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GENERAL AND MULTIPLICATIVE NON-PARAMETRIC MODELS WITH INTERVAL RATIO DATA: APPLICATION TO THE BANKING INDUSTRY

Esmaeel Farzadi[1], Abdollah Hadi-Vencheh[1,*], Yong Tan[2] and Zahra Gheleg Beigi[3]

Abstract. The empirical literature has tried to propose relevant data envelopment analysis (DEA) models to evaluate the efficiency level of the decision-making unit (DMU) in the presence of interval ratio data; however, the use of variable production frontier in the evaluation suffers from a number of limitations. The current study fills in the gap in the previous literature by proposing relevant DEA models based on interval arithmetic, through which the shortcomings of the previous existing studies have been overcome. The findings show that extra variable changes are not needed by the proposed model and a fixed, unified production frontier can be used to measure the DMUs' efficiency with interval data. The potential application of the proposed model is illustrated through a numerical example in the banking industry.

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1. Introduction

Data envelopment analysis (DEA), developed by Charnes *et al.* [13] is used to measure the relative efficiency of a homogenous set of units – decision making units (DMUs). The DMUs typically consume multiple inputs to produce multiple outputs. The ability to handle multiple inputs and outputs has made DEA an effective non-parametric approach for evaluating the relative efficiency of DMUs [1, 3]. This method has been widely applied to various sectors of the economy and the empirical literature has made numerous attempts to further develop this method from different perspectives [4, 36]. The basic characteristic of the original DEA models is its precise form of the input and output data. However, in real-world practices, the observations of inputs and outputs sometimes are interval ratios rather than crisp numbers [31].

Besides the non-parametric analysis, the accounting and financial ratios have been widely used in the empirical studies to examine the performance of a firm from the finance perspective, and these ratios were also compared with other firms within an industry for performance ranking purposes [22, 53]. In addition, this information is also applied to other business operation practices, such as bankruptcy forecasting and mergers and acquisitions. Because of the importance of these ratios, as illustrated above, it would be interesting to further study and

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[1] Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran.

[2] School of Management, University of Bradford, Bradford BD7 1SR, UK.

[3] Department of Mathematics, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Isfahan, Iran.

*Corresponding author: ahadi@khuisf.ac.ir

characterize these financial ratios. The empirical literature has attempted to develop relevant models for this issue. These include the partial adjustment model developed by Lev [40] and further applied by Lee and Wu [38], David and Peles [15], Wu and Ho [56], Gallizo *et al.* [26], Lemmon and Zender [39], Faulkender *et al.* [24] and among others.

The financial ratios can be classified into two types, namely, normative and positive, and they were initially identified by Whittington [55]. The former normally compares a firm's ratio with the industry benchmark, such as the mean or median of the values across different firms within the same industry [9, 25]. In terms of the latter, it is mainly used for forecasting purposes, *i.e.*, business failure forecasting. Beaver [10, 11] and Altman [2] are the pioneers in this area through using different forms of discriminant analysis. This specific technique was popularized in the literature to forecast business failure [5, 33, 59]. However, using this technique to estimate business failure probabilities leads to less robust and accurate results [25]. In the banking sector specifically, the operational issues related to bank performance and stability, or even the market structure, have been mainly characterized by relevant interval ratios, such as bank profitability, cost management, bank liquidity, bank capitalization, and labour productivity, as well as market structure measured by concentration ratio. All these indicators are widely used in the empirical literature in estimating and comparing bank performance, market conditions and bank stability, while very few attempts have been made to use these ratio data in DEA. Using these ratio data in the data envelopment is of vital importance due to the facts that, (1) it can be used to compare the performance evaluation results with the one purely depending on the financial ratios; and (2) it can also be used to check whether the robustness efficiency results will be consistent between this and the one using the crisp numbers. Emrouznejad and Anouze [16] show that the presence of ratio variables will make DEA produce inconsistent results. There are mainly two problems in the DEA and general non-parametric corporate performance (GNCP) models when dealing with ratio data for the inputs and outputs: (1) the convexity property of the production possibility set; (2) the proportionality property of the production possibility set. A multiplicative model was proposed by Emrouznejad and Cabanda [17] and Emrouznejad *et al.* [20] to non-parametrically measure corporate performance. In their studies, the information and communication technology opportunity index was developed through the model. Olesen *et al.* [44] proposed a model to measure the efficiency of DMUs with ratio inputs and outputs. Mozaffari *et al.* [43] proposed a method that found efficient hyperplanes for output-oriented DEA models with ratio data. Gerami *et al.* [27] extended the Network DEA models with the ratio data to evaluate the performance of supply chains.

The traditional CCR [13] and BCC models [8] mainly focus on the investigation of efficiency using precise data and an assumption has been made that all the inputs and outputs are exactly known. However, in reality, the assumption does not hold and some input and output variables are not known. The unknown variables take different forms, including interval data, ordinal data and ratio data, a DEA model dealing with these variables is called imprecise DEA [14].

There are a number of research articles using advanced imprecise DEA models to deal with the interval data. Esmaeili [21] developed a method based on the enhanced Russell measure. In comparison, a cross-efficiency method based on the DEA model is introduced by Wu *et al.* [57] to deal with the issue of interval data, together with a new TOPSIS method which was proposed to rank the decision-making units. When dealing with the variables with interval data, it is quite common that some of the variables would take negative values for which the traditional DEA model cannot accommodate. Empirical studies used various ways to cope with this issue in the DEA model, including data transformation [30, 32, 34, 42, 49]; use of the absolute value of inputs as outputs and the absolute value of outputs as inputs [48]; range directional measure model [29, 45]; Modified slack-based measure [50]; and Semi-oriented radial measure model [18, 19]. Ye *et al.* [60] proposed a model according to interval data envelopment analysis (IDEA) that consider undesirable output to evaluate interval efficiencies of environmental pollution management.

Zerafat Angiz *et al.* [61] argue that some approaches under the DEA model dealing with uncertain data suffered from a number of shortcomings, including: (1) the α-level and fuzzy ranking approaches in the defuzzification are not considered; (2) the fuzzy coefficients are not treated in a direct way (although the equality and inequity signs are fuzzified in the tolerance approach). Therefore, they developed a fuzzy DEA model to examine the efficiency of DMUs l and include α-level in the model under the fuzzy environment. To deal with the imprecise data of input and output variables, Zerafat Angiz *et al.* [62] argue that the fuzzy DEA model proposed by Zerafat Angiz *et al.* [61] based on the α-cut, suffers from the disadvantage of being unable to include all information about uncertainty in the model. Thus, they introduced the concept of local α-cut and a multi-objective linear programming was developed to measure the performance of DMUs under uncertainty. The slack-based measure efficiency from the slack-based DEA model can be decomposed into radial, scale and mix-efficiency, the last of which requires data certainty related to inputs and outputs variables (crisp data); however, having such crisp data is not the case in reality. Puri and Yadac [46] addressed this issue by proposing and introducing a fuzzy input mix-efficiency and it was evaluated by the α-cut approach. A fuzzy correlation coefficient method using the expected value approach was further developed by the authors to calculate the expected intervals, as well as the expected fuzzy values of coefficients between fuzzy inputs and fuzzy outputs. Finally, the authors also propose a method to rank DMUs for the fuzzy input mixed efficiency. The authors applied this method to the banking sector.

The empirical literature also deals with the uncertainty data using the rough DEA model [51, 58]. A rough DEA model was used by Xu *et al.* [58] which integrates the classical DEA with rough set theory and the model was further applied for efficiency performance evaluation in the supply chain of the Chinese manufacturing industry. Shiraz *et al.* [51] further extend the work of Xu *et al.* [58] by proposing a fuzzy rough DEA model, in which the classical DEA, rough set theory and fuzzy set theory are integrated together. This new proposed model has the advantage of being able to better accommodate the uncertainty. The authors developed a way to measure the DMUs' relative efficiency under the possibility approach incorporating the fuzzy rough expected value operator.

A growing number of research articles on evaluating the efficiency of DMUs considered the role of undesirable outputs in the production process [28, 63]. Khoshro *et al.* [37] proposed a non-radial DEA model by considering undesirable outputs. An alternative two-stage data envelopment analysis model with undesirable input was proposed by Li *et al.* [41]. A customized DEA model to deal with the issue of uncertain data and undesirable outputs at the same time was developed by Khalili Damghani *et al.* [35]. Besides this, the study further contributes to existing research by using interval values to determine the efficiency scores of DMUs, deriving the most economic scale size for the efficient DMUs and providing practical benchmarks for the inefficient DMUs.

A multi-component DEA approach with imprecise data was proposed and developed by Puri *et al.* [47]. The model further advanced the previous empirical studies by incorporating the undesirable outputs as well as the shared resources in the production process. In addition, a new common set of weights method was developed by the authors to determine the unique weights for measuring the overall interval efficiencies as well as the decomposed components of the overall efficiencies. The method was based on the interval arithmetic and unified production frontier, and the two-level mathematical programming approach does not only preserve the linearity of DEA and exhibits stronger discrimination power among the DMUs, but also provides validity to the overall interval efficiency as well as interval efficiency components.

The current paper significantly contributes to the literature and fills in the gap of the empirical studies in the following ways: (1) it is among the few studies which attempt to evaluate efficiency in the banking industry through proposing models to use interval ratio data in the efficiency analysis; (2) the current paper shows that some shortcomings were contained in Emrouznejad *et al.* [20] DEA models when computing the efficiency intervals of DMUs. Therefore, through innovation in the model proposal, this study can be regarded as the first research providing the most accurate modelling framework in dealing with interval ratio data in DEA. Our model in this study is applied to the banking sector, although it can be further used in other economic or business applications. The findings from the models proposed by Emrouznejad *et al.* [20] show that some DMUs are efficient, while others are not efficient at all. However, our proposed new models, using the same dataset as that of Emrouznejad *et al.* [20], show that the number of efficient DMUs was reduced considerably.

The rest of the paper is structured as below: the models proposed by Emourznejad *et al.* [20] are reviewed in Section 2, followed by Section 3 reviewing Smirlis *et al.* [52] models. An improvement of GNCP and multiplicative non-parametric corporate performance (MNCP) models is presented in Section 4. Section 5 provides a comparison between our proposed interval DEA models and the one from Emrouznejad *et al.* [20] by using a numerical example. Finally, the conclusions are provided in Section 6.

2. Review of non-parametric interval data model for corporate performance

2.1. GNCP model with interval data

Suppose there are $\boldsymbol{n}$ DMUs and $\mathbf{DMU}_{\boldsymbol{j}}$, $\boldsymbol{j = 1, 2, \ldots, n}$, will be assessed based on $\boldsymbol{m}$ ratios $\boldsymbol{r_{ij} (i = 1, \ldots, m)}$. The assumption needs to be made that the ratios are ordered so that higher levels take precedence over lower levels. The following model, which is called GNCP, was proposed by Emrouznejad *et al.* [20], and is used to measure the efficiency of $\mathbf{DMU}_{\boldsymbol{p}}$, $\boldsymbol{p = 1, 2, \ldots, n}$, through ratio data:

$$\begin{aligned}
\tilde{R}_p = {} & \max Z_p \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \tilde{r}_{ij} \geq Z_p \tilde{r}_{ip}; \qquad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j = 1; \\
& \lambda_j \geq 0; \qquad\qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{1}$$

The weight of each DMU used to construct the effective facet of the DMU is the interval ratio.

Through solving the following models, one can obtain the upper and lower bounds of DMU_p's efficiency:

Optimistic viewpoint

$$\begin{aligned}
R_p^L = {} & \max Z_p \\
\text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j r_{ij}^L + \lambda_p r_{ip}^U \geq Z_p r_{ip}^U; \qquad i = 1, \ldots, m; \\
& \sum_{j=1}^{n} \lambda_j = 1; \\
& \lambda_j \geq 0; \qquad\qquad\qquad j = 1, \ldots, n.
\end{aligned}$$

Pessimistic viewpoint

$$\begin{aligned}
R_p^U = {} & \max Z_p \\
\text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j r_{ij}^U + \lambda_p r_{ip}^L \geq Z_p r_{ip}^L; \qquad i = 1, \ldots, m; \\
& \sum_{j=1}^{n} \lambda_j = 1; \\
& \lambda_j \geq 0; \qquad\qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{2}$$

2.2. The application of MNCP model to interval data

The following model, which is called MNCP, was proposed by Emrouznejad *et al.* [20], and is used to measure the efficiency of DMU through ratio data. The model can be formulated as below to deal with the interval data:

$$
\begin{aligned}
&\max\ h_p \\
&\text{s.t.}\ \prod_{j=1}^{n} r_{ij}^{\lambda_j} \geq h_p r_{ip}, \qquad i=1,\ldots,m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0, \qquad j=1,\ldots,n
\end{aligned} \tag{3}
$$

where $\tilde{r}_{ij} \in [\tilde{r}_{ij}^{L}, \tilde{r}_{ij}^{U}]$ is the interval ratio. The solution to model (4) below will get the upper and lower bounds efficiency of DMU_p.

Optimistic viewpoint

$$
\begin{aligned}
k_p^L =&\ \max h_p \\
\text{s.t.}&\ \prod_{\substack{j=1 \\ j\neq p}}^{n} r_{ij}^{L\lambda_j} * r_{ip}^{U\lambda_p} \geq h_p r_{ip}^{U}, \qquad i=1,\ldots,m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0, \qquad j=1,\ldots,n.
\end{aligned}
$$

Pessimistic viewpoint

$$
\begin{aligned}
k_p^U =&\ \max h_p \\
\text{s.t.}&\ \prod_{\substack{j=1 \\ j\neq p}}^{n} r_{ij}^{U\lambda_j} * r_{ip}^{L\lambda_p} \geq h_p r_{ip}^{L}, \qquad i=1,\ldots,m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0, \qquad j=1,\ldots,n.
\end{aligned} \tag{4}
$$

A linear programming model can be obtained through applying the following alternation to model (3):

$$
h_p r_{ip} = e^{-s_i} \prod_{j=1}^{n} r_{ij}\lambda_j; \qquad i=1,\ldots,m, \quad j=1,\ldots,n.
$$

In addition, $h_p \exp(\varepsilon \sum_{i=1}^{m} s_i)$ is substituted by the objective function in model (2), in which $\boldsymbol{s_i} \geq 0$ and the slacks and the non-Archimedean infinitesimal is represented by ε. Through this we are able to obtain the following model:

$$
\begin{aligned}
H_p =&\ \max g_p + \varepsilon \sum_{i=1}^{m} s_i \\
\text{s.t.}&\ \sum_{j=1}^{n} \lambda_j \tilde{\rho}_{ij} - s_i = g_p + \tilde{\rho}_{ip}; \qquad i=1,\ldots,m, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j, s_i \geq 0; \qquad i=1,\ldots,m, \quad j=1,\ldots,n
\end{aligned} \tag{5}
$$

where $\tilde{\rho}_{ij} \in [\rho_{ij}^L, \rho_{ij}^U]$ are the interval ratios. The construction of the following pair of models is to evaluate the upper and lower bounds efficiency of DMU_p:

Optimistic viewpoint

$$\begin{aligned}
H_p^L = \ & \max g_p + \varepsilon \sum_{i=1}^{m} s_i \\
\text{s.t.} \ & \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \rho_{ij}^L + \lambda_p \rho_{ip}^U - s_i = g_p + \rho_{ip}^U; \qquad i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j, s_i \geq 0; \qquad i = 1,\ldots,m, \quad j = 1,\ldots,n.
\end{aligned}$$

Pessimistic viewpoint

$$\begin{aligned}
H_p^U = \ & \max g_p + \varepsilon \sum_{i=1}^{m} s_i \\
\text{s.t.} \ & \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j \rho_{ij}^U + \lambda_p \rho_{ip}^L - s_i = g_p + \rho_{ip}^L; \qquad i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j, s_i \geq 0; \qquad i = 1,\ldots,m, \quad j = 1,\ldots,n
\end{aligned} \tag{6}$$

where $\rho_{ij}^L = \ln(r_{ij}^L)$ and $\rho_{ij}^U = \ln(r_{ij}^U)$.

Note that the optimal target values of the above models are represented by $\boldsymbol{H_p^{L^*}}$ and $\boldsymbol{H_p^{U^*}}$. In addition, DMU_p's interval efficiency in model (4) can be represented by $[\boldsymbol{k_p^L}, \boldsymbol{k_p^U}]$. Also note the pessimistic and optimistic viewpoint models (6) generate the optimal solution values, which are denoted by $\boldsymbol{g_p^{L^*}}$ and $\boldsymbol{g_p^{U^*}}$, respectively. The ratio data in the actual problem is usually a value between 0 and 1. If the ratio value is less than 1, we cannot directly use the model (6), because the logarithm of these values is converted to negative, and we need the model to be positive (6). Emrouznejad and Cabanda suggested that logarithmic conversion requires data scaling, after which the model can be run and the negative values can be avoided. Therefore, if necessary, the same transformation can be used here. Similar to the interval DEA model, all evaluation unit models (3) provide bounded intervals for efficiency scores, which can be divided into the following three groups according to the level of efficiency:

Efficient group: all DMUs that are efficient in pessimistic views (obviously in optimistic views) are included in this group, that is, $\boldsymbol{E^{++} = \{\mathrm{DMU}_j : k_j^U = 1\}}$.

High-efficiency group: the optimistic views these DMUs as efficient, while the pessimistic views them as inefficient, *i.e.*, $\boldsymbol{E^{+} = \{\mathrm{DMU}_j : k_j^L = 1, k_j^U > 1\}}$.

Inefficient group: all DMUs that are inefficient in optimistic view (obviously in pessimistic view) are included in this group, *i.e.*, $\boldsymbol{E^{-} = \{\mathrm{DMU}_j : k_j^L > 1\}}$.

3. Models in related to interval DEA

3.1. Revisiting Smirlis *et al.*'s models

The basic assumption in DEA analysis is that s different outputs will be produced using m different inputs by n production units. To be more specific, we use j to represent the production unit, i to stand for the specific input and r to denote the specific output. Compared to the traditional DEA, the interval DEA assumes that either the input values or the output values are not known for its precise form, the only information available is that the values are within certain intervals;

The following pair of linear programming models were proposed by Smirlis *et al.* [52], from which the upper and lower bounds of the efficiency interval of each DMU can be obtained

$$\begin{aligned}
\min\ h_o^L = &\sum_{i=1}^m v_i x_{io}^L - w_o \\
\text{s.t.} \quad &\sum_{i=1}^m v_i x_{ij}^U - \sum_{r=1}^s u_r y_{rj}^L - w_o \geq 0; \qquad j = 1,\ldots,n, \quad j \neq o, \\
&\sum_{i=1}^m v_i x_{io}^L - \sum_{r=1}^s u_r y_{ro}^U - w_o \geq 0; \\
&\sum_{r=1}^s u_r y_{ro}^U = 1, \\
&u_r, v_i \geq \varepsilon; \qquad r = 1,\ldots,s, \quad i = 1,\ldots,m.
\end{aligned} \tag{7}$$

$$\begin{aligned}
\min\ h_o^U = &\sum_{i=1}^m v_i x_{io}^U - w_o \\
\text{s.t.} \quad &\sum_{i=1}^m v_i x_{ij}^L - \sum_{r=1}^s u_r y_{rj}^U - w_o \geq 0; \qquad j = 1,\ldots,n, \quad j \neq o, \\
&\sum_{i=1}^m v_i x_{io}^U - \sum_{r=1}^s u_r y_{ro}^L - w_o \geq 0; \\
&\sum_{r=1}^s u_r y_{ro}^L = 1, \\
&u_r, v_i \geq \varepsilon; \qquad r = 1,\ldots,s, \quad i = 1,\ldots,m.
\end{aligned} \tag{8}$$

In models (7) and (8), DMU_o represents the DMU being evaluated, $v_i (i = 1,\ldots,m)$ and $u_r (r = 1,\ldots,s)$ are decision variables, and ε is non-Archimedes infinitesimal. h_o^L and h_o^U represent the best relative efficiency of DMU_o under the most favorable and unfavorable conditions, respectively. Through the evaluation of models (7) and (8), we can see that there is a level of difference in the set of constraints used for efficiency evaluation on one DMU compared to that of another. In addition, we notice that there are different sets of constraints used to measure the upper and lower bounds of the DMU. In reality, the process of efficiency evaluation is affected by the production frontiers, therefore, using different sets of constraints will make the efficiency evaluation among different DMUs less comparable. In order to confirm our argument, we use a simple example, which is shown in Table 1. We assume that there are three DMUs and the lower bound of DMU A's interval efficiency can be calculated by using $\{(1,2),(4,4),(6,6)\}$ according to model (7); the dotted line in Figure 1a represents the efficiency boundary. For DMU B, the lower bound of interval efficiency under model (7) would be calculated using $\{(2,1),(3,5),(6,6)\}$; the dotted line in Figure 1b stands for the efficiency boundary. For DMU C, the calculation of the lower bound of interval efficiency under the same model uses $\{(2,1),(4,4),(5,7)\}$; the dotted line of Figure 1c is the efficiency boundary. In comparison, under model (8), the upper bound of interval efficiency for

Table 1. Data for three DMUs with one input and one output.

DMU	Input	Output
A	$[1,2]$	$[1,2]$
B	$[3,4]$	$[4,5]$
C	$[5,6]$	$[6,7]$

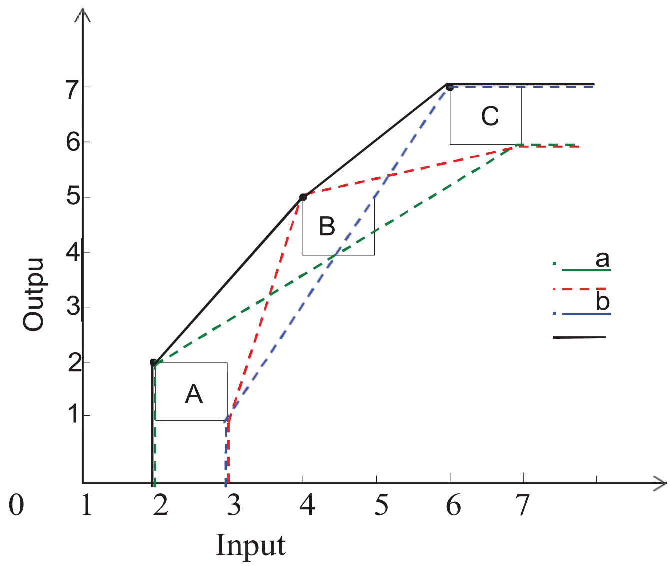

Figure 1. Efficiency frontiers based on models (7) and (8).

these three DUM*s* will be calculated using $\{(2,1),(3,5),(5,7)\}$, $\{(1,2),(4,4),(5,7)\}$ and $\{(1,2),(3,5),(6,6)\}$. Obviously, six efficiency boundaries are used rather than only one, as in the model proposed by Smirlis *et al.* [52].

Due to the fact that the ratio of actual production to maximum production in the production frontier measures the efficiency level, it would be impossible to compare the efficiency level among different DMUs if there is not a fixed or uniformed production boundary. Furthermore, it is assumed that a single actual production frontier is commonly shared among n DMUs. Taking into the consideration that each DMU aims to maximize the output production using the minimum amount of inputs, the optimal activity of each DMU will be the basis of the actual production frontier. The solid line in Figure 1 represents the real efficiency frontier, we obtain this on the basis of the dataset $\{(1,2),(3,5),(5,7)\}$.

Consequently Emrouznejad *et al.* [20] use n different production frontiers for each model therefore we improve GNCP and MNCP models in the next section.

4. Improvement of GNCP and MNCP models

In this section, new GNCP and MNCP models are developed, in which different production frontiers are avoided for efficiency evaluation. Based on the interval arithmetic, we propose the new interval DEA models and we use the same constraints in all cases. More specifically, the fixed and unified production frontier is included in the constraints [6, 7, 23, 54].

4.1. GNCP model with interval data

Let $\varphi_j(j = 1, \ldots, n)$ be the DMU$_j$'s efficiency $(j = 1, \ldots, n)$ in accordance with the concept of DMU$_j$'s efficiency in variable returns to scale (VRS) model is defined as:

$$\varphi_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij} - w_o}, \qquad j = 1, \ldots, n.$$

If we have only m ratios, then: $\varphi_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{w_o}, j = 1, \ldots, n.$

Substituting interval data and using the rules for interval data, we have:

$$\varphi_j = \frac{\sum_{i=1}^{m} u_i \left[r_{ij}^L, r_{ij}^U\right]}{w_o} = \frac{\left[\sum_{i=1}^{m} u_i r_{ij}^L, \sum_{i=1}^{m} u_r r_{ij}^U\right]}{[w_o, w_o]} = \left[\frac{\sum_{i=1}^{m} u_i r_{ij}^L}{w_o}, \frac{\sum_{i=1}^{m} u_i r_{ij}^U}{w_o}\right], \qquad j = 1, \ldots, n.$$

It is evident that $\varphi_j(j = 1, \ldots, n)$ should be an interval number, which we denote by

$$\left[\varphi_j^L, \varphi_j^U\right](j = 1, \ldots, n) \text{ and let } \varphi_j = \left[\varphi_j^L, \varphi_j^U\right] = \left[\frac{\sum_{i=1}^{m} u_i r_{ij}^L}{w_o}, \frac{\sum_{i=1}^{m} u_i r_{ij}^U}{w_o}\right] \subseteq (0, 1], \qquad (j = 1, \ldots, n).$$

The lower and upper bounds of the efficiency interval for DMU$_o$ in the output-oriented mode can be obtained by developing the following fractional programming models.

$$\begin{aligned}
\min\ \theta_o^L &= \frac{w_o}{\sum_{i=1}^{m} u_i r_{io}^U} \\
\text{s.t. } \theta_j^L &= \frac{w_o}{\sum_{i=1}^{m} u_i r_{ij}^U} \geq 1, && j = 1, \ldots, n \\
u_i &\geq 0, && i = 1, \ldots, m.
\end{aligned} \tag{9}$$

$$\begin{aligned}
\min\ \theta_o^U &= \frac{w_o}{\sum_{i=1}^{m} u_i r_{io}^L} \\
\text{s.t. } \theta_j^L &= \frac{w_o}{\sum_{i=1}^{m} u_i r_{ij}^U} \geq 1, && j = 1, \ldots, n \\
u_i &\geq 0, && i = 1, \ldots, m
\end{aligned} \tag{10}$$

where the index o shows the DMU under assessment, $u_i(i = 1, \ldots, m)$ are decision variables. We use the transformation proposed by Charnes and Cooper [12] to convert these fractional programming models to the linear models as below:

$$\begin{aligned}
\min\ & \theta_o^L = w_o \\
\text{s.t. } & w_o - \sum_{i=1}^{m} u_i r_{ij}^U \geq 0, && j = 1, \ldots, n \\
& \sum_{i=1}^{m} u_i r_{io}^U = 1 \\
& u_i \geq 0, && i = 1, \ldots, m.
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\min\ \theta_o^U = w_o \\
&\text{s.t. } w_o - \sum_{i=1}^{m} u_i r_{ij}^U \geq 0, \qquad j = 1, \ldots, n \\
&\qquad \sum_{i=1}^{m} u_i r_{io}^L = 1 \\
&\qquad u_i \geq 0, \qquad\qquad i = 1, \ldots, m.
\end{aligned} \tag{12}$$

In models (11) and (12), under the most favorable conditions, the best relative efficiency is represented by θ_o^L, and under the most unfavorable conditions, it is denoted as θ_o^U. The efficiency interval is thus formed as $[\theta_o^L, \theta_o^U]$.

Now, the efficient DMU is determined using the following criteria:

Definition 1. DMU_o is defined as DEA efficient if $\theta_o^{L^*} = 1$. Alternatively, for $\theta_o^{L^*} > 1$, it is defined as inefficient.

For evaluating DMU_o, we write dual of models (11) and (12) as follows:

$$\begin{aligned}
R_o^L = &\max \theta \\
\text{s.t.} \quad &\sum_{j=1}^{n} \lambda_j r_{ij}^U \geq \theta r_{io}^U; \qquad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0; \qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{13}$$

$$\begin{aligned}
R_o^U = &\max \theta \\
\text{s.t.} \quad &\sum_{j=1}^{n} \lambda_j r_{ij}^U \geq \theta r_{io}^L; \qquad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0; \qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{14}$$

Definition 2. DMU_o is efficient if $R_o^{L^*} = 1$. Alternatively, for $R_o^{L^*} > 1$, it is inefficient.

4.2. The application of MNCP model to interval data

We assume that there are n DMUs, the efficiencies of which need to be assessed in the presence of interval ratios. We express model MNCP as below:

$$\begin{aligned}
\tilde{k}_o = &\max h_o \\
\text{s.t.} \quad &\prod_{j=1}^{n} \tilde{r}_{ij}^{\lambda_j} \geq h_o \tilde{r}_{io}, \qquad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} \lambda_j = 1; \\
&\lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n.
\end{aligned} \tag{15}$$

The interval ratios are represented by $\tilde{r}_{ij} \in [r_{ij}^L, r_{ij}^U]$.

The solution to the pair models below is able to obtain the upper and lower bounds of DMU$_o$'s efficiency:

Optimistic viewpoint

$$\begin{aligned} k_o^L &= \max h_o \\ \text{s.t.} \quad & \prod_{j=1}^{n} r_{ij}^{U\lambda_j} \geq h_o r_{io}^U, \qquad i=1,\ldots,m; \\ & \sum_{j=1}^{n} \lambda_j = 1; \\ & \lambda_j \geq 0, \qquad j=1,\ldots,n. \end{aligned}$$

Pessimistic viewpoint

$$\begin{aligned} k_o^U &= \max h_o \\ \text{s.t.} \quad & \prod_{j=1}^{n} r_{ij}^{U\lambda_j} \geq h_o r_{io}^L, \qquad i=1,\ldots,m; \\ & \sum_{j=1}^{n} \lambda_j = 1; \\ & \lambda_j \geq 0, \qquad j=1,\ldots,n. \end{aligned} \tag{16}$$

We apply the alternations as mentioned above and formulate model (15) as below:

$$\begin{aligned} H_o = \ & \max g_o + \varepsilon \sum_{i=1}^{m} s_i \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \tilde{\rho}_{ij} - s_i = g_o + \tilde{\rho}_{io}; \qquad i=1,\ldots,m, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j, s_i \geq 0; \qquad i=1,\ldots,m, j=1,\ldots,n \end{aligned} \tag{17}$$

where $\tilde{\rho}_{ij} \in [\rho_{ij}^L, \rho_{ij}^U]$ are the interval ratios. The following models are developed to measure the upper and lower bounds of the DMU$_o$'s efficiency:

Optimistic viewpoint

$$\begin{aligned} H_o^L = \ & \max g_o + \varepsilon \sum_{i=1}^{m} s_i \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \rho_{ij}^U - s_i = g_p + \rho_{io}^U; \qquad i=1,\ldots,m, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j, s_i \geq 0; \qquad i=1,\ldots,m, \quad j=1,\ldots,n. \end{aligned}$$

Pessimistic viewpoint

$$\begin{aligned} H_o^U = \ & \max g_o + \varepsilon \sum_{i=1}^{m} s_i \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \rho_{ij}^U - s_i = g_o + \rho_{io}^L; \qquad i=1,\ldots,m, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j, s_i \geq 0; \qquad i=1,\ldots,m, \quad j=1,\ldots,n. \end{aligned} \tag{18}$$

Efficient DMU is determined by using the following criteria:

TABLE 2. Banking data application under models (2), (13) and (14).

Bank	CS		RO		ROA		Models (14) and (13)		Model (2)	
	r_{1j}^S	r_{1j}^U	r_{2j}^L	r_{2j}^U	r_{3j}^S	r_{3j}^U	R_J^S	R_J^U	R_J^S	R_J^U
1	14.84	15.4	23.1	26.1	3.44	4	1.390524	1.49231	1.04112	1.49231
2	14.17	19.2	36.2	41.8	5.9	6.9	1.000000	1.0000	1.0000	1.0000
3	11.91	12.6	31.1	33.2	3.71	4.2	1.259036	1.34409	1.09039	1.34409
4	9.93	13.5	24.7	34.9	3.3	3.46	1.197708	1.69233	1.03724	1.69233
5	12.7	13.6	24.2	26.06	3.1	3.54	1.495410	1.60539	1.14025	1.60539
6	13.2	15	20.22	22.31	2.6	3.34	1.502122	1.69520	1.10387	1.69520
7	9.5	13.64	13.42	24.2	1.83	2.3	1.540701	2.41721	1.16131	2.41721
8	11.5	11.9	22.04	26.4	2.53	3.1	1.583333	1.76835	1.24735	1.76835
9	9.24	11.8	27.83	32	2.57	3.7	1.306250	1.4993	1.13135	1.4993
10	8.4	9.1	23.4	37.44	2.1	3.15	1.116453	1.78635	1.0000	1.78635
11	11.36	12.1	32.3	33.43	3.8	3.9	1.250374	1.29416	1.08265	1.29416
12	18.8	29.13	13.76	17.1	3.2	4.01	1.000000	1.02135	1.0000	1.02135
13	7.44	13.7	17.9	33.3	2.4	2.47	1.255255	2.33536	1.05186	2.33536
14	10.74	11.7	23.7	28.55	2.8	3.07	1.464098	1.76367	1.23016	1.76367
15	13.8	14	20.6	32.1	2.9	4.41	1.302181	1.63052	1.04932	1.63052
16	9.4	10.2	28.84	31.4	2.71	3.2	1.331210	1.44949	1.15287	1.44949
17	8.7	9.69	18.3	22.12	1.6	2.14	1.889693	2.24215	1.51814	2.24215
18	14.5	14.98	16.7	22.03	2.4	3.3	1.510476	1.69722	1.10828	1.69722
19	13.4	14.6	20.04	33.4	2.68	4.9	1.251497	1.67813	1.00695	1.67813
20	7.57	10.8	12.9	14.61	1.1	1.4	2.159382	2.82247	1.56642	2.82247

Definition 3. The DMU is defined as DEA efficient if $K_o^{L^*} = 1$. Alternatively, for $K_o^{L^*} > 1$, it is defined as inefficient

5. Model applications to the banking industry

In this section, the proposed interval DEA models are applied to the banking industry, through which the discriminatory power is demonstrated. In this example, $\varepsilon = 10^{-10}$ is assumed.

Example. In this section, we use a dataset containing a set of regional banks using our proposed model. We assume that each of the 20 banks in the sample uses three interval ratios. We use the dataset of Emrouznejad *et al.* [20] for comparison purpose and this is shown in Table 2.

There are three variables included in the numerical example, which are CS, representing customer satisfaction, ROE stands for Return on Equity and ROA is return on assets.

Applying the interval DEA models (13) and (14) we proposed to the dataset, we report the findings in the columns 8 and 9 in Table 2. For comparison purposes, the findings of interval DEA model (2) are shown in the last column of Table 2.

The interval efficiencies and respective classifications derived from the proposed multiplicative ratio model (16) are reported in Table 3. The values from the interval DEA models (4) are reported in the last column of Table 3, therefore both of these two can be compared.

Unlike Emrouznejad *et al.* [17], the proposed model does not employ a variable frontier and DMUs are classified into two groups. That is, efficient and inefficient. Whereas, in Emrouznejad *et al.* [17], DMUs are classified in three groups.

TABLE 3. Banking data application under models (4) and (16).

Bank	CS		RO		ROA		Model (16)		Model (4)	
	ρ_{1j}^L	ρ_{1j}^U	q_{2j}^L	q_{2j}^U	q_{3j}^L	q_{3j}^U	k_j^L	k_j^U	k_j^L	k_j^U
1	1.17	1.18	1.36	1.41	0.53	0.60	1.14424	1.17043	1.04112	1.49231
2	1.15	1.28	1.55	1.62	0.77	0.83	1.0000	1.06184	1.0000	1.0000
3	1.07	1.10	1.49	1.52	0.56	0.62	1.10517	1.13883	1.09039	1.34409
4	0.99	1.13	1.39	1.54	0.51	0.53	1.08329	1.25860	1.03724	1.69233
5	1.10	1.13	1.38	1.41	0.49	0.54	1.18406	1.22012	1.14025	1.60539
6	1.12	1.17	1.30	1.34	0.41	0.52	1.17784	1.23433	1.10387	1.69520
7	0.97	1.13	1.12	1.38	0.26	0.36	1.19533	1.44773	1.16131	2.41721
8	1.06	1.07	1.34	1.42	0.40	0.49	1.22140	1.26991	1.24735	1.76835
9	0.96	1.07	1.44	1.50	0.40	0.56	1.12750	1.19722	1.13135	1.4993
10	0.92	0.95	1.36	1.57	0.32	0.49	1.05127	1.29693	1.0000	1.78635
11	1.05	1.08	1.50	1.52	0.57	0.59	1.10517	1.12750	1.08265	1.29416
12	1.27	1.46	1.13	1.23	0.50	0.60	1.00000	1.16240	1.0000	1.02135
13	0.87	1.13	1.25	1.52	0.38	0.39	1.10517	1.44773	1.05186	2.33536
14	1.03	1.06	1.37	1.45	0.44	0.48	1.18530	1.28402	1.23016	1.76367
15	1.13	1.14	1.31	1.50	0.46	0.64	1.12750	1.22204	1.04932	1.63052
16	0.97	1.00	1.45	1.49	0.43	0.50	1.13883	1.18350	1.15287	1.44949
17	0.93	0.99	1.26	1.34	0.20	0.33	1.32313	1.42356	1.51814	2.24215
18	1.16	1.17	1.22	1.34	0.38	0.51	1.17784	1.23173	1.10828	1.69722
19	1.12	1.16	1.30	1.52	0.42	0.69	1.08096	1.23433	1.00695	1.67813
20	0.87	1.03	1.11	1.16	0.04	0.14	1.37206	1.55516	1.56642	2.82247

6. Conclusion

The current paper aims to illustrate the shortcomings of the DEA models dealing with interval data proposed by Emrouznejad *et al.* [20] and improved models were developed to assess the interval DEA efficiencies of DMUs using a fixed and unified production frontier. The use of variable production frontier, extra variable changes and scale transformation in calculating the DMUs' efficiency intervals are the root shortcomings of Emrouznejad *et al.* [20]'s models. To overcome these limitations, improved DEA models dealing with interval data based on the interval arithmetic to assess the efficiency intervals of the DMUs are presented in the current paper. In comparison to the models Emrouznejad *et al.* [20], our improved models benefit from the advantage of being easily understandable and more reliable. First, our model is more compact because scale transformation and variable change are avoided. Second, our proposed interval DEA models possess the benefits of the original DEA model and do not impose any additional constraints. This advantage makes our proposed model easier to be understood and it produces more robust results. The findings from the numerical example of 20 banks show that our proposed model reduces the number of efficiency DMUs compared to the ones of the method proposed by Emrouznejad *et al.* [20]. Despite of the prominent contributions, which have been introduced in this paper, there are still some limitations can be considered as follows.

The proposed approach assumes that all inputs and outputs are positive values, which may conflict with many settings of real-life, particularly in banking sector. For example, outputs of profit and earning per share can take negative values for some DMUs. Undesirable outputs play a key role in evaluating efficiency measures of DMUs that some of their desirable outputs can produce undesirable outputs. However, the utilized data set in this paper ignored the existence of undesirable outputs of the evaluated banks, such as non-performing loans (NPLs) producing by loans which can contribute in evaluating efficiency accurately. Consequently, the proposed approach did not take into consideration the effect of undesirable outputs on its efficiency measurement. In efficiency evaluation of banking sector, deposits can be considered as intermediate output produced in a first stage, which become as input to a second stage. In order to remedy the raised limitations of this paper, we explore some extensions for future research, are as follows.

The proposed approach in this paper can be extended to deal with negative data. Therefore, a newly model considering negative outputs can be introduced. To extend the application area of the proposed models, undesirable outputs can be considered into the proposed approach. Consideration of the undesirable outputs should be performed under the weak disposability or strong disposability regarding the nature of the application that it handles. Due to the nature of some operational processes of the banking sector and many settings of real life, which can be dealt as linking activities (*i.e.*, intermediate products), the proposed approach has to be improved to evaluate the divisional and overall efficiencies of banking sector.

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