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# THE EFFECTS OF GREEN HOUSE GAS COSTS ON OPTIMAL PRICING AND PRODUCTION LOTSIZE IN AN IMPERFECT PRODUCTION SYSTEM

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**Abstract.** The present article deals with an imperfect production system considering costs for green-house gas (GHG) to determine the optimal reserve selling price, sales teams' efforts and production lotsize. As per government guidelines, the manufacturer used to adopt green practices in supply chain management to meet the customers' satisfaction regarding fair prices and quality of the products. In this connection, a mathematical model is formulated and analyzed considering various cost factors and interval values of the key parameters. Finally, numerical illustrations are considered to justify the proposed model.

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## 1. Introduction

It is common belief that all items produced by any mechanical systems are not perfect quality and consequently a certain percent of the whole products is an imperfect quality products. Generally speaking, the manufacturing industries are in front of severe competition for achievement of economical benefits. This competition is increasing day to day because many alternatives are available in global marketing systems. As a result, manufacturing companies are compelled to introduce many innovative approaches to sustain their businesses keeping in mind a brand image of quality and fair prices. In this manufacturing system, imperfect quality items are reworked as new as original one in a parallel system or after the end of regular production. This approach is implemented to reduce the production cost as well as reduce the utility of nonrenewal natural resources.

In 21st Century, the environment is being polluted by green house gases (GHG) due to enormous increase of population and advancement of civilization. The GHG mainly includes carbon-di-oxide, carbon monoxide methene sulphur oxides, etc. The emission of carbon-di-oxide is generated more in GHG from industries, transportation and power plants. To save and keep healthy our environment, green technologies are used to reduce carbon emission. Consequently, more investments in technologies and marketing management system are required to make sustainable business atmosphere. Several innovative approaches are applied on the consumers to be a Grey Consumers who are ignorant about green value of the products on the environment. The perceptive research on green products takes much time and deliberate effort of the consumers. As the prices of the green product is comparatively higher than nongreen products because of its more investment in technology and

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packaging system which is not generally preferred by the consumers. In such cases, more awareness programmes in green consuming and information on the environmental consequences of the products are needed to sustain healthy business environment.

Quite often, more items displayed in a supermarket attracts the customers to buy more as there are more options to choose the products. Besides stock level, price and quality of the product play important roles to boost the sale. Considering these issues, imperfect quality products are reworked at a cost to make it as new as original one and the selling price is determine based on the costs involved in the whole system and the production lotsize. The objective of the present article is to obtain the optimal selling price and production lotsize considering the costs of raw materials, costs for carbon emission in production and transportation system, cost of disposal of waste products, labour and energy costs, tool/die costs, inventory cost while demand of the products is dependent of stock level and selling price. Moreover, the selling price varies with present state of price and level of the stock. In this model, all key parameters are considered as closed intervals and then the average profit function is maximized to obtain optimum selling prices and production quantity.

Generally speaking, sales teams' efforts/initiatives play an imperative role to introduce any green products in a new market or a society. The sales teams' efforts include many sales promotion methods like demonstrations, exhibitions, eco-friendly brand awareness, boosting customer satisfaction, charitable promotion techniques, free shipping and returns, free service within warranty period, referral discounts, branded gifts or bundles, cashback promotion and many more. These sales promotions are the most affective methods to boosting the sales as well as the customer satisfaction. As a result, there are no alternatives to capture a market of a green products without sales promotion methods mentioned above. Although a significant fund is required to implement the above efforts, the companies/firms would increase sales volumes that results in more profit by boosting customer satisfaction and intensifying brand awareness of the green products. The pictorial representation of the proposed model is given in Figure 1.

The rest of the is organized as follows: The Section 2 includes literature survey. The Section 3 provides assumptions and notations of the proposed model. The Section 4 formulates the mathematical model and its analysis. The Section 5 illustrates numerical examples and Section 6 provides managerial insights of the model. Finally, conclusions have been drawn in Section 7.

## 2. Brief literature

In last decades, many academic researchers and industrial practitioners are deliberately searching alternative advanced techniques to diminish GHG emission from manufacturing industries, power generating systems, transportation systems and human activities due to rapid increase in population and human civilization. As a result, the inspiring and preventing policies are taken by the government organizations to reduce GHG emission and develop sustainability of the human civilization [36]. Ghosh *et al.* [10] suggested a dual-channel supply chain model where the demand of the customers was carbon emission sensitive stochastic pattern based on governments' cap-and-trade guidelines and consumers' low carbon preferences. Mishra *et al.* [23] proposed a price-sensitive demand to obtain optimal strategies related to price setting, investment in green technologies and preservation systems for inventory. Later, Sarkar *et al.* [35] suggested a production inventory model for imperfect production system considering the impacts of carbon emission and quality of the products for fixed life time products. Han *et al.* [11] determined the optimal degree of carbon emission reduction and selling price considering carbon taxes imposed by the government. In this direction, a large number of research work have been done. Among those, the works done by Marchenko [19], Tseng and Hung [42], Sarkar *et al.* [34], Yang *et al.* [45], Bai *et al.* [3], Daryanto *et al.* [8], Sinha and Modak [38], Manna *et al.* [18], Mandal and Pal [17], Marquez *et al.* [20], Mishra *et al.* [24], Radovanovic *et al.* [30] and Vandana *et al.* [43] are worth mentioning.

In practice, all machinery systems undergo "out of control" state after a certain time and the systems starts to produce imperfect quality items which is a certain percent of the whole products. The manufacturing systems shifts from "in-control" to "out-of-control" state not only for its machinery parts. This fact occurs due to inefficient labours and hastily production rate. To maintain the quality of the products and proper utilize of the


FIGURE 1. The schematic diagram of the production-inventory systems.

resources, these imperfect quality products are reworked at a cost to make it as new as original one in a parallel or after completion of regular production system. Therefore, supply chain coordination, quality and fair price of the products are important factors to sell the products to the customers efficiently. Heydari *et al.* [12] studied supply chain coordination structure where uncertain demand were affected by the money-back guarantee offered to the customers. Lee and Lee [15] investigated a correlated multi-item continuous review inventory problem to obtain optimal values of the reorder point and order quantity. Avramidis [2] developed a pricing model with finite inventory and semi-parametric demand uncertainty. Somogyi [39] lightened the importance of local monopoly power in price setting based on a Bertrand–Edgeworth duopoly with exogenous capacity constraints and a significant degree of product differentiation. Tasnadi [40] obtained analytically a symmetric mixed-strategy equilibrium of the production-in-advance form of this game for a bulky region of transitional volumes. Oller [27] analyzed a duopoly model of divisionalisation and Cournot competition in two firms where demand and selling price are inter-connected. He showed that both firms were coexisted in equilibrium only when the cost variation is small enough and division are preferred sequentially.

Since the large piles of goods displayed in an enterprise fascinates the customers to buy more, the organizations prerequisite to regulate the inventory level as well as optimal selling prices for the products in order to inspire the customers to buy more and more products [9,22,28,32,33,37,41]. As a result, the research scholars, academicians and investigators are being deliberately inclined to develop the sustainable supply chain models assuming stock and price sensitive demand pattern for deterministic as well as uncertain cases. In this line of research works, Lim [16] developed stock-dependent inventory model in which demand depends on a monomial function whose shape and scale parameters are uncertain in nature. Cardenas-Barron *et al.* [6] investigated the EOQ model for nonlinear stock-sensitive demand and nonlinear holding cost for retailer's viewpoint where the supplier offered trade-credit facility to the retailer. Zang *et al.* [46] analysed the retailer's strategy to obtain optimal reorder level, investment for preservation technology for perishable items and order lot-size for maximum profit of the enterprise taking into account of stock dependent nonlinear holding cost and a stock dependent demand rate.

The sales teams efforts have been used for decades as most effective method to attract the customers to buy more. Its main objective is to entice the channel members of a supply chain to drive sales, make available information about the brand and fair prices, and remind the eco-friendly effect on the environment of the green products. Cardenas-Barran and Sana [5] extended a production-inventory model of two layer supply chain system where demand of the end customers is dependent on sales teams' initiatives. They analyzed the profit functions of both the manufacturer and retailer at maximum level to obtain the optimal production rate, production lot size, backlogging and the initiatives of sales teams. Nasir and Bal [26] studied the impact of sales promotional efforts on consumer buying behavior in an emerging market and they exposed that the consumers are more inclined to price discounts offers on buying behavior in terms of brand switching, stockpiling, purchase acceleration, and product trial. Khan and Warraich [14] surveyed the buying behaviour of the consumers on four promotional efforts: price discount, buy 1 get 1, coupons, sweepstakes and games, They revealed that more consumers are likely to purchase at price discount offers. The works of Chaudhuri *et al.* [7], Raji *et al.* [31], Buyukdag *et al.* [4], Venegas and Ventura [44], Heydari *et al.* [13], Nasir and Bal [26], Qazi *et al.* [29] and Al-Sahli [1] are noteworthy in this line of research works.

Most research articles investigated the stock and price dependent demand pattern where price is a decision variable for fixed key parameters. Few studies considered the key parameters as interval valued function. The proposed article considers the cost parameters as interval valued function and the price and on-hand inventory follow interconnected differential equations. The rate of change of selling price increases with its current state of price and it decreases with on-hand inventory level. This features occur with time and level of stock due to higher demand or to clear the stock at optimum time. This new investigation based on mathematical analysis bridges the research gap between the existing literature and the proposed model. Additionally, the initial selling price and production lot size are assumed to be decisions variables considering various costs related to production and GHG emission. As far as the author's knowledge goes, such type of mathematical investigation in production-inventory literature is not studied earlier.

## 3. Assumption and notation

The following assumptions and notations are considered to depict the proposed model.

### 3.1. Assumption

(i) The manufacturing system is an imperfect system. The imperfect items are produced in this system and reworked at a cost after end of the regular production or in a parallel production system. The reworked items are perfect items as new as original one.

(ii) The manufacturing firm produces the items instantly according to the lotsize required to place in display of its own retail channel. The manufacturing system follows just-in-time (JIT) philosophy and consequently the cycle length is considered for retail channel only.
(iii) The effect of GHG emission is considered to calculate optimal selling prices. The initial reserve price is a decision variable.
(iv) The cycle length of retail channel follows both the finite and infinite time horizon.
(v) The manufacturing and transportation systems use green technologies to reduce GHGs emission which result in higher production cost at early stage but lower production cost in long term projects. The cost parameters related to GHG emission depends upon advance technology. More investment in green technology reduces the costs of GHG emission.
(vi) The governmental guidelines of pollution control board and the cost for carbon cap and trade are followed by the manufacturer.
(vii) The cycle length of the retail channel follows both the finite and infinite time horizon.

### 3.2. Notation

| | |
|---|---|
| $[x^l,\ x^u]$ | This is a closed interval where $x^l(>0)$ is lower bound of $x$ and $x^u(>0)$ is upper bound of $x$. |
| $\rho\epsilon[0,1]$ | The shadow index of inflation of currency. |
| $\mu\epsilon[0,1]$ | The index of inflation of quantity parameter. |
| $D(t)$ | The demand rate of the products at time $t$. |
| $I(t)$ | The on-hand inventory at time $t$. |
| $P(t)$ | The sales price at time $t$. |
| $E(t)$ | The sales effort at time $t$. |
| $\dot{I}(t)$ | The derivative of $I(t)$ with respect to time $t$. |
| $\dot{P}(t)$ | The derivative of $P(t)$ with respect to time $t$. |
| $P_0$ | The initial reserve selling price of the product. |
| $Q$ | The production lot size for the cycle $T$. |
| $T=[T^l,\ T^u]$ | The cycle length of the retail channel of the manufacturer. |
| $R=[R^l,\ R^u]$ | The production rate. |
| $d_0=[d_0^l,\ d_0^u]$ | The approximated value of the capacity of the market that is a constant part of the demand rate. |
| $d_1=[d_1^l,\ d_1^u]$ | The proportional constant of the increment of demand rate related to the onhand inventory at time $t$. |
| $d_2=[d_2^l,\ d_2^u]$ | The proportional constant of the decrement of demand rate related to the onhand inventory at time $t$. |
| $d_3=[d_3^l,\ d_3^u]$ | The proportional constant of the increment of demand rate related to the sales effort at time $t$. |
| $E_0=[E_0^l,\ E_0^u]$ | The initial sales effort level at time $t=0$. |
| $\gamma=[\gamma^l,\ \gamma^u]$ | The scale parameter of cost per unit volume of sales effort. |
| $\alpha=[\alpha^l,\ \alpha^u]$ | The scale parameter of growth rate of the selling price. |
| $\beta=[\beta^l,\ \beta^u]$ | The scale parameter of diminishing rate of the selling price. |
| $\theta=[\theta^l,\ \theta^u]$ | The percentage of the imperfect products during manufacturing process. |
| $C_{10}=[C_{10}^l,\ C_{10}^u]$ | The cost of GHG emission per unit time of regular production run time of the whole production lot size. |
| $C_{11}=[C_{11}^l,\ C_{11}^u]$ | The cost of GHG emission per unit time of production run time of reworking of the imperfect products. |
| $C_{20}=[C_{20}^l,\ C_{20}^u]$ | The fixed cost of disposal of the waste products during manufacturing. |
| $C_{21}=[C_{21}^l,\ C_{21}^u]$ | The cost of disposal of the waste per unit item of the products during manufacturing. |

| | |
|---|---|
| $C_{30} = [C_{30}^l,\ C_{30}^u]$ | The cost of GHG emission per unit vehicle for transportation of the products to the customers. |
| $C_T = [C_T^l,\ C_T^u]$ | The capacity per vehicle to transport the products. |
| $C_{40} = [C_{40}^l,\ C_{40}^u]$ | The cost of GHG emission per unit time during preservation of the products at retail channel. |
| $C_{41} = [C_{41}^l,\ C_{41}^u]$ | The cost of GHG emission per unit time during stock of inventory. |
| $C_{42} = [C_{42}^l,\ C_{42}^u]$ | The cost of inventory per unit per unit time for other costs of energy consumption. |
| $C_{50} = [C_{50}^l,\ C_{50}^u]$ | The cost of raw materials per unit item. |
| $C_{51} = [C_{51}^l,\ C_{51}^u]$ | The cost of technology, energy and labours to produce the whole product. |
| $C_{52} = [C_{52}^l,\ C_{52}^u]$ | The cost of tool/die costs per unit item. |

### 3.3. Preliminary concept and definition

Let an interval number $A\epsilon[a^l, a^u]$ where $a^l\epsilon R^+$ and $a^u\epsilon R^+$ are lower and upper bounds of the number$A$. $B\epsilon[b^l, b^u]$ where $b^l\epsilon R^+$ and $b^u\epsilon R^+$ are lower and upper bounds of the number $B$. Then, the interval valued function is $f(\rho) = (a^l)^{1-\rho}(a^u)^\rho$ where $\rho\epsilon[0, 1]$ and $f(\rho)\epsilon[a^l, a^u]$. Now, the addition, subtraction, scalar multiplication, multiplication and division properties of the interval are as follows:

**Addition property.** Let $A = [a^l, a^u]$ and $B = [b^l, b^u]$. Then $A + B = [a^l + b^l, a^u + b^u]$ and the intervalued function would be $f(\rho) = (a^l + b^l)^{1-\rho}(a^u + b^u)^\rho$ where $\rho\epsilon[0, 1]$.

**Subtraction property.** Let $A = [a^l, a^u]$ and $B = [b^l, b^u]$. Then $A - B = [a^l - b^u, a^u - b^l]$ provided $a^l - b^u > 0$, $(a^u - b^l) > (a^l - b^u)$ and the intervalued function would be $f(\rho) = (a^l - b^u)^{1-\rho}(a^u - b^l)^\rho$ where $\rho\epsilon[0, 1]$.

**Scalar multiplication property.** Let $A = [a^l, a^u]$, then $\gamma A = [\gamma a^l, \gamma a^u]$ when $\gamma \geq 0$ and $\gamma A = [\gamma a^u, \gamma a^l]$ when $\gamma < 0$. The interval valued function for $\gamma A$ would be $\{f(\rho) = (\gamma a^l)^{1-\rho}(\gamma a^u)^\rho, \forall\gamma \geq 0\}$ and $\{f(\rho) = -(|\gamma|a^u)^{1-\rho}(|\gamma|a^l)^\rho, \forall\gamma < 0\}$ where $\rho\epsilon[0, 1]$.

**Multiplication property.** Let $A = [a^l, a^u]$ and $B = [b^l, b^u]$. Then $A \times B = [a^l b^l, a^u b^u]$ and the intervalued function would be $f(\rho) = (a^l b^l)^{1-\rho}(a^u b^u)^\rho$ where $\rho\epsilon[0, 1]$.

**Division property.** Let $A = [a^l, a^u]$ and $B = [b^l, b^u]$. Then $A/B = [a^l/b^u, a^u/b^l]$ and the intervalued function would be $f(\rho) = (a^l/b^u)^{1-\rho}(a^u/b^l)^\rho$ where $\rho\epsilon[0, 1]$.

Therefore, the interval values of the key parameters are as follows: $\hat{d}_0 = (d_0^l)^{1-\mu}(d_0^u)^\mu$, $\hat{d}_1 = (d_1^l)^{1-\mu}(d_1^u)^\mu$, $\hat{d}_2 = (d_2^l)^{1-\mu}(d_2^u)^\mu$, $\hat{d}_3 = (d_3^l)^{1-\mu}(d_3^u)^\mu$, $\hat{E}_0 = (E_0^l)^{1-\mu}(E_0^u)^\mu$, $\hat{\alpha} = (\alpha^l)^{1-\mu}(\alpha^u)^\mu$, $\hat{\beta} = (\beta^l)^{1-\mu}(\beta^u)^\mu$, $\hat{\theta} = (\theta^l)^{1-\mu}(\theta^u)^\mu$, $\hat{T} = (T^l)^{1-\mu}(T^u)^\mu$, $\hat{R} = (R^l)^{1-\mu}(R^u)^\mu$, $\hat{\gamma} = (\gamma^l)^{1-\rho}(\gamma^u)^\rho$, $\hat{C}_{10} = (C_{10}^l)^{1-\rho}(C_{10}^u)^\rho$, $\hat{C}_{11} = (C_{11}^l)^{1-\rho}(C_{11}^u)^\rho$, $\hat{C}_{20} = (C_{20}^l)^{1-\rho}(C_{20}^u)^\rho$, $\hat{C}_{21} = (C_{21}^l)^{1-\rho}(C_{21}^u)^\rho$, $\hat{C}_{30} = (C_{30}^l)^{1-\rho}(C_{30}^u)^\rho$, $\hat{C}_T = (C_T^l)^{1-\rho}(C_T^u)^\rho$, $\hat{C}_{40} = (C_{40}^l)^{1-\rho}(C_{40}^u)^\rho$, $\hat{C}_{41} = (C_{41}^l)^{1-\rho}(C_{41}^u)^\rho$, $\hat{C}_{42} = (C_{42}^l)^{1-\rho}(C_{42}^u)^\rho$, $\hat{C}_{50} = (C_{50}^l)^{1-\rho}(C_{50}^u)^\rho$, $\hat{C}_{51} = (C_{51}^l)^{1-\rho}(C_{51}^u)^\rho$, $\hat{C}_{52} = (C_{52}^l)^{1-\rho}(C_{52}^u)^\rho$.

## 4. Model formulation

In this model, an imperfect production system is considered where imperfect products are used in recycling process to make those as new as perfect one's. In this manufacturing system, transportation systems and storage systems for the inventory of finished products generate GHG. Carbon emission is a major issue in GHG effect on the environment. In this context, the proposed article formulates a mathematical model considering the possible parameters of the following factors: (i) Carbon emission cost due to preservation of finished products. (ii) Carbon emission cost due to transportation of delivery of the items to the customers. (iii) Disposal cost due to manufacturing the items. (iv) Carbon emission cost due to manufacturing the items. (v) Production cost per unit item which includes cost of raw materials, labour cost, cost of technology, tool or die costs, etc.

In this production system, the manufacturer estimates the total demand of the market where demand depends on selling price and stock-level of the products. Here, the production lotsize is $Q$ which are produced beforehand of the delivery of the products to the customers. Since the demand is varying with sales price and on-hand inventory, the sales price depends on stock-level and its existing price. Therefore, the demand function at time "$t$" is

$$D(t) = \hat{d}_0 + \hat{d}_1 I(t) - \hat{d}_2 P(t) \tag{1}$$

where $I(t)$ and $P(t)$ are on-hand inventory and sales price at time "$t$" respectively; $\hat{d}_0$ is deterministic market size; $\hat{d}_1$ and $\hat{d}_2$ are demand sensitive parameters to on-hand stock and sales price of the product respectively. The governing differential equations of on-hand inventory and sales price are as follows:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\hat{d}_0 - \hat{d}_1 I(t) + \hat{d}_2 P(t) \tag{2}$$

$$\text{with } I(0) = Q \text{ and } I(\hat{T}) = 0$$

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = \hat{\alpha} P(t) - \hat{\beta}\frac{\mathrm{d}I(t)}{\mathrm{d}t} \tag{3}$$

$$\text{with } P(0) = P_0$$

where $\hat{\alpha}$ denotes continuous growth rate in respect of $P(t)$ and $\hat{\beta}$ denotes continuous decay rate in respect of the change of stock level. Let $Q$ items per lotsize are produced. As the production system is imperfect system, the perfect items (good quality) are $(1-\hat{\theta})Q$ and imperfect items are $\hat{\theta}Q$ where $0 \leq \hat{\theta} \leq 1$. The GHG emission cost during production is

$$\hat{C}_1(Q) = \hat{C}_{10}\Big(Q/\hat{R}\Big) + \hat{C}_{11}\ \hat{\theta}\Big(Q/\hat{R}\Big) \tag{4}$$

where $\hat{R}$ is production rate, $\hat{C}_{10}$ is carbon emission cost during production lot size $Q$ and $\hat{C}_{11}$ is carbon emission cost during recycling. The disposal cost due to waste products during production is

$$\hat{C}_2(Q) = \hat{C}_{20} + \hat{C}_{21} Q. \tag{5}$$

The emissions of GHG from transportation systems is a continuing hazard to the global climate change which threatens to change many natural systems in random ways. Carbon dioxide and Carbon monoxide are the major sources of GHG which are generated from the combustion of gasoline, natural gas, and other fuels. Several economic models with climate variation situations have been used to monetize greenhouse gases. The social cost of carbon is estimated by integrated models of climate change with economic growth and the forthcoming cost of the damage characteristics to climate change. Consequently, Carbon Cap-and-Trade regulations are established at a socially optimal level such that the cost of mitigation does not overdo the social costs of the pollution. The GHG emission cost due to transportation may be reduced by using renewable energies like battery electrification, biomass derived fuels, solar energy, wind energy and synthetic fuels. The GHG emission cost due to transportation of delivery items to the customers is

$$\hat{C}_3(Q) = \hat{C}_{30} Q/\hat{C}_T. \tag{6}$$

The holding cost due to preservation of the inventory is

$$\hat{C}_4(Q, P_0) = \hat{C}_{40} T + \Big(\hat{C}_{41} + \hat{C}_{42}\Big)\int_0^T I(t)\,\mathrm{d}t. \tag{7}$$

The production cost per unit item is

$$\hat{C}_5(Q) = \hat{C}_{50} + \hat{C}_{51}/Q + \hat{C}_{52} Q. \tag{8}$$

Here, $\hat{C}_5(Q)$ is a convex function of $Q$ and it attains minimum value at $Q = \sqrt{\hat{C}_{51}/\hat{C}_{52}}$. Solving equations (2) and (3), we have

$$I(t) = a_1 e^{m_1 t} + b_1 e^{m_2 t} - \Big(\hat{d}_0/\hat{d}_1\Big) \tag{9}$$

$$P(t) = a_2 e^{m_1 t} + b_2 e^{m_2 t} \tag{10}$$

where $m_1 = \frac{1}{2}(-(\hat{d}_1 + \hat{\beta}\ \hat{d}_2 - \hat{\alpha}) + \sqrt{(\hat{d}_1 + \hat{\beta}\ \hat{d}_2 - \hat{\alpha})^2 + 4\hat{\alpha}\hat{d}_1});$ $m_2 = \frac{1}{2}(-(\hat{d}_1 + \hat{\beta}\ \hat{d}_2 - \hat{\alpha}) - \sqrt{(\hat{d}_1 + \hat{\beta}\ \hat{d}_2 - \hat{\alpha})^2 + 4\hat{\alpha}\ \hat{d}_1});$ $a_1 = x_{11}P_0 - x_{12}Q - x_{13}$; $b_1 = -y_{11}P_0 + y_{12}Q + y_{13}$; $a_2 = x_{21}P_0 - x_{22}Q - x_{23}$; $b_2 = -y_{21}P_0 + y_{22}Q + y_{23}$; $x_{11} = \hat{d}_2/(m_1 - m_2)$; $x_{12} = (m_2 + \hat{d}_1)/(m_1 - m_2)$; $x_{13} = \hat{d}_0(m_2 + \hat{d}_1)/(\hat{d}_1(m_1 - m_2))$; $y_{11} = \hat{d}_2/(m_1 - m_2)$; $y_{12} = (m_1 + \hat{d}_1)/(m_1 - m_2)$; $y_{13} = \hat{d}_0(m_1 + \hat{d}_1)/(\hat{d}_1(m_1 - m_2))$; $x_{21} = x_{11}(m_1 + \hat{d}_1)/\hat{d}_2$; $x_{22} = x_{12}(m_1 + \hat{d}_1)/\hat{d}_2$; $x_{23} = x_{13}(m_1 + \hat{d}_1)/\hat{d}_2$; $y_{21} = y_{11}(m_2 + \hat{d}_1)/\hat{d}_2$; $y_{22} = y_{12}(m_2 + \hat{d}_1)/\hat{d}_2$; $y_{23} = y_{13}(m_2 + \hat{d}_1)/\hat{d}_2$. Here, $I(\hat{T}) = 0$ provides

$$Q = z_{11}P_0 + z_{12} - z_{13} \tag{11}$$

where $m_1 \neq m_2$ as $(\hat{d}_1 + \hat{\beta}\hat{d}_2 - \hat{\alpha})^2 + 4\hat{\alpha}\hat{d}_1 \neq 0$; $z_{11} = (e^{m_1\hat{T}} - e^{m_2\hat{T}})\hat{d}_2/((m_2 + \hat{d}_1)e^{m_1\hat{T}} - (m_1 + \hat{d}_1)e^{m_2\hat{T}})$; $z_{12} = (m_2 - m_1)\hat{d}_0/(\hat{d}_1((m_2 + \hat{d}_1)e^{m_1\hat{T}} - (m_1 + \hat{d}_1)e^{m_2\hat{T}})$; $z_{13} = -\hat{d}_0/\hat{d}_1$ since $(m_2 + \hat{d}_1)e^{m_1\hat{T}} \neq (m_1 + \hat{d}_1)e^{m_2\hat{T}}$.

Therefore, the average profit per cycle is

$$\begin{aligned}
\mathrm{AP}(P_0, Q) &= \frac{1}{\hat{T}}\left\{\int_0^{\hat{T}} P(t)D(t)\,\mathrm{d}t - \hat{C}_1(Q) - \hat{C}_2(Q) - \hat{C}_3(Q) - \hat{C}_{40}T - (C_{41} + C_{42})\int_0^{\hat{T}} I(t)\,\mathrm{d}t - \hat{C}_5(Q)Q\right\} \\
&= \frac{1}{\hat{T}}\Bigg[\hat{d}_1\left\{\frac{a_1a_2}{2m_1}\left(e^{2m_1\hat{T}} - 1\right) + \frac{a_1b_2 + a_2b_1}{m_1 + m_2}\left(e^{(m_1+m_2)\hat{T}} - 1\right) + \frac{b_1b_2}{2m_2}\left(e^{2m_2\hat{T}} - 1\right)\right\} \\
&\quad - \hat{d}_2\left\{\frac{a_2^2}{2m_1}\left(e^{2m_1\hat{T}} - 1\right) + \frac{b_2^2}{2m_2}\left(e^{2m_2\hat{T}} - 1\right) + \frac{2a_2b_2}{m_1 + m_2}\left(e^{(m_1+m_2)\hat{T}} - 1\right)\right\} \\
&\quad - \left(\hat{C}_{41} + \hat{C}_{42}\right)\left\{\frac{a_1}{m_1}\left(e^{m_1\hat{T}} - 1\right) + \frac{b_1}{m_2}\left(e^{m_2\hat{T}} - 1\right) - \frac{\hat{d}_0}{\hat{d}_1}\hat{T}\right\} \\
&\quad - \hat{C}_{10}\left(\frac{Q}{\hat{R}}\right) - \hat{C}_{11}\left(\frac{\theta Q}{\hat{R}}\right) - \hat{C}_{20} - \hat{C}_{21}Q - \hat{C}_{30}\left(\frac{Q}{\hat{C}_T}\right) - \hat{C}_{40}\hat{T} - \hat{C}_{50}Q - \hat{C}_{51} - \hat{C}_{52}Q^2\Bigg].
\end{aligned} \tag{12}$$

Substituting the values of $a_i$, $b_i (i = 1, 2)$ and $Q$ in the above, we have

$$\mathrm{AP}(P_0, T) = V_1(T)P_0^2 + V_2(T)P_0 + V_3(T) \tag{13}$$

where $V_1(\hat{T}) = \frac{\hat{d}_1}{2\hat{T}m_1}(e^{2m_1\hat{T}} - 1)(x_{11} - x_{12}z_{11})(x_{21} - x_{22}z_{11}) + \frac{\hat{d}_1}{\hat{T}(m_1+m_2)}(e^{(m_1+m_2)\hat{T}} - 1)\{(x_{11} - x_{12}z_{11})(-y_{21} + y_{22}z_{11}) + (x_{21} - x_{22}z_{11})(-y_{11} + y_{12}z_{11})\} + \frac{\hat{d}_1}{2\hat{T}m_2}(e^{2m_2\hat{T}} - 1)\ (y_{12}z_{11} - y_{11})(z_{11}y_{22} - y_{21}) - \frac{\hat{d}_2}{2\hat{T}m_1}(e^{2m_1\hat{T}} - 1)(x_{21} - x_{22}z_{11})^2 - \frac{\hat{d}_2}{2\hat{T}m_2}(e^{2m_2\hat{T}} - 1)(y_{22}z_{11} - y_{21})^2 - \frac{2\hat{d}_2}{\hat{T}(m_1+m_2)}(e^{(m_1+m_2)\hat{T}} - 1)\ (x_{21} - x_{22}z_{11})(z_{11}y_{22} - y_{21}) - \frac{\hat{C}_{52}}{\hat{T}}z_{11}^2$; $V_2(\hat{T}) = -\frac{\hat{d}_1}{2\hat{T}m_1}(e^{2m_1\hat{T}} - 1)\{(x_{11} - x_{12}z_{11})(z_{22}(z_{12} - z_{13}) + x_{23}) + (x_{21} - x_{22}z_{11})(x_{12}(z_{12} - z_{13}) + x_{13})\} + \frac{\hat{d}_1}{\hat{T}(m_1+m_2)}(e^{(m_1+m_2)\hat{T}} - 1)\{(x_{11} - x_{12}z_{11})(y_{22}(z_{12} - z_{13}) + y_{23})\ - (-y_{21} + y_{22}z_{11})(x_{12}(z_{12} - z_{13}) + x_{13}) + (x_{21} - x_{22}z_{11})(y_{12}(z_{12} - z_{13}) + y_{13})\ - (y_{12}(z_{12} - z_{13}) + y_{13})(x_{22}(z_{12} - z_{13}) + x_{23})\} + \frac{\hat{d}_1}{2\hat{T}m_2}(e^{2m_2\hat{T}} - 1)\{(y_{12}z_{11} - y_{11})(y_{22}(z_{12} - z_{13}) + y_{23}) + (z_{11}y_{12} - y_{21})(y_{12}(z_{12} - z_{13}) + y_{13})\} + \frac{\hat{d}_2}{\hat{T}m_1}(e^{2m_1\hat{T}} - 1)(x_{21} - x_{22}z_{11})(x_{22}(z_{12} - z_{13}) + x_{23}) - \frac{\hat{d}_2}{\hat{T}m_2}(e^{2m_2\hat{T}} - 1)(z_{11}y_{22} - y_{21})(y_{22}(z_{12} - z_{13}) + y_{23}) - \frac{2\hat{d}_2}{\hat{T}(m_2+m_1)}(e^{(m_2+m_1)\hat{T}} - 1)\{(x_{21} - x_{22}z_{11})(y_{22}(z_{12} - z_{13}) + y_{23}) - (z_{11}y_{22} - y_{21})(x_{22}(z_{12} - z_{13}) + x_{23})\} - \frac{(\hat{C}_{41}+\hat{C}_{42})}{\hat{T}m_1}(e^{m_1\hat{T}} - 1)(x_{11} - x_{12}z_{11}) - \frac{(\hat{C}_{41}+\hat{C}_{42})}{\hat{T}m_2}(e^{m_2\hat{T}} - 1)(-y_{11} + y_{12}z_{11}) - \frac{(\hat{C}_{10}+\hat{\theta}\hat{C}_{11})z_{11}}{\hat{T}\hat{R}} - \{\frac{\hat{C}_{50}}{\hat{T}}z_{11} + \frac{2\hat{C}_{52}}{\hat{T}}z_{11}(z_{12} - z_{13})\}\ - (\hat{C}_{21} + \frac{\hat{C}_{30}}{\hat{C}_T})\frac{z_{11}}{\hat{T}}$; $V_3(\hat{T}) = \frac{\hat{d}_1}{2\hat{T}m_1}(e^{2m_1\hat{T}} - 1)(x_{12}(z_{12} - z_{13}) + x_{13})(x_{22}(z_{12} - z_{13}) + x_{23}) - \frac{\hat{d}_1}{\hat{T}(m_1+m_2)}(e^{(m_1+m_2)\hat{T}} - 1)\ \{(x_{12}(z_{12} - z_{13}) + x_{13})(y_{22}(z_{12} - z_{13}) + y_{23}) + (x_{22}(z_{12} - z_{13}) + x_{23})(y_{12}(z_{12} - z_{13}) + y_{13})\}\ + \frac{\hat{d}_1}{2\hat{T}m_2}(e^{2m_2\hat{T}} - 1)(y_{12}(z_{12} - z_{13}) + y_{13})(y_{22}(z_{12} - z_{13}) + y_{23}) - \frac{\hat{d}_2}{2\hat{T}m_1}(e^{2m_1\hat{T}} - 1)(x_{22}$

$(z_{12}-z_{13})+x_{23})^2-\frac{\hat{d}_2}{2\hat{T}m_2}(e^{2m_2\hat{T}}-1)(y_{22}(z_{12}-z_{13})+y_{23})^2+\frac{2\hat{d}_2}{\hat{T}(m_1+m_2)}(e^{(m_1+m_2)\hat{T}}-1)\ (x_{22}(z_{12}-z_{13})+x_{23})(y_{22}(z_{12}-z_{13})+y_{23})+\frac{(\hat{C}_{41}+\hat{C}_{42})}{\hat{T}m_1}(e^{m_1\hat{T}}-1)(x_{12}(z_{12}-z_{13})+x_{13})-\hat{C}_{40}-\frac{(\hat{C}_{41}+\hat{C}_{42})}{\hat{T}m_2}(e^{m_2\hat{T}}-1)(y_{12}(z_{12}-z_{13})+y_{13})+\frac{(\hat{C}_{41}+\hat{C}_{42})\hat{d}_0}{\hat{d}_1}-\frac{(\hat{C}_{10}+\hat{\theta}C_{11})}{\hat{T}\hat{R}}(z_{12}-z_{13})-\frac{\hat{C}_{50}}{\hat{T}}(z_{12}-z_{13})-\frac{\hat{C}_{51}}{\hat{T}}-\frac{\hat{C}_{52}}{\hat{T}}(z_{12}-z_{13})^2-\frac{\hat{C}_{20}}{\hat{T}}-\frac{\hat{C}_{21}}{\hat{T}}(z_{12}-z_{13})-\frac{\hat{C}_{30}}{\hat{T}\hat{C}_T}(z_{12}-z_{13})$. When time horizon is finite, *i.e.*, $\hat{T}$ is fixed; then $V_1(\hat{T})$, $V_2(\hat{T})$ and $V_3(\hat{T})$ are constant. Therefore, the average profit function $\mathrm{AP}(P_0)$ is a quadratic function of $P_0$ only. Now, our objective is to maximize $\mathrm{AP}(P_0)$ where $P_0$ is a decision variable and $\mathrm{AP}(P_0)$ is maximized using the following Lemma 1.

**Lemma 1.** *Average profit* $\mathrm{AP}(P_0)$ *attains its maximum value at* $P_0=P_0^*=-\frac{V_2}{2V_1}$ *if* $V_1<0$ *and* $V_2>0$ *hold simultaneously and then the optimum production lotsize is* $Q^*=z_{11}P_0^*+z_{12}-z_{13}$.

*Proof.* If the time horizon ($T$) is finite, the average profit $\mathrm{AP}(P_0)$ is a function of $P_0$ only. Differentiating $\mathrm{AP}(P_0)$ with respect to "$P_0$", we have

$$\frac{\mathrm{d\,AP}}{\mathrm{d}P_0}=2V_1(T)P_0+V_2(T)$$
$$\frac{\mathrm{d}^2\mathrm{AP}}{\mathrm{d}P_0^2}=2V_1(T).$$

For optimum values of $\mathrm{AP}(P_0)$, $\frac{\mathrm{d\,AP}}{\mathrm{d}P_0}=0$ provides $P_0^*=-\frac{V_2(T)}{2V_1(T)}$ and $\mathrm{AP}(P_0^*)$ is maximum when $V_1(T)<0$ holds. If $V_1(T)<0$ holds, then $P_0^*$ would be feasible when $V_2(T)>0$, and the optimum production lotsize is $Q^*=z_{11}P_0^*+z_{12}-z_{13}$. □

When time horizon is infinite, the average profit function $\mathrm{AP}(P_0,T)$ is function $Q$ and $T$, *i.e.*, $Q$ and $T$ are decision variables. Then, $\mathrm{AP}(P_0,T)$ would be maximized using the following Lemma 2.

**Lemma 2.** *Average profit* $\mathrm{AP}(P_0,T)$ *attains its maximum value at* $P_0=P_0^*=-\frac{V_2}{2V_1}$ *and* $T=\{T^*:[V_2^2\frac{\partial V_1}{\partial T}-2V_1V_2\frac{\partial V_2}{\partial T}+4V_1^2\frac{\partial V_3}{\partial T}]=0\}$ *if* $V_1<0$, $V_2>0$, $\frac{\partial^2\mathrm{AP}}{\partial T^2}<0$ *and* $(\frac{\partial^2\mathrm{AP}}{\partial T^2})(\frac{\partial^2\mathrm{AP}}{\partial P_0^2})-(\frac{\partial^2\mathrm{AP}}{\partial P_0\partial T})^2>0$ *hold simultaneously and then the optimum production lotsize is* $Q^*=z_{11}P_{0i}^*+z_{12}-z_{13}$.

*Proof.* If the time horizon ($T$) is infinite, *i.e.*, $T$ is a decision variable, the average profit $\mathrm{AP}(P_0,T)$ is a function of $P_0$ and $T$. Differentiating $\mathrm{AP}(P_0,T)$ with respect to $P_0$ and $T$ we have

$$\frac{\partial\mathrm{AP}}{\partial P_0}=2V_1(T)P_0+V_2(T)$$
$$\frac{\partial^2\mathrm{AP}}{\partial P_0^2}=2V_1(T)$$
$$\frac{\partial\mathrm{AP}}{\partial T}=\frac{\partial V_1}{\partial T}P_0^2+\frac{\partial V_2}{\partial T}P_0+\frac{\partial V_3}{\partial T}$$
$$\frac{\partial^2\mathrm{AP}}{\partial P_0\partial T}=2\frac{\partial V_1}{\partial T}P_0+\frac{\partial V_2}{\partial T}$$
$$\frac{\partial^2\mathrm{AP}}{\partial T^2}=\frac{\partial^2 V_1}{\partial T^2}P_0^2+\frac{\partial^2 V_2}{\partial T^2}P_0+\frac{\partial^2 V_3}{\partial T^2}.$$

For optimum values of $\mathrm{AP}(P_0,T)$, $\frac{\partial\mathrm{AP}}{\partial P_0}=0$ and $\frac{\partial\mathrm{AP}}{\partial T}=0$ provide

$$P_0^*=-\frac{V_2(T)}{2V_1(T)} \tag{14}$$

$$V_2^2\frac{\partial V_1}{\partial T}-2V_1V_2\frac{\partial V_2}{\partial T}+4V_1^2\frac{\partial V_3}{\partial T}=0. \tag{15}$$

Here, the equation (15) is a nonlinear function of $T$ only. It can be solved by Newton-Raphson method. It may be unique or multi-valued for $T^*\epsilon\ R^+$. Let $T_i^*(i=1,2,\dots,n)\epsilon\ R^+$ are obtained from equation (15). Then, the required values of the optimal values of selling prices are $P_{0i}^* = -\frac{V_2(T_i^*)}{2V_1(T_i^*)}$. Now, $[\frac{\partial^2 \mathrm{AP}}{\partial P_0^2}]_{(P_{0i}^* T_i^*)} = 2V_1(T_i^*) < 0$ if $V_1(T_i^*) < 0$ hold. The profit function $\mathrm{AP}(P_{0i}^*, T_i^*)$ attain maximum values when $V_1(T_i^*) < 0$, $V_2(T_i^*) > 0$, $[\frac{\partial^2 \mathrm{AP}}{\partial T^2}]_{(P_{0i}^* T_i^*)} < 0$ and $[(\frac{\partial^2 \mathrm{AP}}{\partial T^2})(\frac{\partial^2 \mathrm{AP}}{\partial P_0^2}) - (\frac{\partial^2 \mathrm{AP}}{\partial P_0 \partial T})^2]_{(P_{0i}^* T_i^*)} > 0$ hold simultaneously. If $\mathrm{AP}(P_0, T)$ is a multimodal function, the required maximum value of AP is the greatest value among the optimal values of $\mathrm{AP}(P_{0i}^*, T_i^*)$ and its corresponding optimum production lotsize is $Q^* = z_{11}P_{0i}^* + z_{12} - z_{13}$. □

If $mT \ll 1$, then we may approximate $e^{mT} \approx 1 + mT$ and substituting it in equation (13), we have

$$\mathrm{AP}(P_0, T) \approx V_1^{\mathrm{app}}(T)P_0^2 + V_2^{\mathrm{app}}(T)P_0 + V_3^{\mathrm{app}}(T) \tag{16}$$

where $V_1^{\mathrm{app}}(T) \approx u_{11} - u_{12}/T$; $V_2^{\mathrm{app}}(T) \approx u_{21} - u_{22}/T$; $V_3^{\mathrm{app}}(T) \approx u_{31} - u_{32}/T$; $u_{11} \approx \hat{d}_1(x_{11} - x_{12}z_{11})(x_{21} - x_{22}z_{11}) + \hat{d}_1\{(x_{11} - x_{12}z_{11})\ (-y_{21} + y_{22}z_{11}) + (x_{21} - x_{22}z_{11})(-y_{11} + y_{12}z_{11})\} + \hat{d}_1(y_{12}z_{11} - y_{11})(z_{11}y_{22} - y_{21}) - \hat{d}_2(x_{21} - x_{22}z_{11})^2 - \hat{d}_2(y_{22}z_{11} - y_{21})^2 - 2\hat{d}_2(x_{21} - x_{22}z_{11})(z_{11}y_{22} - y_{21})$; $u_{12} = \hat{C}_{52}z_{11}^2$; $u_{21} \approx -\hat{d}_1\{(x_{11} - x_{12}z_{11})(z_{22}(z_{12}-z_{13})+x_{23}) + (x_{21}-x_{22}z_{11})(x_{12}(z_{12}-z_{13})+x_{13})\} + \hat{d}_1\{(x_{11}-x_{12}z_{11})(y_{22}(z_{12}-z_{13})+y_{23}) - (-y_{21} + y_{22}z_{11})(x_{12}(z_{12}-z_{13})+x_{13}) + (x_{21}-x_{22}z_{11})(y_{12}(z_{12}-z_{13})+y_{13}) - (y_{12}(z_{12}-z_{13})+y_{13})(x_{22}(z_{12}-z_{13})+x_{23})\} + \hat{d}_1\{(y_{12}z_{11}-y_{11})\ (y_{22}(z_{12}-z_{13})+y_{23}) + (z_{11}y_{12}-y_{21})(y_{12}(z_{12}-z_{13})+y_{13})\} + \hat{d}_2(x_{21}-x_{22}z_{11})(x_{22}(z_{12}-z_{13})+x_{23}) - 2\hat{d}_2(z_{11}y_{22}-y_{21})(y_{22}(z_{12}-z_{13})+y_{23}) - \hat{d}_2\{(x_{21}-x_{22}z_{11})(y_{22}(z_{12}-z_{13})+y_{23}) - (z_{11}y_{22}-y_{21})(x_{22}(z_{12}-z_{13})+x_{23})\} - (\hat{C}_{41}+\hat{C}_{42})(x_{11}-x_{12}z_{11}) - (\hat{C}_{41}+\hat{C}_{42})(-y_{11}+y_{12}z_{11})$; $u_{22} = \frac{(\hat{C}_{10}+\hat{\theta}\hat{C}_{11})z_{11}}{\hat{R}} + \hat{C}_{50}z_{11} + 2\hat{C}_{52}z_{11}(z_{12}-z_{13}) + (\hat{C}_{21} + \frac{\hat{C}_{30}}{\hat{C}_T})z_{11} - \hat{C}_{40}$; $u_{31} \approx \hat{d}_1(x_{12}(z_{12}-z_{13})+x_{13})(x_{22}(z_{12}-z_{13})+x_{23}) - \hat{d}_1\ \{(x_{12}(z_{12}-z_{13})+x_{13})(y_{22}(z_{12}-z_{13})+y_{23}) + (x_{22}(z_{12}-z_{13})+x_{23})(y_{12}(z_{12}-z_{13})+y_{13})\} + \hat{d}_1(y_{12}(z_{12}-z_{13})+y_{13})(y_{22}(z_{12}-z_{13})+y_{23}) - \hat{d}_2(x_{22}\ (z_{12}-z_{13}) + x_{23})^2 - \hat{d}_2(y_{22}(z_{12}-z_{13})+y_{23})^2 + 2\hat{d}_2\ (x_{22}(z_{12}-z_{13})+x_{23})(y_{22}(z_{12}-z_{13})+y_{23}) + (\hat{C}_{41}+\hat{C}_{42})(x_{12}(z_{12}-z_{13})+x_{13}) - (\hat{C}_{41}+\hat{C}_{42})(y_{12}(z_{12}-z_{13})+y_{13}) + \frac{(\hat{C}_{41}+\hat{C}_{42})\hat{d}_0}{\hat{d}_1}$; $u_{32} = \frac{(\hat{C}_{10}+\hat{\theta}\hat{C}_{11})}{\hat{R}}(z_{12}-z_{13}) + \hat{C}_{50}(z_{12}-z_{13}) + \hat{C}_{51} + \hat{C}_{52}(z_{12}-z_{13})^2 + \hat{C}_{20} + \hat{C}_{21}(z_{12}-z_{13}) + \frac{\hat{C}_{30}}{\hat{C}_T}(z_{12}-z_{13})$. Using the above in equations (11), (13) and (14), we have optimal solution as follows:

$$\begin{aligned}
P_0^* &= -\left(\frac{u_{21}T_i^* - u_{22}}{u_{11}T_i^* - u_{12}}\right)\\
Q^* &= z_{11}P_0^* + z_{12} - z_{13}\\
T_1^* &= \frac{-S_2 + \sqrt{S_2^2 - 4S_1S_3}}{2S_1}\\
T_2^* &= \frac{-S_2 - \sqrt{S_2^2 - 4S_1S_3}}{2S_1}\\
S_1 &= u_{21}^2u_{12} - 2u_{11}u_{21}u_{22} + 4u_{11}^2u_{32}\\
S_2 &= 2u_{11}u_{22}^2 - 8u_{11}u_{12}u_{32}\\
S_3 &= u_{22}^2u_{12} - 2u_{12}u_{22} + 4u_{12}^2u_{32}.
\end{aligned}$$

### 4.1. Extension of the existing model when sales effort is applied

Green marketing is becoming more widespread as more educated people are afraid from GHG regarding environmental issues. The sales efforts in a green marketing system promote to use the environmentally friendly products, services, and initiatives. Although green marketing is apparently expensive than traditional nongreen products, the manufacturer of the green products can earn more due to increasing demand of the customers. Sales efforts taken by sales management of a firm are the processes of developing a sales power, managing sales procedures and executing sales techniques that permit a business to steadily success, and even exceeds its sales goals. In this case, sales effort $E(t)$ is applied to convince the customers about green products and quality of the products and consequently the demand rate increases and cost of sales effort increase with level of sales effort. Here, cost of sales effort is $\hat{\gamma}E^2(t)$. Then, the governing differential equations are as follows:

$$\begin{aligned} D(t) &= \hat{d}_0 + \hat{d}_1 I(t) - \hat{d}_2 P(t) + \hat{d}_3 E(t) && (17)\\ \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= -\hat{d}_0 - \hat{d}_1 I(t) + \hat{d}_2 P(t) - \hat{d}_3 E(t) && (18)\\ \text{with } I(0) &= Q \text{ and } I(\hat{T}) = 0 \ E(0) = E_0 \\ \frac{\mathrm{d}P(t)}{\mathrm{d}t} &= \hat{\alpha}P(t) - \hat{\beta}\frac{\mathrm{d}I(t)}{\mathrm{d}t} && (19)\\ \text{with } P(0) &= P_0. \end{aligned}$$

Now, the equation (19) can be rewritten as

$$P(t) = \frac{1}{\hat{\alpha}}\left(\frac{\mathrm{d}P(t)}{\mathrm{d}t} + \hat{\beta}\frac{\mathrm{d}I(t)}{\mathrm{d}t}\right)\cdot \tag{20}$$

From equation (18), we have

$$E(t) = \frac{1}{\hat{d}_3}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\frac{\mathrm{d}P}{\mathrm{d}t} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\frac{\mathrm{d}I}{\mathrm{d}t}\right)\cdot \tag{21}$$

The average profit function is as follows:

$$\begin{aligned} \mathrm{AP} = \frac{1}{\hat{T}}\Bigg\{&\int_0^{\hat{T}} P(t)D(t)\,\mathrm{d}t - (\hat{C}_{41} + \hat{C}_{42})\int_0^{\hat{T}} I(t)\,\mathrm{d}t - \hat{\gamma}\int_0^{\hat{T}} E^2(t)\,\mathrm{d}t \\ &- \hat{C}_{10}Q/\hat{R} - \hat{C}_{11}\hat{\theta}Q/\hat{R} - \hat{C}_{20} - \hat{C}_{21}Q - \hat{C}_{30}Q/\hat{C}_T - \hat{C}_{40}\hat{T} - \hat{C}_{50}Q - \hat{C}_{51} - \hat{C}_{52}Q^2\Bigg\} \end{aligned} \tag{22}$$

Using equations (18), (20) and (21) in the above equation (22) and rewriting, we have

$$\begin{aligned} \mathrm{AP} &= \frac{1}{\hat{T}}\Bigg\{\int_0^{\hat{T}} -\frac{1}{\hat{\alpha}}\left(\frac{\mathrm{d}P(t)}{\mathrm{d}t} + \hat{\beta}\frac{\mathrm{d}I(t)}{\mathrm{d}t}\right)I(t)\,\mathrm{d}t - \left(\hat{C}_{41} + \hat{C}_{42}\right)\int_0^{\hat{T}} I(t)\,\mathrm{d}t \\ &\quad - \frac{\hat{\gamma}}{\hat{d}_3^2}\int_0^{\hat{T}}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\frac{\mathrm{d}P}{\mathrm{d}t} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\frac{\mathrm{d}I}{\mathrm{d}t}\right)^2\mathrm{d}t \\ &\quad - \left(\hat{C}_{10}Q/\hat{R} + \hat{C}_{11}\hat{\theta}Q/\hat{R}\right)/\hat{T}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{20}/\hat{T}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{21}Q/\hat{T}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{30}Q/\left(\hat{C}_T\hat{T}\right)\int_0^{\hat{T}}\mathrm{d}t \\ &\quad - \hat{C}_{40}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{50}Q/\hat{T}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{51}/\hat{T}\int_0^{\hat{T}}\mathrm{d}t - \hat{C}_{52}Q^2/\hat{T}\int_0^{\hat{T}}\mathrm{d}t\Bigg\} \\ &= \int_0^{\hat{T}}\phi\left(\dot{I}, I, \dot{P}, P, t\right)\mathrm{d}t \end{aligned} \tag{23}$$

where

$$\phi\left(\dot{I}, I, \dot{P}, P, t\right) = \frac{1}{\hat{T}}\Bigg\{-\frac{1}{\hat{\alpha}}\left(\dot{P}(t) + \hat{\beta}\dot{I}(t)\right)I(t) - \left(\hat{C}_{41} + \hat{C}_{42}\right)I(t) - \frac{\hat{\gamma}}{\hat{d}_3^2}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\dot{P} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\dot{I}\right)^2$$

$$- \left(\hat{C}_{10}Q/\hat{R} + \hat{C}_{11}\hat{\theta}Q/\hat{R}\right)/\hat{T} - \hat{C}_{20}/\hat{T} - \hat{C}_{21}Q/\hat{T} - \hat{C}_{30}Q/\left(\hat{C}_T\hat{T}\right)$$
$$\left. - \hat{C}_{40} - \hat{C}_{50}Q/\hat{T} - \hat{C}_{51}/\hat{T} - \hat{C}_{52}Q^2/\hat{T}\right\}. \tag{24}$$

The necessary partial derivatives of $\phi(\dot{I}, I, \dot{P}, P, t)$ are as follows:

$$\frac{\partial\phi}{\partial\dot{I}} = \frac{1}{\hat{T}}\left\{-\frac{1}{\hat{\alpha}}\left(\dot{P} + 2\hat{\beta}\dot{I}\right) - \frac{2\hat{\gamma}}{\hat{d}_3^2}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\dot{P} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\dot{I}\right)\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\right\} \tag{25}$$

$$\frac{\partial\phi}{\partial I} = \frac{1}{\hat{T}}\left\{-\left(\hat{C}_{41} + \hat{C}_{42}\right) + \frac{2\hat{\gamma}\hat{d}_1}{\hat{d}_3^2}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\dot{P} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\dot{I}\right)\right\} \tag{26}$$

$$\frac{\partial^2\phi}{\partial\dot{I}^2} = -\frac{2\hat{\beta}}{\hat{T}\hat{\alpha}} - \frac{2\hat{\gamma}}{\hat{T}\hat{d}_3^2}\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)^2 \tag{27}$$

$$\frac{\partial^2\phi}{\partial\dot{I}\partial I} = \frac{2\hat{\gamma}\hat{d}_1}{\hat{T}\hat{d}_3^2}\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right) \tag{28}$$

$$\frac{\partial^2\phi}{\partial I^2} = -\frac{2\hat{\gamma}\hat{d}_1^2}{\hat{T}\hat{d}_3^2} \tag{29}$$

$$\frac{\partial\phi}{\partial\dot{P}} = \frac{1}{\hat{T}}\left\{-\frac{1}{\hat{\alpha}}\dot{I} - \frac{2\hat{\gamma}}{\hat{d}_3^2}\left(-\hat{d}_0 - \hat{d}_1 I + \frac{\hat{d}_2}{\hat{\alpha}}\dot{P} + \left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\dot{I}\right)\left(\frac{\hat{d}_2}{\hat{\alpha}}\right)\right\} \tag{30}$$

$$\frac{\partial\phi}{\partial P} = 0 \tag{31}$$

$$\frac{\partial^2\phi}{\partial\dot{P}^2} = -\frac{2\hat{\gamma}}{\hat{T}\hat{d}_3^2}\left(\frac{\hat{d}_2}{\hat{\alpha}}\right)^2 \tag{32}$$

$$\frac{\partial^2\phi}{\partial\dot{P}\partial P} = 0 \tag{33}$$

$$\frac{\partial^2\phi}{\partial P^2} = 0. \tag{34}$$

**Lemma 3.** *The average profit function* AP *attains maximum value in* $[0, \hat{T}]$.

*Proof.* Let $I(t)$ and $P(t)$ are along the curve $C_1$ and $C_2$ respective such that AP will be maximum in $[0, \hat{T}]$ along these optimal paths. Then we consider $I_\epsilon = I + \epsilon\eta_1(t)$ and $P_\epsilon = P + \epsilon\eta_2(t)$ where $\eta_1(t)$ and $\eta_2(t)$ are arbitrary differentiable functions of $t$, and $\dot{I}_\epsilon = \dot{I} + \epsilon\dot{\eta}_1(t)$ and $\dot{P}_\epsilon = \dot{P} + \epsilon\dot{\eta}_2(t)$. Now, $\frac{\mathrm{d}\,\mathrm{AP}(\epsilon)}{\mathrm{d}\epsilon} = 0$ for extremum value of $\mathrm{AP}(\epsilon)$. Therefore,

$$\begin{aligned}
\frac{\mathrm{d}\,\mathrm{AP}(\epsilon)}{\mathrm{d}\epsilon} &= \int_0^{\hat{T}}\left\{\eta_1\frac{\partial\phi}{\partial I} + \dot{\eta}_1\frac{\partial\phi}{\partial\dot{I}}\right\}\mathrm{d}t + \int_0^{\hat{T}}\left\{\eta_2\frac{\partial\phi}{\partial P} + \dot{\eta}_2\frac{\partial\phi}{\partial\dot{P}}\right\}\mathrm{d}t \\
&= \int_0^{\hat{T}}\eta_1\frac{\partial\phi}{\partial I}\,\mathrm{d}t + \left[\eta_1\frac{\partial\phi}{\partial\dot{I}}\right]_0^{\hat{T}} - \int_0^{\hat{T}}\eta_1\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{I}}\right)\mathrm{d}t + \int_0^{\hat{T}}\eta_2\frac{\partial\phi}{\partial P}\,\mathrm{d}t + \left[\eta_2\frac{\partial\phi}{\partial\dot{P}}\right]_0^{\hat{T}} - \int_0^{\hat{T}}\eta_2\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{P}}\right)\mathrm{d}t \\
&= \int_0^{\hat{T}}\eta_1\left[\frac{\partial\phi}{\partial I} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{I}}\right)\right]\mathrm{d}t + \int_0^{\hat{T}}\eta_2\left[\frac{\partial\phi}{\partial P} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{P}}\right)\right]\mathrm{d}t.
\end{aligned}$$

Since $I(t)$ and $P(t)$ are fixed at the end points, the arbitrary functions $\eta_1$ and $\eta_2$ would vanish at the end points, *i.e.*, $\eta_1(0) = 0 = \eta_1(\hat{T})$ and $\eta_2(0) = 0 = \eta_2(\hat{T})$. Thus the necessary condition $\frac{\mathrm{d\,AP}(\epsilon)}{\mathrm{d}\epsilon}\ |_{\epsilon=0}= 0$ provides as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{I}}\right) - \frac{\partial\phi}{\partial I} = 0 \tag{35}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\phi}{\partial\dot{P}}\right) - \frac{\partial\phi}{\partial P} = 0. \tag{36}$$

Again differentiating $\frac{\mathrm{d\,AP}(\epsilon)}{\mathrm{d}\epsilon}$ with respect to $\epsilon$, we have

$$\frac{\mathrm{d}^2\mathrm{AP}(\epsilon)}{\mathrm{d}\epsilon^2} = \int_0^{\hat{T}}\left[\eta_1^2\frac{\partial^2\phi}{\partial I^2} + 2\eta_1\dot{\eta}_1\frac{\partial^2\phi}{\partial I\partial\dot{I}} + \dot{\eta}_1^2\frac{\partial^2\phi}{\partial\dot{I}^2}\right]\mathrm{d}t + \int_0^{\hat{T}}\left[\eta_2^2\frac{\partial^2\phi}{\partial P^2} + 2\eta_2\dot{\eta}_2\frac{\partial^2\phi}{\partial P\partial\dot{P}} + \dot{\eta}_2^2\frac{\partial^2\phi}{\partial\dot{P}^2}\right]\mathrm{d}t.$$

Now using equations (27)–(29), (32)–(34) in the above second order derivative, we have

$$\begin{aligned}
\frac{\mathrm{d}^2\mathrm{AP}}{\mathrm{d}\epsilon^2}\ |_{\epsilon=0} &= -\int_0^{\hat{T}}\left[\left(\frac{2\hat{\beta}}{\hat{T}\hat{\alpha}} + \frac{2\hat{\gamma}}{\hat{T}\hat{d}_3^2}\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)^2\right)\dot{\eta}_1^2 - \frac{4\hat{\gamma}\hat{d}_1}{\hat{T}\hat{d}_3^2}\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\eta_1\dot{\eta}_1 + \frac{2\hat{\gamma}\hat{d}_1^2}{\hat{T}\hat{d}_3^2}\eta_1^2\right]\mathrm{d}t\\
&\quad - \int_0^{\hat{T}} 2\dot{\eta}_2^2\left(\frac{\hat{\gamma}\hat{d}_2^2}{\hat{T}\hat{d}_3^2\hat{\alpha}^2}\right)\mathrm{d}t\\
&= -\frac{2}{\hat{T}}\int_0^{\hat{T}}\left[\left(\frac{\hat{\beta}}{\hat{\alpha}}\right)\dot{\eta}_1^2 + \frac{\hat{\gamma}}{\hat{d}_3^2}\left(\left(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1\right)\dot{\eta}_1 - \hat{d}_1\eta_1\right)^2 + \frac{\hat{\gamma}}{\hat{d}_3^2}\left(\frac{\hat{d}_2}{\hat{\alpha}}\right)^2\dot{\eta}_2^2\right]\mathrm{d}t\\
&< 0 \quad \text{as } \hat{T} > 0\ \hat{\alpha} > 0,\ \hat{\gamma} > 0,\ \hat{\beta} > 0,\ \hat{d}_3 > 0.
\end{aligned} \tag{37}$$

Thus the sufficient condition $\frac{\mathrm{d}^2\mathrm{AP}}{\mathrm{d}\epsilon^2}\ |_{\epsilon=0}< 0$ indicates that the average profit function AP has a maximum value along the optimal path $I(t)$ and $P(t)$ obtained from the differential equations (35) and (36). Hence the proof. □

Now, simplifying the equations (36) and (35), we have respectively as follows:

$$A_1\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + A_2\frac{\mathrm{d}^2 P}{\mathrm{d}t^2} + A_3\frac{\mathrm{d}I}{\mathrm{d}t} = 0 \tag{38}$$

$$B_1\frac{\mathrm{d}^2 P}{\mathrm{d}t^2} + B_2\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} + B_3 I = B_4 \tag{39}$$

where $A_1 = -(\frac{1}{\hat{\alpha}} + \frac{2\hat{\gamma}\hat{d}_2}{\hat{\alpha}\hat{d}_3^2}(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1))$; $A_2 = -\frac{2\hat{\gamma}\hat{d}_2^2}{\hat{\alpha}^2\hat{d}_3^2}$; $A_3 = \frac{2\hat{\gamma}\hat{d}_1\hat{d}_2}{\hat{\alpha}\hat{d}_3^2}$; $B_1 = -(\frac{1}{\hat{\alpha}} + \frac{2\hat{\gamma}\hat{d}_2}{\hat{\alpha}\hat{d}_3^2}(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1) + \frac{2\hat{\gamma}\hat{d}_1\hat{d}_2}{\hat{\alpha}^2\hat{d}_3^2})$; $B_2 = -(\frac{2\hat{\beta}}{\hat{\alpha}} + \frac{2\hat{\gamma}}{\hat{d}_3^2}(\frac{\hat{d}_2\hat{\beta}}{\hat{\alpha}} - 1)^2)$; $B_3 = \frac{2\hat{\gamma}\hat{d}_1^2}{\hat{d}_3^2}$; $B_4 = -\frac{2\hat{\gamma}\hat{d}_1\hat{d}_0}{\hat{d}_3^2} - (\hat{C}_{41} + \hat{C}_{42})$. Solving equations (38) and (39), we have

$$I(t) = g_1 e^{m_1 t} + g_2 e^{m_2 t} + B_4/B_3 \tag{40}$$

$$P(t) = P_0 e^{\hat{\alpha}t} - \hat{\beta}\left[\left(\frac{m_1 g_1}{m_1 - \hat{\alpha}}\right)\left(e^{m_1 t} - e^{\hat{\alpha}t}\right) + \left(\frac{m_2 g_2}{m_2 - \hat{\alpha}}\right)\left(e^{m_2 t} - e^{\hat{\alpha}t}\right)\right] \tag{41}$$

$$\begin{aligned}
E(t) = \frac{1}{\hat{d}_3}\Big[&-\hat{d}_0 - \hat{d}_1\left(g_1 e^{m_1 t} + g_2 e^{m_2 t} + B_4/B_3\right)\Big]\\
&+ \hat{d}_2\left(P_0 e^{\hat{\alpha}t} - \hat{\beta}\left[\left(\frac{m_1 g_1}{m_1 - \hat{\alpha}}\right)\left(e^{m_1 t} - e^{\hat{\alpha}t}\right) + \left(\frac{m_2 g_2}{m_2 - \hat{\alpha}}\right)\left(e^{m_2 t} - e^{\hat{\alpha}t}\right)\right]\right)\\
&- g_1 e^{m_1 t} - g_2 e^{m_2 t}
\end{aligned} \tag{42}$$

$$g_1 = u_{21}P_0 + u_{22} \tag{43}$$
$$g_2 = u_{31}P_0 + u_{32}. \tag{44}$$

Using $E(0) = \hat{E}_0$, we have

$$Q = u_{11}P_0 + u_{12} \tag{45}$$

where $m_1 = \frac{A_3B_1+\sqrt{A_3^2B_1^2+4A_2B_3(A_1B_1-A_2B_2)}}{2(A_2B_2-A_1B_1)}$; $m_2 = \frac{A_3B_1-\sqrt{A_3^2B_1^2+4A_2B_3(A_1B_1-A_2B_2)}}{2(A_2B_2-A_1B_1)}$; $u_{11} = \frac{\hat{d}_2(e^{m_2\hat{T}}-e^{m_1\hat{T}})}{\hat{d}_1(e^{m_2\hat{T}}-e^{m_1\hat{T}})+m_1e^{m_2\hat{T}}-m_2e^{m_1\hat{T}}}$; $u_{12} = \frac{B_4(m_1e^{m_2\hat{T}}-m_2e^{m_1\hat{T}}+m_2-m_1)-B_3(\hat{d}_3\hat{E}_0-\hat{d}_0)(e^{m_2\hat{T}}-e^{m_1\hat{T}})}{B_3(\hat{d}_1(e^{m_2\hat{T}}-e^{m_1\hat{T}})+m_1e^{m_2\hat{T}}-m_2e^{m_1\hat{T}})}$; $u_{21} = \frac{u_{11}e^{m_2\hat{T}}}{e^{m_2\hat{T}}-e^{m_1\hat{T}}}$; $u_{22} = \frac{B_4(1-e^{m_2\hat{T}})}{B_3(e^{m_2\hat{T}}-e^{m_1\hat{T}})} + \frac{u_{12}e^{m_2\hat{T}}}{e^{m_2\hat{T}}-e^{m_1\hat{T}}}$; $u_{31} = \frac{u_{11}e^{m_1\hat{T}}}{e^{m_1\hat{T}}-e^{m_2\hat{T}}}$; $u_{32} = \frac{B_4(1-e^{m_1\hat{T}})}{B_3(e^{m_1\hat{T}}-e^{m_2\hat{T}})} + \frac{u_{12}e^{m_1\hat{T}}}{e^{m_1\hat{T}}-e^{m_2\hat{T}}}$. Using the optimal paths defined in equations (40)–(42) in equation (23) and integrating, we have

$$\mathrm{AP}(P_0) = V_{e1}P_0^2 + V_{e2}P_0 + V_{e3} \tag{46}$$

where $V_{e1} = -(\frac{m_1}{m_1+\hat{\alpha}})(\frac{e^{(m_1+\hat{\alpha})T}-1}{\hat{T}})u_{21} - (\frac{m_2}{m_2+\hat{\alpha}})(\frac{e^{(m_2+\hat{\alpha})T}-1}{\hat{T}})u_{31} + \frac{\hat{\beta}m_1^2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{2m_1\hat{T}}-1}{2m_1}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}u_{21}^2 + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}u_{21}u_{31} + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}u_{21}u_{31} + \frac{\hat{\beta}m_2^2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{2m_2\hat{T}}-1}{2m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}u_{21}^2 - \frac{\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_1\hat{T}}-1}{2m_1})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})^2u_{21}^2 - \frac{\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_2\hat{T}}-1}{2m_2})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})^2u_{31}^2 - \frac{\hat{\gamma}\hat{d}_2^2}{\hat{T}d_3^2}(\frac{e^{2\hat{\alpha}\hat{T}}-1}{2\hat{\alpha}})\{1 + \frac{\hat{\beta}^2m_1^2}{(m_1-\hat{\alpha})^2}u_{21}^2 + \frac{\hat{\beta}^2m_2^2}{(m_2-\hat{\alpha})^2}u_{31}^2 + \frac{2\hat{\beta}m_2}{(m_2-\hat{\alpha})}u_{31} + \frac{2\hat{\beta}m_1}{(m_1-\hat{\alpha})}u_{21} + \frac{2\hat{\beta}^2m_1m_2}{(m_1-\hat{\alpha})(m_2-\hat{\alpha})}u_{21}u_{31}\} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})u_{21}u_{31} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})\{\hat{d}_2u_{21} + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{21}^2 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{21}u_{31}\} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})\{\hat{d}_2u_{31} + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{21}u_{31} + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{31}^2\} - \frac{1}{\hat{T}}\hat{C}_{52}u_{11}^2$;

$V_{e2} = -(\frac{m_1}{m_1+\hat{\alpha}})(\frac{e^{(m_1+\hat{\alpha})T}-1}{\hat{T}})u_{22} - (\frac{m_2}{m_2+\hat{\alpha}})(\frac{e^{(m_2+\hat{\alpha})T}-1}{\hat{T}})u_{32} + \frac{2\hat{\beta}m_1^2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{2m_1\hat{T}}-1}{2m_1}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}u_{21}u_{22} + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}(u_{22}u_{31} + u_{21}u_{32}) + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}(u_{22}u_{31} + u_{21}u_{32}) + \frac{2\hat{\beta}m_2^2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{2m_2\hat{T}}-1}{2m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}u_{21}u_{22} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_1\hat{T}}-1}{2m_1})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})^2u_{21}u_{22} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_2\hat{T}}-1}{2m_2})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})^2u_{31}u_{32} - \frac{\hat{\gamma}\hat{d}_2^2}{\hat{T}d_3^2}(\frac{e^{2\hat{\alpha}\hat{T}}-1}{\hat{\alpha}})\{\frac{\hat{\beta}^2m_1^2}{(m_1-\hat{\alpha})^2}u_{21}u_{22} + \frac{\hat{\beta}^2m_2^2}{(m_2-\hat{\alpha})^2}u_{31}u_{32} + \frac{\hat{\beta}m_2}{(m_2-\hat{\alpha})}u_{32} + \frac{\hat{\beta}m_1}{(m_1-\hat{\alpha})}u_{22} + \frac{\hat{\beta}^2m_1m_2}{(m_1-\hat{\alpha})(m_2-\hat{\alpha})}(u_{22}u_{31} + u_{21}u_{32})\} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{m_1\hat{T}}-1}{m_1})u_{21} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{m_2\hat{T}}-1}{m_2})u_{31} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{\hat{\alpha}\hat{T}}-1}{\hat{\alpha}})\{\hat{d}_2 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{21} + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{31}\} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2})(u_{22}u_{31} + u_{21}u_{32}) + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})\{\hat{d}_2u_{22} + \frac{2\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{21}u_{22} + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}(u_{21}u_{32} + u_{22}u_{31})\} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})\{\hat{d}_2u_{32} + \frac{2\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{31}u_{32} + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}(u_{22}u_{31} + u_{21}u_{32})\} - \{(\frac{\hat{C}_{10}+\hat{\theta}\hat{C}_{11}}{\hat{T}\hat{R}}) + \frac{\hat{C}_{21}}{\hat{T}} + \frac{\hat{C}_{30}}{\hat{T}\hat{C}_T} + \frac{\hat{C}_{50}}{\hat{T}}\}u_{11} - \frac{2\hat{C}_{52}}{\hat{T}}u_{11}u_{12}$;

$V_{e3} = \frac{\hat{\beta}m_1^2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{2m_1\hat{T}}-1}{2m_1}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}u_{22}^2 + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})\}u_{22}u_{32} + \frac{\hat{\beta}m_1m_2}{\hat{T}(m_1-\hat{\alpha})}\{(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}u_{22}u_{32} + \frac{\hat{\beta}m_2^2}{\hat{T}(m_2-\hat{\alpha})}\{(\frac{e^{2m_2\hat{T}}-1}{2m_2}) - (\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})\}u_{22}^2 - \frac{\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_1\hat{T}}-1}{2m_1})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})^2u_{22}^2 - \frac{\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{2m_2\hat{T}}-1}{2m_2})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})^2u_{32}^2 - \frac{\hat{\gamma}\hat{d}_2^2}{\hat{T}d_3^2}(\frac{e^{2\hat{\alpha}\hat{T}}-1}{2\hat{\alpha}})\{\frac{\hat{\beta}^2m_1^2}{(m_1-\hat{\alpha})^2}u_{22}^2 + \frac{\hat{\beta}^2m_2^2}{(m_2-\hat{\alpha})^2}u_{32}^2 + \frac{2\hat{\beta}^2m_1m_2}{(m_1-\hat{\alpha})(m_2-\hat{\alpha})}u_{22}u_{32}\} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{m_1\hat{T}}-1}{m_1})u_{22} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{m_2\hat{T}}-1}{m_2})u_{32} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})(\frac{e^{\hat{\alpha}\hat{T}}-1}{\hat{\alpha}})\{\hat{d}_2 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{21} + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{31}\} - \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})(\frac{e^{(m_1+m_2)\hat{T}}-1}{m_1+m_2})u_{22}u_{32} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_1+\hat{\alpha})\hat{T}}-1}{m_1+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}})\{\frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{22}^2 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{22}u_{32}\} + \frac{2\hat{\gamma}}{\hat{T}d_3^2}(\frac{e^{(m_2+\hat{\alpha})\hat{T}}-1}{m_2+\hat{\alpha}})(\hat{d}_1 + \frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}})\{\frac{\hat{d}_2\hat{\beta}m_2}{m_2-\hat{\alpha}}u_{32}^2 + \frac{\hat{d}_2\hat{\beta}m_1}{m_1-\hat{\alpha}}u_{22}u_{32}\} - (\frac{\hat{C}_{41}+\hat{C}_{42}}{\hat{T}})(\frac{e^{m_1\hat{T}}-1}{m_1})u_{22} - (\frac{\hat{C}_{41}+\hat{C}_{42}}{\hat{T}})(\frac{e^{m_2\hat{T}}-1}{m_2})u_{32} - (\hat{C}_{41}+\hat{C}_{42})\frac{B_4}{B_3} - \frac{\hat{\gamma}}{\hat{d}_3^2}(\hat{d}_0 + \frac{\hat{d}_1B_4}{B_3})^2 - \{(\frac{\hat{C}_{10}+\hat{\theta}\hat{C}_{11}}{\hat{T}\hat{R}}) + \frac{\hat{C}_{21}}{\hat{T}} + \frac{\hat{C}_{30}}{\hat{T}\hat{C}_T} + \frac{\hat{C}_{50}}{\hat{T}}\}u_{12} - \frac{\hat{C}_{52}}{\hat{T}}u_{12}^2 - \frac{\hat{C}_{20}}{\hat{T}} - \frac{\hat{C}_{51}}{\hat{T}} - \hat{C}_{40}$.

As time horizon is finite, *i.e.*, $\hat{T}$ is fixed; then $V_{e1}(\hat{T})$, $V_{e2}(\hat{T})$ and $V_{e3}(\hat{T})$ are constant. In this case, the average profit function $\mathrm{AP}(P_0)$ is a quadratic function of $P_0$ where $P_0$ is a decision variable and $\mathrm{AP}(P_0)$ is maximized using the following lemma.

**Lemma 4.** *Average profit* $\mathrm{AP}(P_0)$ *has maximum value at* $P_0 = P_0^* = -\frac{V_{e2}}{2V_{e1}}$ *if* $V_{e1} < 0$ *and* $V_{e2} > 0$ *hold simultaneously and then the optimum production lotsize is* $Q^* = u_{11}P_0^* + u_{12}$.

*Proof.* Here, the average profit $\mathrm{AP}(P_0)$ is a function of $P_0$ only. Differentiating $\mathrm{AP}(P_0)$ with respect to "$P_0$", we have

$$\frac{\mathrm{d\,AP}}{\mathrm{d}P_0} = 2V_{e1}(T)P_0 + V_{e2}(T)$$
$$\frac{\mathrm{d}^2\mathrm{AP}}{\mathrm{d}P_0^2} = 2V_{e1}(T).$$

For optimum values of $\mathrm{AP}(P_0)$, $\frac{\mathrm{d\,AP}}{\mathrm{d}P_0} = 0$ provides $P_0^* = -\frac{V_{e2}(T)}{2V_{e1}(T)}$ and $\mathrm{AP}(P_0^*)$ is maximum when $V_{e1}(T) < 0$ holds. If $V_{e1}(T) < 0$ holds, then $P_0^*$ would be feasible when $V_{e2}(T) > 0$, and the optimum production lotsize is $Q^* = u_{11}P_0^* + u_{12}$. □

## 5. Numerical example

**Example 1.** The interval values of the key parameters in appropriate units are considered as follows: $T = [3, 5]$, $R = [500, 700]$, $d_0 = [180, 200]$, $d_1 = [0.15, 0.25]$, $d_2 = [0.5, 1.0]$, $\alpha = [0.1, 0.3]$, $\beta = [0.2, 0.3]$, $\theta = [0.05, 0.2]$, $C_{10} = [2.0, 3.0]$, $C_{11} = [2.5, 4.0]$, $C_{20} = [5, 10]$, $C_{21} = [0.05, 2]$, $C_{30} = [10, 15]$, $C_T = [10, 12]$, $C_{40} = [0.2, 0.3]$, $C_{41} = [0.5, 0.8]$, $C_{42} = [0.7, 1.0]$, $C_{50} = [20.0, 25.0]$, $C_{51} = [200, 250]$, $C_{52} = [0.01, 0.02]$, $\mu = 0.25$ and $\rho = 0.25$. Then, the required optimal solution is as follows: optimum reserve selling price $P_0^* = 53.54$, Optimum production lot size $Q^* = 1287.8875$ and maximum average profit $\mathrm{AP} = 20193.34$. This is global maximum value because the Figure 2 shows that the average profit function is clearly concave function of $P_0$ and the selling price at retail channel is a monotonic increasing function (Fig. 3) of time that is quite rational in a given economy. The Figure 4 shows that the level of stock gradually decreases due of demand of the customers.

**Example 2.** When the cycle time horizon is infinite, the $P_0$ and $T$ are decision variables and the values of all the parameters are considered as same as Example 1. In this case, the near optimal solution is $T^* = 2.0$, $P_0^* = 146.62$, $Q^* = 817.56$ and $\mathrm{AP}^* = 20419.61$. In this case, average profit function shows concave function (Fig. 5) of time horizon ($T$) and reserve selling price $P_0$.

**Example 3.** When the cycle time horizon is infinite and AP is approximated considering $|m_1T| \ll 1$ and $|m_2T| \ll 1$, and the $P_0$ and $T$ are decision variables and the values of all the parameters are considered as same as Example 1. In this case, the near optimal solution is $T^* = 1,68$, $P_0^* = 200.56$, $Q^* = 669.97$ and $\mathrm{AP}^* = 23545.70$. In this case, average profit function shows concave function (Fig. 6) of time horizon ($T$) and reserve selling price $P_0$.

**Example 4.** Let us consider the interval values of the key parameters in appropriate units when sales effort is applied as follows: $T = [1, 2]$, $R = [500, 700]$, $d_0 = [180, 200]$, $d_1 = [0.15, 0.25]$, $d_2 = [0.5, 1.0]$, $d_3 = [0.2, 0.8]$, $\gamma = [0.2, 0.5]$, $E_0 = [200, 300]$, $\alpha = [0.01, 0.03]$, $\beta = [0.2, 0.3]$, $\theta = [0.05, 0.2]$, $C_{10} = [2.0, 3.0]$, $C_{11} = [2.5, 4.0]$, $C_{20} = [5, 10]$, $C_{21} = [0.05, 2]$, $C_{30} = [10, 15]$, $C_T = [10, 12]$, $C_{40} = [0.2, 0.3]$, $C_{41} = [0.5, 0.8]$, $C_{42} = [0.7, 1.0]$, $C_{50} = [20.0, 25.0]$, $C_{51} = [200, 250]$, $C_{52} = [0.01, 0.02]$ and $\rho = 0.25$. Then, the required optimal solution is as follows: optimum reserve selling price $P_0^* = 262.94$, Optimum production lot size $Q^* = 139.35$ and maximum average profit $\mathrm{AP} = 5268.90$. This is global maximum value because the Figure 7 shows that the average profit function is clearly concave function of $P_0$ and the selling price at retail channel is a monotonic increasing function (Fig. 8) of time that is quite rational in a given economy. The Figure 9 shows that the level of stock gradually decreases due of demand of the customers and sales effort level increases with time (Fig. 10) to capture the market efficiently.

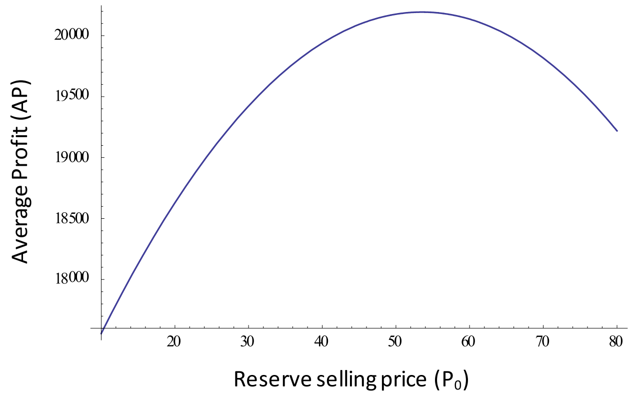


FIGURE 2. Average profit *versus* reserve selling price.

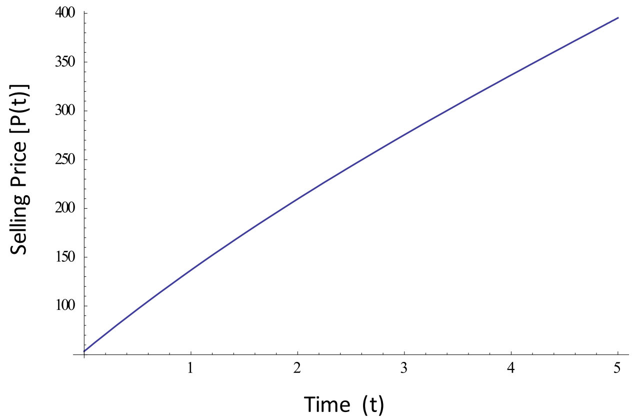


FIGURE 3. Selling price at retail channel *versus* time.

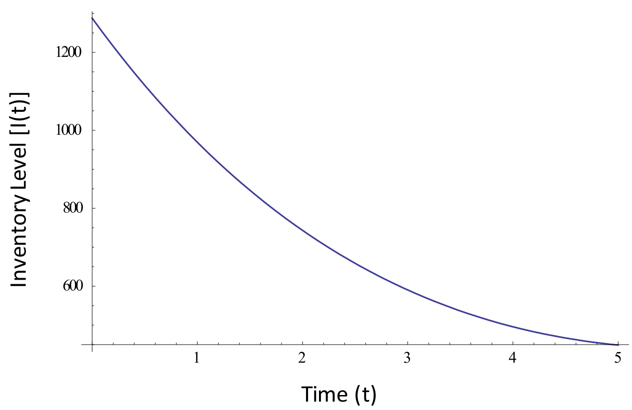


FIGURE 4. Inventory level at retail channel *versus* time.

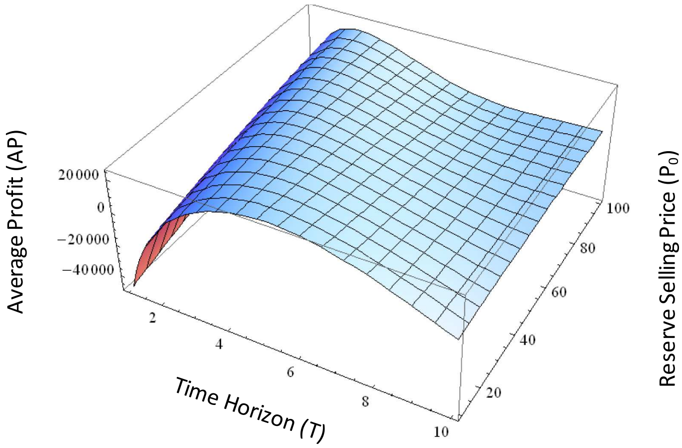


FIGURE 5. Average profit *versus* time horizon and reserve selling price.

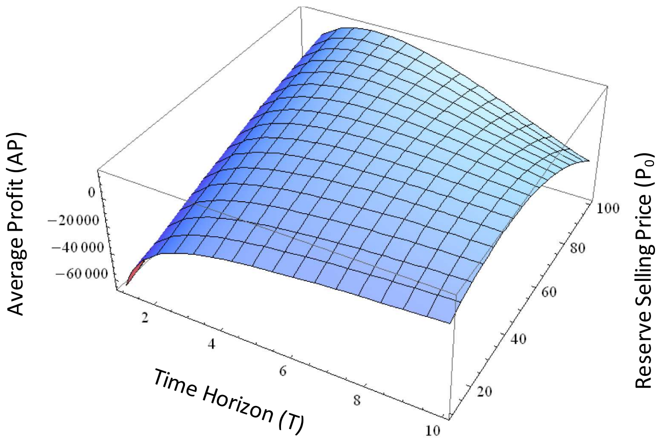


FIGURE 6. Average profit *versus* time horizon and reserve selling price for approximate value of average profit function.

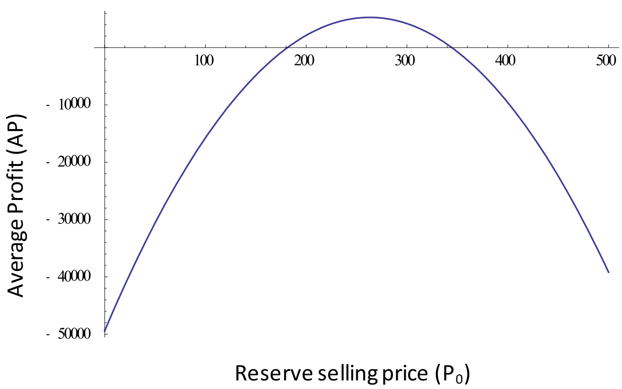


FIGURE 7. Average profit *versus* reserve selling price when sales effort is applied.

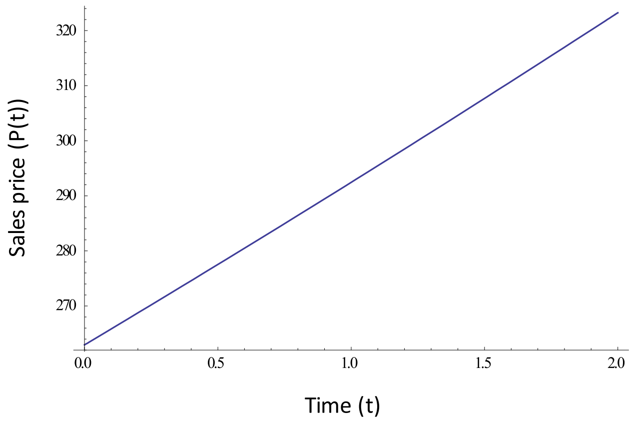


FIGURE 8. Sales price at retail channel *versus* time when sales effort is applied.

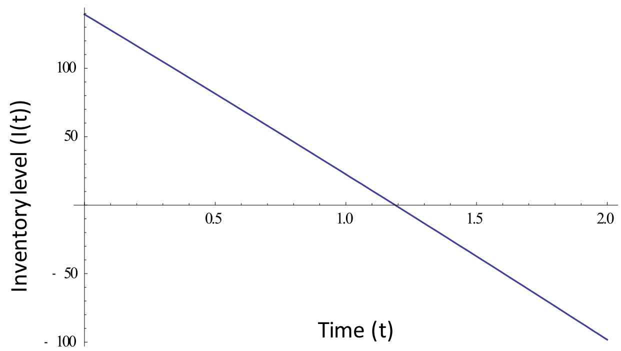


FIGURE 9. Inventory level at retail channel *versus* time when sales effort is applied.

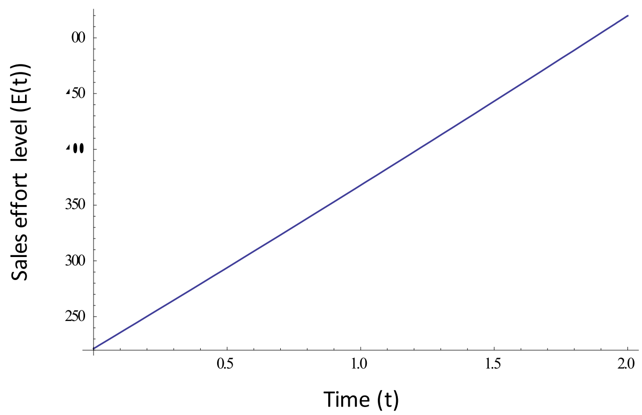


FIGURE 10. Sales effort *versus* time.

## 6. Managerial insights

In spite of the omnipresence of environmental narratives of the green products in the green marketing literature remarkably few pragmatic studies guide the businesses to integrate and functioning green marketing in normal business practice. As a result, green technology and green marketing be unsuccessful to achieve its potential for enlightening the quality of life for consumers, and nor it benefits the ecosystem. The green technologies/policies for production, transportation and consumption are assertive sustainable development to the society, and managers of the firms are compelled to recognize the need of green marketing throughout the organization, and build sustainability into the performance of their channel members, products and services. Every successful firm should emphasize on pricing strategy, branded quality of the products, corporate social responsibilities and sales promotion activities, among many other factors. As price and quality of the products are vital factors to attract the consumers of a product, the environmentally-conscious pricing strategy fixes a price that mirrors the ecological ingredients of a product, donations to environmentally responsible firms, and sales promotion methods that engage end-users to support green initiatives like life-cycle costing, carbon offset pricing and competitive pricing.

Keeping in mind the above issues, the findings of the model presented in numerical analysis helps the manager of the manufacturing firm to determine the optimal sales prices and optimal production lotsizes considering several costs related to production, inventory of the finished products and carbon cap-and-trade for GHG emission. Since the product cost of an item depends on its manufacturing costs like cost of raw-materials, cost of technology, costs of labour, energy, tool/die, disposal of wastes, rework cost of imperfect quality products, carbon emission costs due to production and transportation, among others. Consequently, the selling price varies with these costs directly. Quite often, the cost parameters of GHG emission diminished with advanced green technology that requires more investment at initial stage. Although advance green technology and green supply chain need more investment, it reduces the costs of GHG emission directly and reduces the manufacturing costs indirectly in a long run project. On the other hand, sales efforts are applied in marketing management to attract the customers to buy more. Thus, the green technology for supply chain management should be adopted by the industrialists to keep safe the environment from damage due to pollution caused by GHG and to sustain their businesses in a competitive marketing system.

## 7. Conclusions

A modern business management cannot be sustained without an operative ecological system as a source of natural resources. The GHGs is a key factor of ecological degradation. The GHGs are generally engendered for all living beings and these are emitted from industries, transportation systems, rapid population exploration, poverty, soil degradation, traffic, mismanagement and misuses of open access resources because of unfocussed property rights, etc. In the last decades, the governmental policies are implemented to control pollution caused by industries, transportation systems and other factors caused by human beings.

As the government charges cost to control pollution caused by manufacturing industries, the manufacturing costs include material costs, energy costs, labor costs, the cost for technology, tool/die cost, transportation cost, and cost for GHGs. As a result, the selling price increases indirectly for the cost of advanced technologies that reduce the emission of GHGs. Moreover, advanced technologies for energy generation like wind energy, solar energy, and hydro energy save non-renewable energy. Therefore, the inclusion of renewable energy systems in whole energy systems is urgent to save our human civilization from pollution and the scarcity of non-renewable energies. As the cost for the set up of renewable energy systems is high at starting level, the setting of the selling price of the green products is a vital issue to attract the end customers to buy more. In this context, an awareness program about green products conducted by marketing management is a way to boost selling the products at a fair price. The objective of the proposed model is to find out optimal selling prices and production lot size trading of the several costs related to manufacturing and transporting the products to the customers so that the average profit of the manufacturer is maximum. In this model, the selling prices are determined based on the stock of the product and several costs related to manufacturing the items. Moreover, the 21 imperfect quality products are reworked at a cost to save the material cost as well as natural resources. This model helps the management of a manufacturing enterprise to determine selling prices and production lot size keeping in mind the sustainability of his/her business as well as the sustainability of a clean environment and available natural resources.

The proposed model may be extended immediately for a dynamical system incorporating the control variables like advertisement and awareness efforts for gray consumers who are ignorant about green product and green technologies. The proposed model is analysed for continuous cases only which is a limitation of the model. This limitation may be relaxed further considering discrete variables. This model can be expanded for uncertain market demand. Furthermore, production disruption due to supply disruption of raw-materials and machine breakdown is necessary to incorporate in forms of uncertainty that can improve the implication of the model in practice and make future research more meaningful.

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