THE STRATEGIC INTERACTION BETWEEN BUSINESS MODE AND STORE BRAND INTRODUCTION IN A PLATFORM-BASED SUPPLY CHAIN

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Abstract. The phenomenon of store brands introduced by large-scale platforms is becoming common, and manufacturers should carefully choose the business mode of selling national brand (NB) products via platforms. Considering big-data marketing, we examine the sales mode selection in a platform-based supply chain based on the strategic interaction between the business mode decision and store brand decision. By continuous dynamic game theory, the strategies and performance under different modes are solved. We find that given the store brand (SB) decision, the business mode adopted by the manufacturer depends only on the commission rate, which is high for reselling and low for agency selling. Given the business mode, under reselling, whether the platform introduces SB depends only on the brand preference, i.e., SB is introduced when the preference for NB is low. While under agency, he introduces SB in the general or relatively passive situation, depending both on the commission rate and the brand preference. In addition, four equilibrium sales modes are obtained based on the strategic interaction. In order to achieve a win-win-win situation in profitability among the manufacturer, the platform and the platform-based supply chain, the manufacturer should adopt agency selling and the platform forgoes introducing SB when both the commission rates and NB preference are low.

1. Introduction

In the digital age, online platforms have become the wind vane leading the development of the world economy, which promotes the platform-based supply chain to open the road of digitization and intelligence [47]. Affected by the pandemic, online shopping is the preferred method for 68% of consumers, while the proportion is less than 30% before the pandemic in 2019 [55]. Online platforms represented by Amazon, eBay, Walmart, Etsy, etc. have further risen strongly in the post-epidemic era in line with the trend of global economic development [50]. Actually, the global online retail sales reached $4.938 trillion in 2021, with a year-on-year increase of 16.3% change, accounting for 19.0% of the total retail sales [27]. According to eMarketer, the global online retail sales is expected to stabilize in 2022, and despite the gradual slowdown in growth, the sales and its share in total retail sales are still increasing steadily.

However, practice demonstrates that large-scale online platforms growing up based on the logic of bilateral market in the era of digital economy are different from the giants in traditional industries. The distinction...
is that the advantages and leading position of platforms are not indestructible, that is, latecomers are more likely to give them huge impact and carve up their existing market share [15]. For example, in the Southeast Asian market, in addition to Shopee and Lazada, local platforms such as Tokopedia, Bukalapak, Tiki, Blibli and so on have joined the market competition and grown strongly in the face of huge opportunities of online retailing [63]. In addition, at the end of 2021, relying on Tiktok’s huge traffic pool, ByteDance established Fanno for the European market, which launched an impact on the existing large-scale platforms [51]. Undoubtedly, platforms need to sort out diverse shopping data, build effective data management and analysis mechanisms, and finally abstract the data into big data marketing services that match consumers [18, 23, 47]. Obviously, the big-data marketing service is a marketing activity based on big data analysis. Based on this, the platform can provide customized products or services according to individual needs, thus achieving precise marketing and improving operational efficiency and competitiveness [20, 42, 68].

Meanwhile, with the continuous and in-depth development of online platforms, manufacturers physical sales barriers have been broken. Online platforms have become important partners for manufacturers to sell their national brand (NB) products due to unprecedented connection with consumers. In reality, manufacturers sell NB products via platforms, such as Amazon and JD.com, mainly through two business modes: reselling or agency selling [1]. The major distinction between the two business modes lies in the different ownership of pricing power of the NB product. Specifically, under agency selling, the pricing power of the NB product belongs to the manufacturer because the platform only acts as an intermediary connecting manufacturers and consumers and collects commissions from manufacturers for selling NB products. While under reselling, the platform has the pricing power of the NB product because the platform first purchases NB products from manufacturers and then sells them to consumers [43, 74]. Therefore, considering the ownership of pricing power, the manufacturer’s business mode decision affects the platform’s profitability, which in turn affects the platform’s investment in the big-data marketing service of the NB product to a certain extent.

In addition, consumers have gradually altered their demand for products with the trend of upgrading consumption [28]. On one hand, they are more daring to try and willing to accept new brands and products. On the other hand, they advocate de-logo and pursue the essence of products. Accordingly, the online platform can timely grasp the changes of consumers shopping preferences and behaviors with the help of big data analysis. In other words, the platform has the motivation to introduce the store brand (SB) to strengthen consumers stickiness by providing them with more cost-effective products. This measure also helps the platform to enhance the discourse power of dialogue between itself and manufacturers, and further consolidate the bilateral network effect and competitiveness [2, 9, 21]. Besides Amazon and JD.com, which have introduced their store brands early and achieved great success, some emerging platforms have accelerated the layout of their store brands and joined the new track of store brands [26]. For example, in 2020, Suning.com launched Epin, a store brand that offers products with high value-added, appearance value and cost-effective. In fact, Aipin, which has been on the market for only one year, saw a 190% increase in sales on Nov. 11, 2021 compared to Aug. 18. Since store brands are bound to seize the market share of national brands, the manufacturer may change the quality of NB product and the business mode of selling NB product via the platform, and then indirectly affecting the platform’s big-data marketing service of the NB product.

To sum up, we aim to investigate the strategic interaction between the manufacturer’s business mode decision (reselling or agency selling) and the platform’s store brand decision (introduced or not), i.e., the selection of sales modes for a platform-based supply chain. In particular, our research questions are as follows: (1) given the sales mode, how should the manufacturer decide the quality and price of the NB product? How should the platform determine the big-data marketing service and price of the NB product? In addition, if SB is introduced, how should the platform decide the quality, big-data marketing service and price of the SB product? (2) Given the SB decision, which business mode does the manufacturer adopt? Which business mode does the platform want the manufacturer to adopt? Which business mode is more beneficial to the platform-based supply chain? (3) Given the business mode, does the platform introduce store brands? Is the manufacturer willing? How does the store brand affect the platform-based supply chain? (4) What is the equilibrium sales mode? Do there exist a win-win-win situation among the manufacturer, the platform and the platform-based supply chain?
To answer the above questions, we consider a platform-based supply chain consisting of a manufacturer (denoted by she), a platform (denoted by he) and consumers. According to the interaction between the manufacturer and the platform, we develop four theoretical sales modes, i.e., mode NR (the manufacturer adopts reselling and the platform does not introduce store brand), mode NA (the manufacturer adopts agency selling and the platform does not introduce store brand), mode SR (the manufacturer adopts reselling and the platform introduces store brand) and mode SA (the manufacturer adopts agency selling and the platform introduces store brand). We analyze the optimal operation strategies and corresponding profits of the manufacturer and the platform under four sales modes. Our main findings are included as follows.

First, Given the SB decision, for the manufacturer, the business mode she adopts only depends on the commission rate. That is, she adopts agency selling if the commission rate is low, otherwise, she adopts reselling. For the platform, he prefers that the manufacturer adopts reselling (agency selling) if the commission rate is low (high). Furthermore, when the platform introduces SB, consumers preference for the two brands will affect his preference for the business mode if the commission rate is moderate. That is to say, he prefers reselling (agency selling) if consumers prefer SB (NB). For the platform-based supply chain, reselling (agency selling) is better if the commission rate is sufficient large (low). Especially, when the platform introduces SB, agency selling is more beneficial for this supply chain if and only if both the commission rate and the preference for NB are large enough.

Second, Given the business mode, for the platform, under reselling, he introduces (forgoes introducing) SB if the preference of consumers for NB is low (high). Under agency selling, he introduces SB in the general situation, i.e., the commission rate is very low, or both the commission rate and preference for NB are relatively low, or introduces it in the relatively passive situation, i.e., both the commission rate and preference for NB are extremely high. For the manufacturer, she does not want SB to be introduced by the platform either under reselling or agency selling. For the platform-based supply chain, the existence of store brand is always unfavorable to the supply chain under reselling. Compared with reselling, the introduction of store brand is better if the commission rate and the preference for NB are very high under agency selling.

What’s more, as for the equilibrium sales mode, the equilibrium sales mode is NR (SR) if the commission rate is high and the preference for NB is low (high), the equilibrium sales mode is NA (SA) if the commission rate is low and the preference for NB is low (high). It is worth noting that the equilibrium sales mode NR under that condition is the most profitable situation for the manufacturer. Furthermore, the equilibrium sales mode NA under that condition is a win-win-win situation in profitability among the manufacturer, platform and the platform-based supply chain.

Our contributions are explained as follows. First, considering that the platform’s store brand is bound to seize the market share of the manufacturer’s national brand, the interaction between the platform’s store brand decision and the manufacturer’s business mode decision becomes more complex. Therefore, our research is of theoretical and practical significance to explore the sales mode selection of the platform-based supply chain. Second, to the best of our knowledge, our research is the first to quantitatively study the big-data marketing service strategy under the interaction between the platform’s store brand decision and the manufacturer’s business mode decision. Third, our research discusses the performance of the manufacturer, the platform and the platform-based supply chain under the specific scenario and identify the win-win-win situation. What’s more, our research further investigates whether the three parties can achieve win-win-win situation under the equilibrium sales mode. Thus, our findings not only reveal the reasons why enterprises adopt different sales modes in reality, but also provide references and suggestions for better operation of enterprises and improvement of platform theory system.

The remainder of this paper is organized as follows. Section 2 reviews a literature review closely related to this paper. Section 3 presents a description of the problem and relevant model assumptions. Section 4 analyzes the optimal operation strategies and corresponding profits of the manufacturer and the platform under the four sales modes. Moreover, the impact of different key parameters on corporate goodwill and operation strategies are discussed, and the relationships of the steady state goodwill and corresponding strategies under different mode combinations are compared. Section 5 investigates the business mode decision and the store brand decision.
under the specific scenario, as well as the equilibrium sales mode. Section 6 contains a model extension that considers positive production costs to confirm the robustness of our main results. Finally, the key conclusions and managerial insights are summarized in Section 7.

2. Literature review

The streams of literature closely related to our research involve three aspects, namely, store brand, business mode and big-data marketing service.

2.1. Store brand

There is extensive literature on the store brand. Firstly, a portion of the research focused on how to position SBs, including horizontal positioning and vertical positioning. The former mainly examined the issue of how to match SBs functions with consumers tastes and preferences [19, 44, 52, 60, 62], while the latter primarily pays close attention to the issue of the quality or pricing difference between SB and NB [22, 30, 38, 67, 78]. Secondly, the previous studies of SB also focused on the competition between SB and NB to explore whether or when retailers introduce SBs and how SBs affect manufacturers. Intuitively, the introduction of SB always favors retailers to the detriment of manufacturers, which has been confirmed by empirical studies [25,32] and analytical studies [5,33,57]. Nevertheless, some studies obtained the counter-intuitive conclusion that SB is beneficial to manufacturers, meaning that the introduction of SB could be a win-win situation for both manufacturers and retailers. For instance, Chintagunta et al. [17] and Ailawadi and Harlam [3] demonstrated empirically that manufacturers may benefit from the move of retailers to introduce SBs based on the premise of relatively powerful retailers’ sales data. Ru et al. [58] found that the presence of SB increases the wholesale price and demand for NB, which contributes to the manufacturer’s profitability. Cheng et al. [16] revealed the impact of SB on the interaction and performance among members of a three-echelon supply chain and found that all members are likely to benefit when the SB is sufficiently competitive and eventually introduced.

What’s more, the introduction of SBs can shake up and alter the relationship between manufacturers and retailers and aggravate the complexity of their interaction [24,40]. Actually, in view of the fact that SBs inevitably affect the sales of manufacturers’ NB, it is common for NB manufacturers to take some action to interfere with the retailer’s potential SB introduction strategy, even though the effect may be positive. Yu et al. [72] investigated a NB manufacturer’s responses to a retailer’s SB based on consumers fairness concern, and found that the use of direct consumer incentive by the manufacturer could mitigate their fairness concern for NB and effectively prevent the retailer from introducing SB. Amrouche and Yan [7] showed that the NB manufacturer can effectively deal with SB’s entry and generate high revenue by using the partnership strategy of revenue sharing and the aggressive strategy of advertising. Nasser et al. [53] demonstrated that the threat of SB can be reduced by designing an entire NB portfolio on pricing and quality positioning if consumers have heterogeneity in product quality. Karray and Martn-Herrn [36] show that the NB manufacturer can hinder or benefit from the introduction of SB by adjusting the timing of NB’s pricing and advertising strategies. Most recently, with the development of e-commerce, manufacturers online sales channel has become a major means to prevent the introduction of SB, which is also reflected in the latest research on SB focusing on the strategic interaction between manufacturers channel choice and retailers introduction of store brands [10,35,39–41]. They found that under certain conditions, manufacturers channel encroachment, i.e., the introduction of online sales channels, can prevent retailers from introducing SB or mitigate the negative impact of retailers SBs once SBs has entered.

Actually, in view of manufacturers sell NB products via platforms through online channels, the change in manufacturer’s business mode is a powerful response to the adverse effects of SBs. Therefore, different from the above-mentioned literature, our study focuses on the manufacturer’s business mode decision in the face of the platform’s SB decision.
2.2. Business mode

At present, reselling and agency selling are the two prevalent business modes for manufacturers to conduct online sales through platforms, which has prompted the industry and academia to explore the business mode decision that focus on which business mode is more conducive to better operation. Abhishek et al. [1] studied the question of when e-retailers business modes should shift from traditional reselling to agency selling, investigated that agency selling is more effective than reselling and e-retailers preference for agency selling increases as competition increases. Liu et al. [48] explored the business mode strategies of a monopoly manufacturer when faced with two competing online retail platforms, showed that which business mode the manufacturer adopts depends on the competition between platforms and the cost of order fulfilment. Pu et al. [56] added the consideration of direct selling to reselling and agency selling, examined the choice of business mode for a manufacturer selling products online on the basis of retaining the offline channel, and found that the corresponding cost plays a key role in which business mode adopted by the manufacturer. In addition, some scholars have also compared the two business modes of reselling and agency selling by considering factors such as information sharing [74], product quality [76], dynamic advertising [49], competition effect [73] and channel selection [64].

In particular, more relevant to our research are studies involving both business mode decision and SB, which are relatively rare. Li et al. [42] studied the impact of the platform’s SB introduction decision on the manufacturer’s business mode considering the investment effect, found that after SB was introduced, the manufacturer could obtain a high return by taking agency selling, while a high investment efficiency was likely to be achieved by choosing reselling. Zhang and Hou [75] analyzed the preferences of the manufacturer and the e-tailer over reselling and agency selling in the presence of SB, taking into account the brand advantage of NB, and showed that they both preferred agency selling when the percentage cost and brand advantage were moderate, which allowed for incentive alignment and even optimal environmental performance. The former focused on the choice of the manufacturer’s business mode when SB exists, while the latter discussed the manufacturer’s mode decision to respond to the introduction of SB, but ignored product quality and big-data marketing service. In contrast, we use the price, product quality and big-data marketing service of NB and SB as endogenous variables to examine the business mode decision of the manufacturer selling NB product via the platform that may introduces a SB.

2.3. Big-data marketing service

Affected by the rapid development of digital and intelligent technology, online platforms developed by relying on technology and data use their own advantages to carry out big-data marketing services as one of the core tools to sustain operations and maintain competitiveness [45,70]. Currently, most research on big-data marketing either focused on qualitative research on the definition, opportunities and challenges, and applications of big-data marketing [4,11,37,61], or concentrated on quantitative research to explore the system design to improve the accuracy and effectiveness of big-data marketing services [12,13,59]. Only a few studies have quantitatively analyzed the impact of big-data marketing services on enterprises strategies and performances by establishing mathematical models. Ghoshal et al. [29] analyzed the data sharing coalition decision of two competing companies based on big data personalized recommendations. Considering the participation of third-party Internet recycling platforms, Xiang and Xu [71] studied the impact of big-data marketing on members decisions in a closed-loop supply chain under three scenarios considering the involvement of third-party internet recycling platforms and found that the cost-sharing contract would stimulate the platform to put more efforts into big-data marketing. Cao et al. [13] explored the choice of sales mode for the platform based on data-driven marketing and found that the platform prefers reselling to agency selling when data-driven marketing is more efficient. Liu et al. [47] studied the platform’s preference for reselling and agency selling based on the influence of data-driven marketing, and found that the high efficiency of data-driven marketing led to a stronger preference for agency selling.

The related studies of this paper in the literature are shown in Table 1. Firstly, the subject of SB introduction in our research is an online platform rather than a traditional brick-and-mortar retailer. Hence, the question we investigate is the strategic interaction between the manufacturer’s business mode decision and the platform’s
Table 1. The related studies of the relevant literature.

<table>
<thead>
<tr>
<th>Articles</th>
<th>Supply chain structure</th>
<th>Brand strategy</th>
<th>Business mode</th>
<th>Big-data marketing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li et al. [43]</td>
<td>MR</td>
<td>NB/ NB+SB</td>
<td>Reselling</td>
<td>N/A</td>
</tr>
<tr>
<td>Wang et al. [67]</td>
<td>MR</td>
<td>NB/ NB+SB</td>
<td>Reselling</td>
<td>N/A</td>
</tr>
<tr>
<td>Zhang and Hou [76]</td>
<td>MP</td>
<td>NB+SB</td>
<td>Reselling/agency selling</td>
<td>N/A</td>
</tr>
<tr>
<td>Liu et al. [48]</td>
<td>MP</td>
<td>NB</td>
<td>Reselling/agency selling</td>
<td>Data-driven marketing</td>
</tr>
<tr>
<td>Li et al. [39]</td>
<td>MP</td>
<td>NB/ NB+SB</td>
<td>Reselling/agency selling</td>
<td>Investment effect</td>
</tr>
</tbody>
</table>

Notes. MR - a manufacturer and a retailer; MP - a manufacturer and a platform.

Table 2. Four settings of business mode and store brand.

<table>
<thead>
<tr>
<th>Store brand</th>
<th>Business mode</th>
<th>Agency selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not introduced</td>
<td>NR</td>
<td>NA</td>
</tr>
<tr>
<td>Introduced</td>
<td>SR</td>
<td>SA</td>
</tr>
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</table>

store brand decision, which is in line with the reality of the platform context and can enrich the research of SB introduced by platforms. Secondly, considering the important role of big-data marketing in gaining insight into consumers shopping behavior and preferences, we conduct a quantitative study on the big-data marketing service in a platform-based supply chain.

3. Model

We consider a platform-based supply chain (PSC) in which a manufacturer (M, denoted by she) produces a national brand (NB) product and sells it with quality $q_N(t)$ and sales price $p_N(t)$ to consumers via a platform (P, denoted by he). The platform can analyze data such as search, browsing and purchase information to accurately grasp the personalized potential demand. That is, he can adopt accurate big-data marketing service $b_N(t)$ to actively provide consumers with customized product recommendations, thereby stimulating demand and increasing the order volume. In addition, in order to diversify profit sources, the platform considers whether to introduce store brand (SB). If he introduces store brand, he sells not only M’s NB product, but also his own SB product, and determines the sales price $p_S(t)$, quality $q_S(t)$ and big-data marketing service $b_S(t)$ of the SB product accordingly. After the store brand decision of the platform, the manufacturer can choose to sell the NB product to consumers via the platform through two business modes: reselling (R) or agency selling (A). To study the manufacturer’s business mode decision (reselling or agency selling) under the condition of the platform’s store brand decision (Introduced or not), four modes are established, as shown in Table 2, and the corresponding structures of the four modes are illustrated in Figure 1.

Case 1. SB Not introduced, merely NB product is sold in PSC. Virtually, the preference of consumers for a product, more often than not, hinges on quality of product itself rather than marketing means of enterprises. According to the Nerlove-Arrow goodwill model (Nerlove and Arrow [54]), we assume that the goodwill of NB is positively related to the quality of NB product, which signifies that the higher quality of NB product, the better the reputation of NB among consumers, so as to enhance the goodwill of NB. The dynamic evolution of
goodwill of NB can be expressed as:

\[ \dot{G}_N(t) = \alpha_N q_N(t) - \delta_N G_N(t), G_N(0) = G_{N0} \]  

where, \( G_N(t) \) represents the goodwill of NB at time \( t \), \( G_{N0} \geq 0 \) represents the initial goodwill of NB. Here, \( \alpha_N > 0 \) is the influence coefficient of the quality of NB product on goodwill of NB. The higher the value of \( \alpha_N \), the greater the promotion of quality to goodwill, and the more consumers believe that the higher the product quality under the brand with high goodwill and great reputation. Moreover, \( \delta_N > 0 \) is the natural decay rate of goodwill of NB caused by consumers brand forgetting if quality does not affect goodwill.

In the term of demand, due to the rise of the new generation of consumers, in addition to price factor, they also regard the added value beyond the basic value of products as important, such as online shopping experience (including the fit between recommended content and their real needs, the convenience of information browsing, the level of product search cost, etc.) and goodwill (including positioning, popularity, reputation, etc.). Following the relevant assumptions of demand function [46], we assume that besides the price factor, the demand of NB product is also affected by the platform’s big-data marketing service and the goodwill of NB, which can be constructed as:

\[ D_N(t) = a - \lambda_N p_N(t) + \gamma_N b_N(t) + \beta_N G_N(t) \]  

where, \( a > 0 \) denotes the basic market size. \( \lambda_N \in [0, 1], \gamma_N > 0, \beta_N > 0 \) are the influence coefficient of sales price, big-data marketing service and goodwill of NB on the demand of NB product, respectively.

Case 2. SB Introduced, both NB and SB product are sold in PSC. In this case, the goodwill of NB is still related to its own quality, \textit{i.e.}, the dynamic evolution of NB’s goodwill is consistent with equation (1). Meanwhile, same as NB, the goodwill of SB is also positively related to its quality, hence, the dynamic evolution of goodwill of SB can be expressed as:

\[ \dot{G}_S(t) = \alpha_S q_S(t) - \delta_S G_S(t), G_S(0) = G_{S0} \]  

where, \( G_S(t) \) represents the goodwill of SB at time \( t \), \( G_{S0} \geq 0 \) represents the initial goodwill of SB. \( \alpha_S > 0 \) is the influence coefficient of the quality of SB product on goodwill of SB. \( \delta_S > 0 \) is the natural decay rate of goodwill of SB.

As for demand, if SB is introduced into the platform, customers can choose which brands to buy (NB or SB) on the basis of their preference [34]. Nowadays, consumers emerge different preferences for complex brands: part of them firmly pursue national brands and pay more attention to the brand goodwill and reputation, while others prefer store brands and more emphasis on the product itself and differentiation. Therefore, taking into account the differences in the brand preferences of consumers and following the relevant assumptions of demand
function [8], we assume that the introduction of SB merely subdivides the market and diverts part of the basic market demand of NB. Simultaneously, the price, quality and big-data marketing service level of one brand not affect the demand of another brand under the specific brand preference. Assume that \( \chi \) proportion of consumers choose to buy NB product, and \( 1 - \chi \) proportion of consumers choose to buy SB product, the demand of NB product and SB product are:

\[
D_N(t) = \chi [a - \lambda_N p_N(t) + \gamma_N b_N(t) + \beta_N G_N(t)] \\
D_S(t) = (1 - \chi) [a - \lambda_S p_S(t) + \gamma_S b_S(t) + \beta_S G_S(t)]
\]

(4)

where, \( \chi \in [0, 1] \) not only represents the proportion of consumers purchasing NB product, but also reflects the preference for NB. \( \lambda_S \in [0, 1], \gamma_S > 0, \beta_S > 0 \) are the influence coefficient of sales price, big-data marketing service and goodwill of SB on the demand of SB product, respectively. It is worth noting that the price sensitivity of consumers to SB product is usually higher than that of NB product, thus we let \( 0 \leq \lambda_N < \lambda_S \leq 1 \). This setting is prevalent in the literature of operations and marketing [10, 76] and is in line with the actual situation.

In addition to the relevant assumptions of the above two cases, we assume that the manufacturer and the platform pursue their own profit maximization in an infinite planning period, and they have a same discount rate \( \rho \). In order to describe the law of diminishing marginal cost of quality investment and big-data marketing service investment [14], we assume that the quality cost functions of M’s NB product and P’s SB product are \( C_{q_N}(t) = \frac{1}{2} q_N(t)^2 \) and \( C_{q_S}(t) = \frac{1}{2} q_S(t)^2 \) respectively; the big-data marketing service cost functions of the platform for NB product and SB product are \( C_{b_N}(t) = \frac{1}{2} b_N(t)^2 \) and \( C_{b_S}(t) = \frac{1}{2} b_S(t)^2 \) respectively.

The notation used in this paper is summarized in Table 3. Throughout the analysis, we use subscripts M and P to represent the manufacturer and the platform in PSC respectively. And we use superscript R and S to represent the business mode adopted by the manufacturer respectively, and further add S and N to the superscript to indicate whether the platform introduces or does not introduce SB. Thus, we omit the implication of the notation when this information is clearly visible from the context.

3.1. Reselling

Under reselling, the manufacturer produces the NB product and sells it to the platform at the wholesale price, which does not determine the sales price of NB product. This business mode is extremely common in the study of SB, such as Li et al. [40], Cheng et al. [16], Shi and Geng [65], Zhang et al. [77] and so on. If the platform does not introduce SB, purely sells the manufacturer’s NB product, he makes the most of big-data marketing service for the NB product to locate groups and stimulate the demand of consumers, finally sells them to consumers at the sales price. If SB is introduced, besides the above tasks, the platform also requires to properly position SB product and determines the corresponding sales price, quality and big-data marketing service level.

The game sequence under reselling is: (i) the manufacturer announces the wholesale price \( w(t) \) and the quality \( q_N(t) \) of its NB product; (ii) if the platform decides not to introduce SB, it determines the sales price \( p_N(t) \) and big-data marketing service level \( b_N(t) \) of NB product; if the platform decides to introduce SB, it decides the sales prices \( p_N(t) \) and \( p_S(t) \) of the NB and SB, big-data marketing service levels \( b_N(t) \) and \( b_S(t) \) of the NB and SB, and the quality \( q_S(t) \) of SB, respectively; (iii) the customer decides whether to buy and which brand of product to buy, either NB or SB.

Under mode NR, the target functions of the manufacturer and the platform are:

\[
J_{M}^{NR} = \int_{0}^{+\infty} e^{-\rho t} \left[ w(t) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_{P}^{NR} = \int_{0}^{+\infty} e^{-\rho t} \left[ (p_N(t) - w(t)) D_N(t) - \frac{1}{2} b_N(t)^2 \right] dt.
\]

(5)

Under mode SR, the target functions of the manufacturer and the platform are:

\[
J_{M}^{SR} = \int_{0}^{+\infty} e^{-\rho t} \left[ w(t) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_{P}^{SR} = \int_{0}^{+\infty} e^{-\rho t} \left[ (p_N(t) - w(t)) D_N(t) + p_S(t) D_S(t) - \frac{1}{2} q_S(t)^2 - \frac{1}{2} b_N(t)^2 - \frac{1}{2} b_S(t)^2 \right] dt.
\]

(6)
3.2. Agency selling

Under agency selling, unlike reselling, the power to determine the sales price of NB product is controlled by
the manufacturer, and the platform is entirely the link between the manufacturer and consumers, charging a
commission based on the sales of NB product [47, 64]. With regard to the commission, the platform usually
releases its own detailed commission rules in advance. This means that the platform divides the products
into different categories, charging the same commission rate for the identical category, and charging different
commission rates for different categories [31, 48]. Consequently, in this business mode, the manufacturer produces
NB product and sells it with sales price to customers through the platform. And she needs to pay a certain
percentage of commission to the platform after sales, at a given commission rate $\theta$, $\theta \in (0, 1)$. The assumption
that the platform adopts a given commission rate is consistent with the literature [42, 69, 75]. If SB is not
introduced, the platform only uses big-data marketing service for NB product to promote the demand of
consumers. If SB is introduced, he determines the corresponding sales price, quality and big-data marketing
service level of SB product.

The game sequence under agency selling is: (i) the manufacturer announces the sales price $p_N(t)$ and the
quality $q_N(t)$ of its NB product; (ii) if the platform decides not to introduce SB, it determines the big-data
marketing service level $b_N(t)$ of NB product; if the platform decides to introduce SB, it decides the big-data
marketing service levels $b_N(t)$ and $b_S(t)$ of NB and SB, the sales prices $p_S(t)$ and quality $q_S(t)$ of SB, respectively;
(iii) the customer chooses to purchase NB product or SB product.

Under mode NA, the target functions of the manufacturer and the platform are:

$$J_M^{NA} = \int_0^{+\infty} e^{-\rho t} [(1-\theta) p_N(t)D_N(t) - \frac{1}{2} q_N(t)^2] \, dt$$
$$J_P^{NA} = \int_0^{+\infty} e^{-\rho t} [\theta p_N(t)D_N(t) - \frac{1}{2} b_N(t)^2] \, dt. \tag{7}$$

Under mode SA, the target functions of the manufacturer and the platform are:

$$J_M^{SA} = \int_0^{+\infty} e^{-\rho t} [(1-\theta) p_N(t)D_N(t) - \frac{1}{2} q_N(t)^2] \, dt$$
$$J_P^{SA} = \int_0^{+\infty} e^{-\rho t} [\theta p_N(t)D_N(t) + p_S(t)D_S(t) - \frac{1}{2} q_S(t)^2 - \frac{1}{2} b_N(t)^2 - \frac{1}{2} b_S(t)^2] \, dt. \tag{8}$$

Note that the manufacturer producing NB product, as well as the platform introducing SB product, will
incur various costs, such as research and development, supplier selection of raw materials and so on [36, 58, 66].
Without losing generality and ease of analysis, we assume that the costs of NB and SB are fixed constants, and
set them to zero following the literature [6, 40, 77]. In addition, all information is overt and transparent in our
research, that is, the decision-making interaction between M and P is based on information symmetry.

4. Analysis

This section analyzes the optimal operation strategies (including pricing strategy, quality strategy and big-
data marketing service strategy) and corresponding profits of the manufacturer and the platform under different
modes in Table 1, which are mode NR, mode NA, mode SR and mode SA respectively. Moreover, the impact
of different key parameters on corporate goodwill and operation strategies are discussed, and the relationships
of the steady state goodwill and corresponding strategies under mode combination (NR, NA), (NR, SR), (NA,
SA) and (SR, SA) are compared. For clarity, all proofs are relegated to Appendix A.

4.1. SB Not introduced

We first discuss the optimal strategies and profits under the reselling and the agency selling, respectively,
with the circumstance that the platform purely sells NB product.
4.1.1. Mode NR

In this mode, in accordance with the target functions of M and P in 3.1.1 and the constraint of dynamic evolution of NB’s goodwill in equation (1), the differential game model between M and P of the platform-based supply chain can be expressed as:

\[
\begin{align*}
\max_{w(\cdot), q_N(\cdot)} J_M^{NR}(t) &= \int_0^{\infty} e^{-\rho t} \left[ w(t) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
\text{s.t.} & \quad \max_{p_N(\cdot), b_N(\cdot)} J_P^{NR}(t) = \int_0^{\infty} e^{-\rho t} \left[ (p_N(t) - w(t)) D_N(t) - \frac{1}{2} b_N(t)^2 \right] dt \\
G_N(t) &= \alpha_N q_N(t) - \delta G_N(t), G_N(0) = G_{N0}
\end{align*}
\]

By solving the differential game model, we obtain Proposition 4.1, which characterizes the equilibrium strategies and corresponding profits of M and P and the evolution path of NB’s goodwill under mode NR.

**Proposition 4.1.** Under mode NR, the equilibrium outcomes are: \( w^{NR}(t) = \frac{\bar{\alpha}_N}{2 \bar{\alpha}_N} G^{NR}_N(t) + \frac{a}{2 \bar{\alpha}_N}, q^{NR}_N(t) = 2 \alpha_N f_1 G^{NR}_N(t) + \alpha_N f_2, p^{NR}_N(t) = \frac{\bar{\alpha}_N}{2 \lambda_N (2 \lambda_N - \gamma_N)} G^{NR}_N(t) + \frac{a(3 \alpha_N - \gamma_N)}{2 \lambda_N (2 \lambda_N - \gamma_N)} G^{NR}_N(t) + \frac{\bar{\alpha}_N}{2 \lambda_N (2 \lambda_N - \gamma_N)} G^{NR}_N(t) + \frac{\gamma_N \beta_{NR}}{2 \lambda_N (2 \lambda_N - \gamma_N)} G^{NR}_N(t) + \frac{\gamma_N^2}{2 \lambda_N (2 \lambda_N - \gamma_N)}, \) \( G^{NR}_N(t) = \left( G_{N0} - \frac{a \bar{\alpha}_N^2}{\delta - 2 \alpha_N f_1} \right) e^{-\left( \delta - 2 \alpha_N f_1 \right) t} + \frac{a \bar{\alpha}_N^2}{\delta - 2 \alpha_N f_1} \), \( V_M^{NR}(t) = f_1 G^{NR}_N(t)^2 + f_2 G^{NR}_N(t) + \frac{1}{\rho} \left( \frac{a}{\delta - 2 \alpha_N f_1} \right)^2, \) \( V_P^{NR}(t) = g_1 G^{NR}_N(t)^2 + g_2 G^{NR}_N(t) + \frac{1}{\rho} \left( \frac{a}{\delta - 2 \alpha_N f_1} \right)^2, \) where, \( f_1 = \frac{(\rho + 2 \delta - 4 a \bar{\alpha}_N)(\rho + 2 \delta - 2 M)}{4 \alpha_N^2}, f_2 = \frac{a \bar{\alpha}_N}{2 \lambda_N (2 \lambda_N - \gamma_N)} G^{NR}_N(t) + \frac{\alpha \bar{\alpha}_N}{2 \lambda_N (2 \lambda_N - \gamma_N)}, \) \( g_1 = \frac{1}{8 (2 \lambda_N - \gamma_N)} (\rho + 2 \delta - 4 a \bar{\alpha}_N f_1), g_2 = \frac{a \bar{\alpha}_N}{4 (\rho + 2 \delta - 4 a \bar{\alpha}_N f_1)} \).
Table 4. The impact of key parameters on goodwill and strategies.

<table>
<thead>
<tr>
<th></th>
<th>$G_N^{NR}(t)$</th>
<th>$w_N^{NR}(t)$</th>
<th>$p_N^{NR}(t)$</th>
<th>$q_N^{NR}(t)$</th>
<th>$b_N^{NR}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
</tr>
<tr>
<td>$\beta_N$</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
<td>↗</td>
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</tr>
<tr>
<td>$\gamma_N$</td>
<td>↗</td>
<td>↗</td>
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<td>↗</td>
</tr>
<tr>
<td>$\lambda_N$</td>
<td>↘</td>
<td>↘</td>
<td>↘</td>
<td>↘</td>
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</tr>
</tbody>
</table>

Notes. ↗ represents positive correlation; ↘ represents negative correlation.

Proposition 4.1 reveals that the product quality, big-data marketing service, price and the profit of enterprises are positively related to the goodwill of NB product, which implies that goodwill has increasingly become one of the key factors affecting the decision-making and profitability of enterprises.

To be specific, firstly, product quality is positively correlated with goodwill. This demonstrates that consumers cognition that the brand with good goodwill must have the product with high quality urges the manufacturer to increase investment in quality improvement and strictly grasp quality. In this way, the manufacturer can meet consumers psychological expectations for quality, maintain or further improve goodwill in their mind.

Secondly, with regard to the big-data marketing service of the platform, he is more willing to increase big-data marketing investment for products with great reputation. The platform obtains the shopping psychology and potential demand of consumers through big data technology, and stimulates demand through accurate big data marketing services for the purpose of increasing revenue.

Finally, from the perspective of pricing, due to consumers usually have a higher acceptance of brands with high positioning and good reputation, which means that they have a higher willingness to pay for products with large brands and are willing to pay for brand premium. As a result, enterprises with great goodwill tend to raise prices for high returns. On the other hand, enterprises need to bear costs for quality improvement and big-data marketing service. The higher goodwill, the higher quality and service level required by consumers, and the higher corresponding cost, which also leads to the increase of prices.

Furthermore, according to Proposition 4.1, we can deduce the impact of different key parameters on goodwill and operation strategies under mode NR, as shown in Table 4.

From Table 4, it is clear that the goodwill, price (including wholesale price and sales price), quality and big-data marketing service level are all improved with the large market base scale, the high promotion of quality to goodwill, the positive response to big-data marketing service and goodwill, and the low sensitivity to price. Concretely, a higher market size means the larger basic demand and potential demand of the product, as well as the possibility that the potential demand can be transformed into actual demand and the possibility of enterprise profit is relatively high. Meanwhile, the positive response of consumers to quality and goodwill can motivate the manufacturer to improve product quality, drive good reputation and great goodwill with high quality, thereby stimulating demand. Additionally, the positive response of consumers to big-data marketing also motivate the platform to provide personalized and customized big-data marketing service, thus tapping into potential demand and eventually increasing sales. However, the above behaviors of consumers not only promote enterprises to improve quality and big-data marketing service level, but also aggravate the double marginal effect to a certain extent, that is, increase the wholesale price and sales price. Here, since consumers are sensitive to product price, that is, both higher product price and higher price sensitivity reduce their enthusiasm and possibility to consume, it leads to negative decision-making.
4.1.2. Mode NA

According to the target functions in 3.2.1 and the constraint of equation (1), the differential game model in mode NA can be expressed as:

\[
\begin{align*}
\max_{p_N(t), q_N(t)} & \quad J_N^N(t) = \int_0^{+\infty} e^{-\rho t} \left[ (1 - \theta) p_N(t) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
\text{s.t.} & \quad \max_{b_N(t)} J_P^N(t) = \int_0^{+\infty} e^{-\rho t} \left[ \theta p_N(t) D_N(t) - \frac{1}{2} b_N(t)^2 \right] dt \\
& \quad \tilde{G}(t) = \alpha_N q_N(t) - \delta G_N(t), G_N(0) = G_{N0}.
\end{align*}
\]

In this mode, we gain the equilibrium results in Proposition 4.2.

**Proposition 4.2.** Under mode NA, the equilibrium outcomes are:

\[
p_N^N(t) = \frac{\beta_N}{2(\lambda_N - \theta \gamma_N^2)} G_N^N(t) + \frac{a}{2(\lambda_N - \theta \gamma_N^2)}, \quad q_N^N(t) = 2\alpha_N m_1 G_N^N(t) + \alpha_m m_2, \quad b_N^N(t) = \frac{\theta \gamma_N \beta_N G_N^N(t)}{2(\lambda_N - \theta \gamma_N^2)} + \frac{\theta \gamma_N a}{2(\lambda_N - \theta \gamma_N^2)},
\]

\[
V_P^N(t) = n_1 G_N^N(t)^2 + n_2 G_N^N(t) + \frac{1}{\rho} \left( \frac{\theta (2\lambda_N - 3\gamma_N^2 N)^2}{8(\gamma_N \theta \gamma_N^2)} + \alpha_N^2 m_2 n_2 \right)
\]

where, \(m_1 = (\rho+2\delta) - \sqrt{(\rho+2\delta)^2 - 2(1-\theta)K}\), \(K = \frac{4a\beta_N}{\lambda_N - \theta \gamma_N^2}\), \(m_2 = \frac{a\beta_N (1-\theta)}{(\lambda_N - \theta \gamma_N^2)(\rho+\delta - 2\alpha_N^2)}\), \(n_1 = \frac{\theta (2\lambda_N - 3\gamma_N^2 N)^2}{8(\gamma_N \theta \gamma_N^2)} + \frac{\alpha_N^2 m_2}{(\rho+\delta - 2\alpha_N^2)}\), \(n_2 = \frac{\theta (2\lambda_N - 3\gamma_N^2 N)^2 \alpha_N}{4(\lambda_N - \theta \gamma_N^2)^2 (\rho+\delta - 2\alpha_N^2)}\).

It can be seen from Proposition 4.2 that the commission rate \(\theta\) charged by the platform to the manufacturer positively influences the M’s sales price strategy and the P’s big-data marketing service strategy, and negatively influences the quality strategy and the goodwill of NB. In consequence, an increase in \(\theta\) will impel the platform to elevate big-data marketing service level and pressure the manufacturer to increase sales price, however, and simultaneously abate the manufacturer’s enthusiasm to improve product quality, which is not conducive to NB’s goodwill.

Indeed, for the manufacturer, who have the power to set the sales price under agency selling, she cannot help but cost-saving and profit-increasing in the face of the high commission. Here, cost-saving implies to curtail the investment of quality improvement, although this damage the brand goodwill, and profit-increasing means to increase the sales price and obtain high income as much as possible. As for the platform, high commission rate signifies that he would get more revenue from the unit sales of NB product. Furthermore, the profit will be more supposing the sales volume is considerable, this is more evident than ever. Thus, the platform has enormous intention and motivation to boost the service level of big data marketing with the stimulation of high commission rate.

In view of the fact that enterprises operate continuously in unlimited time, the goodwill and corresponding strategies will reach a steady-state level when time tends to infinity. To ensure the existence of steady state, we always consider \(\delta - 2\alpha_N^2 f_1 > 0\) and \(\delta - 2\alpha_N^2 m_1 > 0\). Let \(t \to \infty\), Table 5 displays the steady-state goodwill of NB and the corresponding steady-state optimal strategies under the reselling and the agency selling, which with the circumstance that SB is not introduced by platform.

In the light of Table 5, we gain the relationships among the optimal steady-state results of mode NR and mode NA, namely Proposition 4.3.

**Proposition 4.3.** (NR vs. NA) When SB is not introduced, we have:

1. \(p_{N\infty}^{NA} \leq p_{N\infty}^{NR} \) if \(0 \leq \theta \leq \theta_0\) and \(p_{N\infty}^{NR} < p_{N\infty}^{NA} \) if \(\theta_0 < \theta \leq 1\), where \(\theta_0 = \frac{2\delta (\rho+\delta) \lambda_N - (2\lambda_N - \gamma_N^2) \alpha_N^2 \beta_N^2}{(3\lambda_N - \gamma_N^2)(2\delta (\rho+\delta) \gamma_N^2 - \alpha_N^2 \beta_N^2)}\).
2. \(G_{N\infty}^{NR} < G_{N\infty}^{NA} \), \(q_{N\infty}^{NR} < q_{N\infty}^{NA} \) and \(b_{N\infty}^{NA} < b_{N\infty}^{NR} \) if \(0 \leq \theta \leq \frac{1}{2}\), \(G_{N\infty}^{NA} < G_{N\infty}^{NR} \), \(q_{N\infty}^{NA} < q_{N\infty}^{NR} \) and \(b_{N\infty}^{NA} < b_{N\infty}^{NR} \) if \(\frac{1}{2} < \theta \leq 1\).
4.2. SB Introduced

We analyze the optimal strategies and profits under two business modes with the condition that the platform sells not only NB product, but also SB product in this section.

4.2.1. Mode SR

In mode SR, the differential game model which as per the target functions of M and P in 3.1.2 and the constraint of goodwill dynamic evolutions in equations (1) and (3) are:

Proposition 4.3 reflects that when the commission rate is low (high), the manufacturer and the platform provide consumers with NB product, which possesses more expensive sales price and preferable big-data marketing service under mode NR (NA), and with the superior quality and worthier goodwill under mode NA (NR).

As we all know, the product structure of the market is single if SB is not introduced, which indicates that NB product are circulating purely in this supply chain, and consumers only have the choice of whether to purchase it. Additionally, under agency selling, the manufacturer grasps the right of final sales pricing of NB product and pays a certain percentage fee to the platform, while the profit of the platform originates entirely from the commission charged from the manufacturer.

Based on the above, when the commission rate is low, for the platform, a lower commission rate diminishes his revenue from unit NB product under agency selling. On the other hand, he cannot but bear the cost of big-data marketing service, which mainly refers to the cost incurred by circulating and updating consumption data, ensuring data security and privacy, etc. Therefore, he is inevitable to reduce the level of the big-data marketing service for NB product to maintain profitability with a low commission rate. As for the manufacturer, the low commission rate has limited effect on her profit. This implies that she can easily drive the growth of demand at a cheap price, promote the goodwill of NB with high quality, and finally bring NB product with high quality-price ratio for consumers.

Conversely, when the commission rate is high, the platform is motivated to increase the investment in data-marketing service to further improve the sales of NB product and obtain the high commission, i.e., the high profit. Looking at the manufacturer, her profit would be decreased greatly with a high commission rate. This is obvious. And at this moment, the manufacturer is bound to set an expensive sales price to augment her income as much as possible. Concurrently, she is awfully negative about enhancing product quality and unable to upgrade the goodwill through quality advantage due to the high commission. Consequently, compared with agency selling, reselling adopted by the manufacturer and the retailer can enable consumers to obtain NB product with high quality-price ratio.
Proposition 4.4. Under mode SR, the equilibrium outcomes are:

\[
w^S_R(t) = \frac{\gamma a \alpha x_4}{(\alpha + \gamma x_4)} G_N^S(t) + \alpha x_4, \quad \nu^S_R(t) = \frac{\beta N}{2(\alpha + \gamma x_4)} G_N^S(t) + \frac{\alpha (\alpha + \gamma x_4)}{2(\alpha + \gamma x_4)} b^S_R(t) = \frac{\alpha x_4}{2(\alpha + \gamma x_4)} G_N^S(t) + \frac{\beta a}{2(\alpha + \gamma x_4)} G_N^S(t) + \frac{\alpha x_4}{2(\alpha + \gamma x_4)} b^S_R(t) = \frac{\alpha x_4}{2(\alpha + \gamma x_4)} G_N^S(t), \quad \psi^S_R(t) = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} G_N^S(t) + \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} G_N^S(t) = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} G_N^S(t), \quad \chi^S_R(t) = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} G_N^S(t) + \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} G_N^S(t) \text{ and } \gamma y_1 G_N^S(t)^2 + \gamma y_2 G_N^S(t)^2 + \psi y_4 G_N^S(t)^2 + \frac{1}{\rho} \left( \frac{\gamma x_4^2}{8(\alpha + \gamma x_4)^2} + \alpha x_3 y_3 + \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)} \right),
\]

where,

\[
x_1 = \frac{(p^2+\delta)-\sqrt{(p^2+\delta)^2-4\alpha N}}{4\alpha N}, \quad N = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)}, \quad x_3 = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)}, \quad \gamma y_1 = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)}, \quad \gamma y_2 = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)}, \quad \gamma y_4 = \frac{(1-\gamma)\alpha x_3}{2(\alpha + \gamma x_4)}.
\]

On the basis of Proposition 4.4, we find that, as the proportion of purchasing NB product \( \chi \) increases, as well as the proportion of purchasing SB product \( 1 - \chi \) decreases, the goodwill, price, quality and big-data marketing service level of NB product increase accordingly, while those of SB product decrease.

This is a realistic result and is consistent with intuition. In particular, a large proportion of buying NB product indicates that consumers prefer NB rather than SB, that is, the SB will not pose a great threat to NB. What is more, the more consumers buy NB product, the less attention the manufacturer pays to SB. Hence, when NB is not highly recognized, the manufacturer is supposed to focus on improving the quality of NB product to create the dual advantages of quality and goodwill, furtherly, upgrade NB’s position in the eyes of consumers. And she can also interact with the platform more initiative and elevate her bargaining power, that is, increase the wholesale price. Simultaneously, in order to obtain high profit, the platform would turn his attention to NB product with large demand and focused on providing big-data marketing service to consumers. As for SB product, due to low sales volume, the platform is insufficient to improve the quality and big-data marketing service, and keeps a low sales price with it.

However, when the proportion of buying NB product is relatively small, the manufacturer has no choice but to reduce the wholesale price and attract the platform to pay more attention to NB product. On other side, she would adventure the risk of damaging goodwill to cut the investment of quality improvement in order to diminish the cost pressure. For the platform, he will naturally cut the input in big-data marketing service of NB product and increase that of SB product supposing most consumers are inclined to buy SB product. Furthermore, the platform is confident to improve the quality level of SB product with a large consumer base of SB, and then create good reputation and high goodwill. At the same time, it is also conducive to improving consumers willingness to pay for SB product.
4.2.2. Model SA

The differential game model of mode SA In the light of the target functions in 3.2.2 and the constraint of equations (1) and (3) are:

\[
\begin{align*}
\max_{p(t),q(t)} J_M^{SA}(t) &= \int_0^{+\infty} e^{-\rho t} \left[ (1-\theta) p(t) D_{N}(t) - \frac{1}{2} q(t)^2 \right] dt \\
\text{s.t.} & \left\{ \begin{array}{l}
\max_{p(t),q(t)} J_P^{S}(t) = \int_0^{+\infty} e^{-\rho t} \left[ \theta p(t) D_{N}(t) + p(t) D_{S}(t) + \frac{1}{2} b(t)^2 \right] dt \\
G_{N}(t) = \alpha_N q(t) - \delta G_{N}(t), G_N(0) = G_{N0} \\
G_{S}(t) = \alpha_S q(t) - \delta G_{S}(t), G_S(0) = G_{S0}.
\end{array} \right.
\end{align*}
\]

Proposition 4.5. Under mode SA, the equilibrium outcomes are: \( p_N^{SA}(t) = \frac{\delta_{\beta}}{2(\lambda_N-\delta-\gamma_N^2)} G_N^{SA}(t) + \frac{\delta_{\beta}}{2(\lambda_N-\delta-\gamma_N^2)}, \)
\( q_N^{SA}(t) = 2\alpha_N k_1 G_N^{SA}(t) + \alpha_N k_3, \)
\( b_S^{SA}(t) = \frac{\gamma_N^2 \alpha}{2(\lambda_N-\delta-\gamma_N^2)} G_N^{SA}(t) + \frac{\gamma_N^2 \alpha}{2(\lambda_N-\delta-\gamma_N^2)}, \)
\( \frac{\gamma_N^2 \alpha}{2(\lambda_N-\delta-\gamma_N^2)} G_N^{SA}(t) + \frac{\gamma_N^2 \alpha}{2(\lambda_N-\delta-\gamma_N^2)}. \)

The equilibrium strategies and corresponding profits of mode SA are shown in Proposition 4.5.

Proposition 4.6. (SR vs. SA) When SB is introduced, we have:

1. \( G_{N\infty}^{SR} \leq G_{N\infty}^{SA}, \)
2. \( \theta_N^{SR} \leq \theta_N^{SA} \)
3. \( 0 < \theta_N^{SR} \leq \theta_N^{SA} \)

From the first and second results in Proposition 4.6, we can discover that when the commission rate is low (high), the enterprises provide NB product with higher sales price and preferable big-data marketing service under mode SR (SA), and with the superior quality and worthier goodwill under mode SA (SR), which is similar to Proposition 4.3. Thus, combined with Proposition 4.3, we find that: under specific SB strategy (introduced or not) of the platform, when the commission rate is small, agency selling adopted by the manufacturer is more conducive to consumers to obtain NB product with high quality and great goodwill at a low price. On the contrary, when the commission rate is large, reselling is more beneficial for consumers to buy NB product with high-quality-price ratio.

The third result in Proposition 4.6 portrays the relationship of pricing, quality strategy and big-data marketing service strategy of SB product between reselling and agency selling when SB is introduced. It is interesting that the relevant decision-making level of the platform for SB product under the two business modes remains the same. In other words, no matter whether the business mode of NB product is reselling or agency selling, the decision of the platform on SB product is not affected by the business modes of NB product.
Table 6. The steady-state results of NB under mode SR and mode SA.

<table>
<thead>
<tr>
<th></th>
<th>Mode SR</th>
<th>Mode SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_{N\infty})</td>
<td>(\frac{25(\rho+\delta)(2\lambda_N-\chi_S^2)}{\alpha_S \delta(\rho+\delta)})</td>
<td>(\frac{25(\rho+\delta)(1-\chi_S^2)}{\alpha_S \delta(\rho+\delta)})</td>
</tr>
<tr>
<td>(w_{\infty})</td>
<td>(\frac{\lambda_N(2\lambda_N-\gamma_S^2)}{\alpha_S^2(\rho+\delta)})</td>
<td>(-)</td>
</tr>
<tr>
<td>(p_{N\infty})</td>
<td>(\frac{\lambda_N(2\lambda_N-\gamma_S^2)}{\alpha_S^2(\rho+\delta)})</td>
<td>(\frac{a\delta(\rho+\delta)}{\alpha_S \delta(\rho+\delta)})</td>
</tr>
<tr>
<td>(q_{N\infty})</td>
<td>(\frac{\lambda_N(2\lambda_N-\gamma_S^2)}{\alpha_S^2(\rho+\delta)})</td>
<td>(\frac{a\delta(\rho+\delta)}{\alpha_S \delta(\rho+\delta)})</td>
</tr>
<tr>
<td>(b_{N\infty})</td>
<td>(\frac{\lambda_N(2\lambda_N-\gamma_S^2)}{\alpha_S^2(\rho+\delta)})</td>
<td>(\frac{a\delta(\rho+\delta)}{\alpha_S \delta(\rho+\delta)})</td>
</tr>
</tbody>
</table>

Table 7. The steady-state results of SB under mode SR and mode SA.

<table>
<thead>
<tr>
<th></th>
<th>Mode SR</th>
<th>Mode SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_{S\infty})</td>
<td>(\frac{(1-\chi)\alpha_S^2 \beta_S}{(1-\chi)\alpha_S^2 \beta_S})</td>
<td>(\frac{(1-\chi)\alpha_S^2 \beta_S}{(1-\chi)\alpha_S^2 \beta_S})</td>
</tr>
<tr>
<td>(p_{S\infty})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
</tr>
<tr>
<td>(q_{S\infty})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
</tr>
<tr>
<td>(b_{S\infty})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
<td>(\frac{\delta(\rho+\delta)(2\lambda_S-\gamma_S^2)}{\alpha_S^2 \delta(\rho+\delta)})</td>
</tr>
</tbody>
</table>

Proposition 4.7. (NR vs. SR and NA vs. SA) In reselling, we have \(w_{N\infty}^{SR} < w_{N\infty}^{NR}\), \(q_{N\infty}^{SR} < q_{N\infty}^{NR}\), \(G_{N\infty}^{SR} < G_{N\infty}^{NR}\), \(p_{N\infty}^{SR} < p_{N\infty}^{NR}\), \(b_{N\infty}^{SR} < b_{N\infty}^{NR}\); in agency selling, we have \(p_{N\infty}^{NA} < p_{N\infty}^{NA}\), \(G_{N\infty}^{NA} < G_{N\infty}^{NA}\), \(q_{N\infty}^{NA} < q_{N\infty}^{NA}\), \(b_{N\infty}^{NA} < b_{N\infty}^{NA}\).

Proposition 4.7 demonstrates that in reselling, the price, quality, big-data marketing and goodwill of NB product when SB is introduced are lower than those without SB. Similarly, the conclusion also lends itself to agency selling. This means that Given the business mode (reselling or agency selling), the introduction of store brand of the platform can mitigate the double marginal effect, i.e., reduce the wholesale price and sales price of NB product. However, at the same time, it abates the enthusiasm of enterprises to improve product quality and strengthen big-data marketing due to low price.

5. The sales mode decision

With the profits in Proposition 4.1, 4.2, 4.4 and 4.5, the optimal sales mode decision can be derived. However, it is troublesome to calculate the explicit form of the region due to the complex calculation, we illustrate the results through numerical examples in a bid to facilitate intuitive analysis. Note that we compare the steady-state profit under the four modes, since enterprises are able to operate stably when time tends to infinity. Without losing generality, in combination with the relevant settings in this paper, we set the basic parameters of the system as follows: \(a = 60, \rho = 0.1, \delta = 0.8, G_{N0} = 0.0, G_{S0} = 0.0, \alpha_N = 0.7, \beta_N = 0.4, \gamma_N = 0.4, \lambda_N = 0.3, \alpha_S = 0.7, \beta_S = 0.3, \lambda_S = 0.4\). For the sake of providing some references and suggestions for enterprises to operate preferably, this section discusses the business mode decision and the store brand decision under the specific scenario, as well as the equilibrium sales mode, which affected by the commission rate (\(\theta\)) and the preference for NB (\(\chi\)).

5.1. The business mode decision

We discuss the manufacturer’s choice of business mode from two cases: whether store brand is introduced (SR vs. SA) or not (NR vs. NA). Figure 2 depicts the impact of and on the profit of M, P and the whole PSC.
THE STRATEGIC INTERACTION BETWEEN BUSINESS MODE AND STORE BRAND INTRODUCTION

5.1.1. The business mode decision without SB (NR vs. NA)

Since the platform does not introduce SB, consumers merely have the option to decide whether to purchase NB product. In this case, the business mode decisions of the manufacturer, platform and the whole platform-based supply chain are only affected by the commission rate (as shown in Fig. 2).

Especially, firstly, for the manufacturer, when the commission rate is less than half (Region I in Fig. 2a), she would like to adopt agency selling. This is because that the manufacturer has higher initiative under agency selling without SB, and Figure 3 shows the corresponding regions that benefit the three parties under agency selling.

Similarly, Figure 4 respectively demonstrates the impact of these two factors with SB, and Figure 5 shows the corresponding regions of the three parties benefited by agency selling.
selling by reason of mastering the final pricing right of NB product. Naturally, a lower commission rate entails that she just pays fewer commission to the platform. That is, she can finally obtain higher income after sales. Conversely, when the commission rate is greater than half (region II in Fig. 2a), she would like to adopt reselling. This is because although the manufacturer can control the price under the agency selling, the high commission makes her to pay most of her profit to the platform as intermediary fee, which is an arduous but fruitless task, or even a thankless task for the manufacturer. Therefore, she might as well adopt reselling to hands over the pricing right of NB product to the platform and charges the wholesale price from him.

Secondly, for the platform, when the commission rate is small (Region I in Fig. 2b), he craves for reselling took by the manufacturer, and when the commission rate is large (Region II in Fig. 2b), he hopes she to adopt agency selling. The reason behind this may be that the platform purely acts as an intermediary for the manufacturer to sell the NB product to consumers, and his income comes entirely from the commission charged from the manufacturer under agency selling. Moreover, the platform exists the cost of big-data marketing service generated to balloon sales volume of the NB product, which will further compress his profit space. Thereby, when the commission rate reaches a certain size, he aspires that the manufacturer would choose agency selling. And yet, when the commission rate is relatively small and fails to reach his expectation, he desires the manufacturer to take reselling rather than agency selling. In this way, he can master more voice and initiative by determining the final sales price of NB product, thus enhancing his profitability.

What’s more, for the whole platform-based supply chain, when the commission rate is sufficient large (region II in Fig. 2c), reselling adopted by the manufacturer is more conducive to the overall benefit of the platform-based supply chain. When the commission rate is not particularly high (Region I in Fig. 2c), agency selling is more conducive to the overall benefit and is the more ideal business mode of this supply chain.

In the light of Figure 2, we obtain the profitability of the manufacturer, the platform and the whole platform-based supply chain under agency selling contrasted with reselling, as shown in Figure 3. We observe that if reselling is adopted by the manufacturer when SB is not introduced, and there is no win-win-win area for the three parties, which is shown as the region of lose-lose-lose in Figure 3. Whereas, they can achieve win-win-win situation under agency selling when the commission rate is moderate (Region II in Fig. 3). This signifies that by conferring the commission rate in line with the expectations of the manufacturer and the platform, agency selling can not only satisfy them, i.e., obtain higher profits, but also help the entire supply chain become bigger and stronger, so as to enhance the operating environment and the overall competitiveness of this supply chain. This phenomenon is consistent with reality. That is, most manufacturers regard platform as a link between them and consumers, and adopt agency selling with the platform. As an illumination, part of manufacturers in platforms such as Taobao and JD.com sell their products on agency selling in the form of official flagship stores, and pay commissions to the platform pursuant to the sales.

In addition, contrasted with reselling, although the agency selling reduce the profit of one member of the supply chain, it is beneficial to the whole supply chain to get higher profit when the commission rate is small or relatively large. Specifically, in the face of a small commission rate (Region in Fig. 3), it is more beneficial for the manufacturer to adopt agency selling, whereas the profit of the platform under this business mode is lower than that of reselling. On the contrary, faced with a relatively large commission rate (Region in Fig. 3), if the manufacturer take agency selling, it will be more beneficial to the platform, but not to herself. Obviously, since the specific business mode is determined by the manufacturer, this leads to the fact that although choosing agency selling in the latter case is more beneficial from the overall point of view, she inevitably not chooses it with high commission rate.

Beyond that, when the commission rate is awfully large (Region in Fig. 3), only the platform can obtain more benefit from agency selling. In contrast, in the face of the high commission, although the benefit of the platform will be harmed, reselling is not only beneficial to the manufacturer, but also helps the supply chain to get high return. Undoubtedly, reselling is the natural choice of the manufacturer in this situation.
5.1.2. The business mode decision with SB (SR vs. SA)

In the case of introducing SB into the platform, like in absence of SB in Figure 2a, the manufacturer’s business mode decision is also only affected by the commission rate (illustrated in Fig. 4a). Thus, combining these two Figures, we can discover that no matter whether the platform introduces SB, the business mode adopted by the manufacturer only depends on the commission rate. In other words, when the commission rate is low, she adopts agency selling; otherwise, she adopts reselling.

As for the platform and the whole supply chain in this case, unlike without SB, their profit depends on both the commission rate and the preference for NB (illustrated in Figs. 4b and c). Specifically, for the platform, when the commission rate is low, especially lower than his expectation (Region in Fig. 4b), regardless of the consumer’s preference for the two brands, considering the low income caused by the low commission, he prefers that the manufacturer adopts reselling to control the pricing power of NB product. When the commission rate is high and meets the platform’s expectation (Region in Fig. 4b), he always hopes to adopt agency selling because the income from the high commission is high enough. It is worth noting that when the commission rate is general, that is, not too high or not too low (Region II in Fig. 4b), the preference for the two brands will affect the platform’s preference for the business mode adopted by the manufacturer. That is to say, when consumers prefer SB, due to the low commission rate and small NB sales, the platform collects less commission from the manufacturer under reselling. So, he prefers the manufacturer to take reselling. Conversely, he hopes that the manufacturer will adopt agency selling when consumers prefer NB. The reason behind this is although the commission rate is still not very high, the larger demand for NB allows him to charge more commissions under the agency selling. In addition, for the whole supply chain, see Figure 4c, we find that when the commission rate and the preference for NB products are not particularly large (Region in Fig. 4c), it is more beneficial for the supply chain under agency selling. Otherwise, reselling is more conducive to the positive development of the supply chain as a whole.

Similarly, according to Figure 4, we can get the profitability of the manufacturer, the platform, and the entire platform-based supply chain in agency selling compared with reselling after the introduction of SB, as shown in Figure 5. We find that there is no lose-lose-lose region, but only win-win-win region when the platform introduces SB. The former means that if the manufacturer adopts reselling, this business mode will not maximize the profits of the three parties. The latter means that if the manufacturer adopts agency selling and the commission rate is moderate (region II in Fig. 5), the three parties can achieve win-win-win results under this business mode. That is, each member of the three parties can get higher profits than reselling. This finding is roughly consistent with Figure 3. The difference is that when SB is introduced, as consumers preference for NB increases, the bottom line of commission rate on sales of the NB product gradually decreases, which is compared with the case without SB (region II in Fig. 3). The reasons behind this lie in two aspects. On the one hand, the manufacturer tends to reduce the commission rate paid to the platform under agency selling because the introduction of SB will carve up the market share of NB product and damage her interest. On the other hand, when consumers are not
interested in SB, the platform has to accept a relatively low commission rate in order to maximize his profit in the face of high demand on NB product.

5.2. The store brand decision

Next, we investigate the platform’s choice of whether to introduce SB under two business modes: reselling (NR vs. SR) or agency selling (NA vs. SA). Here, the impact of and on the profit of M, P and PSC under reselling is presented in Figure 6 and that under agency selling is presented in Figure 8. Meanwhile, the corresponding regions of three parties profitability without SB under reselling and agency selling are depicted in Figures 7 and 9, respectively.

5.2.1. The SB decision under reselling (NR vs. SR)

For the platform, he owns the final pricing power of the NB product under reselling. According to Figure 6a, it is evident that the store brand decision of the platform depends only on the brand preference of consumers. Specifically, consumers high preference for NB means that they have low recognition and low demand for SB. At this time, the platform’s initiative to introduce SB will increase his burden and be futile. Therefore, he might as well forgo introducing SB and concentrate on selling the NB product when consumers prefer NB (region II in Fig. 6a). On the contrary, if consumers have a high degree of recognition and acceptance of SB, the platform’s store brand gives play to the purpose that he really wants to achieve, that is, to achieve profit growth (region in Fig. 6a). However, from Figures 6b and c, it can be seen that both the manufacturer and the platform-based supply chain hope that the platform not introduce SB. In other words, the introduction of the platform’s SB always damages the profits of them.
Figure 8. The profit relationship with or without SB under agency selling. (a) The platform. (b) The manufacturer. (c) The whole supply chain.

Integrating the profits of the manufacturer, the platform and the platform-based supply chain, as shown in Figure 7, it is obvious that they can all obtain high returns and achieve a win-win-win situation under reselling when consumers’ preference for NB is high and the platform give up introducing SB.

5.2.2. The SB decision under agency selling (NA vs. SA)

Under agency selling, the platform introduces SB in two situations. One is the general situation, i.e., the commission rate is very low, or both the commission rate and preference for NB are relatively low (Region I in Fig. 8a). The other is a relatively passive situation, i.e., both the commission rate and preference for NB are extremely high (region III in Fig. 8a).

Specifically, the reason behind the former may be that a low commission rate means that the platform can obtain less profit from selling the NB product. A low preference for NB means that consumers are more likely to buy the SB product. In this case, he has a greater probability of obtaining higher income by introducing SB, which is in line with common sense.

For the latter, intuitively, the platform does not introduce SB when consumers have a high preference for NB. However, since he only acts as an intermediary for selling the NB product under agency selling if he does not introduce SB. That is to say, the source of his income completely depends on the commission. Hence, he tends to charge a high commission to the manufacturer for high return. On the contrary, from the manufacturer’s point of view, once the commission rate exceeds the upper limit she can accept, she largely gives up selling her NB product through the platform, which makes the platform unable to profit from selling NB product, so he has to passively introduce SB for sales. For example, it is reported by the website of Amazon global selling that Amazon has implemented a new charging structure since January 2022, in which the sales commission charged for extended warranty, protection plan and service contract remained unchanged, still as high as 96%. In order to enhance its competitiveness and profit from this category, Amazon launched the “New Security” extended warranty service plan for electronic and electrical products in China in 2015, with the help of Assurant, until Amazon withdrew from the Chinese market.

In addition, for the manufacturer, like reselling, she still hopes that the platform only serves as a bridge between herself and consumers to sell the NB product under agency selling, that is, to forgo the introduction of SB (as show in Fig. 8b). However, for the whole platform-based supply chain, different from reselling, although consumers high NB preference is unfavorable to the introduction of SB, the high commission under agency selling makes it difficult for the manufacturer and the platform to achieve cooperation. Therefore, compared with reselling, the introduction of SB into the platform can better maintain and ensure the normal operation of the supply chain when the commission rate is extremely high under agency selling (Region in Fig. 8c).

Based on the profits of the three parties, considering the loss of the manufacturer’s profit due to NB’s market share is cannibalize by SB, although the introduction of SB under agency selling may be beneficial to the overall platform-based supply chain, the win-win-win situation will never be realized when SB exists.
Conversely, according to Figure 9, the manufacturer, platform and the platform-based supply chain can achieve win-win-win situation under agency selling when the commission rate is moderate and the consumers preference for NB is high if SB is not introduced by the platform (as show in Region ). This is because if the commission rate charged by the platform to the manufacturer when selling NB products under agency selling reaches a certain level, he is willing to give up the SB to achieve a win-win situation in the supply chain. And consistent with intuition, as consumers preference for NB increases, the platform correspondingly reduces the bottom line of commission rate, which is the minimum value that make him give up the introduction of SB.

5.3. The final equilibrium result

Taking into account the game sequence of members in the platform-based supply chain, as stated in Section 3, the platform first decides whether to introduce store brand, and the manufacturer determines the business mode according to the platform’s store brand decision. Therefore, the equilibrium sales mode of the manufacturer and platform when the platform first decides the store brand decision is illustrated in Figure 10.

Through the above analysis of the manufacturer’s profit under the specific mode (NR vs. NA and SR vs. SA), we draw the conclusion that for the manufacturer, no matter whether the platform introduces SB, the business mode she adopts only depends on the commission rate. That is, when the commission rate is low (Region and Region in Fig. 10), she adopts agency selling. Otherwise (Region and Region in Fig. 10), she adopts reselling. Building upon the manufacturer’s optimal business mode decision, we examine the platform’s optimal store brand decision under the restriction of business mode.

Reselling. The manufacturer adopts reselling when the commission rate is high, i.e., higher than 50%. It can be seen from Figures 2b and 4b that the platform always hopes the manufacturer to sell on agency selling under this commission rate range, which is opposite to the actual choice of the manufacturer. This means that no matter whether the platform introduces store brands or not, he cant maximize his profit. In other words, the platform can only achieve a relatively large profit through measurement under the restriction of agency

\[ \text{Figure 9. Regions of three parties profitability without SB under agency selling.} \]

\[ \text{Figure 10. The final equilibrium result.} \]
sellers. Specifically, when the preference for NB is low (region III in Fig. 10), the platform introduces store brand, that is, the equilibrium sales mode is SR. On the contrary, when the preference for NB is high (region IV in Fig. 10), the platform forsakes the introduction of store brand, that is, the equilibrium sales mode is NR. Interestingly, considering that the manufacturer always wants the platform not to introduce store brand under reselling (Fig. 6b), the implementation of mode NR in Region IV can enable her to achieve the most profitable situation. Even if the commission rate is high enough, as shown in Figures 2c and 4c, the implementation of mode NR in region is also beneficial to the overall realization of the most favorable situation of the platform-based supply chain.

Agency selling. The manufacturer adopts agency selling when the commission rate is low, i.e., lower than 50%. When the commission rate is particularly low, the platform always prefers the manufacturer to take reselling (as illustrated in Figs. 2b and 4b), which is also contrary to the manufacturer’s choice in reality. Thus, in this case, the platform tends to introduce store brand to cannibalize the market share of the NB product in order to obtain more revenue. Moreover, with the decrease of the preference for NB, the possibility of introducing store brand into the platform gradually increases, that is, the equilibrium sales mode is SA (region I in Fig. 10). However, when the commission rate is not particularly low, the preference of the platform to the manufacturer’s business mode depends on consumer’s brand preference. Specifically, with the increase of the preference for NB, the preference of the platform to the business mode will gradually be shifted from reselling to agency selling (as illustrated in Fig. 4b). In other words, the higher the preference for NB, the lower the willingness of the platform to introduce store brand, that is, the equilibrium sales mode is NA (region II in Fig. 10). It is worth noting that, referring to Figures 5 and 9, we find that region II is a “win-win-win” area, which means that the implementation of mode NA in this region can realize the win-win-win situation in profitability among the manufacturer, platform and the platform-based supply chain.

6. Extension

In the benchmark model, we ignore production costs in both NB and SB. This setting is better to focus the research on the strategic interaction between the manufacturer and the platform. Nonetheless, in practice, production costs are not negligible because they are bound to exist whenever product production is involved, such as raw materials, machinery and equipment, labor, etc. Thus, in this section, we assume that both NB and SB have positive production costs, denoted by \(c_N\) and \(c_S\), respectively. The other settings are consistent with the benchmark model to confirm the robustness. Accordingly, under mode NR, the target functions of the manufacturer and the platform are:

\[
J_M^{NR} = \int_0^\infty e^{-\rho t} \left[ (w(t) - c_N) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_P^{NR} = \int_0^\infty e^{-\rho t} \left[ (p_N(t) - w(t)) D_N(t) - \frac{1}{2} b_N(t)^2 \right] dt.
\]  

Under mode SR, the target functions of the manufacturer and the platform are:

\[
J_M^{SR} = \int_0^\infty e^{-\rho t} \left[ (w(t) - c_N) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_P^{SR} = \int_0^\infty e^{-\rho t} \left[ (p_N(t) - w(t)) D_N(t) + (p_S(t) - c_S) D_S(t) \right] dt.
\]

Under mode NA, the target functions of the manufacturer and the platform are:

\[
J_M^{NA} = \int_0^\infty e^{-\rho t} \left[ (1 - \theta) (p_N(t) - c_N) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_P^{NA} = \int_0^\infty e^{-\rho t} \left[ \theta (p_N(t) - c_N) D_N(t) - \frac{1}{2} b_N(t)^2 \right] dt.
\]

Under mode SA, the target functions of the manufacturer and the platform are:

\[
J_M^{SA} = \int_0^\infty e^{-\rho t} \left[ (1 - \theta) (p_N(t) - c_N) D_N(t) - \frac{1}{2} q_N(t)^2 \right] dt \\
J_P^{SA} = \int_0^\infty e^{-\rho t} \left[ \theta (p_N(t) - c_N) D_N(t) + (p_S(t) - c_S) D_S(t) \right] dt.
\]
Table 8. Equilibrium results with positive production costs when SB is not introduced.

<table>
<thead>
<tr>
<th>Mode SR</th>
<th>Mode SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_N(t)$</td>
<td>$G_N(t)$</td>
</tr>
<tr>
<td>$\frac{G_N(0) - \frac{a^2 f_2}{\delta - 2a^2 f_1}}{\alpha_2 f_2} e^{-(\delta - 2a^2 f_1)t} +$</td>
<td>$\frac{G_N(0) - \frac{a^2 m_2}{\delta - 2a^2 m_1}}{\alpha_2 m_2} e^{-(\delta - 2a^2 m_1)t} +$</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>$\frac{w(t)}{2a_N} + \frac{e_N}{\beta_N G_N(t)}$</td>
</tr>
<tr>
<td>$p_N(t)$</td>
<td>$\frac{2a_N G_N(t)}{2(2a_N - \gamma N)} + \frac{a(3a_N - \gamma N)}{2(2a_N - \gamma N)} + \frac{\beta_N G_N^A(t)}{2(\lambda_N - \theta_N^2)} + \frac{a}{2(\lambda_N - \theta_N^2)} + \frac{(\lambda_N - 2\theta_N^2)c_N}{2(\lambda_N - \theta_N^2)}$</td>
</tr>
</tbody>
</table>

Notes. $\hat{f}_1 = f_1; \hat{g}_1 = g_1; \hat{m}_1 = m_1; \hat{n}_1 = n_1; \hat{f}_2 = \frac{(a - \lambda_N c_N)\beta_N}{2(2a_N - \gamma N)(\rho + \delta - 2a_N f_1)}; \hat{g}_2 = \frac{(a - \lambda_N c_N)\beta_N}{2(2a_N - \gamma N)(\rho + \delta - 2a_N m_1)}; \hat{n}_2 = \frac{\theta \beta_N (2a_N - 3\theta_N^2)(a - \lambda_N c_N)}{4(\lambda_N - \theta_N^2)(\rho + \delta - 2a_N m_1)} + \frac{2a_N m_1}{(\rho + \delta - 2a_N m_1)}$.

\[ \frac{2a_N f_1 G_N^R(t) + \alpha_N \hat{f}_2}{2(2a_N - \gamma N)} + \frac{\gamma_N \beta_N G_N^R(t)}{2(2a_N - \gamma N)} + \frac{\gamma_N (a - \lambda_N c_N)}{2(2a_N - \gamma N)} + \frac{\theta \gamma_N \beta_N G_N^A(t)}{2(\lambda_N - \theta_N^2)} + \frac{\theta \gamma_N (a - \lambda_N c_N)}{2(\lambda_N - \theta_N^2)} \]

Figure 11. The profit relationship between reselling and agency selling without SB. (a) The manufacturer. (b) The platform. (c) The whole supply chain.

Proposition 6.1. Considering the positive production cost, the equilibrium results when SB is not introduced or is introduced are shown in Tables 8 and 9, respectively.

This section still uses numerical examples to verify the results for intuitive analysis. Figures 11 and 12 show the impact of $\theta$ and $\chi$ on the profits of M, P and PSC without and with and with SB for different production costs, respectively. We find that production costs have no impact on M’s business mode choice and P’s and PSC’s expectations of the business mode regardless of the introduction of SB, which is consistent with the benchmark model.

The impact of $\theta$ and $\chi$ on the profit of M, P and PSC under reselling and agency selling for different production costs are presented in Figures 13 and 14, respectively. First, Figures 13a and 14a illustrate that the
Table 9. Equilibrium results with positive production costs when SB is introduced.

<table>
<thead>
<tr>
<th>Mode AR</th>
<th>Mode AA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_N(t) \left( G_{N0} - \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) e^{-\left(\delta - 2\alpha_2^2 \delta_1\right)t} + \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) + \frac{a^2}{2} \frac{\lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a^2}{2} (\lambda_N - \alpha_1) \delta_3 \frac{\lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}</td>
<td></td>
</tr>
<tr>
<td>$w(t) \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
<tr>
<td>$p(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) + \frac{a^2}{2} \frac{\lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a^2}{2} (\lambda_N - \alpha_1) \delta_3 \frac{\lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2}</td>
<td></td>
</tr>
<tr>
<td>$G(t) \left( G_{N0} - \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) e^{-\left(\delta - 2\alpha_2^2 \delta_1\right)t} + \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
<tr>
<td>$ps(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
<tr>
<td>$qs(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
<tr>
<td>$b(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
<tr>
<td>$V(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
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<tr>
<td>$V(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
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<tr>
<td>$V(t) \left( \frac{\beta_N G_{N0}^2 h(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3} \right) = \frac{\beta_N G_{N0}^2 A(t)}{2 \lambda_N (\alpha_1 - \lambda_N) c_{SN}^2} + \frac{a}{2} \frac{\alpha_2^2 \delta_3}{2 - \alpha_2^2 \delta_3}</td>
<td></td>
</tr>
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</table>

Notes.

\( \hat{x}_1 = x_1; \hat{k}_1 = k_1 \)

\( \hat{y}_1 = y_1; \hat{y}_2 = y_2 \)

\( \hat{l}_1 = l_1; \hat{l}_2 = l_2 \)

Figure 12. The profit relationship between reselling and agency selling with SB. (a) The manufacturer. (b) The platform. (c) The whole supply chain.
platform's SB decision is influenced by production costs. Specifically, in reselling, the SB decision of the platform relies on the brand preference of consumers, in which the lower the NB preference, the higher the possibility of introducing SB. It is clear from Figure 13a that the increased production cost exacerbates the platform’s concern about NB preference when weighing whether to introduce SB. In other words, the high production cost reduces the threshold of NB preference that the platform can accept when introducing SB. Similarly, in agency selling, the possibility of introducing SB in the platform decreases with the increase in production costs (e.g., Fig. 14a). Secondly, regardless of the business mode, the manufacturer’s attitude towards SB is still always opposed in the face of different production costs (as shown in Figs. 13b and 14b). Finally, for the whole supply chain, production costs do not affect the expectation for SB under the reselling (as shown in Fig. 13c). While in agency selling (Fig. 14c), production costs change the threshold of the commission rate and NB preference that the PSC expects to be acceptable for SB introduction. That is, high production costs can make the conditions under which PSC expects SB to be introduced stringent. Nonetheless, the increase in production costs affects the platform and the whole supply chain in terms of the commission rate and NB preference when SB is introduced, but it does not affect the overall trend. As a result, the win-win-win situation in profitability among the manufacturer, the platform and the platform-based supply chain still exists under certain conditions. Therefore, the main results in the benchmark model do not change when positive production costs are considered.

7. Conclusion

This study considers a platform-based supply chain consisting of a manufacturer, a platform and consumers. To study the strategic interaction between the manufacturer’s business mode decision and the platform’s store
brand decision, four modes are established, which are mode NR, mode NA, mode SR and mode SA respectively. Using differential game theory and continuous dynamic programming theory, the optimal operation strategies (including pricing strategy, quality strategy and big-data marketing service strategy) and corresponding profits of the manufacturer and the platform under different modes are obtained. Through comparative analysis and numerical examples, the main results and insights are as follows.

Firstly, the impact of goodwill is obtained. Goodwill is a key factor affecting enterprise decision-making and profitability. To be specific, the price, quality, big-data marketing service level and the profit of enterprises are positively related to the goodwill. Therefore, considering the positive impact of goodwill, enterprises should take the initiative to enhance the goodwill and reputation of the brand by improving product quality and big data marketing service level. In this way, enterprises can make consumers willing to buy the product of the brand at a high price, and finally gain both fame and wealth.

Secondly, the goodwill of brands and the optimal strategies of the manufacturer and the platform under specific scenarios are analyzed. (1) For the store brand, the relevant strategies for the SB product are not affected by the manufacturer’s business mode decision. (2) For the national brand, both the business mode decision and the SB decision affect the strategies related to NB product. Accordingly, the management insight is that regardless of whether the platform introduces SB or not, the manufacturer should implement agency selling to facilitate consumers to obtain the NB product with high quality and goodwill at a low price if the commission rate is low; otherwise, reselling should be adopted to provide more quality-price ratio NB products. In addition, since the introduction of SB can reduce the double marginal effect, the platform should introduce SB in moderation within the capacity.

Thirdly, the business mode decision and the store brand decision under the specific scenario are investigated. (1) Given the SB decision, the business mode adopted by the manufacturer only depends on the commission rate. Thus, the manufacturer should adopt agency selling if the commission rate is low; otherwise, she should choose reselling. (2) Given the business mode, the factors considered by the platform whether to introduce SB or not differ in different business modes. So, for the platform, he should introduce (forgo) SB under reselling if the preference of NB is low (high). Under agency selling, on the other hand, he needs to trade off both the commission rate and the NB preference when deciding the SB strategy.

Finally, the equilibrium sales mode of the manufacturer and platform when the platform first decides the store brand decision is provided. Although we obtained four equilibrium sales modes for the strategic interaction of the manufacturer and the platform under different conditions, only two equilibrium scenarios. For one, when both the commission rate and NB preference are high, mode-NR is the equilibrium mode, which allows the manufacturer to achieve the most profitable situation. For the other, when the commission rate is low and NB preference is high enough, the equilibrium sales mode is mode NA, which can achieve a “win-win-win” situation in profitability among the manufacturer, the platform, and the platform-based supply chain.

APPENDIX A. PROOF OF PROPOSITION 1

For clarity, the time variable is omitted below. Suppose $V_{M}^{NR}, V_{P}^{NR}$ are the optimal value functions of the manufacturer and the platform, and $\frac{\partial V_{M}^{NR}}{\partial G_{N}}, \frac{\partial V_{P}^{NR}}{\partial G_{N}}$ are the corresponding first derivative of goodwill, respectively. According to the theory of continuous dynamic programming, the optimal value function of the manufacturer and the platform satisfies the following Hamilton-Jacobi-Bellman (HJB) equation can be expressed as

$$
\rho V_{M}^{NR} = \max_{w(\cdot),q_{N}(\cdot)} \left\{ w\left(a - \lambda_{N} p_{N} + \gamma_{N} b_{N} + \beta_{N} G_{N}\right) - \frac{1}{2} q_{N}^{2} + \frac{\partial V_{M}^{NR}}{\partial G_{N}} \left(\alpha_{N} q_{N} - \delta G_{N}\right) \right\} \tag{A.1}
$$

$$
\rho V_{P}^{NR} = \max_{p_{N}(\cdot),b_{N}(\cdot)} \left\{ (p_{N} - w)\left(a - \lambda_{N} p_{N} + \gamma_{N} b_{N} + \beta_{N} G_{N}\right) - \frac{1}{2} b_{N}^{2} + \frac{\partial V_{P}^{NR}}{\partial G_{N}} \left(\alpha_{N} q_{N} - \delta G_{N}\right) \right\}. \tag{A.2}
$$

According to equation (A.2) and the first-order optimality conditions, we get

$$
p_{N} = \frac{a + \beta_{N} G_{N} + (\lambda_{N} - \gamma_{N})w}{2\lambda_{N} - \gamma_{N}}, b_{N} = \frac{\gamma_{N}(a + \beta_{N} G_{N} - \lambda_{N} N\cdot w)}{2\lambda_{N} - \gamma_{N}}. \tag{A.3}
$$
Substituting equation (A.3) into equation (A.1) and using the first-order optimality conditions, we get

\[ \rho V_M^{NR} = \frac{(a+\beta_N G_N)^2}{4(2\lambda_N-\gamma_N^2)} + \frac{1}{2} \alpha_N \frac{\partial V_M^{NR}}{\partial G_N} - \delta \rho \frac{\partial V_M^{NR}}{\partial G_N} G_N \]

\[ \rho V_P^{NR} = \frac{(a+\beta_N G_N)^2}{8(2\lambda_N-\gamma_N^2)} + \alpha_N \frac{\partial V_M^{NR}}{\partial G_N} \frac{\partial V_P^{NR}}{\partial G_N} - \delta \rho \frac{\partial V_M^{NR}}{\partial G_N} G_N. \]

(A.4)

(A.5)

According to the above structures, the optimal value functions can be assumed that \( V_M^{NR} = f_1 G_N^2 + f_2 G_N + f_3, \)

\( V_P^{NR} = g_1 G_N^2 + g_2 G_N + g_3, \)

where, \( f_i, g_i, (i = 1, 2, 3) \) are undetermined coefficients. Substituting them into equations (A.5) and (A.6) and according to the identity relationship, we get

\[ f_1 = \frac{\rho + 2\delta}{4\alpha_N} - \frac{1}{4\alpha_N} \sqrt{(\rho + 2\delta)^2 - \frac{2a\beta_N^2}{2\lambda_N-\gamma_N^2}}, \]

\[ f_2 = \frac{a\beta_N}{2(2\lambda_N-\gamma_N^2)}((\rho + 2\delta) - 2a\beta_N f_1), \]

\[ g_1 = \frac{\beta_N^2}{8(2\lambda_N-\gamma_N^2)((\rho + 2\delta) - 4\alpha_N f_1)}, \]

\[ g_2 = \left( \frac{a\beta_N}{2(2\lambda_N-\gamma_N^2)} \right) \left( \frac{a\beta_N}{4\alpha_N} \right) + 2a\beta_N g_1, \]

\[ f_3 = \frac{a^2}{4\rho(2\lambda_N-\gamma_N^2)} + \frac{a\beta_N^2}{2\rho}, \]

\[ g_3 = \left( \frac{a^2}{8\rho(2\lambda_N-\gamma_N^2)} \right) + \frac{a\beta_N^2}{2\rho}. \]

By substituting equation (B.1) into equations (A.3) and (A.4), the optimal strategies and optimal value functions of members under mode NR can be obtained.

**APPENDIX B. PROOF OF PROPOSITION 2**

Suppose \( V_M^{NA}, V_P^{NA} \) are the optimal value functions, and \( \frac{\partial V_M^{NA}}{\partial G_N}, \frac{\partial V_P^{NA}}{\partial G_N} \) are the corresponding first derivative, respectively. The HJB equation of the manufacturer and the platform can be expressed as

\[ \rho V_M^{NA} = \max_{p_N(\cdot); q_N(\cdot)} \left\{ (1 - \theta) p_N (a - \lambda_N p_N + \gamma_N b_N + \beta_N G_N) - \frac{1}{2} q_N^2 + \frac{\partial V_M^{NA}}{\partial G_N} (\alpha_N q_N - \delta G_N) \right\} \]

\[ \rho V_P^{NA} = \max_{b_N(\cdot)} \left\{ \theta p_N (a - \lambda_N p_N + \gamma_N b_N + \beta_N G_N) - \frac{1}{2} b_N^2 + \frac{\partial V_P^{NA}}{\partial G_N} (\alpha_N q_N - \delta G_N) \right\}. \]

(B.1)

(B.2)

According to equation (B.3) and the first-order optimality conditions, we get

\[ b_N = \theta \gamma_N p_N. \]

(B.3)

Substituting equation (B.4) into equation (B.2) and using the first-order optimality conditions, we get

\[ p_N = \frac{a + \beta_N G_N}{2(\lambda_N - \theta \gamma_N^2)}, q_N = \alpha_N \frac{\partial V_M^{NA}}{\partial G_N}. \]

(B.4)

Substituting equations (B.4) and (B.5) into equations (B.2) and (B.3), we get

\[ \rho V_M^{NA} = \frac{(1 - \theta)(a + \beta_N G_N)^2}{4(\lambda_N - \theta \gamma_N^2)} + \frac{1}{2} \alpha_N \frac{\partial V_M^{NA}}{\partial G_N} - \delta \rho \frac{\partial V_M^{NA}}{\partial G_N} G_N \]

\[ \rho V_P^{NA} = \frac{\theta(2\lambda_N - 3\theta \gamma_N^2)(a + \beta_N G_N)^2}{8(\lambda_N - \theta \gamma_N^2)} + \alpha_N \frac{\partial V_M^{NA}}{\partial G_N} \frac{\partial V_P^{NA}}{\partial G_N} - \delta \rho \frac{\partial V_M^{NA}}{\partial G_N} G_N. \]

(B.5)

(B.6)

According to the above structures, the optimal value functions can be assumed that \( V_M^{NA} = m_1 G_N^2 + m_2 G_N + m_3, \)

\( V_P^{NA} = n_1 G_N^2 + n_2 G_N + n_3, \)

where, \( m_i, n_i, (i = 1, 2, 3) \) are undetermined coefficients. Substituting them into equations (B.6) and (B.7) and according to the identity relationship, we get

\[ m_1 = \frac{\rho + 2\delta}{4\alpha_N} - \frac{1}{4\alpha_N} \sqrt{(\rho + 2\delta)^2 - \frac{2a\beta_N^2}{2\lambda_N-\gamma_N^2}}, m_2 = \frac{a\beta_N (1 - \theta)}{2(\lambda_N - \theta \gamma_N^2)((\rho + 2\delta) - 2a\beta_N m_1)}, \]

\[ n_1 = \frac{\theta(2\lambda_N - 3\theta \gamma_N^2)(a + \beta_N G_N)^2}{8(\lambda_N - \theta \gamma_N^2)((\rho + 2\delta) - 4\alpha_N m_1)}, n_2 = \frac{1}{2} \rho \frac{\partial V_M^{NA}}{\partial G_N} \left( \frac{\theta(2\lambda_N - 3\theta \gamma_N^2)a\beta_N}{4(\lambda_N - \theta \gamma_N^2)} + 2\alpha_N n_1 m_2 \right), \]

\[ n_3 = \frac{(1 - \theta)a^2}{4\rho(\lambda_N - \theta \gamma_N^2)} + \frac{a\beta_N^2}{2\rho}, n_3 = \frac{(1 - \theta)a^2}{8\rho(\lambda_N - \theta \gamma_N^2)} + \frac{a\beta_N^2}{2\rho}. \]
By substituting equation (C.1) into equations (B.4) and (B.5), the optimal strategies and optimal value functions of members under mode NA can be obtained.

**APPENDIX C. PROOF OF PROPOSITION 3 (NR vs. NA)**

Based on the results of mode NR and mode NA, let \( t \to \infty \), we obtain the corresponding steady-state results. Compare the steady-state price of NB, we get

\[
p_{N\infty}^{NR} - p_{N\infty}^{NA} = a \delta (\rho + \delta) \left( \frac{3\gamma_1 - \gamma_2^2}{2b(\rho + \delta)(\lambda_N - \gamma_2^2) - \lambda_N \alpha_N^2 \beta_N^2} - \frac{1}{2b(\rho + \delta)(\lambda_N - \theta \gamma_2^2) - (1 - \theta) \alpha_N^2 \beta_N^2} \right) .
\]  

(C.1)

According to equation (C.2), there is \( p_{N\infty}^{NR} - p_{N\infty}^{NA} < 0 \) when \( \theta > \frac{2b(\rho + \delta)\gamma_N^2 - (2\lambda_N - \gamma_2^2)\alpha_N^2 \beta_N^2}{(3\lambda_N - \gamma_2^2)(2b(\rho + \delta)\gamma_N^2 - \alpha_N^2 \beta_N^2) - \gamma_2^2} \), and there is \( p_{N\infty}^{NR} - p_{N\infty}^{NA} > 0 \) when \( \theta \leq \frac{2b(\rho + \delta)\gamma_N^2 - (2\lambda_N - \gamma_2^2)\alpha_N^2 \beta_N^2}{(3\lambda_N - \gamma_2^2)(2b(\rho + \delta)\gamma_N^2 - \alpha_N^2 \beta_N^2) - \gamma_2^2} \). Similarly, comparing the steady-state goodwill, the steady-state quality and big-date marketing service of NB, we get

\[
G_{N\infty}^{NR} - G_{N\infty}^{NA} = a \alpha_N^2 \beta_N \left( \frac{1}{2b(\rho + \delta)(\lambda_N - \gamma_2^2) - \alpha_N^2 \beta_N^2} - \frac{1}{2b(\rho + \delta)(\lambda_N - \theta \gamma_2^2) - (1 - \theta) \alpha_N^2 \beta_N^2} \right) .
\]  

(C.2)

\[
G_{N\infty}^{NR} - G_{N\infty}^{NA} = a \alpha_N^2 \beta_N \left( \frac{1}{2b(\rho + \delta)(\lambda_N - \gamma_2^2) - \alpha_N^2 \beta_N^2} - \frac{1}{2b(\rho + \delta)(\lambda_N - \theta \gamma_2^2) - (1 - \theta) \alpha_N^2 \beta_N^2} \right) .
\]  

(C.3)

\[
G_{N\infty}^{NR} - G_{N\infty}^{NA} = a \alpha_N^2 \beta_N \left( \frac{1}{2b(\rho + \delta)(\lambda_N - \gamma_2^2) - \alpha_N^2 \beta_N^2} - \frac{1}{2b(\rho + \delta)(\lambda_N - \theta \gamma_2^2) - (1 - \theta) \alpha_N^2 \beta_N^2} \right) .
\]  

(C.4)

According to equations (C.3)–(C.18), there is \( G_{N\infty}^{NR} - G_{N\infty}^{NA} > 0, q_{N\infty}^{NR} - q_{N\infty}^{NA} > 0 \) and \( b_{N\infty}^{NR} - b_{N\infty}^{NA} > 0 \) when \( \theta > \frac{1}{2} \), and there is \( G_{N\infty}^{NR} - G_{N\infty}^{NA} \leq 0, q_{N\infty}^{NR} - q_{N\infty}^{NA} \leq 0 \) and \( b_{N\infty}^{NR} - b_{N\infty}^{NA} \leq 0 \) when \( \theta \leq \frac{1}{2} \).

Since the proofs of Propositions 4.4, 4.5 and 6.1 are similar to Propositions 4.1 and 4.2, and the proofs of Propositions 4.6 and 4.7 are similar to Proposition 4.3, we omit them here.

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**REFERENCES**


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