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A MULTI-STAGE STOCHASTIC PROGRAMMING APPROACH FOR AN INVENTORY–ROUTING PROBLEM CONSIDERING LIFE CYCLE

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Abstract. Food waste and proper methods to deal with it are one of the main challenges of supply chain network management. The majority of studies on how to use mathematical models in the supply chain have focused on goods that are at their peak of freshness as soon as they are produced and deteriorate over time. While some products experience an increase in value at the start of their life cycle, this value eventually reaches its maximum level, and after this point, these products experience a decline in value before being eliminated from the consumption cycle. The objective of this study is to develop a comprehensive inventory–routing model suitable for supply chain networks where products exhibit an increase and decrease in value over time. By considering the randomness and dynamic uncertainty of market demands and the fact that each period has effects on the next period, The proposed model employs a multi-stage stochastic programming (MSSP) approach. By doing so, the model ensures a balanced flow between different components of the network while considering non-deterministic demand under various scenarios that are shown in a tree of scenarios. The utilization of MSSP leads to more reliable solutions compared to deterministic models, making it possible for chain stores to make well-informed decisions in their inventory management and distribution strategies. Ultimately, this approach results in cost savings for chain stores handling such products. This research makes a significant contribution to the existing literature by demonstrating the effectiveness of the proposed model on actual data and highlighting the benefits of using stochastic programming in supply chain optimization.

Mathematics Subject Classification. 90B06, 90C15.

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1. Introduction

Nowadays, due to consumer interest in food quality and the promotion of healthy diets, the supply chain of perishable products has become particularly important, so it is clear that there are steps to be taken for this governing atmosphere to be optimized [1]. Perishable products are generally divided into two groups: the first group includes dairy and blood-based products, which can be used up to the expiration date, and one can say that the quality of these products is almost untouched. But in the second group, which mostly includes vegetables, fruits, and meat, the quality of the product is constantly decreasing over time [2]. Thus, in multiple

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Keywords. Inventory-routing problem, perishability, multi-stage stochastic programming, supply chain network design, product life cycle.

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product systems, due to the variety of the product families, it is important to note the differences in product perishing from the beginning of the supply chain design because it will affect the level of profit and system stability [3]. Considering a life cycle for the products will able the managers to make their decisions with a better view so that the costs and wastes can be minimized [4,5]. Cerutti *et al.* stated that in the fruit sector, life cycle assessment is a suitable tool for evaluating the environmental effects of fruit production and consumption, and for greater accuracy, the entire supply chain, from production to consumption, should be considered [6].

One of the effects of perishing in fruits and vegetables is the change in the selling price and storing cost [7–10]. In other words, the longer the storing time, the lower the selling price, and on the other hand, to reduce the speed of this process, the product should be stored in special conditions, which will result in higher costs. While in most research, at least one of these two (selling price and holding cost) is considered constant.

Since the quality of the products affects the sales and, as a result, the whole network efficiency, continual evaluation of the quality of foodstuffs and readiness for response to any scenarios are the main basis for system efficiency [11,12]. Regarding the estimation of the quality of the available products, numerous quality functions have been presented, depending on whether time is in the discrete or continuous mode, but all of them see the quality only in decreasing trend. In the first group in which time is considered continuous, the common basic assumption is that all products right after production will experience a consistent decrease in quality, which may be linear or exponential [13]. In some models, up to a certain point in time, the quality of the product is preserved, and after that point, the decreasing process starts [14, 15]. But in the second group in which time is considered to be discrete, for each period, a certain decreasing quality is considered for the product [16]. Regardless of the time dimension, other quality functions have also been presented that include more variables. For example, Zhang *et al.* Using the temperature as the key factor of destruction, they have estimated the rate of spoilage of products [17]. Rong *et al.* considering the temperature factor and whether there is microbial growth in the process of spoiling that product, they developed mathematical equations to estimate the speed of spoilage, which they used in a product distribution model [18]. Some others, like Zanoni and Zavanella reduced costs and increased the quality of products by developing perishability relationships focused on temperature and time factors [19].

In this study, we review a multi-stage stochastic programming (MSSP) model consisting of three levels: wholesales center, warehouses, and chain store branches, to create an integrated model for the inventory–routing problem with backorders considered. Based on assumptions, any store during any period is likely to have demands that will affect the flow of delivered goods to that store during the following period. Thus, the use of MSSP can be an effective tool for this problem. Also, the reason that among all the vegetables and fruits, we selected tomato for this study is that this product, in comparison to other products, goes through its life cycles relatively faster and is one of the most used vegetables in Iran. This study can be helpful to organizations such as chain stores, which must also supply their branches' demands. If, initially, by storing products, their quality and value will initially increase, but if they remain in the warehouse, this process will be reversed, and the products' quality and value will decrease.

Several questions are raised in this study, such as: How can a mathematical model describe changes in product prices? How to create an integrated inventory–routing problem for products with fluctuating value over time? What is the most appropriate way to describe the uncertainty of demands in the discussed issue? What is the ideal policy for determining the optimal amount of input to the stores as well as the level of their inventory? The goal of this study is to provide adequate answers to the questions addressed by proposing a novel model for managing the supply chain network.

The study will continue as follows: in Section 2, the subject literature of the works close to the goals of this study is reviewed. In Section 3, the problem and its assumptions have been explained, and in Section 4, the new mathematical model is presented. In Section 5, the case study is reviewed, and in Section 6, the results of the numerical solution of the problem are stated with the answers verified. Finally, in Section 7, we conclude our study.

2. Literature review

In this chapter, literature on the supply and distribution of products is reviewed in three domains: Usable uncertainty approaches with regards to the designed model, research conducted related to designing and managing the supply chain with emphasis on using routing, and models that have attended to the concept of perishability.

2.1. Supply chain management

An integrated view of the supply chain network will cause essential decisions, such as inventory stock amount determination, decisions on site and number of facility construction, and model routing methods not to be made separately, leading to decision effectiveness and functioning enhancement [20]. Using this strategy in supply chain management manifests in inventory–routing problems, which seek to determine the optimized inventory stocks and specifications and prioritize the methods to meet the customers. The important feature of this kind of problem is Np-hard, which forces the researchers to either use simple assumptions such as limited periods or solve the problem on a small scale or use meta-heuristic and heuristic algorithms methods and define the model less confusingly, for example, the research conducted to define the constraints of resolving sub-tour more simply [21,22]. Lots of research has been conducted about combining routing with other problems. For example, Abdelmaguid *et al.* [23] developed an inventory–routing model in a multi-period state to use the concept of backlash and stocking products. In this model, each customer is allowed to store products for the next period, more than their demanded amount, up to their maximum capacity, or even receive less than their demanded amount. In most models, the distribution capacity in potential spots for constructing or activating distribution centers has a specific amount and here the decision is whether there should be any facilities at the specified site be activated or not. Ahmadi-Javid and Seddighi [24] have proposed a model for location-routing-inventory problem, which determines the optimal distributor capacity on its own concerning the supplier, distributor, and customer. Considering the time window for sending the products to the customers, Amorim and Almada-Lobo [25] have presented a bi-objective model for minimizing the costs and maximizing the freshness of the sent products to the customers, for a two-echelon problem, including the supplier and the customer. In addition, Vahdani *et al.* [26] proposed a nonlinear model for a production and distribution system of perishable products in which customer's demand follows a specific probability distribution function. With the assumption that the Pickup and Delivery procedure in the product distribution among the customers is simultaneous, Hosseini-Motlagh *et al.* [27] by considering the customer's demand to be fuzzy, have proposed a new model for a two-echelon location-routing problem. Regarding the use of special inventory control strategies, Rafie-Majd *et al.* [28] presented a model in which customers demand follows normal distribution function and inventory supply strategy for the distributors is (Q, R); and have solved the problem consisting of three levels of the supplier, distributor, and customer by using the Lagrangian relaxation Technique. Another ability that recent organization managers are trying to use, is to assign the delivery of the product to other corporates. In a model presented by Soysal *et al.* [29], for the distribution of perishable products between suppliers and customers, the 3PL feature is used as a routing–inventory model. Another interesting feature is assigning the freedom of selection or discount to the customers; for example, Afsar *et al.* [30] presented a routing model intended to increase the profit and consists of two levels: distributor and customer. In this model, the distributor divides its geographical area into particular regions and assigns a specific price for the distribution process for each region. Now the customers of each region can decide whether the assigned price is suitable for them or not. In a model presented by Biuki *et al.* [31] at the first level, the suppliers are ranked using the PROMETHEE method and with regards to 12 criteria of stability; in the next level, a multi-objective model is defined with several layers consisting of a supplier, producer, distributor, and customer, considering uncertainty on demand. Recently some actions have been made to decrease the use of fossil fuels and road traffic, which try to present a model in which drones and EVs are in use [32, 33]. In addition, Morales Chavez *et al.* [34], for producing biofuel from agricultural wastes, presented a multi-objective model which wanted to increase profit, reduce environmental effects such as CO_2 emission, and increase positive aspects of social life, like employment, for solving a location-routing-inventory problem.

2.2. Time dependent criteria

Numerous types of research have been conducted concerning the relationship between product quality and product age and related changes in price or preserving cost that are as follows. Coelho and Laporte [7] presented a bi-objective model containing one supplier and a group of customers in the form of an inventory–routing problem for a perishable product distribution network. In the model presented by Hiassat *et al.* [35], considering the preserving cost of products to be constant and defining a shelf life, the concept of the perishability of products is put up to use. Rohmer *et al.* [36] presented a model of inventory–routing which included three levels the supplier, warehouse, and customers. In this model, fresh products from the supplier's side are moved to a warehouse, and from there, the customers' demands are met using a transportation system. Ghasemkhani *et al.* [9] presented an inventory–routing model for perishable products in a way that the model only includes a warehouse and a few numbers of customers; also, the demand is considered to be uncertain. Alkaabneh *et al.* [8] presented a deterministic inventory–routing model for perishable products, which only includes suppliers and customers. In the presented model, the selling price and holding cost of products will change depending on the time that the products are preserved. Yavari *et al.* [37] considering any interruptions in roads, connecting warehouses, and retailers due to heavy traffic or maintenance or such, presented a location–inventory–routing model for perishable products. In this model, retailers' demand depends on the price of the products, and that price is a function of the durability of the product. Mahata and Debnath [38] presented an inventory control problem for two supplier and customer levels in order to determine the optimal amount of ordering and maximize the customer's profit. The movement of goods from the supplier's warehouse to the customer's warehouse causes perishability, and the retailer's warehouse has preservation technology to slow down the rate of deterioration. They also assumed that the holding cost depends on how long the goods are stored in the warehouse. Priyamvada *et al.* [12] propose a problem to optimize the selling price and the cycle length after criticizing why in the warehouse department of goods, only the initial rate of perishability is taken into account and importance is not given to the selling price of the goods and how many periods the goods have been in the warehouse.

2.3. Perishable supply chain

Research has been conducted on the use of the concept of perishability in different industries. In the supply chain of fruits, since the quality of the product depends on the time of harvest, weather, and the soil, which can be different, Grillo *et al.* [10] proposed a linear model in which, at first, the product is taken from the supplier and then based on the quality of the product, is packaged into different groups with different prices inside the packaging units and finally delivered to the customers. Al Theeb *et al.* [39] designed a cold supply chain for a network consisting of two levels: supplier and customer. In this model, two types of products are considered, including refrigerated and frozen products, in which heterogeneous vehicles in a closed route with the possibility of partial delivery are obliged to supply the customers' demands that have certain amounts with partial delivery. In addition, Esteso *et al.* [3], have shown that the economic performance of the supply chain for fruits is enhanced, considering the planting-to-harvest stages, capacity-related constraints in a multi-period mode, and product perishability. Yousefi *et al.* [40] presented a multi-product, multi-period model consisting of four levels of the supplier, producer, distributor, and customer, for global planning of the dairy supply chain, in which operational and financial decisions on regulating the strategy of sales on credit, are combined. In the presented model, for the first time, the amount of sales on credit has been considered to be a decision variable, and for dealing with the uncertainty of the exchange rate and the quality and quantity of returned products, fuzzy CVaR criteria have been used. In dairy products, assuming that perishability follows probability distribution function, Jouzdani and Govindan [41] estimated the life span of the products using Weibull distribution in a location–inventory–routing model consisting of two levels of distributor and customer, depending on if, at the time of transporting, the refrigerator of the vehicle was on or off, separate parameters have been used in the Weibull distribution. Also, in this model, three objectives are followed: reduce the current costs, reduce the road traffic and reduce fuel consumption. Game theory and Stackelberg principles are very often used in supply chain network management. Considering that dynamic pricing and cooperation among supply chain members lead to more rational decision-making by managers, Zarouri *et al.* [42] demonstrated in a problem for distributing a perishable product involving a manufacturer, a customer, and two production and sales periods that in the case of cooperation, the most profit is obtained due to a reduction in the sales price. Maiti [43], with the aim of identifying optimal tactics for producing high-quality products at lower prices, developed a model involving a manufacturer and retailer trading one product, where demand for each period depends on selling price and quality. The quality of goods in the first period affects demand in the next period, and returned items from the first period are repurposed into new items for sale in the second period. The manufacturer employs game theory as a Stackelberg leader to manage pricing strategies over time, either maintaining fixed rates or not. In the distribution of fresh goods to account for the length of the lead time during which the demand changes, Yang and Peng [44] presented a model for the supply chain network that includes a supplier and a retailer using Stackelberg's concepts. In this model, the quantity of goods is produced using a bi-directional risk-sharing contract, where the supplier decides the production quantity first, and the responsible retailer decides the order afterward. The proposed contract achieves Pareto improvement for both members by splitting the supply chain profit between them.

2.4. Uncertainty

A lot of research has been done regarding the solution methods to deal with uncertainty, especially when there is equality sign in the constraints. Based on the existing data status, when we want to use scenario-based approaches and the model is defined to be multi-period, it is recommended to use Multi-Stage Stochastic Programming. We use a scenario tree to show the state of scenarios in this model. The most important feature of this kind of problem is dividing the decisions into several stages in which the decision in each stage tries to compensate for the adverse effect of the previous stage [45]. Also, if the model is designed to be mono-staged, the Two-Stage Stochastic Programming approach can be used [46]. In a state where data uncertainty is of deep uncertainty, Bertsimas and Thiele presented a new approach for a multi-period inventory control problem in a state where the demand is uncertain. They closed the uncertainty on the period rather than taking Constraint-wise [47]. In the end, if the type of uncertainty of the considered parameters is fuzzy, a solving method is presented using a credibility measure [48]. In the fuzzy state, despite the type of signs, there would be a Robust Possibilistic Programming approach. In this approach, the model itself specifies the optimized amounts of necessity or possibility of constraints rather than the decision maker. This solving method is used in a two-purpose problem to maximize Career Opportunities and minimize production waste, days lost due to work losses, and the number of potentially hazardous products [49].

According to the conducted studies, researchers have so far combined various models of products distribution with the concepts of perishability and uncertainty. However, the main difference between this study and other research is as follows (Tab. 1):

- Presenting an integrated inventory–routing–pricing model in the supply chain of perishable products.
- Using assumptions that follow reality such as constraints related to warehouses and vehicle capacities, and the possibility of backorders. It is also considered that if a store's inventory is greater than its next-period demand, no products will be delivered to that store in the next period. Additionally, there is a possibility that a vehicle tends to stop at other distribution centers along its route to reload products.
- The number of products each vehicle can transport is a function of its capacity and the amount of product the origin warehouse has.
- Considering that the time is defined as discrete, a three-stage life cycle is considered for the product (the tomato), including unripe, ripe, and mashed.
- Although the sales price varies in the actual world based on what stage the product is in, this issue is rarely explored in the literature, and often, a decreasing process for price change is taken into account [50]. But in this study, the sale price increases when the product goes from unripe to ripe and decreases when the product goes from ripe to mash.
- The product's holding cost has a relationship with its price.

- Regarding the fact that most of the research conducted in the inventory–routing area considers the customers' demands to be deterministic [51], in this study, MSSP is used to deal with modeling the uncertainty of demands because the model is multi-period and the equality sign in constraints is used.
- Implementation of the presented model in a case study.

3. Problem description

Timing and optimizing the number of orders are two difficulties faced by fruit and vegetable stores. Since if they order less than their need, there will be a shortage and they risk losing consumers, and if they order more than their demand, they will have to trash their items due to perishability, which is a significant loss regardless. Now, if in this described environment, the selling price of some perishable goods does not decrease as other perishable goods do, and if there is a possibility of fluctuations in the selling price and holding cost of the goods due to the transition from unripe to ripe and from ripe to overripe, the problem becomes more complex. Therefore, the manager of a chain store must keep track of how much to purchase, how to utilize the warehouses, and how to distribute products based on the demands of each branch. All these factors must be considered for a given period of time.

In this study, the supply chain network described in Figure 1 is composed of a wholesale center that sells fresh products, store branches, and warehouses that belong to the store and are obliged to control the quality and delivery of the products to the store branches by vehicle. At the beginning of each period, first a decision is made about which central warehouses should receive the unripe tomato from the wholesale center, and then what amount of the product should be delivered to the branches inside the urban area. Note that each vehicle should return to the warehouse of origin; since warehouses and vehicles belong to the same organization, the vehicles can use other warehouses as well.

The main assumptions of this study are:

- A beneficiary of this model is the supply chain network.
- Periods are discrete.
- The model is a single product.
- The location of distribution centers and stores are already specified.
- Each distributing warehouse has a specific capacity, and the wholesale center has no constraints on product supply capacity.
- Wholesale center and warehouses only distribute unripe tomatoes.
- The branches of the chain store are only capable of preserving unripe and ripe tomatoes.
- During each period, each local store can stock the products until their capacity restriction is met at which point the holding cost will be calculated.
- The present phase of the product's life cycle will be altered by storing it, as seen in Figure 2. Therefore, the selling price will change. Figure 3 depicts how this price adjustment occurs.
- The holding cost of each product is a function of its sale price.
- If a local store's demand cannot be completely satisfied during a period, the remaining amount will be added to the demand for the next period. (Backorder)
- If a store's inventory exceeds their demand for the next period, no product will be delivered to that store in the next period.
- This model's routing features include a closed route, homogenous vehicles, several warehouses, and the inability to make partial deliveries.
- The probability defined on the scenario tree's nodes are considered to be independent.
- The demand value of each node includes one of the possible values randomly.
- According to Figure 4, eight scenarios are considered for the three-stage model to be defined.
- At each node, demand changes within a certain range, and the average of this range is the demand value.

TABLE 1. A comparison of this study to previously relevant research.

Year	Researcher	Time-dependent		Decision				Linear model	Backlog		Vehicle fleets		Capacity						Data		Number of objective functions		TIME		Route structure		Network elements					Multi-layer
		Holding cost	Sale price	Supplier selection	Routing	Inventory	Location		Lost sale	Backorder	Heterogeneous	Homogeneous	Vehicle	Customer	Recycle	Distributer	Manufacture	Supplier	Uncertainty	Certainty	Multi O.F.	Single O.F.	Multi period	Single period	Close	Open	Recycle	Customer	Distributer	Manufacturer	Supplier	
2014	Coelho and Laporte [7]	✓	✓		✓	✓		✓			✓		✓	✓				✓		✓		✓	✓		✓			✓			✓	
2017	Grillo *et al.* [10]		✓			✓	✓	✓									✓			✓	✓		✓					✓		✓	✓	✓
2018	Rafie-Majd *et al.* [28]				✓	✓	✓				✓		✓			✓			✓			✓	✓		✓			✓	✓		✓	✓
2018	Soysal *et al.* [29]				✓	✓		✓		✓		✓	✓	✓					✓			✓	✓		✓			✓			✓	✓
2019	Rohmer *et al.* [36]				✓	✓		✓				✓	✓			✓				✓		✓	✓		✓			✓	✓		✓	✓
2019	Ghasemkhani *et al.* [9]		✓		✓	✓		✓			✓		✓	✓		✓			✓			✓	✓		✓			✓	✓			
2020	Alkaabneh *et al.* [8]	✓	✓		✓	✓		✓			✓		✓	✓				✓		✓		✓	✓		✓			✓			✓	
2020	Biuki *et al.* [31]			✓	✓	✓	✓	✓		✓	✓		✓			✓	✓	✓	✓		✓		✓		✓			✓	✓	✓	✓	✓
2020	Al Theeb *et al.* [39]				✓	✓		✓	✓		✓		✓	✓						✓		✓	✓		✓			✓	✓			
2020	Yavari *et al.* [37]		✓		✓	✓	✓	✓	✓			✓	✓	✓						✓		✓	✓		✓			✓	✓			
2021	Yousefi *et al.* [40]					✓		✓									✓		✓			✓	✓					✓	✓	✓	✓	✓
2021	Morales Chavez *et al.* [34]				✓	✓	✓	✓			✓		✓		✓		✓		✓		✓		✓		✓		✓	✓		✓	✓	✓
2021	Afsar *et al.* [30]				✓			✓				✓	✓							✓		✓		✓	✓			✓	✓			✓
2021	Jouzdani and Govindan [41]				✓	✓	✓	✓			✓		✓			✓			✓		✓		✓			✓		✓	✓			✓
2022	Mahata and Debnath [38]	✓				✓														✓		✓	✓						✓		✓	
2022	Priyamvada *et al.* [12]	✓	✓			✓														✓		✓	✓								✓	
	This study	✓	✓		✓	✓		✓		✓		✓	✓			✓			✓			✓	✓		✓				✓		✓	✓

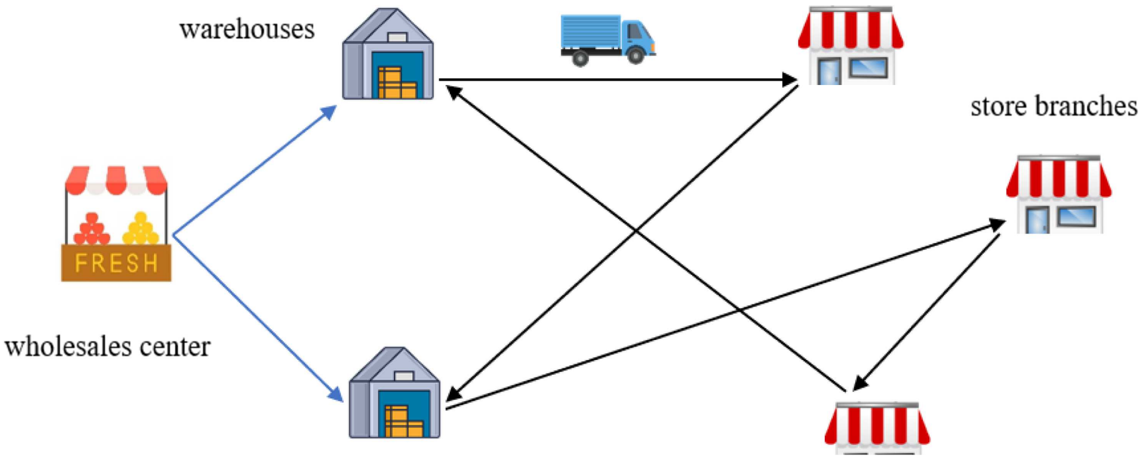


Figure 1. A schematic view of the intended supply chain network.

Figure 2. The supposed life cycle of tomato.

The main goal of this model is to increase the chain stores' profit, which is derived from subtracting costs from income. The costs that are taken into account are the cost of using the store's warehouse, the cost of shipping with vehicles, the cost of keeping the product on hand, and the cost of the backorder penalty during different time periods. Also, the income in each period equals the amount of sold tomatoes, considering the price at the time of sale.

Considering that the defined model is multi-staged and there is no dependency on demand between periods, the MSSP approach is used. meaning that, in each period, the decision maker makes a decision followed by one of the scenarios. Now the next decision is made in such a way that the probable adverse effects of the previous decisions are compensated, and this procedure continues until the final decisions are made. Each of the three time periods depicted in Figure 4 corresponds to two working days. Thus, it can be stated that for each execution, this model is studied for one week. Each node in this tree represents the possible demand for each store, which is based on the existing data history in two states. The branches of each node represent the next scenario in the next period. Each scenario's probability is equal to the multiplication of the probabilities on the constructive branches of that scenario (the assumption of independence is veridical). For example, the probability of the first scenario equals the multiplication of p_1, p_3, and p_7. Note that the total summation of probabilities of nodes in each period is 1. By "scenario" we mean the entire path of one of the existing branches which starts from the origin node (n_0) and ends at one of the leaf nodes (n_7 to n_{14}). For example, the first scenario consists of nodes n_0, n_1, n_3, and n_7.

One of the specific constraints used in MSSP models is the non-anticipative constraint. For defining these constraints, one of the following approaches is used: split variable or compact model. In the split variable method, a set of variables that are related to a node and different scenarios must be equal in each period, so

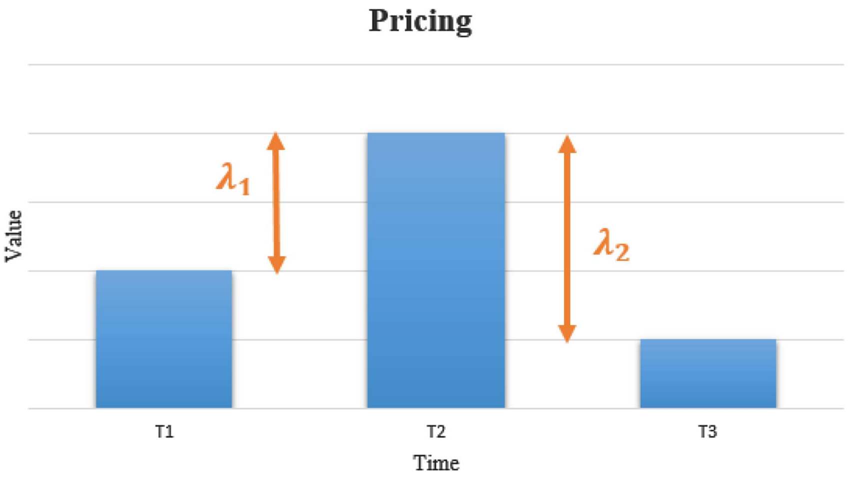


FIGURE 3. Depiction of the price function.

naturally, the scenarios index will be added to the decision-making variables and the size of the problem will increase. But in the compact model method, the relation between the decision-making variables is defined per node, and there would be no need to add the scenarios index and increase the aspects of the problem, which, as a result, will shorten the solving time of the problem. In this study, the compact model is in use.

4. MATHEMATICAL MODEL

In this part, the model explained in the previous chapter is mathematically modeled. It should be noted that due to the homogeneity of vehicles, no index is considered for the vehicles; with route tracing, we can find out from what origin the vehicle has started and what nodes have been passed along its route.

Sets

DC	Set of warehouses
CU	Set of local stores throughout the city
N'	Set of all the points in the product distribution (The union of two sets DC, CU) $i, j, k, l \in N'$
N	Set of random event nodes $\{n_0, n_1, \ldots, n_{14}\}$
$a(n)$	A node before the intended node $n \in N$
$b(n)$	A node after the intended node $n \in N$
Ω	Set of defined scenarios $\{\Omega_1, \Omega_2, \ldots, \Omega_8\}$
S_Ω	Set of random events nodes present in the scenario Ω

Parameters

c_{ij}	Transportation cost from $i \in$ DC node to $j \in$ CU node (currency)
π_i	Backorder penalty for the local store $i \in$ CU (currency)
r	Rate of bank interest
IV	Unripe product price (currency)
λ_1	Price increase amount due to the change of state from unripe to ripe (currency)
λ_2	Price decrease amount due to the change of state from ripe to mash (currency)
h	Holding cost for each unit of product when unripe, $h = r(\mathrm{IV})$ (currency)
h'	Holding cost for each unit of product when ripe, $h = r(\mathrm{IV} + \lambda_1)$ (currency)
h''	Holding cost for each unit of product when overripe, $h = r(\mathrm{IV} + \lambda_1 + \lambda_2)$ (currency)

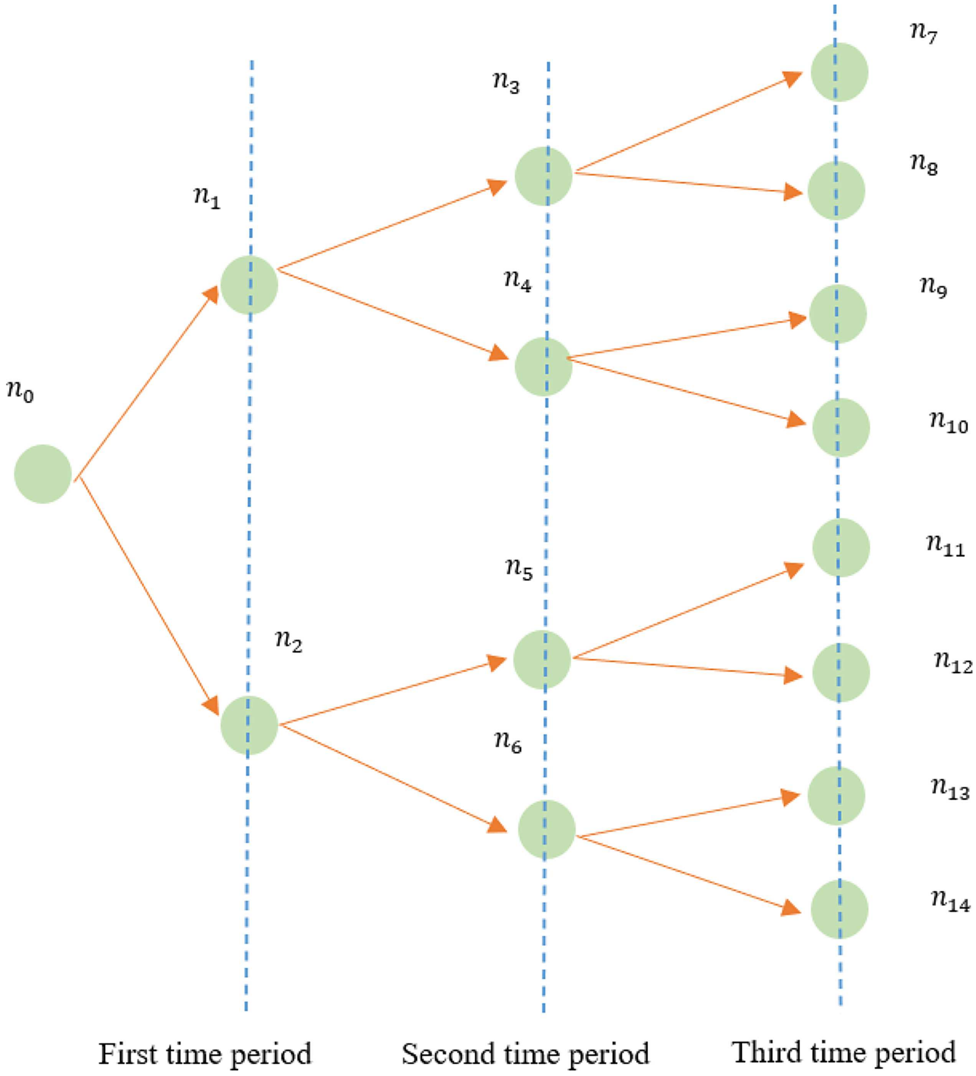


FIGURE 4. Tree of scenarios.

cap_i	Stores holding capacity $i \in$ CU (KG)
f_{in}	Cost of using warehouse $i \in$ DC at the time of random node occurrence $n \in N$ (currency)
ψ_i	Warehouse capacity $i \in$ DC (KG)
d_{in}	Local store demand $i \in$ CU at the time of random node occurrence $n \in N$ (KG)
M_{big}	A big number
ε	A too small number
q	Vehicle capacity (KG)
π_Ω	Scenario probability $\Omega \in \mathbf{\Omega}$

Decision making variables

X_{ijn}	Equals 1 if the vehicle in the random node of event $n \in N$ goes from node $i \in N'$ to node $j \in N'$
YP_{in}	Equals 1 if warehouse $i \in$ DC at the random node of the event $n \in N$ is used. If else equals 0

W_{in}	Equals 1 if the variable value G_{in} is positive. If else equals 0
Y_{ijn}	The amount of transported product by vehicle at the random node of the event $n \in N$ from node $i \in N'$ to node $j \in N'$ (KG)
Inv_{in}	Stocked products in the local store $i \in \mathrm{CU}$ at the random node of the event $n \in N$ (KG)
B_{in}	The backorder amount of local store $i \in \mathrm{CU}$ at the random node of the event $n \in N$ (KG)
G_{in}	The maximum between 0 and the amount difference of the stocked products in the local store $i \in \mathrm{CU}$ with the next period demand at the random node of the event $n \in N$ (KG)
Q_{in}	Amount of the income of the local store $i \in \mathrm{CU}$ at the random node of the event $n \in N$ (currency)
HC_{in}	Holding cost for each unit of product for the store $i \in \mathrm{CU}$ at the random node of the event $n \in N$ (currency)
θ_Ω	Profit of the supply chain network if the considered scenarios $\Omega \in \boldsymbol{\Omega}$ happen

Objective function and constraints

$$\text{Max } Z = \sum_{\Omega} \pi_\Omega \theta_\Omega \tag{1}$$

$$\begin{aligned}\theta_\Omega = {} & \sum_{S_\Omega \geq 2} \sum_{i \in \mathrm{CU}} Q_{i,S_\Omega} \\ & - \left(\begin{array}{c} \displaystyle\sum_{S_\Omega \leq 3} \sum_{i \in \mathrm{DC}} f_{i,S_1} YP_{i,S_1} + \sum_{S_\Omega \geq 2} \sum_{i \in \mathrm{DC}} \sum_{j \in \mathrm{CU}} c_{ij} X_{i,j,S_1} \\ \displaystyle + \sum_{S_\Omega \geq 2} \sum_{i \in \mathrm{CU}} \pi_i B_{i,S_1} + \sum_{S_\Omega \geq 2} \sum_{i \in \mathrm{CU}} \mathrm{HC}_{i,S_1} \end{array} \right),\end{aligned} \tag{2}$$

$$\sum_j Y_{ijn} \leq \psi_i YP_{i,a(n)}, \qquad \forall n \geq n_1, i \in \mathrm{DC}, \tag{3}$$

$$\sum_{i \in \mathrm{CU}} Y_{ijn} = 0, \qquad \forall n \geq n_1, j \in \mathrm{DC}, \tag{4}$$

$$\sum_l X_{lin} = \sum_k X_{ikn}, \qquad \forall i \in N', n \geq n_1, \tag{5}$$

$$Y_{ijn} \leq q X_{ijn}, \qquad \forall i, j \in N', n \geq n_1, \tag{6}$$

$$\sum_l Y_{lin} - \sum_k Y_{ikn} \geq 0, \qquad \forall n \geq n_1, i \in \mathrm{CU}, \tag{7}$$

$$\sum_{j \neq i} X_{ijn} \leq YP_{i,a(n)}, \qquad \forall n \geq n_1, i \in \mathrm{DC}, \tag{8}$$

$$X_{ijn} = 0, \qquad \forall n \geq n_1, i, j \in N' : i = j, \tag{9}$$

$$\sum_i X_{ijn} \leq 1, \qquad \forall n \geq n_1, j \in \mathrm{CU}, \tag{10}$$

$$\sum_l Y_{lin} - \sum_k Y_{ikn} = d_{i,n} + Inv_{i,n} - B_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2\}, \tag{11}$$

$$G_{i,n} = \mathrm{Max}\big\{0, \big(Inv_{i,n} - d_{i,b(n)}\big)\big\}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2, \ldots, n_6\}, \tag{12}$$

$$W_{i,n} \leq M_{\mathrm{big}} G_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2, \ldots, n_6\}, \tag{13}$$

$$W_{i,n} \geq \varepsilon G_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2, \ldots, n_6\}, \tag{14}$$

$$\begin{aligned}\sum_l Y_{lin} - \sum_k Y_{ikn} = {} & \big(1 - W_{i,a(n)}\big) \\ & \times \big[d_{i,n} + Inv_{i,n} - B_{in} - Inv_{i,a(n)} + B_{i,a(n)}\big],\end{aligned} \qquad \forall i \in \mathrm{CU}, n = \{n_3, n_4, \ldots, n_{14}\}, \tag{15}$$

$$W_{i,n}Inv_{i,b(n)} = G_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2, \ldots, n_6\}, \tag{16}$$

$$Inv_{i,n}B_{i,n} = 0, \qquad \forall n = \{n_1, n_2, \ldots, n_{14}\}, i \in \mathrm{CU}, \tag{17}$$

$$Inv_{i,n} \leq cap_i, \qquad \forall n = \{n_1, n_2, \ldots, n_{14}\}, i \in \mathrm{CU}, \tag{18}$$

$$B_{i,n} \leq d_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2\}, \tag{19}$$

$$B_{i,n} \leq \big(1 - W_{i,a(n)}\big)d_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_3, n_4, \ldots, n_{14}\}, \tag{20}$$

$$Q_{i,n} = (d_{i,n} - B_{i,n})(\mathrm{IV}), \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2\}, \tag{21}$$

$$\begin{aligned} Q_{i,n} = {} & W_{i,a(n)}[d_{i,n}(\mathrm{IV} + \lambda_1)] + \big(1 - W_{i.a(n)}\big) \\ & \times \begin{bmatrix} \big(d_{i,n} - B_{i,n} + B_{i,a(n)} - Inv_{i,a(n)}\big)(\mathrm{IV}) \\ +Inv_{i,a(n)}(\mathrm{IV} + \lambda_1) \end{bmatrix}, \qquad \forall i \in \mathrm{CU}, n = \{n_3, n_4, n_5, n_6\}, \end{aligned} \tag{22}$$

$$\begin{aligned} Q_{i,n} = {} & W_{i.a(n)} \begin{bmatrix} W_{i,a(a(n))}(d_{i,n}(\mathrm{IV} + \lambda_1 + \lambda_2)) \\ +\big(1 - W_{i,a(a(n))}\big)(d_{i,n}(\mathrm{IV} + \lambda_1)) \end{bmatrix} \\ & + \big(1 - W_{i,a(n)}\big)\Big[\big(d_{i,n} - B_{i,n} + B_{i,a(n)} - Inv_{i,a(n)}\big)(\mathrm{IV}) \\ & + Inv_{i,a(n)}(\mathrm{IV} + \lambda_1)\Big], \qquad \forall i \in \mathrm{CU}, n = \{n_7, n_8, \ldots, n_{14}\}, \end{aligned} \tag{23}$$

$$\mathrm{HC}_{i,n} = hInv_{i,n}, \qquad \forall i \in \mathrm{CU}, n = \{n_1, n_2\}, \tag{24}$$

$$\mathrm{HC}_{i,n} = W_{i,a(n)}(h'Inv_{i,n}) + (1 - W_{i,a(n)})(hInv_{i,n}), \qquad \forall i \in \mathrm{CU}, n = \{n_3, n_4, n_5, n_6\}, \tag{25}$$

$$\begin{aligned} \mathrm{HC}_{i,n} = {} & W_{i,a(n)}\big(W_{i,a(a(n))}h''Inv_{i,n} + \big(1 - W_{i,a(a(n))}\big)h'Inv_{i,n}\big) \\ & + \big(1 - W_{i,a(n)}\big)(hInv_{i,n}), \qquad \forall i \in \mathrm{CU}, n = \{n_7, n_8, \ldots, n_{14}\}, \end{aligned} \tag{26}$$

$$YP_{in} \in \{0, 1\}, \qquad \forall n \in N, i \in \mathrm{DC}, \tag{27}$$

$$X_{ijn} \in \{0, 1\}, \qquad \forall n \in N, i, j \in N', \tag{28}$$

$$W_{in} \in \{0, 1\}, \qquad \forall n \in N, i \in \mathrm{CU}, \tag{29}$$

$$Inv_{i,n}, B_{i,n}, G_{i,n}, Q_{i,n}, \mathrm{HC}_{i,n} \geq 0, \qquad \forall n \in N, i \in \mathrm{CU}, \tag{30}$$

$$Y_{ijn} \geq 0, \qquad \forall n \in N, i, j \in N'. \tag{31}$$

The objective function (1) tends to maximize the profit of the whole network in each possible scenario. By calculating the store's income by product sale, cost of using warehouses, transportation between networks levels, holding cost, and product backorder penalty; organization profit in each scenario is calculated. According to relations (3), the maximum amount of product departure from each warehouse $i \in \mathrm{DC}$ at the time of random node of the event occurrence equals the warehouse's capacity. Relations (4) state that at the random node of the event $n \in N$, each vehicle at the end of its route must turn back empty to the warehouse $i \in \mathrm{DC}$. Relations (5) demonstrate flow balance constraints in the routing problem. Relations (6) state that the maximum amount of the products which a vehicle can carry at the random node of the event $n \in N$ from node $i \in N'$ to node $j \in N'$ equals its capacity. Relations (7) is used to resolve the sub-tour. Relations (8) guarantees that in the random node of the event $n \in N$, a vehicle can depart from warehouse $i \in \mathrm{DC}$, only if that warehouse was activated during the occurrence of that node. Relations (9) prevent any loopholes in the route. Relations (10) demonstrates partial non-delivery. Relations (11) specifies the number of delivered products to each store in the 1st period. Relations (12) examines whether the preserved products at the time of occurrence of the considered random node $n \in N$ can meet the demand of its next node or not. Relations (13) and (14) show how the Auxiliary variables gain value. Relations (15) specifies the number of delivered products to each local store from the 2nd period forward. If the number of preserved products is more than the demand of the next period, Relation (16) tends to the relation between inventory variables. Relations (17) states that in the case of product stocking, there would be no backorder and *vice versa*. Relations (18) determines the maximum amount of product stocking in each period. Relations (19) and (20) determine the maximum amount of product backorder in each period. In relations (20)–(23), the income of the local stores is calculated regarding the increase and decrease of products

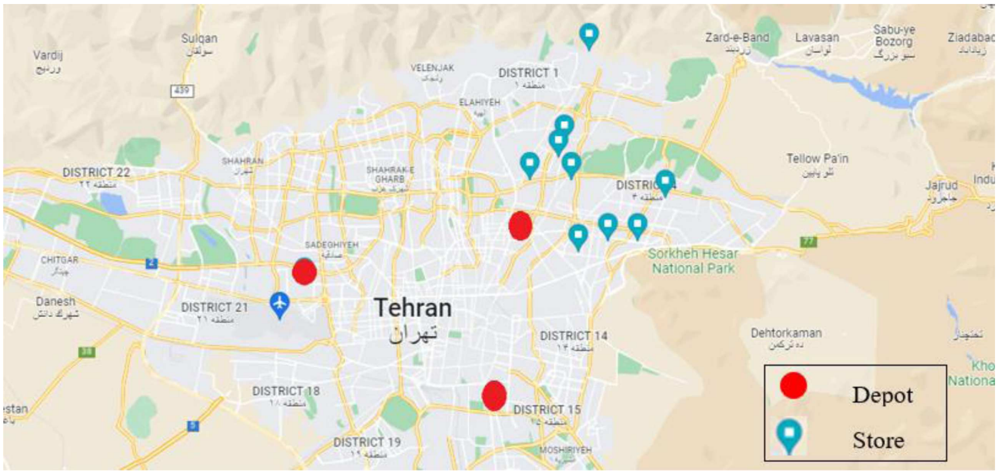


FIGURE 5. Location of the chain store branches.

price over time. In relations (24)–(26), the holding costs of the stores are calculated regarding the increase and decrease of products price over time. Relations (27)–(31) demonstrate the type of variables in the existing relations.

5. CASE STUDY

Chain stores are a group of stores operating under a single brand and sharing the same management and marketing strategy. Chain stores offer several benefits to society. One such advantage is the availability of goods at a cheaper sale price, which is made possible through distribution process improvements and bulk ordering from suppliers. Additionally, chain stores provide customers with a diverse range of products, increasing their options and convenience. Nevertheless, certain stores focus solely on a certain kind of merchandise. For instance, "Bagh e Move Irani" is one of Iran's most popular chain stores. Due to the vastness of Tehran and the fact that the store's offices are in this city, the management has divided their work into four sections: north, west, south, and east. To make it easier for stores to purchase the fruit they need, Tehran's main fruit market supplies all of its branches. According to Figure 5, this model is applied to the stores in the eastern part of Tehran, which include nine stores that are related to three warehouses that are in Tehran.

Table 2 shows the distances between the members of this network, which are measured in kilometers. Experts believe that the transportation cost per ton is 500 thousand IRTs which means that the transportation cost per Kg would be 500 IRTs and because in this problem the considered currency is a thousand Tomans (1000 IRTs), thus the transportation cost per Kg between the members of the network would be the product of 0.5 and the mentioned values in Table 2.

The product preservation capacity for each branch of the store is 300 Kg and the backorder penalty per kilo equals 2 units of currency and the bank interest rate for calculating preservation cost is 0.05% a day. According to Figure 6, Unripe tomatoes are sold throughout the stores in 7 units of currency which with the change of state to ripe, the price will go up by 60%, and by changing state to mash, the price will fall by 70%; thus, the price of the product when ripe and mash equals to 11.2 and 3.36 units of currency, respectively.

Based on the existing data, the assumption is that the demand of each store at each of the considered random nodes has a 50% chance of falling in the range of (30–50) and another 50% chance to fall in the range of (60–80) which according to Figure 7 the mean amount of these ranges, is considered as the demand of each store.

TABLE 2. The distance between distribution centers and stores.

	d_1	d_2	d_3	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
d_1	0	16.9	13.5	12.5	8.4	7.6	5.7	6.5	11.4	9	6	4.4
d_2	16.9	0	19.2	26.8	27.2	20.4	20	20.5	26.7	22.9	21.6	18.9
d_3	13.5	19.2	0	22.7	17.3	16.3	15.6	16.1	20.9	17	15.4	13
c_1	12.5	26.8	22.7	0	6.2	7.3	9.4	8.8	15.5	15.8	14.6	13.5
c_2	8.4	27.2	17.3	6.2	0	1.1	3.7	2.6	9.9	11.5	8.7	7.7
c_3	7.6	20.4	16.3	7.3	1.1	0	2.5	2	9.2	9.8	8.6	7.5
c_4	5.7	20	15.6	9.4	3.7	2.5	0	3.3	9.7	9.8	8.6	7.1
c_5	6.5	20.5	16.1	8.8	2.6	2	3.3	0	9.2	9.8	6.4	5.3
c_6	11.4	26.7	20.9	15.5	9.9	9.2	9.7	9.2	0	3.5	7.3	7.6
c_7	9	22.9	17	15.8	11.5	9.8	9.8	9.8	3.5	0	3.8	4.1
c_8	6	21.6	15.4	14.6	8.7	8.6	8.6	6.4	7.3	3.8	0	2.7
c_9	4.4	18.9	13	13.5	7.7	7.5	7.1	5.3	7.6	4.1	2.7	0

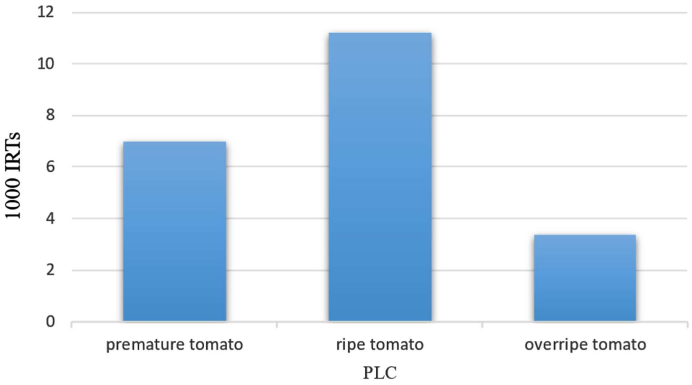


FIGURE 6. Product price change diagram.

The distribution capacity of warehouses for each store is 550, 450, and 400 units of products, the cost of using them at different periods is 200, 210, and 180 units of currency, respectively, and the capacity of the transport vehicles is 1000 units of products.

6. RESULTS

6.1. Numerical results

The selected problem was solved using GAMS 24.1.2 software and CPLEX solver on a computer system of Intel Core i5 4200M 2.50 GHz and 12 GB RAMDDR3 under Windows 7. The solving time of this problem with GAMS software has been 190 s.

The profit of the organization in different scenarios is shown in Table 3, which is also depicted in Figure 8.

According to Table 3 and Figure 8, in which the horizontal axis represents the scenarios, and the vertical axis represents the profit gained by the organization, in scenarios where the demand is decreased, the profit gained is also decreased.

To examine the results of the model, regarding the fact that there are too many scenarios, we only tend to examine the values of the variables in the 2nd scenario which includes (n_0, n_1, n_3, n_8) nodes. In this scenario,

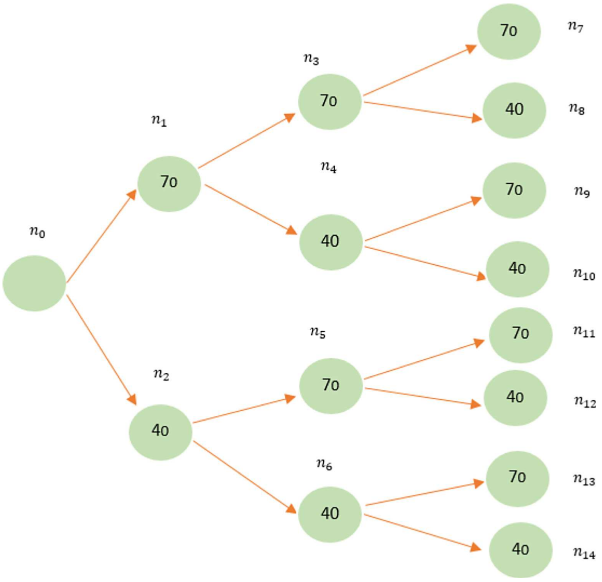


Figure 7. Amount of the store's demand.

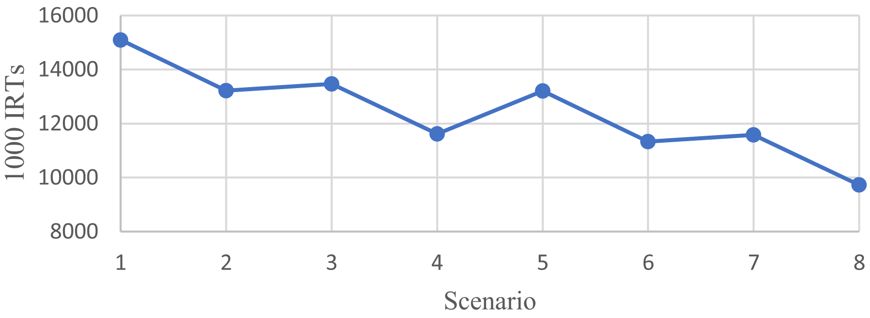


Figure 8. The gained profit in each scenario.

the demand in the 1st and 2nd periods is 70 and in the 3rd period, equal 40 units of products. Figure 9 depicts the routing method and Table 3 shows the income, backlash, and inventory of each store at different periods.

Based on Table 4, regarding the changes in product value and holding costs, the model tries to preserve products for each store as much as their maximum demand for the next period. Because the store will not order new loads of products through the transportation fleet for the next period and will only include the holding cost at its lower rate because the holding cost for unripe products is lower than that for ripe products, and as a result, the system costs are reduced. On the other hand, by preserving an unripe product for one time period, the product will change its state to be ripe, and thus the sale price will increase. Other capabilities of this model include determining the optimal number of vehicles, according to the results, at each time period, only one vehicle is used.

6.2. Model validation

In this chapter, WS stands for "wait and see". In this method, it is assumed that we can wait until the time of occurrence of a random event. In such conditions, the uncertainty will change to definite certainty, and it is

Table 3. The gained profit in each scenario.

Scenario	Path	Profit
1	(n_0, n_1, n_3, n_7)	15 098.41
2	(n_0, n_1, n_3, n_8)	13 218.51
3	(n_0, n_1, n_4, n_9)	13 472.03
4	(n_0, n_1, n_4, n_{10})	11 615.23
5	(n_0, n_2, n_5, n_{11})	13 208.51
6	(n_0, n_2, n_5, n_{12})	11 328.61
7	(n_0, n_2, n_6, n_{13})	11 582.13
8	(n_0, n_2, n_6, n_{14})	9725.33

Table 4. Stores status in the 2nd scenario.

Period	Store	Demand (kg)	Inventory (kg)	Backorder (kg)	Income (1000 Toman)
1	c_1	70	40	0	490
	c_2	70	40	0	490
	c_3	70	40	0	490
	c_4	70	40	0	490
	c_5	70	40	0	490
	c_6	70	40	0	490
	c_7	70	40	0	490
	c_8	70	40	0	490
	c_9	70	40	0	490
2	c_1	$70 - 40 = 30$	0	30	448
	c_2	$70 - 40 = 30$	30	0	652
	c_3	$70 - 40 = 30$	40	0	652
	c_4	$70 - 40 = 30$	40	0	652
	c_5	$70 - 40 = 30$	40	0	652
	c_6	$70 - 40 = 30$	40	0	652
	c_7	$70 - 40 = 30$	40	0	652
	c_8	$70 - 40 = 30$	40	0	652
	c_9	$70 - 40 = 30$	40	0	652
3	c_1	$40 + 30 = 70$	0	0	490
	c_2	$40 - 30 = 10$	0	0	406
	c_3	$40 - 40 = 0$	0	0	448
	c_4	$40 - 40 = 0$	0	0	448
	c_5	$40 - 40 = 0$	0	0	448
	c_6	$40 - 40 = 0$	0	0	448
	c_7	$40 - 40 = 0$	0	0	448
	c_8	$40 - 40 = 0$	0	0	448
	c_9	$40 - 40 = 0$	0	0	448

assumed that there is sufficient information about the future and the actual value of uncertain parameters is specified since it is obvious which scenario will occur. EV stands for "Expected Value". In this case, instead of each random parameter, its average or mean value is placed. Now in the deterministic model, the uncertain parameter takes a specific value, each time per scenario, and first-stage variable values are also equal to the value gained from the EV approach. The sum of the product of the objective function's value gained through this method, in each scenario's probability, equals EEV or Expected result of using the EV solution.

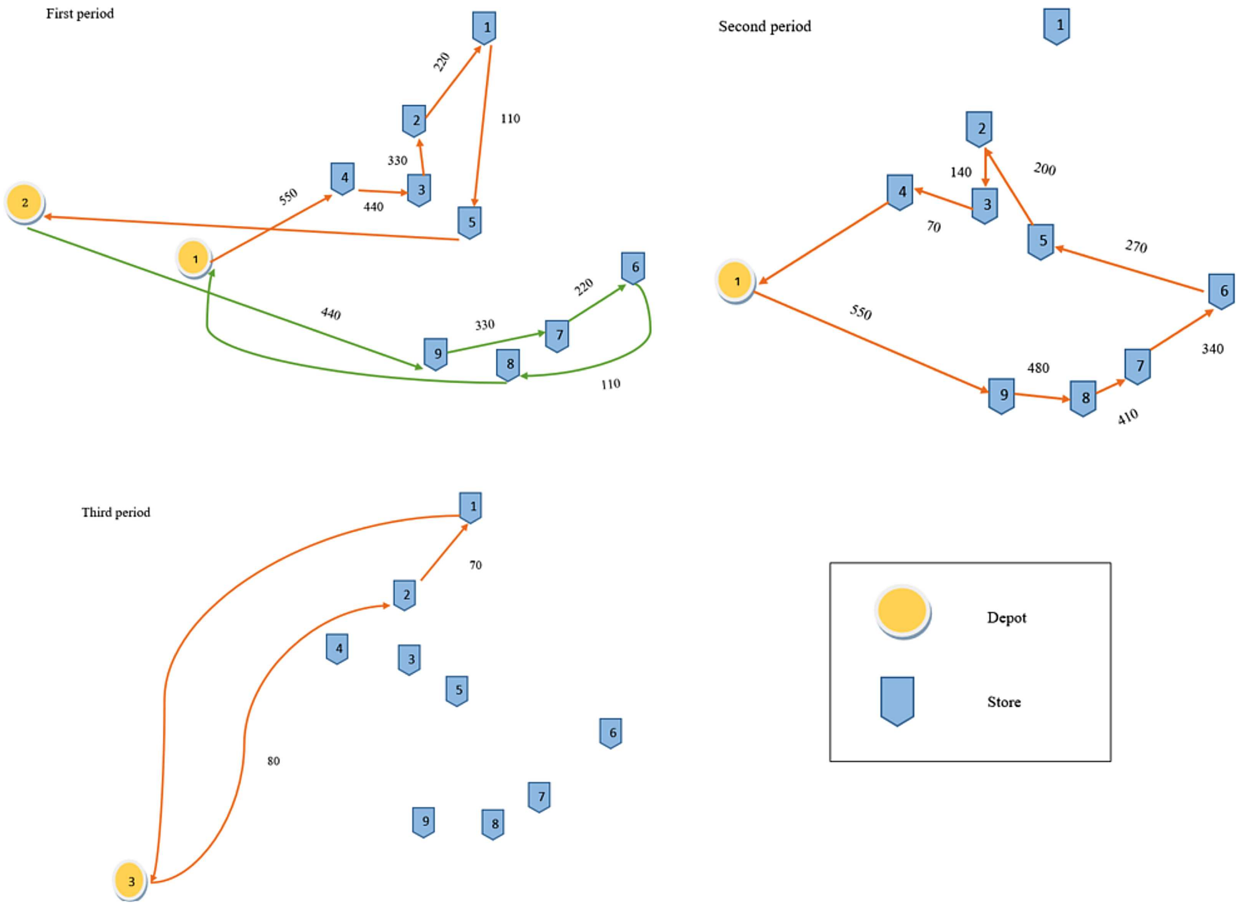


Figure 9. Routing in the 2nd scenario.

SP stands for "Stochastic Programming" which is achieved by solving the uncertain model. Two criteria are used for examining the performance of the model: Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS). EVPI index is the gained profit if there is sufficient information and awareness of the occurrence of which scenario. In other words, it shows how much it is worth to achieve detailed information. The larger this index, the more reasonable the effort to obtain information becomes. The VSS index puts the gained profit of using the uncertain model in contrast to the case wherein the problem, instead of the uncertain parameters, their mean value is replaced. In this case, the higher the value of this index, the more efficient the model is. Relations (32) and (33) show how these indexes are calculated.

$$\mathrm{EVPI} = \mathrm{WS} - \mathrm{SP} \tag{32}$$

$$\mathrm{VSS} = \mathrm{SP} - \mathrm{EEV}. \tag{33}$$

Due to the weather conditions in Iran, the price of tomatoes will face changes throughout the year. For example, in fall and winter, due to the cold, the price will increase, and in spring and summer, due to the ideal weather conditions and harvest, the price will decrease. For this reason, by considering three values for the sale price parameter, based on Table 5 values, the behavior of the model is achieved.

Based on Table 5 and Figure 10, in general, with an increase in the unripe tomato price, the EVPI index value will increase, and it has a little dependency on the backorder penalty value. Meaning that in each price

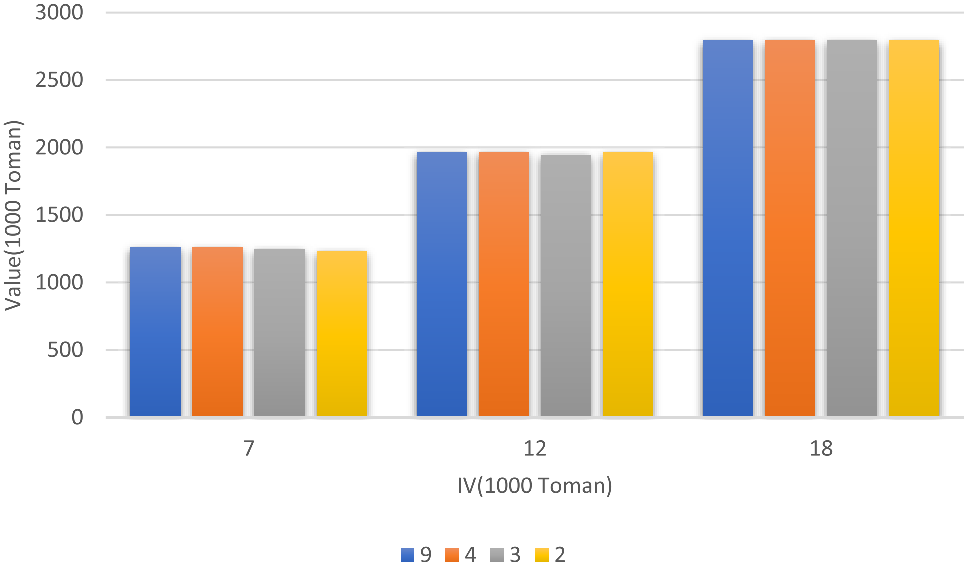


FIGURE 10. EVPI criterion behavior under the effect of the sales price.

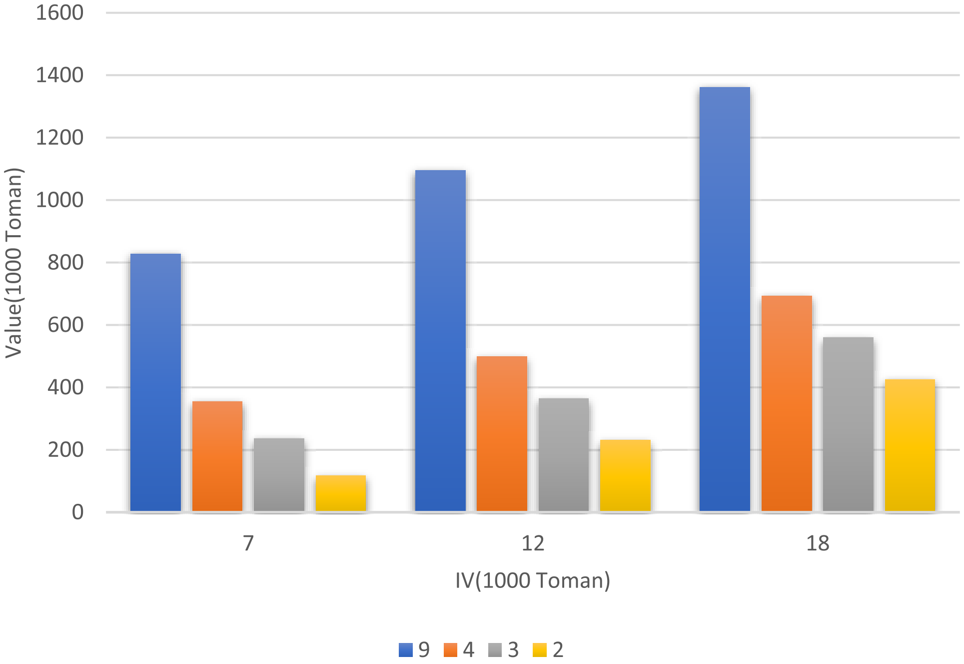
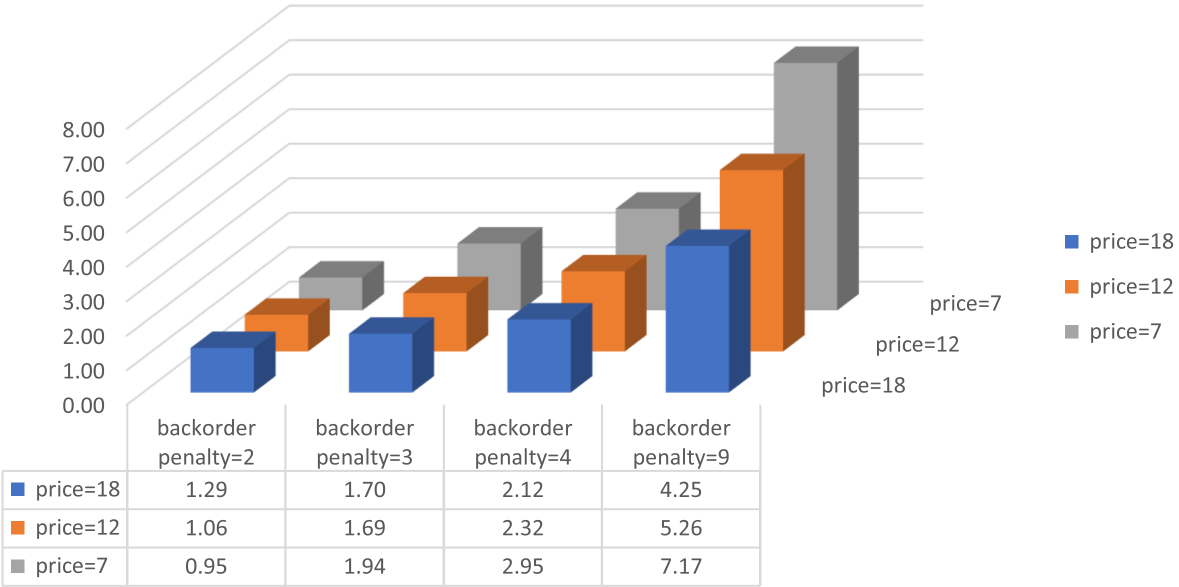


FIGURE 11. VSS criterion behavior under the effect of the sales price.



	backorder penalty=2	backorder penalty=3	backorder penalty=4	backorder penalty=9
■ price=18	1.29	1.70	2.12	4.25
■ price=12	1.06	1.69	2.32	5.26
■ price=7	0.95	1.94	2.95	7.17

FIGURE 12. VSS criterion behavior under the effect of the sales price.

range, for different backorder penalty values, its behavior is almost unchanged. Also, in every possible value, this index has a positive number, which shows the value of an investment for more accurate forecasting of future demands. Based on Table 5 and Figure 11, when the price of unripe tomatoes goes up, the VSS index goes up, but what's interesting is that it depends a lot on the value of the backorder penalty. In other words, in each price range, the greater the penalty for the lack of goods in the stores for the organization (such as the reputation of the brand and its history that may be destroyed due to the lack of goods), the value of using the uncertainty model instead of the deterministic model will be higher.

By looking at Figure 12 shows the amount of improvement of the objective function for using the achieved answer of the uncertain model (SP) in contrast to the gained result of the expected mean value method (EEV), we can see that generally by an increase in the backorder penalty, the use of the uncertain model is more profitable for the decision makers than using the mathematical expectations of the parameters.

6.3. Discussion

In this study, we propose a novel method for increasing the profit of the entire supply chain network of chain stores. This method deals with the challenges of balancing inventory levels and following goods between elements of chain stores based on varying the product's demand, sale price, which is related to the product's quality and the weather conditions, and holding cost, which is also related to the product's quality. Our goal was to optimize the routing and inventory parts by considering the dynamic uncertainty of the product's demands and the life cycle of the product, which means the product experiences an increase and decrease in its value over time, which is the cause of variations in sale price and holding cost. For preparing models to cope with the randomness of demands, we use the "compact model" approach against the "split variable" approach.

By comparison, the current situation, which is like the EEV conditions problem, with the result of the uncertainty model (SP), the percentage increase in profit, and other important measures like VSS and EVPI, are shown in Table 5. The effectiveness of this model has been demonstrated using a real case study and considering the effect of seasonal changes on the selling price of products. Among the important decisions in this model is ordering enough products to be stored one period in stores and then sold by higher price because

Table 5. Uncertain model evaluation indexes.

Backorder penalty	Raw price	7	12	18
9	EVPI	1263.96	1967.79	2795.9
	VSS	828.17	1095.69	1361.83
	WS	13 635.89	23 908.12	36 236.07
	EEV	11 543.76	20 844.64	32 078.34
	SP	12 371.93	21 940.33	33 440.17
	Percentage increase in profit	4%	5%	7%
4	EVPI	1259.75	1967.34	2795.18
	VSS	354.47	498.25	693.25
	WS	13 635.84	23 907.67	36 235.35
	EEV	12 021.62	21 442.08	32 746.92
	SP	12 376.09	21 940.33	33 440.17
	Percentage increase in profit	2%	2%	3%
3	EVPI	1244.51	1946.03	2795.5
	VSS	235.7	364.41	559.42
	WS	13 635.60	23 886.36	36 235.67
	EEV	12 155.39	21 575.92	32 880.75
	SP	12 391.09	21 940.33	33 440.17
	Percentage increase in profit	2%	2%	2%
2	EVPI	1229.95	1964.05	2795.34
	VSS	117.18	230.65	425.59
	WS	13 636.04	23 904.38	36 235.51
	EEV	12 288.91	21 709.68	33 014.58
	SP	12 406.09	21 940.33	33 440.17
	Percentage increase in profit	1%	1%	1%

of changing from unripe to ripe. In this way, organizational decision-making will achieve an optimized direction depending on the changes in demand amount. According to the calculated EVPI and VSS indexes, we can understand that it is necessary to plan for estimating the demand of the branches of the chain store more accurately and also to use the uncertain model in the cold seasons where the price of the products increases or in the case that a shortage in products will damage the brand and reputation of the store.

7. Conclusion

Transportation of food products is always challenging due to their perishability, which if not managed properly, will cause damage. Damage that results from the costs of product waste, decrease in price, inventory holding cost, and customer dissatisfaction. On the other hand, it should be noted that not all of the products have a decreasing quality process over time. Many products will also face a price increase. For example, turning an unripe tomato into a ripe one.

In this study, an inventory–routing problem in the form of a linear mixed integer model is presented to manage the flow of products between the levels of different networks that include a wholesale market, supply warehouses, and stores, in a way that the mentioned product shows its perishability in a life cycle. At each stage of this life cycle, the product has different sales and holding costs. On the other hand, due to the changes in the amount of consumption during the planning horizon, a multi-stage stochastic planning method is used to deal with this uncertainty. To demonstrate the potency of predicting the impact of seasonal fluctuations on product prices, the model is put to the test using a case study. The model can help organizations make important decisions such as ordering enough products to meet future demand and increase profits. Calculated EVPI and VSS measures show the need for more careful demand estimation and use of uncertain model in cold seasons or when shortages might damage the brand. Overall, this model can help improve profitability and guide organizations towards more informed decisions.

Due to the use of MSSP in this study, the number of scenarios grows exponentially as the number of periods increases, which, combined with the problem's routing term, makes the problem complex and takes a long time to solve. Of course, the best way to handle the complexity brought on by the rise in the number of scenarios is to use the "compact model" method rather than the "split variable" method, as was described in this study. In order to reduce the complexity caused by the routing term, if homogeneous vehicles are used, we can avoid using vehicle indexes and instead trace the path that each vehicle took by using common binary variables. Despite these efforts to address complexity, it is important to note that the proposed method may still have limitations in scalability and may not be practical for very large-scale problems.

Therefore, it is reasonable to anticipate that further studies in the future will focus on MSSP and routing issues together. In addition, in this study, the concept of disruption was not used, which can be used as a trafficking factor of the existing roads, taking into account the time window for sending customers' goods. Also, the development of the proposed model for a longer planning horizon in such a way that the possibility of using the concepts of the time value of money seems reasonable, the use of waste to sell to industrial factories, or trying to define the model in multi-product mode are other attractive areas to continue this study.

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