EFFECTS OF PREPAYMENT POLICY ON EQUILIBRIUM OF THE RETAILER-DOMINATED CHANNEL CONSIDERING MANUFACTURER EFFORT

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Abstract. Although upstream manufacturers with small- and medium-sized are gradually willing to invest green efforts for stimulating market demand, they have been encountering the challenge of securing sufficient working capital to develop the green supply chain. Thus, this paper systematically incorporates two types of prepayment policies including risk-free (RF) and risk-taking (RT) into a retailer’s dominated channel. Via deriving operational and financing equilibrium of the green supply chain, a series of interesting findings can be offered. Specifically, (1) this paper identifies a threshold value regarding the manufacturer’s own capital, and proposes two scenarios for assisting the retailer to decide whether offers the manufacturer prepayment policy. (2) The effectiveness of RF for the capital-constrained manufacturer depends on how well its green effort can be implemented. That is, provided that the quality effect is large enough, the manufacturer can get more upfront capital from the retailer, which may entirely cover its total production and green effort costs. (3) Under RT, if the manufacturer’s capital is relatively lower, RT enables the manufacturer to obtain sufficient capital and the retailer is willing to share partial of the manufacturer’s default risk. Otherwise, RT may not be the best prepayment policy for the retailer.

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1. Introduction

For the last few years, due to the increasingly serious environmental crisis, the upstream manufacturer has devoted to exerting green efforts for behaving more responsible towards the social and environmental issues. That is, the manufacturer is required to invest advanced technologies for improving the publicized product greenness indicators [10, 11, 19]. For example, NIKE has made materials recyclable by recycling production waste and plastic bottles to extract materials when producing Flyknit fabrics. As a result, consumers with more environmentally conscious can be attracted by the manufacturer offering products with higher greenness, resulting in the increase of the market demand [20].
Apparently, the manufacturer has to spend considerable cost for conducting its green efforts, such as developing R&D expenditures and procuring new machinery [8, 21]. And it is no doubt that more investment cost is required for the manufacturer to exert more green efforts. However, most of upstream enterprises are small- and medium-sized, thus facing the challenge of securing sufficient working capital for supporting their green efforts investment [7, 30]. For instance, the European steel industry has drawn up 60 decarbonization projects, which are expected to effectively reduce 81.5 million tons of CO₂ emissions directly and indirectly caused by the steel production processes. Nevertheless, since the capital expenditure of these projects is expected to be as high as 31 billion euros, steel enterprises lacked of sufficient funds and delayed the implementation of these projects.

Meanwhile, over the past few decades, bargaining power has gradually transferred from the upstream manufacturer to the downstream retailer, leading giant retailers to dominating the entire channel, which can be easily identified in retailing, apparel, smartphone industries [2, 3]. And in the context of green supply chain management, the dominated retailer is more likely to require the manufacturer to exert green efforts for attracting more customers with environmentally conscious. For example, Starbucks, one of the dominant retailers in coffee industry, requires its suppliers to adopt digital high-tech equipment for constructing the agile digital factories, aiming to implement energy saving and achieve green management. As a result, the challenge between the manufacturer’s limited own capital and the high cost of green efforts becomes more acute.

As the dominated role of the supply chain, it is possible for the retailer to grant partial prepayment to capital-constrained manufacturers for alleviating its capital pressure [9, 14]. For instance, in 2013, in order to ensure sufficient supply of large-size LCD screens, Samsung Electronics invested 110$ million in Sharp which was in deep financial trouble. Nevertheless, in the context of the stochastic demand, when the prepayment ratio is relatively low, it may still not solve the manufacturer’s financial dilemma; while when the prepayment ratio is rather high, it is possible for the manufacturer failing to pay off the retailer provided that the realized demand is lower. Therefore, it is important for retailers to develop an effective prepayment strategy to balance the earned revenue and the default risk.

In response these urgent issues, this paper proposes a retailer-dominated channel including a capital-constrained manufacturer and a capital-sufficient retailer, where two types of prepayment policies are involved. That is, (1) risk-free prepayment (RF): the retailer only pay in advance for the increased sales stimulated by the manufacturer’s green effort, while the rest of purchase cost will be paid off as soon as receiving finished products from the manufacturer. (2) Risk-taking prepayment (RT): as soon as placing the order, the retailer pay off the purchase cost to the manufacturer. Therefore, this paper aims to solve the equilibrium of the supply chain under these two types of prepayment policies, systematically discuss the similarities and differences between them in terms of solving the operational dilemma of the capital-constrained manufacturer, and provide decision supports for the retailer’s prepayment policy selection.

Based on derived theoretical results and managerial insights, this study contributes to the literature in three ways. (1) Based on a threshold value regarding the manufacturer’s own capital, this paper proposes two scenarios for assisting the retailer to decide whether offers the manufacturer prepayment policy. That is, in the higher capital scenario, there is no need for the retailer to offer prepayment policy. Otherwise, the retailer should trade off between RF and RT to enhance its operational performance via alleviating the manufacturer’s capital pressure. (2) The effectiveness of RF for the capital-constrained manufacturer depends on how well its green effort can be implemented. Specifically, if the quality effect is large enough, the manufacturer can get more upfront capital from the retailer, which may entirely cover its total production and green effort costs; otherwise, although the manufacturer can still improve its performance due to its increased working capital, the manufacturer cannot cover its optimal policy via RF. (3) Under RT, the manufacturer’s own capital is the key for the retailer to offer RT, which also leads to two regions. That is, if the manufacturer’s capital is relatively lower, RT enables the manufacturer to obtain sufficient capital and the retailer is willing to share partial of the manufacturer’s default risk. Otherwise, RT will not have a huge impact on the manufacturer’s operational decision-making, but will make the retailer take unnecessary demand risks. Therefore, RT may not be the best prepayment policy for the retailer.
The rest of the paper is arranged as follows. In Section 2, we provide a brief review of the related literature concerning green supply chain management, retailer dominance channel and operational policy under trade credit financing. Then, the problems being solved are presented in Section 3, and the equilibrium without prepayment policy is presented is Section 4. Section 5 provides the mathematical model under two different prepayment policies. Section 6 summarizes the whole paper and proposes the further research directions.

2. Literature review

In this study, we base on the retailer-dominated channel to discuss operational and financing equilibrium of the green supply chain. Therefore, this section briefly reviews a number of recent studies on green supply chain management, retailer dominance channel and operational policy under trade credit financing to identify key gaps in this study.

The first research area related to this paper is the green supply chain management, where plenty of researches have addressed optimal solutions of the green supply chain. Murali et al. [20] formulated a consumer-driven model focusing on the self-labeling competition between two firms, and incorporated voluntary ecolabels and mandatory environmental regulation on green product development. In a two-echelon supply chain, Chen et al. [5] assumed that the cooperation between a manufacturer and a retailer can be achieved to jointly invest green R&D, and then quantified its effects towards the economic, environmental and social performances of the supply chain. Mahdi et al. [19] attempted to coordinate a sustainable supply chain under competition, where a manufacturer focuses on the carbon emissions reduction and two retailers invest in green efforts. Paul et al. [23] presents a retailer’s green inventory model that takes into account investments in eco-friendly operations, variable holding costs, and green-sensitive demand. Derived results reveal that green retailing is a cost-effective way for the retailer to reduce their carbon footprint and protect the environment.

This paper is also closed to operational policies under the retailer-dominated channel, which has been applied in various dimensions, such as supply chain financing [16], government subsidy [18], retailer competition [3]. Then, several scholars have incorporated the green supply chain into the retailer-dominated channel. In a sustainable supply chain, Chen et al. [4] discussed impacts of power relationship and coordination, where customer demand is sensitive to carbon emission. Dash Wu et al. [6] attempted to address how carbon emission reduction affects supply chain operations and financing decisions, pointing out that a win-win can be achieved as long as manufacturers invest in emission reduction. Based on a two-level supply chain model, Pan et al. [22] addressed the impact of fairness concerns on pricing, profitability and utility of supply chain members. Nevertheless, since the follower manufacturer often lacks sufficient capital, it is very unlikely for itself to achieve the optimal production input and green effort at the same time. In this case, there is a lack of relevant research on what kind of support the leading retailer should offer the manufacturer, and how this support can help operational performances of both parties.

The third research steam is operational policies of the supply chain under trade credit financing. On one hand, existing researches pay more attention to how manufacturers optimize ordering decisions of downstream capital-constrained retailers through delay in payment policy under various dimensions, such as partial back-order [25], two levels of trade-credit [26], deteriorating items [1], expiration date [17]. On the other hand, several scholars have also discussed influences of prepayment strategy on operational policies of the supply chain. Under the deterministic demand setting for green products, Ghosh et al. [8] incorporated both advanced payment and trade credit policy to achieve the supply chain coordination model with capital constrained retailers. Paul et al. [24] examined effects of multiple prepayments and green efforts towards the equilibrium of an economic production quantity model, where the market demand relies on selling-price and green level of products. Under downstream delayed payment and upstream partial prepayment, Kumar et al. [15] examined how food preservation impacts optimal operational policies of a food supply chain model for a deteriorating item. In the context of multiple advance and delayed payments policies, Ghosh et al. [9] formulated an economic order quantity model to derive the retailer’s optimal order cycle for perishable items with fixed lifetime.
Table 1. Summary of related literature.

<table>
<thead>
<tr>
<th>Papers</th>
<th>Green effort</th>
<th>Capital constraint</th>
<th>Prepayment</th>
<th>Delay in payment</th>
<th>Market demand</th>
<th>Retailer dominated</th>
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</thead>
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<tr>
<td>Chen et al. [4]</td>
<td>✓</td>
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<tr>
<td>Pervin et al. [25]</td>
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<tr>
<td>Lin and Xiao [16]</td>
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<tr>
<td>Murali et al. [20]</td>
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<tr>
<td>Chen et al. [5]</td>
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<tr>
<td>Roy et al. [26]</td>
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<td>Mahdi et al. [19]</td>
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<tr>
<td>Paul et al. [23]</td>
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<tr>
<td>Liu et al. [18]</td>
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<tr>
<td>Chakraborty and Mandal [3]</td>
<td>✓</td>
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<td>Asim et al. [1]</td>
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<tr>
<td>Lin et al. [17]</td>
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<tr>
<td>Ghosh et al. [8]</td>
<td>✓</td>
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<tr>
<td>Kumar et al. [15]</td>
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<tr>
<td>Ghosh et al. [9]</td>
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<tr>
<td>Paul et al. [24]</td>
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<tr>
<td>Present paper</td>
<td>✓</td>
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</table>

Notes. (1) terms “Manu.” and “Ret.” refer to the manufacturer and the retailer, respectively; (2) terms “Det.” and “Sto.” refer to deterministic and stochastic, respectively.

However, existing prepayment terms do not take into account the default risk of the manufacturer arising from the stochastic demand. In this case, it is unwise for the leading retailer to offer the manufacturer full prepayment. Therefore, this paper incorporated two types of prepayment policies into a retailer-dominated channel to examine their effects on the equilibrium of the supply chain. For simplicity, the following table is offered to easily distinguish this paper from previous related studies.

3. Model description

This paper formulates a two-level supply chain consisting of two risk neutral members, i.e., a single manufacturer with limited capital $K_m$ and a single retailer with sufficient capital. In this paper, the retailer acts as a leader, and offers the manufacturer its anticipated the wholesale price $w$. Then, the follower manufacturer is in charge of designing and manufacturing the products, taking charging of determining production input quantity $q$ and effort degree $e$.

The retailer’s unit selling price $p$ is exogenous, which is normalized to 1, and the manufacturer’s unit production cost is $c$. And referring to [28] and [12], in the retailer-dominated channel, the retailer bears no responsibility for any inventory risk, while the manufacturer bears all the inventory risk. Further, the market demand $\varepsilon$ is stochastic, following a probability distribution function $f(\varepsilon)$ and a cumulative distribution function $F(\varepsilon)$. Then, let $h(\varepsilon)$ indicate the hazard function, where $h(\varepsilon) = f(\varepsilon) / F(\varepsilon)$, and $h(\varepsilon)$ is assumed to increase with $\varepsilon$ via referring to [29] and [27].

As [20] noted, the manufacturer is able to assume efforts $e$, whose corresponding cost equals $\frac{1}{2}ke^2$. As a result, market demand is positively related to marketing effort, i.e., $D = \varepsilon + ae$. Nevertheless, due to owning insufficient working capital, the manufacturer may fail to raise enough funding to afford both production cost and efforts for implementing the optimal solution. The corresponding model can be defined as the manufacturer’s effort (ME) model, see Section 4.

To mitigate capital pressure of the manufacturer, this paper proposes two types of prepayment policies offered by the retailer. (1) Risk-free prepayment (RF): the manufacturer simply pays off $wae$ to the manufacturer in advance, leading the manufacturer’s total working capital to equalling $wae + K_m$. Since $ae$ denotes the increased market demand due to the manufacturer’s green efforts, and this part of products can be successfully sold to
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Table 2. Notations throughout the paper.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$</td>
<td>the limited capital of the manufacturer (in $)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>the stochastic market demand (in year)</td>
</tr>
<tr>
<td>$f(\varepsilon)$</td>
<td>a probability distribution function of the market demand</td>
</tr>
<tr>
<td>$F(\varepsilon)$</td>
<td>a cumulative distribution function of the market demand</td>
</tr>
<tr>
<td>$D$</td>
<td>the stimulated demand due to the manufacturer’s effort, equaling $\varepsilon + ae$</td>
</tr>
<tr>
<td>$k$</td>
<td>the coefficient of the manufacturer’s green effort</td>
</tr>
<tr>
<td>$p$</td>
<td>the retailer’s unit selling price (in $)</td>
</tr>
<tr>
<td>$c$</td>
<td>the manufacturer’s unit production cost (in $)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>the wholesale price determined by the retailer (in $)</td>
</tr>
<tr>
<td>$q$</td>
<td>the manufacturer’s production input quantity</td>
</tr>
<tr>
<td>$e$</td>
<td>the manufacturer’s exerted effort degree</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Profit functions</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\Pi_m(q, e)$</td>
<td>the manufacturer’s profit function</td>
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<tr>
<td>$\Pi_r(w)$</td>
<td>the retailer’s profit function</td>
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</tbody>
</table>

customers. Under the circumstance, the prepayment can be regarded as risk-free prepayment contract. (2) Risk-taking prepayment (RT): as soon as placing the order, the retailer bases on $wq$ to pay off the purchase cost, which the manufacturer’s total working capital equals $wq + K_m$. In this case, since the manufacturer is simply responsible for limited default risk, the retailer’s prepayment may exceed its expected revenue when the realized demand is relatively lower.

For simplicity, let the subscripts $i = 0, 1, 2$ be the index to ME, RF and RT model, respectively. In addition, the notations adopted in this paper are presented in Table 2.

4. Equilibrium without prepayment policy

In this part, the retailer is assumed not to offer prepayment policy, and thus the manufacturer must complete its effort and production process with its own capital. Then, the decision sequences of the retailer-dominated channel can be listed below: (1) the retailer offers the manufacturer its anticipated wholesale price. (2) After observing the retailer’s pricing policy, the manufacturer announces both production input quantity and effort degree. As a result, backward induction can be used to obtain the equilibrium of the supply chain.

According to the VMI setting, the manufacturer’s sale revenue relies on the final realized demand, i.e., $w_{\text{min}}(D, q)$. And the manufacturer’s total cost includes production cost and investment cost, which should not exceed its own capital. To sum up, the manufacturer’s maximization problem can be listed below.

$$\begin{align*}
\text{P1} \quad & \text{Max } \Pi_m(q, e) = w_{\text{min}}(D, q) - cq - \frac{ke^2}{2}, \\
& \text{s.t., } cq + \frac{ke^2}{2} \leq K_m.
\end{align*}$$

(1a)

(1b)

By addressing properties of $\Pi_m(q, e)$, the following results can be formulated.

**Proposition 4.1.** For the given $w$, let $e_0^a = \frac{a(w-c)}{k}$, $q_0^a$ satisfy $F(q - ae_0^a) = \frac{\varepsilon}{w}$, $e_0^b$ satisfy $a = \left(\frac{k}{c} + a\right) F\left(\frac{K_m}{\varepsilon} - \frac{k(e_0^b)^2}{2c} - ae_0^b\right)$, $L_0 = cq_0^a + \frac{k(e_0^a)^2}{2} - K_m$, the following hold true.

1. If $L_0 \leq 0$, then $(q_0^a, e_0^a) = (q_0^a, e_0^a)$.
If \( L_0 > 0 \), then \((q^*_0, e^*_0) = (K_m c - k (e^*_b)^2, e^*_b)\).

Proof. see Appendix A for the details. □

From Proposition 4.1, it is apparent that the manufacturer’s limited working capital constraints the manufacturer’s optimal response, resulting in different relationships between wholesale price and production input quantity. Specifically, (1) if \( L_0 \leq 0 \), the manufacturer can achieve its global optimal performance within its capital. And since \( \frac{de}{aw} = \frac{a^2}{k} + \frac{c}{w^2 f(q - ae^*_0)} > 0 \), a positive relationship between wholesale price and effort degree (production input quantity) can be identified. (2) If \( L_0 > 0 \), the manufacturer’s optimal solution is irrelevant to wholesale price.

Further, based on these findings, there exists a unique solution \((w^*_0)\) satisfying \(L_0 = 0\), see Figure 1, where \(w^*_0 > c\). That is, for \(w \leq w^*_0\), the manufacturer can achieve its optimal performance \((q^*_0, e^*_0)\); otherwise, the manufacturer has to run out of its capital to conduct the supply process. In other words, due to lacking sufficient working capital, the manufacturer will keep the same response when the retailer adopts a wholesale price more than \(w^*_0\).

Additionally, observing the manufacturer’s optimal response, the retailer’s profit equals \((1 - w) \min (\varepsilon, q - ae)\). Based on the manufacturer’s response provided in Proposition 4.1, the retailer’s profit maximization can be presented below:

\[
\Pi_r(w) = \begin{cases} 
(1 - w) \min (\varepsilon + ae^*_0, q^*_0), & \text{if } w \leq w^*_0, \\
(1 - w) \min (\varepsilon + ae^*_b, q^*_b), & \text{if } w > w^*_0.
\end{cases}
\]

Thus, by addressing properties of equations (2a)–(2b), the following results can be formulated.

Proposition 4.2. Let \(w^*_0\) satisfy \(\int_{q^*_0}^{q^*_0 - ae^*_0} F(\varepsilon) \, d\varepsilon = \frac{(1 - w)c}{w^2 h(q^*_0 - ae^*_0)}\), the following hold true.

(1) If \(w^*_0 \geq 1\), then \(w^*_0 = w^*_0\).

(2) If \(w^*_0 < 1\), then \(w^*_0 = \min \{w^*_0, w^*_0\}\).

Proof. see Appendix B for the details. □
From Proposition 4.2, the threshold value $w^*_0$ plays the key role of reflecting effects of the manufacturer’s capital constraint and determining the optimal wholesale price. That is, provided that $w^*_0 \geq 1$, the manufacturer actually owns the sufficient working capital, and is unlikely to require the retailer’s prepayment.

For simplicity, since $K_m = cq_0^a + \frac{k(e^a_0)^2}{2}$ for $w = w^*_0$, let $K_b^a$ and $K_a^b$ denote values of $cq_0^a + \frac{k(e^a_0)^2}{2}$ when $w = w^*_0$ and $w = 1$, where $K_a^a \leq K_b^a$. By referring to Proposition 4.2, in the case of $w^*_0 \geq 1$, it yields $w^*_0 = w^*_0$ for $K_m \geq K_a^a \geq K_b^a$; otherwise, we have $w^*_0 = w^*_0$ for $K_b^b > K_m > K_a^a$ (i.e., $w^*_0 \leq w^*_0$), and $w^*_0 = w^*_0$ for $K_a^a > K_m$ (i.e., $w^*_0 > w^*_0$). To sum up, based on the relationship between $K_m$ and $K_a^a$, two cases in Proposition 4.2 can be regarded as the higher and lower capital scenarios, respectively, see Figure 2.

(1) In higher capital scenario (i.e., $K_m \geq K_a^a$), the retailer can control wholesale price to be not more than $w^*_0$, and the manufacturer performs as without capital constraint. That is, the manufacturer can assume the optimal production policy and marketing effort cost $(q^a_0, e^a_0)$. Under the circumstance, there is no need for the retailer to offer prepayment policy.

(2) In lower capital scenario (i.e., $K_m < K_a^a$), the retailer cannot announce its optimal wholesale price, above which the manufacturer cannot guarantee to fulfil the corresponding production quantity and effort. These findings indicate that offering prepayment to the manufacturer may be mutually beneficial to the supply chain.

Further, in order to explicitly present effects of the manufacturer’s limited capital on the operational performance of the retailer, the following numerical example is offered. That is, let initial values of parameters are
given below: \( \lambda = 0.3, a = 3, c = 0.5, k = 2 \), where let \( K_m \) equals 0.3, 0.6 and 1.2, respectively. Then, from Figure 3, by varying values of parameters, it is easy to find that the retailer’s profit is positively associated with \( a \), while negatively related to \( c, \lambda, k \). In particular, as the manufacturer’s own capital increases, it is likely to increase its input quantity and effort level, thereby benefiting operational performance of the retailer. This conclusion can well explain why the leading retailer prefers to choose the manufacturer with sufficient funds to cooperate. In other words, if the retailer can offer the manufacturer the prepayment strategy to improve the manufacturer’s capital flow, it is highly likely to achieve a win-win situation for upstream and downstream firms.

5. **Equilibrium under prepayment policies**

In this section, two types of payment policies will be offered by the retailer to increase the manufacturer’s working capital, i.e., RF, and RT. Corresponding decision sequences are the same as those under Section 3, which can also be solved by backward induction.

5.1. **Equilibrium under risk-free prepayment contract**

In this case, the manufacturer assumes risk-free prepayment contract, i.e., \( wae \), leading the manufacturer’s total working capital to equalling \( wae + K_m \). According to the VMI setting, at the end of selling period, two cases can be proposed. (1) If \( D \geq q (\varepsilon > q - ae) \), then all finished products can be successfully sold to customers, and the manufacturer’s selling revenue equals \( wq \); (2) If \( q > D \), then the manufacturer’s final realized quantity equal \( ae + w\varepsilon \). And it should be noted that under both cases, the final realized market demand is always more than the retailer’s prepayment, thus there is no default risk for the manufacturer. Further, considering the manufacturer’s production and effort cost, the manufacturer’s maximization problem can be listed below.

\[
P2 \quad \text{Max } \Pi_m (q, e) = w\min (D, q) - c_q \frac{ke^2}{2},
\]

\[
s.t., \quad c_q + \frac{ke^2}{2} \in [0, K_m + wae].
\]

It is surprising to find that equation (3a) is the same as equation (1a). That is, this prepayment doesn’t alter the selling revenue of the manufacturer, but increase the manufacturer’s working capital. Thus, by referring to Proposition 4.1, the following proposition can be obtained.

**Proposition 5.1.** For the given \( w \), let \( (q^a_1, e^a_1) = (q^a_0, e^a_0) \), \( (q^b_1, e^b_1) \) satisfy \( e^b_1 = \frac{wae - ca}{k} \) and \( q^b_1 = \frac{K_m}{c} + \frac{(ca)^2 - (ca)^2}{2ke} \), \( L_1(w) = c_q q^a_1 + \frac{k(e^a_1)^2}{2} \leq K_m + wae^a_1 \), the following hold true.

1. If \( L_1(w) \leq 0 \), then \( (q^a_1, e^a_1) = (q^a_0, e^a_0) \).
2. If \( L_1(w) > 0 \), then \( (q^b_1, e^b_1) = (q^b_1, e^b_1) \).

**Proof.** see Appendix C for the details.

From Proposition 5.1, this type of prepayment could exert two kinds of effects on the manufacturer’s production and effort cost policy. Specifically, (1) due to the retailer’s prepayment, the manufacturer can apply more capital to conduct the optimal supply process. In other words, the manufacturer is more likely to achieve global optimal solution. (2) In addition, under both cases, the manufacturer’s production and effort cost is positively related to wholesale price. That is, by offering partial prepayment, the retailer can more effectively control the manufacturer’s production response via adjusting the pricing policy.

Further, it can be summarized that wholesale price exerts two different types of effects on the manufacturer’s production response. That is, along with the increase of \( w \), the manufacturer can own more working capital; but simultaneously the manufacturer may consume more capital due to its stimulated production incentive. In addition, the corresponding theoretical result concerning \( L_1(w) \) can be presented below:
Corollary 5.2. Let $w_{11}^i$ and $w_{12}^i$ satisfy $L_1(w) = 0$, where $w_{12}^i > w_{11}^i \geq w_0^i$. And the following hold true for when $w_0^i < 1 < w_{12}^i$.

(1) If $w_{11}^i > 1 \geq w_0^i$, then $L_1(w) \leq 0$ for $w \in [0, 1]$.
(2) If $1 > w_{11}^i \geq w_0^i$, then $L_1(w) < 0$ for $w \in [0, w_{11}^i]$ and $L_1(w) \leq 0$ for $w \in [w_{11}^i, 1]$.

Proof. see Appendix D for the details.

From Corollary 5.2, the value of $w_{11}^i$ can be applied to measure the quality effect. That is, (1) when quality effect is relatively higher ($w_{11}^i > 1$), the manufacturer is able to pay off its production cost due to RF. Under the circumstance, the retailer’s offered prepayment can entirely solve the manufacturer’s capital constraint. (2) When quality effect is relatively lower ($1 > w_{11}^i$), the risk-free prepayment cannot always provide sufficient capital for the manufacturer, reflecting the disadvantage of RF.

Further, observing the manufacturer’s optimal response, the retailer’s profit equals $(1 - w) \min (\varepsilon + ae_1^*, q_1^*)$. Two cases can be proposed based on Corollary 5.2. Based on the manufacturer’s responses, the retailer’s profit function can be presented below:

$$
\Pi_r(w) = \begin{cases} 
(1 - w) \min (\varepsilon + ae_1^*, q_1^*) & \text{if } w \leq w_1^i, \\
(1 - w) \min (\varepsilon + ae_0^*, q_1^*) & \text{if } w > w_1^i.
\end{cases}
$$

(4a)

By addressing properties of equations (4a)–(4b), the following results can be formulated.

Proposition 5.3. Under RF, let $w_1^i$ satisfy $\int_0^{z_1(w)} F(\varepsilon) d\varepsilon = \frac{a^2(1-w)(w-1)F(z_1(w)) + a^2(c-2w+1)}{k} + \frac{(ac-aw)^2}{2ke}$, the following hold true when $(1 - \frac{1}{c}) \bar{F}(\frac{K_m}{c}) + 2 > 0$.

(1) If $w_{11}^i \geq 1$, then $w_1^i = w_0^i$.
(2) If $1 > w_{11}^i$,
   (i) If $w_0^i \leq w_{11}^i$, then $w_1^i = w_0^i$.
   (ii) If $w_0^i > w_{11}^i$, then $w_1^i = w_1^i$.

Proof. see Appendix E for the details.

For simplicity, since $K_m = cq_0^i + \frac{k(e_0^*)^2}{2} - wae$ for $w = w_{11}^i$, let $K_m^i$ and $K_1^i$ denote values of $cq_0^i + \frac{k(e_0^*)^2}{2} - wae$ when $w = w_1^i$ and $w = 1$. By referring to Proposition 5.3, in the case of $w_{11}^i \geq 1$, it yields $w_1^i = w_0^i$ for $K_m \geq K_b^i \geq K_a^i$. Otherwise, we have $w_1^i = w_0^i$ for $K_b^i > K_m > K_a^i$, and $w_1^i = w_1^i$ for $K_1^i > K_m$. To sum up, if $K_m \geq K_a^i$, the manufacturer can assume the optimal production policy and effort cost $(q_1^*, e_1^*)$. Otherwise, the retailer cannot announce its optimal wholesale price, above which the manufacturer cannot guarantee to fulfill the corresponding production quantity and invested efforts.

Compared with Proposition 4.2, in the lower capital scenario (i.e., $K_m < K_0^i$), two types of results can be presented when RF is offered by the retailer, see Figure 4. (1) RF complete valid region ($K_m \geq K_a^i$): in this case, since quality effect is large enough, the retailer is possibly to gain its expected revenue. As a result, the retailer is able to pay off the manufacturer considerable large payment, which entirely covers the manufacturer’s cost. (2) RF partial valid region ($K_m < K_1^i$): in this case, when the quality effect is not obviously large, the manufacturer cannot cover its optimal policy via the retailer’s prepayment. Nevertheless, the manufacturer can still improve its performance due to its increased working capital.

Similar to Section 4, this part still uses the exponential distribution to depict the demand function, and the relevant parameters also remain the same. By letting $K_m$ equal 0.2, 0.4 and 1, respectively, the retailer’s corresponding profit under RF can be seen in Figure 5. That is, under RF, the correlation characteristics between the retailer’s profit and parameters are consistent with those under ME. And combined with Figure 3 and Figure 5, it can be seen that: (1) Due to the existence of RF, the manufacturer with a relatively lower capital can increase its own working capital to some degree, which greatly optimizes its production strategy.
5.2. Equilibrium under risk-taking prepayment contract

In this case, the manufacturer assumes default risk prepayment, i.e., \( wq \), which the manufacturer’s total working capital equals \( wq + K_m \). According to the VMI setting, at the end of selling period, two cases can be proposed: (1) If \( D \geq q \), then \( \Pi_m (q, e) = \left( wq + K_m - cq - \frac{ke^2}{2} \right)^+ \); (2) If \( q > D \), then \( \Pi_m (q, e) = \left( wD + K_m - cq - \frac{ke^2}{2} \right)^+ \). To sum up, the manufacturer’s maximization problem can be listed below.

\[
P3 \quad \text{Max} \quad \Pi_m (q, e) = \left[ w \min (q, D) + K_m - cq - \frac{ke^2}{2} \right]^+ - K_m, \quad (5a)
\]
\[
\text{s.t., } cq + \frac{ke^2}{2} \in [K_m + wae, K_m + wq]. \quad (5b)
\]

Under RT, the threshold demand equals \( \bar{\xi}_p = \frac{cq + \frac{ke^2}{2} - K_m}{ae} - ac \), and the constraint equation \((5b)\) can ensure that \( \bar{\xi}_p > 0 \). That is, the realized demand is not less than \( \bar{\xi}_p \), the manufacturer can get rid of default risk;
Corollary 5.5. Let \( e_2^0 = \frac{aw-ac}{k} \) and \( q_2^a \) satisfy \( F(q_c) = \frac{c}{w} \tilde{F}(\tilde{\varepsilon}_p) \), where \( q_c = q - ac_2^0 \), the following hold true.

(1) If \( cq^a_2 + \frac{k(e_2^0)^2}{2} \in [K_m + wae_2^0, K_m + wq^a_2] \), then \( (q_2^a, e_2^0) = (q_2^0, e_2^0) \).

(2) If \( cq^a_2 + \frac{k(e_2^0)^2}{2} > K_m + wq^a_2 \), then \( (q_2^a, e_2^0) = (0, 0) \).

(3) If \( cq^a_2 + \frac{k(e_2^0)^2}{2} < K_m + wae_2^0 \), then \( (q_2^a, e_2^0) = (q_2^0, e_2^0) \).

Proof. see Appendix F for the details.

Proposition 5.4. Let \( e_2^0 = \frac{aw-ac}{k} \) and \( q_2^a \) satisfy \( F(q_c) = \frac{c}{w} \tilde{F}(\tilde{\varepsilon}_p) \), where \( q_c = q - ac_2^0 \), the following hold true.

Proof. see Appendix F for the details.

Corollary 5.5. Let \( q_2^a \) satisfy \( F(q_c - \frac{a^2(1-c^2)}{2k}) = c\tilde{F}(cq - K_m - \frac{a^2(1-c^2)}{2k}) \), the following hold true.

(1) If \( q_2^a \geq K_m + \frac{a^2(1-c^2)}{2k} \), then \( q_2^a, q_c \) and \( \tilde{\varepsilon}_p \) are positive with \( w \) for \( w \in [w_2^1, 1] \), where \( w_2^1 \) satisfies \( K_m + wae_2^0 \).

(2) Otherwise, RT is not applicable.

Proof. see Appendix G for the details.

From Corollary 5.5, it is no doubt that the manufacturer’s production input quantity can be stimulated by the retailer’s increased wholesale price. And provided that \( q_2^a \geq K_m + \frac{a^2(1-c^2)}{2k} \), RT can be available for the retailer, and the corresponding announced wholesale price should be not below \( w_2^1 \).

Regarding the retailer’s profit, (1) If \( D \geq q > \frac{cq+k_2^a-K_m}{w} \), then \( \Pi_r(w) = (1-w)q \); (2) If \( q > D > \frac{cq+k_2^a-K_m}{w} \), then \( \Pi_r(w) = (1-w)D \); (3) If \( q > D > \frac{cq+k_2^a-K_m}{w} \), then \( \Pi_r(w) = D + K_m - cq - \frac{ke_2^2}{2} \).

Based on the manufacturer’s response, the retailer’s maximization problem can be listed below.

\[
P4 \Pi_r(w) = \min(q,D) - \max(w \min(q,D),cq + \frac{ke_2^2}{2} - K_m) \tag{6a}
\]

\[
s.t., \ w \in [w_2^1, 1]. \tag{6b}
\]

Since \( \max(w \min(q,D),cq + \frac{ke_2^2}{2} - K_m) = w \max(q_c, \tilde{\varepsilon}_p) + wae \) and \( \max(q_c, \tilde{\varepsilon}_p) = \int_{\tilde{\varepsilon}_p} F(\varepsilon) d\varepsilon + \tilde{\varepsilon}_p \). Further, referring to [13], let the failure rate of the demand distribution \( h(\varepsilon) \) be both increasing and convex on its support. The following proposition can be obtained via analysing P4.

Proposition 5.6. Under RT, let \( w_2^1 \) satisfy \( \tilde{\varepsilon}_p = \frac{1 - \tilde{F}(\tilde{\varepsilon}_p)}{1 - \tilde{F}(\tilde{\varepsilon}_p)} = (1-w) \tilde{F}(q_c), \) and \( w_2^0 \) satisfy \( \frac{c-h(q_c)\tilde{\varepsilon}_p}{wH(q_c)-cH(\tilde{\varepsilon}_p)} + \frac{(1-w)\tilde{F}(q_c)w-H(\tilde{\varepsilon}_p)}{w[H(q_c)-cH(\tilde{\varepsilon}_p)]} - \int_{\tilde{\varepsilon}_p} F(\varepsilon) d\varepsilon + \frac{(-2w+c+1)a^2}{k} = \tilde{\varepsilon}_p \), the following hold true if \( -\int_{\tilde{\varepsilon}_p(w_2^1)} F(\varepsilon) d\varepsilon + \frac{(-2w_2^1+c+1)a^2}{k} - \tilde{\varepsilon}_p(w_2^0) < 0 \):
Figure 6. Equilibrium under RF prepayment policy.

Figure 7. Retailer’s preferences towards RF and RT prepayment policies.

(1) If \( w^2_a \geq w^2_l \), then \( w^*_2 = w^2_a \).
(2) If \( w^2_a < w^2_l \), then \( w^*_2 = w^2_l \).

Proof. see Appendix H for the details.

For simplicity, let \( K^a_2 = cq^a_2 + \frac{k(q^a_2)^2}{2} - wa^a_2 \) for \( w^*_2 = w^2_a \) and \( K_2 = cq^a_2 + \frac{k(q^a_2)^2}{2} - wa^a_2 \) for \( w^*_2 = w^1_l \), respectively. Then, two types of scenarios can be offered to examine effects of RT, see Figure 6. That is, (1) RT valid region (\( K_m \leq K^a_2 \), i.e., \( w^*_2 \geq w^2_l \)) in this case, the prepayment policy can successfully stimulate the manufacturer’s production incentive and mitigate its capital pressure. As a result, the retailer is willing to share partial of the manufacturer’s risk via offering a higher wholesale price. (2) RT fail region (\( K_m > K^a_2 \), i.e., \( w^*_2 > w^1_l \)) in this case, the retailer is reluctant to share the manufacturer’s default risk. And other prepayment policy such as RF is very likely to be a better solution for the manufacturer and the retailer.

Further, in order to explicitly present the retailer’s preferences towards RF and RT, the same parameters are offered as those in Section 4. By varying values of parameters, the derived result can be presented in Figure 7. Specifically,

(1) In terms of \( K_m \), three cases can be discussed. There is no doubt that when the manufacturer’s own higher capital, there is no need for the retailer to offer prepayment policy. Further, when the retailer’s capital is at the middel level, the retailer prefers to offer RF for controlling default risk. And if the manufacturer urgently need working capital, RT is more appropriately for the retailer to offer.

(2) In the lower capital scenario, it can be found that along with the increases of \( c, \lambda \) and \( k \), the retailer will becomes more conservative to control the default risk via offering RF. Further, when confronted with higher values of \( a \), the manufacturer is more likey to turn to use prepayment policy.
6. Conclusion

In this paper, a retailer-dominated channel is proposed, consisting of a single manufacturer with limited capital and a single retailer with sufficient capital. And then two types of prepayment policies are offered by the retailer to enhance operational performances of the supply chain. The interesting findings can be presented below.

(1) When the prepayment policy is not considered, the retailer always prefers to cooperate with the manufacturer with sufficient capital. That is, since the manufacturer with sufficient capital can achieve the optimal product input and green effort, consumers with more environmentally conscious can be attracted which enables the retailer to obtain more benefits. In specific cases, it can be inferred that the retailer may choose the manufacturer with higher capital as its partner even if the manufacturer with less capital has lower production cost or green effort cost. Further, this paper effectively defines the threshold value regarding the manufacturer’s own capital in this case. When the manufacturer’s own funds are below the threshold (i.e., lower capital scenario), the retailer cannot announce its optimal wholesale price, above which the manufacturer cannot guarantee to fulfil the corresponding production quantity and marketing effort.

(2) The effectiveness of RF for the capital-constrained manufacturer depends on how well its green effort can be implemented. Specifically, if the quality effect is large enough (i.e., RF complete valid region), the manufacturer can get more upfront capital from the retailer, which may entirely cover its total production and green effort costs; otherwise (i.e., RF partial valid region), although the manufacturer can still improve its performance due to its increased working capital, the manufacturer cannot cover its optimal policy via RF. In other words, the retailer should apply RF to the capital-constrained manufacturer who can implement effective green efforts; otherwise, RF will not greatly improve the manufacturer’s capital level, nor will it have an impact on the retailer’s operational performance. At the same time, capital-constrained manufacturers should focus on improving their green production levels and optimizing green input costs to attract the retailer granting RF.

(3) Two types of scenarios can be offered to examine effects of RT. That is, when the manufacturer’s own capital is rather low, the leading retailer tends to set a higher wholesale price through RT. In this case, the manufacturer can obtain sufficient prepaid funds, and significantly improve its production input/effort level, which finally benefit the retailer’s expected profit. Otherwise, RT will not have a huge impact on the manufacturer’s operational decision-making, but will make the retailer take unnecessary demand risks. Therefore, RT may not be the best prepayment policy for the retailer.

The following limitations exist in the proposed models of this paper. Firstly, although this paper proves the existence and uniqueness of the optimal solution in different prepayment models, it fails to theoretically prove the retailer’s preferences for different prepayment strategies due to the complexity of the analytical solution. Secondly, in this paper, we only use the exponential distribution to depict the market demand for numerical analysis, lacking the application of more functional forms to portray the impact of parameters on supply chain operation decisions. Finally, since this paper doesn’t discuss the case of retailer’s capital constraint, the application of the relevant findings should be further examined.

This paper can be further extended in the following directions. Firstly, the capital-constrained manufacturer can also seek financing support from the bank to optimize its operational decisions. In this case, how the manufacturer chooses between prepayment and bank credit remains to be studied. Further, when there is uncertainty in a manufacturer’s product production process, it becomes more difficult to match supply and demand. In this case, the retailer should reconsider how to optimize its prepayment strategies to trade off risk control and performance improvement. Finally, in a multi-manufacturer scenario, manufacturers may adjust pricing strategies or green effort levels to compete for retailer orders. As a result, it is in doubt how the resulting adjustment in operational strategy affect the retailer’s prepayment strategy choices.
Appendix A. Proof of Proposition 1

Regarding equation (1a), we have \( \frac{\partial^2 \Pi_m(q,e)}{\partial q^2} = -wf(q - ae) < 0 \) and \( \frac{\partial^2 \Pi_m(q,e)}{\partial e^2} = -wa^2f(q - ae) - k < 0 \). Further, since \( \frac{\partial^2 \Pi_m(q,e)}{\partial q \partial e} = waf(q - ae) \), it yields \( \frac{\partial^2 \Pi_m(q,e)}{\partial q^2} \frac{\partial^2 \Pi_m(q,e)}{\partial e^2} - \left( \frac{\partial^2 \Pi_m(q,e)}{\partial q \partial e} \right)^2 = kwf(q - ae) > 0 \), indicating that \( \Pi_m(q,e) \) is jointly concave with \( q \) and \( e \). Thus, by simultaneously letting \( \frac{\partial \Pi_m(q,e)}{\partial q} \) and \( \frac{\partial \Pi_m(q,e)}{\partial e} \) equalling zero, we have \( e_0^a = \frac{a(w-e)}{k} \) and \( q_0^a \) satisfy \( F(q - ae_0^a) = \frac{w}{e} \). Further, considering the constraint \( cq + \frac{ke^2}{2} \leq K_m \), two situations can be classified via letting \( L_0 = cq_0^a + \frac{k(e_0^a)^2}{2} - K_m \).

1. If \( L_0 \leq 0 \), then \((q_0^a, e_0^a) = (q_0^b, e_0^b)\).
2. If \( L_0 > 0 \), then the optimal solution should satisfy the condition \( cq + \frac{ke^2}{2} = K_m \) (i.e., \( q = \frac{K_m}{c} - \frac{ke^2}{2c} \)), and thus \( \Pi_m(e) = w\min \left( e, \frac{K_m}{c} - \frac{ke^2}{2c} - ae \right) + wa(e - K_m) \). Since \( \frac{d^2 \Pi_m(e)}{de^2} = \frac{-wk}{c} \bigg( \frac{K_m}{c} - \frac{ke^2}{2c} - ae \bigg) - w\left( \frac{ke}{c} + a \right)^2 f \left( \frac{K_m}{c} - \frac{ke^2}{2c} - ae \right) < 0 \), thus let \( \frac{d \Pi_m(e)}{de} = 0 \), then \( a = \left( \frac{ke}{c} + a \right) \bigg( \frac{K_m}{c} - \frac{ke^2}{2c} - ae \bigg) \), and thus \( e_0^b = \frac{K_m}{c} - \frac{k(e_0^b)^2}{2c} \).

Appendix B. Proof of Proposition 2

1. Regarding equation (2a), we have \( \frac{d^2 \Pi_r(w)}{dw^2} = -\frac{2}{w^3} t(q - ae_0^b) + \frac{(1-w)e^2}{w^3 f(q - ae_0^b)} t'(q - ae_0^b) - \frac{2a^2}{k} < 0 \), where \( t(x) = \frac{1}{e\sqrt{x}} \), and \( t'(x) < 0 \). Then, the retailer can achieve its maximum value at \( w_0^b \) via \( \frac{d \Pi_r(w)}{dw} = 0 \), i.e., \( \int_{q_0^b - ae_0^b}^{w_0^b} F(e) \frac{d\varepsilon}{\varepsilon} = \left( \frac{1-w}{w^2} \right) t(q_0^b - ae_0^b) + \frac{a^2(e-2e+1)}{k} \). Further, due to \( \frac{d^2 \Pi_r(w)}{dw^2} < 0 \) \( w = 1 < 0 \), it yields \( w_0^b < 1 \).
2. Regarding equation (2b), we have \( \frac{d \Pi_m(w)}{dw} = w\min \left( e, ae_0^b, q_0^b \right) < 0 \), and thus \( \Pi_r(w) \) decreases with \( w \) for \( w > w_0^b \).
3. Considering the relationship between \( w_0^b \) and 1, two cases to occur.
   (i) If \( w_0^b \geq 1 \), then it yields that \( w_0^b = w_0^b \).
   (ii) If \( w_0^b < 1 \), then if \( w_0^b > w_0^b \), then \( w_0^b = w_0^b \); otherwise, then \( w_0^b = w_0^b \).

Appendix C. Proof of Proposition 3

Since equation (3a) equals equation (1a), it is apparent that \( \Pi_m(q,e) \) is jointly concave with \( q \) and \( e \), and we have \((c_1^a, q_1^a) = (e_0^a, q_0^a)\). Further, considering the constraint \( cq + \frac{ke^2}{2} \leq K_m + wa(e - e_0^b) \), two cases can be classified via letting \( L_1(w) = cq_1^b + \frac{k(c_1^b)^2}{2} - K_m - wa(e - e_0^b) \).

1. If \( L_1(w) \leq 0 \), then \( (c_1^a, q_1^a) = (q_0^b, e_0^b) \).
2. If \( L_1(w) > 0 \), then the optimal solution should satisfy the condition \( cq + \frac{ke^2}{2} = K_m + wa(e - e_0^b) \), and thus \( \Pi_m(q,e) = w\min \left( e, \frac{K_m + wa(e - e_0^b) - ke_0^b}{2c} - ae \right) + wa(e - K_m) \). Since \( \frac{d^2 \Pi_m(e)}{de^2} = \frac{-wk}{c} \bigg( \frac{K_m + wa(e - e_0^b) - ke_0^b}{2c} - ae \bigg) - w\left( \frac{we}{c} - \frac{ke}{c} - a \right)^2 f \left( \frac{K_m + wa(e - e_0^b) - ke_0^b}{2c} - ae \right) < 0 \), thus let \( \frac{d \Pi_m(e)}{de} = 0 \), then \( a = \frac{a(w-e)}{k} \) and \( q_1^b = \frac{K_m}{c} + \frac{(a(w-e))^2}{2kc} \).

Appendix D. Proof of Corollary 1

Let \( L_1(w) = \left( cq_1^b + \frac{k(c_1^b)^2}{2} \right) - K_m - wa(e - e_0^b) \). Then, \( \frac{dL_1(w)}{dw} = \frac{c^2}{w^3 f(q - ae_0^b)} + \frac{a^2(w-e)}{k} = 0 \), \( \frac{d^2 L_1(w)}{dw^2} = -\frac{2a^2}{k} + \frac{c^2t'(q_1^b - ae)}{w^2 f(q_1^b - ae)} - \frac{ct'(q_1^b - ae)}{w^2} < 0 \), and thus \( L_1(w) \) is concave with respect to \( w \). Further, let \( w_{11}^l \) and \( w_{12}^l \) satisfy \( L_1(w) = 0 \), where \( w_{11}^l < w_{12}^l \). For simplicity, this paper focuses on the case of \( w_{12}^l > 1 \).
Thus, this part focuses on the case of \( \omega (1) \) and \( \omega (2) \) if \( 1 < m \).

Further, since \( \frac{dL_0(w)}{dw} = \frac{\omega^2}{w^2f(q-ae_0^2)} + \frac{\omega^2}{k} > 0 \), it yields that \( \frac{dL_0(w)}{dw} > \frac{dL_1(w)}{dw} \). And due to \( w_0^* \) satisfies \( L_0(w) = 0 \), it yields that \( w_0^* < w_{11}^* < w_{12}^* \). And based on Proposition 4.2, when \( w_0^* \geq 1 \), it yields \( w_{11}^* > w_{01}^* \). Thus, this part focuses on the case of \( w_0^* < 1 \):

1. If \( w_{11}^* > 1 > w_{01}^* \), then \( L_1(w) < 0 \) for \( w \in [0, 1] \).
2. If \( 1 > w_{11}^* > w_{01}^* \), then \( L_1(w) < 0 \) for \( w \in [0, w_{11}^*] \) and \( L_1(w) > 0 \) for \( w \in [w_{11}^*, 1] \).

Appendix E. Proof of Proposition 4

1. Regarding equation (4a): Since equation (4a) equals equation (2a), by referring to Appendix B, then \( \Pi_r(w) \) increases with \( w \) for \( w \leq w_0^* \) and decreases with \( w \) for \( w > w_0^* \).

2. Regarding equation (4b): it yields that \( \frac{d\Pi_r(w)}{dw} = -z_2(w) F(z_1(w)) - \frac{2a^2}{k} \), and \( \frac{d^2\Pi_r(w)}{d\omega^2} = z_2(w) + \frac{2a^2}{k} \), where \( z_1(w) = \frac{(ac-aw)^2}{2kc} + \frac{K_m}{c} \), \( z_2(w) = \frac{a^2(2c+1-3\omega)}{kc} - \frac{a^2(ac-aw)}{k} \left(1 - \frac{1}{c}\right) h(z_1(w)) \), and \( \frac{dz_2(w)}{dz} = -3a^2 + \frac{(c-3\omega)ac-aw)^2}{k^2c^2} + \frac{1}{K_m} \), and \( \frac{d^2z_2(w)}{d\omega^2} = \frac{2a^2}{k} > w \). Further, due to \( z_2(w)_{|w=c} = \frac{2a^2(1-c)}{k} > 0 \) and \( z_2(w)_{|w=\frac{2c+1}{3}} = -\left[\frac{(ac-aw)}{k}\right]^2 (1 - \frac{1}{c}) h(z_1(w)) < 0 \), there exists \( w_0^* \) satisfying \( z_2(w) = 0 \), resulting the following two cases.

(i) For \( w \in [w_0^*, \frac{2c+1}{3}] \), it yields that \( z_2(w) < 0 \) and \( \frac{d\Pi_r(w)}{dw} < 0 \).

(ii) For \( w \in [c, w_0^*] \), it yields that \( z_2(w) > 0 \) and \( \frac{d\Pi_r(w)}{dw} < 0 \), indicating that \( \frac{d\Pi_r(w)}{dw} \) decreases with \( w \). Without loss of generality, this paper assumes \( (1 - \frac{1}{c}) \) \( \frac{K_m}{c} \) \( + 2 \geq 0 \), indicating that then \( \frac{d\Pi_r(w)}{dw} \) is continuous with respect to \( w \). As a result, let \( \frac{d\Pi_r(w)}{dw} = 0 \), then \( w_0^* \) satisfies \( \frac{d\Pi_r(w)}{dw} = \frac{d\Pi_r(w)}{dw} |_{w=w_0^*} = \Pi_r(w) |_{w=w_0^*} = \Pi_r(w) |_{w=w_0^*} \), indicating that equations (4a) and (4b) is continuous.

(i) If \( w_{11}^* > 1 > w_{01}^* \), then \( L_1(w) < 0 \) for \( w \in [0, 1] \), and thus \( w_1^* = w_0^* \).

(ii) If \( 1 > w_{11}^* > w_{01}^* \), then \( L_1(w) < 0 \) for \( w \in [0, w_{11}^*] \) and \( L_1(w) > 0 \) for \( w \in [w_{11}^*, 1] \).

(i) If \( w_0^* < w_{11}^* \), then \( w_1^* < w_{11}^* \), and \( w_1^* = w_0^* \).

3. It is easy to find that \( \frac{d\Pi_r(w)}{dw} = \frac{d\Pi_r(w)}{dw} |_{w=w_0^*} = \Pi_r(w) |_{w=w_0^*} = \Pi_r(w) |_{w=w_0^*} \), indicating that equations (4a) and (4b) is continuous.

Appendix F. Proof of Proposition 5

1. \( \Pi_m(q, e) = w \int_{p}^{\infty} F(x) dx - K_m \) and \( \frac{\partial \Pi_m(q, e)}{\partial q} = (aw - ek) \frac{\partial \Pi_m(q, e)}{\partial q} \), resulting in two cases.

(i) If \( aw \leq ek \), then \( \frac{\partial \Pi_m(q, e)}{\partial q} |_{q=e_0} = 0 \), and \( \frac{\partial \Pi_m(q, e)}{\partial e} |_{e=e_0} = -kF(\tilde{e}_p) + (aw - ek) \frac{\partial \Pi_m(q, e)}{\partial e} < 0 \). Further, let \( q_0^* = \frac{(aw - ek)}{w} \), and \( \frac{\partial \Pi_m(q, e)}{\partial e} |_{e=e_0} > 0 \), it yields that \( \frac{\partial \Pi_m(q, e)}{\partial e} > 0 \). Under the circumstances, we have \( [\frac{\partial \Pi_m(q, e)}{\partial e}]^2 - (\frac{\partial \Pi_m(q, e)}{\partial q})^2 \).
\( \Pi_m(q, e) \) is jointly concave with \( q \) and \( e \). Thus, by simultaneously letting \( \frac{\partial \Pi_m(q, e)}{\partial q} \) and \( \frac{\partial \Pi_m(q, e)}{\partial e} \), we have \( c_2^e = \frac{aw-ce}{k} \) and \( q_2^e \) satisfy \( \bar{F}(q - ae) = \frac{\bar{F}(\bar{e})}{w} \).

(2) Further, considering the constraint \( cq + k_2^e \in [K_m + wae, K_m + wq] \), three cases can be classified.

(i) If \( cq + k_2^e \in [K_m + wae, K_m + wq] \), then \( (q_2^e, e_2^e) = (q_2^e, e_2^e) \).

(ii) If \( cq + k_2^e > K_m + wq \), then the optimal solution should satisfy the condition \( \frac{1}{w-c}(\frac{k_2^2}{2} - K_m) = q \), and thus \( \Pi_m(e) = -K_m \). In this case, the transaction between the manufacturer and the retailer will not be conducted.

(iii) If \( cq + k_2^e < K_m + wae \), then \( \Pi_m(q, e) \) equals equation (3a), indicating that \( (q_2^e, e_2^e) = (q_1^e, e_1^e) \).

### Appendix G. Proof of Corollary 2

Based on \( w\bar{F}(q_e) = c\bar{F}(\bar{e}_p) \), then \( \frac{dq}{dw} = \frac{ck[\bar{w}(\bar{F}(\bar{e}_p)) + wa(\bar{F}(\bar{e}_p))]}{w[wh(q_e)-ch(\bar{e}_p)]} \), and \( \frac{d\bar{e}_p}{dw} = \frac{w-H(\bar{e}_p)}{w[wh(q_e)-ch(\bar{e}_p)]}. \) For simplicity, let lower and upper bound of \( w \) are \( w^l = c \) and \( w^u = 1 \), respectively. Then, the discussion is conducted for \( w \in [w^l, w^u] \).

(1) Supposing that \( \frac{dq}{dw} < 0 \) (i.e., \( h(\bar{e}_p) \bar{e}_p > w \)) for \( w \in (w, w^l), \) where \( w^l \leq w \leq w^u \). It can be inferred that \( \frac{d\bar{e}_p}{dw} < 0 \) (i.e., \( h(q_e) \bar{e}_p > c \)) for \( w \in (w, w^l). \) Thus, the following inferences can be obtained.

(i) Assuming that for \( w \in (w-1, w), \) we have \( \frac{dq}{dw} \geq 0 \) (i.e., \( h(\bar{e}_p) \bar{e}_p < w \)) and \( \frac{d\bar{e}_p}{dw} < 0 \), where \( w^l \leq w-1 \leq w \). Then, it can be inferred that for \( w \in (w-1, w), \) \( \frac{d\bar{e}_p}{dw} < 0 \) and \( h(\bar{e}_p) \bar{e}_p \) decreases with \( w \). Further, regarding \( w - H(\bar{e}_p) \), we have \( \frac{d(w-H(\bar{e}_p))}{dw} = 1 - \frac{H'(\bar{e}_p)}{\bar{e}_p} > 0 \). Then, it yields that \( h(\bar{e}_p) \bar{e}_p \) will still be less than \( w \) along with the increase of \( w \). Thus, this inferences contradicts with the assumption of \( \frac{dq}{dw} < 0 \) (i.e., \( h(\bar{e}_p) \bar{e}_p > w \)) for \( w \in (w, w^l) \).

(ii) Based on the above contradiction, it can be further inferred that for \( w \in (w^l, w), \) \( \frac{dq}{dw} < 0 \) and \( \frac{d\bar{e}_p}{dw} < 0 \). Nevertheless, since the condition of \( w^l = c \) results in \( q_e = \bar{e}_p, e = 0 \) and \( q = 0 \), indicating that \( \frac{dq}{dw} = 0 \), which contrasts with the inference that \( \frac{dq}{dw} < 0 \). Further, since \( \frac{dq}{dw} > 0 \), it yields that \( \frac{d\bar{e}_p}{dw} > 0 \) for \( w \in (w^l, w^u) \).

(iii) To sum up, since the assumption of \( \frac{dq}{dw} < 0 \) cannot hold, we have \( \frac{dq}{dw} > 0 \) for \( w \in (w^l, w^u) \). Further, since \( \frac{dq}{dw} > 0 \), it yields that \( \frac{d\bar{e}_p}{dw} > 0 \) for \( w \in (w^l, w^u) \).

(2) Since \( \frac{d\bar{e}_p}{dw} = \frac{c-h(q_e)}{w[wh(q_e)-ch(\bar{e}_p)]} > 0 \), and \( \frac{d(c-h(q_e))}{dw} = h'(q_e)\frac{dq}{dw} + h(q_e)\frac{d\bar{e}_p}{dw} > 0 \). That is, \( c - h(q_e) \bar{e}_p \) increases with \( w \), indicating that \( c \geq h(q_e) \bar{e}_p \) for \( w \in (w^l, w^u), \) and \( \frac{d\bar{e}_p}{dw} > 0 \).

(3) By letting \( L_2(w) = cq_a^2 + \frac{(c^2-1)^2}{2} - K_m - wae_a \) and \( L_3(w) = cq + \frac{k(c^2-1)^2}{2} - K_m - wq_a \), it yields that \( \frac{dL_2(w)}{dw} = (c-w)\frac{dq}{dw} - q_e < 0 \) and \( \frac{dL_3(w)}{dw} = 0 \). Thus, we have \( L_3(w) < 0 \).

Further, due to \( \frac{dL_2(w)}{dw} = \frac{c dq}{dw} - \frac{a^2(c-w)}{k} = \frac{ck[w-H(\bar{e}_p)]+wa^2(wh(q_e)-ch(\bar{e}_p))}{w[wh(q_e)-ch(\bar{e}_p)]} > 0 \) and \( L_2(w)_{w=e} < 0 \), there are two cases to occur based on the value of \( L_2(w)_{w=1} \):

(i) If \( q_2^e \geq K_m + \frac{a^2(1-c^2)}{2k} \), where \( q_2^e \) satisfies \( F(q - \frac{a^2(1-c^2)}{k}) = c\bar{F} \left( cq - K_m - \frac{a^2(1-c^2)}{2k} \right) \), then \( q_2^e, q_e \) and \( \bar{e}_p \) are positive with \( w \) for \( w \in [w^l, 1] \), where \( w^l \) satisfies \( q_2^e + \frac{(c^2-1)^2}{2} = K_m + wae_a \).

(ii) Otherwise, RT is not applicable.

### Appendix H. Proof of Proposition 6

Since \( \Pi_r(w) = \int_{\bar{e}_p}^{\bar{F}(\bar{e}_p)} \bar{F}(\xi)\,d\xi + \int_{\bar{F}(\bar{e}_p)}^{\bar{F}(\bar{e}_p)} \bar{F}(\xi)\,d\xi + \bar{e}_p, \) it yields \( \frac{d\Pi_r(w)}{dw} = L_41 - \frac{dq}{dw} \bar{F}(\bar{e}_p) + \frac{2w+1}{k} c^2(1-c) a^2 - \bar{e}_p, \) where \( L_41 = \int_{\bar{F}(\bar{e}_p)}^{\bar{F}(\bar{e}_p)} \bar{F}(\xi)\,d\xi + \frac{2w+1}{k} c^2(1-c) a^2 - \bar{e}_p, \) and the following results can be obtained.
(1) In terms of $w \bar{F}(q_e) = c \bar{F}(\bar{z}_p)$, by taking the first- and second-order derivative with respect to $w$, it yields that $w \left[ h(\bar{z}_p) \frac{dF}{dw} - h(q_e) \frac{dq}{dw} \right] + 1 = 0$ and $h(\bar{z}_p) \frac{dF}{dw} - h(q_e) \frac{dq}{dw} + w \left[ h'(\bar{z}_p) \left( \frac{dF}{dw} \right)^2 - h'(q_e) \left( \frac{dq}{dw} \right)^2 \right] + w\left[ h(\bar{z}_p) \frac{d^2F}{dw^2} - h(q_e) \frac{d^2q}{dw^2} \right] = 0$. Thus, it can be inferred that $h(\bar{z}_p) \frac{dF}{dw} - h(q_e) \frac{dq}{dw} < 0$. Further, it yields $h'(\bar{z}_p) \left( \frac{dF}{dw} \right)^2 - h'(q_e) \left( \frac{dq}{dw} \right)^2 < 0$, indicating that $h(\bar{z}_p) \frac{d^2F}{dw^2} > h(q_e) \frac{d^2q}{dw^2}$ and $0 > \frac{d^2F}{dw^2} > \frac{d^2q}{dw^2}$.

(2) Let $L_{43} = w \left[ \bar{F}(\bar{z}_p) - 1 \right] + (1 - w) \bar{F}(q_e)$, and $\frac{dL_{43}}{dw} = \bar{F}(\bar{z}_p) - 1 - f(\bar{z}_p) w \frac{d\bar{F}}{dw} - \bar{F}(q_e) - (1 - w) f(q_e) \frac{dq}{dw} < 0$. Further, when $w = w_2^1$, it yields that $\bar{z}_p = 0$ and $L_{43|w=w_2^1} = (1 - w) \bar{F}(q_e) > 0$. Additionally, when $w = 1$, we have $L_{43|w=1} = \bar{F}(\bar{z}_p) - 1 \leq 0$. Therefore, there exists a unique solution $w_2^0$ satisfying $L_{43} = 0$, i.e., $w \left[ 1 - \bar{F}(\bar{z}_p) \right] = (1 - w) \bar{F}(q_e)$. Under the circumstances, (i) for $w \in (w_2^0, 1]$, we have $L_{43} < 0, w \frac{d\bar{F}}{dw} \left[ \bar{F}(\bar{z}_p) - 1 \right] < \frac{dq}{dw} \left[ \bar{F}(\bar{z}_p) - 1 \right]$ and $L_{41} \leq \frac{dq}{dw} < 0$. Additionally, when $w = 1$, we have $L_{43|w=1} = \bar{F}(\bar{z}_p) - 1 \leq 0$. Therefore, there exists a unique solution $w_2^0$ satisfying $L_{43} = 0$, i.e., $w \left[ 1 - \bar{F}(\bar{z}_p) \right] = (1 - w) \bar{F}(q_e)$. Under the circumstances, (i) for $w \in [w_2^0, 1]$, we have $L_{43} \geq 0, w \frac{d\bar{F}}{dw} \left[ \bar{F}(\bar{z}_p) - 1 \right] < \frac{dq}{dw} \left[ \bar{F}(\bar{z}_p) - 1 \right]$, and $L_{42} = w \frac{d^2\bar{F}}{dw^2} \left[ \bar{F}(\bar{z}_p) - 1 \right] + (1 - w) \frac{d^2q}{dw^2} \bar{F}(q_e) < \frac{d^2q}{dw^2} < 0$. Thus, $\frac{d\bar{L}}{dw} < 0$ and $\frac{d^2\bar{L}}{dw^2}$ decreases with $w$.

(3) To sum up, $\frac{d\bar{L}(w)}{dw}$ decreases with $w$. Thus, let $w_2^a$ satisfy $\frac{d\bar{L}(w)}{dw} = 0$:

(i) if $w_2^0 \geq w_2^a$, then $w_2^a = w_2^0$.
(ii) if $w_2^0 < w_2^a$, then the optimal wholesale price $w_2^* = w_2^a$.

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REFERENCES


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