INTERIOR-POINT ALGORITHM FOR LINEAR PROGRAMMING BASED ON A NEW DESCENT DIRECTION

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Abstract. We present a full-Newton step feasible interior-point algorithm for linear optimization based on a new search direction. We apply a vector-valued function generated by a univariate function on a new type of transformation on the centering equations of the system which characterizes the central path. For this, we consider a new function $\psi(t) = t^{7/4}$. Furthermore, we show that the algorithm finds the $\epsilon$-optimal solution of the underlying problem in polynomial time, namely $O\left(\sqrt{n\log\left(\frac{n+\frac{1}{\epsilon^2}}{\epsilon}\right)}\right)$ iterations. Finally, a comparative numerical study is reported in order to analyze the efficiency of the proposed algorithm.

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1. Introduction

Consider the standard linear programming problem:

$$\min\{c^T x : Ax = b, x \geq 0\}, \quad \text{(LP)}$$

and its dual problem:

$$\max\{b^T y : A^T y + s = c, s \geq 0\}, \quad \text{(LD)}$$

where $b = (b_1, b_2, \ldots, b_m)^T \in \mathbb{R}^m$, $c = (c_1, c_2, \ldots, c_n)^T \in \mathbb{R}^n$ and $A$ is a $m \times n$ given matrix.

The linear optimization (LO) is one of the most active research areas in mathematical programming. There are many approaches for solving LO. Among them, recent interior-point methods IPMs which gain much more attention than others. For a survey, we refer to [12,17,23].

Primal-dual interior-point method IPM is one of the most efficient numerical methods for solving large classes of optimization problems and highly efficient in both theory and practice [25,31]. The success of these methods for solving LO leads researchers to extend it naturally to other important problems such as semidefinite programs.

Keywords. Linear programming, interior-point methods, descent direction, primal-dual algorithm.

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(SDP) [14], quadratic programs (QP) [3], complementarity problems (CP) [1] and conic optimization problems (COP).

The determination of the search directions plays a key role in case of the IPMs. Recently, primal-dual algorithms based on the technique of kernel functions have received much attention from researchers. Indeed, several works have been published giving rise to new research directions offering polynomial complexity for short and long step algorithms. In this sense, Bai et al. [4] presented a large class of eligible kernel functions, which is fairly general and includes the classical logarithmic functions and the self-regular functions, as well as many non-self-regular functions as special cases. For some other related kernel function, we refer to [5–7, 13, 21, 22, 24].

Another technique for defining other search directions for LO was given by Darvay [8]. This technique is based on an algebraic equivalent transformation on the centering equations of the central path. The new search directions are obtained by applying Newton’s method to the resulting system. Achache [2] generalized this method for convex quadratic optimization (CQO), while Wang and Bai [28–30] for semidefinite optimization (SDO), second-order cone optimization (SOCO) and symmetric optimization (SO).

In 2018, Darvay and Takács [9] proposed another technique for obtaining a new descent direction for solving LO. The technique is based on applying the function \( \psi(t) = t^2 \) on both sides of a specific algebraic equivalent transformation of the centering equations defined by Zhang and Xu [32]. They showed both the theoretical and numerical effectiveness of their method compared to other existing techniques. Furthermore, the same authors in [26] extended this approach to symmetric optimization problems. Later, Kheirfam [18] extended the method to \( P^*(k) \)-horizontal linear complementarity problems, while Guerra [16] applied it to the SDO case. For more related papers about Darvay and Takács’ technique, we refer to (see e.g., [11, 19, 20]).

The purpose of this work is to introduce a new short-step algorithm for LO. The algorithm is derived by modifying the technique proposed by Darvay and Takács [9] for LO, using a new function \( \psi(t) = t^7 \). This modification leads to a new efficient descent direction for the considered algorithm. Moreover, we prove that the new algorithm deserves a polynomial complexity, namely \( 10\sqrt{\pi} \log \left( \frac{n+\frac{1}{15}}{\epsilon} \right) \) iterations bound. Finally, we compare the performance of our algorithm with that of Darvay and Takács’ algorithm [9] on a set of LO problems, specifically on some selected problems from the Netlib test collection [15], in order to evaluate the effectiveness of our proposed algorithm.

The outline of the paper is as follows: In Section 2, we introduce a fundamental concepts and give the classical Newton direction. In Section 3, we deal with the new search direction and the description of the corresponding algorithm. In Section 4, we study the complexity analysis of the algorithm where, we prove its convergence and give the most number of iterations necessary to the optimality condition. Section 5 contains a comparative numerical experimentations and comments. Finally, a conclusion ends Section 6.

2. Classical Newton search direction for LO

In this section, we recall the notion of the central path with its properties and we drive the classical Newton search direction for LO.

Throughout the paper, we make the following assumptions on (LP) and (LD):

- \( \text{Rank}(A) = m < n \).
- The interior-point condition (IPC) holds for the primal and dual problems, which means that there exists \((x^0, y^0, s^0)\) such that:
  \[
  Ax^0 = b, \quad A^T y^0 + s^0 = c, \quad x^0 > 0, \quad s^0 > 0. \tag{IPC}
  \]

The optimal solution of the primal-dual pair can be given with the following system of equations:

\[
\begin{aligned}
Ax &= b, \quad x \geq 0, \\
A^T y + s &= c, \quad s \geq 0, \\
x^0 s &= 0
\end{aligned}
\]
Where $xs$ denotes the coordinatewise product of the vectors $x$ and $s$, hence $xs = (x_1s_1, x_2s_2, \ldots, x_n s_n)^T \geq 0$.

The first two equations of system (1) are named feasibility conditions and the last one is called complementarity condition.

The key idea of primal-dual path following IPMs is to replace the complementarity condition $xs = 0$ in (1) by the parameterized equation $xs = \mu e$, where $e$ is the all-one vector of length $n$ and $\mu$ is a positive parameter. Thus, we consider the following system:

$$\begin{cases} 
Ax = b, \ x > 0, \\
A^Ty + s = c, \ s > 0, \\
xs = \mu e, \ \mu > 0.
\end{cases}$$

(2)

It is proved in [27] that under our assumptions there is a unique solution $(x(\mu), y(\mu), s(\mu))$ to the system (2) for any barrier parameter $\mu > 0$. We call $(x(\mu), y(\mu), s(\mu))$ the $\mu$-center of (LP) and (LD). The set of all $\mu$-center is called the central path. The limit of the central path as $\mu$ tends to zero exists and since the limit point satisfies (1), it gives the optimal solution for (LP) and (LD).

3. NEW SEARCH DIRECTION

In this section, we reconsider the technique introduced by Darvay and Takács [9] with our new function $\psi(t) = t^{\frac{k}{2}}$ in order to obtain better theoretical and numerical results. Note that for $x, s > 0$ and $\mu > 0$, from the third equation of system (2) we deduce that:

$$xs = \mu e \iff \frac{xs}{\mu} = e \iff \sqrt{\frac{xs}{\mu}} = e \iff \frac{xs}{\mu} = \sqrt{xs}.$$  

Where, $\frac{xs}{\mu}$ denotes the coordinatewise product of the vectors $x$ and $s$ divided by $\mu > 0$, hence $\frac{xs}{\mu} = \left(\frac{x_1s_1}{\mu}, \frac{x_2s_2}{\mu}, \ldots, \frac{x_n s_n}{\mu}\right)^T \geq 0$ and $\sqrt{\frac{xs}{\mu}}$ is the vector obtained by taking square roots of the components of $\frac{xs}{\mu}$.

Now, the perturbed central path can be equivalently stated as follows:

$$\begin{cases} 
Ax = b, \\
A^Ty + s = c, \\
\sqrt{\frac{xs}{\mu}} = \frac{xs}{\mu}.
\end{cases}$$

(3)

In accordance with the Darvay idea [9], we consider the invertible function $\psi$ defined and continuously differentiable on the interval $(k^2, \infty)$, where $0 \leq k < 1$, such that $2t\psi''(t^2) - \psi'(t) > 0$, $\forall t > k^2$. We apply the equivalent algebraic transformation method to (3), we get:

$$\begin{cases} 
Ax = b, \\
A^Ty + s = c, \\
\psi\left(\sqrt{\frac{xs}{\mu}}\right) = \psi\left(\frac{xs}{\mu}\right).
\end{cases}$$

(4)

This last system (4) can be written in the form $f(x, y, s) = 0$, where:

$$f(x, y, s) = \begin{pmatrix} Ax - b \\
A^Ty + s - c \\
\psi\left(\sqrt{\frac{xs}{\mu}}\right) - \psi\left(\frac{xs}{\mu}\right)
\end{pmatrix}.$$  

(5)
Applying Newton’s method to this system we get: $x_+ = x + \Delta x$, $y_+ = y + \Delta y$, $s_+ = s + \Delta s$, where $(\Delta x, \Delta y, \Delta s)$ is the solution of the linear system:

$$
\begin{cases}
A\Delta x = 0, \\
A^T \Delta y + \Delta s = 0, \\
\frac{1}{\mu}(s\Delta x + x\Delta s) = \frac{-\psi(x) + \psi(\sqrt{xs})}{\psi'(x) - \sqrt{xs} \psi'(\sqrt{xs})}.
\end{cases}
$$

(6)

Defining the scaled vector $v$ and the scaled search directions $d_x$ and $d_s$ according to

$$
v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v \Delta x}{x} \quad \text{and} \quad d_s = \frac{v \Delta s}{s}.
$$

Hence, we obtain:

$$
\frac{1}{\mu}(s\Delta x + x\Delta s) = v(d_x + d_s),
$$

(7)

and

$$
d_x d_s = \frac{\Delta x \Delta s}{\mu}.
$$

(8)

Obviously, with these notations, the scaled feasible Newton system of (6) can be expressed as:

$$
\begin{cases}
\overline{A}d_x = 0, \\
\overline{A}^T \Delta y + d_s = 0, \\
d_x + d_s = p_v.
\end{cases}
$$

(9)

Where:

$$
p_v = \frac{2\psi(v) - 2\psi(v^2)}{2v \psi'(v^2) - \psi'(v)},
$$

$$
\overline{A} = \frac{1}{\mu} \text{diag} \left( \frac{x}{v} \right).
$$

Here, $\text{diag}(\frac{x}{v})$ is a diagonal matrix, which contains on its main diagonal the elements of the vector $\frac{x}{v}$ respectively in the original order.

For different $\psi$ functions, we obtain different values of $p_v$, vector that lead to new search directions.

We mention that $\psi(t) = t$ yields $p_v = \frac{2v^2 - 2v^3}{2v^2 - 3v}$ and we obtain that this direction is similarly to the algorithm defined in [10]. Recently, Darvay and Takács in [9] observed that $\psi(t) = t^2$ yields $p_v = \frac{v - v^3}{2v^2 - 3v}$ which define a new search direction.

In this paper, we shall consider $\psi : \left((\frac{1}{2})^{\frac{2}{3}}, \infty\right) \rightarrow \mathbb{R}$, such that $\psi(t) = t^{\frac{2}{3}}$. Then:

$$
p_v = \frac{8v - 8v^{\frac{11}{3}}}{14v^{\frac{2}{3}} - 7v}.
$$

(10)

The condition $2t\psi'(t^2) - \psi'(t) > 0, \forall t > k^2$ is satisfied in this case, where $k^2 = \left(\frac{1}{2}\right)^{\frac{2}{3}}$.

We give a proximity measure to the central path as follows:

$$
\delta(v) = \delta(xs, \mu) = \frac{||p_v||}{2} = \frac{4}{7} \left\| v - v^{\frac{11}{3}} \right\|_{2v^{\frac{2}{3}} - 3v}.
$$

(11)

where $||.||$ denotes the Euclidean norm.
3.1. The algorithm prototype

The generic representation of this algorithm is given as follows:

**Primal-dual algorithm for LO**

**Input**
- a proximity parameter $0 < \beta < 1$;
- an accuracy parameter $\epsilon > 0$;
- a fixed barrier update parameter $\theta$, $0 < \theta < 1$;
- a strictly feasible $(x^0, y^0, s^0)$ such that $\delta(x^0 s^0, \mu^0) < \beta$ and $v^0 = \sqrt{\frac{x^0 s^0}{\mu^0}} > \frac{\epsilon}{27}$ where
  \[ \mu^0 = (x^0)^T s^0; \]

**begin**
- $x = x^0; y = y^0; s = s^0; \mu = \mu^0$;
- while $x^T s \geq \epsilon$ do
  - $\mu = (1 - \theta)\mu$;
  - solve the system (9) via (6) to obtain $(\Delta x, \Delta y, \Delta s)$;
  - take $x = x + \Delta x; y = y + \Delta y; s = s + \Delta s$;

**end**

In order to facilitate the analysis of the convergence of the algorithm, we define the vector $q_v \in \mathbb{R}^n$ by:

\[ q_v = d_x - d_s, \]

this implies that:

\[ d_x^T d_s = 0 \implies \|p_v\| = \|q_v\|. \]

From system (9), we have $p_v = d_x + d_s$ then

\[ d_x = \frac{p_v + q_v}{2}, \quad d_s = \frac{p_v - q_v}{2}, \]

which gives:

\[ d_x d_s = \frac{p_v^2 - q_v^2}{4}. \quad (12) \]

In the next section, we present some results related to algorithm complexity analysis.

### 4. Analysis of the algorithm

In the following lemma, we state a condition which ensures the feasibility of the generated point after a full Newton step $x_+$ and $s_+$, where $x_+ = x + \Delta x$ and $s_+ = s + \Delta s$.

**Lemma 4.1.** Let $\delta = \delta(x s, \mu) < 1$ and $v > \frac{1}{27}\epsilon$. Then

\[ x_+ > 0 \text{ and } s_+ > 0. \]

**Proof.** For each $0 \leq \alpha \leq 1$ denote $x_+(\alpha) = x + \alpha \Delta x$ and $s_+(\alpha) = s + \alpha \Delta s$. Hence,

\[ x_+(\alpha)s_+(\alpha) = xs + \alpha(x \Delta s + s \Delta x) + \alpha^2 \Delta x \Delta s. \quad (13) \]

Now, in view of (7) and (8) we have:

\[ \frac{1}{\mu} x_+(\alpha)s_+(\alpha) = v^2 + v(d_x + d_s) + \alpha^2 d_x d_s, \quad (14) \]
also from (9) and (12), we can write:

\[
\frac{1}{\mu}x_+(\alpha)s_+(\alpha) = (1 - \alpha)v^2 + \alpha(v^2 + vp_v) + \alpha^2 \left( \frac{p_v^2}{4} - \frac{q_v^2}{4} \right). \tag{15}
\]

In addition, from (10) we obtain:

\[
v^2 + vp_v = v^2 + \frac{8v^2 - 8v^2}{14v^2 - 7e} = \frac{6v^2}{14v^2 - 7e}. \tag{16}
\]

Let’s consider the function: \( f(x) = \frac{6x^{1/2} + x^2}{14x^{1/2} - 7} \), for \( x > \frac{1}{2} \). We have \( f(x) \geq f(1) \), so \( f(x) \geq 1 \). Using this result, we get:

\[
v^2 + vp_v \geq e, \tag{17}
\]

this implies that:

\[
\frac{1}{\mu}x_+(\alpha)s_+(\alpha) \geq (1 - \alpha)v^2 + \alpha e + \alpha^2 \left( \frac{p_v^2}{4} - \frac{q_v^2}{4} \right)
\geq (1 - \alpha)v^2 + \alpha e + \alpha^2 \left( \frac{p_v^2}{4} - \frac{q_v^2}{4} \right) - \alpha^2 \frac{p_v}{4}
\geq (1 - \alpha)v^2 + \alpha e + \alpha(\alpha - 1) \frac{p_v^2}{4} - \alpha^2 \frac{q_v^2}{4}
\geq (1 - \alpha)v^2 + \alpha \left[ e - \left( \frac{p_v^2}{4} + \frac{q_v^2}{4} \right) \right].
\]

In addition, we have

\[
\left\| \left( 1 - \alpha \right) \frac{p_v^2}{4} + \alpha \frac{q_v^2}{4} \right\|_\infty \leq (1 - \alpha) \frac{\|p_v\|_\infty^2}{4} + \alpha \frac{\|q_v\|_\infty^2}{4}
\leq (1 - \alpha) \frac{\|p_v\|^2}{4} + \alpha \frac{\|q_v\|^2}{4} = \delta^2,
\]

where \( \| \cdot \|_\infty \) marks the Chebychev norm (or \( l_\infty \) norm).

Also, from \( \delta < 1 \) we get:

\[
\left\| \left( 1 - \alpha \right) \frac{p_v^2}{4} + \alpha \frac{q_v^2}{4} \right\|_\infty < 1,
\]

then

\[
e - \left[ \left( 1 - \alpha \right) \frac{p_v^2}{4} + \alpha \frac{q_v^2}{4} \right] > 0.
\]

Hence, \( x_+(\alpha)s_+(\alpha) > 0 \) for each \( 0 \leq \alpha \leq 1 \), which means that the linear functions of \( \alpha \), \( x_+(\alpha) \) and \( s_+(\alpha) \) do not change sign on the interval \([0, 1]\). Therefore \( x_+(0) = x > 0 \) and \( s_+(0) = s > 0 \) give \( x_+(1) = x_+ > 0 \) and \( s_+(1) = s_+ > 0 \). This means that the full-Newton step is strictly feasible. □
We state the following lemma which will be useful in the next part of the analysis.

**Lemma 4.2.** [10, Lemma 5.2] Let \( f : [d, \infty) \rightarrow (0, \infty) \) be a decreasing function with \( d > 0 \). Furthermore, let us consider the positive vector \( v \) of length \( n \) such that \( \min(v) > d \). Then

\[
\| f(v) (e - v^2) \| \leq f(\min(v)) \| e - v^2 \| \leq f(d) \| e - v^2 \|.
\]

Now, in Lemma 4.3 we show the quadratic convergence of the full-Newton step.

**Lemma 4.3.** Suppose that \( \delta = \delta(x, \mu) < \frac{1}{2^7} \) and \( v > \frac{1}{2^7} e \), then:

\[
v_+ > \frac{1}{2^7} e \quad \text{and} \quad \delta(x_+ s_+, \mu) \leq 18 \delta^2,
\]

which proves that the full Newton step ensures a local quadratic convergence of the proximity measure.

**Proof.** We know from Lemma 4.1 that \( x_+ > 0 \) and \( s_+ > 0 \), then \( v_+ = \frac{x_+ s_+}{\mu} \) is well defined. From \( \delta = \delta(x, \mu) < \frac{1}{2^7} \), we get

\[
\sqrt{1 - \delta^2} > \sqrt{1 - \frac{1}{2^7}} > \frac{1}{2^7}.
\]

Let \( \alpha = 1 \). Then from (15) it follows that:

\[
v_+^2 = \frac{x_+ s_+}{\mu} = v^2 + v p_v + \frac{p_v^2}{4} - \frac{q_v^2}{4}.
\]

From (17) and \( \left( \frac{\nu_+^2}{4} > 0 \right) \) we obtain:

\[
v_+^2 = v^2 + v p_v + \frac{p_v^2}{4} - \frac{q_v^2}{4} > e - \frac{q_v^2}{4},
\]

hence

\[
\min(v_+^2) \geq 1 - \frac{\|q_v\|_\infty^2}{4} \geq 1 - \frac{\|q_v\|^2}{4} \geq 1 - \delta^2,
\]

and this relation yields

\[
\min(v_+) \geq \sqrt{1 - \delta^2}.
\]

From (18) and (20) we obtain that: \( v_+ > \frac{1}{2^7} e \).
This completes the first part of the proof.

Now, we introduce the notation:

\[
\delta(v_+) = \delta(x_+ s_+, \mu)
= \frac{\|p_{v_+}\|}{2}
= 4 \left\| \frac{(v_+ - v_+^1)}{(14v_+^2 - 7e)(e - v_+^2)}(e - v_+^2) \right\|.
\]
We have:

\[
e - v_+^2 = e - x + s_+ = e = \left(v^2 + \nu v_0 + \frac{p_0^2}{4} - \frac{q_0^2}{4}\right)
\]

\[
= \frac{q_0^2}{4} - \left(v^2 + \nu v_0 + \frac{p_0^2}{4} - e\right)
\]

\[
= \frac{q_0^2}{4} - \frac{p_0^2}{4} \left[4(v^2 + \nu v_0) + \frac{p_0^2}{4} + e - \frac{4}{p_0^2}\right].
\]

Applying (10) and (16), we obtain:

\[
e - v_+^2 = \frac{q_0^2}{4} - \frac{p_0^2}{4} \left[100v^{12} - 60v^{15} + 9v^2 - (14v^7 - 7e)^2\right]
\]

\[
= \frac{q_0^2}{4} - \frac{p_0^2}{4} \left[112(v - v^{14})^2 - 12v^{12} + 164v^{15} - 103v^2 - (14v^7 - 7e)^2\right]
\]

\[
= \frac{q_0^2}{4} - \frac{p_0^2}{4} \left[7e + 164v^{15} - 12v^{12} - 103v^2 - (14v^7 - 7e)^2\right].
\]

We have 0 \leq \frac{164v^{15} - 12v^{12} - 103v^2 - (14v^7 - 7e)^2}{16(v - v^{14})^2} \leq 7e, \forall v > \frac{1}{2\pi} e, so these imply that:

\[
\|e - v_+^2\| \leq \left\|\frac{q_0^2}{4}\right\| + \left\|\frac{p_0^2}{4}\right\| \left[7e + \frac{164v^{15} - 12v^{12} - 103v^2 - (14v^7 - 7e)^2}{16(v - v^{14})^2}\right]
\]

\[
\leq \left\|\frac{q_0^2}{4}\right\|^2 + 7\left\|\frac{p_0^2}{4}\right\|^2 = 8\delta^2.
\]

Let’s consider the function: \(f(t) = \frac{(t - t^{14})}{(14t^{14} - 7)(1 - t^2)}\), for all \(t > \frac{1}{2\pi}\), \(t \neq 1\). Because \(f'(t) < 0\), so \(f\) is decreasing.

Hence, in view of Lemma 4.2, we obtain:

\[
\delta(x+s+,\mu) < 4\sqrt{1 - \delta^2} - (1 - \delta^2)^{\frac{11}{14}}\frac{1}{(14(1 - \delta^2)^{\frac{8}{7}} - 7)\delta^2}\|e - v_+^2\|.
\]

Then, from (21) and (22) we deduce:

\[
\delta(x+s+,\mu) < 32\sqrt{1 - \delta^2} - (1 - \delta^2)^{\frac{11}{14}}\frac{1}{(14(1 - \delta^2)^{\frac{8}{7}} - 7)\delta^2}
\]

Now, if we take \(f(\delta) = 32\sqrt{1 - \delta^2} - (1 - \delta^2)^{\frac{11}{14}}\frac{1}{(14(1 - \delta^2)^{\frac{8}{7}} - 7)\delta^2}\), then we obtain that \(f(\delta) < f(\frac{1}{2\pi}) < 18\), and we conclude that

\[
\delta(x+s+,\mu) \leq 18\delta^2.
\]

This completes the proof. □
Lemma 4.4. Let $\delta = \delta(xs, \mu)$. Then, the duality gap satisfies

$$(x_+)^T s_+ \leq \mu(n + 6\delta^2),$$

for all $n \in \mathbb{N}^*$.

Proof. From (16) we have:

$$v^2 + vp_v = \frac{6v^{15} + v^2}{14v^{\frac{7}{4}} - 7e} = \frac{6v^{15} + v^2 + (14v^{\frac{7}{4}} - 7e) - (14v^{\frac{7}{4}} - 7e)}{14v^{\frac{7}{4}} - 7e} = e + \frac{14v^{\frac{7}{4}} - 7e}{14v^{\frac{7}{4}} - 7e} \frac{164}{4} \left[ \frac{4(6v^{15} + v^2 - 14v^{\frac{7}{4}} + 7e)(14v^{\frac{7}{4}} - 7e)}{64(v - v^{\frac{15}{4}})} \right] = e + \frac{164}{4} \left[ \frac{84v^{15} - 14v^{\frac{7}{4}} + 14v^{\frac{7}{4}} - 14v^{\frac{7}{4}} - 7e - 14v^{\frac{7}{4}} - 7e}{16(v - v^{\frac{15}{4}})} \right] = e + \frac{164}{4} \left[ \frac{96(v - v^{\frac{15}{4}})^2 + 164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2}{16(v - v^{\frac{15}{4}})^2} \right] = e + \frac{164}{4} \left[ 6e + \frac{164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2}{16(v - v^{\frac{15}{4}})^2} \right].$$

We have $6e + \frac{164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2}{16(v - v^{\frac{15}{4}})^2} \leq 6e$, because

$$\frac{164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2}{16(v - v^{\frac{15}{4}})^2} \leq 0, \text{ for all } v \geq 0.15e.$$

We can demonstrate this last inequality easily. Indeed, let us suppose that there exists $v > \frac{1}{2} e$ such that:

$$\frac{164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2}{16(v - v^{\frac{15}{4}})^2} > 0$$

then

$$164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} - (14v^{\frac{7}{4}} - 7e)^2 > 0$$

which is equivalent to

$$164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}} > (14v^{\frac{7}{4}} - 7e)^2$$

therefore

$$\frac{164v^{\frac{7}{4}} - 103v^2 - 12v^{\frac{7}{4}}}{(14v^{\frac{7}{4}} - 7e)^2} > e,$$
contradiction, because we know that: \( \forall v > \frac{1}{2^7} e, \frac{164v^{15} - 103v^2 - 12v^{17}}{(14v^2 - 7e)^2} \leq e, \) so:

\[
\frac{164v^{15} - 103v^2 - 12v^{17}}{16(v - v^{17})^2} \leq 0, \quad \forall v > \frac{1}{2^7} e,
\]

this gives:

\[
v^2 + vp_v \leq e + 6\frac{p_v^2}{4} \tag{23}
\]

Finally, using (15) and (23), we obtain:

\[
x_T^T s_+ \leq \mu(n + 6\delta^2).
\]

The next lemma shows that the algorithm is well defined.

**Lemma 4.5.** Let \( \delta = \delta(x, s; \mu) < \frac{1}{2^7}, v > \frac{1}{2^7} e \) and \( \mu_+ = (1 - \theta)\mu, \) where \( 0 < \theta < 1, \) then:

\[
v_+ = \sqrt{\frac{x_+ s_+}{\mu_+}} > \frac{1}{2^7} e,
\]

and

\[
\delta(x_+ s_+, \mu_+) < \frac{4}{7} \sqrt{1 - \delta^2} \left( (1 - \theta)\frac{\nu}{2} - (1 - \delta^2)\frac{\nu}{2} \right)(8\delta^2 + \nu) \frac{2(1 - \theta)^2}{2 - 2\sqrt{1 - \theta}(1 - \delta^2)\frac{\nu}{2} - (1 - \theta)^2} + (1 - \theta)^\frac{1}{2} (1 - \delta^2)^\frac{1}{2}.
\]

Moreover, if \( \delta < \frac{1}{8} \) and \( \theta = \frac{1}{10\sqrt{n}}, \) then:

\[
\delta(x_+ s_+, \mu_+) < \frac{1}{8}.
\]

**Proof.** Using Lemma 4.3 we have \( v_+ > \frac{1}{2^7} e. \) From \( v_+ = \sqrt{\frac{x_+ s_+}{\mu_+}} \) it follows that

\[
v_+ = \sqrt{\frac{x_+ s_+}{\mu_+}} = \sqrt{\frac{x_+ s_+}{\mu(1 - \theta)}} = \frac{1}{\sqrt{1 - \theta}} v_+ > \frac{1}{2^7} e. \tag{24}
\]

This last inequality follows from \( 0 < \theta < 1 \Rightarrow \frac{1}{\sqrt{1 - \theta}} > 1. \)

Now, from the definition of \( \delta, \) we write:

\[
\delta(v_+) = \frac{\|p_{v_+}\|}{2} = \frac{1}{2} \left\| \frac{8v_+ - 8v_+^{17}}{14v_+^{17} - 7e} \right\| = 4 \left\| \frac{(v_+ - v_+^{17})}{(14v_+^{17} - 7e)(e - v_+^{17})} \right\|, \tag{25}
\]
Let us compute the three expressions of the previous norm, from (24) we obtain:
\[
\begin{align*}
\hat{v}_{++} - \hat{v}_{++} &= v_{++} (e - v_{++}^2) \\
&= \frac{v_+}{\sqrt{1 - \theta}} \left( e - \frac{v_+^2}{(1 - \theta)^{\frac{\theta}{2}}} \right) \\
&= \frac{v_+}{(1 - \theta)^{\frac{\theta}{2}}} \left( 1 - \theta \right) \hat{z} e - v_+^2. \\
(14v_{++}^2 - 7e) (e - v_{++}^2) &= \left[ \frac{14v_+^2 - 7(1 - \theta)^{\frac{\theta}{2}} e}{(1 - \theta)^{\frac{\theta}{2}}} \right] \left[ \frac{(1 - \theta) e - v_+^2}{(1 - \theta)} \right] \\
&= \frac{14(1 - \theta) v_+^2 - 14v_+^2 - 7(1 - \theta) \frac{\theta}{2} e + 7(1 - \theta) \hat{z} v_+^2}{(1 - \theta)^{\frac{\theta}{2}}}.
\end{align*}
\]

From (27) and (28), we have
\[
(14v_{++}^2 - 7e) (e - v_{++}^2) = \left[ \frac{14v_+^2 - 7(1 - \theta)^{\frac{\theta}{2}} e}{(1 - \theta)^{\frac{\theta}{2}}} \right] \left[ \frac{(1 - \theta) e - v_+^2}{(1 - \theta)} \right] = \frac{14(1 - \theta) v_+^2 - 14v_+^2 - 7(1 - \theta) \frac{\theta}{2} e + 7(1 - \theta) \hat{z} v_+^2}{(1 - \theta)^{\frac{\theta}{2}}}.
\]

Then, from (26), (28) and (29), we obtain:
\[
\begin{align*}
\frac{(v_+ - v_{++}^2)(e - v_{++}^2)}{(14v_{++}^2 - 7e) (e - v_{++}^2)} &= \left[ \frac{v_+(1 - \theta)^{\frac{\theta}{2}} \left( 1 - \theta \right) \hat{z} e - v_+^2}{(1 - \theta)} \right] \left[ \frac{(1 - \theta) e - v_+^2}{(1 - \theta)} \right] \\
&= \frac{v_+ \left( 1 - \theta \right) \hat{z} e - v_+^2 \left( 1 - \theta \right) e - v_+^2}{\sqrt{1 - \theta} \left( 14(1 - \theta) v_+^2 - 14v_+^2 - 7(1 - \theta) \frac{\theta}{2} e + 7(1 - \theta) \hat{z} v_+^2 \right)}.
\end{align*}
\]

Let us consider the function
\[
f(x) = \frac{x \left( 1 - \theta \right) \hat{z} - x \hat{z}}{\sqrt{1 - \theta} \left( 14(1 - \theta) x \hat{z} - 14x \hat{z} - 7(1 - \theta) \frac{\theta}{2} e + 7(1 - \theta) \hat{z} x \hat{z} \right)}, \quad x > \frac{1}{2 \hat{z}}.
\]

We have \( f'(x) < 0 \), then the function \( f \) is decreasing for all \( x > \frac{1}{2 \hat{z}} \). From Lemma 4.2, (25) and (30), we deduce that
\[
\delta(x_{++}, s_{++}, \mu_{++}) < 4 \frac{\sqrt{1 - \theta^2} \left( 1 - \theta \right) \hat{z} - \sqrt{1 - \theta^2} \hat{z}}{\sqrt{1 - \theta} \left( 14(1 - \theta) \sqrt{1 - \theta^2} - 14\sqrt{1 - \theta^2} \hat{z} - 7(1 - \theta) \frac{\theta}{2} + 7(1 - \theta) \hat{z} (1 - \delta^2) \right)}.
\]
According to (21), we get:
\[
\| (1 - \theta) e - v_+^2 \| \leq \| e - v_+^2 \| + \| \theta e \| \leq 8\delta^2 + \theta \sqrt{n}.
\] (32)
Hence, by using (31) and (32) we get
\[
\delta(x^+s^+, \mu^+) < \frac{4}{7} \left[ \sqrt{1 - \delta^2 (1 - \theta)^{\frac{7}{2}} (1 - \theta^2)^{\frac{7}{2}}} \right] \left( 8\delta^2 + \theta \sqrt{n} \right),
\]
which proves the first part of the lemma.
Now, suppose that \( \delta < \frac{1}{8} \) and \( \theta = \frac{1}{10\sqrt{n}} \). Let’s consider the function
\[
f(\delta) = \frac{\sqrt{1 - \delta^2 (1 - \theta)^{\frac{7}{2}} (1 - \theta^2)^{\frac{7}{2}}} - \frac{11}{14}}{2(1 - \theta)^{\frac{7}{2}} (1 - \delta^2)^{\frac{7}{2}} - 2\sqrt{1 - \theta (1 - \delta^2)^{\frac{11}{2}} - (1 - \theta)^{\frac{11}{2}} + (1 - \theta)^{\frac{11}{2}} (1 - \delta^2)}}
\]
we obtain that \( f'(\delta) > 0 \), so \( f \) is increasing for each \( \delta < \frac{1}{8} \), then
\[
f(\delta) \leq f\left( \frac{1}{8} \right),
\] (33)
where
\[
f\left( \frac{1}{8} \right) = \frac{a\sqrt{1 - \theta^\frac{7}{2}} - a^\frac{11}{14}}{2a^\frac{7}{2}\sqrt{1 - \theta^3} - 2a^\frac{11}{2}\sqrt{1 - \theta - \sqrt{1 - \theta^\frac{11}{2}}} + 2a^2\sqrt{1 - \theta^\frac{11}{2}}},
\]
such as \( a = \sqrt{1 - \frac{1}{8^2}} \), \( \theta = \frac{1}{10\sqrt{n}} \). Using \( n \geq 1 \) we get \( \theta \leq \frac{1}{10} \), which is equivalent to:
\[
f\left( \frac{1}{8} \right) \leq 0.89.
\] (34)
Moreover we have:
\[
8\delta^2 + \theta \sqrt{n} = 8\delta^2 + \frac{1}{10} < \frac{1}{8} + \frac{1}{10} = \frac{9}{40},
\] (35)
Finally, using (33), (34) and (35), we get:
\[
\delta(x^+s^+, \mu^+) < \frac{4}{7} f(\delta)(8\delta^2 + \theta \sqrt{n}) \Leftrightarrow \delta(x^+s^+, \mu^+) < \frac{4}{7} \times 0.89 \times \frac{9}{40} = 0.1144
\]
\[
\Leftrightarrow \delta(x^+s^+, \mu^+) < \frac{1}{8},
\]
which completes the second part of the proof.

The next lemma gives a bound on the number of iterations.

**Lemma 4.6.** Assume that the pair \((x^0, s^0)\) is strictly feasible, \( \mu^0 = \frac{(x^0)^T s^0}{n} \) and \( \delta(x^0 s^0, \mu^0) < \frac{1}{3^2} \). Moreover, let \( x^k \) and \( s^k \) be the vectors obtained after \( k \) iterations. Then the inequality \((x^k)^T s^k < \epsilon\) is satisfied when
\[
k \geq \frac{1}{\theta} \log \left[ \frac{\mu^0(n + \frac{3}{\sqrt{2}})}{\epsilon} \right]
\]
Proof. After $k$ iterations we have $\mu^k = (1 - \theta)^k \mu^0$. From Lemma 4.4 and $\delta(x, \mu) < \frac{1}{2^4}$, we get

$$(x^k)^T s^k \leq \mu^k \left[ n + \frac{3}{\sqrt{2}} \right] = \mu^0 (1 - \theta)^k \left[ n + \frac{3}{\sqrt{2}} \right].$$

Hence, the inequality $(x^k)^T s^k < \varepsilon$ holds if

$$\mu^0 (1 - \theta)^k \left[ n + \frac{3}{\sqrt{2}} \right] \leq \varepsilon$$

$$\iff \log (1 - \theta)^k + \log \mu^0 \left[ n + \frac{3}{\sqrt{2}} \right] \leq \log \varepsilon$$

$$\iff -k \log (1 - \theta) \geq \log \frac{\mu^0 \left[ n + \frac{3}{\sqrt{2}} \right]}{\varepsilon}.$$

As $\theta \leq -\log (1 - \theta)$, we see that the last inequality is valid only if

$$k \theta \geq \log \frac{\mu^0 \left[ n + \frac{3}{\sqrt{2}} \right]}{\varepsilon}$$

$$\iff k \geq \frac{1}{\theta} \log \frac{\mu^0 \left[ n + \frac{3}{\sqrt{2}} \right]}{\varepsilon}.$$

This completes the proof. \hfill \Box

Theorem 4.7. Suppose that $x^0 = s^0 = e$. If we consider the default values for $\theta$ and $\beta$, we obtain that the algorithm 3.1 requires no more than

$$10 \sqrt{n} \log \left( n + \frac{3}{\sqrt{2}} \right)$$

iterations. The resulting vectors satisfy $(x^k)^T s^k < \varepsilon$.

Proof. Since $x^0 = s^0 = e$. Replacing $\mu^0 = \frac{(x^0)^T s^0}{n}$ by 1 and $\theta$ by $\frac{1}{10\sqrt{n}}$ in Lemma 4.6, the result holds. \hfill \Box

Remark 4.8. Note that in the approach of Darvay and Takács [9], the default values for $\theta$ and $\beta$ are equals to $\frac{1}{12\sqrt{n}}$ and $\frac{1}{10}$, respectively. Moreover, the complexity of their algorithm is

$$12 \sqrt{n} \log \left( n + \frac{4}{\varepsilon} \right)$$

5. NUMERICAL EXPERIMENTS

In this section, we present comparative numerical tests of Algorithm 3.1 with two different choices of the function $\psi$. We consider our new function $\psi(t) = t^2$ proposed in this paper and the one of Darvay and Takács [9] defined by $\psi(t) = t^2$. 
Table 1. Comparative results for Example 5.1.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\psi(t) = t^2$</th>
<th>$\psi(t) = t^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T(s)$</td>
<td>$T(s)$</td>
</tr>
<tr>
<td>0.2</td>
<td>51 0.010870</td>
<td>51 0.015540</td>
</tr>
<tr>
<td>0.5</td>
<td>19 0.007314</td>
<td>18 0.004542</td>
</tr>
<tr>
<td>0.7</td>
<td>17 0.005312</td>
<td>14 0.003982</td>
</tr>
<tr>
<td>0.9</td>
<td>17 0.005221</td>
<td>14 0.003890</td>
</tr>
</tbody>
</table>

Our objective is to evaluate the efficiency of Algorithm 3.1 by using these two functions differently and to see the appropriateness of our function with respect to that of Darvay and Takács.

In the numerical tests, we consider three fixed size examples and one variable size example. After that, we solve some problems from the Netlib test collection [15]. The used language is Matlab R2009b with the following data:

- The accuracy parameter $\epsilon = 10^{-4}$.
- $\theta = 0.2, 0.5, 0.7, 0.9$ for the first three fixed size examples.
- $\theta = 0.7$ for the variable size example and the Netlib problems.

The obtained results will be presented in comparative tables where we note by:

- $k$: the number of iterations necessary for optimality.
- $T(s)$: the execution time in seconds.

5.1. Examples with fixed size

Let us consider the following LO problems:

**Example 5.1.**

$$ A = \begin{pmatrix}
-1 & 1 & 1 & -1 & 0 & 0 \\
0 & 2 & -3 & 2 & 0 & 1 \\
-3 & 2 & 1 & 0 & 0 & 1 \\
3 & 5 & 4 & 0.5 & 0 & 0 & 0
\end{pmatrix}, \ b = (1 \ 2 \ 1 \ 12.5)^T, $n$
$$

$c = (2 \ -9 \ -2 \ -0.5 \ 0 \ 0 \ 0)^T$.  

The starting points are:

- $x^0 = (1 \ 1 \ 1 \ 1 \ 1 \ 1)^T$,
- $y^0 = (-1 \ -1 \ -1 \ -1)^T$,
- $s^0 = (1 \ 1 \ 1 \ 1 \ 1 \ 1)^T$.

The optimal solutions are:

- $x^* = (0.8726 \ 1.5605 \ 0.4968 \ 0.1847 \ 0.0000 \ 0.0000 \ 0.0000)^T$,
- $y^* = (-2.1145 \ -1.1401 \ -0.6306 \ -0.6688)^T$,
- $s^* = (0.0000 \ 0.0000 \ 0.0000 \ 0.0000 \ 2.1145 \ 1.1401 \ 0.6306)^T$.

In Table 1, we summarize some numerical results of Example 5.1 for different choice of the parameter $\theta$.

**Example 5.2.**

$$ A = \begin{pmatrix}
1 & 0 & -4 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
5 & 3 & 1 & 0 & -1 & 3 & 0 & 1 & 0 & 0 & 0 \\
4 & 5 & -3 & 3 & -4 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 2 & 1 & -5 & 0 & 0 & 1 & 0 & 0 \\
-2 & 1 & 1 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 0 \\
2 & -3 & 2 & -1 & 4 & 5 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}, $n$
$$

$b = (3 \ 12 \ 7 \ -2 \ 6 \ 10)^T, \ c = (-9 \ -4 \ 4 \ -7 \ -2 \ -6 \ 0 \ 0 \ 0 \ 0 \ 0)^T$.  


The starting points are:

\[ x^0 = (1 1 1 1 1 1 1 1 1 1 1 1 1 1 1)^T, \]
\[ y^0 = (-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1)^T, \]
\[ s^0 = (1 1 1 1 1 1 1 1 1 1 1 1 1 1 1)^T. \]

The optimal solutions are:

\[ x^* = (1.1331 1.0394 1.2487 1.5490 1.1690 1.0455 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000)^T, \]
\[ y^* = (-1.1100 -0.7942 -0.8447 -0.7029 -0.5467 -0.8167)^T, \]
\[ s^* = (0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.1100 0.7942 0.8447 0.7029 0.5467 0.8167)^T. \]

In Table 2, we summarize some numerical results of Example 5.2 for different choice of the parameter \( \theta \).

### Table 2. comparative results for Example 5.2.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \psi(t) = t^2 )</th>
<th>( \psi(t) = t^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( T(s) )</td>
<td>( k )</td>
</tr>
<tr>
<td>0.2</td>
<td>53</td>
<td>0.017075</td>
</tr>
<tr>
<td>0.5</td>
<td>20</td>
<td>0.009839</td>
</tr>
<tr>
<td>0.7</td>
<td>18</td>
<td>0.006380</td>
</tr>
<tr>
<td>0.9</td>
<td>17</td>
<td>0.006038</td>
</tr>
</tbody>
</table>

In Table 3, we summarize some numerical results of Example 5.3 for different choice of the parameter \( \theta \).
Table 3. Comparative results for Example 5.3.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\psi(t) = t^2$</th>
<th>$\psi(t) = t^\frac{7}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$T(s)$</td>
<td>$k$</td>
</tr>
<tr>
<td>0.2</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>0.5</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>0.7</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>0.9</td>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>

**Comment 1**

The numerical results show that the number of iterations and the execution time necessary for optimality of the Algorithm depends on the values of the parameter $\theta$. It is quite surprising that $\theta = 0.9$ give the best number of iteration and the minimal time.

The number of iterations required for optimality by means of our function $\psi(t) = t^\frac{7}{4}$ is much lower than that of the function $\psi(t) = t^2$. Also the computation time is slightly improved in $\psi(t) = t^\frac{7}{4}$ compared to $\psi(t) = t^2$.

### 5.2. Example with variable size

**Example 5.4. (Cube example)**

Let $n = 2m$, where $m$ is the number of constraints, $n$ is the number of variables.

$$A(i,j) = \begin{cases} 1 & \text{if } j = i \text{ or } j = i + m \\ 0 & \text{otherwise.} \end{cases}$$

$$b(i) = 2 \quad \text{for } i = 1, \ldots, m$$

$$c(i) = \begin{cases} -1 & \text{if } i = 1, \ldots, m \\ 0 & \text{if } i = m + 1, \ldots, n \end{cases}$$

The starting points are:

$$x^0(i) = 1 \quad \text{for } i = 1, \ldots, n$$

$$s^0(i) = \begin{cases} 1 & \text{if } i = 1, \ldots, m \\ 2 & \text{if } i = m + 1, \ldots, n \end{cases}$$

$$y^0(i) = -2 \quad \text{for } i = 1, \ldots, m.$$  

The optimal solutions are:

$$x^*(i) = \begin{cases} 2 & \text{if } i = 1, \ldots, m \\ 0 & \text{if } i = m + 1, \ldots, n \end{cases}$$

$$y^*(i) = -1 \quad \text{for } i = 1, \ldots, m.$$  

$$s^*(i) = \begin{cases} 0 & \text{if } i = 1, \ldots, m \\ 1 & \text{if } i = m + 1, \ldots, n. \end{cases}$$

The optimal value is:

$$z^* = -2m.$$  

The obtained results are summarized in the Table 4.
Table 4. Comparative results for the cube example.

<table>
<thead>
<tr>
<th>(m, n)</th>
<th>(\psi(t) = t^2)</th>
<th>(\psi(t) = t^\frac{2}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>k (T(s))</td>
<td>k (T(s))</td>
<td></td>
</tr>
<tr>
<td>(50, 100)</td>
<td>22 0.148007</td>
<td>18 0.134152</td>
</tr>
<tr>
<td>(100, 200)</td>
<td>23 0.974729</td>
<td>19 0.845180</td>
</tr>
<tr>
<td>(300, 600)</td>
<td>24 24.048228</td>
<td>20 17.570769</td>
</tr>
<tr>
<td>(400, 800)</td>
<td>25 50.886173</td>
<td>21 42.042491</td>
</tr>
<tr>
<td>(500, 1000)</td>
<td>25 97.205637</td>
<td>21 80.990312</td>
</tr>
<tr>
<td>(1000, 2000)</td>
<td>26 743.577535</td>
<td>22 689.511535</td>
</tr>
</tbody>
</table>

Table 5. Numerical results for Netlib set problems.

<table>
<thead>
<tr>
<th>Problem</th>
<th>(m, n)</th>
<th>(\psi(t) = t^2)</th>
<th>(\psi(t) = t^\frac{2}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k (T(s))</td>
<td>k (T(s))</td>
<td></td>
</tr>
<tr>
<td>afiro</td>
<td>(27, 51)</td>
<td>22 0.0941</td>
<td>19 0.0828</td>
</tr>
<tr>
<td>sc50a</td>
<td>(50, 78)</td>
<td>16 0.2909</td>
<td>11 0.1956</td>
</tr>
<tr>
<td>sc50b</td>
<td>(50, 78)</td>
<td>16 0.3073</td>
<td>11 0.1325</td>
</tr>
<tr>
<td>blend</td>
<td>(74, 114)</td>
<td>24 0.9493</td>
<td>20 0.6779</td>
</tr>
<tr>
<td>share2b</td>
<td>(96, 162)</td>
<td>27 2.3279</td>
<td>23 1.9154</td>
</tr>
<tr>
<td>scsd1</td>
<td>(77, 760)</td>
<td>23 60.3563</td>
<td>19 48.7907</td>
</tr>
<tr>
<td>bandm</td>
<td>(305, 472)</td>
<td>29 71.4231</td>
<td>26 51.2125</td>
</tr>
<tr>
<td>Scsd6</td>
<td>(141, 1350)</td>
<td>23 377.7166</td>
<td>19 302.2165</td>
</tr>
</tbody>
</table>

5.3. Netlib problems

The obtained numerical results for some Netlib problems are summarized in Table 5.

Comment 2

The numerical results show the efficiency of our new function for problems with large size as well as for the Netlib collection. Indeed, the number of iterations and the computation time are naturally reduced in the case of \(\psi(t) = t^\frac{2}{3}\) compared to the case of \(\psi(t) = t^2\). The obtained results confirm and consolidate our theoretical purposes.

6. Conclusion

We have described a new primal-dual path-following method to solve linear programs. Our approach is a reconsideration of [9] with using a new function \(\psi(t) = t^\frac{2}{3}\). We showed that the obtained algorithm solves the problem in polynomial time. Moreover, we provided some numerical experiments that prove the efficiency of our proposed algorithm. Future research might extended the algorithm for other optimization problems such as semidefinite problems, quadratic problems and complementarity problems.

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References


