xi) = \sum_{\xi\in\Xi} P_\xi \int_{L_{qh}}^{L_{qh}+k^+_{qh}-k^-_{qh}} \mathrm{M}_{qh\xi}(x)\,\mathrm{d}(x). \tag{9}$$

Then, the objective function in rebalancing and allocating medical supplies among all regions is given by

$$\begin{aligned}\Psi = {} & \sum_{q\in\mathcal{Q}}\sum_{s\in\mathcal{S}} \int_{L_{qs}}^{L_{qs}-a_{qs}} \mathbb{M}_{qs}(x)\,\mathrm{d}(x) + \sum_{q\in\mathcal{Q}}\sum_{d\in\mathcal{D}}\sum_{\xi\in\Xi} P_\xi \int_{L_{qd}}^{L_{qd}+b_{qd}} \mathbb{M}_{qd\xi}(x)\,\mathrm{d}(x) \\ & + \sum_{q\in\mathcal{Q}}\sum_{h\in\mathscr{H}}\sum_{\xi\in\Xi} P_\xi \int_{L_{qh}}^{L_{qh}+k^+_{qh}-k^-_{qh}} \mathbb{M}_{qh\xi}(x)\,\mathrm{d}(x).\end{aligned} \tag{10}$$

4.3. Marginal-utility-oriented optimization (MUOO) model

This collaborative medical supply rebalancing and allocating problem can be formulated as MUOO model $\mathscr{W}$.

$$\begin{aligned}&\mathscr{W}: \\ &\text{Max } \Psi = \sum_{q\in\mathcal{Q}}\sum_{s\in\mathcal{S}} \int_{L_{qs}}^{L_{qs}-a_{qs}} \mathrm{M}_{qs}(x)\,\mathrm{d}(x) + \sum_{q\in\mathcal{Q}}\sum_{d\in\mathcal{D}}\sum_{\xi\in\Xi} P_\xi \int_{L_{qd}}^{L_{qd}+b_{qd}} \mathrm{M}_{qd\xi}(x)\,\mathrm{d}(x) \\ &\qquad + \sum_{q\in\mathcal{Q}}\sum_{h\in\mathscr{H}}\sum_{\xi\in\Xi} P_\xi \int_{L_{qh}}^{L_{qh}+k^+_{qh}-k^-_{qh}} \mathrm{M}_{qh\xi}(x)\,\mathrm{d}(x)\end{aligned} \tag{11}$$

s.t.

$$\sum_{s\in\mathcal{S}} a_{qs} + \sum_{h\in\mathscr{H}} k^-_{qh} + \sum_{c\in\mathscr{C}} e_{qc} = \sum_{h\in\mathscr{H}} k^+_{qh} + \sum_{d\in\mathcal{D}} b_{qd} \qquad \forall\, q\in\mathcal{Q}, \tag{12}$$

$$a_{qs} \le L_{qs} \qquad \forall\, q\in\mathcal{Q},\ s\in\mathcal{S}, \tag{13}$$

$$k^-_{qh} \le L_{qh} \qquad \forall\, q\in\mathcal{Q},\ h\in\mathscr{H}, \tag{14}$$

$$L_{qh} + k^+_{qh} \le \max_\xi M_{qh\xi} \qquad \forall\, q\in\mathcal{Q},\ h\in\mathscr{H},\ \xi\in\Xi, \tag{15}$$

$$L_{qd} + b_{qd} \le \max_\xi M_{qd\xi} \qquad \forall\, q\in\mathcal{Q},\ d\in\mathcal{D},\ \xi\in\Xi, \tag{16}$$

$$e_{qc} \le O_{qc} \qquad \forall\, q\in\mathcal{Q},\ c\in\mathscr{C}, \tag{17}$$

$$k^+_{qh}k^-_{qh} = 0 \qquad \forall\, q\in\mathcal{Q},\ h\in\mathscr{H}, \tag{18}$$

$$a_{qs},\ b_{qd},\ k^+_{qh},\ k^-_{qh},\ \text{and } e_{qc} \text{are non-negative variables} \qquad \forall\, q\in\mathcal{Q},\ s\in\mathcal{S},\ d\in\mathcal{D},\ h\in\mathscr{H},\ c\in\mathscr{C}. \tag{19}$$

Objective function (11) maximizes the expected total utility by rebalancing and allocating medical supplies among all regions. Constraint (12) ensures the balance between outgoing and incoming medical supplies. Constraint (13) ensures that the quantity of outgoing medical supplies is not greater than the stock level in region s. Constraints (14) and (15) guarantee that the quantity of outgoing medical supplies is not greater than the stock level and maximum required demand in region h, respectively. Constraint (16) restricts the quantity of incoming medical supplies to not exceed the maximum required demand in region d. Constraint (17) guarantees that the quantity of outgoing medical supplies is not greater than the supply from supplier c. Constraint (18) determines that either sending or receiving medical supplies could happen in region h. Constraint (19) defines the decision variables.

4.4. Linearization approach

The proposed MUOO model is nonlinear because of the objective function (11) and constraint (18). This study develops a linearization approach using the IBM CPLEX optimizer to obtain the global optimum. Four auxiliary parameters are introduced to linearize the proposed MUOO model as follows:

$i_{qs}^{(-y)}$ Quantity of sent medical supplies is y in region s.
$j_{qd}^{(+y)}$ Quantity of received medical supplies is y in region d.
$r_{qh}^{(-y)}$ Quantity of sent medical supplies is y in region h.
$r_{qh}^{(+y)}$ Quantity of received medical supplies is y in region h.

Then, the partial discrete solution space can be easily constructed as follows:

$$i_{qs}^{(-y)} = 0, \ldots, y, \ldots, L_{qs} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{20}$$
$$j_{qd}^{(+y)} = 0, \ldots, y, \ldots, \max_{\xi} M_{qd\xi} - L_{qd} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{21}$$
$$r_{qh}^{(-y)} = 0, \ldots, y, \ldots, L_{qh} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{22}$$
$$r_{qh}^{(+y)} = 0, \ldots, y, \ldots, \max_{\xi} M_{qh\xi} - L_{qh} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}. \tag{23}$$

Four auxiliary binary variables are introduced with the above auxiliary parameters to select the final solution.

$$\mathscr{i}_{qs}^{(-y)} = \begin{cases} 1 & \text{if the quantity of send medical item } q \text{ is } y \text{ in region } s \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}. \tag{24}$$
$$\mathscr{j}_{qd}^{(+y)} = \begin{cases} 1 & \text{if the quantity of received medical item } q \text{ is } y \text{ in region } d \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}. \tag{25}$$
$$\mathscr{r}_{qh}^{(-y)} = \begin{cases} 1 & \text{if the quantity of send medical item } q \text{ is } y \text{ in region } h \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}. \tag{26}$$
$$\mathscr{r}_{qh}^{(+y)} = \begin{cases} 1 & \text{if the quantity of received medical item } q \text{ is } y \text{ in region } h \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}. \tag{27}$$

In addition, four additional auxiliary binary variables are defined and given by

$$\alpha_{qs}^{(-y)} = \begin{cases} 1 & \text{if } a_{qs} > i_{qs}^{(-y)} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{28}$$
$$\beta_{qd}^{(+y)} = \begin{cases} 1 & \text{if } b_{qd} > j_{qd}^{(+y)} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{29}$$
$$\gamma_{qh}^{(-y)} = \begin{cases} 1 & \text{if } k_{qh}^{-} > r_{qh}^{(-y)} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{30}$$
$$\gamma_{qh}^{(+y)} = \begin{cases} 1 & \text{if } k_{qh}^{+} > r_{qh}^{(+y)} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}. \tag{31}$$

Thus, the proposed MUOO model can be reformulated as the following mathematical model $\mathscr{W}^*$ with a large positive value $\mathbb{B}$:

$\mathscr{W}^*$:

$$
\begin{aligned}
\text{Max } \Psi^* = {} & \sum_{q\in\mathscr{Q}}\sum_{s\in\mathscr{S}}\sum_{y=0}^{L_{qs}} \mathbb{M}_{qs}\left(L_{qs} - \dot{\imath}_{qs}^{(-y)}\right)\alpha_{qs}^{(-y)} + \sum_{q\in\mathscr{Q}}\sum_{d\in\mathscr{D}}\sum_{\xi\in\Xi} P_\xi \sum_{y=0}^{\max_\xi M_{qd\xi}-L_{qd}} \mathbb{M}_{qd\xi}\left(L_{qd} + \dot{\mathscr{j}}_{qd}^{(+y)}\right)\beta_{qd}^{(+y)} \\
& + \sum_{q\in\mathscr{Q}}\sum_{h\in\mathscr{H}}\sum_{\xi\in\Xi} P_\xi \sum_{y=0}^{L_{qh}} \mathbb{M}_{qh\xi}\left(L_{qh} - r_{qh}^{(-y)}\right)\gamma_{qh}^{(-y)} \\
& + \sum_{q\in\mathscr{Q}}\sum_{h\in\mathscr{H}}\sum_{\xi\in\Xi} P_\xi \sum_{y=0}^{\max_\xi M_{qh\xi}-L_{qh}} \mathbb{M}_{qh\xi}\left(L_{qh} + r_{qh}^{(+y)}\right)\gamma_{qh}^{(+y)}
\end{aligned} \tag{32}
$$

s.t.
Constraints (12)–(17) and (19)

$$\frac{a_{qs} - \dot{\imath}_{qs}^{(-y)}}{\mathbb{B}} + 1 \geq \alpha_{qs}^{(-y)} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{33}$$

$$a_{qs} - \dot{\imath}_{qs}^{(-y)} \leq \alpha_{qs}^{(-y)}\mathbb{B} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{34}$$

$$\frac{b_{qd} - \dot{\mathscr{j}}_{qd}^{(+y)}}{\mathbb{B}} + 1 \geq \beta_{qd}^{(+y)} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{35}$$

$$b_{qd} - \dot{\mathscr{j}}_{qd}^{(+y)} \leq \beta_{qd}^{(+y)}\mathbb{B} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{36}$$

$$\frac{k_{qh}^{-} - r_{qh}^{(-y)}}{\mathbb{B}} + 1 \geq \mu_p^{(-i)} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{37}$$

$$k_{qh}^{-} - r_{qh}^{(-y)} \leq \mu_p^{(-i)}\mathbb{B} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{38}$$

$$\frac{k_{qh}^{+} - r_{qh}^{(+y)}}{\mathbb{B}} + 1 \geq \gamma_{qh}^{(+y)} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{39}$$

$$k_{qh}^{+} - r_{qh}^{(+y)} \leq \gamma_{qh}^{(+y)} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{40}$$

$$\sum_{y=0}^{L_{qs}} \dot{\imath}_{qs}^{(-y)} = 1 \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{41}$$

$$a_{qs} = \sum_{y=0}^{L_{qs}} i_{qs}^{(-y)}\dot{\imath}_{qs}^{(-y)} \qquad \forall\, q \in \mathscr{Q},\ s \in \mathscr{S}, \tag{42}$$

$$\sum_{y=0}^{\max\limits_{\xi} M_{qd\xi}-L_{qd}} \dot{\mathscr{j}}_{qd}^{(+y)} = 1 \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{43}$$

$$b_{qd} = \sum_{y=0}^{\max\limits_{\xi} M_{qd\xi}-L_{qd}} j_{qd}^{(+y)}\dot{\mathscr{j}}_{qd}^{(+y)} \qquad \forall\, q \in \mathscr{Q},\ d \in \mathscr{D}, \tag{44}$$

$$\sum_{y=0}^{L_{qh}} \mathscr{r}_{qh}^{(-y)} = 1 \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{45}$$

$$k_{qh}^{-} = \sum_{y=0}^{L_{qh}} r_{qh}^{(-y)}\mathscr{r}_{qh}^{(-y)} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{46}$$

$$\sum_{y=0}^{\max_\xi M_{qh\xi}-L_{qh}} \varkappa_{qh}^{(+y)} = 1 \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{47}$$

$$k_{qh}^{+} = \sum_{y=0}^{\max_\xi M_{qh\xi}-L_{qh}} r_{qh}^{(+y)} \varkappa_{qh}^{(+y)} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{48}$$

$$\sum_{y=0}^{L_{qh}} \varkappa_{qh}^{(-y)} + \sum_{y=0}^{\max_\xi M_{qh\xi}-L_{qh}} \varkappa_{qh}^{(+y)} \leq 1 \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{49}$$

$$k_{qh}^{-} \leq \sum_{y=0}^{L_{qh}} \varkappa_{qh}^{(-y)} \mathbb{B} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}, \tag{50}$$

$$k_{qh}^{+} \leq \sum_{y=0}^{\max_\xi M_{qh\xi}-L_{qh}} \varkappa_{qh}^{(+y)} \mathbb{B} \qquad \forall\, q \in \mathscr{Q},\ h \in \mathscr{H}. \tag{51}$$

With the auxiliary parameters and variables, the objective function Ψ in (11) can be reformulated as Ψ^* in (32). Constraints (33) and (34) guarantee that the objective function Ψ^* counts the utility only while a_{qs} is not smaller than $i_{qs}^{(-y)}$ for medical item q in region s. Constraints (35) and (36) ensure that Ψ^* counts the utility only when b_{qd} is not smaller than $j_{qd}^{(+y)}$ for medical item q in region d. Constraints (37) and (38) ensure that Ψ^* counts the utility only when k_{qh}^{-} is not smaller than $r_{qh}^{(-y)}$ for medical item q in region h. Constraints (39) and (40) guarantee that Ψ^* counts the utility only when k_{qh}^{+} is not smaller than $r_{qh}^{(+y)}$ for medical item q in region h. Constraints (41) and (42) help select the unique optimal solution in region s. Constraints (43) and (44) help select the unique optimal solutions in region d. Constraints (45)–(48) help select the unique optimal solutions in region h. Constraints (49)–(51) guarantee that sending or receiving medical supplies could happen in region h.

5. Computational studies

We conduct an illustrative example to validate the proposed MUOO model and linearization approach in handling the collaborative medical supply rebalancing and allocating problem in fighting against COVID-19. The numerical analysis is based on the situation reported in May 2020.

5.1. Dataset description

According to WHO situation reports, as of May 2020, more than 150 countries were suffering from COVID-19 (see Fig. 3). In particular, this study considers the case of the United States in that period with the main features addressed in the aforementioned problem, which will be detailed later. Accordingly, this research focused on rebalancing medical supplies among different states in the United States and allocating medical supplies from suppliers across the United States without considering racial discrimination or political factors. The datasets used in this study are from WHO, the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (CSSE [12]), and journal papers.

According to reports from CSSE, by May 9, 2020, more than one million cases had been confirmed in the United States. The United States also had the highest number of deaths. In addition, the United States also faced an unevenly distributed prevalence of infection, rapidly increasing the imbalance between supply and demand and hospitalization rates. Hence, medical supplies need to be appropriately rebalanced and allocated among 50 states and Washington DC in the United States (excluding territories) to match the demand and supply. To estimate the number of infected cases, we suppose that the number of newly confirmed cases is going down daily following a regression rate from May 9, 2020 (see Fig. 4), denoted by φ. Then, the number of

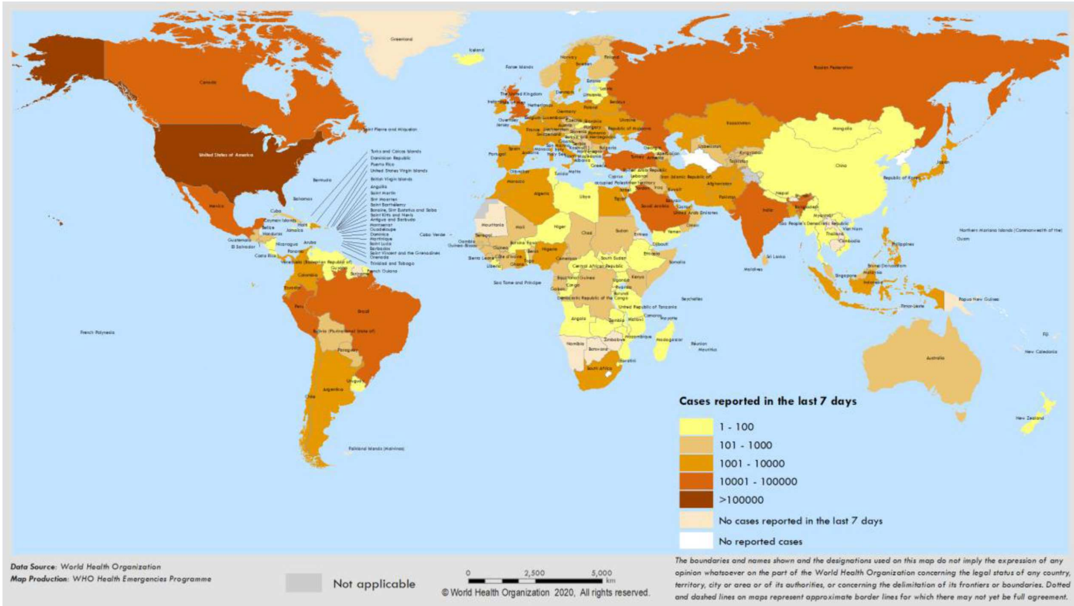


FIGURE 3. Number of confirmed COVID-19 cases reported by country, territory, and area from May 3 to May 9, 2020.

infected cases in the following two weeks can be calculated because the incubation period of the virus is around two weeks. In addition, the utility function considered in this study is based on the unmet demand value of the corresponding unit of medical supplies in each state.

The medical item considered in this study is the ventilator. The available ventilators in each region can be found in Rubinson *et al.* [40]. In addition, there are 8900 available ventilators in the national stockpile, as reported by the Johns Hopkins Center for Health Security [47]. The number of ventilators from companies and donation centers is unknown and randomly generated, which will not yield different results. With the above essential parameters, the proposed model $\mathscr{W}^*$ is implemented in CPLEX Optimization Studio (Version 12.6). All the experiments are run on a computer with an Intel(R) Xeon(R) CPU E5-2450L 0 @1.8 GHz under the Windows 10 Pro system.

5.2. Computational results

As mentioned in Wang *et al.* [48], 18.5% of the COVID-19 cases were considered critical, requiring ventilator support. Critical cases requiring ventilators can be estimated based on the estimated number of COVID-19 cases in each region. This section discusses rebalancing and allocating ventilators among all regions with the help of companies and donation centers.

In Figure 5, the quantities of received and sent ventilators are presented without considering any supply from companies and donation centers. In addition, the stock levels and possible quantities of demand for ventilators are also given. There are only 12 regions (*i.e.*, the states of New York, New Jersey, Illinois, Massachusetts, Pennsylvania, Maryland, Connecticut, Georgia, Michigan, Virginia, Rhode, and Nebraska) identified as demand points. Among them, New York received the highest number of ventilators due to the extremely high number of infected COVID-19 cases, followed by New Jersey, Illinois, Massachusetts, and Pennsylvania. Thirty-nine regions are considered supply points, although some experienced relatively severe epidemic situations. Texas has the highest number of ventilators. The total number of forwarded ventilators is 17 815 between the supply and demand regions.

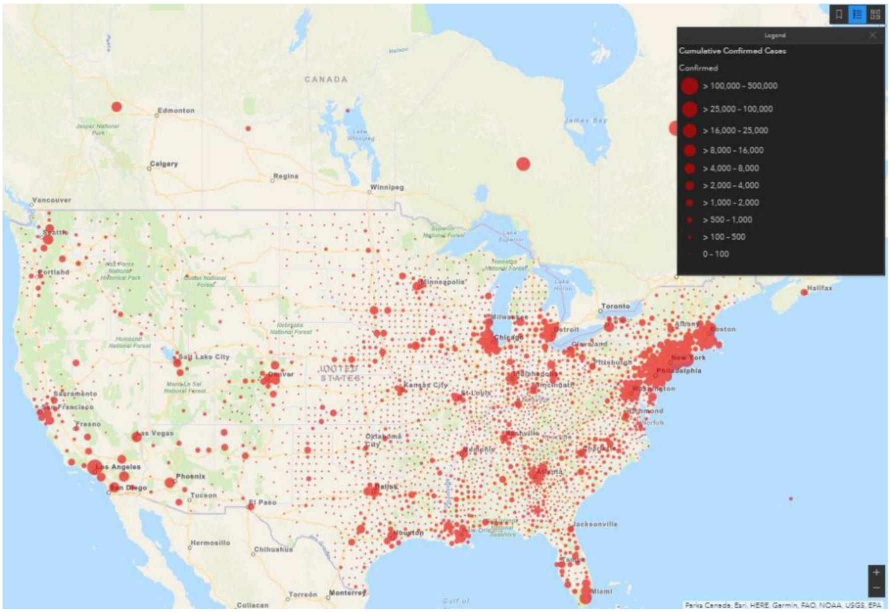


FIGURE 4. Number of confirmed COVID-19 cases in the United States (CSSE, May 9, 2020).

When the 8900 ventilators from the national stockpile were first used to meet the demand, these ventilators were allocated to the 51 regions together with the rebalancing scheme. The quantities of delivered and sent ventilators are provided in Figure 6. These additional ventilators increased the number of demand points from 12 to 17. Moreover, the demand points receive more ventilators because these ventilators are available to support the 51 regions. The total number of forwarded ventilators is 26 161, which is higher than that in Figure 5.

5.3. Sensitivity analysis

A sensitivity analysis is conducted to verify the proposed model and method to investigate the consequences of varying critical parameters.

5.3.1. Sensitivity to ventilator supply from the suppliers

Different quantities of ventilators from the companies and donation centers are tested to analyze how additional ventilators (*i.e.*, e_{qc}) impact the final decisions and objective function. As presented in Tables 2–11, the outgoing and incoming ventilators in the 51 regions are provided when the number of ventilators from the companies and donation centers ranges from 0 to 100 000. The number of ventilators from the companies and donation centers considerably influences the quantities of outgoing and incoming ventilators in these 51 regions.

As the number of ventilators from the companies and donation centers increases from 0 to 100 000, the number of outgoing ventilators decreases, and the number of incoming ventilators increases. On the one hand, the number of incoming ventilators in New York shows an upward tendency as more ventilators are available from the suppliers, as shown in Figure 7(a). A similar situation can be identified in New Jersey, Illinois, Massachusetts, and Pennsylvania. On the other hand, the number of outgoing ventilators in Missouri shows a downward tendency because more ventilators from the suppliers support the worst-hit states, as shown in Figure 7(b). A similar situation can be identified in some states, such as Kentucky, Oregon, and Montana.

Moreover, some regions change from supply to demand points with the growing quantity of ventilators from the suppliers. Figure 8 presents the specific quantities of outgoing and incoming ventilators in North Carolina for different quantities of ventilators from the suppliers. A similar situation can be identified in Florida, Ohio,

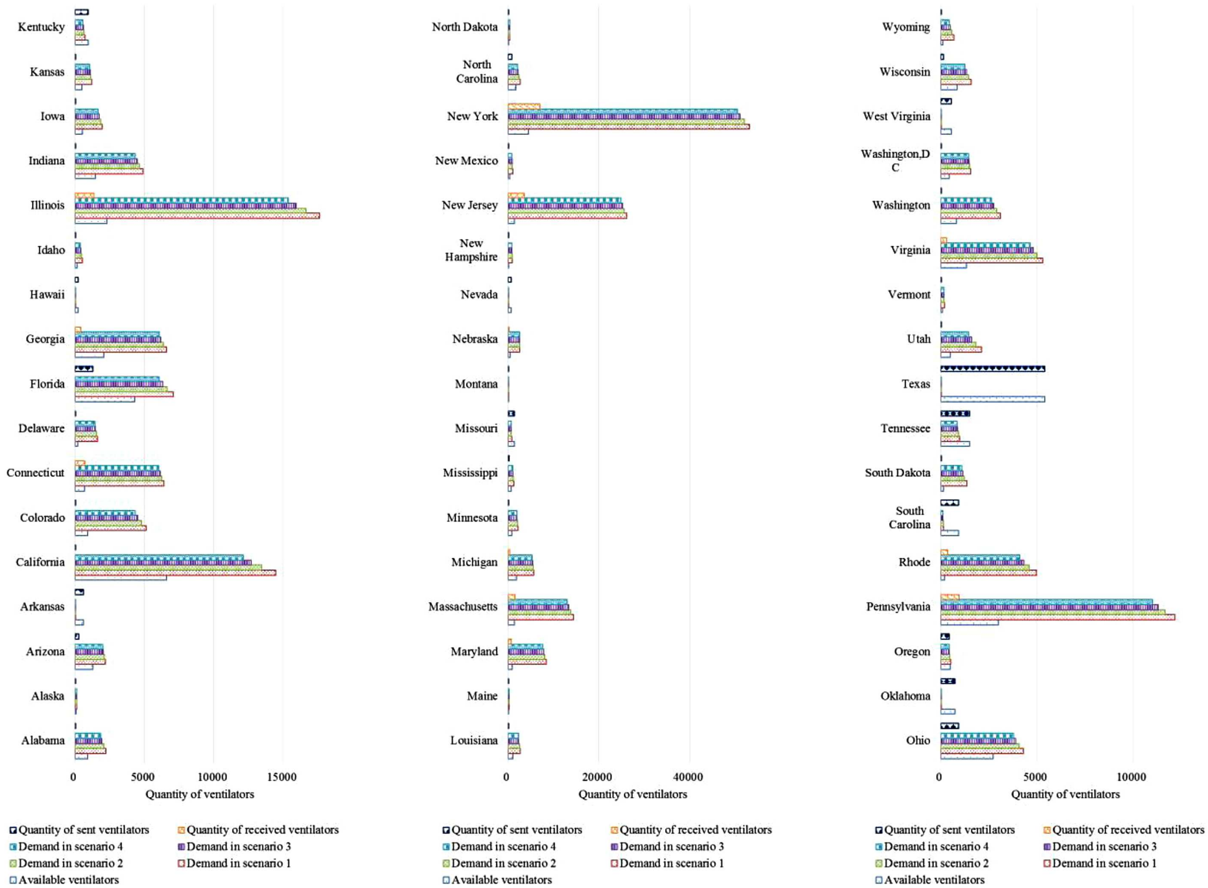


FIGURE 5. Rebalancing strategy of ventilators among 51 regions in the United States.

TABLE 2. Rebalancing and allocating strategy when the quantity of medical supplies is 10 000.

Region	a_{qs}/k_{gh}^{-}	b_{qd}/k_{gh}^{+}	Region	a_{qs}/k_{gh}^{-}	b_{qd}/k_{gh}^{+}	Region	a_{qs}/k_{gh}^{-}	b_{qd}/k_{gh}^{+}
Alabama	0	5	Louisiana	0	6	Ohio	0	669
Alaska	0	53	Maine	0	82	Oklahoma	0	740
Arizona	0	151	Maryland	1596	0	Oregon	0	390
Arkansas	0	633	Massachusetts	2860	0	Pennsylvania	1886	0
California	800	0	Michigan	760	0	Rhode	912	0
Colorado	616	0	Minnesota	128	0	South Carolina	0	949
Connecticut	1334	0	Mississippi	0	135	South Dakota	56	0
Delaware	176	0	Missouri	0	1437	Tennessee	0	1517
Florida	0	883	Montana	0	85	Texas	0	5419
Georgia	897	0	Nebraska	539	0	Utah	0	3
Hawaii	0	241	Nevada	0	753	Vermont	0	1
Idaho	0	1	New Hampshire	0	1	Virginia	720	0
Illinois	2926	0	New Jersey	6076	0	Washington	288	0
Indiana	522	0	New Mexico	0	2	Washington, DC	174	0
Iowa	120	0	New York	11 956	0	West Virginia	0	550
Kansas	0	3	North Carolina	0	704	Wisconsin	0	78
Kentucky	0	949	North Dakota	0	1	Wyoming	0	1

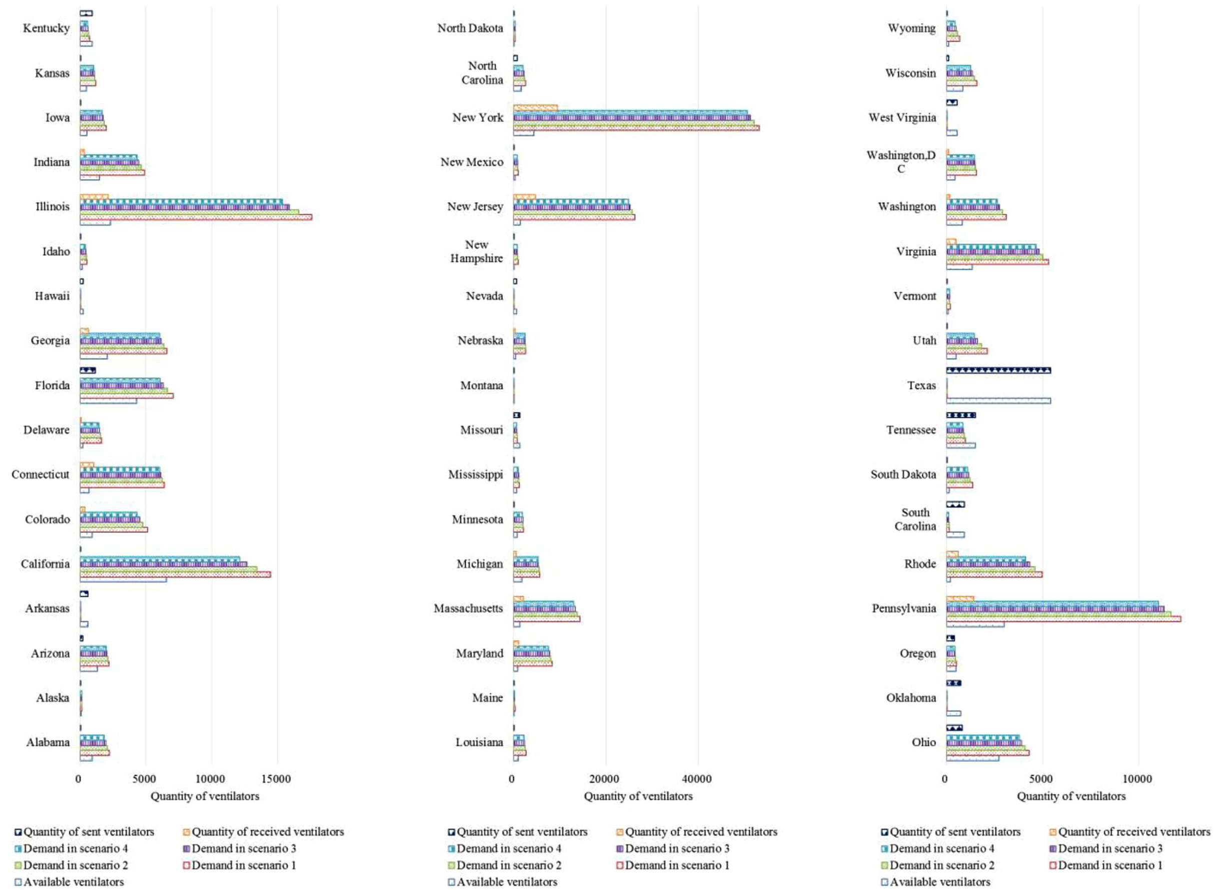


FIGURE 6. Rebalancing and allocating strategy of ventilators among 51 regions in the United States.

TABLE 3. Rebalancing and allocating strategy when the quantity of medical supplies is 20 000.

Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}
Alabama	168	0	Louisiana	171	0	Ohio	0	519
Alaska	0	44	Maine	0	71	Oklahoma	0	740
Arizona	0	79	Maryland	1976	0	Oregon	0	360
Arkansas	0	633	Massachusetts	3510	0	Pennsylvania	2392	0
California	1200	0	Michigan	980	0	Rhode	1104	0
Colorado	836	0	Minnesota	208	0	South Carolina	0	949
Connecticut	1653	0	Mississippi	0	112	South Dakota	126	0
Delaware	256	0	Missouri	0	1437	Tennessee	0	1517
Florida	0	668	Montana	0	81	Texas	0	5419
Georgia	1127	0	Nebraska	660	0	Utah	0	3
Hawaii	0	241	Nevada	0	753	Vermont	0	0
Idaho	0	1	New Hampshire	50	0	Virginia	920	0
Illinois	3773	0	New Jersey	7440	0	Washington	420	0
Indiana	702	0	New Mexico	52	0	Washington, DC	234	0
Iowa	200	0	New York	14,396	0	West Virginia	0	550
Kansas	72	0	North Carolina	0	615	Wisconsin	0	35
Kentucky	0	940	North Dakota	0	1	Wyoming	42	0

TABLE 4. Rebalancing and allocating strategy when the quantity of medical supplies is 30 000.

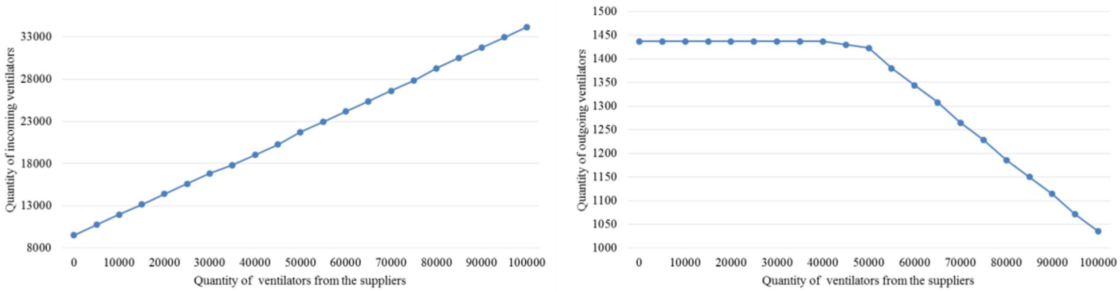
Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}
Alabama	231	0	Louisiana	252	0	Ohio	0	437
Alaska	0	41	Maine	0	55	Oklahoma	0	740
Arizona	165	0	Maryland	2318	0	Oregon	0	345
Arkansas	0	633	Massachusetts	4160	0	Pennsylvania	2806	0
California	1560	0	Michigan	1160	0	Rhode	1296	0
Colorado	1034	0	Minnesota	280	0	South Carolina	0	949
Connecticut	1914	0	Mississippi	0	62	South Dakota	189	0
Delaware	328	0	Missouri	0	1437	Tennessee	0	1517
Florida	462	0	Montana	0	70	Texas	0	5419
Georgia	1334	0	Nebraska	759	0	Utah	126	0
Hawaii	0	241	Nevada	0	753	Vermont	0	1
Idaho	38	0	New Hampshire	95	0	Virginia	1100	0
Illinois	4389	0	New Jersey	8556	0	Washington	528	0
Indiana	864	0	New Mexico	88	0	Washington, DC	288	0
Iowa	264	0	New York	16 836	0	West Virginia	0	550
Kansas	108	0	North Carolina	0	535	Wisconsin	0	9
Kentucky	0	902	North Dakota	0	1	Wyoming	69	0

TABLE 5. Rebalancing and allocating strategy when the quantity of medical supplies is 40 000.

Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}
Alabama	301	0	Louisiana	333	0	Ohio	352	0
Alaska	0	40	Maine	0	39	Oklahoma	0	740
Arizona	210	0	Maryland	2698	0	Oregon	0	337
Arkansas	0	633	Massachusetts	4745	0	Pennsylvania	3220	0
California	1960	0	Michigan	1340	0	Rhode	1560	0
Colorado	1254	0	Minnesota	360	0	South Carolina	0	949
Connecticut	2175	0	Mississippi	66	0	South Dakota	259	0
Delaware	408	0	Missouri	0	1437	Tennessee	0	1487
Florida	602	0	Montana	0	66	Texas	0	5419
Georgia	1564	0	Nebraska	858	0	Utah	216	0
Hawaii	0	241	Nevada	0	753	Vermont	0	1
Idaho	56	0	New Hampshire	140	0	Virginia	1300	0
Illinois	5236	0	New Jersey	9796	0	Washington	636	0
Indiana	1044	0	New Mexico	124	0	Washington, DC	348	0
Iowa	352	0	New York	19 032	0	West Virginia	0	550
Kansas	148	0	North Carolina	0	446	Wisconsin	92	0
Kentucky	0	845	North Dakota	0	1	Wyoming	99	0

and Arizona. The numbers of demand and supply points, given any quantity of ventilators from the suppliers, are shown in Figure 9. The number of demand points presents an opposite trend compared with the number of supply points when the number of ventilators from the suppliers ranges from 0 to 100 000. The reason is that many states change from supply to demand points because of the increased number of available ventilators from the suppliers.

We present the influence of the number of ventilators from the suppliers on the forwarded ventilators between the supply and demand regions. The quantities of incoming and outgoing ventilators at demand and supply points, respectively, are illustrated in Figure 10(a). Initially, the difference between the quantities of incoming and outgoing ventilators is 8900, caused by the national stockpile supply. After that, the total incoming ventilators increase, whereas the total outgoing ventilators decrease. Thus, it is determined that the supply from the suppliers is necessary because the moderately affected regions do not need to share too many ventilators with other states. Finally, we show the relationship between the objective function and the number of ventilators



(a) (b)

FIGURE 7. Influence of different quantities of supplied ventilators on the outgoing and incoming ventilators in the states. (a) New York and (b) Missouri.

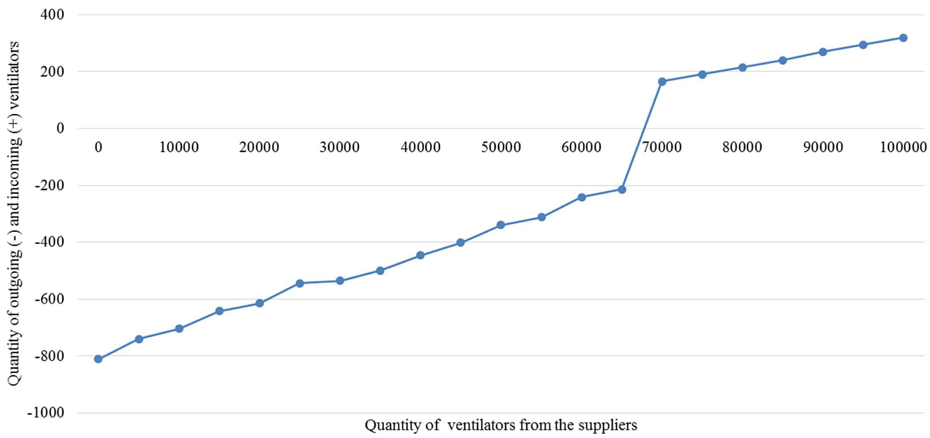


FIGURE 8. Influence of different quantities of supplied ventilators on the identification of supply and demand points.

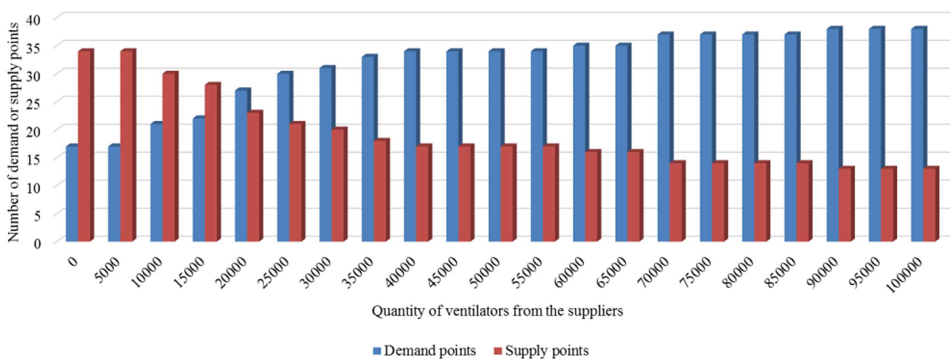


FIGURE 9. Influence of different quantities of supplied ventilators on the numbers of supply and demand points.

Table 6. Rebalancing and allocating strategy when the quantity of medical supplies is 50 000.

Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+	Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+	Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+
Alabama	371	0	Louisiana	432	0	Ohio	440	0
Alaska	0	29	Maine	0	24	Oklahoma	0	740
Arizona	265	0	Maryland	3078	0	Oregon	0	304
Arkansas	0	633	Massachusetts	5460	0	Pennsylvania	3726	0
California	2360	0	Michigan	1560	0	Rhode	1800	0
Colorado	1474	0	Minnesota	440	0	South Carolina	0	949
Connecticut	2494	0	Mississippi	99	0	South Dakota	329	0
Delaware	488	0	Missouri	0	1423	Tennessee	0	1396
Florida	742	0	Montana	0	55	Texas	0	5419
Georgia	1794	0	Nebraska	979	0	Utah	306	0
Hawaii	0	241	Nevada	0	753	Vermont	0	1
Idaho	78	0	New Hampshire	195	0	Virginia	1500	0
Illinois	6006	0	New Jersey	10 912	0	Washington	768	0
Indiana	1242	0	New Mexico	168	0	Washington, DC	408	0
Iowa	432	0	New York	21 716	0	West Virginia	0	550
Kansas	188	0	North Carolina	0	339	Wisconsin	128	0
Kentucky	0	750	North Dakota	0	1	Wyoming	129	0

Table 7. Rebalancing and allocating strategy when the quantity of medical supplies is 60 000.

Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+	Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+	Region	a_{qs}/k_{qh}^-	b_{qd}/k_{qh}^+
Alabama	448	0	Louisiana	522	0	Ohio	528	0
Alaska	0	27	Maine	0	9	Oklahoma	0	740
Arizona	315	0	Maryland	3496	0	Oregon	0	267
Arkansas	0	633	Massachusetts	6110	0	Pennsylvania	4186	0
California	2800	0	Michigan	1760	0	Rhode	2064	0
Colorado	1694	0	Minnesota	528	0	South Carolina	0	949
Connecticut	2813	0	Mississippi	132	0	South Dakota	406	0
Delaware	576	0	Missouri	0	1344	Tennessee	0	1305
Florida	896	0	Montana	0	51	Texas	0	5419
Georgia	2047	0	Nebraska	1089	0	Utah	405	0
Hawaii	0	241	Nevada	0	753	Vermont	0	1
Idaho	98	0	New Hampshire	245	0	Virginia	1720	0
Illinois	6776	0	New Jersey	12 276	0	Washington	888	0
Indiana	1422	0	New Mexico	208	0	Washington, DC	468	0
Iowa	520	0	New York	24 156	0	West Virginia	0	550
Kansas	232	0	North Carolina	0	241	Wisconsin	168	0
Kentucky	0	750	North Dakota	26	0	Wyoming	162	0

from the suppliers, as depicted in Figure 10(b). The profit increases as the number of ventilators from suppliers is increased from 0 to 100 000. The first unit of consumption yields more utility than the subsequent units, which verifies the effectiveness of the law of diminishing marginal utility.

The above numerical results suggest that the distribution of the preliminary stocked ventilators is unbalanced and insufficient to support the affected regions. Thus, allocating ventilators by suppliers is imperative with ventilator rebalancing operations in the affected regions. Besides, the quantity of ventilators from suppliers considerably influences the demand and supply point identification (see Fig. 8). Furthermore, the quantity of ventilators from suppliers affects the shipment between demand and supply points. Therefore, governments should encourage donations and facilitate the expanded production of emergency supplies.

Table 8. Rebalancing and allocating strategy when the quantity of medical supplies is 70 000.

Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}	Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}	Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}
Alabama	518	0	Louisiana	621	0	Ohio	608	0
Alaska	0	18	Maine	0	9	Oklahoma	0	740
Arizona	365	0	Maryland	3876	0	Oregon	0	252
Arkansas	0	633	Massachusetts	6760	0	Pennsylvania	4646	0
California	3200	0	Michigan	1940	0	Rhode	2304	0
Colorado	1936	0	Minnesota	608	0	South Carolina	0	949
Connecticut	3074	0	Mississippi	162	0	South Dakota	476	0
Delaware	656	0	Missouri	0	1265	Tennessee	0	1252
Florida	1036	0	Montana	0	40	Texas	0	5419
Georgia	2277	0	Nebraska	1199	0	Utah	495	0
Hawaii	0	241	Nevada	0	753	Vermont	4	0
Idaho	120	0	New Hampshire	300	0	Virginia	1920	0
Illinois	7623	0	New Jersey	13 516	0	Washington	1008	0
Indiana	1602	0	New Mexico	248	0	Washington, DC	534	0
Iowa	600	0	New York	26 596	0	West Virginia	0	550
Kansas	272	0	North Carolina	165	0	Wisconsin	216	0
Kentucky	0	698	North Dakota	46	0	Wyoming	192	0

Table 9. Rebalancing and allocating strategy when the quantity of medical supplies is 80 000.

Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}	Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}	Region	a_{qs}/k_{qh}^{-}	b_{qd}/k_{qh}^{+}
Alabama	588	0	Louisiana	711	0	Ohio	688	0
Alaska	0	14	Maine	0	9	Oklahoma	0	740
Arizona	420	0	Maryland	4256	0	Oregon	0	204
Arkansas	0	633	Massachusetts	7410	0	Pennsylvania	5152	0
California	3600	0	Michigan	2180	0	Rhode	2544	0
Colorado	2156	0	Minnesota	688	0	South Carolina	0	949
Connecticut	3393	0	Mississippi	192	0	South Dakota	546	0
Delaware	736	0	Missouri	0	1186	Tennessee	0	1161
Florida	1176	0	Montana	0	40	Texas	0	5419
Georgia	2507	0	Nebraska	1320	0	Utah	585	0
Hawaii	0	241	Nevada	0	753	Vermont	13	0
Idaho	140	0	New Hampshire	350	0	Virginia	2120	0
Illinois	8393	0	New Jersey	14 756	0	Washington	1140	0
Indiana	1782	0	New Mexico	292	0	Washington, DC	594	0
Iowa	680	0	New York	29 280	0	West Virginia	0	550
Kansas	312	0	North Carolina	215	0	Wisconsin	256	0
Kentucky	0	660	North Dakota	66	0	Wyoming	222	0

5.3.2. Sensitivity to stock level of ventilators

The initial stock level of ventilators at each state also influences the identification of demand and supply points and the number of outgoing and incoming ventilators. Three states (New York, Texas, and Connecticut) from three categories were tested to investigate the influence of different initial stock levels of ventilators. The results are provided in Figure 11. Note that no additional ventilators were considered in this subsection.

First, the outgoing and incoming ventilators in the state of New York are presented in Figure 11(a). New York is always considered a demand point to receive ventilators from other states because of its huge demand for ventilators. The number of incoming ventilators to support infected patients decreases as stocked ventilators increase from 1000 to 10 000. Furthermore, the objective function, in Figure 12, decreases as stocked ventilators increase from 1000 to 10 000 because the high stock levels of ventilators result in small utility functions in the state of New York.

TABLE 10. Rebalancing and allocating strategy when the quantity of medical supplies is 90 000.

Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}
Alabama	658	0	Louisiana	801	0	Ohio	776	0
Alaska	0	8	Maine	23	0	Oklahoma	0	740
Arizona	470	0	Maryland	4636	0	Oregon	0	189
Arkansas	0	633	Massachusetts	8125	0	Pennsylvania	5612	0
California	4040	0	Michigan	2380	0	Rhode	2784	0
Colorado	2398	0	Minnesota	776	0	South Carolina	0	949
Connecticut	3683	0	Mississippi	222	0	South Dakota	623	0
Delaware	816	0	Missouri	0	1114	Tennessee	0	1085
Florida	1330	0	Montana	0	24	Texas	0	5419
Georgia	2760	0	Nebraska	1430	0	Utah	675	0
Hawaii	0	241	Nevada	0	753	Vermont	24	0
Idaho	160	0	New Hampshire	400	0	Virginia	2340	0
Illinois	9163	0	New Jersey	16 120	0	Washington	1260	0
Indiana	1980	0	New Mexico	332	0	Washington, DC	660	0
Iowa	768	0	New York	31 720	0	West Virginia	0	550
Kansas	356	0	North Carolina	270	0	Wisconsin	296	0
Kentucky	0	603	North Dakota	86	0	Wyoming	255	0

TABLE 11. Rebalancing and allocating strategy when the quantity of medical supplies is 100 000.

Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}	Region	a_{qs}/k^-_{qh}	b_{qd}/k^+_{qh}
Alabama	735	0	Louisiana	900	0	Ohio	856	0
Alaska	0	9	Maine	0	1	Oklahoma	0	740
Arizona	525	0	Maryland	5054	0	Oregon	0	199
Arkansas	0	633	Massachusetts	8840	0	Pennsylvania	6118	0
California	4440	0	Michigan	2600	0	Rhode	3048	0
Colorado	2618	0	Minnesota	856	0	South Carolina	0	949
Connecticut	4002	0	Mississippi	255	0	South Dakota	693	0
Delaware	904	0	Missouri	0	1035	Tennessee	0	994
Florida	1484	0	Montana	0	20	Texas	0	5419
Georgia	2990	0	Nebraska	1551	0	Utah	774	0
Hawaii	0	240	Nevada	0	753	Vermont	35	0
Idaho	182	0	New Hampshire	455	0	Virginia	2540	0
Illinois	10 010	0	New Jersey	17 484	0	Washington	1392	0
Indiana	2160	0	New Mexico	376	0	Washington, DC	720	0
Iowa	848	0	New York	34 160	0	West Virginia	0	550
Kansas	396	0	North Carolina	320	0	Wisconsin	340	0
Kentucky	0	612	North Dakota	108	0	Wyoming	285	0

Next, the state of Texas is used to test the consequences of varying the quantity of stocked ventilators. The outgoing and incoming ventilators in the state of Texas are presented in Figure 11, and the objective function is provided in Figure 12 when the number of stocked ventilators increases from 1000 to 10 000. Texas is always considered a supply point owing to the small number of infected patients. The number of outgoing ventilators increases as the number of stocked ventilators increases from 1000 to 10 000 to support infected patients in other states. In addition, the objective function increases as the quantity of stocked ventilators increases from 1000 to 10 000 because more ventilators yield higher utility.

Next, we investigate the consequences of changes in the ventilator stock level in Connecticut. Contrary to New York and Texas, Connecticut is initially considered a demand point because of its low ventilator stock levels. Connecticut changes from a demand to supply point when the quantity of stocked ventilators is increased from 1000 to 10 000, as shown in Figure 11(c). Moreover, the trend in the objective function caused by changes in the stocked ventilators differs from that in New York and Texas. The objective function first decreases and

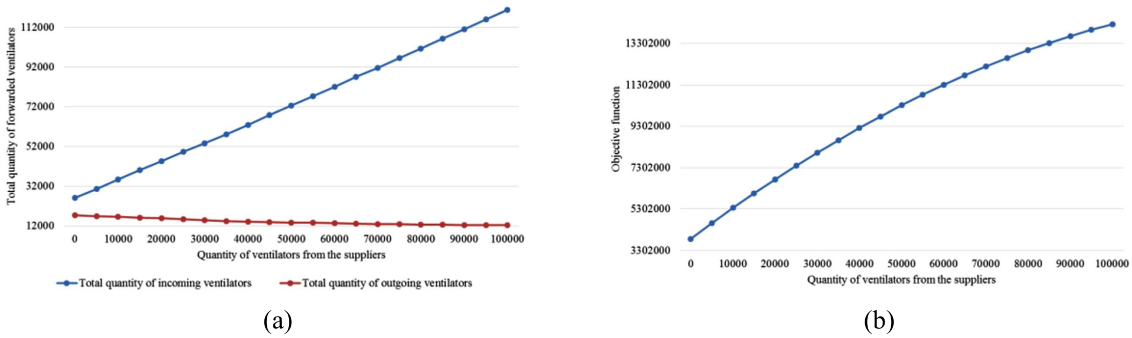


FIGURE 10. Influence of different quantities of supplied ventilators on the forwarded ventilators and objective function.

TABLE 12. Probability types considered in this study.

Ξ	Type I	Type II	Type III	Type IV	Type V
$\xi = 1$	$P_1 = 0.6$	$P_1 = 0.2$	$P_1 = 0.2$	$P_1 = 0.1$	$P_1 = 0.1$
$\xi = 2$	$P_2 = 0.2$	$P_2 = 0.6$	$P_2 = 0.3$	$P_2 = 0.1$	$P_2 = 0.1$
$\xi = 3$	$P_3 = 0.1$	$P_3 = 0.1$	$P_3 = 0.3$	$P_3 = 0.6$	$P_3 = 0.2$
$\xi = 4$	$P_4 = 0.1$	$P_4 = 0.1$	$P_4 = 0.2$	$P_4 = 0.2$	$P_4 = 0.6$

then increases when the stocked ventilators increase from 1000 to 10 000 (see Fig. 12). The reason is that the utility function is sensitive to the quantity of stocked ventilators in Connecticut. After increasing the quantity of stocked ventilators, the ventilators can be shared with other states to increase utility.

Using stocked medical supplies is the first choice to support the affected regions. Medical supply reserves need to be reasonably and vigorously implemented before a pandemic outbreak to respond quickly to the pandemic. In practice, the prevalence of the infection is dissimilar and unevenly distributed, making medical supply rebalancing necessary. Therefore, all regions must work together to maximize the total marginal utility of medical supplies in response to an epidemic such as COVID-19.

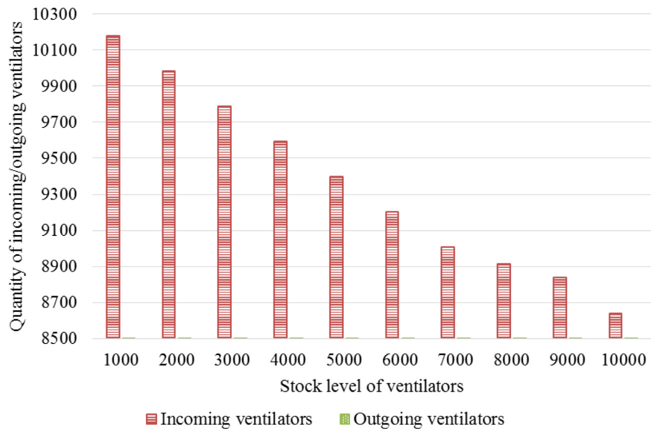
5.3.3. Sensitivity to scenarios

The possible numbers of infected cases in each scenario also significantly influence the final rebalancing and allocating strategy. The rebalancing and allocating plans were compared for five different probability types (*i.e.*, I, II, III, IV, and V, presented in Tab. 12) to validate the reliability of the proposed model and method. Probability type III is the same as in the above case study.

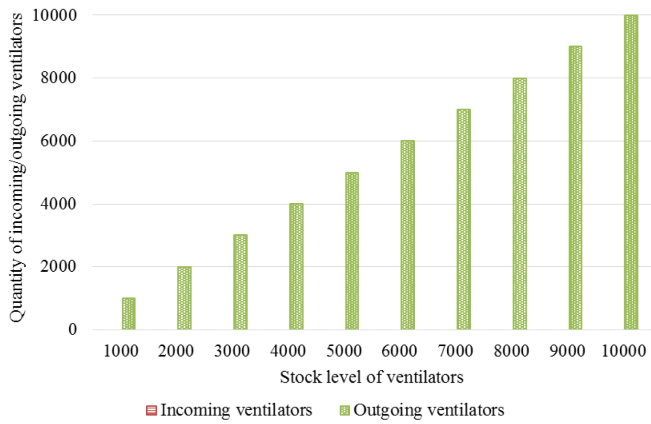
The incoming and outgoing ventilators in the two representative states (*i.e.*, New York and Florida) based on five different probability types are provided in Figure 13. In Figure 13(b), the demand point (*i.e.*, New York) receives more ventilators, given probability type V. Simultaneously, the supply point (*i.e.*, Florida) shares more ventilators with others given the same probability type V (Fig. 13(b)). In addition, the objective function under the five probability types in Figure 14 presents a downward trend. The probability type also greatly influences rebalancing and allocating ventilators among the 51 regions. Hence, the probability type must be estimated accurately before applying the collaborative medical supply rebalancing and allocating process.

5.3.4. Sensitivity to objective functions

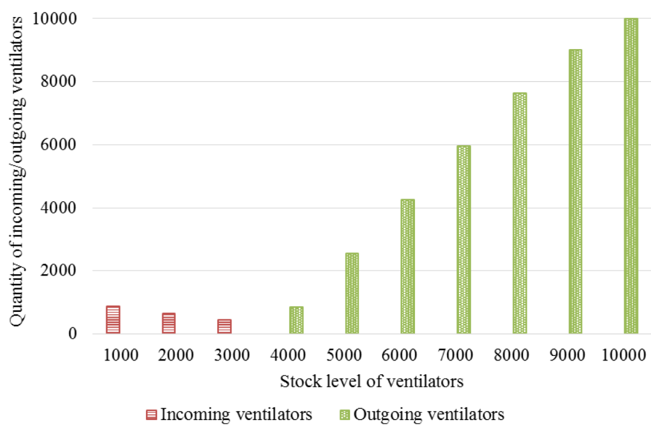
To illustrate the performances of different methods, the proposed MUOO model is compared with the traditional common objective functions (*i.e.*, minimization of unsatisfied demand and maximization of weighted



(a)



(b)



(c)

FIGURE 11. Incoming/outgoing ventilators at different stock levels in New York, Texas, and Connecticut. (a) New York. (b) Texas. (c) Connecticut.

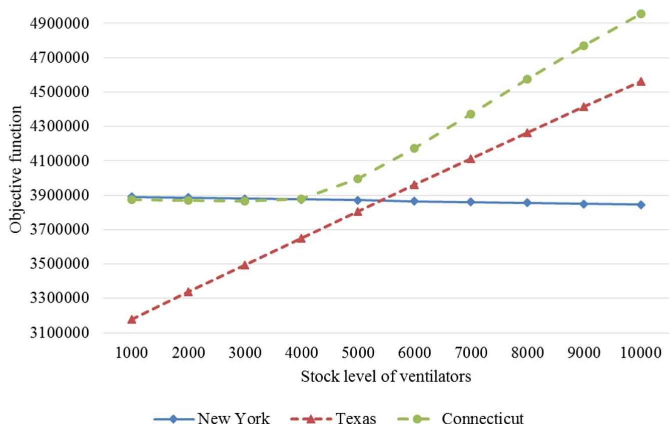


FIGURE 12. Objective function at different stock levels in New York, Texas, and Connecticut.

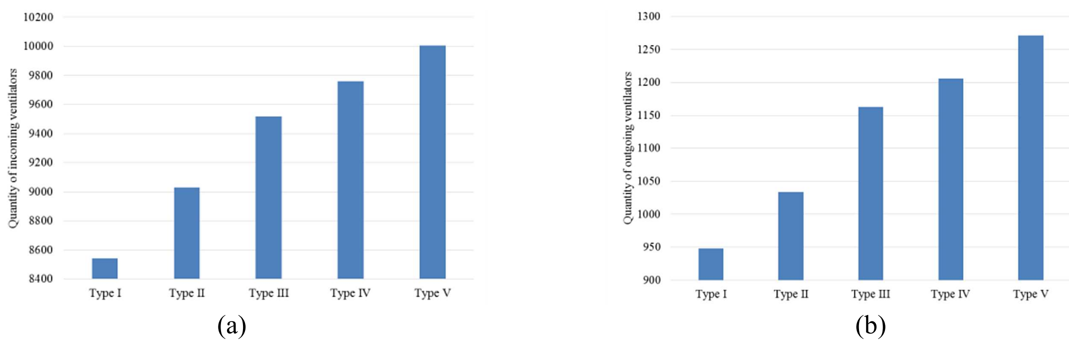


FIGURE 13. Quantities of incoming and outgoing ventilators under different probability types. (a) New York and (b) Florida.

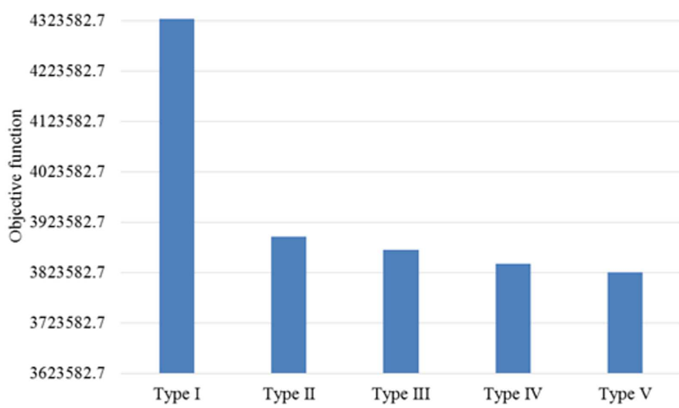


FIGURE 14. Objective function under different probability types.

coverage rate (minimization of unmet demand proportion) used by Hosseini *et al.* [27] and Gao and Cao [20]. Because this study did not consider weights at affected regions, we assume equal weight at these affected regions. For the unsatisfied demand-based objective functions with the same weight at demand points, the total unsatisfied demand remains unchanged given any decision variables; therefore, there is no need to present the numerical results. Hence, this study mainly investigates the numerical results of the weighted coverage rate-based objective function. Given the same input parameters, the ventilator rebalancing and allocating strategies using different methods are presented in Figure 15.

As presented in Figure 15, the coverages of ventilators at 51 regions without and with an additional supply of ventilators from the national stockpile are provided. Forty-two states (excluding Maryland, Connecticut, Florida, California, Pennsylvania, Massachusetts, Illinois, New Jersey, and New York) maintain a high coverage rate of ventilators in four scenarios without additional ventilator supplies from the national stockpile (see Fig. 15(a)). With the additional supply of ventilators from the national stockpile, the number of states with high ventilator coverage increases to 43 in four scenarios (see Fig. 15(b)). Different coverage rates of ventilators are obtained upon comparing the weighted coverage rate-based objective function with the proposed function. The coverage rates of ventilators through the proposed MUOO model can be easily calculated with the numerical results presented in Figures 5 and 6. In Figure 15, low-demand states (*e.g.*, Montana) maintain a relatively high coverage rate because they contribute more to increasing the total weighted coverage rates. However, high-demand states (*e.g.*, New York) did not receive any ventilators because of their lower contribution to the objective function (total weighted coverage rates), which is unreasonable because their urgent demands are not met with high priorities. As presented in Figures 5 and 6, the high-demand states (*e.g.*, New York) can maintain higher coverage rates than the low-demand states (*e.g.*, Montana) because the marginal utility values of ventilators are emphasized in the proposed MUOO model.

6. Conclusions and future research

6.1. Summary of key conclusions

As an exploratory work, this study investigated collaborative medical supply rebalancing and allocating problems under uncertainty in response to COVID-19. An MUOO model was proposed to formulate the problem and address various non-negligible issues, including fairness, the law of diminishing marginal utility, and uncertainty. Fairness is represented by maximizing the expected total utility (*i.e.*, maximizing fairness). A linearization approach was developed to overcome the nonlinearity of the proposed MUOO model. Finally, a case study of 51 regions in the United States validated the proposed MUOO model and linearization approach. Furthermore, profound managerial implications were drawn from the case study, representing the main contributions and benefits of this study.

6.2. Implications for theory and practice

The main managerial insights for researchers and managers are outlined as follows:

(1) *The proposed MUOO model and linearization approach perform better than traditional approaches.* The proposed objective function is more appropriate than the unmet demand-based fairness measurements because the diminishing marginal utility better describes the nature of humans (*i.e.*, the satisfaction of infected patients and medical staff). Besides, the proposed MUOO model incorporates uncertainty to reflect better the practical situations caused by pandemics. In addition, this study developed a linearization approach to obtain the global optimum of the nonlinear MUOO model, which is worthy of reference.

(2) *A collaborative medical supply rebalancing and allocating process is imperative to guide relief operations, considering scarce medical supplies immediately after a large-scale epidemic.* As presented in Figures 5 and 6, the incoming and outgoing ventilators are positive in the 51 regions in the United States, indicating that the previous ventilator stock levels among the regions are unbalanced. Rebalancing and allocating scarce medical supplies is imperative to solve the unbalanced medical supplies caused by the infection's dissimilarly

FIGURE 15. Rebalancing and allocating strategy of ventilators with maximization of coverage (*i.e.*, minimizing unmet demand proportion). (a) Without additional supply and (b) with additional supply.

and uneven distribution characteristics. The proposed MUOO can rebalance and allocate medical supplies to maximize the total utility in response to an epidemic such as COVID-19.

(3) *All institutions must collaborate to rebalance and allocate scarce medical supplies to maximize fairness.* The support of ventilators from suppliers is the key factor in relieving the detrimental situation caused by COVID-19. As presented in Tables 2–11, if ventilators from the suppliers can be used to support the infected cases, the states receive more ventilators and need to share fewer ventilators with others. Simultaneously, the objective function (*i.e.*, total utility) increases with a growing supply of ventilators from the suppliers (see Fig. 9), which guarantees fairness in rebalancing and allocating scarce medical supplies among different regions.

(4) *A higher stock level promotes fewer incoming ventilators at the demand points and more outgoing ventilators at the supply points.* Furthermore, some demand points change to supply points with increasing ventilator stock levels because these ventilators can be shared with regions in other states that are worst-hit or moderately affected (see Figs. 10 and 11). Therefore, increasing the stock of ventilators in response to infectious diseases, such as COVID-19, is encouraged.

6.3. Limitations and future studies

Despite these novelties and contributions, this study has several limitations. The demand for medical supplies relies on the estimated number of infected cases in the next few days. This study only explores the issue of a single-period collaborative medical supply rebalancing and allocating problem in response to COVID-19. Furthermore, deriving the optimal solution may be challenging when the problem size is large. Therefore, potential future research works will (i) investigate the prediction model on the number of infected cases in the next few days, (ii) provide a cyclic rolling horizon-based updating framework to construct a reliable multi-period medical supply rebalancing and allocating planning in response to disasters, and (iii) propose the corresponding metaheuristic method to solve large-sized problems.

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