EMERGENCY ORDER RESPONSE STRATEGY UNDER SUPPLY CHAIN COLLABORATION*

HAO TAN¹,² AND XIAO FU²,*

Abstract. This paper studies the production and ordering response strategy with uncertain demand in two-tier supply chain composed of a social planner and a manufacturer. In light of the uncertain market demand, the manufacturer needs advance production before receiving the regular order from social planner to make quick response, and reserve some additional production capacity in case of emergency orders. We derive the conditions whereby the manufacturer can benefit from the emergency production and emergency orders and the social planner is willing to place the emergency orders and upgrade the optimal decisions with Pareto improvement. Besides, we find that, (1) the unit cost for emergency order is either too high or low, which might be worse off if the emergency order opportunity is provided to social planner; (2) even if the centralized case is better than the decentralized case in terms of the total cost, the social planner prefers decentralized case to centralized case. Finally, we use the real data to prove the theoretical analysis and show that the emergency supply mechanism can almost meet domestic demand. At the same time, we determine the optimal unit cost for emergency order based on total cost and respective costs for both parties.

Mathematics Subject Classification. 91A80.

Received February 18, 2023. Accepted July 15, 2023.

1. Introduction

The outbreak of the COVID-19 pandemic has been posing sustained pressures and difficulties in managing all economic entities in general and medical products supply chains in particular. Bartik et al. conducted a survey of more than 5800 small businesses between March 28 and April 4, 2020, which shows the median business spending more than $100,000 per month had only about 2 weeks of cash on hand at the time of the survey [23]. Due to the economic downturn of business and the increasing of confirmed cases, it has severely inflicted insufficient supplies of acute healthcare materials, equipment, and resources, such as personal protective equipments (PPEs), intensive care unit (ICU) beds, hand sanitizers, and mechanical ventilators [18]. The World Health Organization (WHO) estimates a monthly consumption of 89 million masks, 76 million gloves, and 1.6 million goggles for the world, and the data continues to increase [2]. Faced with such a dilemma, how to improve it becomes a challenge for the government and WHO. In the conventional supply chain, with the sudden advent of the pandemic, the

Keywords. Supply chain management, social planner, emergency order, additional capacity.

* Supplementary Material is only available in electronic form at https://www.rairo-ro.org/10.1051/ro/2023107.
¹ School of Mathematical Sciences, Xiamen University, Xiamen 361005, P.R. China.
² Zhejiang Informatization Development Institute, Hangzhou Dianzi University, Hangzhou 310018, P.R. China.
*Corresponding author: fuxiao@hdu.edu.cn

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social planner (e.g., the government or a non-profit organization) starts to order medical products from the manufacturer and then coordinates product distribution to the customer. At the same time, as observed in practice, the manufacturer also initiates a production scheme to accommodate orders and demands. However, the demand for medical products (e.g., masks, rubbing alcohol) increases tremendously, and the purchase process of some medical products is complex, vaccines in particular. Moreover, at the early stage of the pandemic, the demand information is not clear and cannot be fully captured by the stakeholders of the supply chain, making the ordering and production decision becomes more challenging. So, we put forward the idea of advanced production before receiving orders under the conventional supply model to overcome these difficulties. At the same time, we combine the initiative proposed by UNICEF that each purchase process informs the next process, either directly or indirectly, and forms an integral part of sustainable development.

In this paper, we investigate the optimal ordering, production, and capacity reservation decisions for a supply chain consisting of a social planner (e.g., the government or a non-profit organization such as the Red Cross) and a manufacturer faced with the outbreak of the COVID-19 pandemic. The social planner is responsible for ordering medical products (e.g., medical masks) from the manufacturer to satisfy the surging but highly uncertain demand. Similarly, the manufacturer optimally decides the production scheme and further determines the reserved production capacity. To be specific, we model the problem in a two-stage stochastic framework, where the social planner attempts to minimize the cost, whereas the manufacturer attempts to maximize the revenue. At stage 1, to anticipate the upcoming order and uncertain demand, the manufacturer first decides on the advance production and decides whether he tells the social planner about the advance production or not. And then (or maybe simultaneously) the social planner places an order with the manufacturer. After receiving the order from the social planner, the manufacturer may expand the production capacity with uncertain demand and inform the overall capacity of the social planner. At stage 2, the demand uncertainty is realized. Depending on the first-period production and ordering quantities and the realized demand, the social places the emergency order to the manufacturer to satisfy the unmet demand (if any), and thus, the manufacturer conducts emergency production correspondingly. Moreover, if the manufacturer’s emergency production cannot meet the demand, then the social planner purchases products from other regions/countries to fill the demand gap. By using this framework, we address the production and capacity expansion problems for the manufacturer and the ordering problem for the social planner with uncertain demand. The main contributions of this paper are summarized as follows: (1) We characterize the optimal production, ordering, and capacity expansion schemes for the supply chain in both centralized case and decentralized case, and provide five strategies based on the optimal decisions. (2) We derive the conditions whereby the manufacturer can benefit from the emergency orders and the social planner is willing to place the emergency orders and upgrade the optimal decisions with Pareto improvement. (3) By comparing different strategies with the different unit costs for emergency orders, we determine the optimal strategy for both parties or the whole supply chain. (4) The practical data is used to determine the optimal unit cost for emergency orders and strategy, and the ordering and delivery plans of medical supplies are formulated for six countries including China, America, Russia, and so on.

The remainder of this paper is organized as follows. Section 2 reviews the related literature and discusses the difference between our work from other works. We introduce the research problem in Section 3 and analyze the problem in decentralized and centralized supply chains in Section 4. And then we propose five strategies in the decentralized case and centralized case and compare the respective cost and total cost of both parties in different strategies in Section 5. Besides, we use the real data to do some experiments and validate the theoretical results in Section 6. Finally, we conclude the paper and propose topics for future research in Section 7. Proofs of all the results are provided in the appendix.

2. Literature review

This work is closely related to the research on relevant problems of the COVID-19 pandemic. In this area, a number of the paper focus on developing predictive models to forecast the dynamic impacts of different methods on controlling the pandemic (see e.g., [12, 14, 15, 25]). However, the main purpose of our work is to investigate
the optimal operations of the supply chain formed by a manufacturer and a social planner. Some papers analyze the impacts of the pandemic on the operations of supply chains. For example, Hosseini and Ivanov [9] develop a model that incorporates a multi-layer Bayesian network model to quantify the impacts of pandemic disruptions on the performance of the operations of supply chains. Alkahtani et al. [1] employ a non-linear supply chain management model to investigate the optimal production decisions when the production rate is changeable and demand is uncertain. Paul et al. [17] investigate the recovery policy for the supply chain during the COVID-19 pandemic. Tirkolaee et al. [21] propose a Multi-objective Mixed-Integer Linear Programming model to address the location, distribution, production, distribution, and recovery decisions for a closed-loop supply chain during the COVID-19 outbreak. We recommend Ivanov and Dolgui [11] for a state-of-art review of the research and models used to address operation problems in the supply chain under COVID-19. Our work differs from those studies because we focus on the production and ordering decisions and the interactions between the supplier (the manufacturer) and the buyer (the social planner) when faced with demand uncertainty.

Our work is also related to the research on production and ordering decisions in the operation of the supply chain. For example, Chen and Yang [3] analyze the optimal production and ordering decisions in a two-echelon supply chain considering random yield and uncertain demand. In their paper, the emergency backup order is launched when the production is insufficient to meet the realized demand. Giri et al. [7] consider a similar problem for a three-echelon supply chain with random production yield and demand. However, in our work, we focus on the production and ordering operations in a supply chain under sustained emergency events or disruptions. Giri and Bardhan [6] analyze a two-echelon supply chain with uncertain demand and random yield with disruption in the supply chain. They characterize the optimal production and ordering quantity when the emergency order is launched. Xu et al. [24] propose production-inventory models for different disruption scenarios and characterize the optimal decisions to minimize the expected cost. Schmitt et al. [19] analyze the impact of supply disruptions using a large-scale agent-based model and show the effect of order-up-to policy as an adaptive tool to mitigate the impact of disruption. For more research in this stream, we recommend Paul et al. [16] and Duong and Chong [5] for extensive reviews. In addition to the specific decision-making context, our work also considers the capacity decision, which has not been analyzed in these papers.

There is a stream of literature that analyze relevant problems of capacity reservation. For example, Jin and Wu [13] propose a deductible capacity reservation mechanism for a two-level supply chain where customers can reserve production capacity with a deductible fee from the purchasing price. Serel [20] studies a multi-period capacity reservation contract between a manufacturer and a long-term supplier when the uncertain quantity of an import item is available in the spot market. Inderfurth and Kelle [10] consider a simple and easy-to-implement capacity reservation – base stock policy and compare it to either capacity reservation or spot market sourcing option. They examine the joint effect of demand and spot market price uncertainty on the performances of different policies. These papers focus on the risk hedging function of capacity reservation when the price is uncertain and the capacity reservation decision is made by downstream customers or retailers. However, in our context, the capacity reservation decision is made by the manufacturer. Like our work, Zheng et al. [26, 27] also consider the capacity reservation decision and the emergency ordering quantity commitment made by the manufacturer. However, these works differ from ours in two main aspects:

(i) They consider a make-to-order context such that the manufacturer makes production decisions in the presale season after receiving orders from the downstream.
(ii) They consider the downstream with the aim of making a profit, while the downstream (social planner) in our supply chain is not acting for the purpose of making a profit, which embodies humanitarianism.
(iii) They only focus on either the decentralized or centralized decision context, whereas we have a detailed analysis of both contexts.
(iv) In the context of the decentralized case, they only discuss upstream or downstream dominates of the supply chain, whereas we analyze and compare these two cases in supply chains at the same time.
3. Problem description

Since many manufacturers are more familiar with demand information than downstream, such as social planner [4], we consider the operation of a manufacturer-dominated supply chain which consists of a social planner (the buyer) and a manufacturer (the supplier) in the background that the epidemic is breaking out. The social planner, with the aim of minimizing cost, optimally determines the order quantity from the manufacturer to satisfy the demand, whereas the manufacturer decides the quantity to produce so as to minimize his cost. Specifically, we model the supply chain’s decisions in a two-stage problem: at stage 1, faced demand uncertainty, the manufacturer first produces $Q_m$ units of products in advance to anticipate the upcoming order and demand, while incurring a unit cost $c_m$, and he determines whether to inform the social planner of the advance production quantity $Q_m$. The social planner decides the ordering quantity $Q_s$, while incurring a unit ordering cost $w_s$ with uncertain demand. After receiving the order quantity from the social planner, the manufacturer then decides the additional production capacity for the emergency order $\beta$ to fill the quantity gap (when $Q_s > Q_m$) and/or uncertain demand with a unit capacity cost $c_\beta$.

At stage 2, the social planner observes the demand. Throughout this text, we use $X$ to denote the uncertain demand and $x$ to be its realization. The corresponding probability density function (pdf) and cumulative probability function (cdf) are denoted by $f(x)$ and $F(x)$, respectively. If the demand is larger than that in the order (i.e., $x > Q_s$), the social planner needs to place an emergency order with $Q_{se}$ units to the manufacturer to fill the demand gap, while incurring a unit cost $w_e$. The manufacturer then produces based on the advanced production quantity $Q_m$, the regular ordering quantity $Q_s$. Specifically, if $Q_m \geq x > Q_s$, no emergency production is needed; otherwise, the manufacturer has to produce for emergency order with a unit cost $c_e$. However, the manufacturer is subjected to production capacity constraints the additional capacity $\beta$ determined at stage 1 and the emergency order may also be insufficient to satisfy the demand gap. In this regard, the social planner needs to purchase more $Q_o$ units of products from other regions or countries with a unit cost of $w_o$ to satisfy the demand. For the manufacturer, the unit residual value of unsold products in the second stage is $r_m$. For the demand gap that is unsatisfied by the manufacturer at stage 2, we assume that there is a unit penalty cost $b_m$.

Throughout the paper, for brevity, we normalize the residual value of the additional capacity to zero. In particular, if the manufacturer does not consider expanding the additional production capacity $\beta$, the supply chain decisions are similar to stage 1. However, the manufacturer needs to inform the social planner of the advance production quantity $Q_m$ in order to refrain from meeting the regular ordering quantity from the social planner.

Throughout the sequence of the decision process, we make the following assumptions.

**Assumption 1.** $0 < r_s < c_m < c_e < b_m < w_s < w_e < w_o$.

Assumption 1 ensures that, instead of sourcing from other regions and/or countries, the social planner prefers to buy products from the manufacturer. Moreover, this assumption also guarantees that the manufacturer is motivated to satisfy the regular order or emergency order from the social planner.

**Assumption 2.** $0 < r_m \leq r_s, c_e + c_\beta < w_s$.

Assumption 2 guarantees the manufacturer revenues in emergency orders and facilitates advanced production of the manufacturer. The first inequality represents that salvage of the social planner’s unsold product is more powerful than that of the manufacturer. The second inequality represents that cost of regular production is higher than the total production cost in an emergency transaction.

**Assumption 3.** $0 < c_\beta < c_m - r_m$.

Assumption 3 means that the cost of additional production capacity is smaller than the loss of unsold product in stage 1, which ensures that the manufacturer prefers to reserve additional capacity to respond to the emergency production.
4. Model analysis

In this section, we analyze the problem in both the decentralized case (where the manufacturer and the social planner make decisions independently) and the centralized case (where the social planner and manufacturer jointly decide the advanced production quantity and capacity). In the decentralized case, we first consider that the manufacturer does not provide the emergency order opportunity and only provides his advance production quantity, and study the optimal advance production quantity for the manufacturer and the optimal regular ordering quantity for the social planner (Sect. 4.1). Secondly, we then add the emergency supply mechanism into the supply chain and characterize the optimal advance production quantity and additional production capacity for the manufacturer and the optimal regular ordering quantity for the social planner by using a two-stage stochastic model. A backward induction method is adopted to solve the problem. As a result, we first find out the optimal emergency ordering quantity in the second stage for the social planner. Based on the analysis in the second stage, we figure out the advanced production quantity for the manufacturer and the regular ordering quantity for the social planner in the first stage in the decentralized case (Sect. 4.2). In the centralized case, we discuss the advanced production quantity and the additional production capacity for the whole supply chain without or with additional capacity (Sect. 4.3), respectively.

In the decentralized case, the manufacturer and social planner, as independent economic entities, make respectively optimal choices based on their utility maximization principles. Next, we study their cost framework and find the optimal production and ordering decisions to minimize their costs.

4.1. Decentralized case – without additional capacity

In this section, we analyze the decentralized case without considering additional capacity where the manufacturer and the social planner make their decisions respective. The detailed sequence of the decision process is shown in Figure 1.

Stage 1: Faced with uncertain market demand, the manufacturer produces $Q_m$ units in advance at time $t_1$ and informs the social planner of the advanced production quantity $Q_m$ at time $t_2$, the social planner places an order with $Q_s$ units to the manufacturer at time $t_3$.

When the real demand is determined, the social planner needs to place emergency orders on suppliers from other regions or countries as necessary. Combing with the decision process of Figure 1, the expected costs of the manufacturer and social planner are shown in equations (1) and (2), respectively. In order to be consistent with the social planner’s cost model, and to measure uncertain social planner’s order with expected form, the manufacturer’s expected cost model $\Pi_{M0}(Q_m)$ is shown in equation (1).
The expected cost of the manufacturer:

$$\Pi_{M_0}(Q_m) = c_m Q_m - w_s Q_s - r_m (Q_m - Q_s) + b_m \int_{Q_m}^{+\infty} (x - Q_m) f(x) \, dx \quad (1)$$

where the first item of equation (1) represents the cost of advance production quantity, the second item represents the revenue from the social planner’s order, the third item represents the residual value of unsold products, the fourth item represents the penalty cost if actual demand exceeding the manufacturer’s advance production quantity.

The expected cost of the social planner:

$$\Pi_{S_0}(Q_s) = w_s Q_s - r_s \int_{0}^{Q_s} (Q_s - x) f(x) \, dx + w_o \int_{Q_s}^{+\infty} (x - Q_s) f(x) \, dx$$

subject to: $0 \leq Q_s \leq Q_m \quad (2)$

where the first item of equation (2) represents the cost of regular order, the second item represents the residual value of unsold products, the third item represents the cost associated with purchasing products from suppliers in other regions and/or countries.

Next, we study the impact of the decision variables on the expected cost of the manufacturer and social planner and obtain the optimal advance production $Q^*_m$ and the optimal regular order $Q^*_s$.

**Lemma 1.** In a decentralized decision without considering the additional production capacity, $\Pi_{M_0}(Q_m)$ is a convex function with respect to $Q_m$, and $\Pi_{S_0}(Q_s)$ is a convex function with respect to $Q_s$. The optimal advanced production $Q^*_m = F^{-1}(\frac{w_s - w_o}{b_m + r_m - c_m})$. When $b_m \geq \frac{(w_o - r_s)(c_m - r_m)}{w_s - r_s}$, the optimal regular ordering quantity $Q^*_s = F^{-1}(\frac{w_o - w_s}{b_m - r_m})$. When $b_m < \frac{(w_o - r_s)(c_m - r_m)}{w_s - r_s}$, the optimal regular ordering quantity $Q^*_s = F^{-1}(\frac{b_m + r_m - c_m}{b_m})$.

Lemma 1 implies that the unit penalty cost $b_m$ determines the optimal advance production $Q^*_m$ and the optimal regular ordering quantity $Q^*_s$. If $b_m + r_m$ is close to the unit production cost of advanced production $c_m$ and $w_o$ is very large, that is $\frac{b_m + r_m - c_m}{b_m} \ll \frac{w_o - w_s}{w_s - r_s}$, the manufacturer is reluctant to expand production, and the social planner will purchase more products from other regions or countries. If $b_m < w_s$ is comparatively large and $w_o$ is close to the unit cost $w_s$ for the order placed at stage 1, $\frac{b_m + r_m - c_m}{b_m} \gg \frac{w_o - w_s}{w_s - r_s}$, the manufacturer will have plenty of residual product. In order to solve the dilemma, we consider the manufacturer to expand the additional production capacity.

**4.2. Decentralized case – with additional capacity**

In this section, we focus on the decentralized case considering additional capacity where the manufacturer and the social planner make their decisions respective. The detailed sequence of the decision process is shown in Figure 2.

Stage 1: Faced with uncertain market demand, the manufacturer produces $Q_m$ units in advance at time $t_1$ and considers whether to inform the social planner of the advance production quantity $Q_m$ at time $t_2$. The social planner places an order with $Q_s$ units to the manufacturer at time $t_3$, and then the manufacturer expands additional production capacity $\beta$ to accommodate the social planner’s order and future demand and then informs the social planner of the overall capacity at time $t_4$; Stage 2: After learning about the real market demand, the social planner decides to place emergency order to the manufacturer and even to the supplier from other regions and/or countries if necessary at time $t_5$. The manufacturer then produces the emergency order (if necessary) at time $t_6$.

We start from the problems at stage 2 where the demand uncertainty is realized. In this stage, the social planner optimally decides the emergency orders placed to the manufacturer ($Q_m$) and the supplier from other regions or countries ($Q_s$) if necessary. For the manufacturer, he optimally decides the emergency production...
quantity (i.e., $Q_{me}$) and is subjected to the additional production capacity. In this regard, we establish the second-stage solution depending on whether the advance production of the manufacturer meets the regular order from the social planner.

**Lemma 2.** When the advance production of the manufacturer satisfies the regular order from the social planner, the manufacturer’s optimal emergency production quantity $Q_{me}^*$ and the social planner’s optimal emergency orders placed to the manufacturer $Q_{se}^*$ and other regions/countries $Q_o^*$ are characterized as follows:

$$Q_{me}^* = \begin{cases} \beta & \text{if } Q_s \leq Q_m \leq Q_m + \beta \leq x \\ x - Q_m & \text{if } Q_s \leq Q_m \leq x < Q_m + \beta \\ 0 & \text{if } 0 \leq x < Q_s \leq Q_m \leq Q_m + \beta \end{cases}, \quad Q_{se}^* = \begin{cases} Q_m + \beta - Q_s & \text{if } Q_s \leq Q_m \leq Q_m + \beta \leq x \\ x - Q_s & \text{if } Q_s \leq Q_m \leq x < Q_m + \beta \\ x - Q_s & \text{if } Q_s \leq x < Q_m \leq Q_m + \beta \\ 0 & \text{if } 0 \leq x < Q_s \leq Q_m \leq Q_m + \beta \end{cases}$$

and

$$Q_o^* = \begin{cases} x - Q_m - \beta & \text{if } Q_s \leq Q_m \leq Q_m + \beta \leq x \\ 0 & \text{if } Q_s \leq Q_m \leq x < Q_m + \beta \\ 0 & \text{if } Q_s \leq x < Q_m \leq Q_m + \beta \\ 0 & \text{if } 0 \leq x < Q_s \leq Q_m \leq Q_m + \beta \end{cases}.$$

**Lemma 3.** When the advance production of the manufacturer cannot satisfy the regular order from the social planner, the manufacturer’s optimal emergency production quantity $Q_{me}^*$ and the social planner’s optimal emergency orders placed to the manufacturer $Q_{se}^*$ and other regions/countries $Q_o^*$ are characterized as follows:

$$Q_{me}^* = \begin{cases} \beta & \text{if } Q_m < Q_s \leq Q_m + \beta \leq x \\ x - Q_m & \text{if } Q_m < Q_s \leq x < Q_m + \beta \\ Q_s - Q_m & \text{if } Q_m \leq x < Q_s \leq Q_m + \beta \\ Q_s - Q_m & 0 \leq x < Q_m < Q_s \leq Q_m + \beta \end{cases}, \quad Q_{se}^* = \begin{cases} Q_m + \beta - Q_s & \text{if } Q_m < Q_s \leq Q_m + \beta \leq x \\ x - Q_s & \text{if } Q_m < Q_s \leq x < Q_m + \beta \\ 0 & \text{if } Q_m \leq x < Q_s \leq Q_m + \beta \\ 0 & \text{if } 0 \leq x < Q_m < Q_s \leq Q_m + \beta \end{cases}$$

and

$$Q_o^* = \begin{cases} x - Q_m - \beta & \text{if } Q_m < Q_s \leq Q_m + \beta \leq x \\ 0 & \text{if } Q_m < Q_s \leq x < Q_m + \beta \\ 0 & \text{if } Q_m \leq x < Q_s \leq Q_m + \beta \\ 0 & \text{if } 0 \leq x < Q_m < Q_s \leq Q_m + \beta \end{cases}.$$
Based on the second stage solution, we then analyze the first stage problem; that is, to determine the optimal ordering decision for the social planner and the optimal production and capacity reservation schemes for the manufacturer when the demand is uncertain.

4.2.1. Considering informing advance production quantity

When the manufacturer informs the social planner of advanced production $Q_m$, the quantity of regular orders from the social planner is within advanced production. So, we discuss the impact of decision variables $Q_s, Q_m$ on the manufacturer’s and social planner’s expected cost based on Lemma 2.

First of all, we characterize the manufacturer’s expected cost at stages 1 and 2 in (3) based on $Q_{me}$ and $Q_{se}$ of Lemma 2.

$$
\Pi_{M_1}(Q_m, \beta) = c_m Q_m - w_s Q_s + c_\beta \beta + c_e \left[ \int_{Q_m}^{Q_m + \beta} (x - Q_m) f(x) \, dx + \int_{Q_m}^{+\infty} \beta f(x) \, dx \right] 
- w_e \int_{Q_s}^{Q_m} (x - Q_s) f(x) \, dx + \int_{Q_m + \beta}^{+\infty} (Q_m + \beta - Q_s) f(x) \, dx 
- r_m \left[ \int_{0}^{Q_s} (Q_m - Q_s) f(x) \, dx + \int_{Q_s}^{Q_m} (Q_m - x) f(x) \, dx \right] 
+ b_m \int_{Q_m + \beta}^{+\infty} (x - Q_m - \beta) f(x) \, dx
$$

where the first item of equation (3) represents the cost from advance production quantity, the second item represents the revenue from the social planner’s regular order, the third item represents the cost of reserving additional production capacity, the fourth item represents the emergency production cost, the fifth item represents the revenue from emergency orders, and the seventh item represents the penalty cost of actual demand exceeding the manufacturer’s overall supply capacity.

Secondly, we will discuss how the manufacturer’s expected cost changes concerning the social planner’s ordering quantity $Q_s$, advanced production quantity $Q_m$, and additional production capacity $\beta$. The result is shown below.

**Lemma 4.** On the premise that the manufacturer informs the social planner of his advanced production quantity, $\Pi_{M_1}(Q_m, \beta)$ is a concave function with respect to $Q_s$, and a convex function concerning $Q_m, \beta$. It reaches the maximum value when the ordering quantity $Q_s^* = F^{-1} \left( \frac{w_e - w_s}{w_m - r_m} \right)$ and reaches the minimum value when the optimal advance production quantity $Q_m^{1*} = F^{-1} \left( \frac{c_e + c\beta - c_m}{c_e - r_m} \right)$ and optimal additional capacity $\beta^{1*} = F^{-1} \left( \frac{w_s + b_m - c_e - c_\beta}{w_e + b_m - c_e} \right) - F^{-1} \left( \frac{c_e + c\beta - c_m}{c_e - r_m} \right)$.

From Lemma 4, it is shown that the manufacturer needs to strike a balance between the gain and loss from the emergency order and unused production capacity. When the optimal additional capacity $\beta^{1*}$ is too large, the manufacturer will suffer a loss when the reserved additional capacity for the emergency order is not used up. When the optimal additional capacity $\beta^{1*}$ is too small, the manufacturer also loses revenue from the emergency order. And if $\frac{w_e + b_m - c_e - c_\beta}{w_e + b_m - c_e} < \frac{c_e + c\beta - c_m}{c_e - r_m}$, the optimal additional capacity is negative, the revenue of the manufacturer decreases with the additional capacity $\beta$, as $\beta > 0$, which implies the manufacturer will be worsened while expanding additional capacity. Besides, the optimal overall capacity $R^{1*} = Q_m^{1*} + \beta^{1*}$ is irrelevant to the regular order from the social planner.
At the same time, we characterize the social planner’s expected cost at stages 1 and 2 in (4) based on $Q_{se}^*$ and $Q_o^*$ of Lemma 2.

$$
\Pi_{S_1}(Q_s, Q_m) = w_s Q_s - r_s \int_0^{Q_s} (Q_s - x) f(x) \, dx + w_e \left[ \int_{Q_s}^{R} (x - Q_s) f(x) \, dx + \int_{R}^{+\infty} (R - Q_s) f(x) \, dx \right] + w_o \int_{R}^{+\infty} (x - R) f(x) \, dx
$$

subject to: $0 \leq Q_s \leq Q_m$.

In equation (4), the first term is the ordering cost at stage 1, the second term represents the residual value of the unsold product at stage 2, the third term represents the cost of placing an emergency order to the manufacturer, and the fourth term represents the cost associated with purchasing products from suppliers in other regions and/or countries.

Next, we study the impact of decision variables on the expected cost of the social planner, and obtain the optimal regular order $Q_s^{1*}$.

**Lemma 5.** In the decentralized decision considering informing advance production quantity, $\Pi_{S_1}(Q_s, Q_m)$ is a convex function with respect to $Q_s$. When $w_e \leq (r_s + \frac{(w_e - w_s)(c_e - r_m)}{c_m - r_m - c_\beta})$, the optimal regular ordering quantity $Q_s^{1*} = F^{-1}(\frac{w_e - w_s}{w_e - r_s})$. When $w_e > (r_s + \frac{(w_e - r_s)(c_e - r_m)}{c_m - r_m - c_\beta})$, the optimal regular ordering quantity $Q_s^{1*} = F^{-1}(\frac{w_e + c_\beta - r_m}{c_e - r_m})$.

Lemma 5 implies that the optimal regular order from the social planner is within different $w_e$ value ranges. If the unit cost for the emergency order $w_e$ is large, the regular order from the social planner is restricted by the advance quantity of the manufacturer. The optimal order regular $Q_s^{1*}$ is the same as the optimal advance quantity $Q_{o}^{1*}$, which means that the social planner maybe put more cost into emergency order. This enables the manufacturer to benefit from emergency orders. If the unit cost for the emergency order $w_e$ is close to $w_s$, decisions for both parties are optimal. However, the manufacturer is prone to generate redundant capacity. So, it is very important for both parties to set an optimal $w_e$.

**4.2.2. Considering not informing advance production quantity**

In this section, we discuss the optimal decisions for both parties when the manufacturer does not inform directly social planner of advance production $Q_m$. The specific analysis and its results are as follows.

**4.2.2.1. Social planner’s optimal ordering.** As we described, in stage 2, the social planner places emergency orders to the manufacturer and/or the suppliers from other regions/countries only when the demand realized cannot be satisfied by the order quantity at stage 1. Therefore, we characterize the social planner’s expected cost at stages 1 and 2 in equation (5) based on $Q_{se}^*$ and $Q_o^*$ of Lemmas 2 and 3.

$$
\Pi_{S_2}(Q_s, R) = w_s Q_s - r_s \int_0^{Q_s} (Q_s - x) f(x) \, dx + w_e \left[ \int_{Q_s}^{R} (x - Q_s) f(x) \, dx + \int_{R}^{+\infty} (R - Q_s) f(x) \, dx \right] + w_o \int_{R}^{+\infty} (x - R) f(x) \, dx
$$

subject to: $0 \leq Q_s \leq R$

where $R = Q_m + \beta$ represents the maximum product quantity that the manufacturer can produce, that is the overall capacity. Since the additional capacity $\beta$ is determined by the manufacturer at stage 1 after receiving the regular order $Q_s$, the overall capacity is larger than the regular ordering quantity.

In equation (5), the first term is the order cost at stage 1, the second term denotes the residual value of the unsold product at stage 2, the third term represents the cost of placing an emergency order to the manufacturer,
and the fourth term represents the cost associated with purchasing products from suppliers in other regions and/or countries. We characterize the optimal ordering quantity $Q_s^\star$ as follows.

**Lemma 6.** In a decentralized decision, $\Pi_{S_2}(Q_s, R)$ is a convex function with respect to $Q_s$, achieving the minimum value at the optimal ordering quantity $Q_s^\star = F^{-1}(\frac{w_c-w_s}{w_c-r_s})$ and decreases monotonically with respect to the overall capacity $R$.

Lemma 6 shows that the social planner hopes that the manufacturer can provide maximum overall capacity, if possible, to decrease the purchase cost from other regions or countries. Further, in the optimal regular order $Q_s^\star$, for the manufacturer, uncertainty is reflected in $r_s$ and the cost equation (5), but the cost structure associated with $Q_s$ is clear for the manufacturer. So, the manufacturer can infer $Q_s^\star \in (F^{-1}(\frac{w_c-w_s}{w_c-r_m}), F^{-1}(\frac{w_c-w_s}{w_c-r_m}))$ since the unit residual value of unsold $r_s \in (r_m, c_m)$.

4.2.2.2. Manufacturer’s optimal production. In supply chain transactions, an emergency order can be triggered in two situations.

1. The manufacturer’s advanced production quantity cannot meet the social planner’s ordering quantity, and there is also the possibility of failing to meet the real market demand.
2. The manufacturer’s advanced production quantity meets the social planner’s ordering quantity, but cannot meet the actual market demand.

For situation 1, the impact of decision variables on the manufacturer’s revenue is investigated below based on $Q_m^\star$ and $Q_s^\star$ of Lemma 3. In order to be consistent with the social planner’s cost model, and to use the expected form to measure an uncertain social planner’s order, the manufacturer’s expected cost model $\Pi_{M_2}(Q_m, \beta)$ is shown in equation (6).

$$\Pi_{M_2}(Q_m, \beta) = c_mQ_m - w_sQ_s + c_\beta \beta$$
$$+ c_e \left[ \int_0^{Q_s} (Q_s - Q_m)f(x) \, dx + \int_{Q_s}^{Q_m+\beta} (x - Q_m)f(x) \, dx + \int_{Q_m+\beta}^{+\infty} \beta f(x) \, dx \right]$$
$$- w_e \left[ \int_{Q_s}^{Q_m+\beta} (x - Q_s)f(x) \, dx + \int_{Q_m+\beta}^{+\infty} (Q_m + \beta - Q_s) f(x) \, dx \right]$$
$$+ b_m \int_{Q_m+\beta}^{+\infty} (x - Q_m - \beta)f(x) \, dx,$$

the first item of equation (6) represents the cost from advance production quantity, the second item represents the revenue from the social planner’s order, the third item represents the cost of reserving additional production capacity, the fourth item represents the emergency production cost, and the fifth item represents the revenue from emergency order, the sixth item represents the penalty cost when actual demand exceeding the manufacturer’s overall supply capacity.

Next, we will examine how the manufacturer’s expected cost changes with respect to the social planner’s ordering quantity $Q_s$, advanced production quantity $Q_m$, and additional production capacity $\beta$, and find the upper bound of the optimal value $Q_m$. The result is as below.

**Lemma 7.** On the premise that the manufacturer’s advance production cannot meet the social planner’s regular order, $\Pi_{M_2}(Q_m, \beta)$ is a concave and convex function with respect to $Q_s$ and $\beta$, respectively. And $\Pi_{M_2}(Q_m, \beta)$ is monotonically decreasing with respect to $Q_m$. It reaches the maximum value at order quantity $Q_m^\star = F^{-1}(\frac{w_c-w_s}{w_c-c_\beta})$; and reaches the minimum value at optimal additional production capacity $\beta^\star = F^{-1}(\frac{w_e+b_m-c_\beta-c_\beta}{w_e+b_m-c_\beta}) - Q_m$ when the regular of the social planner $Q_s$ is not greater than $F^{-1}(\frac{w_e+b_m-c_\beta-c_\beta}{w_e+b_m-c_\beta})$. (see the appendix for the derivation process)
From Lemma 7, when the manufacturer’s advance production does not know the regular order of the social planner, the manufacturer can only determine the optimal overall capacity \( R^2 = F^{-1}(\frac{w_e + b_m - c_e - c_\beta}{w_e + b_m - c_e}) \), but cannot decide the optimal production plan. Through the optimal overall capacity \( R^2 \), we can infer that the optimal value of \( Q_m \) is in the range of \((0, R^2]\). Besides, if the regular order of the social planner satisfies the inequality \( Q_s \geq F^{-1}(\frac{w_e + b_m - c_e - c_\beta}{w_e + b_m - c_e}) > Q_{s'} > Q_m \), the additional capacity for an emergency order is \( \beta = 0 \). This implies that the manufacturer does not benefit from the emergency production.

For situation 2, the expected cost of the manufacturer and corresponding analysis is the same as equation (3) and Lemma 4 of Section 4.1. So, we no longer analyze here.

**Corollary 1.** In the decentralized decision, combining the optimal decision of the social planner and manufacturer, the following results are obtained,

1. The optimal regular ordering quantity \( Q^2_s \in (Q_s, \tilde{Q}) \);
2. In situation 1, \( Q^2_m < F^{-1}(\frac{w_e - w_s}{w_e - r_s}) \); In situation 2, when \( w_e \leq (r_s + \frac{w_s - r_s}{c_m - r_m - c_\beta}) \), \( Q^*_m \geq Q^2_s \).

Corollary 1-1 implies that the optimal regular order \( Q^*_s \) is not the worst regular order for the manufacturer, but the value of \( r_s \) has a great impact on the manufacturer’s revenue, whose value is close to \( r_m \) or \( c_e \) and cannot benefit the manufacturer. Corollary 1-2 ensures that the establishment of situation 1 and situation 2 is the optimal decision.

Since the advanced production of the manufacturer is prior to receiving the regular order the from the social planner, the manufacturer doesn’t judge the size relationship between \( Q_m \) and \( Q_s \). But the manufacturer can get the value range of \( Q^2_s \in (F^{-1}(\frac{w_e - w_s}{w_e - r_s}), F^{-1}(\frac{w_e - w_s}{w_e - r_m})) \). For the convenience of writing, we abbreviate it as \( Q^2_s \in (Q_{\text{min}}, Q_{\text{max}}) \) and denote \( F^{-1}(\frac{w_e - w_s}{w_e - r_m + c_\beta - c_m}) \) as \( \bar{Q} \). When \( \bar{Q} \geq Q_{\text{max}} \), we can determine the advanced advance production \( Q^2_m = \bar{Q} \). When \( \bar{Q} < Q_{\text{max}} \), due to the uncertainty of regular order \( Q_s \) and advance production \( Q_m \) for the manufacturer, we define \( Q_s, Q_m \) as two independent random variables \( Y, Z \). Their corresponding density functions are as follows,

\[
  f_1(y) = \frac{1}{\int_{Q_{\text{min}}}^{Q_{\text{max}}} f(y) \, dy} f(y), \quad f_{2(z)} = \frac{1}{\int_{\bar{Q}}^{\infty} f(z) \, dz} f(z), \quad \tilde{Q} \leq z \leq +\infty.
\]

So, the probability of \( Q^*_s > Q^*_m \) is

\[
p_2 = \begin{cases} 
  \int_{Q_{\text{min}}}^{\bar{Q}} \int_{Q_{\text{min}}}^{\bar{Q}} f_1(y) f_2(z) \, dy \, dz & \text{if } Q_{\text{min}} \leq \bar{Q} \\
  \int_{Q_{\text{min}}}^{\bar{Q}} \int_{Q_{\text{min}}}^{\bar{Q}} f_1(y) f_2(z) \, dy \, dz & \text{if } Q_{\text{min}} > \bar{Q}
\end{cases}
\]

and the probability of \( Q^*_s \leq Q^*_m \) is \( p_1 = 1 - p_2 \). Thus, the manufacturer’s expected cost function is shown in equation (7).

\[
  \Pi_{M_2}(Q_m, \beta) = p_1 \Pi_{M_1}(Q_m, \beta) + p_2 \Pi_{M_2}(Q_m, \beta)
\]

where the first item of equation (7) represents the expected cost of the manufacturer if \( Q^*_s > Q^*_m \). The second item represents the expected cost of the manufacturer if \( Q^*_s \leq Q^*_m \).

Next, we determine the optimal advance production by comparing the value of \( \bar{Q} \) and \( Q_{\text{max}} \).

**Theorem 1.** When \( w_e > c_e - b_m + \frac{p_1c_\beta(c_e - r_m)}{c_m - c_\beta - p_2c_e - p_1r_m} \), \( \Pi_{M_2}(Q_m, \beta) \) is a convex function with respect to \( Q_m, \beta \), respectively. The manufacturer’s optimal advance production \( Q^*_{m_0} \) and additional capacity \( \beta^* \) are characterized as follows:

\[
  \begin{cases} 
  F^{-1}(\frac{c_e + c_\beta - c_m}{c_e - r_m}), & \text{if } Q_{\text{max}} \leq \bar{Q} \\
  F^{-1}(\frac{c_e + c_\beta - c_m}{p_1(c_e - r_m)}), & \text{if } Q_{\text{max}} > \bar{Q}
\end{cases}
\]

where \( c_e - b_m + \frac{p_1c_\beta(c_e - r_m)}{c_m - c_\beta - p_2c_e - p_1r_m} \) is the additional capacity for an emergency order, \( \beta = 0 \). This implies that the manufacturer does not benefit from the emergency production.
Theorem 1 implies that, when the upper bound of the regular order is not less than the optimal advance production of situation 2, the received regular order is always fewer than the advance production. So, the optimal production plan is the same as situation 2. When the upper bound of the regular order is larger than the optimal advance production of situation 2, the optimal advance production $Q_m^s$ increases as $p_2(\leq 1)$ decreases in order to reduce the risk of not meeting the regular order and the cost of emergency production. However, if $p_2$ is too small and $\frac{w_e+b_m-c_e-c_3}{w_s+b_m-c_e} < \frac{c_e+c_3-c_m}{p_2(c_e-r_m)}$, the overall production is very large and exceeds the optimal overall capacity. In this way, the manufacturer cannot benefit from reserving the additional capacity for responsive production, which is used to satisfy the emergency order.

**Theorem 2.** In a decentralized decision and the optimal decisions of both parties, if the manufacturer provides an emergency order opportunity to the social planner:

(1) The optimal overall capacity of the manufacturer always meets the order regular from the social planner;
(2) Whether the manufacturer informs the social planner of the advance production quantity or not, when $w_e \leq c_e - b_m + \frac{(c_e-r_m)\beta}{c_m-c_3-r_m}$, the manufacturer cannot benefit from emergency orders; when $w_e > c_e - b_m + \frac{p_2\beta(c_e-r_m)}{c_m-c_3-p_1c_e-p_2r_m}$, the manufacturer can benefit from emergency orders.

Theorem 2-1 shows that the optimal overall capacity of the manufacturer is always larger than the optimal regular ordering quantity from the social planner since $\frac{w_e-r_s}{w_s-r_s} > \frac{\beta}{w_e+b_m-c_e}$. Theorem 2-2 implies that, if $w_e \leq c_e - b_m + \frac{(c_e-r_m)\beta}{c_m-c_3-r_m}$ and the manufacturer provides the emergency orders, the revenue of the manufacturer decreases rather than increases. Even worse, if the demand is highly uncertain, the cost of providing an emergency order opportunity may be greater than without providing it, that is $\Pi_{M1}(Q_m^s, \beta^1) > \Pi_{M1}(Q_m^r, 0)$. Then $\beta^1 = F^{-1}(\frac{w_e+b_m-c_m}{w_s+b_m})$. And the manufacturer always provides the emergency order opportunity for the social planner under the condition of $w_e > c_e - b_m + \frac{p_2\beta(c_e-r_m)}{c_m-c_3-p_1c_e-p_2r_m}$. It depends on the unit cost for the emergency order $w_e$ and additional capacity $\beta$ for the emergency order. Therefore, it is critical to set an appropriate $w_e$ and $\beta$ for the emergency order. And if both the manufacturer and social planner are better off when the emergency order is required, namely, Pareto improvement.

According to Corollary 1, Theorems 1 and 2, in the decentralized decision, the information transmission between the social planner and the manufacturer is not perfect, so the manufacturer cannot be accurately informed of the ordering quantity, but can infer the interval of $Q_m^s$ based on the cost structure and parameter. Next, we analyze the optimal production of the manufacturer according to the relationship between the unit cost of the emergency order cost $w_e$ and the unit cost of producing $c_e$, as shown in Figure 3.

As can be seen from Figure 3, when $w_e < c_e - b_m + \frac{p_2\beta(c_e-r_m)}{c_m-c_3-p_1c_e-p_2r_m}$, the optimal production plan does not exist, like an infeasible area. It implies that low unit production costs for emergency orders could dampen the manufacturer’s enthusiasm for emergency supply. And if $c_m - c_3 - p_1c_e - p_2r_m > 0$ with $c_e$ increases, the optimal advance production exceeds the optimal overall capacity and $\beta^3 < 0$. According to the optimal production plan, the revenue of the manufacturer will reduce if an emergency order is still provided. When $w_e \geq c_e - b_m + \frac{p_2\beta(c_e-r_m)}{c_m-c_3-p_1c_e-p_2r_m}$, the feasible area is divided by $w_e = c_e + \frac{(w_s-c_e)(c_e-r_m)}{c_m-c_3}$ into $Q_{max} \leq Q$ and $Q_{max} > Q$, and have the corresponding optimal production plan. Both the manufacturer and social planner can benefit from emergency supplies and the manufacturer is less likely to suffer from hoarding.

### 4.3. Centralized case

The centralized decision is associated with uniformity, contributes to reliability and compliance, and is therefore considered as a more effective way of decision [8]. In the centralized decision, the manufacturer shares market demand information, and the social planner and manufacturer jointly decide the production process. We denote the advanced production quantity and additional production capacity of the manufacturer as $Q_m^c$ and $\beta^*$. Due to the sharing of market demand information, the manufacturer’s advance production quantity can be consistent with the social planner’s ordering quantity, thereby reducing the risk of hoarding,
and at the same time, the production capacity can be adjusted in advance. Therefore, we build the expected cost model of the entire supply chain without considering additional and considering additional capacity, respectively.

4.3.1. Without additional capacity

When the manufacturer does not reserve additional capacity, he only provides a single supply to the social planner, that is the advanced production quantity $Q^C_m$. So, the specific model is shown in equation (8).

$$\Pi_{C1}(Q^C_m) = c_m Q^C_m - r_s \int_0^{Q^C_m} (Q^C_m - x) f(x) \, dx + (b_m + w_o) \int_{Q^C_m}^{+\infty} (x - Q^C_m) f(x) \, dx$$

(8)

where the first item represents the order production cost in stage 1, the second item represents the residual value of unsold products, and the third item represents the manufacturer’s shortage cost and the cost of the social planner purchasing medical supplies from foreign markets.

Next, we will examine how $\Pi_{C1}(Q^C_m)$ changes relative to advance production quantity $Q^C_m$. The results are as follows.

**Lemma 8.** When the manufacturer does not provide the emergency supply in a centralized case, $\Pi_{C1}(Q^C_m)$ is a convex function with respect to $Q^C_m$, and achieves the minimum at the optimal advanced production quantity $Q^C_m = F^{-1}\left(\frac{w_o + b_m - c_m}{w_o + b_m - r_s}\right)$.

Lemma 8 implies that the supply of the centralized case is more than that of the decentralized case without considering the additional capacity. In particular, since $w_o - r_s \geq c_m - r_s$, the advance production quantity $Q^C_m$ under the centralized case is not less than the optimal regular orders $Q^*_s$ in the decentralized case. It not only meets the social planner’s regular order, but also increases the manufacturer’s revenue. Besides, if $\frac{c_p}{w_o + b_m - c_i} > \frac{c_m - r_s}{w_o + b_m - r_s}$, the supply of the centralized case without considering the additional capacity is more than that of the decentralized case while considering the additional capacity. It decreases the risk of insufficient supply and even achieves zero risk for the manufacturer, but increases the risk of the social planner’s oversupply. So, it is necessary to preserve the additional capacity.
4.3.2. With additional production capacity

When the manufacturer expands the additional capacity, he not only provides the regular supply, but also the emergency supply to the social planner, that is the advanced production quantity and additional production capacity $Q^C_M, \beta^C$. So, the specific model is shown in equation (9).

$$
\Pi_{C_2}(Q^C_M, \beta^C) = c_m Q^C_m - r_s \int_0^{Q^C_m} (Q^C_m - x) f(x) \, dx + c_\beta \beta^C
$$

$$
+ c_c \left[ \int_{Q^C_m}^{Q^C_m + \beta^C} (x - Q^C_m) f(x) \, dx + \int_{Q^C_m + \beta^C}^{+\infty} \beta^C f(x) \, dx \right]
$$

$$
+ (b_m + w_o) \int_{Q^C_m + \beta^C}^{+\infty} (x - Q^C_m - \beta^C) f(x) \, dx,
$$

where the first item represents the cost of producing the order in stage 1, the second item represents the residual value of unsold products, the third item represents the cost of maintaining additional production capacity, the fourth item represents the cost of producing emergency order, and the fifth item represents the manufacturer’s shortage cost and the purchase cost of social planner purchasing medical supplies from foreign markets.

Next, we examine how $\Pi_{C_2}(Q^C_M, \beta^C)$ changes relative to advance production quantity $Q^C_M$ and additional production capacity $\beta^C$. The results are as follows.

**Lemma 9.** When the manufacturer can provide the emergency supply in a centralized case, $\Pi_{C_2}(Q^C_M, \beta^C)$ is a convex function with respect to $Q^C_M$ and $\beta^C$. When $c_m - r_s > c_\beta$, it reaches the minimum value with the optimal advanced production quantity $Q^C_M^* = F^{-1}\left(\frac{c_m + c_\beta - cm}{c_m - r_s}\right)$ and the optimal additional production capacity $\beta^{C_2}_o = F^{-1}\left(\frac{w_o + b_m - c_e - c_\beta}{w_o + b_m - c_e}\right)$.

Comparing Lemma 4 with Lemma 9, it can be found that since $w_o > w_e$, the optimal overall production capacity in the centralized decision $Q^C_M^* + \beta^{C_2}_o$ is greater than that in the decentralized decision $Q^C_M^* + \beta^C$. And since $r_s \geq r_m$, the manufacturer’s optimal advanced production quantity is $Q^C_M^* \geq Q^C_M$. This is the result of the centralized decision and unified production by the social planner and manufacturer, and there is a strong desire to meet market demand. However, if $c_\beta$ is too small and $r_s$ is close to $c_m$, the optimal additional production capacity in the centralized decision $\beta^{C_2}_o$ may be less than that in the decentralized decision $\beta^C$, which could shrink the emergency revenue of the manufacturer.

**Theorem 3.** In the centralized decision,

1. When $w_e = \frac{c_e - r_s}{c_m - c_\beta - r_s} w_s + r_s$ and $r_m = r_s$, both parties can minimize their own expected cost in stage 1;
2. When the residual values of both parties are equal, that is $r_s = r_m$, the social planner’s additional emergency order range increases from $[0, \beta^C]$ (or $[0, \beta^C]$) to $[0, \beta^{C_2}_o]$, and the manufacturer’s emergency production benefit increases;
3. Only when $w_e = \frac{c_e - r_s}{c_m - c_\beta - r_s} (c_e - r_s) + c_e - b_m < w_o$ will the alliance consisting of the social planner and manufacturer expand additional production capacity for emergency production.

According to Lemmas 9 and 6, if $w_e = \frac{w_o - w_e}{w_o - w_m} w_m + w_m$ and $c_m + c_\beta - cm$ are equal to $c_e + c_\beta - cm$, the optimal advance production quantity in the centralized decision is the same as the expected optimal decision quantity of both parties in the decentralized decision, namely $Q^C_M^* = Q^C_M = Q^C_M$. It means that the optimal centralized decisions are the same as the optimal decentralized decisions in stage 1. And at the same time, the overall production capacity of the manufacturer in the centralized decision is greater, which is consistent with the social planner’s expectation in Lemma 6. Thus, it is more beneficial to the social planner. Besides, it can be seen from Theorem 2 that in the decentralized decision, the maximum value of the social planner’s emergency additional order is $\beta^C$ (or $\beta^C$).
Table 1. Optimal values of decision variables under different supply frameworks.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Decision player</th>
<th>Condition</th>
<th>Optimal values of decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Social Planner</td>
<td>$b_m \geq \frac{(w_0 - r_x)(c_m - r_m)}{(w_s - r_s)}$</td>
<td>$Q_w^* = F^{-1} \left( \frac{w_0 - w_s}{w_0 - r_x} \right)$</td>
</tr>
<tr>
<td>Case 2</td>
<td>Social Planner</td>
<td>$w_s \leq \frac{(w_a - r_x)(c_e - r_m)}{c_m - r_m - c_\beta} + r_s$</td>
<td>$Q_w^* = F^{-1} \left( \frac{w_s - w_a}{w_s - r_s} \right)$</td>
</tr>
<tr>
<td>Manufacturer</td>
<td></td>
<td></td>
<td>$Q_w^* = F^{-1} \left( \frac{b_m - c_m}{b_m - r_m} \right)$</td>
</tr>
<tr>
<td>Case 3</td>
<td>Manufacturer</td>
<td>$w_a \geq \frac{p_1 (c_e - r_m) c_\beta}{c_m - r_m - p_1 c_e - c_\beta} + c_e - b_m$</td>
<td>$Q_w^* = F^{-1} \left( \frac{w_a - w_0 - c_e - c_\beta}{w_a - b_m - c_e - c_\beta} \right)$</td>
</tr>
<tr>
<td>Case 4</td>
<td>Coalition</td>
<td>$w_a \geq \frac{(c_e - r_m) c_\beta}{c_m - r_x} + c_e - b_m$</td>
<td>$Q_w^* = F^{-1} \left( \frac{w_a + b_m - c_e - c_\beta}{w_a + b_m - c_e - c_\beta} \right)$</td>
</tr>
<tr>
<td>Case 5</td>
<td>Coalition</td>
<td>$c_m - r_s &gt; c_\beta$</td>
<td>$Q_w^* = F^{-1} \left( \frac{b_m - c_m}{b_m - r_s} \right)$</td>
</tr>
</tbody>
</table>

and the maximum value of the centralized decision is $\beta^{C^2}$. The manufacturer’s optimal advance production quantity is $Q_w^C = Q_w^*$ when $r_s = r_m$, then $\beta^{C^2} \geq \beta^2$. And when $\beta^{C^2} > 0$, that is, $w_a + b_m - c_e - c_\beta > \frac{c_e + c_\beta - c_m}{c_e - r_x}$, the entire alliance will initiate emergency supply production, otherwise, the whole emergency order will be met from the backup channel.

4.4. Optimal values of decision variables

Through the above analysis of the decentralized case and centralized case, we can obtain the optimal advance production quantity and the optimal additional production capacity of the manufacturer, and the optimal regular ordering quantity of the social planner in each case. In order to clarify the optimal values under different supply frameworks, we define decentralized case without expanding the additional capacity, decentralized case to expand the additional capacity and inform the advance production quantity of the social planner, and decentralized case to expand the additional capacity without informing the advance production quantity of the social planner, centralized case without expanding the additional capacity, centralized case to expand the additional capacity as Case 1, Case 2, Case 3, Case 4, Case 5, respectively. The results are summarized in Table 1.

From Table 1, we find those parameters $w_s$, $c_e$ and $b_m$ play important roles in decision variables. Next, we discuss the influence of their different values on the choice of strategy.
5. Strategy choice under optimal decision variables

In Section 4, we theoretically analyze the optimal cost situations in decentralized and centralized case, and finally obtain their optimal values of decision variables with the minimum cost under certain conditions. In the following, the optimal values of decision variables are brought into the respective expected cost models, and the cost is calculated, and for different strategies corresponding to each case, it is analyzed and judged which strategy is better.

**Strategy 1 – Π₁ (Case 1).** According to equations (1) and (2), combined with Lemma 1, we can get the minimum expected cost of the social planner and manufacturer in the decentralized decision, respectively. The expected cost of the manufacturer:

\[ \Pi_{M_0} \left( Q_{m}^{\ast} \right) = - (w_s - r_m) Q_s^{\ast} + b_m \int_{Q_{m}^{\ast}}^{\infty} x f(x) \, dx. \]

The expected cost of the social planner:

\[ \Pi_{S_0} \left( Q_{s}^{\ast} \right) = \left[ (w_s - w_o) Q_s^{\ast} + (w_o - r_s) Q_s^{\ast} F \left( Q_s^{\ast} \right) \right] + r_s \int_{Q_{s}^{\ast}}^{\infty} x f(x) \, dx + w_o \int_{Q_{s}^{\ast}}^{\infty} x f(x) \, dx. \]

**Strategy 2 – Π₂ (Case 2).** According to equations (3) and (4), combined with Lemmas 4 and 5, we can get the minimum expected cost of the social planner and manufacturer in the decentralized decision, respectively. The expected cost of the manufacturer:

\[ \Pi_{M_1} \left( Q_{m}^{\ast}, \beta^{\ast} \right) = c_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx - w_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx - r_m \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast}} x f(x) \, dx + b_m \int_{Q_{m}^{\ast} + \beta^{\ast}}^{\infty} x f(x) \, dx + Q_s^{\ast} \left[ (w_e - w_s) - (w_e - r_m) F \left( Q_s^{\ast} \right) \right]. \]

The expected cost of the social planner:

\[ \Pi_{S_1} \left( Q_{s}^{\ast}, R^{\ast} \right) = \left[ (w_s - w_e) Q_s^{\ast} + (w_e - r_s) Q_s^{\ast} F \left( Q_s^{\ast} \right) \right] + (w_e - w_o) R^{\ast} \left[ 1 - F \left( R^{\ast} \right) \right] + r_s \int_{Q_{s}^{\ast}}^{Q_{s}^{\ast}} x f(x) \, dx + w_e \int_{Q_{s}^{\ast}}^{R^{\ast}} x f(x) \, dx + w_o \int_{R^{\ast}}^{\infty} x f(x) \, dx. \]

**Strategy 3 – Π₃ (Case 3).** According to equations (5) and (6), combined with Lemma 6 and Theorem 1, we can get the minimum expected cost of the social planner and manufacturer in the decentralized decision, respectively. The expected cost of the manufacturer:

When \( Q_{m}^{\ast} \geq Q_{s}^{\ast} \), we have

\[ \Pi_{M_1} \left( Q_{m}^{\ast}, \beta^{\ast} \right) = c_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx - w_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx + b_m \int_{Q_{m}^{\ast} + \beta^{\ast}}^{Q_{m}^{\ast}} x f(x) \, dx + Q_s^{\ast} \left[ (w_e - w_s) - (w_e - r_m) F \left( Q_s^{\ast} \right) \right]. \]

When \( Q_{m}^{\ast} < Q_{s}^{\ast} \), we have

\[ \Pi_{M_2} \left( Q_{m}^{\ast}, \beta^{\ast} \right) = c_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx - w_e \int_{Q_{m}^{\ast}}^{Q_{m}^{\ast} + \beta^{\ast}} x f(x) \, dx + b_m \int_{Q_{m}^{\ast} + \beta^{\ast}}^{\infty} x f(x) \, dx + Q_s^{\ast} \left[ (w_e - w_s) - (w_e - c_e) F \left( Q_s^{\ast} \right) \right] + (c_m - c_e - c_\beta) Q_{m}^{\ast}. \]
The expected cost of the social planner:

\[ \Pi_{S2}(Q^*_2, R^*_2) = r_s \int_0^{Q^*_2} x f(x) \, dx + w_e \int_{Q^*_2}^{R^*_2} x f(x) \, dx + w_o \int_{R^*_2}^\infty x f(x) \, dx + (w_e - w_o) R^2 \left[ 1 - F(R^2) \right]. \]

Next, in the decentralized case, we compare the expected cost of both parties in different cases. The results are shown in Theorem 4.

**Theorem 4.** In the decentralized case, if both parties' optimal decision variables exist, we have the following conclusions:

1. When \( w_o \leq r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta} \), the social planner expected cost of strategy 2 is less than that in strategy 1, that is \( \Pi_{S2}(S) \leq \Pi_{S1}(S) \); When \( w_e > r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta} \) and \( b_m > \frac{(w_e-r_s)(c_m-r_m)}{w_e-r_s} \), \( w_2 \in (r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta}, w_o) \) such that when \( w_e > w_2 \), we have \( \Pi_{S2}(S) > \Pi_{S1}(S) \).
2. The social planner expected cost in strategy 3 is less than or equal to that in strategy 2, \( \Pi_{S3}(S) \leq \Pi_{S2}(S) \) and always less than that of strategy 1, \( \Pi_{S3}(S) < \Pi_{S1}(S) \).
3. When \( (1+\frac{(c_m-r_m)}{w_e+b_m-c_e})c_\beta \geq c_m-r_m \) and \( \frac{c_m-r_m-c_\beta}{c_m-r_m} \leq \frac{c_m-r_m}{b_m} \leq 0.5 \), there is \( w_1 \in (w_s, r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta}) \) such that when \( w_e < w_1 \), the manufacturer expected cost of strategy 2 is more than that in strategy 1, that is \( \Pi_{M2}(M) > \Pi_{M1}(M) \) and when \( w_e \geq w_1 \), the manufacturer expected cost of strategy 2 is less than or equal to that in strategy 1, that is \( \Pi_{M2}(M) \leq \Pi_{M1}(M) \).
4. When \( w_e \leq c_e + \frac{(w_e-c_e)(c_e-r_m)}{c_m-r_m-c_\beta} \), the manufacturer expected cost in strategy 2 is equal to that of strategy 3, that is \( \Pi_{M2}(M) = \Pi_{M3}(M) \).
   When \( c_e + \frac{(w_e-c_e)(c_e-r_m)}{c_m-r_m-c_\beta} < w_e \leq r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta} \), the manufacturer expected cost in strategy 2 is more than that of strategy 3, that is \( \Pi_{M2}(M) > \Pi_{M3}(M) \).
   When \( r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta} < w_e \leq r_s + \frac{(w_e-r_s)(p_2(c_e-r_m))}{c_m-c_\beta-p_2c_\beta-p_1c_e} \), it is \( w_3 \in (r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta}, r_s + \frac{(w_e-r_s)(p_2(c_e-r_m))}{c_m-c_\beta-p_2c_\beta-p_1c_e}) \) such that when \( w_e = w_3 \), the manufacturer expected cost of strategy 2 is equal to that of strategy 3, \( \Pi_{M2}(M) = \Pi_{M3}(M) \).
   When \( w_e > r_s + \frac{(w_e-r_s)(p_2(c_e-r_m))}{c_m-c_\beta-p_2c_\beta-p_1c_e} \), the manufacturer expected cost of strategy 2 is less than that of strategy 3, that is \( \Pi_{M2}(M) < \Pi_{M3}(M) \).

By comparing the manufacturer and social planner’s expected cost in strategy 1, strategy 2, and strategy 3, we determine the magnitude relationship of different strategies in decentralized case.

Theorem 4-1 shows that for the manufacturer, strategy 2 providing an emergency supply is not always better than strategy 1 without an emergency supply. When \( w_e < w_o < r_s + \frac{(w_e-r_s)(c_e-r_m)}{c_m-r_m-c_\beta} \), the advanced production quantity in strategy 2 can meet the regular order from the social planner and reduce purchase quantity from other regions or countries by the emergency supplies. So, the expected cost of strategy 2 is less than that of strategy 1. However, when the unit cost for emergency orders \( w_e \) is too large and \( w_e > w_2 \), the advance production quantity \( Q_m \) of the manufacturer can only meet a small part of regular orders \( Q_s \) in strategy 2 and unit cost for emergency orders \( w_e \) is too large. Compared to strategy 2, the advance production quantity of strategy 1 can completely meet the regular order from the social planner and reduce purchase quantity from other regions or countries, resulting in the expected cost of strategy 1 lower than that in strategy 2.

Theorem 4-2 shows that for the social planner, strategy 3 providing an emergency supply is always better than strategy 1 without an emergency supply and is not worse than strategy 2 with an emergency supply. It means the social planner prefers strategy 3 to strategies 1 and 2.

Theorem 4-3 shows that for the manufacturer, strategy 2 providing emergency supplies is not always better than strategy 1 without emergency supplies. In particular, when \( w_e < w_1 \), the manufacturer’s unit emergency order is less than the unit’s regular order. Even if the total order quantity with emergency supplies is greater than
that that without emergency supplies, the total revenue is smaller. This will dampen enthusiasm for emergency supplies.

Theorem 4-4 shows that for the manufacturer, strategy 3 providing emergency supplies is not always better than strategy 2 with emergency supplies. Although the manufacturer dominates at stage 1 in strategy 2 and the social planner dominates at stage 1 in strategy 3. However, when \( c_ε + \frac{(w_e-c_ε)(c_ε-r_m)}{c_m-r_m-r_β} < w_e < w_3 \), the expected cost of strategy 2 is less than the expected cost of strategy 3. This is because when the regular order quantity is the same, the remaining advanced production quantity of strategy 3 is higher than that of strategy 2 and emergency production is reduced, so the revenue of emergency supply of strategy 3 is larger and its expected cost is lower.

Therefore, if we can set a suitable \( w_e \), strategy 3 is better than strategies 1 and 2 for both the manufacturer and social planner. Next, we continue to discuss the total expected cost in the centralized case the and respective expected costs for both parties.

**Strategy 4 – Π_4 (Case 4).** According to equation (8), combined with Lemma 8, we can get the minimum expected cost in the centralized decision, respectively.

\[
Π_{C_1} \left( Q_m^C \right) = r_s \int_0^{Q_m^C} xf(x) dx + (w_o + b_m) \int_{Q_m^C}^{+∞} xf(x) x.
\]

**Strategy 5 – Π_5 (Case 5).** According to equation (9), combined with Lemma 9, we can get the minimum expected cost in the centralized decision, respectively.

\[
Π_{C_2} \left( Q_m^C, β^C \right) = r_s \int_0^{Q_m^C} xf(x) dx + c_ε \int_{Q_m^C}^{+∞} xf(x) dx + (w_o + b_m) \int_{Q_m^C}^{+∞} β^C x f(x) dx.
\]

Next, we compare the total expected cost of strategies 4 and 5 in the centralized case and choose a better strategy for the expected cost analysis on both sides.

**Theorem 5.** If the optimal decision variables exist, we have the following conclusions:

1. In the centralized case, the total expected cost of strategy 5 with emergency supplies is lower than that of strategy 4 without emergency supplies.
2. When \( w_e < c_ε + \frac{(w_e-c_ε)(c_ε-r_m)}{c_m-r_m-r_β} \) and \( r_m = r_s \), the total expected cost of strategy 5 in the centralized case is lower than that of strategy 2 or 3 in the decentralized case.
3. It is \( w_4 \in (r_s + \frac{(w_e-r_s)(c_s-r_s)}{c_m-r_s-r_β}, w_o) \) such that when \( w_e > w_4 \), the social planner’s expected cost in strategy 5 in the centralized case is always higher than that of strategy 3 in the decentralized case, that is \( Π_5(S) < Π_3(S) \).
4. When the residual values of both parties are equal, \( r_m = r_s \), the manufacturer’s expected cost of strategy 5 under the centralized case is lower than that of strategy 2 under the decentralized case, that is \( Π_2(M) > Π_5(M) \).

Theorem 5-1 shows that strategy 5 is better than strategy 4 in terms of the total expected cost. However, if the unit cost of producing \( c_ε \) is too high, it can be induced that \( w_e + b_m - c_ε - c_β < \frac{c_ε + c_β - c_m}{c_m - r_m} \) and the additional capacity \( β < 0 \). Since \( β \) is greater than 0 in actual transactions, strategy 5 may be worse than strategy 4 in terms of the total expected cost. Theorem 5-2 illustrates that the total expected cost of strategy 5 is better than that of strategies 2 and 3 with the optimal decision on both sides. This is because the centralized case is more unified than the decentralized case. Theorem 5-3 shows that the social planner’s expected cost of strategy 5 is not always better off than that of strategy 3 that as the social planner dominated at stage 1. However, if the unit cost of emergency orders \( w_e \) is too high, the optimal regular order from the social planner exceeds the advance production, which means the social planner needs to invest more cost into the emergency order. So, setting a suitable \( w_e \) is very incentive to build a supply chain alliance. Theorem 5-4 explains that when the
residual values of both parties are equal, the manufacturer’s expected cost of strategy 5 is always better than that in strategy 3 which has the manufacturer-dominated in stage 1 since the emergency supplies of strategy 5 are greater than that in strategy 3. This means that the manufacturer prefers strategy 5 to strategy 3 when he provides the emergency supply. Combining Theorems 5-1, 5-2 and 5-3, we know that the centralized case that provides emergency supplies can outperform the decentralized case in terms of overall cost, but the individual cost to both parties is not always better off.

Next, we will use numerical experiments to do a more comprehensive comparison of various strategies and analyze the impact of unit cost for emergency orders \( w_e \) on the strategy that provides the emergency supply mechanism. Please refer to Section 6 for the specific results and analysis above.

6. Numerical experiment and result analysis

In this section, we first explore the impact of regular order quantity \( Q_s \) on the expected cost in strategy 3 in which the manufacturer does not inform directly the social planner of the advance production \( Q_m \). Secondly, we analyze the advance production quantity \( Q_m \) and additional production capacity \( \beta \) on the manufacturer and social planner’s expected cost when the emergency supplies are in the supply chain. Then, by comparing expected cost of both parties and the expected total cost in different strategies under different values of unit cost for emergency supply \( w_e \), we can determine the best strategy. Besides, considering the different national conditions of different countries during the epidemic, different strategies are selected [8], and the corresponding ordering plan of medical supplies is given. Finally, we discuss the effect of backup channel purchase cost \( w_o \), demand uncertainty \( \sigma \), and additional production capacity maintenance cost \( c_{\beta} \) on cost for both parties. Based on the real vaccine price (https://www.unicef.org/supply/covid-19-vaccine) and parameters of market demand distribution [28], the parameters are set as \( w_s = 30, w_e = 50, w_o = 105, c_m = 8, c_e = 10, c_{\beta} = 1, r_m = 1, r_s = 3, b_m = 20, u = 671370 \) and \( \sigma = 347 \).

6.1. The impact of regular ordering quantity

In the decentralized case, when the manufacturer informs the social planner of the advanced production quantity \( Q_m \), the manufacturer dominates at stage 1 and hopes that the regular order quantity \( Q_s \) can be close to his advanced production quantity in order to reduce the oversupply losses. And the social planner hopes that the regular ordering quantity is as large as possible when the optimal advance production quantity is less than the optimal order regular quantity, while the expected cost of the social planner can always be minimized when the optimal advance production quantity is more than or equal to the optimal order regular quantity. However, when the manufacturer does not inform the social planner of the advanced production quantity \( Q_m \), the social planner dominates in stage 1 and the expected cost of the social planner can always be minimized. Even if the manufacturer has the optimal plan \( Q_m^3, \beta^3 \), the regular ordering quantity \( Q_s \) has a great impact on the expected cost of the manufacturer. So, we only analyze how the manufacturer’s expected cost of strategy 3 varies with the difference \( Q_s \). The result is shown in Figure 4.

It can be seen from Figure 4 that the manufacturer’s expected cost reaches the local maximum value at \( Q_s' = 671289, Q_s'' = 671369 \) and the local minimum value at \( Q_s^3 = 671321 \), while the social planner’s expected cost decreases and then increases with \( Q_s \) before achieving local extreme values at \( Q_s^2 = 671304 \). Obviously, the optimal decisions of both parties are not consistent, but the optimal decision of the social planner is between \( Q_s' \) and \( Q_s'' \). At the same time, we easily find that the total expected cost of both parties achieves the minimum at \( Q_s = Q_m^3 = 671321 \). So, according to Lemma 6, the manufacturer can make the decision of both parties consistent by increasing the unit cost for emergency orders \( w_e \) or decreasing the unit cost for emergency production \( c_e \) in the decentralized case. So, even if the social planner dominates at stage 1, the manufacturer can adjust the price or cost parameters to minimize his own expected cost.
6.2. The impact of advance production quantity and additional production capacity

Since different strategies provide emergency supply mechanisms, the manufacturer’s overall production capacity has a similar impact on the social planner’s expected cost and manufacturer’s expected revenue, and the pros and cons of decisions are the same as in Section 6.2, only decentralized decision is analyzed. At the same time, we reset the mean and variance of market demand as $u = 100, \sigma = 20$ [27] to reduce run time assuming that it does not affect the result. Next, under the social planner’s optimal regular ordering quantity, the impact of different overall production capacities of the manufacturer on the expected costs and expected revenues of both parties is analyzed, and the results are shown in Figure 5.

It can be seen from Figure 5 that the social planner’s expected cost decreases with the increase of overall production capacity, but when the overall production capacity reaches $R^* = 137$, the expected cost drops slowly, indicating that the manufacturer’s increase in overall production capacity is less effective at this time. As for the manufacturer, when the overall production capacity is too low, the manufacturer’s expected cost is negative, which is because the out-of-stock cost is too high. When the overall production capacity increases to $R^* = 137$ and continues to increase production capacity, there will be the risk of overcapacity, so the revenue will also decrease. At the same time, it can be found that the social planner’s minimum cost is about twice the
manufacturer’s maximum revenue, and that the manufacturer’s revenue is equal to the total cost of the supply chain. It indicates the market demand is basically met when the overall production capacity is $R^3$.

### 6.3. The impact of unit cost for emergency orders on strategies

From Section 4.4, we know that the unit cost for emergency orders determines the optimal production and ordering decision and has a great impact on the expected cost of both parties and the total expected cost in different strategies, especially in the decentralized case. Next, we first discuss the impact of $w_e$ on the expected cost of both parties under different strategies. The result is shown in Figure 6.

Figure 6 shows that the social planner’s expected cost increases with $w_e$ and the manufacturer’s expected cost decreases with $w_e$ when the supply chain has an emergency supply mechanism. And we find that the social planner’s expected cost in the centralized case ($\Pi_5$) is not always less than that in the decentralized case ($\Pi_3$), which is because the optimal advance quantity of the manufacturer deviates from the optimal regular ordering quantity from the social planner. And the manufacturer’s expected cost in the centralized case ($\Pi_5$) is also not always lower than that in the decentralized case ($\Pi_2$), which is because the optimal advance quantity of the centralized case is higher than that in the decentralized case and it reduces the revenue of emergency orders. Only when $w_e$ is about 58, both parties’ expected costs of $\Pi_3$ are no worse than that in other strategies. In the decentralized case, the manufacturer’s expected cost of $\Pi_2$ that the manufacturer dominates at stage 1 is better than that of $\Pi_3$ that the social planner dominates in stage 1, and the social planner’s expected cost is the opposite. Besides, when $w_e > 83$, $Q_1^{1*}$ is much larger than $Q_1^{3*}$ but the real regular order quantity is equal to $Q_1^r$, which increases the cost of emergency orders. So, the social planner’s expected cost of $\Pi_2$ is greater than that in $\Pi_1$ without emergency supplies. When $w_e < 38$, since $w_e$ is too smaller, the manufacturer cannot benefit from the emergency production. So, the manufacturer’s expected costs of $\Pi_2$ and $\Pi_3$ are higher than that of $\Pi_1$. Finally, when $w_e > 100$, since the probability of estimating ($Q_s^* > Q_m^*$) is too large, the optimal advance production quantity of $\Pi_3$ is higher than that of $\Pi_3$, that is not adjusting the advance production quantity. Since the revenue of emergency production is reduced, the manufacturer’s expected cost of $\Pi_3$ is higher than that of $\Pi_5$.

Secondly, we discuss the impact of $w_e$ on the total expected cost of different strategies. The result is shown in Figure 7.

Figure 7 shows that the total expected cost of $\Pi_5$ is better than other strategies, which fully embodies the advantages of centralized decision-making and emergency supplies. In the decentralized case, when $w_e \in (52, 68)$, since the more additional capacity for emergency supplies, the total expected cost of $\Pi_5$ that the social planner dominates in stage 1 and the manufacturer adjusts the optimal advance production quantity is less than that of other strategies. However, as $w_e$ increases, the total expected cost of $\Pi_1$ and $\Pi_3$ increases and is higher than that of $\Pi_4$, that is without emergency supply in centralized case, and $\Pi_3$ is because the manufacturer meets the social
planner’s regular ordering quantity to the greatest extent and reduces his own revenue of emergency supplies; \( \Pi_3 \) is because the social planner has to pay more for emergency order since the unit cost for an emergency order is too high. But when \( w_e > 68 \), \( \Pi_2 \) performs better and gradually stabilizes, means the manufacturer dominating stage 1 has a great impact on the total expected cost.

6.4. Ordering and delivery plans of medical supplies

In the early stage of the outbreak of COVID-19, there are many medical supplies in urgent need except vaccines such as medical protective clothing, isolation clothing, and isolation masks. We further study the ordering scheme for confirming the patient’s need for medical supplies with the same price and cost parameters. From Section 6.2, we know that \( \Pi_2 = \Pi_3 \) and both strategies are better off than \( \Pi_1 \) since \( w_e < 52 \). So, we select strategy 2 or 3 in the decentralized case and strategy 5 in the centralized case. The following is based on the data provided by WHO’s official website (https://covid19.who.int/info/). First of all, we fit the logistic model to the data of confirmed cases in China, the United States, Russia, Australia, Japan, and Germany [22]. Since the data of confirmed cases is too large, in order to make it a better fit, we take the logarithm of the number of confirmed cases to obtain a logarithmic Logistic model, as shown below:

\[
\log(P(t)) = \frac{K}{1 + \left( \frac{K}{P_0} - 1 \right) e^{-rt}}.
\]

Then, we select different decisions based on the national conditions of different countries, set the decentralized case in every country except China, and the centralized case in China. Give each country the latest delivery date for both regular orders and emergency orders, and the optimal combined capacity that the manufacturer should have, as shown in Table 2.

From Table 2, it can be seen that the epidemic situation in Japan and Australia is relatively mild, but it is found that except in China, the delivery date of emergency orders in other countries lasts more than 10 days since the order delivery date in stage 1; while in China, there are only 6 days, and the emergency supply in the short term is the most urgent, which is in line with the development of domestic epidemic at that time. The following is a fitting based on the confirmed data of the epidemic, and the fitting effect is shown in Figure 8.

As can be seen from Figure 8, it can well reflect that in the early stage of the epidemic, the outbreak in China was in January, while in other countries it was from March to April. And we find that the confirmed cases corresponding to the ordering date we provide are all below the \( \frac{K}{P_0} \) line (before the outbreak period), indicating that ordering at this time can not only ensure the minimum total cost input, but also effectively control the epidemic. Meanwhile, we find that the latest delivery dates provided by us, except for Japan, are...
Table 2. Fitting parameters, optimal ordering plans, and decisions for each country.

<table>
<thead>
<tr>
<th>Country</th>
<th>Log (K)</th>
<th>Initial value</th>
<th>Rate of increase</th>
<th>Date-1</th>
<th>Date-2</th>
<th>Decision</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Russian</td>
<td>12.90</td>
<td>0.0356</td>
<td>0.07</td>
<td>2020-04-17</td>
<td>2020-05-08</td>
<td>Decentralized decision</td>
<td>365 404</td>
</tr>
<tr>
<td>Germany</td>
<td>12.20</td>
<td>0.0817</td>
<td>0.08</td>
<td>2020-03-24</td>
<td>2020-04-14</td>
<td>Decentralized decision</td>
<td>208 441</td>
</tr>
<tr>
<td>Japan</td>
<td>9.70</td>
<td>0.5907</td>
<td>0.05</td>
<td>2020-04-17</td>
<td>2020-05-30</td>
<td>Decentralized decision</td>
<td>82 275</td>
</tr>
<tr>
<td>Australia</td>
<td>9.20</td>
<td>0.2249</td>
<td>0.06</td>
<td>2020-03-13</td>
<td>2020-03-30</td>
<td>Decentralized decision</td>
<td>3342</td>
</tr>
<tr>
<td>USA</td>
<td>14.80</td>
<td>0.5592</td>
<td>0.05</td>
<td>2020-04-08</td>
<td>2020-05-30</td>
<td>Decentralized decision</td>
<td>2070 774</td>
</tr>
<tr>
<td>China</td>
<td>11.30</td>
<td>0.8674</td>
<td>0.14</td>
<td>2020-01-23</td>
<td>2020-01-29</td>
<td>Centralized decision</td>
<td>8655</td>
</tr>
</tbody>
</table>

Notes. Date-1 represents the delivery date of the regular order in stage 1; Date-2 represents the delivery date of the emergency order in stage 2.

Figure 8. Fitting effect of confirmed cases and discrimination of order rationality.

all before the confirmed cases reach the $K_2$ line, which means that prevention and control measures are taken before the outbreak of the epidemic, which can ensure the minimum cost to the social planner. As for Japan, due to the slow development of the epidemic in the first half of 2020, even if the confirmed cases at the time of $K_2$ line reached the social planner’s ordering quantity, the follow-up prevention and control work and the latest delivery date will be advanced to before April 10. Since the epidemic comes all of a sudden, the process of submitting orders first and then trading could lead to a shortage of medical supplies, while the process of advance production, trading, and emergency production provided by us can largely meet the surging demand to alleviate the situation.
Table 3. Numerical result of parameter analysis.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( Q^*_3 )</th>
<th>( R^*_3 )</th>
<th>( Q^*_m )</th>
<th>( \beta^*_3 )</th>
<th>( -\Pi_3(M) )</th>
<th>( -\Pi_3(S) )</th>
<th>( -\Pi_3 )</th>
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<tbody>
<tr>
<td>( w_o )</td>
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<td>96</td>
<td>137</td>
<td>97</td>
<td>40</td>
<td>2420.75</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>96</td>
<td>137</td>
<td>97</td>
<td>40</td>
<td>2420.75</td>
<td>0</td>
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<tr>
<td></td>
<td>100</td>
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<td>( \sigma )</td>
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Notes. \( \% \): represents the cost change rate of social planner and manufacturer respectively under the setting \( w_o = 105 \), \( \sigma = 20 \), \( c_e = 10 \), \( r_s = 3 \) and \( c_\beta = 1 \).

6.5. Sensitivity analysis

In the process of establishing theory, we find several important parameters except for unit emergency order cost \( w_e \), including unit foreign procurement cost \( w_o \), unit cost of emergency production \( c_e \), unit reserved cost of additional production capacity \( c_\beta \), the unit residual value of unsold production \( r_s \) and variance of the density function \( \sigma \). In order to fully reflect the degree of influence of parameters on the results, we reset the mean and variance of market demand as \( u = 100 \) and \( \sigma = 20 \), and select strategy 3 in decentralized case as the object of analysis since other strategies are same as it. These parameters are analyzed below as Table 3.

From Table 3, we find the following conclusions. in the decentralized case, the social planner’s unit foreign procurement cost \( w_o \) is negatively correlated with the own expected cost, but not correlated with the manufacturer’s revenue, and it has little impact on the social planner’s expected cost and the overall expected cost of the supply chain. As \( w_o \) increases, the optimal regular ordering quantity \( Q^*_3 \), the optimal advance production quantity \( Q^*_m \) and the overall production capacity \( R^*_3 \) remain unchanged, which means the emergency supply mechanism can fully meet the social planner’s demand. Secondly, we find that the increase in the uncertainty of market demand will reduce the optimal advance production quantity, but it increases the...
optimal additional production capacity of the manufacturer, which will greatly increase the emergency supplies’ revenue of the manufacturer and each growth range will increase by about 2.28%. However, the increase in the uncertainty of market demand will also increase the risk of overcapacity for the manufacturer. At the same time, for the social planner, the increased uncertainty of demand will increase the quantity of the social planner’s emergency orders, and the cost of purchasing emergency orders will increases, which will lead to an increase in the cost of the entire supply chain of 4.21%. Therefore, it is particularly important for the supply chain to get information about market demand uncertainty. Furthermore, with the increase of $c_\alpha$ and $c_\beta$, $Q_3^*$ remains unchanged, $Q_m$ increases and $\beta^*$ decreases. It shows that the manufacturer will decrease the emergency production since the unit cost for emergency production and the revenue of the manufacturer is greatly affected, especially with the increase of $c_\beta$. It dampens the manufacturer’s enthusiasm for emergency production. Therefore, the increase in advance production and reduction in additional production capacity increases the risk of hoarding costs, which reduces the revenue of the manufacturer. At the same time, due to the insufficient overall production capacity, the social planner may be forced to purchase at a high price, leading to higher costs. Finally, the unit residual value of unsold production $r_s$ is only related to the social planner. As $r_s$ increases, decision variables barely change, and the range increase amplitude of the total expected cost is only about 0.35%, indicating that the unit residual value of unsold production has little impact on the decentralized case.

Based on the above analysis, we know that the uncertainty of market demand and the size of the reserved cost has a greater impact on both parties. So, in practice, we respond to the more accurate forecast of market demand and control emergency supply costs to stimulate emergency production from the manufacturer.

7. Conclusions

Since the outbreak of COVID-19, there is surging demand for medical supplies all over the world, especially mask, protective clothing, and vaccine. However, unbalanced production, the sudden advent of the epidemic, and other factors interrupt the entire supply. To alleviate this situation, we provide the advanced production and emergency supply mechanism and build a manufacturer-dominated supply chain, enabling quick response to emergency orders. Based on the mechanism, we study several strategies in the decentralized case and centralized case, respectively. Firstly, in the decentralized case, we provide strategy 1 without emergency supply, strategy 2 with emergency supply and informing the social planner of advanced production quantity, and strategy 3 with emergency supply but does not inform the social planner of advanced production quantity of social planner. And in the centralized case, we also provide strategy 4 without emergency supplies and strategy 5 with emergency supplies. Secondly, we determine the optimal production and ordering decision variables in different strategies. Under the optimal decision variables, we find that when the unit cost for emergency orders $w_e$ is too high or too low, the supply chain established emergency supplies is not good for the social planner or manufacturer by comparing the expected cost of both parties. At the same time, strategy 5 is better than other strategies in terms of the total expected cost, while the total expected cost of the social planner and manufacturer in strategy 5 is always not lower than that in strategies 2 and 3. Even, the social planner is more inclined to follow the decentralized case. Next, by conducting some numerical experiment combined with the real data, we find that the advanced production and emergency supply mechanism fully meets the market demand and greatly reduce the purchase cost from the alternate channel. And interestingly, the social planner tends to adopt strategy 3 instead of strategy 5 and the total expected cost in strategy 3 is close to that in strategy 5 when $w_e$ is about 63, which indicates the decentralized case is more advantageous in a pandemic environment. Besides, according to our proposed strategies, we develop the ordering and delivery plans for six countries including China, the United States, Japan, and so on. Finally, through sensitivity analysis, the uncertainty of market demand $\sigma$ and unit reservation cost $c_\beta$ shows a great impact on different strategies. So, manufacturers’ accuracy of market demand forecast and control of emergency supply cost all play important roles in the supply chain.
Acknowledgements. This work was supported in part by National Natural Science Foundation of China (Nos. 72272046, 71802065), National Social Science Foundation of China (No. 22FJL020), Humanities and Social Science Fund of Ministry of Education of China (No. 22YJC630024).

Conflict of interest. The authors declared that they have no conflicts of interest to this work.

Data availability. All data included in this study are available upon request by contact with the corresponding author.

References