ONLINE SALES MODE SELECTION OF COMPETING SUPPLIERS IN A FRESH PRODUCT SUPPLY CHAIN

GUOLI WANG* AND YUJIE HAO

Abstract. This paper studies the selection of sales mode in a fresh product supply chain consisting of two competing suppliers and an e-tailer providing online platform. Suppliers can choose an online sales mode and provide freshness-keeping service. Considering the product freshness level, platform commission, supplier’s market share and cross-price coefficient between two channels, we study three sales scenarios and obtain the optimal decision and profit of the two suppliers and the e-tailer. Then, we get some conclusions through theoretical and numerical analysis. The lower the commission rate charged by online platform, the less the e-tailer likes the marketplace mode. The best action of the suppliers is also related to this commission. The lower the commission rate, the more the suppliers tend to choose the marketplace mode. And their best action is different under different sales scenarios. If the two suppliers adopt different modes, the one who chooses the marketplace mode will have more advantage. We also get an interesting conclusion that when suppliers make the same choice, the commission does not affect their competition and decision-making.

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1. Introduction

In recent years, online retail industry and e-commerce are developing rapidly in the world, which changes the consumption mode of buyers and the sales mode of sellers. Online shopping has become a part of consumers’ daily life and online sales is already a major channel for many suppliers. In China, e-tailers reached 11% in 2015, with a total value of more than 3.2 trillion yuan [32]. In 2016, the sales of e-tailers in the United States increased by 14.86%, accounting for nearly half of the sales growth of retail departments in the United States [11]. Online retail transactions exceeded $3.5 trillion in 2019, which is accounting for 14.1% of total worldwide retail transactions. And in 2023, this proportion is expected to exceed 22% [40].

In addition, with the increasing demand of consumers for fresh products, fresh product suppliers have gradually begun to use online platforms for sales in order to seek higher profits. In 2005, the establishment of Yiguo in Shanghai marked the birth of China’s first e-commerce for fresh food. Since then, Meituan, Freshhippo, JD.com and other fresh e-commerce gradually developed. In 2019, the transaction scale of the fresh market reached 1789.7 billion yuan, of which the fresh e-commerce accounted for 141.8 billion yuan. The online market continued to improve, reaching 7.9%, and will reach 29.7% in 2022.

Keywords. Fresh product supply chain, sales mode selection, online platform, competing suppliers, Stackelberg game.

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Suppliers usually sell products through e-commerce in two modes, known as reselling mode and marketplace mode [14, 39]. In the reselling mode, fresh product suppliers wholesale their products to e-tailers (such as Meituan and JD.com), which then set retail prices and sell them to consumers. In contrast, the marketplace mode provides an online marketplace. Fresh product suppliers set prices independently and sell through this online marketplace, and then pay fees to the e-commerce platform [28]. However, can these suppliers benefit from their choice of sales mode? In fact, it has always been the focus of competing suppliers to select the best sales mode so as to obtain greater benefits in the cooperation with e-commerce.

In practice, supply chain members will interact with each other. The sales mode of a supplier using an online platform will have an impact on other suppliers using the same platform, especially when they choose different sales mode [27]. Therefore, the pricing and revenue of a supplier are related to the choice of sales mode of other suppliers. In addition, the pricing and revenue of e-tailers offering online platforms are also influenced by the choice of sales mode of competing suppliers using the platform.

Therefore, the choice of sales mode of fresh product suppliers has a significant impact on the pricing and profit of all supply chain members. This drives us to explore the sales mode selection of competing suppliers from the following perspectives.

1. How do different sales mode combinations in the market affect the pricing and profit of supply chain members?
2. As the leader of fresh product supply chain, which sales mode do suppliers prefer to choose?
3. Which sales mode combination do the e-tailer providing an online platform prefer?

To address these questions, we study a fresh product supply chain consisting of two upstream suppliers and a downstream e-tailer. Suppliers can choose a sales mode (reselling mode or marketplace mode) to sell fresh product through cooperation with the e-tailer. The e-tailer provides an online platform on which both the e-tailer and the two suppliers can sell products directly. According to the selection of sales modes of suppliers, there are the following three alternative scenarios. (a) RR: In this scenario, both suppliers cooperate with the e-tailer in reselling mode; (b) RM: In this scenario, supplier 1 cooperates with the e-tailer in reselling mode while the supplier 2 cooperates with the e-tailer in marketplace mode; (c) MM: In this scenario, both suppliers cooperate with the e-tailer in marketplace mode.

Based on the study of the sales mode selection in the online marketplace, we add the exploration of fresh products and competing suppliers. Our work has the following contributions. The preferences of the suppliers and the e-tailer are affected by the commissions charged by online platform. The lower the commission rate, the more suppliers tend to select the marketplace mode, while the e-tailer prefers the reselling mode. Moreover, the sales scenario also affects the best actions and profits of suppliers and e-tailer. If the two suppliers choose different sales modes, the profit of the one who chooses reselling mode well lower than that of the other. In addition, product’s freshness level affects the profits of the two suppliers and the e-tailer. The higher the freshness level, the higher the profit of e-tailer, and the suppliers’ profit increases only when the freshness-keeping cost is low enough.

The rest of this paper is arranged as follows. We briefly introduce relevant literature in Section 2. Section 3 shows the specific model establishment and solution, and analyzes the influence of exogenous parameters on equilibrium decision and profit of each supply chain member. Section 4 compares and analyzes the scenarios of three different sales mode combinations, and puts forward the corresponding management views. Finally, we sort out the research results and future research directions in Section 5.

2. Literature review

Our research mainly focuses on the pricing and the choices of competing suppliers’ online sales mode in a fresh product supply chain. The following two groups of literature are relevant to this research: (a) fresh supply chain management, (b) selection of online sales mode.
2.1. Fresh supply chain management

Fresh product supply chain management has been extensively studied, including the following areas: pricing strategy [21], supply chain design [4], supply chain coordination [29,33] and logistics and transportation [25,35]. The most relevant field of this paper is the research of supply chain design and product pricing. Yu et al. [37] studied the cold-chain strategy and pricing strategy in a fresh agricultural product supply chain, and further discussed the influence of vertical integration and horizontal integration design on supply chain decision-making in the presence of competing retailers. They found that horizontal integration is harmful to both third-party logistics providers and suppliers, and whether it can improve retailers’ profits depends on the substitutability of products. Vertical integrated supply chain has more advantages in reducing the quality and quantity loss of agricultural product. Cai et al. [5] discussed the influence of freshness-keeping service on order quantity, freshness level, market demand and pricing strategy, and further studied the optimal strategy in decentralized and centralized decision-making model. Lou et al. [18] discussed the logistics strategy in a retailer-led fresh supply chain, in which the retailer decides whether to provide logistics services by himself or logistics service to a third-party logistics provider. Ma et al. [19] discussed the pricing strategy of a multi-echelon fresh supply chain composed of a 3PL, a supplier and a retailer under the condition of asymmetric demand information.

In recent years, the online retail industry has developed rapidly in the world, and scholars have further expanded the application scope of e-commerce. Song et al. [22] explored the omni-channel strategy of fresh product retailers taking into account the natural loss of products and additional loss in the store when facing online and offline customers. They found that when the proportion of offline consumers is low, the omni-channel supply chain has more advantages for both retailers and consumers. He et al. [8] explored the online pre-sale mode of fresh agricultural products by discussing the pricing and order strategies of a competing online retailer and a physical fresh food store. Tang and Yang [23] studied the financing and pricing strategies of a fresh product supply chain consisting of a supplier and a capital-constrained retailer, in which the retailer sells products to consumers in both offline and online channels.

The above literature mainly studies the cooperation mode between fresh product suppliers and reselling retailers or e-commerce. However, in practice, the e-commerce industry has developed rapidly, which has led to e-tailers in the form of agency sales (also known as marketplace form) gradually occupying a large part of the online sales market. Therefore, in our research, we consider the cooperation between fresh product suppliers and e-tailers supporting the above two sales modes, and the impact of these suppliers’ different choices of sales mode on the fresh product supply chain.

2.2. Selection of online sales mode

With the scholars’ attention to the e-commerce industry, a large number of literature have studied two sales modes supported by e-tailers: reselling mode and marketplace mode [3, 26, 30]. These studies mainly focus on two aspects: whether to introduce the marketplace mode and how to choose the sales mode. Yan et al. [31] considered two kinds of online spillover effects, platform cost and consumer utility to explore the impact of introducing marketplace channel into the existing model on the benefit and decision-making of supply chain members. They found that manufacturers’ willingness to introduce marketplace channel increases, while e-tailers’ willingness decreases when considering the spillover effect. In addition, higher platform fees do not necessarily encourage e-tailers to join the marketplace channel. Zhang and Ma [38] discussed how to integrate the logistics service strategy and whether the supplier and platform can further introduce the marketplace mode in the case of cooperation in the reselling mode. Huang et al. [9] studied the choice of online channel sales mode of manufacturer and the decision of e-tailer to introduce private brand when consumers may wait strategically. Tian et al. [24] regarded e-tailer as an online marketplace connecting suppliers and consumers, and discussed the sales strategies of suppliers in this online marketplace.

In addition, competition is an inevitable topic. Chen et al. [6] discussed the sales strategy of the manufacturer and two e-tailers in the case of considering the retail competition. They found that in the absence of retail competition, both e-tailers tend to reselling and achieve “win–win”. However, the mixed sales mode (i.e. one
Table 1. Similarities and differences between this paper and related literature.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Product</th>
<th>Supply chain members</th>
<th>Sales mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang et al. [40]</td>
<td>General product</td>
<td>One manufacturer and one e-tailer</td>
<td>Reselling/marketplace</td>
</tr>
<tr>
<td>Xu et al. [30]</td>
<td>General product</td>
<td>One manufacturer and one online platform</td>
<td>Reselling/marketplace</td>
</tr>
<tr>
<td>Zhang et al. [38]</td>
<td>General product</td>
<td>One supplier and one online platform</td>
<td>Reselling/marketplace</td>
</tr>
<tr>
<td>Li et al. [12]</td>
<td>General product</td>
<td>One manufacturer, one online platform and one 3PL</td>
<td>Reselling/marketplace</td>
</tr>
<tr>
<td>Liu et al. [16]</td>
<td>Fresh product</td>
<td>One supplier and two retailers</td>
<td>Reselling/marketplace</td>
</tr>
<tr>
<td>Liu et al. [15]</td>
<td>Fresh product</td>
<td>One supplier and one e-tailer</td>
<td>Reselling</td>
</tr>
<tr>
<td>Yang et al. [34]</td>
<td>Fresh product</td>
<td>One supplier and one retailer</td>
<td>Reselling</td>
</tr>
<tr>
<td>This paper</td>
<td>Fresh product</td>
<td>Two suppliers and one e-tailer</td>
<td>Reselling/marketplace</td>
</tr>
</tbody>
</table>

e-tailer chooses marketplace mode and the other chooses reselling mode) can lead to Pareto improvement in the case of intense competition. Considering the competition between online and offline channel, Pu et al. [20] discussed the pricing and sales strategies of the manufacturer in online channel under three power structures of offline channel. Liu et al. [16] discussed the e-platform’s choice of sales strategy for fresh products in online channel in order to compete with an offline retailer in consideration of the traceability goodwill of blockchain and product freshness. Liu et al. [17] discussed that a reseller and an agent seller cooperating with the same manufacturer try a new cooperation mode in order to expand the market, and explored pricing and channel selection under a hybrid retail strategy.

Different from the existing research, we extend the research on the sales mode of e-tailer to the field of fresh products and competing suppliers. Considering the level of product freshness, we establish a game model including a fresh product supply chain, in which competing suppliers cooperating with the same e-tailer decide the sales mode to discuss the impact of the combination of different sales modes and sales strategy of fresh product suppliers on the decision-making of supply chain members. Table 1 summarizes the similarities and differences between relevant literature and this paper from multiple perspectives.

Finally, we use the Stackelberg game model to simulate and solve the optimal decisions of two fresh product suppliers and an e-tailer in the market. The Stackelberg game model considers the order of enterprise decision-making and reflects the asymmetric competition between enterprises, typically including a leader and followers. In the Stackelberg game process, considering both the strategies of the leader and followers, the leader first determines the optimal strategy, and then the followers choose their own optimal strategy based on the leader’s strategy. In order to realize the profit maximization or cost minimization of each member in the supply chain model, many scholars have considered the Stackelberg game model. Yu and Xiao [36] established a Stackelberg game model for a supplier-led fresh agricultural product supply chain. By solving and comparing the optimal profits of supply chain members, they studied the impact of outsourcing mode on the equilibrium strategy and profits of supply chain members. Jia and Li [10] established a Stackelberg game model for a closed-loop supply chain with the manufacturer as the leader and the e-tailer as the follower, and analyzed how the manufacturer chooses the best sales mode for new and remanufactured products. Ghosh et al. [7] established a Stackelberg game model for a supplier-led green supply chain and discussed supply chain coordination strategies under different payment strategies.

In summary, we contribute to the literature on sales modes of e-tailers by discussing the choice of online sales modes of competing fresh product suppliers. We assume that fresh product suppliers will bear all the freshness-keeping costs, and analyze the decision-making and benefit of supply chain members under three sales scenarios: both suppliers choose reselling mode (RR), one supplier chooses marketplace mode and the other chooses reselling mode (RM) and both suppliers choose marketplace mode (MM). In addition, we also obtain the law that the decision-making and benefit of the two suppliers and the e-tailer are affected by product freshness level, platform commission, supplier’s market share and the cross-price coefficient between the two channels.
### Table 2. Symbols and descriptions.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(j = RR, RM, MM)</td>
<td>Alternative sales scenarios</td>
</tr>
<tr>
<td>(a)</td>
<td>The potential market size for the fresh product ((a - (1 - \delta)(p_1 + p_2) &gt; 0))</td>
</tr>
<tr>
<td>(\rho)</td>
<td>The market share of supplier 1 ((0 &lt; \rho &lt; 1))</td>
</tr>
<tr>
<td>(k)</td>
<td>Coefficient of freshness-keeping service cost ((k &gt; 0))</td>
</tr>
<tr>
<td>(\theta)</td>
<td>The freshness level of the product ((\theta &gt; 0))</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Cross-price coefficient between two channels ((0 &lt; \delta &lt; 1))</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Consumer sensitivity to freshness level ((\beta &gt; 0))</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Commission rate ((0 &lt; \tau &lt; 1))</td>
</tr>
<tr>
<td>(w_i^j)</td>
<td>Supplier (i)'s wholesale price under scenario (j), (i = 1, 2)</td>
</tr>
<tr>
<td>(p_i^j)</td>
<td>Retail price of supplier (i)'s product under scenario (j), (i = 1, 2)</td>
</tr>
<tr>
<td>(D_i^j)</td>
<td>The market demand of supplier (i)'s product under scenario (j), (i = 1, 2)</td>
</tr>
<tr>
<td>(\pi_i^j)</td>
<td>Supplier (i)'s profit under scenario (j), (i = 1, 2)</td>
</tr>
<tr>
<td>(\pi_e^j)</td>
<td>E-tailer’s profit under scenario (j)</td>
</tr>
</tbody>
</table>

### 3. The model

#### 3.1. Problem description and parameter definition

We consider a fresh product supply chain consisting of two upstream suppliers and a downstream e-tailer. The two suppliers choose a sales mode to sell fresh products in cooperation with the e-tailer. The e-tailer provides an online platform on which both the e-tailer and the two suppliers can directly sell products. In practice, there are usually two alternative sales modes in the online market. In the reselling mode, supplier \(i\) sells fresh product to the e-tailer at a wholesale price \(w_i\), then the e-tailer sells this fresh product at a retail price \(p_i\). In the marketplace mode, supplier \(i\) directly sells product at a retail price \(p_i\) through the online platform, and then shares \(\tau(0 < \tau < 1)\) proportion of revenue with the e-tailer as commission. Commission rates are not easily variable in actual market transactions and are usually set in advance. Therefore, we assume that the parameter \(\tau\) is exogenous, which is widely used in research \([13, 24]\). The notations used in this paper are listed in Table 2.

We use freshness level to express the quality of product. High freshness level means low decay rate of fresh products, resulting in a high market demand. To avoid spoilage and satisfy consumer preferences, the two suppliers will make the freshness-keeping service. We assume that \(k\theta^2\) represents the cost of freshness-keeping service with freshness level \(\theta\) and \(k(k > 0)\) is referred to as the two suppliers’ freshness-keeping service cost factor. The form of quadratic function can reflect the general situation of the change of freshness-keeping cost, that is, the higher the freshness, the more difficult it is to improve the freshness, and the more the cost will increase, which has been used in previous studies \([15, 34]\). A higher \(k\) means that the supplier has to pay more in order to reach the expected freshness-keeping service level.

According to the selection of sales modes of suppliers, there are the following three alternative scenarios, as shown in Figure 1.

1. **RR**: In this scenario, both suppliers cooperate with the e-tailer in reselling mode. First, the two suppliers determine the wholesale price \(w_1^{RR}\) and \(w_2^{RR}\) respectively. Then, the e-tailer determines the retail price \(p_1^{RR}\) and \(p_2^{RR}\) for the two suppliers’ fresh product.
In this scenario, the supplier 1 cooperates with e-tailer in reselling mode while the supplier 2 cooperates with the e-tailer in marketplace mode. First, the supplier 1 determines wholesale price $w_{1 \text{RM}}$. Then, the e-tailer and supplier 2 set retail prices $p_{1 \text{RM}}$ and $p_{2 \text{RM}}$ for their fresh product respectively.

(3) MM: In this scenario, both suppliers cooperate with the e-tailer in marketplace mode. The two suppliers determine the retail price $p_{1 \text{MM}}$ and $p_{2 \text{MM}}$ respectively.

Based on the above description and assumptions, we get the respective demand functions of the product of the two suppliers as follows:

$$D_1 = \rho a - p_1 + \delta p_2 + \beta \theta,$$

$$D_2 = (1 - \rho)a - p_2 + \delta p_1 + \beta \theta,$$

where $p_i$ is the retail price of supplier $i$’s product ($i = 1, 2$). The parameter $a$ represents the potential market size for the fresh product. We assume that $a - (1 - \delta)(p_1 + p_2) > 0$, which ensures that market demand is nonnegative without taking into account the freshness level. Essentially, it maintains the existence of the market. $\rho(0 < \rho < 1)$ is the market share of supplier 1, then the market share of supplier 2 is expressed as $1 - \rho$. The closer $\rho$ approaches 1 means the larger the supplier 1’s market share and the greater his market power. $\delta$ is used to describe the intensity of price competition between two channels. We assume that $0 < \delta < 1$, which ensures that the retail price of a competing product have a negative impact on the market demand for the specified product, and that market demand is less sensitive to the retail price of a competing product than to the retail price of this product. And the closer the $\delta$ is to 1 means more competition between the products of the two suppliers. $\theta(\theta > 0)$ is freshness level of the fresh product, and $\beta(\beta > 0)$ indicates demand sensitivity with respect to freshness level and the freshness of the product has a positive impact on the market demand. Without loss of generality, we normalized production and sales costs to zero for the suppliers and the e-tailer, which is common in previous studies [1, 40].

In this study, similar to Alaei et al. [2], we establish a Stackelberg game model to describe the interaction between upstream and downstream in the fresh supply chain. We set the two suppliers as leaders and e-tailer as follower. In the following, by assuming that the sales modes are predetermined, we discuss the RR, RM and MM scenarios in turn.
3.2. RR scenario

Under the RR scenario, both suppliers cooperate with the e-tailer in reselling mode. That is, the e-tailer wholesales fresh products from the two suppliers and then retails them to consumers. Combined with the demand functions, their profit functions are

\[
\pi_1 = w_1 D_1 - k \theta^2, \\
\pi_2 = w_2 D_2 - k \theta^2, \\
\pi_e = (p_1 - w_1)D_1 + (p_2 - w_2)D_2.
\]

(3) (4) (5)

For the given wholesale prices \( w_1 \) and \( w_2 \), we first determine the equilibrium retail prices \( p_1^* \) and \( p_2^* \) which would maximize the e-tailer’s profit function \( \pi_e^* \). Then, the two suppliers’ wholesale price \( w_1 \) and \( w_2 \) are determined by maximizing the suppliers’ profit functions \( \pi_1^* \) and \( \pi_2^* \). All of our equilibrium solutions are shown in Lemma 1, and the proofs of all lemmas in this paper are provided in the Appendix A.

**Lemma 1.** The Stackelberg equilibrium wholesale prices, retail prices, demands and profits under the RR scenario are as follows:

\[
w_1^* = \frac{(\delta + 2)\beta \theta + a\delta(1 - \rho) + 2\rho a}{4 - \delta^2},
\]

\[
w_2^* = \frac{(\delta + 2)\beta \theta + 2a(1 - \rho) + \delta \rho a}{4 - \delta^2},
\]

\[
p_1^* = \frac{(2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 2\delta^3 + 5\delta)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8},
\]

\[
p_2^* = \frac{(6 - 2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 3\delta^2)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8},
\]

\[
D_1^* = \frac{(\delta + 2)\beta \theta - (\delta \rho - \delta - 2\rho)a}{8 - 2\delta^2},
\]

\[
D_2^* = \frac{(\delta + 2)\beta \theta + (\delta \rho - 2\rho + 2)a}{8 - 2\delta^2},
\]

\[
\pi_1^* = \frac{((1 - \rho)\delta + 2\rho)a + (\delta + 2)\beta \theta^2}{2(\delta^2 - 4)^2} - k \theta^2,
\]

\[
\pi_2^* = \frac{((\delta - 2\rho + 2)a + \beta \theta(\delta + 2))^2}{2(\delta^2 - 4)^2} - k \theta^2,
\]

\[
\pi_e^* = \frac{2\beta \theta(\beta \theta + a)(\delta + 1)(\delta + 2)^2 + (2(\delta - 1)(\delta - 2)\rho(1 - \rho) + 5\delta^2 + 4)a^2}{64 - 4\delta^6 + 36\delta^4 - 96\delta^2}.
\]

We compare the wholesale prices, retail prices and demands of the product of the two suppliers and their respective profits under the RR scenario. The results are as Proposition 1 and the proofs of all propositions in this article are provided in the Appendix A.

**Proposition 1.** Under RR scenario, comparing the product parameters of the two suppliers and their profits, we have:

\[
w_1^* > w_2^*, p_1^* > p_2^*, D_1^* > D_2^*, \pi_1^* > \pi_2^*, \text{ when } \rho > \frac{1}{2}.
\]

\[
w_1^* \leq w_2^*, p_1^* \leq p_2^*, D_1^* \leq D_2^*, \pi_1^* \leq \pi_2^*, \text{ otherwise.}
\]

Proposition 1 shows that the wholesale price, retail price, demand and profit of supplier 1 are higher than that of supplier 2 when the supplier 1 has more than half of the market share. Otherwise, the result is the opposite. This is easy to understand because the higher market share, the higher demand, and the more power
the supplier has to set a high wholesale price, all of which led to a greater profit for this supplier. This high wholesale price then forces the e-tailer to set a high retail price to offset costs. Therefore, in the market where suppliers all choose reselling mode, suppliers will strive to improve market share to obtain higher initiative and profits.

Next, we discuss in detail the influence of exogenous parameters such as the freshness level, the supplier’s market share and the cross-price coefficient.

**Proposition 2.** Under the RR scenario, the influence of the freshness level on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

(1) \( \frac{\partial \pi_{1}^{RR} \ast}{\partial \rho} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} > 0, \frac{\partial \pi_{1}^{RR} \ast}{\partial \rho} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} > 0, \frac{\partial \pi_{1}^{RR} \ast}{\partial \rho} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} > 0; \)

(2) \( \frac{\partial \pi_{1}^{RR} \ast}{\partial \rho} > 0; \) \( \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} < 0, \) when \( a < \left( \frac{\beta^2 - 2(\delta - 2\delta^2)k}{\beta(\delta - 2\delta^2)} \right) \) and \( k > \frac{\beta^2}{2(\delta - 2\delta^2)}; \frac{\partial \pi_{1}^{RR} \ast}{\partial \rho} \geq 0, \) otherwise;

(3) \( \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} < 0, \) when \( a < \left( \frac{\theta(\delta + 2)(2\delta^2 - 2\delta + 8)k - \beta^2}{\beta(2 + (\delta - 2\delta^2))} \right) \) and \( k > \frac{\beta^2}{2(\delta - 2\delta^2)}; \frac{\partial \pi_{2}^{RR} \ast}{\partial \rho} \geq 0, \) otherwise.

According to Proposition 2 (1), all wholesale price, retail price and demand of the two suppliers’ product are positively correlated with freshness level under RR scenario. This is because the improvement of freshness level reduces the decay rate, and freshness sensitive consumers are willing to buy more and to pay more for those products. Therefore, the e-tailer raises retail prices to obtain higher profit, which also stimulates suppliers to increase wholesale prices.

According to Proposition 2 (2), the profit of the e-tailer is positively correlated with freshness level under RR scenario. Combined with the result of Proposition 2 (1), this is mainly due to increasing retail prices and demands. For the two suppliers, when the freshness-keeping cost coefficient \( k \) is large and the market size \( a \) is small, their profits are negatively correlated with product freshness. Otherwise, they are positively correlated. This is easy to understand because there is a cost to keep product fresh. Higher freshness level means higher freshness-keeping cost for suppliers. Therefore, when the cost is higher than the profit brought to the supplier by freshness-keeping, the supplier’s profit will decrease. Freshness-keeping effort at this time is uneconomical for the supplier. At the same time, the demand is limited in the market which has a small market size, resulting in more significant profit constraints for suppliers, so the profits are more sensitive to the cost. Therefore, suppliers will only provide freshness-keeping services to increase profits in small markets when the cost of freshness-keeping is low, while e-tailers have always wanted suppliers to do so.

**Proposition 3.** Under the RR scenario, the influence of the market share of supplier 1 on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

(1) \( \frac{\partial \pi_{1}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} > 0, \frac{\partial \pi_{1}^{RR} \ast}{\partial \pi_{2}^{RR} \ast} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{2}^{RR} \ast} > 0, \frac{\partial \pi_{1}^{RR} \ast}{\partial \pi_{2}^{RR} \ast} > 0; \)

(2) \( \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} > 0, \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} < 0; \) \( \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} > 0, \) when \( \rho > 1/2; \frac{\partial \pi_{2}^{RR} \ast}{\partial \pi_{1}^{RR} \ast} \leq 0, \) otherwise.

According to Proposition 3 (1), the wholesale price, retail price and demand of supplier 1’s product are positively correlated with supplier 1’s the market share in the RR scenario. The larger the market share of supplier 1, the stronger the competitiveness and the larger the transaction volume. As a result, supplier 1 has more incentive to raise wholesale price, which leads to a high retail price. Supplier 2 is the complete opposite.

According to Proposition 3 (2), the profit of supplier 1 is positively related to his market share, while the profit of supplier 2 is negatively related to this market share in the RR case. This is easy to understand that the greater the product price and trading volume of supplier 1, the greater supplier 1’s profit will be. Supplier 2 is the complete opposite. For the e-tailer, the profit is positively related to supplier 1’s market share when the market share is more than half. Otherwise, there is a negative correlation. In other words, the e-tailer has
the smallest profit when the two suppliers have equal market shares. This is because both suppliers have the same market power and competition between them is weak, so it is difficult for e-tailer to benefit. Therefore, suppliers will strive to increase market share to improve profits, and e-tailers prefer the market with uneven market share of upstream suppliers.

**Proposition 4.** Under the RR scenario, the influence of the cross-price coefficient between two channels on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. \( \frac{\partial w^\text{RR}_1}{\partial \delta} > 0, \frac{\partial w^\text{RR}_2}{\partial \delta} > 0; \frac{\partial p^\text{RR}_1}{\partial \delta} > 0, \frac{\partial p^\text{RR}_2}{\partial \delta} > 0; \frac{\partial D^\text{RR}_1}{\partial \delta} > 0, \frac{\partial D^\text{RR}_2}{\partial \delta} > 0; \frac{\partial \pi^\text{RR}_1}{\partial \delta} > 0, \frac{\partial \pi^\text{RR}_2}{\partial \delta} > 0. \)

Therefore, the fiercer the channel competition, the higher the price and demand of product, and the more suppliers and the e-tailer can benefit from it. Based on this, suppliers and e-tailers are more inclined to choose a more competitive market in order to obtain higher benefits.

### 3.3. RM scenario

Under the RM scenario, the supplier 1 cooperates with the e-tailer in reselling mode while the supplier 2 cooperates with the e-tailer in marketplace mode. That is, the e-tailer wholesales fresh products from supplier 1 and then retails them to consumers. At the same time, the e-tailer charges a commission from supplier 2 and allows him to retail products directly on e-commerce platform. Combined with the demand functions, their profit functions are

\[
\begin{align*}
\pi_1 &= w_1D_1 - k\theta^2, \\
\pi_2 &= (1 - \tau)p_2D_2 - k\theta^2, \\
\pi_e &= (p_1 - w_1)D_1 + \tau p_2D_2.
\end{align*}
\]

For the given wholesale price \( w^\text{RM}_1 \), we first determine the equilibrium retail prices \( p^\text{RM}_1 \) and \( p^\text{RM}_2 \) which would maximize the e-tailer’s profit function \( \pi^\text{RM}_e \) and supplier 2’s profit function \( \pi^\text{RM}_2 \), respectively. Then, the supplier 1’s wholesale price \( w^\text{RM}_1 \) is determined by maximizing the profit function \( \pi^\text{RM}_1 \). All of our equilibrium solutions are shown in Lemma 2.

**Lemma 2.** The Stackelberg equilibrium wholesale price, retail prices, demands and profits under the RM scenario are as follows:

\[
\begin{align*}
w^\text{RM}_1 &= \frac{(1 - \tau)\delta((1 - \rho)a + \beta\theta) + (2 - \delta^2\tau)(a\rho + \beta\theta)}{4 - 2\delta^2}, \\
p^\text{RM}_1 &= \frac{((\rho - 1)a - \beta\theta)((\tau + 1)\delta^3 - (\tau + 3)\delta) - (a\rho + \beta\theta)((\tau + 2)\delta^2 - 6)}{(4 - (\tau + 1)\delta^2)(2 - \delta^2)}, \\
p^\text{RM}_2 &= \frac{(a\rho + \beta\theta)(6\delta - (\tau + 2)\delta^3) + (8 - (\tau + 3)\delta^2)((1 - \rho)a + \beta\theta)}{2(\delta^2\tau + \delta^2 - 4)(\delta^2 - 2)}, \\
D^\text{RM}_1 &= \frac{(\delta^2\tau - 2)(a\rho + \beta\theta) + ((\rho - 1)a - \beta\theta)(1 - \tau)\delta}{(2\tau + 2)\delta^2 - 8}, \\
D^\text{RM}_2 &= \frac{(a\rho + \beta\theta)(6\delta - (\tau + 2)\delta^3) + (8 - (\tau + 3)\delta^2)((1 - \rho)a + \beta\theta)}{2(\delta^2\tau + \delta^2 - 4)(\delta^2 - 2)}, \\
\pi^\text{RM}_1 &= \frac{((\delta^2\tau - 2)(a\rho + \beta\theta) + ((\rho - 1)a - \beta\theta)(1 - \tau)\delta)^2}{4(\delta^2\tau + \delta^2 - 4)(\delta^2 - 2)} - k\theta^2,
\end{align*}
\]
Figure 2. Comparisons of the two supplier’s profits under RM scenario.

\[
\pi_2^{RM^*} = \left(1 - \tau\right)\left((\tau + 2)\delta^3 - 6\delta\right)(a\rho + \beta\theta) - ((\rho - 1)a - \beta\theta)((\tau + 3)\delta^2 - 8)\right)^2 - k\theta^2,
\]

\[
\pi_e^{RM^*} = \frac{4(\delta^2\tau + \delta^2 - 4)(\delta^2 - 2)^2}{(4(\tau + 1)\delta^2 - 4)(\delta^2 - 2)} \frac{((\delta^2\tau - 2)(a\rho + \beta\theta) - (a\rho - a - \beta\theta)(\tau\delta - \delta))(6 - (\tau + 2)\delta^2)}{+ (a\rho - a - \beta\theta)((\delta^3 - \delta)(\tau + \delta^3 - 3\delta))} - \frac{\tau((\tau + 2)\delta^3 - 6\delta)(a\rho + \beta\theta) - (a\rho - a - \beta\theta)((\tau + 3)\delta^2 - 8))}{2((\tau + 1)\delta^2 - 4)^2(\delta^2 - 2)^2}.
\]

We compare the retail prices of the product of the two suppliers and their respective profits under RM scenario. The result is as Proposition 5.

Proposition 5. Under RM scenario, comparing the product parameters of the two suppliers and their profits, we have:

\[ p_1^{RM^*} < p_2^{RM^*}, \text{ when } a > a_1 \text{ and } \rho < \rho_1; \quad p_1^{RM^*} \geq p_2^{RM^*}, \text{ otherwise.} \]

With \[ a_1 = \frac{\beta\theta(\delta + 2)(\delta^3 - \delta^2 - 2)}{(3\tau + 4)(\rho - 2\tau - 2)^3 + ((-3\tau + 1)\rho + 3)\delta^3 + ((-2\tau + 12)\rho + 2\tau + 6)\delta + 20}\rho + 8, \quad \rho_1 = \frac{(2\delta^3 - 2\delta^2 - 2\delta)(\tau + 2\delta^3 - 3\delta^2 - 6\delta + 8)}{(3\delta^3 - 3\delta^2 - 2\delta)(\tau + 2\delta^3 - 7\delta^2 - 12\delta + 20)}. \]

Proposition 5 shows that in the market with large scale and small market share of supplier 1, the retail price of the supplier 1’s product is lower than the retail price of supplier 2’s product. Otherwise, the result is the opposite. This is slightly different from the RR scenario. This is because the smaller the market share of supplier 1, the smaller the pricing initiative the supplier has, resulting in a lower retail price. This phenomenon is more obvious when the market size is large.

As can be seen from the Figure 2, when the commission rate is not extremely high, the profit of supplier 2 is higher than that of supplier 1 in the RM scenario. In other words, when the two suppliers choose different sales modes, the profit of the one who chooses the marketplace mode is much higher than that of the other. Based on this, in the market of mixed sales mode, suppliers are more willing to choose the marketplace mode to obtain greater advantages.

Next, we discuss in detail the influence of exogenous parameters such as the freshness level, the supplier’s market share, the cross-price coefficient and the commission rate.
ONLINE SALES MODE SELECTION OF COMPETING SUPPLIERS IN A FRESH PRODUCT SUPPLY CHAIN

Figure 3. The influence of $\rho$ on the profit of the e-tailer.

**Proposition 6.** Under the RM scenario, the influence of the freshness level on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. $\frac{\partial w_{\text{RM}*1}}{\partial \theta} > 0; \frac{\partial p_{\text{RM}*1}}{\partial \theta} > 0; \frac{\partial D_{\text{RM}*1}}{\partial \theta} > 0; \frac{\partial D_{\text{RM}*2}}{\partial \theta} > 0;$
2. $\frac{\partial \pi_{\text{RM}*1}}{\partial \theta} > 0; \frac{\partial \pi_{\text{RM}*2}}{\partial \theta} > 0,$ when $k < k_1; \frac{\partial \pi_{\text{RM}*2}}{\partial \theta} \leq 0,$ otherwise; $\frac{\partial \pi_{\text{RM}*1}}{\partial \theta} \leq 0,$ otherwise; $\frac{\partial \pi_{\text{RM}*2}}{\partial \theta} \leq 0.$

According to Proposition 6 (1), all wholesale price, retail price and demand of product from the two suppliers are positively correlated with freshness level under RM scenario. This result is the same as RR scenario. This is because the improvement of freshness level reduces the rate of decay, and freshness sensitive consumers are willing to buy more products and pay more for them. Therefore, supplier 2 and e-tailer raise retail prices to obtain higher profits, which also stimulates supplier 1 to increase wholesale price.

According to Proposition 6 (2), the profit of the e-tailer is positively correlated with freshness level under RM scenario. This is mainly due to increasing retail prices and demands. For the two suppliers, when the freshness-keeping cost coefficient $k$ is small, their profits are positively correlated with freshness level. This is easy to understand because it costs to keep the product fresh. Higher freshness level means higher freshness-keeping cost for suppliers. Therefore, when the cost is higher than the profit brought to the supplier by freshness-keeping, the supplier’s profit will decrease. Freshness-keeping effort at this time is uneconomical for the supplier. Based on this, suppliers will only provide freshness-keeping services when the cost of freshness-keeping is low, while e-tailers firmly hope that suppliers can provide this service.

**Proposition 7.** Under the RM scenario, the influence of the market share of supplier 1 on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. $\frac{\partial w_{\text{RM}*1}}{\partial \rho} > 0; \frac{\partial p_{\text{RM}*1}}{\partial \rho} > 0; \frac{\partial p_{\text{RM}*2}}{\partial \rho} < 0; \frac{\partial D_{\text{RM}*1}}{\partial \rho} > 0; \frac{\partial D_{\text{RM}*2}}{\partial \rho} < 0;$
2. $\frac{\partial \pi_{\text{RM}*1}}{\partial \rho} > 0, \frac{\partial \pi_{\text{RM}*2}}{\partial \rho} < 0.$

According to Proposition 7 (1), the wholesale price, retail price and demand of product of supplier 1 are positively related to his market share in the RM scenario. The larger the supplier 1’s market share, the greater
transaction volume. As a result, the supplier 1 has more power to raise wholesale price, which leads to a high retail price. Supplier 2 is the complete opposite.

According to Proposition 7 (2), the profit of supplier 1 is positively related to the market share of supplier 1, while the profit of supplier 2 is negatively related to this market share in the RM scenario. This is the same as the RR scenario. Then, we set the default values of \( a = 200, \beta = 5, \theta = 10, k = 10, \delta = 0.3, \tau = 0.2 \), and analyze the impact of supplier 1’s market share on the e-tailer’s profit through function graph. According to Figure 3, we get that the profit of the e-tailer may first decrease and then increase with the market share of supplier 1, and reach the lowest at about half of this market share. This is because when the two suppliers’ market powers are similar, the competition between them is very weak, so it is difficult for e-tailer to benefit. Therefore, suppliers will try to increase their market share to increase profits, no matter whether they choose marketplace mode or reselling mode, and e-tailers are more willing to avoid the market with balanced market share of suppliers.

**Proposition 8.** Under the RM scenario, the influence of the cross-price coefficient between two channels on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. \( \frac{\partial w_{1,RM}^*}{\partial \delta} > 0; \frac{\partial p_{1,RM}^*}{\partial \delta} > 0; \frac{\partial p_{2,RM}^*}{\partial \delta} > 0; \frac{\partial D_{1,RM}^*}{\partial \delta} > 0; \frac{\partial D_{2,RM}^*}{\partial \delta} > 0; \)
2. \( \frac{\partial \pi_{1,RM}^*}{\partial \delta} > 0, \frac{\partial \pi_{2,RM}^*}{\partial \delta} > 0. \)

It can be seen from Proposition 8 and the Figure 4 that all wholesale price, retail price and demand of product and the profit of the two suppliers and the e-tailer are positively related to the cross-price coefficient between two channels \( \delta \). This result is consistent with the RR scenario. Therefore, the fiercer the channel competition, the higher the price and demand of product, and the more suppliers and the e-tailer can benefit from it. Therefore, suppliers, no matter which sales mode they choose, and e-tailers are more willing to enter the more competitive market to obtain higher profits. The function graphs are obtained by setting the default values of \( a = 200, \beta = 5, \theta = 10, k = 10, \rho = 0.5, \tau = 0.2 \).

**Proposition 9.** Under the RM scenario, the influence of the commission rate on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. \( \frac{\partial w_{1,RM}^*}{\partial \tau} < 0; \frac{\partial p_{1,RM}^*}{\partial \tau} > 0; \frac{\partial p_{2,RM}^*}{\partial \tau} > 0; \frac{\partial D_{1,RM}^*}{\partial \tau} < 0; \frac{\partial D_{2,RM}^*}{\partial \tau} > 0; \)
2. \( \frac{\partial \pi_{1,RM}^*}{\partial \tau} < 0, \frac{\partial \pi_{2,RM}^*}{\partial \tau} < 0. \)
According to Proposition 9 (1), in RM scenario, with the increase of commission rate, the wholesale price and demand of supplier 1’s product decrease, the demand of supplier 2’s product increases, and both retail prices increase. This is because supplier 2 pays commission to e-tailer to use the online platform. The higher commission rate increases the cost of supplier 2, forcing him to raise retail prices to reduce losses. The e-tailer then raises the retail price of product wholesale from supplier 1 to achieve price competition, resulting in lower price sensitive demand. Interestingly, the demand for the product of supplier 2 continues to increase with the commission rate, which is enough to show that in the mixed sales market, suppliers who choose the marketplace mode have greater advantages. At the same time, with the increase of commission rate, the e-tailer prefers supplier 2 who cooperates in the marketplace mode. Thus, supplier 1 has to set a low wholesale price to attract the e-tailer to cooperate.

According to Proposition 9 (2), the profits of both suppliers are negatively correlated with the commission rate in the RM scenario. Combined with Proposition 9 (1), it is easy to understand because lower wholesale price and demand will reduce the profit of supplier 1. The higher the commission rate, the higher the platform fee the supplier 2 needs to pay, the lower his profit will be. Then, we set the default values of $a = 200, \beta = 5, \theta = 10, k = 10, \delta = 0.3, \rho = 0.5$, and get the Figure 5 to discuss the impact of the commission rate on the e-tailer’s profit. As can be seen from the figure that the e-tailer’s profit may be positively related to the commission rate. This is easy to understand because the higher the commission rate, the more commission the e-tailer receives, the more benefits he obtains through reselling, the higher his profit will be. Therefore, in the mixed sales market, suppliers will choose the marketplace mode as much as possible to obtain greater advantages and cooperate with e-tailers with lower commission rate to ensure revenue. On the contrary, e-tailers will try to increase the commission rate as much as possible.

3.4. MM scenario

Under the MM scenario, both suppliers cooperate with the e-tailer in marketplace mode. That is, the e-tailer charges a commission from the two suppliers and allows them to retail products directly on e-commerce platform. Combined with the demand functions, their profit functions are

\[
\pi_1 = (1 - \tau)p_1D_1 - k\theta^2, \\
\pi_2 = (1 - \tau)p_2D_2 - k\theta^2, \\
\pi_e = \tau(p_1D_1 + p_2D_2).
\]

We get the retail price, $p_1^{\text{MM}}$ and $p_2^{\text{MM}}$, by maximizing the profits of both suppliers, $\pi_1^{\text{MM}}$ and $\pi_2^{\text{MM}}$, simultaneously. Then, all of our equilibrium solutions are shown in Lemma 3.
Lemma 3. The Stackelberg equilibrium prices, demands and profits under the MM scenario are as follows:

\[
\begin{align*}
    p_1^{MM^*} &= \frac{((1-\rho)\delta+2\rho)a+\beta\theta(\delta+2)}{4-\delta^2}, \\
    p_2^{MM^*} &= \frac{((\delta-2)\rho+2)a+\beta\theta(\delta+2)}{4-\delta^2}, \\
    D_1^{MM^*} &= \frac{((1-\rho)\delta+2\rho)a+\beta\theta(\delta+2)}{4-\delta^2}, \\
    D_2^{MM^*} &= \frac{((\delta-2)\rho+2)a+\beta\theta(\delta+2)}{4-\delta^2}, \\
    \pi_1^{MM^*} &= (1-\tau)\left((2-\delta)\rho+\alpha\delta+\beta\theta(\delta+2)\right)^2 - k\theta^2, \\
    \pi_2^{MM^*} &= (1-\tau)\left((\delta-2)\rho+2\alpha+\beta\theta(\delta+2)\right)^2 - k\theta^2, \\
    \pi_e^{MM^*} &= \tau\left((2\rho(\rho-1)\delta+\delta^2+4)\rho+2\beta\theta(\beta+\alpha)(\delta+2)^2\right). \\
\end{align*}
\]

We compare the retail prices and demands of product of the two suppliers and their respective profits under MM scenario. The results are as Proposition 10.

Proposition 10. Under MM scenario, comparing the product parameters of the two suppliers and their profits, we have:

\[
\begin{align*}
    p_1^{MM^*} > p_2^{MM^*}, D_1^{MM^*} > D_2^{MM^*}, \pi_1^{MM^*} > \pi_2^{MM^*}, \text{ when } \rho > \frac{1}{2}; \\
    p_1^{MM^*} \leq p_2^{MM^*}, D_1^{MM^*} \leq D_2^{MM^*}, \pi_1^{MM^*} \leq \pi_2^{MM^*}, \text{ otherwise.}
\end{align*}
\]

Proposition 10 shows that the retail price, demand and profit of supplier 1 are higher than that of supplier 2 when the supplier 1 has more than half of the market share. Otherwise, the result is the opposite. This result is consistent with RR scenario. Similar to the RR scenario, the higher the market share leads to the higher the demand for product, the more power the supplier has to set a higher retail price and obtain higher revenue. Based on this, in a market where suppliers all choose the marketplace mode, suppliers will try to obtain a higher market share to set retail prices that are more beneficial to them, and then obtain higher profits.

Next, we discuss in detail the influence of exogenous parameters such as the freshness level, the supplier’s market share, the cross-price coefficient and the commission rate.

Proposition 11. Under the MM scenario, the influence of the freshness level on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

1. \(\frac{\partial p_1^{MM^*}}{\partial \delta} > 0, \quad \frac{\partial p_2^{MM^*}}{\partial \delta} > 0, \quad \frac{\partial D_1^{MM^*}}{\partial \delta} > 0, \quad \frac{\partial D_2^{MM^*}}{\partial \delta} > 0;\)
2. \(\frac{\partial \pi_1^{MM^*}}{\partial \delta} > 0, \quad \frac{\partial \pi_2^{MM^*}}{\partial \delta} > 0, \quad \frac{\partial \pi_e^{MM^*}}{\partial \delta} > 0, \quad \text{when } k < \frac{\beta(1-\tau)(\alpha(\rho+\beta+\theta)\delta+2\alpha(1-\rho)+2\beta\theta)}{\theta(\delta+2)(\delta+2)}, \quad \frac{\partial \pi_1^{MM^*}}{\partial \delta} \leq 0, \quad \text{otherwise};\)
3. \(\frac{\partial \pi_2^{MM^*}}{\partial \delta} > 0, \quad \text{when } k < \frac{\beta(1-\tau)(\alpha(\rho+\beta+\theta)\delta+2\alpha(1-\rho)+2\beta\theta)}{\theta(\delta+2)(\delta+2)}, \quad \frac{\partial \pi_2^{MM^*}}{\partial \delta} \leq 0, \quad \text{otherwise.}\)

According to Proposition 11 (1), retail price and demand of the product of two suppliers are positively correlated with freshness level under MM scenario. This is because the improvement of freshness level reduces the rate of decay, and freshness sensitive consumers are willing to buy more products and pay more for them. Therefore, the two suppliers raise retail prices to obtain higher profits.

According to Proposition 11 (2), the profit of the e-tailer is positively correlated with freshness level under MM scenario. This is mainly due to increasing retail prices and demands, which increased commissions. For
the two suppliers, when the freshness-keeping cost coefficient \( k \) is small, their profits are positively correlated with product freshness level. This is easy to understand because it costs to keep the product fresh. Higher freshness level means higher freshness cost for suppliers. Therefore, when the cost is higher than the profit brought to the supplier by freshness-keeping, the supplier’s profit will decrease. Freshness-keeping effort at this time is uneconomical for the supplier. Based on this, like the other two scenarios, suppliers will only provide this service when the cost of freshness-keeping is low, while e-tailers always hope that suppliers can do so.

**Proposition 12.** Under the MM scenario, the influence of the supplier 1’s market share on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

(1) \( \frac{\partial p_{1}^{MM}}{\partial \rho} > 0, \frac{\partial p_{2}^{MM}}{\partial \rho} < 0; \frac{\partial D_{1}^{MM}}{\partial \rho} > 0, \frac{\partial D_{2}^{MM}}{\partial \rho} < 0; \)

(2) \( \frac{\partial \pi_{1}^{MM}}{\partial \rho} > 0, \frac{\partial \pi_{2}^{MM}}{\partial \rho} < 0; \frac{\partial \pi_{e}^{MM}}{\partial \rho} > 0, \) when \( \rho > 1/2; \frac{\partial \pi_{e}^{MM}}{\partial \rho} \leq 0, \) otherwise.

Proposition 12 (1) shows that the retail price and demand of product of supplier 1 are positively related to the supplier 1’s market share in the MM scenario. Product of supplier 2 is the complete opposite. This is the same as the RR scenario.

According to Proposition 12 (2), the profit of supplier 1 is positively related to the supplier 1’s market share, while the profit of supplier 2 is related to with this market share in the MM scenario. For the e-tailer, the profit is positively related to the supplier 1’s market share when this market share is more than half. Otherwise, there is a negative correlation. In other words, the e-tailer has the smallest profit when two suppliers have equal market shares. As in the RR scenario, when the two suppliers are completely symmetrical, it is difficult for e-retail to benefit. This result is completely consistent with the RR scenario. Suppliers will increase market share to increase profits, while e-tailers try to choose the market with uneven market share of suppliers to ensure profits.

**Proposition 13.** Under the MM scenario, the influence of the cross-price coefficient between two channels on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

(1) \( \frac{\partial p_{1}^{MM}}{\partial \delta} > 0, \frac{\partial p_{2}^{MM}}{\partial \delta} > 0; \frac{\partial D_{1}^{MM}}{\partial \delta} > 0, \frac{\partial D_{2}^{MM}}{\partial \delta} > 0; \)

(2) \( \frac{\partial \pi_{1}^{MM}}{\partial \delta} > 0, \frac{\partial \pi_{2}^{MM}}{\partial \delta} > 0; \frac{\partial \pi_{e}^{MM}}{\partial \delta} > 0. \)

Proposition 13 shows that the retail price and demand of product and the profit of the two suppliers and the e-tailer are positively related to the cross-price coefficient between two channels \( \delta \). Therefore, the fiercer the channel competition, the higher the price and demand of product, and the more suppliers and the e-tailer can benefit from it. Similar to the results of the other two scenarios, suppliers and e-tailers are more inclined to choose competitive markets to obtain higher profit.

**Proposition 14.** Under the MM scenario, the influence of the commission rate on the equilibrium decisions and profits of the two suppliers and the e-tailer is as follows:

(1) \( \frac{\partial p_{1}^{MM}}{\partial \tau} = 0, \frac{\partial p_{2}^{MM}}{\partial \tau} = 0; \frac{\partial D_{1}^{MM}}{\partial \tau} = 0, \frac{\partial D_{2}^{MM}}{\partial \tau} = 0; \)

(2) \( \frac{\partial \pi_{1}^{MM}}{\partial \tau} < 0, \frac{\partial \pi_{2}^{MM}}{\partial \tau} < 0; \frac{\partial \pi_{e}^{MM}}{\partial \tau} > 0. \)

According to Proposition 14 (1), all retail prices and demands in MM scenario are not affected by commission rate. This result is different from the RM scenario. This is because both suppliers choose the same sales mode and pay the same fee to use the online platform. Therefore, the exogenous commission rate has no impact on the competition between suppliers, nor on their pricing or demand.

According to Proposition 14 (2), the profits of both suppliers are negatively related to the commission rate in MM scenario, while the profit of the e-tailer is positively related to the commission rate. This is easy to understand because the higher the commission rate, the higher the commission the supplier will pay to the
Thus, with the increase of commission rate, the two suppliers’ profits decrease and the e-tailer’s profit increases. Because of this, in a market where all suppliers choose the marketplace mode, suppliers will give priority to e-tailers with lower commission rate for cooperation, and e-tailers will try to improve the commission rate as much as possible.

4. Model comparison

In this section, we compare and analyze the decisions and profits in the three sales scenarios. For convenience, we set the default value of $a = 200, \beta = 5, \theta = 10, k = 10, \delta = 0.3, \rho = 0.5$ to obtain all function graphs in this section. In the following, we make a comparative analysis of the wholesale prices, retail prices, demands, the two suppliers’ strategic choice and the e-tailer’s mode preference under the three sales scenarios in order. The results are shown in the Observation and Proposition respectively.

**Observation 1.** For the supplier 1’s product, when the commission rate is low enough, its wholesale price is higher in the RM scenario; When the commission rate is high, this wholesale price is higher in the RR scenario.

According to the Figure 6, in the RM scenario, when the e-tailer charges a sufficiently low commission, supplier 2 pays very little for using the e-commerce platform, which leads to the e-tailer preferring to cooperate with supplier 1. Therefore, supplier 1 has more power and opportunity to set a high wholesale price. When the e-tailer charges a high commission, the situation is completely opposite. The e-tailer obtain a high profit by cooperating with supplier 2 in the marketplace mode, which causes the supplier 1 having to set a low wholesale price to attract the e-tailer.

**Proposition 15.** By comparing the equilibrium retail prices in the three scenarios, we have:

\begin{align*}
(1) & \quad p_{1\text{RR}}^* > p_{1\text{RM}}^* > p_{1\text{MM}}^*; \\
(2) & \quad p_{2\text{RR}}^* > p_{2\text{RM}}^* > p_{2\text{MM}}^*.
\end{align*}

According to Proposition 15, the retail prices of product of two suppliers are the highest in the RR scenario and the lowest in the MM scenario. The key driver is double marginalization effect. Compared with the RR and RM scenario, the two suppliers in the MM scenario set prices directly, avoiding the double marginalization effect and obtaining the lowest retail prices. In the RM scenario, one supplier sets retail price directly, and the other raises the price twice through the e-tailer. It is slightly affected by the double marginalization effect, resulting in the retail prices in the RM scenario being between the RR scenario and the MM scenario.
Figure 7. Comparisons of the supplier 1’s profits.

In addition, compared with the RR scenario, the retail price of supplier 1 is reduced in the RM scenario where supplier 2 chooses the marketplace mode but supplier 1 does not change his strategy. It shows that the strategy choice of a member will not only affect his own strategy, but also affect the decisions of other supply chain members.

**Proposition 16.** By comparing the equilibrium demands under the three scenarios, we have:

(1) $D_{1}^{MM*} > D_{1}^{RR*} > D_{1}^{RM*}$;
(2) $D_{2}^{RM*} > D_{2}^{MM*} > D_{2}^{RR*}$.

Proposition 16 indicates that for supplier 1, the demand in the MM scenario is the highest, while that in the RM scenario is the lowest. For supplier 2, the demand in the RM scenario is the highest, while that in the RR scenario is the lowest. First, it can be found that MM scenario always have higher demands than RR scenario. This is because consumers’ willingness to buy products is affected by the retail price, which is consistent with our perception. In most cases, the lower the retail price of a product, the higher the consumer’s willingness to pay and the higher the demand for this product. However, the demand level of the products of the two suppliers under the RM scenario does not fully conform to this rule, which is mainly caused by competition in the market of mixed sales. Referring to Proposition 5, in the mixed sales market where the two suppliers have different choices, the one who chooses the marketplace mode (supplier 2 in this paper) has more advantages in retail price. Therefore, more consumers choose the products of supplier 2 and abandon the products of supplier 1, resulting in a large gap in the demand for the products of the two suppliers.

Besides, compared with RR scenario, one of the suppliers in RM scenario changed its sales strategy, resulting in a great change in the demand for both products. This proves once again that the strategic choice of one supply chain member also has an impact on other supply chain members.

**Observation 2.** For the supplier 1: When the commission rate $\tau$ is low enough, the profit in MM scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the RR scenario is the highest.

According to Figure 7, the determination of supplier 1’s optimal strategy is affected by the commission rate. Supplier 1 obtains the highest profit in the MM scenario if the commission charged by the e-tailer is low enough. At this time, supplier 1 only needs to pay a small amount of fees to use the e-commerce platform, so this supplier prefers to choose the marketplace mode. Supplier 1 obtains the highest profit in the RR scenario if the commission charged by the e-tailer is high enough. In this case, choosing reselling mode allows supplier 1 to avoid high commission costs and obtain higher income, so the supplier prefers the reselling mode.
Observation 3. For the supplier 2: When the commission rate $\tau$ is low enough, the profit in RM scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the RR scenario is the highest.

As can be seen from Figure 8, the determination of the optimal strategy of supplier 2 is also affected by the commission rate. Supplier 2’s behavior is similar to supplier 1’s. Supplier 2 obtains the highest profit in RM scenario if the commission charged by the e-tailer is low enough. Similar to supplier 1, supplier 2 pays a little to the e-tailer to use e-commerce platform, so this supplier prefers the marketplace mode. Besides, as the analysis of Proposition 9, if supplier 1 chooses the reselling mode, supplier 2 can benefit from price competition and then achieve higher profit. Therefore, supplier 2 prefers the RM scenario. Supplier 2 obtains the highest profit in the RR scenario if the commission charged by the e-tailer is high enough. This is because supplier 2 has to pay more to use the online platform, so this supplier prefers the reselling mode.

Observation 4. For the e-tailer: When the commission rate $\tau$ is low enough, the profit in RR scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the MM scenario is the highest.

Figure 9 shows that the determination of e-tailer’s optimal strategy is also affected by the commission rate. If the e-tailer sets a sufficiently low commission, he obtains the highest profit in the RR scenario. This is because
the e-tailer charges little to provide online platform to suppliers, leading to low profits. At this time, e-tailers
prefer the reselling market in which both suppliers choose the reselling mode.

If the e-tailer sets a sufficiently high commission, he obtains the highest profit in the MM scenario. This
is because the e-tailer can increase revenue by charging higher fees for offering online platform to suppliers.
Therefore, in this case, e-tailers prefer the market where both suppliers choose the marketplace mode.

5. Extension

In this section, we establish an Stackelberg game model for the fresh product supply chain that includes
three competing suppliers and one e-tailer in an attempt to extend the research conclusions and management
insights of the above model that includes two competing suppliers to a more general situation. To simplify the
model, we assume that the three competing suppliers are symmetrical, which does not affect our conclusions.
Therefore, we obtain the market demand functions for the products of three competing suppliers as follows:

\begin{align*}
D_1 &= a - p_1 + \delta p_2 + \delta p_3 + \beta \theta, \\
D_2 &= a - p_2 + \delta p_1 + \delta p_3 + \beta \theta, \\
D_3 &= a - p_3 + \delta p_1 + \delta p_2 + \beta \theta.
\end{align*}

Each supplier selects one of the reselling and marketplace models to collaborate with the e-tailer to sell fresh
products. Considering the symmetry of the three competing suppliers, four market structures can be formed:
RRR, RRM, RMM, and MMM. To ensure the rationality of the model and solution process, we assume that
0 < \delta < \frac{1}{2}. And then we model and solve the four market structures separately.

5.1. RRR scenario

In the RRR scenario, all three competing suppliers choose to collaborate with the e-tailer through selling
mode. The decision-making order is: firstly, three suppliers simultaneously decide on the wholesale price of their
own products; Then, the e-tailer determines the retail price for each of the three products. The profit functions
of suppliers and the e-tailer are as follows:

\begin{align*}
\pi_1 &= w_1 D_1 - k \theta^2, \\
\pi_2 &= w_2 D_2 - k \theta^2, \\
\pi_3 &= w_3 D_3 - k \theta^2, \\
\pi_e &= (p_1 - w_1) D_1 + (p_2 - w_2) D_2 + (p_3 - w_3) D_3.
\end{align*}

The Stackelberg equilibrium decisions and profits under the RRR scenario are solved as follows:

\begin{align*}
\hat{w}_{RRR} &= \hat{w}_{RRR} = \hat{w}_{RRR} = \frac{-3(\beta \theta + a)^2}{16(2\delta - 1)(\delta - 1)^2}, \\
\hat{p}_{1RRR} &= \hat{p}_{2RRR} = \hat{p}_{3RRR} = \frac{-(\beta \theta + a)(4\delta - 3)}{8\delta^2 - 12\delta + 4}, \\
\hat{\pi}_{1RRR} &= \hat{\pi}_{2RRR} = \hat{\pi}_{3RRR} = \frac{(\beta \theta + a)^2}{8(\delta - 1)^2} - k \theta^2, \\
\hat{\pi}_e &= \frac{-3(\beta \theta + a)^2}{16(2\delta - 1)(\delta - 1)^2}.
\end{align*}

5.2. RRM scenario

In the RRM scenario, supplier 1 and supplier 2 choose the reselling mode, while supplier 3 chooses the marketplace
mode to collaborate with the e-tailer. The decision-making order is: firstly, both supplier 1 and supplier 2
make decisions on the wholesale price of their own products simultaneously; Then, the e-tailer and supplier 3
simultaneously determine retail price for their respective products. The profit functions of suppliers and the
e-tailer are as follows:
\[\begin{align*}
\pi_1 &= w_1 D_1 - k\theta^2, \\
\pi_2 &= w_2 D_2 - k\theta^2, \\
\pi_3 &= (1 - \tau)p_3 D_3 - k\theta^2, \\
\pi_e &= (p_1 - w_1)D_1 + (p_2 - w_2)D_2 + \tau p_3 D_3.
\end{align*}\]

The Stackelberg equilibrium decisions and profits under the RRM scenario are solved as follows:

\[
\begin{align*}
w_1^{\text{RRM}} &= w_2^{\text{RRM}} = \frac{2(-2 + (\tau + 1)\delta^2 + (\tau + 1)\delta)(\beta\theta + a)}{8 + (\tau - 5)\delta^3 + (\tau + 3)\delta^2 + 12\delta}, \\
p_1^{\text{RRM}} &= p_2^{\text{RRM}} = \frac{-(24 + (\tau^2 - 4\tau - 5)\delta^4 + (\tau^2 + 2\tau - 11)\delta^3 + (14\tau + 18)\delta^2 + (-4\tau + 28)\delta)(\beta\theta + a)}{2(-8 + (\tau - 5)\delta^3 + (\tau + 3)\delta^2 + 12\delta)(-2 + (\tau + 1)\delta^2 + 2\delta)}, \\
p_3^{\text{RRM}} &= \frac{(2\delta^4\tau + 2\delta^4 - \delta^3\tau + 5\delta^3 - 3\delta^2\tau - 9\delta^2 - 8\delta + 8)(\beta\theta + a)}{(-8 + (\tau - 5)\delta^3 + (\tau + 3)\delta^2 + 12\delta)(-2 + (\tau + 1)\delta^2 + 2\delta)}, \\
\pi_1^{\text{RRM}} &= \frac{\pi_2^{\text{RRM}}}{\delta^2\tau + \delta^2 + 2\delta - 2}(\delta^2\tau + 5\delta^3 + \delta^2\tau + 3\delta^2 + 12\delta - 8)^2 - k\theta^2, \\
\pi_3^{\text{RRM}} &= \frac{\pi_3^{\text{RRM}}}{\delta^2\tau + \delta^2 + 2\delta - 2}(\delta^2\tau + 5\delta^3 + \delta^2\tau + 3\delta^2 + 12\delta - 8)^2 - k\theta^2, \\
\pi_e^{\text{RRM}} &= \frac{\pi_e^{\text{RRM}}}{\delta^2\tau + \delta^2 + 2\delta - 2}(\delta^2\tau + 5\delta^3 + \delta^2\tau + 3\delta^2 + 12\delta - 8)^2 - k\theta^2.
\end{align*}\]

5.3. RMM scenario

In the RMM scenario, supplier 1 chooses the reselling mode, while suppliers 2 and supplier 3 choose the wholesale price of his own products; Then, the e-tailer, supplier 2, and supplier 3 simultaneously determine retail price for their respective products. The profit functions of suppliers and the e-tailer are as follows:

\[\begin{align*}
\pi_1 &= w_1 D_1 - k\theta^2, \\
\pi_2 &= (1 - \tau)p_2 D_2 - k\theta^2, \\
\pi_3 &= (1 - \tau)p_3 D_3 - k\theta^2, \\
\pi_e &= (p_1 - w_1)D_1 + \tau p_2 D_2 + \tau p_3 D_3.
\end{align*}\]

The Stackelberg equilibrium decisions and profits under the RMM scenario are solved as follows:

\[
\begin{align*}
w_1^{\text{RMM}} &= \frac{(\beta\theta + a)(2\delta^2\tau + 2\delta\tau - \delta - 2)}{4\delta^2 + 2\delta - 4}, \\
p_1^{\text{RMM}} &= \frac{(-6\delta^3\tau + 4\delta^3 + 6\delta^2\tau + 11\delta^2 - 4\delta\tau - 12)(\beta\theta + a)}{4(\delta^2\tau + \delta^2 + \delta - 2)(2\delta^2 + \delta - 2)}, \\
p_2^{\text{RMM}} &= \frac{p_3^{\text{RMM}}}{4(\delta^2\tau + \delta^2 + \delta - 2)} = \frac{-(\beta\theta + a)(2\delta^3\tau + 4\delta^3 + 2\delta\tau + 9\delta^2 - 2\delta - 8)}{4(\delta^2\tau + \delta^2 + \delta - 2)}, \\
\pi_1^{\text{RMM}} &= \frac{\pi_1^{\text{RMM}}}{8(2\delta^2 + \delta - 2)(\delta^2\tau + \delta^2 + \delta - 2)} - k\theta^2, \\
\pi_2^{\text{RMM}} &= \frac{\pi_2^{\text{RMM}}}{16(2\delta^2 + \delta - 2)(\delta^2\tau + \delta^2 + \delta - 2)^2} - k\theta^2, \\
\pi_3^{\text{RMM}} &= \frac{\pi_3^{\text{RMM}}}{16(2\delta^2 + \delta - 2)(\delta^2\tau + \delta^2 + \delta - 2)^2} - k\theta^2.
\end{align*}\]
In the MMM scenario, all three competing suppliers choose the marketplace mode to collaborate with the e-tailer. And the three suppliers simultaneously decide on the retail price of their own products. The profit functions of suppliers and the e-tailer are as follows:

\[
\pi_1 = (1 - \tau) p_1 D_1 - k\theta^2,
\]
\[
\pi_2 = (1 - \tau) p_2 D_2 - k\theta^2,
\]
\[
\pi_3 = (1 - \tau) p_3 D_3 - k\theta^2,
\]
\[
\pi_e = \tau p_1 D_1 + \tau p_2 D_2 + \tau p_3 D_3.
\]

The Stackelberg equilibrium decisions and profits under the MMM scenario are solved as follows:

\[
p_{1}^{\text{MMM}} = p_{2}^{\text{MMM}} = p_{3}^{\text{MMM}} = \frac{-(\beta \theta + a)}{2\delta - 2},
\]
\[
\pi_{1}^{\text{MMM}} = \pi_{2}^{\text{MMM}} = \pi_{3}^{\text{MMM}} = \frac{(1 - \tau)(\beta \theta + a)^2}{4(\delta - 1)^2} - k\theta^2,
\]
\[
\pi_{e}^{\text{MMM}} = \frac{3\tau(\beta \theta + a)^2}{4(\delta - 1)^2}.
\]

5.4. MMM scenario

Finally, since the analytic solutions are too complex to be directly compared, we set the default values to \(a = 200, \beta = 5, \theta = 10, k = 10, \delta = 0.3\), and analyze and compare the profits of three competing suppliers and the e-tailer under four scenarios through numerical examples.

**Observation 5.** For the supplier 1: When the commission rate \(\tau\) is low enough, the profit in MMM scenario is the highest; When the commission rate \(\tau\) is high enough, the profit in the RRR scenario is the highest.

As shown in Figure 10, except for the RRR scenario, the profit of supplier 1 decreases with the commission rate in the other three scenarios. And when the commission rate is low, the profit of supplier 1 is the highest.
Observation 6. For the supplier 2: When the commission rate $\tau$ is low enough, the profit in RMM scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the RRR scenario is the highest.

As shown in Figure 11, except for the RRR scenario, the profit of supplier 2 decreases with the commission rate in the other three scenarios. And when the commission rate is low, the profit of supplier 2 reaches its maximum level in the RMM scenario. At this point, when comparing the RMM scenario with the RRM scenario, only supplier 2’s choice changes from a resale model to a market model, indicating that choosing a market model is significantly more advantageous for supplier 2 (Note that due to the symmetry of the three suppliers, the scenario where only supplier 2 chooses the marketplace mode is the same as the RRM scenario, which will be discussed below). In addition, comparing the profits of supplier 2 under the RMM and MMM scenarios, it was found that when all suppliers chose the marketplace mode, supplier 2’s profit actually decreased. Therefore, for competitive suppliers, choosing a marketplace mode in a mixed market (i.e., a market with both reselling mode and marketplace mode) is the most advantageous. This conclusion is consistent with the previous viewpoint.

Observation 7. For the supplier 3: When the commission rate $\tau$ is low enough, the profit in RRM scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the RRR scenario is the highest.

As shown in Figure 12, except for the RRR scenario, the profit of supplier 3 decreases with the commission rate in the other three scenarios. And when the commission rate is low, the profit of supplier 3 is the highest in the RRM scenario (i.e., only supplier 3 chooses the marketplace mode), indicating that choosing the marketplace mode is beneficial for supplier 3. In addition, in the RRM, RMM, and MMM scenarios, supplier 3 always chooses the marketplace mode, and his profits decrease in turn in these scenarios. This indicates that the more competitors choose the marketplace mode, the more unfavorable it is for the supplier, and choosing a marketplace mode in a mixed market is more advantageous for competing suppliers. This conclusion is consistent with the previous viewpoint.

Observation 8. For the e-tailer: When the commission rate $\tau$ is low enough, the profit in RRR scenario is the highest; When the commission rate $\tau$ is high enough, the profit in the MMM scenario is the highest.
As shown in Figure 13, except for the RRR scenario, the profit of the e-tailer increases with the commission rate in all other scenarios. And when the commission rate is high, the profit of the e-tailer is the highest in the MMM scenario (i.e. the scenario where all suppliers choose the marketplace mode), indicating that collaborating with suppliers in the marketplace mode is beneficial for the e-tailer. In addition, in the RRR, RRM, RMM, and MMM scenarios, as the commission rate increases, the increase in profit of the e-tailer sequentially increases, indicating that the more suppliers choose the marketplace mode, the more advantageous it is for the e-tailer. This conclusion is consistent with the previous viewpoint.

Observations 5–8 is basically consistent with the previous conclusion, indicating that the research conclusion on the supply chain with two competing suppliers is also applicable to the supply chain with three competing suppliers, and it is highly likely to be extended to a more general situation – the market with multiple competing suppliers. We will also consider verifying the conclusions of this article in this general situation in future research.

6. Conclusions and future research

We study the online sales mode selection of competing suppliers in a fresh product supply chain. Specifically, we consider a fresh product supply chain consisting of two competing fresh suppliers and an e-tailer providing
an online platform. Among them, the two suppliers choose a sales mode to sell fresh products and provide freshness-keeping service in cooperation with the e-tailer. The e-tailer provides an online platform on which both the e-tailer and the two suppliers can directly sell products. Considering the product freshness level, platform commission, supplier’s market share and cross price coefficient between two channels, we study three distribution scenarios and obtain the optimal decisions and profits of the two suppliers and the e-tailer. Then, we derive some new managerial insights through theoretical and numerical analysis.

The freshness level has a positive impact on the parameters of fresh product and the profit of the e-tailer. However, the suppliers can benefit only when the freshness cost is low. The suppliers’ market share is conducive to their own profits, but when both suppliers have similar market shares, the e-tailer cannot obtain high profits. The competition between the two channels always has a positive impact on supply chain performance. The lower the e-tailer’s commission rate, the less he likes the marketplace mode. The best action of suppliers is also related to the commission. The lower the commission rate, the more they tend to choose the marketplace mode. And their best action is different under different distribution scenarios. If they choose different sales modes, the one who chooses the marketplace mode will get higher profit. We also get an interesting discovery that when suppliers make the same choice, the commission does not affect competition and decision-making between them. Finally, the research conclusion on the supply chain with two competing suppliers is also applicable to the supply chain with three competing suppliers, and it is highly likely to be extended to a more general situation – the market with multiple competing suppliers.

In our research, we set the deterministic demand function. However, the demand in the market is not always accurate in practice. So, uncertainty can be added to the demand function in future research. Besides, we only consider the competition of fresh product suppliers. However, the competition among multiple e-tailers is not uncommon in practice. Therefore, we can expand our study by taking into account the competition among e-tailers in the market.

APPENDIX A.

Proof of Lemma 1

Proof. In the RR scenario, we solve the e-tailer’s optimization problem first:

$$\max_{p_1, p_2} \pi_e = (p_1 - w_1)D_1 + (p_2 - w_2)D_2.$$ 

Rewrite as:

$$\max_{p_1, p_2} \pi_e = (p_1 - w_1)(\rho a - p_1 + \delta p_2 + \beta \theta) + (p_2 - w_2)((1 - \rho)a - p_2 + \delta p_1 + \beta \theta).$$

The Hessian Matrix is

$$\begin{bmatrix}
-2 & 2\delta \\
2\delta & -2
\end{bmatrix}.$$

We obtain that the first-order principal minor is negative ($-2 < 0$) and the second-order principal minor is positive ($4 - 4\delta^2 > 0$), indicating that the Hessian matrix is negative definite. Therefore, the e-tailer’s profit function is joint concave for the two retail prices. We can obtain the first-order conditions are $\frac{\partial \pi_e}{\partial p_1} = (2p_2 - w_2)\delta + \rho a + \beta \theta - 2p_1 + w_1$ and $\frac{\partial \pi_e}{\partial p_2} = (2p_1 - w_1)\delta + (1 - \rho)a + \beta \theta - 2p_2 + w_2$.

Then let the first-order conditions be equal to zero, and we get

$$p_1(w_1, w_2) = \frac{a\delta \rho - \beta \delta \theta + \delta^2 w_1 - a\delta - a\rho - \beta \theta - w_1}{2\delta^2 - 2}, \quad (A.1)$$

$$p_2(w_1, w_2) = \frac{a\delta \rho + \beta \delta \theta - \delta^2 w_2 - a\rho + \beta \theta + a + w_2}{2 - 2\delta^2}. \quad (A.2)$$
We substitute the equations (A.1) and (A.2) into the two suppliers’ profit functions (3) and (4) and derive:

\[
\begin{align*}
\max_{w_1} \pi_1 &= \frac{w_1^2}{2} + \frac{(\beta + w_2 \delta + \rho a)w_1}{2} - k\theta^2, \\
\max_{w_2} \pi_2 &= -\frac{w_2^2}{2} + \frac{(\beta + w_1 \delta + (1 - \rho) a)w_2}{2} - k\theta^2.
\end{align*}
\]

By computing the first-order conditions \( \frac{d\pi_1}{dw_1} = \frac{\beta + w_2 \delta + \rho a}{2}w_1 - \frac{w_1^2}{2}, \) \( \frac{d\pi_2}{dw_2} = \frac{\beta + w_1 \delta + (1 - \rho) a}{2}w_2, \) and the second-order conditions \( \frac{d^2\pi_1}{dw_1^2} = -1, \frac{d^2\pi_2}{dw_2^2} = -1, \) the two suppliers’ profit functions are concave for the two wholesale prices, respectively. Let the first-order conditions be equal to zero, and we get:

\[
\begin{align*}
w_{1\star}^R &= \frac{(\delta + 2)\beta \theta + a\delta(1 - \rho) + 2\rho a}{4 - \delta^2}, \\
w_{2\star}^R &= \frac{(\delta + 2)\beta \theta + 2a(1 - \rho) + \delta \rho a}{4 - \delta^2}.
\end{align*}
\]

Then, we have:

\[
\begin{align*}
p_{1\star}^R &= \frac{((2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 2\delta^3 + 5\delta)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8}, \\
p_{2\star}^R &= \frac{(6 - (2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 3\delta^2)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8}, \\
D_{1\star}^R &= \frac{(\delta + 2)\beta \theta - (\delta \rho - \delta - 2\rho)a}{8 - 2\delta^2}, \\
D_{2\star}^R &= \frac{(\delta + 2)\beta \theta + (\delta \rho - 2\rho + 2)a}{8 - 2\delta^2}, \\
\pi_{1\star}^R &= \frac{((1 - \rho)\delta + 2\rho)a + (\delta + 2)\beta \theta)^2}{2(\delta^2 - 4)^2} - k\theta^2, \\
\pi_{2\star}^R &= \frac{(\delta \rho - 2\rho + 2)a + \beta \theta(\delta + 2)^2)^2}{2(\delta^2 - 4)^2} - k\theta^2, \\
\pi_{\pi}^R &= \frac{2\beta \theta(\beta \theta + a)(\delta + 1)(\delta + 2)^2 + (2(\delta - 1)(\delta - 2)^2\rho(1 - \rho) + 5\delta^2 + 4)a^2}{64 - 4\delta^6 + 36\delta^4 - 96\delta^2}.
\end{align*}
\]

Proof of Proposition 1

Proof. Based on Lemma 1, we compare the wholesale prices, retail prices and demands of the product from two suppliers and profits of the two suppliers as follows.

\[
\begin{align*}
w_{1\star}^R - w_{2\star}^R &= \frac{(\delta + 2)\beta \theta + a\delta(1 - \rho) + 2\rho a}{4 - \delta^2} - \frac{(\delta + 2)\beta \theta + 2a(1 - \rho) + \delta \rho a}{4 - \delta^2} = \frac{(2\rho - 1)a}{\delta + 2}; \\
p_{1\star}^R - p_{2\star}^R &= \frac{((2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 2\delta^3 + 5\delta)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8} \\
&\quad - \frac{(6 - (2\delta^3 - 3\delta^2 - 5\delta + 6)\rho - 3\delta^2)a + (6 - 2\delta^3 - 3\delta^2 + 5\delta)\beta \theta}{2\delta^4 - 10\delta^2 + 8} \\
&= \frac{(2\rho - 1)(2\delta + 3)a}{2\delta^2 + 6\delta + 4}; \\
D_{1\star}^R - D_{2\star}^R &= \frac{(\delta + 2)\beta \theta - (\delta \rho - \delta - 2\rho)a}{8 - 2\delta^2} - \frac{(\delta + 2)\beta \theta + (\delta \rho - 2\rho + 2)a}{8 - 2\delta^2} = \frac{(2\rho - 1)a}{2\delta + 4};
\end{align*}
\]
Proof of Proposition 2

Proof. (1) Based on Lemma 1, we analyze the impact of the market share of supplier 1 on wholesale prices, retail prices and demands as follows.

\[
\frac{\partial w_1^{RR}}{\partial \theta} = \frac{\beta(\delta + 2)}{4(1 - \delta)(2 - \delta)^2} > 0, \quad \frac{\partial w_2^{RR}}{\partial \theta} = \frac{\beta(\delta + 2)}{4(1 - \delta)} > 0; \\
\frac{\partial p_1^{RR}}{\partial \theta} = \frac{\beta(3 - 2\delta)}{2\delta^2 - 6\delta + 4} > 0, \quad \frac{\partial p_2^{RR}}{\partial \theta} = \frac{\beta(3 - 2\delta)}{2\delta^2 - 6\delta + 4} > 0; \\
\frac{\partial D_1^{RR}}{\partial \theta} = \frac{\beta}{4 - 2\delta} > 0, \quad \frac{\partial D_2^{RR}}{\partial \theta} = \frac{\beta}{4 - 2\delta} > 0.
\]

(2) Based on Lemma 1, we discuss the impact of freshness level on profits of the e-tailer and suppliers as follows.

\[
\frac{\partial \pi_c^{RR}}{\partial \theta} = \frac{\beta(2\beta \theta + a)}{2(1 - \delta)(2 - \delta)^2} > 0; \\
\frac{\partial \pi_1^{RR}}{\partial \theta} = \frac{\beta(2a\rho + 2\beta \theta + ((1 - \rho)a + \beta \theta)\delta)}{(\delta - 2)^2(\delta + 2)} - 2k\theta; \\
\frac{\partial \pi_2^{RR}}{\partial \theta} = \frac{\beta((a \rho + \beta \theta)\delta + 2((1 - \rho)a + \beta \theta))}{(\delta - 2)^2(\delta + 2)} - 2k\theta.
\]

If \(a < \frac{\beta^2}{\beta(\delta + 2)(\delta - 2)^2k(\delta + 2)}\) and \(k > \frac{\beta^2}{2(\delta - 2)^2}\), then we can obtain \(\frac{\partial \pi_c^{RR}}{\partial \theta} < 0\); otherwise, we obtain \(\frac{\partial \pi_c^{RR}}{\partial \theta} \geq 0\).

If \(a < \frac{\theta(\delta + 2)(2\delta^2 + 6\delta + 4)k - \beta^2}{\beta(\delta + 2)(\delta - 2)^2}\) and \(k > \frac{\beta^2}{2(\delta - 2)^2}\), then we can obtain \(\frac{\partial \pi_1^{RR}}{\partial \theta} < 0\); otherwise, we obtain \(\frac{\partial \pi_1^{RR}}{\partial \theta} \geq 0\).

\[\square\]

Proof of Proposition 3

Proof. (1) Based on Lemma 1, we analyze the impact of the market share of supplier 1 on wholesale prices, retail prices and demands as follows.

\[
\frac{\partial w_1^{RR}}{\partial \rho} = \frac{a(2 - \delta)}{4 - \delta^2} > 0, \quad \frac{\partial w_2^{RR}}{\partial \rho} = \frac{a(\delta - 2)}{4 - \delta^2} < 0; \\
\frac{\partial p_1^{RR}}{\partial \rho} = \frac{a(3 + 2\delta)}{2\delta^2 + 6\delta + 4} > 0, \quad \frac{\partial p_2^{RR}}{\partial \rho} = \frac{-a(3 + 2\delta)}{2\delta^2 + 6\delta + 4} < 0; \\
\frac{\partial D_1^{RR}}{\partial \rho} = \frac{a(2 - \delta)}{2(4 - 2\delta^2)} > 0, \quad \frac{\partial D_2^{RR}}{\partial \rho} = \frac{a(\delta - 2)}{2(4 - 2\delta^2)} < 0.
\]

(2) Based on Lemma 1, we discuss the impact of the market share of supplier 1 on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi_1^{RR}}{\partial \rho} = \frac{a(((\rho - 1)a - \beta \theta)\delta - 2a\rho - 2\beta \theta)}{(\delta + 2)^2(\delta - 2)} > 0; \quad \frac{\partial \pi_2^{RR}}{\partial \rho} = \frac{a((\delta \rho - 2\rho + 2)a + \beta \theta(\delta + 2))}{(\delta + 2)^2(\delta - 2)} < 0;
\]

\[\square\]
\[
\frac{\partial \pi^{\text{RR}}_1}{\partial \rho} = \frac{(2\rho - 1)a^2}{2(\delta + 1)(\delta + 2)^2}.
\]

If \( \rho > \frac{1}{2} \), then we can obtain \( \frac{\partial \pi^{\text{RR}}_1}{\partial \rho} > 0 \); Otherwise, we obtain \( \frac{\partial \pi^{\text{RR}}_1}{\partial \rho} \leq 0 \).

\[\square\]

**Proof of Proposition 4**

Proof. (1) Based on Lemma 1, we analyze the impact of the cross-price coefficient between two channels on wholesale prices, retail prices and demands as follows.

\[
\frac{\partial w_i^{\text{RR}}}{\partial \delta} = \frac{((1 - \rho)a + \beta \theta)(\delta^2 + 4) + (4\rho a + 4\beta \theta)\delta}{(\delta^2 - 4)^2} > 0,
\]

\[
\frac{\partial w_2^{\text{RR}}}{\partial \delta} = \frac{(ap + \beta \theta)(\delta^2 + 4) + ((-4\rho a + 4\beta \theta)\delta}{(\delta^2 - 4)^2} > 0;
\]

\[
\frac{\partial p_1^{\text{RR}}}{\partial \delta} = \frac{(1 - \rho)(2\delta^6 - 5\delta^4 + \delta^2 + 20)a + \rho a(6\delta^5 - 24\delta^3 + 36\delta) + \theta(2\delta^6 + 6\delta^5 - 5\delta^4 - 24\delta^3 + \delta^2 + 36\delta + 20)\beta}{2(\delta^4 - 5\delta^2 + 4)^2} \geq 0,
\]

\[
\frac{\partial p_2^{\text{RR}}}{\partial \delta} = \frac{(2\delta^6 - 6\delta^5 - 5\delta^4 + 24\delta^3 + \delta^2 - 36\delta + 20)\rho + 6\delta^5 - 24\delta^3 + 36\delta)a + \theta(2\delta^6 + 6\delta^5 - 5\delta^4 - 24\delta^3 + \delta^2 + 36\delta + 20)\beta}{2(\delta^4 - 5\delta^2 + 4)^2} > 0;
\]

\[
\frac{\partial D_1^{\text{RR}}}{\partial \delta} = \frac{(1 - \rho)(a + \beta \theta)(\delta^2 + 4) + (4\rho a + 4\beta \theta)\delta}{2(\delta^2 - 4)^2} > 0,
\]

\[
\frac{\partial D_2^{\text{RR}}}{\partial \delta} = \frac{(ap + \beta \theta)(\delta^2 + 4) + ((-4\rho a + 4\beta \theta)\delta}{2(\delta^2 - 4)^2} > 0.
\]

(2) Based on Lemma 1, we discuss the impact of the cross-price coefficient between two channels on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi^{\text{RR}}_1}{\partial \delta} = \frac{-((\rho - 1)a - \beta \theta)\delta - 2a\rho - 2\beta \theta)((\rho - 1)a - \beta \theta)(\delta^2 + 4) + (4\rho a + 4\beta \theta)\delta}{(\delta^2 - 4)^3} > 0,
\]

\[
\frac{\partial \pi^{\text{RR}}_2}{\partial \delta} = \frac{-(ap + \beta \theta)(\delta^2 + 4) + ((-4\rho a + 4\beta \theta)\delta)(ap + \beta \theta)\delta + (-2\rho + 2)a + 2\beta \theta}{(\delta^2 - 4)^3} > 0,
\]

\[
\frac{\partial \pi^{\text{RR}}_3}{\partial \delta} = \frac{(3\delta^6 + 20\delta^5 + 3\delta^4 + 14\delta^3 - 68\delta^2 - 88\delta - 32)(\beta^2 \theta^2 + a^2) + ((3\delta^6 - 20\delta^5 + 43\delta^4 - 14\delta^3 - 68\delta^2 + 88\delta - 32)(\rho - \rho^2) + 10\theta^5 + 7\delta^3 - 44\delta)a^2}{2(\delta - 1)^2(\delta - 2)^3(\delta + 2)^3(\delta + 1)^2} > 0.
\]

\[\square\]

**Proof of Lemma 2**

Proof. Under RM scenario, we solve the optimization problems of the e-tailer and supplier 2 simultaneously:

\[
\max_{p_1} \pi_e = (p_1 - w_1)D_1 + \tau p_2 D_2,
\]

\[
\max_{p_2} \pi_2 = (1 - \tau)p_2 D_2 - k\theta^2.
\]

Rewrite as:

\[
\max_{p_1} \pi_e = (p_1 - w_1)(\rho a - p_1 + \delta p_2 + \beta \theta) + \tau p_2((1 - \rho)a - p_2 + \delta p_1 + \beta \theta),
\]
\[
\max_{p_2} \pi_2 = (1 - \tau)p_2((1 - \rho)a - p_2 + \delta p_1 + \beta \theta) - k\theta^2.
\]

By computing the first-order conditions \( \frac{d\pi_2}{dp_2} = \tau p_2 \delta + \beta \theta + \delta p_2 + \rho a - 2p_1 + w_1 \), \( \frac{d\pi_2}{dp_2} = ((1 - \rho)a - 2p_2 + \delta p_1 + \beta \theta)(1 - \tau) \), and the second-order conditions \( \frac{d^2\pi_2}{dp_2^2} = -2 \), \( \frac{d^2\pi_2}{dp_2^2} = -2(1 - \tau) \), the profit functions of the e-tailer and supplier 2 are concave for the two retail prices. Let the first-order conditions be equal to 0 and we get:

\[
p_1(w_1) = \frac{(\tau + 1)((\rho - 1)a - \beta \theta)\delta - 2\rho a - 2\beta \theta - 2w_1}{(\tau + 1)^2 - 4}, \tag{A.3}
\]

\[
p_2(w_1) = \frac{(-\rho a - \beta \theta - w_1)\delta + (2\rho - 2)a - 2\beta \theta}{(\tau + 1)^2 - 4}. \tag{A.4}
\]

We substitute the equations (A.3) and (A.4) into supplier 1’s profit function (6) and derive:

\[
\max_{w_1} \pi_1 = ((ap + \beta \theta)(\delta^2 \tau - 2) + ((2 - \delta^2) + (1 - \tau)((\rho - 1)a - \beta \theta)\delta))w_1 - k\theta^2.
\]

By computing the first-order condition \( \frac{d\pi_1}{dw_1} = \frac{(ap + \beta \theta)(\delta^2 \tau - 2) + (-2\delta^2 + 4)w_1 + (1 - \tau)((\rho - 1)a - \beta \theta)\delta}{(\tau + 1)^2 - 4} \) and the second-order condition \( \frac{d^2\pi_1}{dw_1^2} = \frac{4 - 2\delta^2}{(\tau + 1)^2 - 4} \), the supplier 1’s profit function is concave for the wholesale price. Let the first-order condition be equal to 0 and we get:

\[
w_1^{RM^*} = \frac{(1 - \tau)\delta((\rho - 1)a + \beta \theta) + (2 - \delta^2)(ap + \beta \theta)}{4 - 2\delta^2}.
\]

Then, we have

\[
P_1^{RM^*} = \frac{((\rho - 1)a - \beta \theta)((\tau + 1)\delta^3 - (\tau + 3)\delta) - (ap + \beta \theta)((\tau + 2)\delta^2 - 6)}{(4 - (\tau + 1)^2)(2 - \delta^2)},
\]

\[
P_2^{RM^*} = \frac{(ap + \beta \theta)((\tau + 2)\delta^3 + (8 - (\tau + 3)\delta^2)((1 - \rho)a + \beta \theta)}{2(\delta^2 \tau + \delta^2 - 4)(\delta^2 - 2)},
\]

\[
\pi_1^{RM^*} = \frac{((\tau + 2)\delta^3 - 6\delta)(ap + \beta \theta) - ((\rho - 1)a - \beta \theta)((\tau + 3)\delta^2 - 8))^2}{4(\delta^2 \tau + \delta^2 - 4)(\delta^2 - 2)^2} - k\theta^2,
\]

\[
\pi_2^{RM^*} = \frac{(1 - \tau)((\tau + 2)\delta^3 - 6\delta)(ap + \beta \theta) - ((\rho - 1)a - \beta \theta)((\tau + 3)\delta^2 - 8))^2}{4(\delta^2 \tau + \delta^2 - 4)(\delta^2 - 2)^2} - k\theta^2,
\]

\[
\pi_e^{RM^*} = \frac{((\tau + 2)\delta^3 - 6\delta)(ap + \beta \theta) - (ap - a - \beta \theta)((\tau + 3)\delta^2 - 8))^2}{4(\delta^2 \tau + \delta^2 - 4)(\delta^2 - 2)^2} - k\theta^2.
\]

\[\square\]
Proof of Proposition 5

Based on Lemma 2, we analyze the impact of the freshness level on wholesale price, retail prices and demands as follows.

\[
p_{1}^{RM} - p_{2}^{RM} = \frac{((3\delta^3 - 3\delta^2 - 2\delta)\rho - 2\delta^3 + \delta^2 + 2\delta)\tau + (4\delta^3 - 7\delta^2 - 12\delta + 20)\rho - 2\delta^3 + 3\delta^2 + 6\delta - 8)a}{2(\delta^2 - 2)(-4 + (\tau + 1)\delta^2)}.
\]

If \( a > a_1 \) and \( \rho < \rho_1 \), we can obtain \( p_{1}^{RM} < p_{2}^{RM} \); Otherwise, we obtain \( p_{1}^{RM} \geq p_{2}^{RM} \).

With \( a_1 = \frac{\beta \theta (\delta + 2)(\delta^2 - \delta + 2)}{(3\delta^4 - 4\delta^3 - \delta^2 + 2\delta - 2)} \), \( \rho_1 = \frac{(2\delta^3 - 3\delta^2 - 2\delta)\tau + 2\delta^3 - 3\delta^2 - 6\delta + 8}{(3\delta^4 - 3\delta^3 - 2\delta^2 + 2\delta - 2)} \).

\[ \square \]

Proof of Proposition 6

(1) Based on Lemma 2, we analyze the impact of the freshness level on wholesale price, retail prices and demands as follows.

\[
\frac{\partial w_{i}^{RM}}{\partial \theta} = \frac{\beta (\delta^2 \tau + (\tau - 1)\delta - 2)}{2\delta^2 - 4} > 0;
\]

\[
\frac{\partial p_{1}^{RM}}{\partial \theta} = -\frac{(-6 + (\tau + 1)\delta^3 + \delta^2(\tau + 2) + (-\tau - 3)\delta)\beta}{(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0;
\]

\[
\frac{\partial p_{2}^{RM}}{\partial \theta} = -\frac{\beta(-8 + (\tau + 2)\delta^3 + (\tau + 3)\delta^2 - 6\delta)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0;
\]

\[
\frac{\partial D_{1}^{RM}}{\partial \theta} = \frac{\beta (\delta^2 \tau + (\tau - 1)\delta - 2)}{-8 + (2\tau + 2)^2} > 0;
\]

\[
\frac{\partial D_{2}^{RM}}{\partial \theta} = -\frac{\beta(-8 + (\tau + 2)\delta^3 + (\tau + 3)\delta^2 - 6\delta)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0.
\]

(2) Based on Lemma 2, we discuss the impact of the freshness level on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi_{1}^{RM}}{\partial \theta} = \beta (\delta^2 \tau + (\tau - 1)\delta - 2)(\tau(\rho + \beta \theta)\delta^2 - (\tau - 1)((\rho - 1)\alpha - \beta \theta)\delta - 2\alpha \rho - 2\beta \theta) - 2k\theta,
\]

\[
\frac{\partial \pi_{2}^{RM}}{\partial \theta} = -\beta(\tau - 1)(-8 + (\tau + 2)\delta^3 + (\tau + 3)\delta^2 - 6\delta)((\rho + \beta \theta)((\tau + 2)\delta^3 - 6\delta)
\]

\[
+((\rho - 1)\alpha - \beta \theta)(8 - (\tau + 3)\delta^2)) - 2k\theta,
\]

\[
\frac{\partial \pi_{c}^{RM}}{\partial \theta} = \frac{(A_{1}\tau^3 + A_{2}\tau^2 + A_{3}\tau + A_{4})(\beta^2 \theta + a_{1}(A_{5}\tau^3 + A_{6}\tau^2 + A_{7}\tau + A_{8}))}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)^2} > 0.
\]

If \( k < k_1 \), then we can obtain \( \frac{\partial \pi_{1}^{RM}}{\partial \theta} > 0 \); Otherwise, we obtain \( \frac{\partial \pi_{1}^{RM}}{\partial \theta} \leq 0 \);

If \( k < k_2 \), then we can obtain \( \frac{\partial \pi_{2}^{RM}}{\partial \theta} > 0 \); Otherwise, we obtain \( \frac{\partial \pi_{2}^{RM}}{\partial \theta} \leq 0 \);

With \( k_1 = \frac{(\alpha \beta \theta)(\delta^2 \tau - 2)(\tau - 1)((\rho - 1)\alpha - \beta \theta)\delta - 2\alpha \rho - 2\beta \theta)}{2(\tau + 1)\delta^2(\delta^2 - 2)} \), \( k_2 = \frac{(\alpha \beta \theta)(\tau + 2)\delta^3 - 6\delta)((\rho - 1)\alpha - \beta \theta)(8 - 3\delta^2)((\tau + 2)\delta^3 - 6\delta - 8)(\tau + 1)\theta}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)^2} \).

\[ \square \]
Proof of Proposition 7

Proof. (1) Based on Lemma 2, we analyze the impact of the cross-price coefficient between two channels on wholesale price, retail prices and demands as follows.

\[
\frac{\partial w^{RM*}_1}{\partial \rho} = \frac{a(\delta^2 \tau - \delta \tau + \delta - 2)}{2\delta^2 - 4} > 0,
\]

\[
\frac{\partial p^{RM*}_1}{\partial \rho} = \frac{(6 + (\tau + 1)\delta^3 + (-\tau - 2)\delta^2 + (-\tau - 3)\delta)a}{(4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0,
\]

\[
\frac{\partial p^{RM*}_2}{\partial \rho} = \frac{-(8 + (\tau + 2)\delta^3 + (-\tau - 3)\delta^2 - 6\delta)a}{2(4 + (\tau + 1)\delta^2)(\delta^2 - 2)} < 0;
\]

\[
\frac{\partial D^{RM*}_1}{\partial \rho} = \frac{a(\delta^2 \tau - \delta \tau + \delta - 2)}{2\delta^2 \tau + 2\delta^2 - 8} > 0,
\]

\[
\frac{\partial D^{RM*}_2}{\partial \rho} = \frac{-(8 + (\tau + 2)\delta^3 + (-\tau - 3)\delta^2 - 6\delta)a}{2(4 + (\tau + 1)\delta^2)(\delta^2 - 2)} < 0.
\]

(2) Based on Lemma 2, we discuss the impact of the market share of supplier 1 on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi^{RM*}_1}{\partial \rho} = \frac{a(-2 + \delta^2 \tau + (-\tau + 1)\delta)((ap + \beta\theta)(\tau \delta^2 - 2) - (\tau - 1)((\rho - 1)a - \beta\theta\delta))}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0,
\]

\[
\frac{\partial \pi^{RM*}_2}{\partial \rho} = \frac{-(8 + (\tau + 2)\delta^3 + (-\tau - 3)\delta^2 - 6\delta)(\tau - 1)a((ap + \beta\theta)((\tau + 2)\delta^3 - 6\delta)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} < 0.
\]

\(\square\)

Proof of Proposition 8

Proof. (1) Based on Lemma 2, we analyze the impact of the cross-price coefficient between two channels on wholesale price, retail prices and demands as follows.

\[
\frac{\partial w^{RM*}_1}{\partial \delta} = \frac{((\rho - 1)a - \beta\theta)\delta^2 + (-4\rho - 4\beta\theta)\delta + (2\rho - 2)a - 2\beta\theta)(\tau - 1)}{2(\delta^2 - 2)} > 0;
\]

\[
\frac{\partial p^{RM*}_1}{\partial \delta} = \frac{((-1 + \rho)a - \beta\theta)((-\tau + 1)\delta^2 + (\tau + 1)\delta^4(\tau + 3) - 2(\tau^2 - 6\tau - 3) - 8\tau - 24)}{2(\tau^2 - 4\tau + 3)\delta^4 - 12(\tau + 1)\delta^2 + 48)} > 0;
\]

\[
\frac{\partial p^{RM*}_2}{\partial \delta} = \frac{((\rho - 1)a - \beta\theta)(-2(\tau + 3)(\tau + 1)\delta^3 + 32(\tau + 1)\delta^3 - 16(\tau + 3)\delta)}{2(\tau + 1)\delta^4(\delta^2 - 2)} > 0;
\]

\[
\frac{\partial D^{RM*}_1}{\partial \delta} = \frac{((\rho - 1)a - \beta\theta)((\tau + 1)\delta^2 + 4) + (-4\rho - 4\beta\theta)\delta)(\tau - 1)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0,
\]

\[
\frac{\partial D^{RM*}_2}{\partial \delta} = \frac{((\rho - 1)a - \beta\theta)((\tau^2 + 3\tau + 2)\delta^6 + 2(\tau^2 - 4\tau + 3)\delta^4 - 12(\tau + 1)\delta^2 + 48)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0.
\]

(2) Based on Lemma 2, we discuss the impact of the cross-price coefficient between two channels on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi^{RM*}_1}{\partial \delta} = \frac{(\tau - 1)((ap + \beta\theta)(\delta^2 \tau - 2) - (\tau - 1)((\rho - 1)a - \beta\theta\delta))((\rho - 1)a - \beta\theta\delta)((\tau + 1) - 8)}{2(-4 + (\tau + 1)\delta^2)(\delta^2 - 2)} > 0,
\]
Proof of Proposition 9

Proof. (1) Based on Lemma 2, we analyze the impact of the commission rate on wholesale price, retail prices and demands as follows.

\[
\frac{\partial \pi_{2}^{RM}}{\partial \delta} = \frac{(\tau - 1)((\alpha \rho + \beta \theta)((\tau + 2)\delta^3 - 6\delta) + ((\rho - 1)a - \beta \theta)(8 - (\tau + 3)\delta^2))((\alpha \rho + \beta \theta)((\tau + 2)(\tau + 1)\delta^6 + 2(\tau^2 - 4\tau - 3)\delta^4 - 12(\tau + 1)\delta^2 + 48))}{2(-4 + (\tau + 1)\delta^2)^3(\delta^2 - 2)^3} > 0.
\]

□

(2) Based on Lemma 2, we discuss the impact of the commission rate on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial w_{1}^{RM}}{\partial \tau} = \frac{\delta(\alpha \rho + \beta \theta - a \rho + \beta \theta + a)}{2\delta^2 - 4} < 0;
\]

\[
\frac{\partial p_{1}^{RM}}{\partial \tau} = \frac{(a \rho + \beta \theta - 2a \rho + 2\beta \theta + 2a)\delta}{(\delta^2 \tau + \delta^2 - 4)^2} > 0;
\]

\[
\frac{\partial p_{2}^{RM}}{\partial \tau} = \frac{(a \rho + \beta \theta - 2a \rho + 2\beta \theta + 2a)\delta^2}{2(\delta^2 \tau + \delta^2 - 4)^2} > 0;
\]

\[
\frac{\partial D_{1}^{RM}}{\partial \tau} = \frac{\delta(\delta^2 - 2)(a \rho + \beta \theta - 2a \rho + 2\beta \theta + 2a)}{2(\delta^2 \tau + \delta^2 - 4)^2} < 0;
\]

\[
\frac{\partial D_{2}^{RM}}{\partial \tau} = \frac{(a \rho + \beta \theta - 2a \rho + 2\beta \theta + 2a)\delta^2}{2(\delta^2 \tau + \delta^2 - 4)^2} > 0.
\]

□

Proof of Lemma 3

Proof. Under MM scenario, we solve the optimization problems of the two suppliers simultaneously:

\[
\begin{align*}
\max_{p_{1}} \pi_{1} &= (1 - \tau)p_{1}D_{1} - k\theta^2, \\
\max_{p_{2}} \pi_{2} &= (1 - \tau)p_{2}D_{2} - k\theta^2.
\end{align*}
\]

Rewrite as:

\[
\begin{align*}
\max_{p_{1}} \pi_{1} &= (1 - \tau)p_{1}(\rho a - p_{1} + \delta p_{2} + \beta \theta) - k\theta^2,
\end{align*}
\]
\[ \max_{p_2} \pi_2 = (1 - \tau) p_2 ((1 - \rho) a - p_2 + \delta p_1 + \beta \theta) - k\theta^2. \]

By computing the first-order conditions \( \frac{d\pi_1}{dp_1} = (1 - \tau) (\beta \theta + \delta p_2 + \rho a - 2 p_1) \), \( \frac{d\pi_2}{dp_2} = (1 - \tau) (\beta \theta + \delta p_1 + (1 - \rho) a - 2 p_2) \), and the second-order conditions \( \frac{d^2\pi_1}{dp_1^2} = 2 (\tau - 1) \), \( \frac{d^2\pi_2}{dp_2^2} = 2 (\tau - 1) \), the two suppliers’ profit functions are concave for the two retail prices, respectively. Let the first-order conditions be equal to 0 and we get:

\[
\begin{align*}
\pi_{1}^{MM^{*}} &= \frac{(1 - \rho) (\delta + 2) a + \beta \theta (\delta + 2)}{4 - \delta^2}, \\
\pi_{2}^{MM^{*}} &= \frac{(\delta - 2) a + \beta \theta (\delta + 2)}{4 - \delta^2}.
\end{align*}
\]

Then, we have:

\[
\begin{align*}
D_{1}^{MM^{*}} &= \frac{(1 - \rho) (\delta + 2) a + \beta \theta (\delta + 2)}{4 - \delta^2}, \\
D_{2}^{MM^{*}} &= \frac{(\delta - 2) a + \beta \theta (\delta + 2)}{4 - \delta^2}, \\
\pi_{1}^{MM^{*}} &= \frac{(1 - \tau) ((2 - \delta) a \rho + a \delta + \beta \theta (\delta + 2))^2}{(\delta^2 - 4)^2} - k\theta^2, \\
\pi_{2}^{MM^{*}} &= \frac{(1 - \tau) ((\delta - 2) a + \beta \theta (\delta + 2))^2}{(\delta^2 - 4)^2} - k\theta^2, \\
\pi_{e}^{MM^{*}} &= \frac{\tau (2 (\rho - 1) (\delta - 2) + \delta^2 + 4) a^2 + 2 \beta \theta (\beta \theta + a) (\delta + 2)^2)}{(\delta^2 - 4)^2}.
\end{align*}
\]

\[ \square \]

**Proof of Proposition 10**

*Proof.* Based on Lemma 3, we compare the retail prices and demands of the product from two suppliers and the profits of the two suppliers as follows.

\[
\begin{align*}
p_{1}^{MM^{*}} - p_{2}^{MM^{*}} &= \frac{a \delta \rho - \beta \delta \theta - a \delta - 2 a \rho - 2 \beta \theta}{\delta^2 - 4} - \frac{- (a \delta \rho + \beta \delta \theta - 2 a \rho + 2 \beta \theta + 2 a)}{\delta^2 - 4} = \frac{(2 \rho - 1) a}{\delta + 2}; \\
D_{1}^{MM^{*}} - D_{2}^{MM^{*}} &= \frac{(\rho - 1) a - \beta \theta \delta - 2 a \rho - 2 \beta \theta}{\delta^2 - 4} - \frac{-(\delta \rho + 2 \rho - 2) a - \beta \theta (\delta + 2)}{\delta^2 - 4} = \frac{(2 \rho - 1) a}{\delta + 2}; \\
\pi_{1}^{MM^{*}} - \pi_{2}^{MM^{*}} &= \frac{(-1 + \tau) ((\rho - 1) a - \beta \theta (\delta - 2 a \rho - 2 \beta \theta)^2}{(\delta^2 - 4)^2} - k\theta^2) \\
&\quad - \frac{(-1 + \tau) ((\delta \rho - 2 \rho + 2) a + \beta \theta (\delta + 2)^2)}{(\delta^2 - 4)^2} - k\theta^2) \\
&= \frac{(-1 + \tau) a (2 \rho - 1) (2 \beta \theta + a)}{\delta^2 - 4}. \\
\end{align*}
\]

If \( \rho > \frac{1}{2} \), we can obtain \( p_{1}^{MM^{*}} > p_{2}^{MM^{*}}, D_{1}^{MM^{*}} > D_{2}^{MM^{*}} \), \( \pi_{1}^{MM^{*}} > \pi_{2}^{MM^{*}} \); otherwise, we obtain \( p_{1}^{MM^{*}} \leq p_{2}^{MM^{*}}, D_{1}^{MM^{*}} \leq D_{2}^{MM^{*}}, \pi_{1}^{MM^{*}} \leq \pi_{2}^{MM^{*}} \). \[ \square \]

**Proof of Proposition 11**

*Proof.* (1) Based on Lemma 3, we analyze the impact of the freshness level on retail prices and demands as follows.

\[
\frac{\partial p_{1}^{MM^{*}}}{\partial \theta} = \frac{\beta}{2 - \delta} > 0,
\]
\[ \frac{\partial p^*_1}{\partial \theta} = \frac{\beta}{2 - \delta} > 0; \]
\[ \frac{\partial D^*_1}{\partial \theta} = \frac{\beta}{2 - \delta} > 0; \]
\[ \frac{\partial D^*_2}{\partial \theta} = \frac{\beta}{2 - \delta} > 0. \]

(2) Based on Lemma 3, we discuss the impact of the freshness level on profits of suppliers and the e-tailer as follows.

\[ \frac{\partial \pi^*_1}{\partial \rho} = \frac{2(\tau - 1)((\rho - 1)a - \beta \theta)\delta - 2a\rho - 2\beta \theta)}{(\delta - 2)^2(\delta + 2)} - 2k\theta, \]
\[ \frac{\partial \pi^*_2}{\partial \rho} = \frac{-2(\tau - 1)((a\rho + \beta \theta)\delta + 2\beta \theta - 2(\rho - 1)a)}{(\delta - 2)^2(\delta + 2)} - 2k\theta, \]
\[ \frac{\partial \pi^*_e}{\partial \rho} = \frac{2\tau \beta (2\beta \theta + a)}{(\delta - 2)^2} > 0. \]

If \( k < \frac{\beta(\tau - 1)((\rho - 1)a - \beta \theta)\delta - 2a\rho - 2\beta \theta)}{\theta(\delta + 2)(\delta - 2)^2} \), then we can obtain \( \frac{\partial \pi^*_1}{\partial \rho} > 0 \); Otherwise, we obtain \( \frac{\partial \pi^*_1}{\partial \rho} \leq 0 \);

If \( k < \frac{\beta(1-\tau)((a\rho + \beta \theta)\delta + 2a(\rho - 1) + 2\beta \theta)}{\theta(\delta + 2)(\delta - 2)^2} \), then we can obtain \( \frac{\partial \pi^*_2}{\partial \rho} > 0 \); Otherwise, we obtain \( \frac{\partial \pi^*_2}{\partial \rho} \leq 0 \).

\[ \square \]

**Proof of Proposition 12**

Proof. (1) Based on Lemma 3, we analyze the impact of the supplier 1’s market share on retail prices and demands as follows.

\[ \frac{\partial p^*_1}{\partial \rho} = \frac{a}{\delta + 2} > 0, \]
\[ \frac{\partial p^*_2}{\partial \rho} = \frac{-a}{\delta + 2} < 0; \]
\[ \frac{\partial D^*_1}{\partial \rho} = \frac{a}{\delta + 2} > 0, \]
\[ \frac{\partial D^*_2}{\partial \rho} = \frac{-a}{\delta + 2} < 0. \]

(2) Based on Lemma 3, we discuss the impact of the supplier 1’s market share on profits of suppliers and the e-tailer as follows.

\[ \frac{\partial \pi^*_1}{\partial \rho} = \frac{2(1 - \tau)a((\rho - 1)a - \beta \theta)\delta - 2a\rho - 2\beta \theta)}{(\delta + 2)^2(\delta - 2)} > 0, \]
\[ \frac{\partial \pi^*_2}{\partial \rho} = \frac{-2(\tau - 1)a((\rho - 1)a + \beta \theta(\delta + 2))}{(\delta + 2)^2(\delta - 2)} < 0, \]
\[ \frac{\partial \pi^*_e}{\partial \rho} = \frac{2\tau(2\rho - 1)a^2}{(\delta + 2)^2}. \]

If \( \rho > 1/2 \), then we can obtain \( \frac{\partial \pi^*_1}{\partial \rho} > 0 \); Otherwise, we obtain \( \frac{\partial \pi^*_1}{\partial \rho} \leq 0 \).

\[ \square \]
Proof of Proposition 13

Proof. (1) Based on Lemma 3, we analyze the impact of the cross-price coefficient between two channels on retail prices and demands as follows.

\[
\frac{\partial p_1^\text{MM}^*}{\partial \delta} = \frac{(-\delta^2 + 4\delta - 4)a_\rho + (\delta^2 + 4)a + \theta(\delta^2 + 4\delta + 4)\beta}{(\delta^2 - 4)^2} > 0,
\]
\[
\frac{\partial p_2^\text{MM}^*}{\partial \delta} = \frac{(\alpha_\rho + \beta_\theta)(\delta^2 + 4) + ((-4\rho + 4)\alpha + 4\beta_\theta)\delta}{(\delta^2 - 4)^2} > 0;
\]
\[
\frac{\partial D_1^\text{MM}^*}{\partial \delta} = \frac{((1 - \rho)\alpha + \beta_\theta)(\delta^2 + 4) + (4\alpha_\rho + 4\beta_\theta)\delta}{(\delta^2 - 4)^2} > 0,
\]
\[
\frac{\partial D_2^\text{MM}^*}{\partial \delta} = \frac{(\alpha_\rho + \beta_\theta)(\delta^2 + 4) + ((-4\rho + 4)\alpha + 4\beta_\theta)\delta}{(\delta^2 - 4)^2} > 0.
\]

(2) Based on Lemma 3, we discuss the impact of the cross-price coefficient between two channels on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi_1^\text{MM}^*}{\partial \delta} = \frac{2(\tau - 1)((\rho - 1)\alpha - \beta_\theta)\delta - 2\alpha_\rho - 2\beta_\theta)((\rho - 1)\alpha - \beta_\theta)(\delta^2 + 4) + (4a_\rho - 4\beta_\theta)\delta}{(\delta^2 - 4)^3} > 0,
\]
\[
\frac{\partial \pi_2^\text{MM}^*}{\partial \delta} = \frac{2(\tau - 1)((\alpha_\rho + \beta_\theta)\delta + (-2\rho + 2)\alpha + 2\beta_\theta)((\alpha_\rho + \beta_\theta)(\delta^2 + 4) + ((-4\rho + 4)\alpha + 4\beta_\theta)\delta}{(\delta^2 - 4)^3} > 0,
\]
\[
\frac{\partial \pi_e^\text{MM}^*}{\partial \delta} = \frac{(\delta^3 + 6\delta^2 + 12\delta + 8)(-4\beta^2\tau\theta^2 - 4a_\beta\tau\theta) - 2a^2(2\rho^2 - \rho)(\delta^3 - 6\delta^2 + 12\delta - 8) + \delta^3 + 12\delta}{(\delta^2 - 4)^3} > 0.
\]

□

Proof of Proposition 14

Proof. (1) Based on Lemma 3, we analyze the impact of the commission rate on retail prices and demands as follows.

\[
\frac{\partial p_1^\text{MM}^*}{\partial \tau} = 0, \frac{\partial p_2^\text{MM}^*}{\partial \tau} = 0; \frac{\partial D_1^\text{MM}^*}{\partial \tau} = 0, \frac{\partial D_2^\text{MM}^*}{\partial \tau} = 0.
\]

(2) Based on Lemma 3, we discuss the impact of the commission rate on profits of suppliers and the e-tailer as follows.

\[
\frac{\partial \pi_1^\text{MM}^*}{\partial \tau} = \frac{-((\rho - 1)\alpha - \beta_\theta)\delta - 2\alpha_\rho - 2\beta_\theta)^2}{(\delta^2 - 4)^2} < 0,
\]
\[
\frac{\partial \pi_2^\text{MM}^*}{\partial \tau} = \frac{-((\rho - 2\rho + 2)\alpha + \beta_\theta(\delta + 2))^2}{(\delta^2 - 4)^2} < 0,
\]
\[
\frac{\partial \pi_e^\text{MM}^*}{\partial \tau} = \frac{(2\delta^2 - 8\delta + 8)(\rho^2 - \rho) + \delta^2 + 4)a^2 + 2(\delta + 2)(\beta^2\theta^2 + a_\beta\theta)}{(\delta^2 - 4)^2} > 0.
\]

□

Solution of RRR model

Proof. In the RRR scenario, we solve the e-tailer’s optimization problem first:

\[
\max_{p_1, p_2, p_3} \pi_e = (p_1 - w_1)D_1 + (p_2 - w_2)D_2 + (p_3 - w_3)D_3.
\]
The Hessian Matrix is

\[
\begin{bmatrix}
-2 & 2\delta & 2\delta \\
2\delta & -2 & 2\delta \\
2\delta & 2\delta & -2 \\
\end{bmatrix}
\]

For \( \delta < \frac{1}{2} \), we obtain that the first-order principal minor is negative \((-2 < 0)\), the second-order principal minor is positive \((4 - 4\delta^2 > 0)\) and the third-order principal minor is negative \((16\delta^3 + 24\delta^2 - 8 < 0)\), indicating that the Hessian matrix is negative definite. Therefore, the e-tailer’s profit function is joint concave for the three retail prices. We can obtain the first-order conditions are

\[
\frac{\partial \pi}{\partial p_1} = (2p_2 + 2p_3 - w_2 - w_3)\delta + \beta\theta + a - 2p_1 + w_1, \\
\frac{\partial \pi}{\partial p_2} = (2p_1 + 2p_3 - w_1 - w_3)\delta + \beta\theta + a - 2p_2 + w_2, \\
\frac{\partial \pi}{\partial p_3} = (2p_1 + 2p_2 - w_1 - w_2)\delta + \beta\theta + a - 2p_3 + w_3.
\]

Then let the first-order conditions be equal to zero, and we get

\[
\begin{align*}
p_1(w_1, w_2, w_3) &= -\frac{\beta\theta + 2\delta w_1 - a - w_1}{4\delta - 2}, \quad (A.5) \\
p_2(w_1, w_2, w_3) &= -\frac{\beta\theta + 2\delta w_2 - a - w_2}{4\delta - 2}, \quad (A.6) \\
p_3(w_1, w_2, w_3) &= -\frac{\beta\theta + 2\delta w_3 - a - w_3}{4\delta - 2}. \quad (A.7)
\end{align*}
\]

We substitute the equations (A.5)–(A.7) into the three suppliers’ profit functions and derive:

\[
\begin{align*}
\max_{w_1} \pi_1 &= -\frac{w_1^2}{2} + \frac{((w_2 + w_3)\delta + \beta\theta + a)w_1}{2} - k\theta^2, \\
\max_{w_2} \pi_2 &= -\frac{w_2^2}{2} + \frac{((w_1 + w_3)\delta + \beta\theta + a)w_2}{2} - k\theta^2, \\
\max_{w_3} \pi_3 &= -\frac{w_3^2}{2} + \frac{((w_1 + w_2)\delta + \beta\theta + a)w_3}{2} - k\theta^2.
\end{align*}
\]

By computing the first-order conditions \( \frac{d\pi_1}{dw_1} = \frac{(w_2 + w_3)\delta + \beta\theta + a - 2w_1}{2}, \frac{d\pi_2}{dw_2} = \frac{(w_1 + w_3)\delta + \beta\theta + a - 2w_2}{2}, \frac{d\pi_3}{dw_3} = \frac{(w_1 + w_2)\delta + \beta\theta + a - 2w_3}{2} \), and the second-order conditions \( \frac{d^2\pi_1}{dw_1^2} = -1, \frac{d^2\pi_2}{dw_2^2} = -1, \frac{d^2\pi_3}{dw_3^2} = -1 \), the three suppliers’ profit functions are concave for the three wholesale prices, respectively. Let the first-order conditions be equal to zero, and we get:

\[
\begin{align*}
u_1^{RRR} &= u_2^{RRR} = u_3^{RRR} = -\frac{(\beta\theta + a)}{2\delta - 2}.
\end{align*}
\]

Then, we have:

\[
\begin{align*}
p_1^{RRR} &= p_2^{RRR} = p_3^{RRR} = -\frac{(\beta\theta + a)(4\delta - 3)}{8\delta^2 - 12\delta + 4}, \\
D_1^{RRR} &= D_2^{RRR} = D_3^{RRR} = \frac{\beta\theta + a}{4 - 4\delta}, \\
\pi_1^{RRR} &= \pi_2^{RRR} = \pi_3^{RRR} = \frac{(\beta\theta + a)^2}{8(\delta - 1)^2} - k\theta^2, \\
\pi_e^{RRR} &= \frac{-3(\beta\theta + a)^2}{16(2\delta - 1)(\delta - 1)^2}.
\end{align*}
\]

\[\square\]
Solution of RRM model

Proof. Under RRM scenario, we solve the optimization problems of the e-tailer and supplier 3 simultaneously:

\[
\begin{align*}
\max_{p_1,p_2} \pi_e &= (p_1 - w_1)D_1 + (p_2 - w_2)D_2 + \tau p_3 D_3, \\
\max_{p_3} \pi_3 &= (1 - \tau)p_3 D_3 - k\theta^2.
\end{align*}
\]

The Hessian Matrix is

\[
\begin{bmatrix}
-2 & 2\delta \\
2\delta & -2
\end{bmatrix}.
\]

We obtain that the first-order principal minor is negative \((-2 < 0)\) and the second-order principal minor is positive \((4 - 4\delta^2 > 0)\), indicating that the Hessian matrix is negative definite. Therefore, the e-tailer’s profit function is joint concave for \(p_1\) and \(p_2\). We can obtain the first-order conditions are \(\frac{\partial \pi_2}{\partial p_1} = ((\tau + 1)p_3 + 2p_2 - w_2)\delta + \beta\theta + a - 2p_1 + w_1, \frac{\partial \pi_2}{\partial p_2} = ((\tau + 1)p_3 + 2p_1 - w_1)\delta + \beta\theta + a - 2p_2 + w_2\) and \(\frac{\partial \pi_3}{\partial p_3} = ((p_1 + p_2)\delta + \beta\theta + a - 2p_3)(1 - \tau).\)

Then let the first-order conditions be equal to zero, and we get

\[
\begin{align*}
p_1(w_1, w_2) &= \frac{(w_1 - w_2)(\tau + 1)\delta^2 + ((-2\beta\theta - 2a)\delta + 2\beta\theta - 2a + 4w_1)\delta - 4\beta\theta - 4a - 4w_1}{-8 + (4\tau + 4)\delta^2 + 8\delta}, \quad (A.8) \\
p_2(w_1, w_2) &= \frac{(w_2 - w_1)(\tau + 1)\delta^2 + ((-2\beta\theta - 2a)\delta + 2\beta\theta - 2a + 4w_2)\delta - 4\beta\theta - 4a - 4w_2}{-8 + (4\tau + 4)\delta^2 + 8\delta}, \quad (A.9) \\
p_3(w_1, w_2) &= \frac{(w_1 + w_2)\delta^2 + (-w_1 - w_2)\delta - 2\beta\theta - 2a}{-4 + (2\tau + 2)\delta^2 + 4\delta}. \quad (A.10)
\end{align*}
\]

We substitute the equations \((A.8)-(A.10)\) into the supplier 1’s and supplier 2’s profit functions, and for given \(\delta < \frac{1}{2}\), by computing the second-order conditions \(\frac{d^2 \pi_1}{dw_1^2} = \frac{4 + (1 - \tau)\delta^4 + (\tau - 3)\delta^2 - 4\delta}{-4 + (2\tau + 2)\delta^2 + 4\delta} < 0, \frac{d^2 \pi_2}{dw_2^2} = \frac{4 + (1 - \tau)\delta^3 + (\tau - 3)\delta^2 - 4\delta}{-4 + (2\tau + 2)\delta^2 + 4\delta} < 0\), the two suppliers’ profit functions are concave for the two wholesale prices, respectively. Let the first-order conditions be equal to zero, and we get:

\[
\begin{align*}
w_1^{\text{RRM}^*} &= w_2^{\text{RRM}^*} = \frac{2(-2 + (\tau + 1)\delta^2 + (\tau + 1)\delta)(\beta\theta + a)}{-8 + (\tau - 5)\delta^3 + (\tau + 3)\delta^2 + 12\delta}.
\end{align*}
\]

Then, we have

\[
\begin{align*}
p_1^{\text{RRM}^*} &= p_2^{\text{RRM}^*} = -2(-24 + (\tau^2 - 4\tau - 5)\delta^4 + (\tau^2 + 2\tau - 11)\delta^3 + (14\tau + 18)\delta^2 + (4\tau + 28)\delta)(\beta\theta + a) \\
&\quad \cdot \frac{2(-8 + (\tau - 5)\delta^2 + (\tau + 3)\delta^2 + 12\delta)(-2 + (\tau + 1)\delta^2 + 2\delta)}{2(\delta^2 + \delta^2 - 2\delta^2 + 2\delta)(\delta^2 - \delta^2 + 2\delta)}, \\
p_3^{\text{RRM}^*} &= (2\delta^2 + 2\delta^2 - \delta^2 + \delta^2 + 2\delta)(\delta^2 - \delta^2 + 2\delta)(\delta^2 + 2\delta + \delta^2)(\delta^2 - \delta^2 + 2\delta) \\
&\quad + (\delta^2 + \delta^2 - \delta^2 + 2\delta)(\delta^2 - \delta^2 + 2\delta), \\
\pi_1^{\text{RRM}^*} &= \pi_2^{\text{RRM}^*} = (2 + (\tau + 1)\delta^2 + (\tau + 1)\delta)(-4 + (\tau - 1)\delta^3 + (\tau + 3)\delta^2 + 4\delta)(\beta\theta + a)^2 \\
&\quad - \frac{2(-2 + (\tau + 1)\delta^2 + 2\delta)(-8 + (\tau - 5)\delta^3 + (\tau + 3)\delta^2 + 12\delta)k\theta^2}{(\tau + 1)\delta^2 + \delta^2 + 2\delta)(\tau + 1)\delta^2 + \delta^2 + 2\delta)(\tau + 1)\delta^2 + \delta^2 + 2\delta)} \\
p_3^{\text{RRM}^*} &= -(\tau - 1)(\beta\theta + a)^2(2\delta^2 + 2\delta^2 - \delta^2 + \delta^2 - 2\delta^2 - 2\delta^2 + 2\delta^2)(\delta^2 - \delta^2 + 2\delta) \\
&\quad - k\theta^2, \\
\pi_e^{\text{RRM}^*} &= \frac{\tau(\tau^2 + \delta^2 - \delta^2 + \delta^2 - 3\delta^2 + 2\delta - 2\delta^2 + 2\delta - 2\delta^2 - 2\delta^2 + 2\delta)}{2(\tau^2 + \delta^2 + \delta^2 - 2\delta^2 - 2\delta^2 + 2\delta - 2\delta^2 + 2\delta)}.
\end{align*}
\]
**Solution of RMM model**

Proof. Under RMM scenario, we solve the optimization problems of the e-tailer, supplier 2 and supplier 3 simultaneously:

\[
\begin{align*}
\max_{p_1} \pi_1 &= (p_1 - w_1)D_1 + (p_2 - w_2)D_2 + \tau p_3 D_3, \\
\max_{p_2} \pi_2 &= (1-\tau)p_2 D_2 - k\theta^2, \\
\max_{p_3} \pi_3 &= (1-\tau)p_3 D_3 - k\theta^2.
\end{align*}
\]

By computing the first-order conditions \( \frac{d\pi_1}{dp_1} = ((p_2 + p_3)(\tau + 1)\delta + \beta\theta + a - 2p_1 + w_1, \quad \frac{d\pi_2}{dp_2} = -((p_1 + p_3)\delta + \beta\theta + a - 2p_2(-1 + \tau)), \quad \frac{d\pi_3}{dp_3} = -(p_1 + p_2)\delta + \beta\theta + a - 2p_3)/(-1 + \tau), \quad \) and the second-order conditions \( \frac{d^2\pi_1}{dp_1^2} = -2, \quad \frac{d^2\pi_2}{dp_2^2} = 2(\tau - 1), \quad \frac{d^2\pi_3}{dp_3^2} = 2(\tau - 1), \) the e-tailer’s, supplier 2’s and supplier 3’s profit functions are concave for the three retail prices, respectively. Let the first-order conditions be equal to 0 and we get:

\[
\begin{align*}
p_1(w_1) &= \frac{((2\tau - 1)\beta\theta - 2a\tau - a + w_1)\delta - 2\beta\theta - 2a - 2w_1}{-4 + (2\tau + 2)\delta^2 + 2\delta}, \\
p_2(w_1) &= \frac{(-\beta\theta - a - w_1)\delta - 2\beta\theta - 2a}{-4 + (2\tau + 2)\delta^2 + 2\delta}, \\
p_3(w_1) &= \frac{(-\beta\theta - a - w_1)\delta - 2\beta\theta - 2a}{-4 + (2\tau + 2)\delta^2 + 2\delta}.
\end{align*}
\]

We substitute the equations (A.11)–(A.13) into the supplier 1’s profit functions, and for given \( \delta < \frac{1}{2} \), by computing the second-order condition \( \frac{d^2\pi_1}{dp_1^2} = -\frac{2\delta^2 - 4\delta + 2}{2\tau(\tau + 1)\delta^2 + \delta} < 0 \), the supplier 1’s profit functions are concave for the wholesale price. Let the first-order condition be equal to zero, and we get:

\[
\begin{align*}
w_1^{\text{RMM}} = \frac{(\beta\theta + a)(2\delta^2\tau + 2\delta\tau - \delta - 2)}{4\delta^2 + 2\delta - 4}.
\end{align*}
\]

Then, we have

\[
\begin{align*}
p_1^{\text{RMM}} &= \frac{-6\delta^3\tau + 4\delta^3 + 6\delta^2\tau + 11\delta^2 - 4\delta\tau - 12)(\beta\theta + a)}{4(\delta^2\tau + \delta^2 + \delta - 2)(2\delta^2 + \delta - 2)}, \\
p_2^{\text{RMM}} = p_3^{\text{RMM}} &= \frac{-((\beta\theta + a)(2\delta^3\tau + 4\delta^3 + 2\delta^2\tau + 9\delta^2 - 2\delta - 8))}{4(\delta^2\tau + \delta^2 + \delta - 2)(2\delta^2 + \delta - 2)},
\end{align*}
\]

\[
\begin{align*}
\pi_1^{\text{RMM}} &= \frac{(\beta\theta + a)^2(2\delta^2\tau + 2\delta\tau - \delta - 2)^2}{8(2\delta^2 + \delta - 2)(\delta^2\tau + \delta^2 + \delta - 2)} - k\theta^2, \\
\pi_2^{\text{RMM}} = \pi_3^{\text{RMM}} &= \frac{-((\beta\theta + a)^2(2\delta^2\tau + 2\delta\tau - \delta - 2)^2)}{16(\delta^2\tau + \delta^2 + \delta - 2)^2} - k\theta^2, \\
\pi_e^{\text{RMM}} &= \frac{\tau(2\delta^3\tau + 4\delta^3 + 2\delta^2\tau + 9\delta^2 - 2\delta - 8)^2(\beta\theta + a)^2}{8(2\delta^2 + \delta - 2)^2(\tau\delta^2 + \delta^2 + \delta - 2)^2} + \frac{(4\delta^4\tau^2 + 4\delta^4\tau + 4\delta^3\tau^2 + 12\delta^3\tau + 2\delta^3 - 2\delta^2\tau + 5\delta^2 - 12\delta\tau - 4)(2\delta^2\tau + 2\delta\tau - \delta - 2)(\beta\theta + a)^2}{16(\delta^2 + \delta - 2)(\tau\delta^2 + \delta^2 + \delta - 2)^2}.
\end{align*}
\]
Solution of MMM model

Proof. Under MMM scenario, we solve the optimization problems of the three suppliers simultaneously:

\[
\begin{align*}
\max_{p_1} \pi_1 &= (1 - \tau)p_1D_1 - k\theta^2, \\
\max_{p_2} \pi_2 &= (1 - \tau)p_2D_2 - k\theta^2, \\
\max_{p_3} \pi_3 &= (1 - \tau)p_3D_3 - k\theta^2.
\end{align*}
\]

By computing the first-order conditions
\[
\begin{align*}
\frac{d\pi_1}{dp_1} &= (1 - \tau)((p_2 + p_3)\delta + \beta\theta + a - 2p_1), \\
\frac{d\pi_2}{dp_2} &= (1 - \tau)((p_1 + p_3)\delta + \beta\theta + a - 2p_2), \\
\frac{d\pi_3}{dp_3} &= (1 - \tau)((p_1 + p_2)\delta + \beta\theta + a - 2p_3),
\end{align*}
\]
and the second-order conditions
\[
\begin{align*}
\frac{d^2\pi_1}{dp_1^2} &= 2(\tau - 1), \\
\frac{d^2\pi_2}{dp_2^2} &= 2(\tau - 1), \\
\frac{d^2\pi_3}{dp_3^2} &= 2(\tau - 1),
\end{align*}
\]
the three suppliers’ profit functions are concave for the three retail prices, respectively. Let the first-order conditions be equal to 0 and we get:

\[
\begin{align*}
p_1^{MMM*} &= p_2^{MMM*} = p_3^{MMM*} = \frac{\beta\theta + a}{2 - 2\delta}.
\end{align*}
\]

Then, we have:

\[
\begin{align*}
D_1^{MMM*} &= D_2^{MMM*} = D_3^{MMM*} = \frac{-\beta\theta - a}{2\delta - 2}, \\
\pi_1^{MMM*} &= \pi_2^{MMM*} = \pi_3^{MMM*} = \frac{(1 - \tau)(\beta\theta + a)^2}{4(\delta - 1)^2} - k\theta^2, \\
\pi_e^{MMM*} &= \frac{3\tau(\beta\theta + a)^2}{4(\delta - 1)^2}.
\end{align*}
\]

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References


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