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AN APPLICATION OF A SMART PRODUCTION SYSTEM TO CONTROL DETERIORATED INVENTORY

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Abstract. Deteriorating products require different handling procedures. Handling procedures includes prevention of the natural deterioration rate of the product. The production of deteriorating products requires prevention technology for those products to use for a long time. Overproduction of deteriorating types of products causes more trouble in preventing deterioration. This study uses a smart production system to control the production of deteriorating products. A controllable production rate controls the production of deteriorating products, and preservation technology reduces the deterioration rate of products. Preservation technology helps extend the life of products, but it requires a specific temperature-controlled environment to work at maximum efficiency. Transportation of these products uses refrigerated transportation to maintain the quality during the transportation time. The purpose of using all these features for deteriorating products is to reduce the deterioration rate, which helps to reduce waste generation from production. Besides, imperfect products from the production system pass through a remanufacturing process to support the waste reduction process. A sustainable supply chain management model under the above-stated strategies is described here. Classical optimization is used to find the global optimum solution of the objective function. Then, the total cost of the supply chain is optimized using unique solutions of production rate, number of deliveries, delivery lot size, system reliability, and preservation investment. Global optimum solutions are established theoretically, and few propositions are developed. Some special cases, case studies, and a comparison graph are provided to validate the results. The beta distribution provides the minimum total cost of the system than uniform, gamma, triangular, and double triangular distribution. Smart production allows 72% system reliability with negligible imperfect products. Besides, the proposed policy gains 22.72% more profit than the existing literature. The model is more realistic through convex 3D graphs, sensitivity analyses, and managerial insights.

Mathematics Subject Classification. 90B05, 90B30, 90C30, 90C31, 90C47.

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1. Introduction

For the development of different business strategies in the modern competitive environment, supply chain management (SCM) plays an essential role [38]. However, the supply chain's role in meeting society's daily needs in the current financial situation and the sustainable development of society is undeniable. The waste reduction approach amends one more dimension to the SCM for sustainability [19]. Sustainable supply chain management (SSCM) stands on the three pillars of sustainability, named economical, social, and environmental benefit. For the environmental benefit, the disposal of deteriorated products is considered here. From the economic and social point of view, remanufacturing, preservation technology [11], refrigerated transportation, flexible, and complex production [41] create jobs, reduce total cost, and increase the company's reputation. This study develops a single-manufacturer single-retailer-based SSCM model. The proposed model aims in minimizing the centralized system-based total supply chain cost.

The short lifespan of mass-produced products is a problem that impacts society, industries, and the environment alike. Industries are attempting to reduce the release of deteriorated products into nature to control pollution from imperfect waste products [48]. Conventionally, such products are sent to landfills or dumping grounds as garbage. The concept of zero waste is still under implementation, but industries can reduce deterioration rates to prevent the production of excess waste [1]. Waste can be further reduced if deteriorating products can be preserved more than its original lifespan. As the lifetime of a product increases, its long-term utility increases. Different disposal investments are necessary for different members of the SSCM. However, sustainability stands for waste reduction, safe environment, and different people-oriented development planning of society [49]. Moreover, remanufacturing of imperfect deteriorating products controls the total cost and balances raw materials and natural resources. All these matters are incorporated in the proposed research.

A smart production system can produce imperfect products when the system goes into an out-of-control state. The perfect products are sent to the market for customers, and the fate of the imperfect products depends on manufacturing planning. Ullah *et al.* [46] developed a two-echelon SCM model in which deteriorating products are transferred to the retailer through refrigerated transportation. Still, there is no guarantee of the freshness of the products. Based on various past studies, some new considerations have been reflected here from the current situation. Here the combination of different situations, viz, smart production, preservation, and refrigerated transportation for transfer fresh items, and reduction of failure rate, *i.e.*, increase of the system reliability are considered for an SSCM.

1.1. Research gap

The following research gaps can be drawn based on the existing literature.

- The use of smart production for controlling deteriorating products is not well-exposed in the literature. A widely used technology for preventing deterioration is preservation. But, preservation works after producing products, especially for the overproduction situation. What if the feature of smart production can control the production of deteriorating products? Does this smart production reduce waste generation from deteriorating products?
- Studies on sustainability and waste reduction approach under a two-echelon SCM model exist [19]. However, the concept of remanufacturing for semi-defective items and separate disposal investment for imperfect items from the manufacturer and retailer side has not been studied.
- The use of preservation technology is not new in literature [22]. But using the preservation in association with smart technology how much preservation investment can be reduced rather than constantly producing deteriorating products?
- Studies on refrigerated transportation exists in literature. But, the application of refrigerated transportation facilities for deteriorated items-based smart production system has rarely been investigated.
- The use of remanufacturing system for imperfect deteriorating products provides additional scope for minimizing waste from the smart production system [10]. Therefore, remanufacturing of imperfect deteriorating

products, system reliability, smart production, and refrigerated transportation under a smart production system are novel contributions to the literature.

1.2. Contribution

This is the first study on SSCM under preservation technology investment, smart production, refrigerated transportation facility, and system reliability issues. For the environmental issues, the waste reduction approach through the disposal of imperfect products makes the model more eco-friendly. Variability of the production rate becomes more realistic and controls the production rate based on market demand. Furthermore, remanufacturing the recovered imperfect products decreases the total SSCM cost. At a glance, the main contributions of this model are as follows:

- The present study considers a smart production system for deteriorating products. The controllable production rate controls the production of deteriorating products such that products with preservation can be used for a maximum lifetime with reduced waste.
- As the production of deteriorating products is controlled now, the preservation technology reduces the deterioration rate of the product. Does the smart production has influence to reduce the preservation cost over a constant production system for deteriorating products?
- Remanufacturing of imperfect products reduces the total SCM cost and maintains the balance of natural resources. Does the smart production affect the imperfect production process?

A mathematical model is proposed, and more importantly, the global optimality of the resulting solution has been proved. This model minimizes the total cost by optimizing the production rate, lot size quantity, number of shipments, and preservation investment.

1.3. Structure of this study

The remainder of this paper is organized such as Section 2 describes the related literature review. Table 1 gives the research gap among previous authors. Section 3 presents the problem purpose, relevant symbols, and associated assumptions. Section 4 describes the model formulation, and Section 5 provides the methodology of the solution. The numerical experiments are described in Section 6, and the sensitivity analysis is given in Section 7. Section 8 provides managerial insights into the study, and Section 9 presents the conclusions.

2. Related literature review

Contributions of previous research and research gaps in the literature are discussed in this section. Furthermore, the novel contribution of this study is stated in the subsections. An SSCM is very much essential for the development of society through economic benefit, environment-friendly approach, and social responsibility. The contributions of previous researchers to SSCM research are described in the first subsection. Further, smart production [3], remanufacturing imperfect items through reworking, system reliability, and preservation technology investment are important keywords related to this model. Existing research about the keywords is discussed in this section. However, a research gap determining Table 1 is provided for a better understanding of the research gap.

2.1. Sustainable supply chain management (SSCM)

With learning from the past socio-economic stagnation of society, sustainability is a vital step of advanced thinking toward a resourcesful planning [44]. Social, economic, and environmental matters are the three pillars of the development indicator [14]. Sarkar and Guchhait [17] discussed emissions reduction policies within a SSCM for environmental purpose. On the other hand, a supply chain is crucial for meeting daily life's needs in a short time before the customer's hands. The combination of sustainability and SCM can deliver the message of development to the society. An SSCM model for fixed lifetime products under quality improvement and carbon emissions strategy was presented by Sarkar *et al.* [39]. In the same direction, Rout *et al.* [35] considered an SSCM

Table 1. Author(s) contribution table.

Author(s)	Smart production	Deterioration	Preservation	Remanufacturing	Refrigerated transport	Waste reduction	System reliability	Model type
Ullah *et al.* [46]	NA	Yes	Yes	NA	Yes	Yes	NA	SCM
Roy and Chaudhari [36]	NA	Yes	NA	NA	NA	NA	NA	EPQ
Rout *et al.* [35]	NA	Yes	NA	NA	NA	NA	NA	SSCM
Emamian *et al.* [13]	NA	NA	NA	NA	NA	NA	NA	SSCM
Angizeh *et al.* [4]	Yes	NA	NA	NA	NA	NA	NA	SM
Rahmani *et al.* [33]	Yes	NA	NA	NA	NA	NA	NA	SP
Cañas *et al.* [8]	Yes	NA	NA	NA	NA	NA	Yes	SM
Heydari *et al.* [16]	NA	NA	NA	Yes	NA	NA	NA	SCM
Ben-Daya *et al.* [6]	NA	NA	NA	Yes	NA	NA	NA	SCM
Zhao *et al.* [50]	NA	NA	NA	Yes	NA	NA	NA	EPQ
Ouaret *et al.* [28]	NA	Yes	NA	Yes	NA	NA	NA	Hybrid
Malik *et al.* [7]	Yes	NA	NA	NA	NA	NA	NA	SCM
Adak and Mahapatra [2]	NA	Yes	NA	NA	NA	NA	Yes	EOQ
Sarkar *et al.* [26]	NA	NA	NA	NA	NA	NA	Yes	EPQ
Li *et al.* [21]	NA	Yes	Yes	NA	NA	NA	NA	EOQ
Wang and Jiang [47]	NA	Yes	Yes	NA	NA	NA	NA	EOQ
Gautam *et al.* [15]	NA	NA	NA	Yes	NA	NA	NA	EPQ
Mishra *et al.* [23]	NA	NA	NA	NA	NA	NA	NA	EPQ
Rahaman *et al.* [32]	NA	Yes	Yes	NA	NA	NA	NA	EPQ
Sebatjane [43]	NA	Yes	Yes	NA	NA	NA	NA	SCM
Pal *et al.* [29]	NA	Yes	NA	NA	NA	NA	NA	EPQ
Yadav *et al.* [30]	NA	Yes	Yes	NA	NA	Yes	NA	SCM
Pervin *et al.* [31]	NA	Yes	NA	NA	NA	NA	NA	EOQ
This paper	Yes	Yes	Yes	Yes	Yes	Yes	Yes	SSCM

Notes: SCM – supply chain management, SSCM – sustainable supply chain management, SSM – sustainable smart manufacturing, SM – smart manufacturing, NA – not applicable, EPQ – economic production quantity, EOQ – economic order quantity.

model of deteriorating items and imperfect production under different carbon emissions regulations. Emamian *et al.* [13] proposed a sustainable integration for production routing using the bee colony optimization (BCO) algorithm. They found that BCO gave a more feasible solution than the non-dominated sorting genetic algorithm (NSGA) II. A multiple product-based SCM model was studied by Sarkar *et al.* [42] where the manufactured used radio frequency identification to collect used products and their information. Different type of transportation policies [37] are used within the supply chain as an extended part of supply chain. These policies are important to explain a situation for supply chain.

It is clear from these studies that strong sustainability is based on an environment-friendly approach and initiative toward social development. These research focused on the different carbon emissions reduction approaches and green investment. The waste reduction approach helps for environmental protection, and separate disposal cost for manufacturer & retailer indicates safe environmental matter. However, to make a smart SSCM, a flexible production rate (FPR) can play an important role in maintaining the market demand, avoiding shortages, and making a good reputation. Thus, the effects of smart production for an SSCM are discussed in the next section.

2.2. Smart production for supply chain management

In different unavoidable situations, there is a need for some extra products in a short time to fulfill customer demand. To control such a situation and to satisfy the variable market demand, smart production is important for the industry. However, in the competitive market, smart production maintains the quality of the product with technology. Moreover, smart production reduces the production cost, controls the labor problem, maintains the quality of the products, and controls the fluctuating market demand. Angizeh *et al.* [4] presented an optimal production planning for smart manufacturers with application to food production. The importance of system reliability and sustainability was missing in their research. Tan *et al.* [45] developed a smart production process for optimum energy profiling using machine learning. Rahmani *et al.* [33] discussed a conceptual framework for a smart production system connecting with the necessity of the environment. But their proposed review did not focus on the sustainability property. Cañas *et al.* [8] discussed a systematic review of smart production based on technological development in Industry 4.0.

From the above research studies, it may be concluded that every research included flexibility in their manufacturing system. Until now, less research has considered smart production's effect on controlling deteriorating products and corresponding preservation cost. The proposed study considers refrigerated transport facilities and disposal for the imperfect items for fresh and recovered items, which are very much helpful for SSCM. Smart production is correlated with preservation investment and remanufacturing for waste reduction. In this sense, how the remanufacturing connects with smart production and SSCM under FPR is discussed in detail in the next section.

2.3. Remanufacturing in supply chain management

Everyone awares of the various threats to the environment. Waste reduction, remanufacturing, disposal of damaged items, and greening matters can be utilized to protect the environment. Minimizing the cost of the reworking of imperfect products and make ready for market with environmental protection is the main purpose of remanufacturing. However, remanufacturing controls the use of natural resources, fulfills the fluctuating market demand, and maintains the stability of the environment. A two-stage reverse supply chain model under remanufacturing was discussed by Heydari *et al.* [16]. In their model, a revenue-sharing contract was proposed to convince the retailer to consider the uncertainty of remanufacturing capacity in deciding the reward amount. Still, a sustainable approach with a smart production concept was missing. Ben-Daya *et al.* [6] presented a production-remanufacturing inventory model for multi-buyer under centralized consignment, but the concept of SSCM through reliability issue were not considered in their model.

Zhao *et al.* [50] studied a production model of both single and multiple categories of end-of-life products under carbon emissions uncertainty, a carbon cap-and-trade policy, and remanufacturing. Ouaret *et al.* [28] developed a hybrid manufacturing-remanufacturing system for random deterioration rate. But their model focus on reliability and smart production strategy. Recently, Kerin and Pham [20] discussed the use of modern technologies within a remanufacturing industry. But the reliability issue was not considered in their research.

Remanufacturing is a better opportunity to close the loop for mitigation of resource depletion, reduction of global warming potential, and safe handling of toxic substances. In addition to its environmental benefits, remanufacturing offers job creation and economic growth opportunities. However, remanufacturing gives the facility lower cost, less downtime, increased equipment resale value, and more sustainable options. Remanufacturing and sustainability are correlated with each other. The above research studies found that very less research focused on the effect of remanufacturing and smart production for the SSCM. Moreover, the remanufacturing strategy with smart production and SSCM needs further investigation for increased reliability. This research gap, discussed in the next section, is crucial to system reliability.

2.4. System reliability for production system

Reliability is the probability that a product, system, or service will perform its intended function adequately for a specified period or operate in a defined environment without failure. System reliability has an inverse relationship with the machine as well as product quality. The lesser the system's reliability is, the more reliable the machine is. Thus, a machine with less system reliability produces better-quality products. Malik *et al.* [7] studied a supply chain model by optimizing production rate for the supply chain players under a Nash game policy. Adak and Mahapatra [2] presented a multi-item inventory model of deteriorating items with partial backlogging and shortages under reliability. Modibbo *et al.* [24] proposed a simulation algorithm for system reliability. They considered the reliability functions of some distributions in their proposed model. Recently, Sarkar *et al.* [26] discussed the effects of consumer service, reliability, and production costs under controlled carbon emissions in a reliable production system.

The previous research details mainly focused on system reliability. However, the development investment as a function of system reliability of the manufacturing–remanufacturing system with an FPR is introduced in this research. Moreover, preservation technology is the main tool for deteriorating items. It is directly correlated with the quality of the products, remanufacturing, and system reliability for SSCM. Hence, this important topic is described in the next section.

2.5. Preservation technology for deteriorating products

From an environmental point of view, disposing of deteriorated products contributes to additional greenhouse gas emissions along with resource depletion and material wastage. Preservation technology is the probable solution to such a situation [18]. It reduces the deterioration of imperfect items, maintains the quality of good products [34], fulfills the customer's needs, and protects the environment. However, preservation technology through different techniques makes running a business for the supply chain by supplying fresh items. Ullah *et al.* [46] discussed a two-echelon SCM model of deteriorated products under preservation technology and waste reduction approach, but in their model, no system reliability and sustainability were considered. The pricing, ordering, and preservation investment model for non-instantaneous deteriorating items was developed by Li *et al.* [21]. Das *et al.* [12] presented an inventory model of price-dependent demand and partial backlogging under the application of preservation technology for deteriorated items. A dynamic investment for preservation technology was studied by Saha *et al.* [22]. An economic production quantity (EPQ) model of deteriorated products with variable demand and lock fuzzy under the preservation technology was presented by Rahaman *et al.* [32]. Sebatjane [43] discussed a food supply chain under the preservation technology when live-growing foods were converted into processed food.

It is found from the above discussion that different researchers considered different preservation strategies to minimize the total cost or maximize the total profit. In this proposed research, preservation technology is considered along with a smart production system for maintaining the quality of the product and reducing deterioration. Furthermore, smart production and separate disposal costs for manufacturers and retailers are considered for environmental protection with those business strategies. From the above discussion, one may conclude that there is a big research gap in the literature on the above-discussed keywords. Therefore, an SSCM with smart production, system reliability, and remanufacturing is considered in this proposed study to fulfill the research gap.

3. Problem definition, notation, and assumptions

In this section, the problem definition of this model, based on the SSCM with a controllable production rate, is first described in Section 3.1. Section 3.2 mentions the notation used to illustrate this model, while Section 3.3 describes the assumptions of the proposed SSCM model.

3.1. Problem definition

A smart, automated machinery system that is capable of making decisions autonomously based on informative data analysis through distributive intelligence and autonomous decisions supervised by skilled people, is called a smart production system (Fig. 1). Three main components of a smart production system are data analytics, internet of things (IoT), and autonomation. At first, all machines should be smart to make decisions based on data analysis.

All input data are analyzed and stored in the cloud. All smart machines are interconnected with each other and with the cloud through IoT. That is, all machines receive data from the cloud and then analyze it to make decision about productivity. But, one of the important factors of this system is distributive intelligence. Even if all machines are connected to each other, one machine can disconnect itself from another system for any unwanted situation. Human intervention is significantly less within a smart production system but requires highly skilled people for human-machine interaction. Skilled people verify every decision by machine to prevent risks.

This study occurs under an SSCM model where the manufacturer uses a smart production system to produce deteriorating products. A single manufacturer and a single retailer are the supply chain players. The manufacturer uses a preservation technology to reduce the current deterioration rate of products. With the increase in preservation investment, deterioration monotonically decreases, and reduction of deterioration monotonically

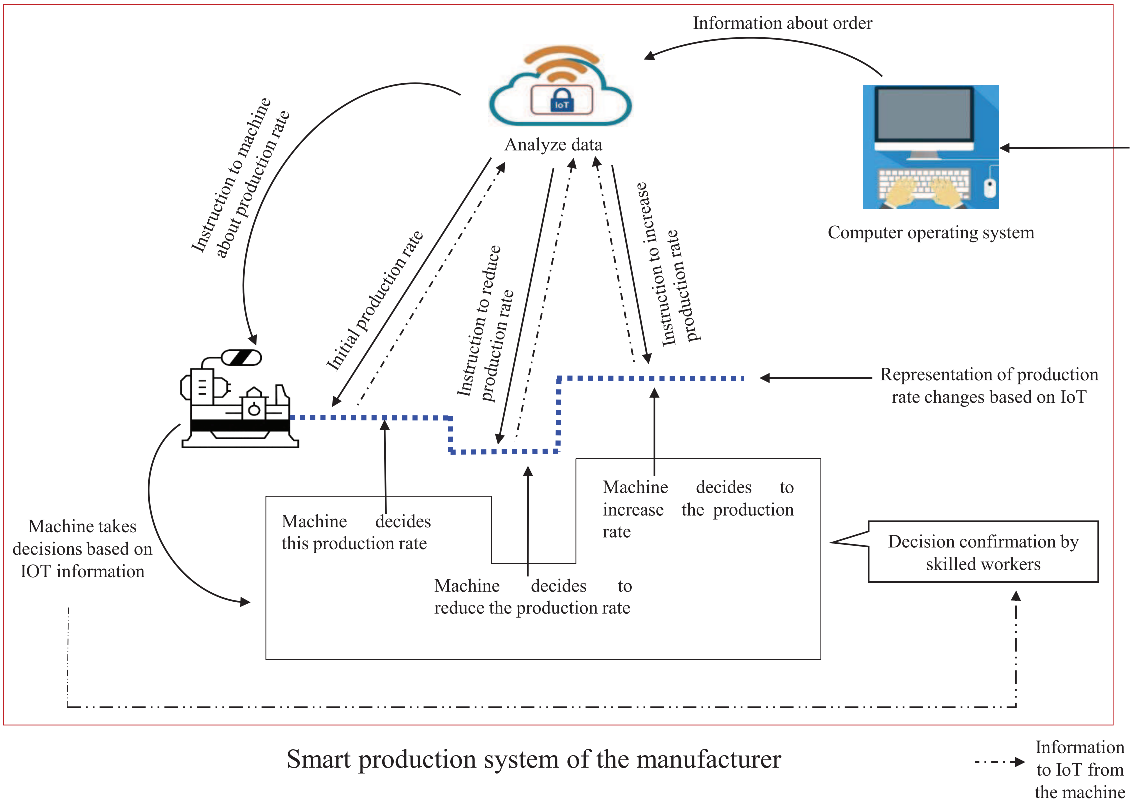

FIGURE 1. Schematic diagram of a smart production system used for deteriorating products.

increases (Fig. 2). The deteriorating products are chilled type such that it requires a refrigerator transportation facility. Then, storing these products requires a refrigeration system too. This incurs a higher holding cost. Thus, the retailer chooses a single-setup-multi-delivery (SSMD) policy to receive products in shipments such that the retailer does not have to store all ordered products at a time. The manufacturer remanufactures imperfect products due to machinery faults instead of land filling due to environmental issues.

The proposed model gives a new direction through a supply chain strategy under remanufacturing and preservation in transportation. A variable production rate is considered instead of a constant production rate to control the fluctuating market demand. Ullah *et al.* [46] considered the production remanufacturing SCM model. However, that research was based on a constant production rate without system reliability. Moreover, the sustainability concept with the variable production rate, waste reduction approach, remanufacturing, and preservation investment makes the model more attractive. The model finds the optimum total cost by optimizing the value of the decision variables production rate, delivery lot size, number of shipments, reliability, and preservation technology.

3.2. Notation

The following notation is used for this model.

Decision variables

p Production rate of the manufacturer (unit/unit time)
q Delivery lot size (units/cycle)

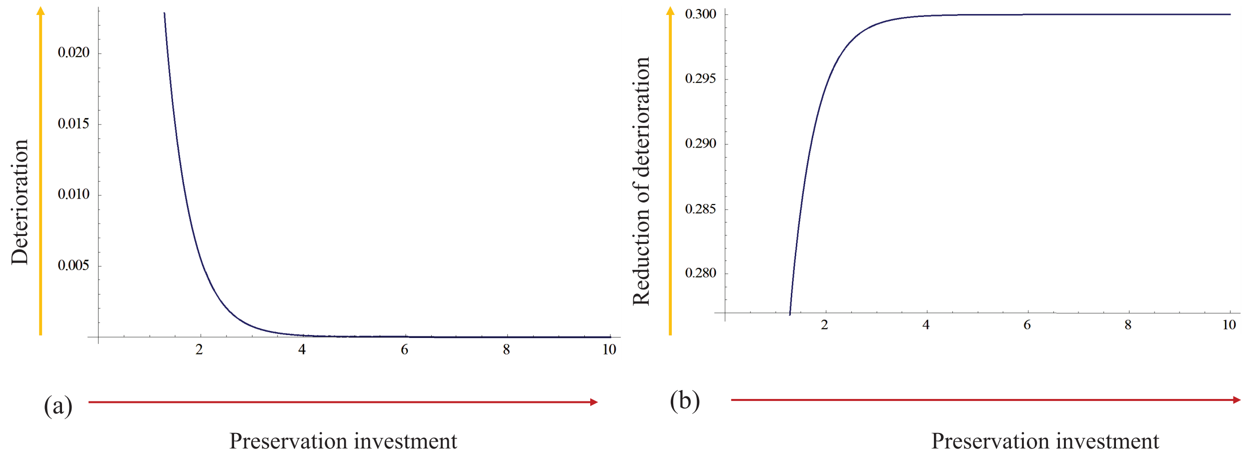

FIGURE 2. (a) Investment in preservation against deterioration. (b) Investment in preservation for reduction of deterioration.

n	Number of lot size deliveries per production batch to the retailer (integer)
α	System reliability parameter $(0 < \alpha < 1)$
κ	Preservation cost, $\kappa \geq 0$ (\$/cycle)

Input parameters

$\alpha_{\max}$	Maximum value of reliability parameter $(0 < \alpha_{\max} < 1)$
$\alpha_{\min}$	Minimum value of reliability parameter $(0 < \alpha_{\min} < 1)$
Λ	Difficulties related to system reliability
C_1	Development cost per unit depending upon system reliability (\$/unit)
C_p	Production cost per unit (\$/unit)
M_c	Material cost per unit (\$/unit)
R_c	Imperfect item's remanufacturing cost per unit(\$/unit)
T	Length of the cycle for production inventory (time unit)
t_1	Manufacturer's production time (time unit)
t_2	Manufacturer's non-production time (time unit)
t_3	Time between two successive deliveries to the retailer
D	Market demand (unit/unit time)
θ	Deterioration rate per unit time, $\theta > 0$ (unit/unit time)
C_l	Unit energy and labor costs, independent of system reliability α (\$/unit)
C_{rt}	Cost of design complexity (\$/unit)
$m(\kappa)$	Reduced rate of deterioration (unit/unit time)
M_l	Lot size manufacturer's production per cycle (units)
A_m	Area of manufacturer's inventory storage
A_r	Area of retailer's inventory storage
l	Distance covered for shipments (km)
C_t	Truck capacity (units/truck)
r	Shape parameter for the reduced deterioration rate
β	Scaling parameter for imperfect production
$C_{\theta m}$	Manufacturer's deterioration cost per unit (\$/unit)
$C_{\theta r}$	Retailer's deterioration cost per unit (\$/unit)
C_d	Unit defective treatment cost of the manufacturer (\$/unit)

I_c	Unit inspection cost of imperfect product of the manufacturer ($/unit)
ξ	Tool/die cost ($/cycle)
S_{cm}	Setup cost for smart production ($/batch)
H_{cm}	Holding cost for manufacturer ($/unit/unit time)
H_{cr}	Holding cost for retailer ($/unit/unit time)
O_{cr}	Ordering cost to the retailer per order ($/order)
F_r	Freezing cost during transportation per unit per km ($/unit/km)
F	Fixed shipment cost for transportation per delivery ($/delivery)
F_t	Unit transportation cost per truck ($/truck unit)
TSC_m	Manufacturer's total cost per cycle ($/cycle)
TSC_r	Retailer's total cost per cycle ($/cycle)
TSC	Total SSCM cost per cycle ($/cycle)

3.3. Assumptions

The following assumptions are used to develop the proposed model.

(1) A single type of deteriorating product is considered for the single-manufacturer-single-retailer two-echelon SSCM. The market demand (D) is constant.
(2) Although a manufacturer uses a smart production system to control the production rate of deteriorating products, the production system produces imperfect products. A development cost C_1 is utilized to improve system reliability in the form $C_1 = c_l + C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}$, where α is the system reliability parameter. The system reliability lies between its maximum and minimum value [26].
(3) Deterioration begins at a random rate θ for the finished deteriorating products. To reduce this deterioration, the manufacturer invests in preservation κ to extend the product lifespan. Following the preservation, the reduced deteriorate rate is $\theta - m(\kappa) = \theta - \theta(1-e^{-r\kappa}) = e^{-r\kappa}$ per unit [46].
(4) Imperfect deteriorated products are considered waste, and both the manufacturer and retailer pay deterioration cost for their disposals, the total of which depends upon their respective averages.
(5) The manufacturer sends deteriorating products through a SSMD policy using truck transportation. The trucks have refrigerator facilities, and the retailer pays for the extra cooling cost during shipping.
(6) The time horizon is finite.

4. Mathematical model

A single-manufacturer-single-retailer is involved within a two-echelon SSCM, where the manufacturer produces deteriorating products. The smart production system of the manufacturer produces imperfect deteriorating products due to system error. Such imperfect production hampers the goodwill of the manufacturer as well as the product quality. These imperfect deteriorating products are then sent for remanufacturing. The finished deteriorating products begin to deteriorate at the rate of θ after a certain time, and the deterioration rate is random. The manufacturer uses preservation technology to reduce the deterioration of finished deteriorating products. The lifespan of the deteriorating products is increased using the preservation method. An investment is required for preservation; even following its implementation, some products deteriorate. Those products are treated as waste, and the retailer and manufacturer agree to dispose of them. Transshipment between the retailer and the manufacturer is possible in road transport, and the trucks contain refrigerating system to prevent deterioration during transport. An SSMD policy is used for transportation. The objective of this study is two-fold: reduction of waste from deteriorating products and reduction of preservation investment. The proposed policy reduces waste from both the production system and supply chain. As the production rate is adjustable within a range, the management does not need to produce products at the same rate and hold them to increase more deteriorated products. Then less preservation cost can preserve the necessary amount of deteriorating products. On the other hand, remanufacturing imperfect products from the production system reduces waste from the

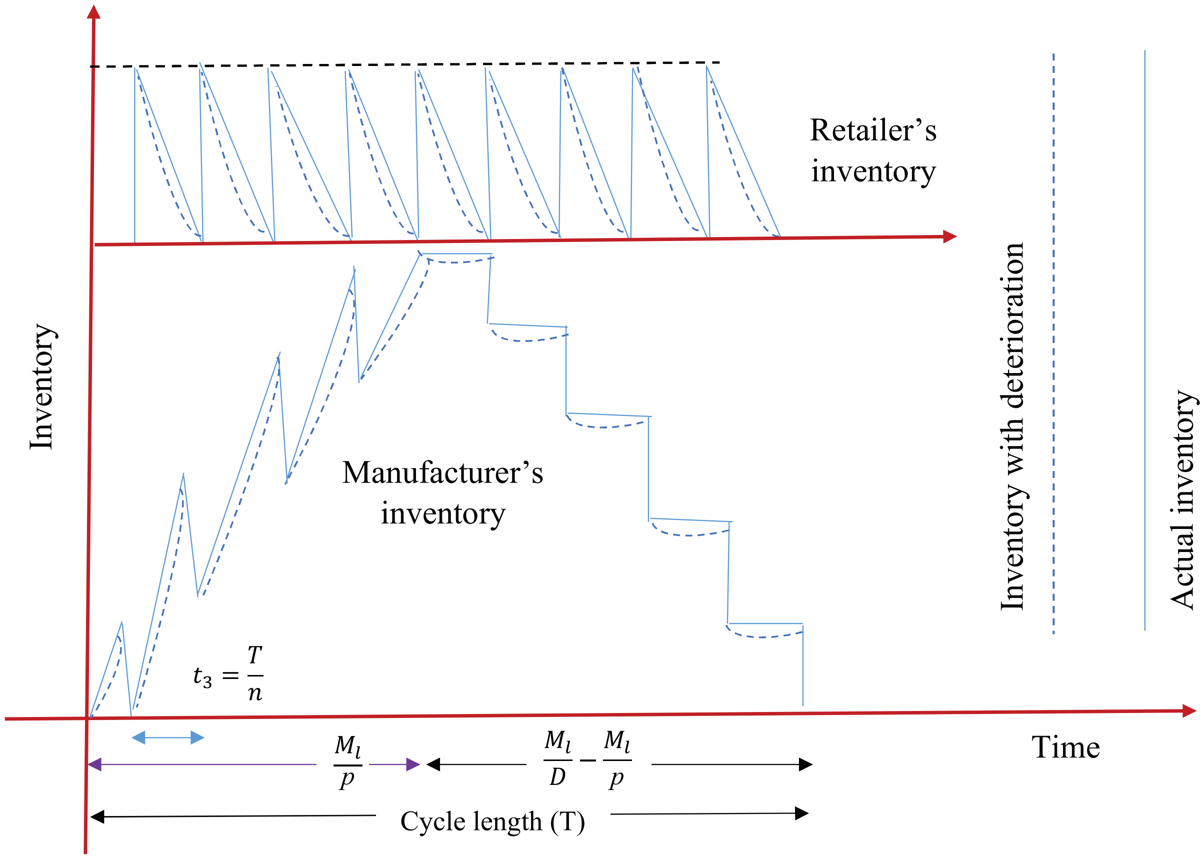

FIGURE 3. Inventory position of manufacturer and retailer.

supply chain and creates opportunities to use remanufactured products again. The total inventory position of the system is given in Figure 3.

4.1. Manufacturer's model

The manufacturer receives an order of M_l quantity per cycle from the retailer. The manufacturer delivers this total ordered quality in n shipments using SSMD, and the delivery lot size to the retailer in each shipment is q. Each finished production batch comprises deteriorated products with the random rate θ. Then, at the end of the cycle, the manufacturer's production lot size is $M_l = nq + \theta A_m$, where A_m is the total inventory area of the manufacturer, and q is the delivery lot size to the retailer in each shipment. Thus, $\delta = \theta A_m \left(i.e.,\ A_m = \frac{\delta}{\theta}\right)$ is the number of deteriorated products per cycle. The deteriorated products at the retailer's end are represented by $\frac{\theta q T}{2}$. $t_1 = M_l/p$ and $t_2 = M_l(1/D - 1/p)$ are the manufacturer's production time and non-production time, respectively. Therefore, $A_m = qT\left(\frac{D}{p} + \frac{n-1}{2} - \frac{Dn}{2p}\right)$ [46].

4.1.1. Setup cost (MSC)

The manufacturer follows a single-setup policy for the business. The smart production system produces all products in one setup per cycle. If S_{cm} is the setup cost for the production batch, then the setup cost per cycle is $\frac{S_{cm}}{T}$, *i.e.*,

$$\text{MSC} = \frac{S_{cm}}{T}\cdot \tag{1}$$

4.1.2. Production cost (MPRC)

The production rate per unit of time is p. The production cost is affected by three factors:

(i) Material is the essential component for production, and the unit material cost of products is M_c.
(ii) Tool/die cost (ξ) is essential when a machine is working and producing products. Then, the tool/die cost of the smart production system is related to production rate as ξp.
(iii) Production cost depends on development cost $C_1(\alpha)$, where α is the system reliability parameter of the machinery system. This development cost comprises two aspects. C_l is the unit cost for the energy and labor of the system. C_{rt} is the unit system design cost, depending on the system reliability ($\alpha, \Lambda \geq 0$). $\alpha \in [\alpha_{\min}, \alpha_{\max}]$, *i.e.*, the system reliability indicator lies between certain limits. System design is inversely proportional to system reliability α. The system design not only controls system reliability but also controls other crucial parameters of a smart production system. Thus, the system design cost can be expressed as $C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}$. Therefore, system development cost is $C_1(\alpha) = C_l + C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}$.

Thus, the production cost per cycle is

$$\text{MPRC} = \frac{nq}{T}\left(M_c + \xi p + \frac{C_1(\alpha)}{p}\right). \tag{2}$$

4.1.3. Holding cost (MHC)

The manufacturer is responsible for the holding cost of products until they are delivered to the retailer. The unit holding cost to the manufacturer is H_{cm} per unit per unit of time. If the area of the inventory is A_m, then the holding cost per cycle is

$$\text{MHC} = \frac{H_{cm}A_m}{T}\cdot \tag{3}$$

4.1.4. Preservation cost (MPC)

Preservation reduces the deterioration rate of deteriorating products and extends their lifespans. κ is the investment for preservation for a production batch. The manufacturer uses this cost to use preservation technology for deteriorating products. Initially, the deterioration rate of products was θ. Deterioration rate decreases after using the preservation and new reduced deterioration rate is inversely proportional to the investment. The reduction of deterioration rate is $m(\kappa) = \theta(1 - e^{-r\kappa})$ and the new reduced deterioration rate is $(\theta - m(\kappa))$. The new reduced deterioration rate is inversely proportional to the reduction of deterioration rate. This implies that investment κ is inversely proportional to reduced deterioration rate. $m(\kappa) = \theta(1 - e^{-r\kappa})$ is a decreasing function of κ. Whenever the manufacturer increases the preservation investment, the corresponding facility of preservation increases. This reduces the deterioration rate of products than initial deterioration rate. Thus, preservation cost per cycle is

$$\text{MPC} = \frac{\kappa}{T}\cdot \tag{4}$$

4.1.5. Deterioration cost (MDTC)

The reduced deterioration rate after using preservation is $(\theta - m(\kappa))$. $(\theta - m(\kappa))$ is a monotonically decreasing function of the preservation investment κ whereas $m(\kappa) = \theta(1 - e^{-r\kappa})$ is a monotonically increasing function of preservation investment κ. Thus, new deterioration cost decreases than initial deterioration cost. The total deterioration cost to the manufacturer is $C_{\theta m}(\theta - m(\kappa))A_m$, where $C_{\theta m}$ is the unit deterioration cost. Therefore, the deterioration cost per cycle is

$$\text{MDC} = \frac{C_{\theta m}(\theta - m(\kappa))A_m}{T}\cdot \tag{5}$$

4.1.6. Remanufacturing cost (MRC)

As products are deteriorating type, any type of machinery faults can produce imperfect products. Thus, imperfect products pass through a remanufacturing process. This remanufacturing process consists of two subprocesses: *inspection process* and *defective treatment of imperfect products*.

(I) Inspection process

Total number of imperfect products is counted by an automated inspection system. Automated inspection machine has high precision level to detect imperfect products. Thus, the inspection error is negligible. The imperfect production depends on the machine's system reliability α. Thus, the total number of imperfect products is found as $\int_0^T \beta e^{\alpha t}\mathrm{d}t = \frac{\beta}{\alpha}\left(e^{\alpha T}-1\right)$. If unit inspection cost is I_c, then total inspection cost for imperfect products is

$$\frac{I_c\beta}{\alpha}\left(e^{\alpha T}-1\right). \tag{6}$$

(II) Defective treatment of imperfect products

A defective treatment is given to those above-inspected imperfect products instead of landfills. Imperfect deteriorating products are upgraded to the required level of satisfaction. For example, a processed cheese (cube shaped) is a deteriorating products and requires refrigerator shipping. Now, during the final stage of production, the shape of the cube is deformed due to a temperature fluctuation in the machinery system. Then those cheeses cannot be sent to the retailer. These imperfect products get a treatment to make it in shape again. If the unit defective treatment cost is C_d, total defective treatment cost is

$$\frac{C_d\beta}{\alpha}\left(e^{\alpha T}-1\right). \tag{7}$$

Hence, the total remanufacturing cost consists of these two costs as $\frac{(I_c+C_d)\beta}{\alpha}\left(e^{\alpha T}-1\right)$. The remanufacturing cost of imperfect products per cycle is

$$\mathrm{MRC} = \frac{(I_c+C_d)\beta}{T\alpha}\left(e^{\alpha T}-1\right) = \frac{R_c\beta}{T\alpha}\left(e^{\alpha T}-1\right). \tag{8}$$

4.1.7. Total cost to the manufacturer

Thus, the average total cost to the manufacturer per cycle is

$$\begin{aligned}\mathrm{TSC}_m &= \mathrm{MSC}+\mathrm{MPRC}+\mathrm{MHC}+\mathrm{MPC}+\mathrm{MDTC}+\mathrm{MDC}+\mathrm{MRC}\\ &= \left(\frac{D}{nq}+\frac{\theta-m(\kappa)}{2n}\right)\left[S_{cm}+\kappa+nq\left\{M_c+\xi p+\frac{C_l+C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}}{p}\right\}+\frac{R_c\beta}{\alpha}\left(e^{\alpha\frac{2qn}{2D+(\theta-m(\kappa))q}}-1\right)\right]\\ &\quad+[H_{cm}+(C_{\theta m}+C_{wm})(\theta-m(\kappa))]q\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right).\end{aligned} \tag{9}$$

4.2. Retailer's model

The retailer fulfills the market demand D by the lot size M_l, where $M_l = nq$, that is, owing to the SSMD policy, they receive the total M_l quantity in n shipments with equal shipment sizes of q. Then, $M_l = DT$ and $M_l = nq$. t_3 represents the time duration of two successive product deliveries. Thus, $t_3 = \frac{T}{n}\cdot$ The market demand during the time t_3 is Dt_3 and the deteriorated products during t_3 is $\frac{(\theta-m(\kappa))qt_3}{2}\cdot$ The total number of deteriorated products during the cycle time is $\frac{(\theta-m(\kappa))qT}{2}\cdot$ Thus, $\frac{(\theta-m(\kappa))qT}{2} = qn-DT$, *i.e.*, $T\left[\frac{(\theta-m(\kappa))q}{2}+D\right] = qn$, *i.e.*, $\frac{1}{T} = \frac{1}{nq}\left(\frac{(\theta-m(\kappa))}{2}+D\right)$. The total inventory area of the retailer is A_r. Therefore, $(\theta-m(\kappa))A_r = nq-DT$, *i.e.*, $\frac{A_r}{T} = \frac{nq}{T(\theta-m(\kappa))}-\frac{D}{(\theta-m(\kappa))}\cdot$ Therefore, average inventory is $\frac{A_r}{T} = \frac{q}{2}\cdot$

4.2.1. Ordering cost (ROC)

To order products from the manufacturer, the retailer invests the ordering cost. The retailer orders products at a time within a cycle. If the ordering cost is O_{cr} per order, then the total ordering cost to the retailer per cycle is

$$\mathrm{ROC} = \frac{O_{cr}}{T}\cdot \tag{10}$$

4.2.2. Holding cost (RHC)

After receiving the deteriorating products from the manufacturer, the retailer puts them up for sale (Fig. 2). The area under the retailer inventory is A_r. The considered type of deteriorating products requires chilled technology for storage. Thus, the retailer chooses to receive products in shipment from the manufacturer in SSMD policy. Thus, the retailer does not need to hold all products at a time. This immediately reduces the inventory area of the retailer as $q < M_l$. The required inventory area will increase if the retailer holds all ordered quantities simultaneously. Thus, the retailer pays less holding cost. The unit holding cost H_{cr} includes refrigerator storage facility expenditure at the retailer's end. The total holding cost per cycle is

$$\mathrm{RHC} = \frac{H_{cr}A_r}{T}\cdot \tag{11}$$

4.2.3. Deterioration cost (RDTC)

As products deteriorate over time, few finished products deteriorate at the retailer's end. Owing to the preservation technique, the reduced rate of deterioration is $\theta - m(\kappa)$, and total deteriorated products at retailer's end are $(\theta - m(\kappa))A_r$. If $C_{\theta r}$ is the unit deterioration cost to the retailer, then the total deterioration cost per cycle is

$$\mathrm{RDC} = \frac{C_{\theta r}(\theta - m(\kappa))A_r}{T}\cdot \tag{12}$$

4.2.4. Transportation cost (RTC)

As the manufacturer supports the retailer in reducing the holding cost of products using the SSMD policy, the retailer is willing to pay the corresponding transportation cost. Total M_l products are subsequently sent to the retailer in n shipments with equal shipment sizes q. Transportation cost involves three cost factors. The fixed transportation cost per delivery is F. Thus, for n shipments, the total fixed transportation cost is nF. The fuel cost is calculated based on the distance. The distance between the manufacturer and retailer is l km, and the truck capacity is C_t. The transportation cost owing to distance is $\frac{nqlF_t}{C_t}$, where F_t is the unit transportation cost per truck. As the truck contains the refrigerating system facility during travel time, the unit refrigerating cost is F_r, and the total refrigerating cost is nqF_rl. Therefore, the total transportation cost per cycle is

$$\mathrm{RTC} = \frac{1}{T}\left(nF + nq\left(\frac{lF_t}{C_t} + F_rl\right)\right). \tag{13}$$

4.2.5. Total cost to the retailer

The average total cost to the retailer per cycle is

$$\begin{aligned}\mathrm{TSC}_r &= \mathrm{ROC} + \mathrm{RHC} + \mathrm{RDTC} + \mathrm{RDC} + \mathrm{RTC}\\ &= \left(\frac{D}{nq} + \frac{(\theta - m(\kappa))}{2n}\right)\left[O_{cr} + nF + nql\left(\frac{F_t}{C_t} + F_r\right)\right] + \frac{q}{2}[H_{cr} + (C_{\theta r} + C_{wr})(\theta - m(\kappa))].\end{aligned} \tag{14}$$

4.3. Total supply chain cost (TSC)

Both the manufacturer and retailer minimize the total supply chain cost instead of personal cost minimization. The joint total cost of the SSCM is given by

$$\begin{aligned}
\mathrm{TSC} &= \mathrm{TSC}_m(p,q,n,\alpha,\kappa) + \mathrm{TSC}_r(p,q,n,\alpha,\kappa)\\
&= \left(\frac{D}{nq} + \frac{\theta - m(\kappa)}{2n}\right)\Bigg[S_{cm} + nqM_c + \kappa + nq\xi p + O_{cr} + nF\\
&+ nql\left(\frac{F_t}{C_t} + F_r\right) + \frac{nqC_l + nqC_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}}{p} + \frac{R_c\beta}{\alpha}\left(e^{\alpha\frac{2qn}{2D+(\theta-m(\kappa))q}} - 1\right)\Bigg]\\
&+ [H_{cm} + (C_{\theta m} + C_{wm})(\theta - m(\kappa))]q\left(\frac{D}{p} + \frac{n-1}{2} - \frac{Dn}{2p}\right) + \frac{q}{2}[H_{cr} + (C_{\theta r} + C_{wr})(\theta - m(\kappa))]. \qquad (15)
\end{aligned}$$

5. Solution methodology

The mathematical model is solved using the classical optimization technique. The continuous decision variables are p, q, α, and κ, and the discrete decision variable n is optimized via a discrete optimization technique. As multiple decision variables exist, a Hessian matrix is used to establish the global optimization of the solution. Solutions are found for the centralized case.

Theorem 1. *Unique solutions of the objective function in Equation (15) are given by the necessary conditions of the classical optimization as* $\frac{\partial \mathrm{TSC}}{\partial p} = 0$, $\frac{\partial \mathrm{TSC}}{\partial q} = 0$, $\frac{\partial \mathrm{TSC}}{\partial \alpha} = 0$, *and* $\frac{\partial \mathrm{TSC}}{\partial \kappa} = 0$.

Proof. First-order derivatives of the objective function in Equation (15) with respect to the decision variables provides the solutions.

$$\begin{aligned}
&\frac{\partial \mathrm{TSC}}{\partial p} = 0, \quad \textit{i.e.,}\\
&\qquad p^* = \sqrt{\frac{\left(\frac{D}{nq} + \frac{\theta e^{-r\kappa}}{2n}\right)(C_l + \Omega) - [H_{cm} + (C_{\theta m} + C_{wm})\theta e^{-r\kappa}]q(Dn - D)}{\left(\frac{D}{nq} + \frac{\theta e^{-r\kappa}}{2n}\right)\xi}}\\
&\frac{\partial \mathrm{TSC}}{\partial q} = 0, \quad \textit{i.e.,}\\
&\qquad q^* = \frac{\left(nl\left(\frac{F_t}{C_t} + F_r\right) - \varrho\right)2D + \sqrt{4D^2\left(nl\left(\frac{F_t}{C_t} + F_r\right) - \varrho\right) + 8D\rho(\theta e^{-r\kappa}\varrho + 2n\varsigma)}}{(2\theta e^{-r\kappa}\varrho + 2n\varsigma)}\\
&\frac{\partial \mathrm{TSC}}{\partial \alpha} = 0, \quad \textit{i.e.,}\\
&\qquad \alpha^* = \frac{\sqrt{\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\frac{2qn}{2D+\theta e^{-r\kappa}q}\right)^2 + 4\left(\frac{\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2}\right)R_c\beta\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}} - 1\right)}}{2\left(\frac{\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2}\right)} - \frac{e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\frac{2qnR_c\beta}{2D+\theta e^{-r\kappa}q}}{2\left(\frac{\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2}\right)}\\
&\frac{\partial \mathrm{TSC}}{\partial \kappa} = 0, \quad \textit{i.e.,}\\
&\qquad \kappa^* = \frac{2n}{\theta r e^{-r\kappa}}\left(\frac{D}{nq} + \frac{\theta e^{-r\kappa}}{2n}\right)\left[1 + \frac{R_c\beta}{\alpha}e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\frac{2q^2n\theta\alpha r}{(2D+\theta e^{-r\kappa}q)^2}\right]\\
&\qquad\quad - (C_{\theta m} + C_{wm})\theta r e^{-r\kappa}q\left(\frac{D}{p} + \frac{n-1}{2} - \frac{Dn}{2p}\right) - \frac{q}{2}(C_{\theta r} + C_{wr})\theta r e^{-r\kappa}\\
&\qquad\quad + \left(\frac{\theta r e^{-r\kappa}}{2n}\right)\left[S_{cm} + M_c + \xi p + O_{cr} + nF + nql\left(\frac{F_t}{C_t} + F_r\right) + \frac{C_l + \Omega}{p} + \frac{R_c\beta}{\alpha}\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}} - 1\right)\right].
\end{aligned} \qquad (16)$$

The value of the objective function in Equation (15) becomes minimum at the values of p^*, q^*, α^*, and κ^*. See Appendix A for the calculations of first-order derivatives. □

Proposition 1. *The variable n is optimized if the following inequalities are satisfied*

$$\mathrm{TSC}(n-1) \geq \mathrm{TSC}(n) \leq \mathrm{TSC}(n+1).$$

Proof. Here, the number of lot size (n^*) deliveries to the retailer per production batch is an integer. Thus, it is optimized via discrete optimization technique, and the optimal value can be determined using the necessary conditions. For the optimal total cost (TSC) for given values of $p = p^*$, $q = q^*$, $\alpha = \alpha^*$ and $\kappa = \kappa^*$, the necessary condition is

$$\mathrm{TSC}(p^*, q^*, \alpha^*, n^* - 1, \kappa^*) \geq \mathrm{TSC}(p^*, q^*, \alpha^*, n^*, \kappa^*) \leq \mathrm{TSC}(p^*, q^*, \alpha^*, n^* + 1, \kappa^*).$$

The optimal value of n is driven using the following equation.

$$\frac{1}{\psi}\log\left[\frac{\alpha}{\phi R_c \beta (e^{-\psi} - 1)}\left(\phi\varphi - \frac{3D\tau}{2P}\right)\right] \leq n^* \leq \frac{1}{\psi}\log\left[\frac{\alpha}{\phi R_c \beta (1 - e^{\psi})}\left(\phi\varphi + \frac{D\tau}{2P}\right)\right],$$

where

$$\phi = \left(\frac{D}{q} + \frac{\theta e^{-r\kappa}}{2}\right),\ \varphi = \left[F + ql\left(\frac{F_t}{C_t} + F_r\right)\right],\ \psi = \alpha\frac{2q}{2D + \theta e^{-r\kappa} q},\ \tau = [H_{cm} + (C_{\theta m} + C_{wm})(\theta - m(\kappa))]q.$$

□

The optimum results of the decision variables are found by the classical optimum technique. Now, the sufficient condition of the global optimality is satisfied by the following Proposition 2. All solutions are obtained in a quasi-closed form. For solving numerically, a mixed-integer programming problem is used (see Appendix for proof of the convexity of the TSC by Hessian calculations).

Proposition 2. *The first principal minor of decision variables p^*, q^*, n^*, α^*, κ^* for the joint total cost function is greater than zero if*

$$\left(\frac{D}{nq} + \frac{\theta e^{-r\kappa}}{2n}\right)\Psi\left[\frac{R_c\beta}{\alpha}\frac{16D^2n^2\alpha^2}{(2D + \theta q e^{-r\kappa})^4} - \frac{R_c\beta}{\alpha}\frac{8Dn\theta\alpha e^{-r\kappa}}{(2D + \theta q e^{-r\kappa})^3}\right]$$
$$> \frac{2D}{nq^2}\left[nl\left(\frac{F_t}{C_t} + F_r\right) + \frac{R_c\beta}{\alpha}\Psi\frac{4Dn\alpha}{(2D + \theta q e^{-r\kappa})^2}\right]$$

Proof. See Appendix B. □

Proposition 3. *The joint total cost function is convex for decision variables p^*, q^*, n^*, α^*, κ^* if $\Delta_1 > 0$, $\Delta_2 > 0$, $\Delta_3 > 0$, $\Delta_4 > 0$.*

Proof. See Appendix C. □

6. Numerical examples

Different examples are proposed here to validate the theoretical model. Using Mathematica 11.3.0 and the parametric values from Malik *et al.* [7], Sarkar *et al.* [26], and Ullah *et al.* [46], the optimum outcomes are obtained and verified.

TABLE 2. Optimum results of Example 1 for five distribution functions with the optimality test results.

Decision variables	Uniform distribution	**Beta distribution**	Gamma distribution	Triangular distribution	Double triangular distribution
p^* (units/unit time)	708.13	**708.13**	708.13	708.13	708.13
q^* (units/cycle)	292.42	**292.37**	292.38	292.42	292.53
n^* (integer)	2	**2**	2	2	2
α^*	0.72	**0.72**	0.72	0.72	0.72
κ^* ($/cycle)	4.17	**3.89**	3.98	4.17	4.69
TSC ($/cycle)	14,011.03	**14,010.91**	14,010.98	14,011.79	14,011.34
Optimality	$H_{11} = 0.010 > 0$	$H_{11} = 0.011 > 0$	$H_{11} = 0.012 > 0$	$H_{11} = 0.017 > 0$	$H_{11} = 0.014 > 0$
Test	$H_{22} = 0.013 > 0$	$H_{22} = 0.013 > 0$	$H_{22} = 0.013 > 0$	$H_{22} = 0.014 > 0$	$H_{22} = 0.012 > 0$
Results	$H_{33} = 0.121 > 0$	$H_{33} = 0.122 > 0$	$H_{33} = 0.122 > 0$	$H_{33} = 0.122 > 0$	$H_{33} = 0.122 > 0$
	$H_{44} = 0.018 > 0$	$H_{44} = 0.019 > 0$	$H_{44} = 0.071 > 0$	$H_{44} = 0.028 > 0$	$H_{44} = 0.057 > 0$

Example 1. The following numerical values of parameters are considered to validate the theoretical model and results. The random deterioration rate θ is assumed to follow a different probability distribution. The value of the input parameters are $O_{cr} = 21$ ($/order), $S_{cm} = 700$ ($/batch), $H_{cr} = 2$ ($/unit/unit time), $H_{cm} = 1$ ($/unit/unit time), $D = 350$ (unit/unit time), $F_t = 0.05$ ($/truck unit), $l = 400$ (km), $F_r = 0.05$ ($/unit/km), $a = 0.1$, $\alpha_{\max} = 0.9$, $\alpha_{\min} = 0.1$, $r = 2.1$, $I_c = 0.5$ ($/unit), $C_d = 1$ ($/unit), $C_{\theta m} = 0.6$ ($/unit), $C_l = 500$ ($/unit), $\Lambda = 0.99$, $M_c = 16$ ($/unit), $\xi = 0.001$ ($/unit), $\beta = 0.5$, $F = 5$ ($/delivery), $C_t = 200$ (units/truck), $b = 0.5$, $C_{rt} = 1.1$ ($/unit), $C_{\theta r} = 2.2$ ($/unit).

Based on the above data, we consider five different experiments depending on the different probability distributions of the deterioration function. Here the random deterioration rate θ is assumed to be uniform distribution with boundaries a & b, *i.e.*, $\theta \sim U[a, b]$, beta distribution with boundaries a & b, *i.e.*, $\theta \sim \beta[a, b]$, gamma distribution with boundaries a & b, *i.e.*, $\theta \sim \gamma[a, b]$, triangular distribution with vertices a, b, & c, and double triangular distribution with vertices a, b, & c. The optimum results of the decision variables with the minimum total cost (the global optimality of the total expected cost function is proved using the Hessian matrix) in different examples are shown in Table 2. The determinant values of all principal minors are in the same sign in each example, which shows that the total supply chain cost is minimum. Hence, the required solution is the global optimum.

Among the different examples with the minimum preservation method investment, lowest total cost is for the beta distribution. The total cost is maximum for the triangular distribution with more preservation investment compared to the other examples, although the system reliability remains the same for all the distributions.

Propositions analytically prove the optimality of results, and numerical experiment establishes analytical results numerically (Tab. 2). In both ways, the convexity of the result is tested through the Hessian matrix, as the determinant value of the Hessian matrix has the same sign. The global minimum total cost in different cases is shown in the convex figure, Figures 4–6.

I. System reliability for smart production: An observation for deteriorating products

The study intends to observe the use of smart production for deteriorating products. The derived results are quite different from the traditional concept of system reliability of a machinery system. It is well-established in literature [26] that the system reliability takes the value of 0.2 or 0.3 for a reliable production system. That means a reliable production system can have system reliability very less, which is around 0.2–0.3, *i.e.*, between 20% and 30%. But, in this study, the optimum system reliability goes up and becomes 0.72, which is quite high compared to earlier studies. Even though the smart production system is used for the production of deteriorating products, it turns out that it increases the optimum system reliability by up to 70%. The number of imperfect products increases when the system reliability increases more than the optimal value. But, imperfect products

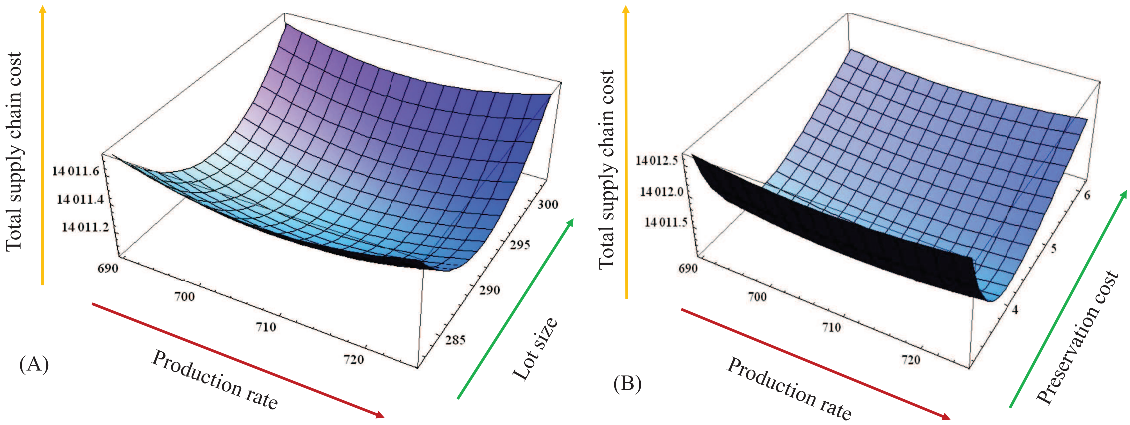

FIGURE 4. (A) Total expected cost versus production rate of manufacturer and production lot size of the retailer. (B) Total expected cost versus production rate and preservation cost of the manufacturer in Example 1.

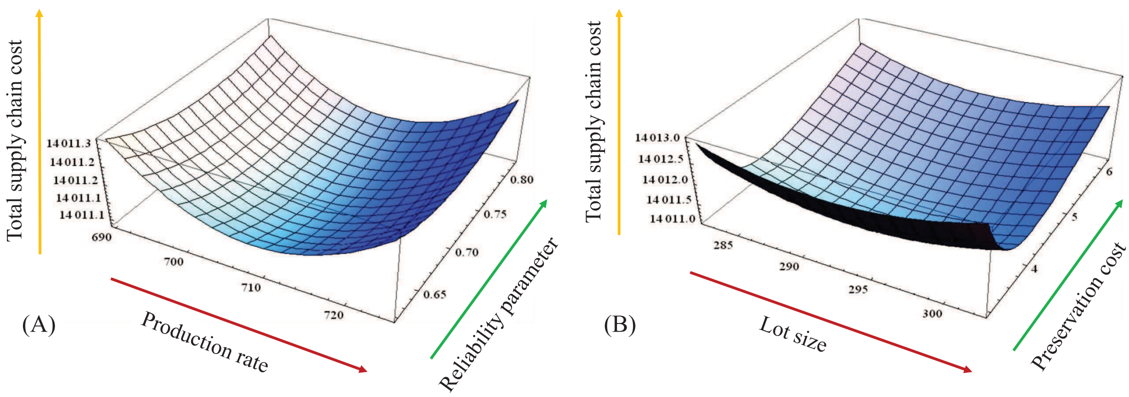

FIGURE 5. (A) Total expected cost versus production rate of manufacturer and reliability. (B) Total expected cost versus production lot size of retailer and preservation cost in Example 1.

from the smart production system for the optimal value of α is 1.72, which is much less than the optimal production rate. This happens because of the smart production system. One of the preferences for using a smart production system is the high precision level of the machine used for smart production. When comparing the system reliability of traditional and smart production systems, even with such high system reliability, smart products work perfectly fine. Thus, it can be concluded from the observation that higher system reliability is allowable for the smart production system with such a less imperfect production.

II. SSMD policy with smart production system

Results of Table 3 shows that constant production rate decreases the shipment number and increases the delivery lot size. The scenario increases the system's total cost than the smart production system. The constant production rate uses single delivery ($n = 1$) and it increases the total cost. Whereas, the smart production controls its production rate and therefore the batch size. Then, the smart production system chooses SSMD as transportation policy and system's total cost reduces.

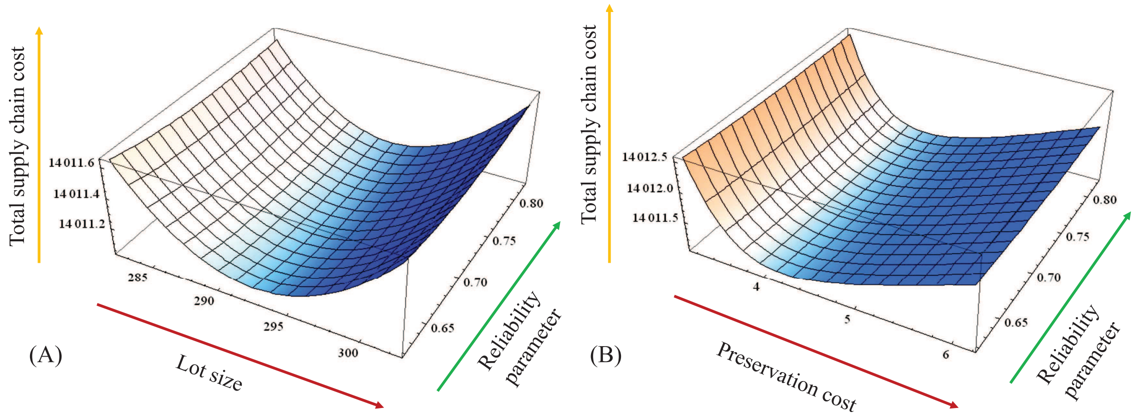

FIGURE 6. (A) Total expected cost versus production lot size of retailer and reliability of the product. (B) Total expected cost versus reliability of product and preservation technology cost in Example 1.

TABLE 3. Optimum results of the special cases of Example 1 for five distribution functions.

Decision variables	Uniform distribution	Beta distribution	Gamma distribution	Triangular distribution	Double triangular distribution
q^* (units/cycle)	451.79	**451.70**	451.73	451.79	451.94
n^* (integer)	1	**1**	1	1	1
α^*	0.80	**0.80**	0.81	0.81	0.80
κ^* ($/cycle)	4.25	**3.98**	4.06	4.26	4.78
TSC ($/cycle)	14,262.61	**14,262.40**	14,262.51	14,262.62	14,263.02
Optimality	$H_{11} = 0.005 > 0$	$H_{11} = 0.005 > 0$	$H_{11} = 0.006 > 0$	$H_{11} = 0.007 > 0$	$H_{11} = 0.004 > 0$
Test	$H_{22} = 0.009 > 0$	$H_{22} = 0.009 > 0$	$H_{22} = 0.009 > 0$	$H_{22} = 0.004 > 0$	$H_{22} = 0.002 > 0$
Results	$H_{33} = 0.048 > 0$	$H_{33} = 0.002 > 0$	$H_{33} = 0.112 > 0$	$H_{33} = 0.182 > 0$	$H_{33} = 0.048 > 0$

III. Effect of constant production rate on preservation investment

Special observations arise depending on the constant production rate of deteriorating products. Table 3 shows results in different experiments depending on the distribution function. When the production rate becomes constant rather than variable, assuming that $p = 701$ units/cycle (which is greater than double the market demand unit per cycle), the values of the other parameters remain the same as those in Example 1. It is seen that the total cost of the production system increases as compared to smart production. The research question for this study was the eligibility of smart production for the reduction of preservation. As a result, the constant production rate increases the preservation investment by 2.31% for beta distribution. Thus, when the smart production system moves to a constant production system, the investment for preservation increases. This implies that smart production helps to reduce preservation cost for deteriorating products. Preservation technology is used for all produced deteriorating products. When excess deteriorating products exist in the system, the corresponding preservation cost has already been used for those extra products. This phenomenon increases the preservation cost, which is avoided by using smart production. As the production rate is adjustable, there is no need to hold these extra products using preservation. Hence, the preservation cost decreases for the smart production system.

Example 2. To validate the theoretical model and results, another example is considered. The value of the input parameters are $O_{cr} = 10$ (\$/order), $S_{cm} = 200$ (\$/batch), $H_{cr} = 1$ (\$/unit/unit time), $H_{cm} = 0.5$ (\$/unit/unit time), $D = 350$ (unit/unit time), $F_t = 0.04$ (\$/ truck unit), $l = 200$ (km), $F_r = 0.0021$ (\$/unit/km), $a = 0.1$, $\alpha_{\max} = 0.9$, $\alpha_{\min} = 0.1$, $r = 4.1$, $C_d = 0.55$ (\$/unit), $I_c = 0.5$ (\$/unit), $C_{\theta m} = 0.5$ (\$/unit), $C_l = 500$ (\$/unit), $\Lambda = 0.99$, $M_c = 10$ (\$/unit), $\xi = 0.001$ (\$/unit), $\beta = 0.5$, $F = 4$ (\$/delivery), $C_t = 200$ (units/truck), $b = 0.5$, $C_{rt} = 1.1$ (\$/unit), $C_{\theta r} = 1.1$ (\$/unit).

The optimum results of the Example 2 are $p = 707.91$ (unit/unit time), $q = 225.97$ (units/cycle), $n = 2$, $\alpha = 0.87$, $\kappa = 1.91$ (\$/cycle), total cost = 4,497.48 (\$/cycle). The global minimum total cost is verified using a Hessian matrix. The values of the principal minors are $H_{11} = 0.00668112 > 0$; $H_{22} = 0.0211679 > 0$; $H_{33} = 0.0761052 > 0$; $H_{44} = 0.000075254 > 0$. Determinant values of all principal minors are positive in sign, which indicates that the required solution is the global optimum and the cost is globally minimum.

IV. Special observation for constant production rate

Results are tested when the production system is constant rather than variable. Assuming that $p = 401$ units/cycle (which is greater than the double of market demand unit per cycle), the values of the other parameters remain the same as those of the input value of Example 2. The optimum results for special case of Example 2 are $q = 225.97$ (units/cycle), $n = 2$, $\alpha = 0.88$, $\kappa = 1.91$ (\$/cycle), total cost = 4,497.51 (\$/cycle). It is seen that the total cost of the production system increases for the constant production rate rather than the smart production system. The values of the principal minors are $H_{11} = 0.00668135 > 0$; $H_{22} = 0.0211688 > 0$; $H_{33} = 0.0760381 > 0$. All the principal minor of the Hessian matrix are positive in sign. This indicates that the obtained solutions are the global optimum and the cost is the global minimum.

6.1. Discussions

From the above numerical experiments and their comparison among the previous research articles, it can be concluded that the TSC is the minimum for the proposed model. All cost amounts are numerically expressed using Mathematica 11.3.0 software. Figure 7 shows a comparison among the TSC of Example 1 of the proposed research with Ullah *et al.* [46], and Sarkar and Chung [38], and Sarkar *et al.* [39]. Ullah *et al.* [46] only considered deterioration and preservation within an SCM for a constant production rate. Thus, no phenomenon for system reliability was discussed there. Their research gave a total cost of \$24,873.47 per cycle. Besides, the variable production concept of Sarkar and Chung [38] gave the total cost \$21,689.59 per cycle, whereas Sarkar *et al.* [39] gave the total cost of \$18,129.17 per cycle. Compared to this previous research, Example 1 of the proposed model gives a total cost of \$14,011.03 per cycle.

The proposed research gives the lowest total cost than the previous research due to the concept of the smart production system. Refrigerated transportation keeps preservative products fresh during transport. This reduces the deterioration rate of products during transportation. Thus, the waste from the system reduces with the help of refrigerated transportation. The waste reduction approach, remanufacturing, and disposal of imperfect products make the model more sustainable. Furthermore, a smart production system with FPR and remanufacturing helps to control imperfect products and reduce waste. Hence, the comparison among the previous research help in the validation of the original research idea.

Moreover, two real case studies are considered here for the strong support of the proposed model.

6.2. Case study 1

Here, a case study is conducted for this study to validate our proposed model. The model is tested on real data and concepts from a company in India. The company shares information about policy and data and uses all this information in this research with full consent.

This data is compared with the data of five different distribution functions of deterioration. The beta distribution yields the minimum cost with the lowest preservation investment, and the gamma distribution yields the maximum total cost with the highest preservation investment. However, the system reliability of the machine is the same for all the distributions. The company is delighted with the results and has decided to reduce total

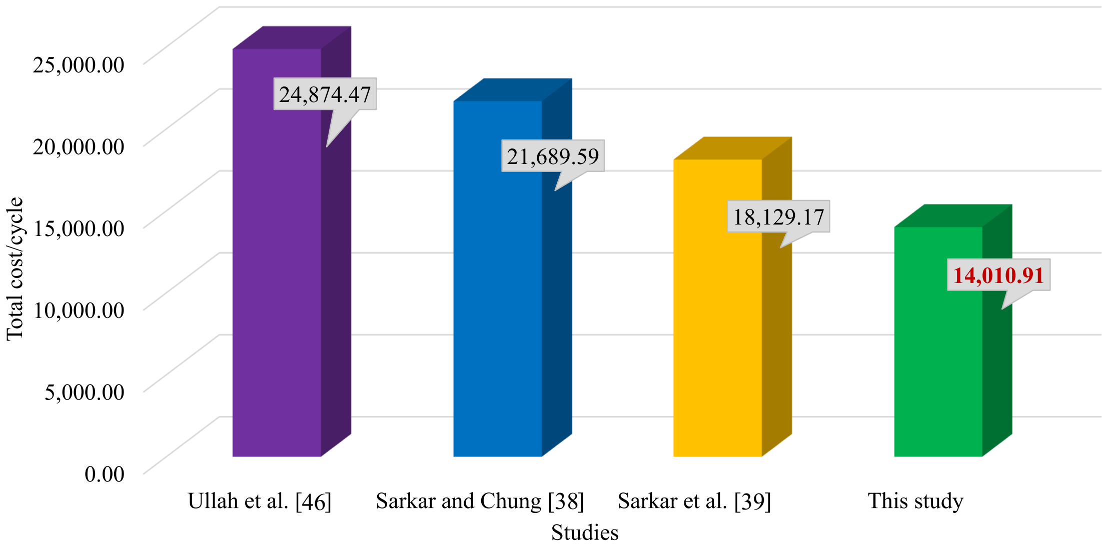

FIGURE 7. Comparison among the total supply chain cost of Example 1 and other studies in the literature review.

cost via a suitable distribution similar to this study. Based on the proposed strategy, the company may change its production planning to increase its savings.

The data is taken from the company such that $O_{cr} = 5$ (\$/order), $S_{cm} = 100$ (\$/batch), $H_{cr} = 1$ (\$/unit/unit time), $H_{cm} = 0.5$ (\$/unit/unit time), $D = 200$ (unit/unit time), $F_t = 0.01$ (\$/ truck unit), $l = 100$ (km), $F_r = 0.001$ (\$/unit/km), $a = 0.1$, $\alpha_{\max} = 0.9$, $\alpha_{\min} = 0.1$, $r = 3.1$, $I_c = 0.4$ (\$/unit), $C_d = 0.55$ (\$/unit), $C_{\theta m} = 0.2$ (\$/unit), $C_l = 190$ (\$/unit), $\Lambda = 0.99$, $M_c = 10$ (\$/unit), $\xi = 0.001$ (\$/unit), $\beta = 0.5$, $F = 2$ (\$/delivery), $C_t = 100$ (units/truck), $b = 0.5$, $C_{rt} = 1.1$ (\$/unit), $C_{\theta r} = 0.11$ (\$/unit).

The optimum results of the case study are $p = 437.14$ (unit/unit time), $q = 121.20$ (units/cycle), $n = 2$, $\alpha = 0.90$, $\kappa = 2.16$ (\$/cycle), total cost = 2,380.61 (\$/cycle). The minimum total expected cost is checked using a Hessian matrix method. The values of the principal minors are $H_{11} = 0.0125166 > 0$; $H_{22} = 0.0318323 > 0$; $H_{33} = 0.0899063 > 0$; $H_{44} = 0.0000822429 > 0$; thus, the required solution is the global optimum, and the cost is minimum.

6.2.1. Special observation of case study: Constant production rate

If the company considers the constant production system rather than variable, assuming that $p = 401$ units/cycle (which is greater than double the market demand unit per cycle), the values of the other parameters remain the same as those of the input value of the case study. The optimum results for special case of case study are $q = 121.19$ (units/cycle), $n = 2$, $\alpha = 0.89$, $\kappa = 2.16$ (\$/cycle), Total cost = 2,381.25 (\$/cycle). It is seen that the total cost of the production system increases. The minimum total expected cost is determined using a Hessian matrix. The values of the principal minors are $H_{11} = 0.0125228 > 0$; $H_{22} = 0.0318518 > 0$; $H_{33} = 0.0891979 > 0$; thus, the required solution is the global optimum, and the cost is minimum.

6.3. Case study 2

Another case study is conducted here to validate the proposed model. For the strongest support of this proposed model, the data for case 2 is collected from another company situated in India. The company manager shared its data and used it in this research with full consent from those concerned.

The data is taken from the company such that $O_{cr} = 4$ (\$/order), $S_{cm} = 90$ (\$/batch), $H_{cr} = 1$ (\$/unit/unit time), $H_{cm} = 0.5$ (\$/unit/unit time), $D = 150$ (unit/unit time), $F_t = 0.01$ (\$/truck unit), $l = 150$ (km), $F_r = 0.001$ (\$/unit/km), $a = 0.1$, $\alpha_{\max} = 0.9$, $\alpha_{\min} = 0.1$, $r = 4.1$, $C_d = 0.55$ (\$/unit), $I_c = 0.3$ (\$/unit), $C_{\theta m} = 0.3$ (\$/unit), $C_l = 195$ (\$/unit), $\Lambda = 0.99$, $M_c = 11$ (\$/unit), $\xi = 0.001$ (\$/unit), $\beta = 0.5$, $F = 4$ (\$/delivery), $C_t = 90$ (units/truck), $b = 0.6$, $C_{rt} = 1.1$ (\$/unit), $C_{\theta r} = 0.11$ (\$/unit).

The optimum results of the case study are $p = 417.45$ (unit/unit time), $q = 73.28$ units, $n = 3$, $\alpha = 0.81$ (units), $\kappa = 1.65$ (unit), Total cost = 1,955.28 (\$/cycle). The minimum total expected cost is checked using a Hessian matrix method. The values of the principal minors are $H_{11} = 0.0274439 > 0$; $H_{22} = 0.0764295 > 0$; $H_{33} = 0.296606 > 0$; $H_{44} = 0.000211015 > 0$; thus, the required solution is the global optimum, and the cost is minimum.

But if they plan their business strategy without a smart production, without preservation, and any refrigerated transportation facility, then their total cost = 2,061.77 (\$/cycle). Their cost increased by 05.45%. Therefore, the company is delighted with the results, and they decided to decrease their total cost via this suitable strategy similar to this study.

6.3.1. *Explanation*

The proposed research is based on SSCM under preservation technology investment, smart production, refrigerated transportation facility, and system reliability. Preservation investment increases the lifespan/expiry date of the products. Smart production controls the fluctuating customer demand. Refrigerated transportation facilities maintain the freshness of the products before deliver to the customer's hand. However, the system reliability issue shows an important effect on the smart production system, which is different from other studies of the traditional or flexible production system. For the environmental issues, the waste reduction approach through the disposal of imperfect items makes the model more eco-friendly. Remanufacturing controls the total cost and creates a balance between raw materials and natural resources. Here, the proposed research mainly focuses on deteriorating items.

At first, different type of cost connected to the proposed production planning and SSCM are discussed, then the mathematical expression for each cost are calculated through different notation. As the proposed study is based on a two-echelon SSCM, two total costs are found for supply chain members, manufacturer and retailer. Then by summing up their total cost, the total cost of the supply chain is formulated. First, optimal solution of the decision variables are derived analytically and then results are obtained numerically to find the total cost. However, the classical optimization technique finds the optimal solution for the decision variables. Moreover, the Hessian matrix is used to establish the global nature of the solution.

From the different examples, case studies, discussions, and explanations, it is concluded that the proposed research provides a new business strategy by focusing on the smart production system. To the best of our knowledge, the research idea is very informative and attractive to the researchers as well as industry managers.

7. Sensitivity analysis

Observations are made based on the sensitivity analysis of cost parameters on the total supply chain cost (TSC), listed in Table 4. Figures 8 and 9 are formulated for clarity based on the sensitivity analysis results. The effects of cost parameters in the surroundings of $(-50\%, -25\%, +25\%, +50\%)$ on the TSC are shown in Table 4. The observations from Table 4 are explained in detail below.

(1) The most sensitive parameter is the refrigerated transportation cost during transportation, which significantly impacts the total cost. Approximately 50% of the changes in the parametric value cause near about 25% of the changes in the total cost. When the refrigerated transportation cost increases, the total cost increases, and vice versa. This implies that the industry manager needs to utilize these based on the priority level of the product.

Table 4. Sensitivity analysis of key parameters of Example 1.

Parameters	Changes (%)	TSC (%)	Parameters	Changes (%)	TSC (%)	Parameters	Changes (%)	TSC (%)
M_c	−50	−19.98	C_l	−50	−01.03	F_r	−50	−24.98
	−25	−10.01		−25	−0.47		−25	−12.49
	+25	+09.99		+25	+0.42		+25	+12.49
	+50	+19.98		+50	+0.79		+50	+24.98
S_{cm}	−50	−01.74	H_{cm}	−50	−0.55	F	−50	−0.02
	−25	−0.80		−25	−0.27		−25	−0.01
	+25	+0.71		+25	+0.26		+25	+0.01
	+50	+01.35		+50	+0.56		+50	+0.02
H_{cr}	−50	−01.15	O_{cr}	−50	−0.04	F_t	−50	−0.13
	−25	−0.55		−25	−0.02		−25	−0.06
	+25	+0.51		+25	+0.02		+25	+0.06
	+50	+01.01		+50	+0.04		+50	+0.13

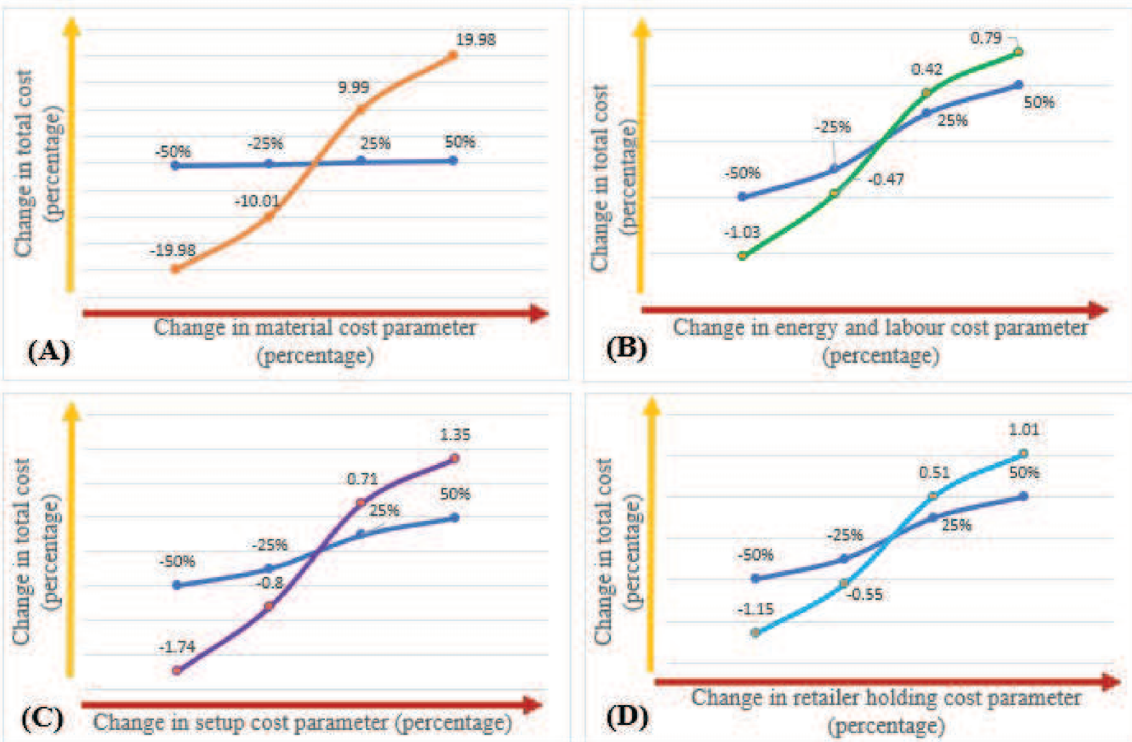

Figure 8. Percentage changes in the total cost versus different parameters' percentage values.

(2) The material cost for the manufacturer's production has a significant impact on the TSC. Owing to small changes in the material cost parameter, TSC changes significantly. The TSC decreases when the material cost decreases and increases when the material cost increases.

(3) Table 4 indicates that the variable transportation cost has a very low impact on the total cost. The total cost increases and decreases negligibly, corresponding to the increase and decrease of the variable transportation cost.

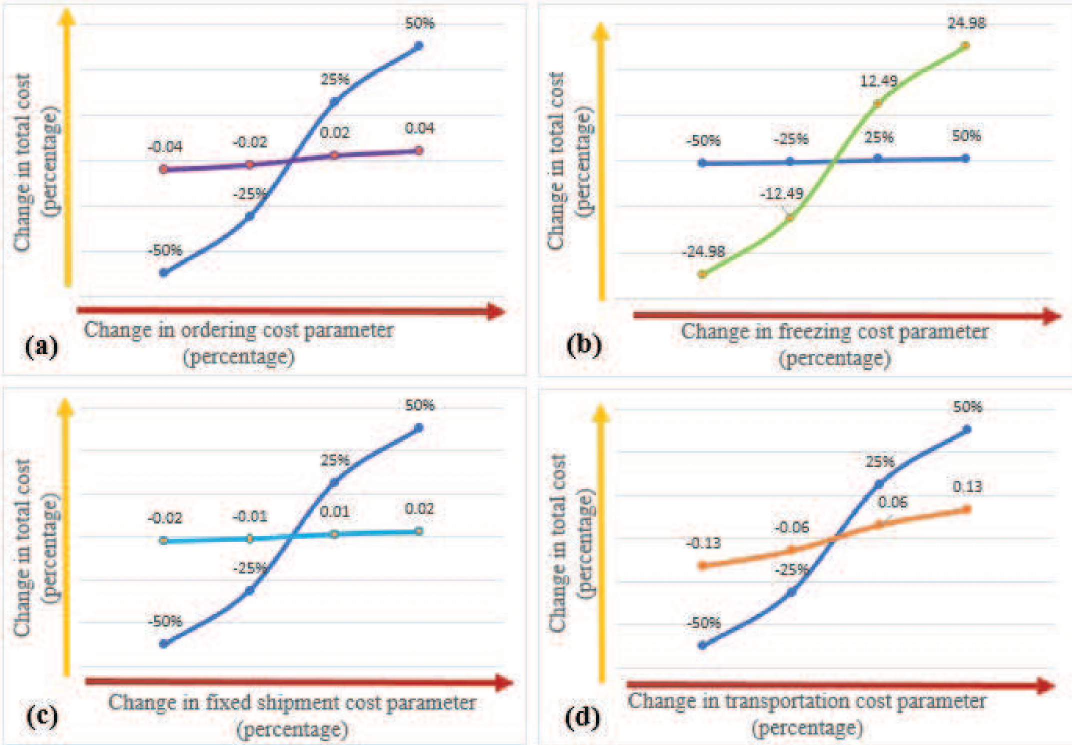

FIGURE 9. Percentage changes in the total cost versus different parameters' percentage values.

(4) The ordering cost to retailers plays a key role in controlling the total cost. It is the most significant cost to the retailer in the supply chain model, but from Table 4, it is shown that it is a less sensitive parameter. The total cost increases and decreases by a negligible amount, corresponding to the increase or decrease of this parameter.

(5) The other cost parameters, setup, investment, holding cost for manufacturer & retailer, energy cost, and unit transportation cost, have near about similar impacts on TSC, as shown in Table 4.

8. Managerial insights

Statistical data and scientific observations are required to start and sustain a business. The implications of moderate research ideas and technologies aid the decision-making of industry managers. This section provides recommendations based on the proposed research for development and profit.

- Smart production for deteriorating products provides a new type of results that can be helpful for the industry. The industry does not need to be concerned about the system reliability to reduce the imperfect production rate of the system. This is a big insight of this study. As the machine has its precision for smart production, the industry can gracefully handle the system reliability. Furthermore, an FPR helps formulate a remanufacturing policy by ensuring that customers trust the company. All of these insights are based on the research conducted in this study.
- From the proposed research, one may decide on the preservation technology, which is the solution to reduce deterioration. Smart production helps to reduce the preservation cost. Thus, the industry can reduce the

corresponding preservation investment using a smart production system. Another important matter discussed in this research is refrigerated transportation, which gives quality items to the customer's hand. This concept can help the production manager to make the right decision about managing deteriorating products.

– For any production system, imperfect products are harmful and increase the total cost. Smart production produces a very less amount of imperfect products even after a high system reliability. This is a notable finding that can help managers deciding the right production system for deteriorating products. Besides, managers should adopt a remanufacturing policy to reduce losses from produced imperfect products. Furthermore, remanufacturing policies encourage customers to trust the company and buy products.
– At present, environmental protection is essential for every industry. The use of smart production can provide one solution to reduce waste in two ways: by producing a reasonable amount of deteriorating products and by producing very few imperfect products. The waste reduction approach through the appropriate disposal of imperfect items is one of the most effective ways to reduce pollution. It should be the focus of every industry manager.
– The manufacturer can have adequate time to produce the next lot of the product. Additionally, as preservation extends the lifespan of deteriorating products, the manufacturer earns higher profits as the waste disposal requirements and product shortages are reduced.
– As smart production plays a vital role, the reliability of the entire system is highly dependent on it. The management must regularly investigate the machine's condition; this improves the development of the machine and increases the reliability index. If the machine reaches an out-of-control state, it begins to produce imperfect items. As imperfect products can decrease brand value, the management returns them for remanufacturing.

Thus, to minimize the total cost of the supply chain, industry managers should focus on smart production, system reliability remanufacturing, preservation, refrigerated transportation facility, and appropriately disposing of imperfect items.

9. Conclusions

The improvement of the production system was the most efficient way to decrease the total supply chain cost under a manufacturing-remanufacturing system. This study investigated the effects of smart production for deteriorating products, preservation technology, and refrigerated transportation within a two-echelon SSCM. Along with cost minimization, which was the primary concern of the industry, a method to reduce waste was provided. The method was related to the smart production, precisely the machine precision of smart production. The preservation method extended the life of deteriorated products, reduced the number of products for disposal, and reduced imperfect production. The results showed that the smart production enabled better cost minimization than a constant production, while preservation reduced the quantity of waste. Quality concerns were addressed via the use of a refrigeration facility during transportation. The goal was to determine the minimum total cost by optimizing the decision variables. Mathematica 11.3.0 software was used for optimal solutions, graphical diagrams, and sensitivity analysis through numerical studies. The total cancellation of imperfect products became a huge loss for the manufacturer. Hence, the proposed strategy of remanufacturing could decrease the total cost. Finally, the proposed model outlined a profitable business strategy considering economic, social, and environmental development. It reduced waste through smart production and remanufacturing, opened new job opportunities for skilled and non-skilled labor, and reduced preservation costs. Skilled labor was used for smart production, whereas both skilled and non-skilled labor had job opportunities in remanufacturing industry. Smart production reduced the preservation cost and the imperfect rate, which was very low, which justifies the economic development. Thus, the proposed study established sustainability within the supply chain and, thus, an SSCM.

Environmental responsibilities and transportation disruption are two of the limitations of this model. Ordering costs for the retailer can be reduced through continuous or discrete investment, as ordering costs can not always be fixed. Quality improvement and setup cost reduction under warranty policy are other important aspects of this research area. It can be further considered to improve the supply chain risk [40]. As the research is based on the deteriorating products under the supply chain model, it can be further considered for deteriorating product management [27] within a cross-dock. For future research extensions, the system's reliability can be considered [26]. This model can be extended by considering different carbon emissions strategies [25]. Variable and stochastic type of demand [9] will be challenging to test the efficiency of a smart production system under circular economy [5].

Abbreviations:

FPR Fixed production rate
SCM Supply chain management
SSCM Sustainable supply chain management
TSC Total supply chain cost
SM Smart manufacturing
EPQ Economic production quantity
EOQ Economic order quantity
SSMD Single-setup-multi-delivery
IoT Internet of things

Appendix A.

Here $\mathrm{TSC}(p,q,\alpha,n,\kappa)=\mathrm{TSC}$ (say); $\Omega = C_{rt}e^{\Lambda\frac{(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{min})}}$; and $\Psi = e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}$

$$\frac{\partial \mathrm{TSC}}{\partial p} = \left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\xi - \frac{C_l + C_{rt}e^{\Lambda\frac{(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}}{p^2}\right] + \left[H_{cm}+(C_{\theta m}+C_{wm})\theta e^{-r\kappa}\right]q\left(\frac{-D}{p^2}+\frac{Dn}{2p^2}\right)$$

$$\begin{aligned}\frac{\partial \mathrm{TSC}}{\partial q} = {}&\left(\frac{-D}{nq^2}\right)\Bigg[S_{cm}+M_c+\kappa+\xi p+O_{cr}+nF+nql\left(\frac{F_t}{C_t}+F_r\right)+\frac{C_l+C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}}{p}\\ &+\frac{R_c\beta}{\alpha}\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}-1\right)\Bigg]+\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[nl\left(\frac{F_t}{C_t}+F_r\right)+\frac{R_c\beta}{\alpha}e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\frac{4Dn\alpha}{(2D+\theta q e^{-r\kappa})^2}\right]\\ &+\left[H_{cm}+(C_{\theta m}+C_{wm})\theta e^{-r\kappa}\right]\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right)+\frac{1}{2}\left[H_{cr}+(C_{\theta r}+C_{wr})\theta e^{-r\kappa}\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathrm{TSC}}{\partial \alpha} = {}&\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\Bigg[\frac{C_{rt}}{p}e^{\Lambda\frac{(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2}-\frac{R_c\beta}{\alpha^2}\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}-1\right)+\frac{R_c\beta}{\alpha}e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\\ &\frac{2qn}{2D+\theta e^{-r\kappa}q}\Bigg]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathrm{TSC}}{\partial \kappa} = {}&\left(\frac{-\theta r e^{-r\kappa}}{2n}\right)\Bigg[S_{cm}+M_c+\kappa+\xi p+O_{cr}+nF+nql\left(\frac{F_t}{C_t}+F_r\right)+\frac{C_l+C_{rt}e^{\frac{\Lambda(\alpha_{\max}-\alpha)}{(\alpha-\alpha_{\min})}}}{p}\\ &+\frac{R_c\beta}{\alpha}\left(e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}-1\right)\Bigg]+\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[1+\frac{R_c\beta}{\alpha}e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\frac{2q^2n\theta\alpha r}{(2D+\theta e^{-r\kappa}q)^2}\right]\\ &-(C_{\theta m}+C_{wm})\theta r e^{-r\kappa}q\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right)-\frac{q}{2}(C_{\theta r}+C_{wr})\theta r e^{-r\kappa}\end{aligned}$$

Appendix B.

Here,

$$\rho=\left[S_{cm}+M_c+\kappa+\xi p+O_{cr}+nF+\frac{C_l+\Omega}{p}+\frac{R_c\beta}{\alpha}(\Psi-1)\right]$$

$$\text{and }\varrho=\left[nl\left(\frac{F_t}{C_t}+F_r\right)+\frac{R_c\beta}{\alpha}\Psi\frac{4Dn\alpha}{(2D+\theta qe^{-r\kappa})^2}\right]$$

$$\varsigma=\left[H_{cm}+(C_{\theta m}+C_{wm})\theta e^{-r\kappa}\right]\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right)+\frac{1}{2}\left[H_{cr}+(C_{\theta r}+C_{wr})\theta e^{-r\kappa}\right]$$

The second-order partial derivatives are

$$\begin{aligned}
\frac{\partial^2\mathrm{TSC}}{\partial p^2}&=\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)2\left[\frac{C_l+\Omega}{p^3}\right]+\left[H_{cm}+(C_{\theta m}+C_{wm})\theta e^{-r\kappa}\right]q\left(\frac{2D}{p^3}-\frac{Dn}{p^3}\right)\\
\frac{\partial^2\mathrm{TSC}}{\partial q^2}&=\left(\frac{2D}{nq^3}\right)\left[S_{cm}+M_c+\kappa+\xi p+O_{cr}+nF+nql\left(\frac{F_t}{C_t}+F_r\right)+\frac{C_l+\Omega}{p}\right.\\
&\quad\left.+\frac{R_c\beta}{\alpha}(\Psi-1)\right]+\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\frac{R_c\beta}{\alpha}\Psi\frac{16D^2n^2\alpha^2}{(2D+\theta qe^{-r\kappa})^4}\right.\\
&\quad\left.-\frac{R_c\beta}{\alpha}\Psi\frac{8Dn\theta\alpha e^{-r\kappa}}{(2D+\theta qe^{-r\kappa})^3}\right]-\left(\frac{2D}{nq^2}\right)\left[nl\left(\frac{F_t}{C_t}+F_r\right)+\frac{R_c\beta}{\alpha}\Psi\frac{4Dn\alpha}{(2D+\theta qe^{-r\kappa})^2}\right]\\
\frac{\partial^2\mathrm{TSC}}{\partial \alpha^2}&=\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\frac{C_{rt}}{p}\Omega\frac{\Lambda^2(\alpha_{\min}-\alpha_{\max})^2}{(\alpha-\alpha_{\min})^4}-\frac{2\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^3}\right.\\
&\quad\left.+\frac{2R_c\beta}{\alpha^3}(\Psi-1)-\frac{2R_c\beta}{\alpha^2}\left(\Psi\frac{2qn}{2D+\theta e^{-r\kappa}q}\right)+\frac{R_c\beta}{\alpha}\left(\Psi\frac{4q^2n^2}{(2D+\theta e^{-r\kappa}q)^2}\right)\right]\\
\frac{\partial^2\mathrm{TSC}}{\partial \kappa^2}&=\left(\frac{\theta r^2e^{-r\kappa}}{2n}\right)\left[S_{cm}+M_c+\kappa+\xi p+O_{cr}+nF+nql\left(\frac{F_t}{C_t}+F_r\right)\right.\\
&\quad\left.+\frac{C_l+\Omega}{p}+\frac{R_c\beta}{\alpha}(\Psi-1)\right]+\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\frac{R_c\beta}{\alpha}\left[\Psi\frac{4q^4n^2\theta^2r^2}{(2D+\theta e^{-r\kappa}q)^4}\right.\\
&\quad\left.+\Psi\frac{4q^2n\theta^2r^2e^{-r\kappa}}{(2D+\theta e^{-r\kappa}q)^3}\right]+\left[(C_{\theta m}+C_{wm})\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right)+\frac{1}{2}(C_{\theta r}+C_{\theta m})\right]\theta r^2e^{-r\kappa}q\\
\frac{\partial^2\mathrm{TSC}}{\partial p\partial q}&=\frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial q\partial p}=\left(-\frac{D}{nq^2}\right)\left[\xi-\frac{C_l+\Omega}{p^2}\right]+\left[H_{cm}+(C_{\theta m}+C_{wm})\theta e^{-r\kappa}\right]\left(\frac{-D}{p^2}+\frac{Dn}{2p^2}\right)\\
\frac{\partial^2\mathrm{TSC}}{\partial p\partial \kappa}&=\frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial \kappa\partial p}=\left(-\frac{\theta e^{-r\kappa}}{2n}\right)\left[\xi-\frac{C_l+\Omega}{p^2}\right]-\left[(C_{\theta m}+C_{wm})\theta re^{-r\kappa}\right]\left(\frac{-D}{p^2}+\frac{Dn}{2p^2}\right)\\
\frac{\partial^2\mathrm{TSC}}{\partial p\partial \alpha}&=\frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial \alpha\partial p}=-\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\frac{\Omega}{p^2}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2}\\
\frac{\partial^2\mathrm{TSC}}{\partial q\partial \kappa}&=\frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial \kappa\partial q}=\left(\frac{-\theta re^{-r\kappa}}{2n}\right)\left[nl\left(\frac{F_t}{C_t}+F_r\right)+\frac{R_c\beta}{\alpha}\Psi\frac{4nD\alpha}{(2D+\theta qe^{r\kappa})^2}\right]-\left(\frac{D}{nq^2}\right)\left[1+\frac{R_c\beta}{\alpha}\right.\\
&\quad\left.\Psi\frac{2q^2n\theta r}{(2D+\theta qe^{-r\kappa})^2}\right]+\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\frac{R_c\beta}{\alpha}\Psi\frac{8q^2n^2\theta rD\alpha}{(2D+\theta qe^{-r\kappa})^4}+\frac{R_c\beta}{\alpha}\Psi\frac{8Dqn\theta r}{(2D+\theta qe^{-r\kappa})^3}\right]\\
&\quad-(C_{\theta m}+C_{wm})\theta re^{-r\kappa}\left(\frac{D}{p}+\frac{n-1}{2}-\frac{Dn}{2p}\right)-\frac{1}{2}(C_{\theta r}+C_{wr})\theta re^{-r\kappa}
\end{aligned}$$

$$\frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} = \frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial \alpha \partial q} = \left(\frac{-D}{nq^2}\right)\left[\frac{\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2} - \frac{R_c\beta}{\alpha^2}(\Psi-1)\right.$$
$$\left. + \frac{R_c\beta}{\alpha}\Psi\frac{2qn}{2D+\theta e^{-r\kappa}q}\right] + \left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[-\frac{R_c\beta}{\alpha^2}\frac{4Dn}{(2D+\theta q e^{-r\kappa})^2} + \Psi\frac{8Dqn^2\alpha}{(2D+\theta e^{-r\kappa}q)^3}\right]$$
$$\frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha} = \frac{\partial^2 TSC(p,q,\alpha,n,\kappa)}{\partial \alpha \partial \kappa} = \left(-\frac{\theta r e^{-r\kappa}}{2n}\right)\left[\frac{\Omega}{p}\frac{\Lambda(\alpha_{\min}-\alpha_{\max})}{(\alpha-\alpha_{\min})^2} - \frac{R_c\beta}{\alpha^2}(\Psi-1) + \frac{R_c\beta}{\alpha}e^{\alpha\frac{2qn}{2D+\theta e^{-r\kappa}q}}\right.$$
$$\left.\frac{2qn}{(2D+\theta q e^{-r\kappa})^2}\right] + \left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\frac{R_c\beta}{\alpha}\Psi\frac{4q^3n^2\theta r}{(2D+\theta e^{-r\kappa}q)^3} - \frac{R_c\beta}{\alpha^2}\Psi\frac{2q^2n\theta r}{(2D+\theta e^{-r\kappa}q)^2}\right]$$

Here $\mathrm{TSC} = \mathrm{TSC}(p,q,\alpha,n,\kappa)$. The first-order principal minor is

$$det(H_{11}) = det\left(\frac{\partial^2 \mathrm{TSC}}{\partial q^2}\right) = \left(\frac{2D}{nq^3}\right)\left[S_{cm} + M_c + \kappa + \xi p + O_{cr} + nF\right.$$
$$\left. + nql\left(\frac{F_t}{C_t}+F_r\right) + \frac{C_l+\Omega}{p} + \frac{R_c\beta}{\alpha}(\Psi-1)\right] + \left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\left[\frac{R_c\beta}{\alpha}\Psi\frac{16D^2n^2\alpha^2}{(2D+\theta q e^{-r\kappa})^4}\right.$$
$$\left. - \frac{R_c\beta}{\alpha}\Psi\frac{8Dn\theta\alpha e^{-r\kappa}}{(2D+\theta q e^{-r\kappa})^3}\right] - \left(\frac{2D}{nq^2}\right)\left[nl\left(\frac{F_t}{C_t}+F_r\right) + \frac{R_c\beta}{\alpha}\Psi\frac{4Dn\alpha}{(2D+\theta q e^{-r\kappa})^2}\right] = \Delta_1 \quad \text{(say)}.$$

The first-order principal minor is greater than zero. If

$$\left(\frac{D}{nq}+\frac{\theta e^{-r\kappa}}{2n}\right)\Psi\left[\frac{R_c\beta}{\alpha}\frac{16D^2n^2\alpha^2}{(2D+\theta q e^{-r\kappa})^4} - \frac{R_c\beta}{\alpha}\frac{8Dn\theta\alpha e^{-r\kappa}}{(2D+\theta q e^{-r\kappa})^3}\right] > \left(\frac{2D}{nq^2}\right)\left[nl\left(\frac{F_t}{C_t}+F_r\right) + \frac{R_c\beta}{\alpha}\Psi\frac{4Dn\alpha}{(2D+\theta q e^{-r\kappa})^2}\right]$$

Hence the proof.

Appendix C.

The second-order principal minor is

$$det(H_{22}) = det\begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q^2} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} \end{pmatrix} = \frac{\partial^2 \mathrm{TSC}}{\partial q^2}\frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} - \left(\frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha}\right)^2 = \Delta_2 \quad \text{(say)}.$$

The second-order principal minor is greater than zero if $\Delta_2 > 0$. The third-order principal minor is

$$det(H_{33}) = det\begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q^2} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \kappa} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa^2} \end{pmatrix}$$
$$= 2\frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q}\frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha}\frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} - \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q}\frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2}\frac{\partial^2 \mathrm{TSC}}{\partial q \partial \kappa} - \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha}\frac{\partial^2 \mathrm{TSC}}{\partial q^2}\frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} + \frac{\partial^2 \mathrm{TSC}}{\partial \kappa^2}$$
$$det(H_{22}) = \Delta_3 \quad \text{(say)}.$$

The third-order principal minor is greater than zero if $\Delta_3 > 0$. The fourth-order principal minor is

$$det(H_{44}) = det\begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q^2} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial p \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial p \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial p \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial p^2} \end{pmatrix}$$

$$= \frac{\partial^2 \mathrm{TSC}}{\partial p \partial q} det \begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial p} \end{pmatrix} - \frac{\partial^2 \mathrm{TSC}}{\partial p \partial \alpha} det \begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q^2} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial \kappa} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial p} \end{pmatrix}$$

$$+ \frac{\partial^2 \mathrm{TSC}}{\partial p \partial \kappa} det \begin{pmatrix} \frac{\partial^2 \mathrm{TSC}}{\partial q^2} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial q \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha^2} & \frac{\partial^2 \mathrm{TSC}}{\partial \alpha \partial p} \\ \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial q} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial \alpha} & \frac{\partial^2 \mathrm{TSC}}{\partial \kappa \partial p} \end{pmatrix} - \frac{\partial^2 \mathrm{TSC}}{\partial p^2} det(H_{33}) = \Delta_4 \quad \text{(say)}$$

Finally, the fourth-order principal minor is greater than zero if $\Delta_4 > 0$.

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References

[1] M. Abuhelwa, W.A. Salah and M.J.K. Bashir, Potential energy production from organic waste and its environmental and economic impacts at a tertiary institution in Palestine. *Environ. Qual. Manag.* (2023). DOI: 10.1002/tqem.21960
[2] S. Adak and G.S. Mahapatra, Effect of reliability on multi-item inventory system with shortages and partial backlog incorporating time dependent demand and deterioration. *Ann. Oper. Res.* **315** (2022) 1551–1571.
[3] P. Alavian, E. Yongsoon, S.M. Semyon and L. Zhang, Smart production systems: automating decision-making in manufacturing environment. *Int. J. Prod. Res.* **58** (2020) 828–845.
[4] F. Angizeh, H. Montero, A. Vedpathak and M. Parvania, Optimal production scheduling for smart manufacturers with application to food production planning. *Comput. Elec. Eng.* **84** (2020) 106609.
[5] A. Debnath and B. Sarkar, Effect of circular economy for waste nullification under a sustainable supply chain management. *J. Clean. Prod.* **385** (2023) 135477.
[6] M. Ben-Daya, R. As'ad and K.A. Nabi, A single-vendor multi-buyer production remanufacturing inventory system under a centralized consignment arrangement. *Comput. Ind. Eng.* **135** (2019) 10–27.
[7] A.I. Malik, B. Sarkar, M.W. Iqbal, M. Ullah, I. Khan and M.B. Ramzan, Coordination supply chain management in flexible production system and service level constraint: A Nash bargaining model. *Comp. Indust. Eng.* **177** (2023) 109002.
[8] H. Cañas, J. Mula, F. Campuzano-Bolarín, and R. Poler, A conceptual framework for smart production planning and control in Industry 4.0. *Comput. Ind. Eng.* **173** (2022) 108659.
[9] J. Chaab and O.C. Demirag, Effects of consumer loyalty and product web compatibility on cooperative advertising and pricing policies in a dual-channel supply chain. *RAIRO: OR* **56** (2022) 2557–2580.
[10] S.Y. Chang, and T.Y. Yeh, A two-echelon supply chain of a returnable product with fuzzy demand. *Appl. Math. Model.* **37** (2013) 4305–4315.
[11] U. Chaudhari, A. Bhadoriya, M.Y. Jani and B. Sarkar, A generalized payment policy for deteriorating items when demand depends on price, stock, and advertisement under carbon tax regulations. *Math. Comput. Simul.* **207** (2023) 556–574.
[12] S.C. Das, A.M. Zidan, A.K. Manna, A.A. Shaikh and A.K. Bhunia, An application of preservation technology in inventory control system with price dependent demand and partial backlogging. *Alex. Eng. J.* **59** (2020) 1359–1369.
[13] Y. Emamian, I.N. Kamalabadi and A. Eydi, Developing and solving an integrated model for production routing in sustainable closed-loop supply chain. *J. Clean. Prod.* **302** (2021) 126997.
[14] C.B. Gabler, V.M. Landers, R. Agnihotri and T.R. Morgan, Environmental orientation on the frontline: a boundaryspanning perspective for supply chain management. *J. Bus. Logist.* (2023). DOI: 10.1111/jbl.12328.
[15] P. Gautam, S. Maheshwari, A. Hasan, A. Kausar and C.K. Jaggi, Optimal inventory strategies for an imperfect production system with advertisement and price reliant demand under rework option for defectives. *RAIRO: OR* **56** (2022) 183–197.
[16] J. Heydari, K. Govindan and R. Sadeghi, Reverse supply chain coordination under stochastic remanufacturing capacity. *Int. J. Prod. Econ.* **202** (2018) 1–11.
[17] B. Sarkar and R. Guchhait, Ramification of information asymmetry on a green supply chain management with the cap-trade, service, and vendor-managed inventory strategies. *Elect. Comm. Res. App.* **60** (2023) 101274.
[18] J. Huo, M. Zhang, D. Wang, A.S. Mujumdar, B. Bhandari and L. Zhang, New preservation and detection technologies for edible mushrooms: a review. *J. Sci. Food Agric.* (2023). DOI: 10.1002/jsfa.12472.

[19] I. Kazancoglu, M. Ozbiltekin-Pala, S.K. Mangla, Y. Kazancoglu, and F. Jabeen, Role of flexibility, agility and responsiveness for sustainable supply chain resilience during COVID-19. *J. Clean. Prod.* **362** (2022) 132431.

[20] M. Kerin and D.T. Pham, A review of emerging industry 4.0 technologies in remanufacturing. *J. Clean. Prod.* **237** (2019) 117805.

[21] G. Li, X. He, J. Zhou and H. Wu, Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. *Omega* **84** (2019) 114–126.

[22] S. Saha, B. Sarkar and M. Sarkar, Application of improved meta-heuristic algorithms for green preservation technology management to optimize dynamical investments and replenishment strategies. *Math. Comp. Simul.* **209** (2023) 426–450.

[23] U. Mishra, J.Z. Wu and B. Sarkar, A sustainable production-inventory model for a controllable carbon emissions rate under shortages. *J. Clean. Prod.* **256** (2020) 120268.

[24] U.M. Modibbo, M. Arshad, O. Abdalghani and I. Ali, Optimization and estimation in system reliability allocation problem. *Reliab. Eng. Syst. Saf.* **212** (2021) 107620.

[25] B. Mridha, S. Pareek, A. Goswami and B. Sarkar, Joint effects of production quality improvement of biofuel and carbon emissions towards a smart sustainable supply chain management. *J. Clean. Prod.* **386** (2023) 135629.

[26] B. Sarkar, H. Seok, T.K. Jana and B.K. Dey, Is the system reliability profitable for retailing and consumer service of a dynamical system under cross-price elasticity of demand? *J. Retail. Consum. Serv.* **75** (2023) 103439.

[27] U. Mishra, J.Z. Wu and B. Sarkar, Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. *J. Clean. Prod.* **279** (2021) 123699.

[28] S. Ouaret, J.P. Kennè and A. Gharbi, Stochastic optimal control of random quality deteriorating hybrid manufacturing/remanufacturing systems. *J. Manuf. Syst.* **49** (2018) 172–185.

[29] B. Pal, A. Mandal and S.S. Sana, Two-phase deteriorated supply chain model with variable demand and imperfect production process under two-stage credit financing. *RAIRO: OR* **55** (2021) 457–480.

[30] D. Yadav, R. Kumari, N. Kumar and B. Sarkar. Reduction of waste and carbon emission through the selection of items with crossprice elasticity of demand to form a sustainable supply chain with preservation technology. *J. Clean. Prod.* **297** (2021) 126298.

[31] M. Pervin, S.K. Roy, P. Sannyashi and G.W. Weber, Sustainable inventory model with environmental impact for non-instantaneous deteriorating items with composite demand. *RAIRO: OR* **57** (2023) 237–261.

[32] M. Rahaman, S.P. Mondal, S. Alam and S.K. De, A study of a lock fuzzy EPQ model with deterioration and stock and unit selling price-dependent demand using preservation technology. *Soft Comput.* **26** (2022) 2721–2740.

[33] M. Rahmani, A. Romsdal, F. Sgarbossa, J.O. Strandhagen and M. Holm, Towards smart production planning and control; a conceptual framework linking planning environment characteristics with the need for smart production planning and control. *Ann. Rev. Control* **53** (2022) 370–381.

[34] T. Ramesh, U. Hariram, A. Srimagal and J.K. Sahu, Applications of light emitting diodes and their mechanism for food preservation. *J. Food Saf.* (2023) e13040.

[35] C. Rout, A. Paul, R.S. Kumar, D. Chakraborty and A. Goswamia, Cooperative sustainable supply chain for deteriorating item and imperfect production under different carbon emission regulations. *J. Clean. Prod.* **272** (2020) 122170.

[36] T. Roy and K.S. Chaudhuri, A production-inventory model under stock-dependent demand, Weibull distribution deterioration and shortage. *Int. Trans. Oper. Res.* **16** (2009) 325–346.

[37] L. Sahoo, Transportation problem in Fermatean fuzzy environment. *RAIRO: OR* **57** (2023) 145–156.

[38] M. Sarkar and B.D. Chung, Flexible work-in-process production system in supply chain management under quality improvement. *Int. J. Prod. Res.* **58** (2020) 3821–3838.

[39] B. Sarkar, M. Sarkar, B. Ganguly and L.E. Cárdenas-Barrón, Combined effects of carbon emission and production quality improvement for fixed lifetime products in a sustainable supply chain management. *Int. J. Prod. Econ.* **231** (2021) 107867.

[40] M. Ullah, I. Asghar, M. Zahid, M. Omair, A. Alarjani and B. Sarkar, Ramification of remanufacturing in a sustainable three-echelon closed-loop supply chain management for returnable products. *J. Clean. Prod.* **290** (2021) 125609.

[41] A.S.H. Kugele and B. Sarkar, Reducing carbon emissions of a multi-stage smart production for biofuel towards sustainable development. *Alex. Eng. J.* **70** (2023) 93–113.

[42] M. Ullah and B. Sarkar, Recovery-channel selection in a hybrid manufacturing-remanufacturing production model with RFID and product quality. *Int. J. Prod. Econ.* **219** (2020) 360–374.

[43] M. Sebatjane, The impact of preservation technology investments on lot-sizing and shipment strategies in a three-echelon food supply chain involving growing and deteriorating items. *Oper. Res. Perspect.* **9** (2022) 100241.

[44] M. Sharma, A. Dhir, H. AlKatheeri, M. Khan and M.M. Ajmal, Greening of supply chain to drive performance through logical integration of supply chain resources. *Bus. Strategy Environ.* (2023). DOI: 10.1002/bse.3340

[45] D. Tan, M. Suvarna, Y.S. Tan, J. Li and X. Wang, A three-step machine learning framework for energy profiling, activity state pblackiction and production estimation in smart process manufacturing. *Appl. Energy* **291** (2021) 116808.

[46] M. Ullah, B. Sarkar and I. Asghar, Effects of preservation technology investment on waste generation in a two-echelon supply chain model. *Mathematics* **7** (2019) 189.

[47] C. Wang and L. Jiang, Inventory policy for deteriorating seasonal products with price and ramp-type time dependent demand. *RAIRO: OR* **49** (2015) 865–878.

[48] B. Sarkar, M. Tayyab, N. Kim and M.S. Habib, Optimal production delivery policies for supplier and manufacturer in a constrained closed-loop supply chain for returnable transport packaging through metaheuristic approach. *Comp. Indust. Eng.* **135** (2019) 987–1003.

[49] Y. Yan, F. Yao and J. Sun, Manufacturer's cooperation strategy of closed-loop supply chain considering corporate social responsibility. *RAIRO: OR* **55** (2021) 3639–3659.

[50] P. Zhao, Q. Deng, J. Zhou, W. Han, G. Gong and C. Jiang, Optimal production decisions for remanufacturing end-of-life products under quality uncertainty and a carbon cap-and-trade policy. *Comput. Ind. Eng.* **162** (2021) 107646.