RAIRO-Oper. Res. 57 (2023) 2363–2392
https://doi.org/10.1051/ro/2023116

RAIRO Operations Research
www.rairo-ro.org

FINANCING AND COORDINATION OF THE AGRICULTURAL SUPPLY CHAIN CONSIDERING GOVERNMENT-ENTERPRISE GUARANTEE

Qihui Lu, Changhua Liao* and Tingting Xu

Abstract. We examine an agricultural supply chain consisting of a core enterprise and a capital-constrained farmer and assess the yield uncertainty of farmer's production. We explore two kinds of financing models: traditional bank financing and government-enterprise guarantee financing. To coordinate the supply chain, a price commitment contract and a revenue-sharing contract are considered. Our results show that no matter in bank or government-enterprise guarantee financing model, we can find the conditions for full coordination of the supply chain with any contract. However, in the government-enterprise guarantee financing model, when a farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, the revenue-sharing contract can simultaneously make the supply chain fully coordinated and achieve Pareto improvement to maximize the profits of the entire supply chain and achieve a win-win situation. In addition, the farmer and enterprise prefer to choose the financing model with a higher promised price and a higher revenue-sharing ratio, and the social welfare under the government-enterprise guarantee financing model is higher than that under the traditional bank financing model.

Mathematics Subject Classification. 90B06.

Received December 8, 2022. Accepted July 29, 2023.

1. Introduction

Agriculture is the basic industry that supports the construction and development of the national economy and the foundation of national security and stability. The Food and Agriculture Organization of the United Nations showed that agriculture is the primary industry for employment in developing countries[1]. However, farmers of developing countries lack sufficient funds to purchase agricultural production necessities, such as seeds, fertilizers, pesticides, and agricultural equipment. The capital constraints of agriculture not only prevent farmers from normal planting but also considerably affect the supply of agricultural products and the stability of the entire supply chain. At the same time, the increasing population in the world forces humans to improve farm yields by using advanced technologies [29]. Only by increasing investment in agriculture can farmers meet market demand. Therefore, financing is urgently needed for farmers, and it is also one of the important methods to solve

Keywords. Agricultural supply chain, Yield uncertainty, traditional bank financing, government-enterprise guarantee financing, coordination contracts.

School of Business Administration (MBA School), Zhejiang Gongshang University, 310018 Hangzhou, China.
*Corresponding author: changhual@163.com

[1] http://www.fao.org/faostat/en/#data/OE

the problem of farmers' capital constraints. Nowadays, various financing methods are emerging. Commercial banks, nonprofit organizations, and governments can all provide financing support to farmers [16, 28, 33].

Traditional bank financing is the most common financing method, which can reduce the financial pressure of farmers to a limited extent. Nevertheless, its loan interest rate is high and it has rigorous financing requirements for the farmers [41]. In developing countries, farmers lack sufficient credit and valuable collateral and may not be able to obtain capital [27]. In addition to the challenges of financial constraints, the agricultural supply chain usually faces yield uncertainty caused by uncontrollable factors, such as extreme weather and natural disasters [1]. Farmers are regarded as high-risk lenders, which exacerbates the difficulty of agricultural financing. To alleviate this problem, guarantor financing methods have emerged. For example, JD, an intermediary e-commerce platform with an exceptionally high reputation can provide guarantor financing services. When farmers' realized income cannot be fully repaid to the bank, the intermediary platform will fill the funding gap, allowing farmers with low or no reputation to be able to access bank loans easily [41]. This financing method has been widely recognized in practice [44]. Moreover, professional guarantee companies that provide guarantees for farmers exist. For example, in 2007, China's New Hope Liuhe Co., Ltd. established an agricultural/livestock guarantee company[2]. We find that government intervention is not considered in the above financing methods, the guarantor bears all the risks, and the profits of weak farmers will be squeezed.

In recent years, to ensure the profit of farmers and reduce the risk of the guarantor, a government-enterprise guarantee financing model has occurred, in which the government and enterprises jointly guarantee farmers' loan. When farmers' realized income cannot be fully repaid to the bank, the government and enterprises together fill the funding gap. For example, Longlin County, Guangxi Province, China, proposed the first "Government-Bank-Enterprise Share" model and introduced the "4321" risk-sharing mechanism. A government platform company was established. It increases credit for all small and medium agricultural enterprises and large farmers who have financing needs. If the lender becomes default, the government platform company, small and medium enterprises, financial institutions, and Guangxi Agricultural Credit Financing Guarantee Co., Ltd. will bear the proportions of 40%, 30%, 20%, and 10%, respectively. We find that the government-enterprise guarantee financing model belongs to the partial credit guarantee (PCG) financing operated by policy guarantee institutions. The guarantee rate of Guangxi Agricultural Credit Financing Guarantee Co., Ltd. is as low as 0.5% a year, the bank loan interest rate is as low as 4.35% a year, and the financing cost of new agricultural business entities is as low as 4.85% a year[3].

From our intuition, the government-enterprise guarantee financing model would not only effectively solve the problem of insufficient financing collateral for a farmer but also greatly reduce the financing costs of the farmer and realize the benefit and risk sharing of the government, core enterprise, bank, and farmer. However, in reality, the farmer's and enterprise's decisions and the performance of the entire agricultural supply chain will be affected by the farmer's financing approach. Everyone seeking to maximize own interests will lead to a double marginal effect. Thus, a situation in which everyone can have more profits than those in the traditional bank financing model is difficult to realize. To improve the total profit of the supply chain and achieve a win-win situation, coordination contracts should be investigated.

We mainly consider price commitment and revenue-sharing contracts. The price commitment contract indicates that the downstream enterprise commits to purchase the upstream enterprise's semifinished product with a committed wholesale price ahead of the upstream enterprise's investment and demand realization [36], and the revenue-sharing contract indicates that the downstream enterprise agrees to share a proportion of its revenue with the upstream enterprise to obtain a lower wholesale price than that in a wholesale price contract [11]. They are widely used in agriculture and have practical significance for their research. For example, in China, Chu Chrysanthemum Institute, which is a popular agricultural product company, sells seedlings and pays farmers in accordance with the commitment price[4], and Guizhou Hongcai Investment Group Co., Ltd. promised to purchase fresh prickly pear fruits at a price of 4 RMB per kilogram and signed a "shareholding cooperation

[2] http://www.newhopeagri.com/lh/info/1353
[3] https://www.sohu.com/a/357626297_665123
[4] http://news.sina.com.cn/o/2004-03-03/05171947899s.shtml

agreement" with agricultural cooperatives at a share ratio of 70%[5]. The design and choice of the contract are worthy of further exploration. Thus, we aimed to study the following questions:

(1) In the traditional bank financing or government-enterprise guarantee financing model, what are the optimal decisions of the farmer and the enterprise?

(2) After adding price commitment and revenue-sharing contract to two financing models, what are the conditions for achieving supply chain coordination? Can we achieve a win-win situation?

(3) How will the farmer and the enterprise choose financing models?

To answer these questions, we first analyze centralized decision. Then, under two scenarios of bankrupt risk and no bankrupt risk of a farmer, we discuss the optimal decentralized decision and study a price commitment contract and a revenue-sharing contract under the traditional bank financing model or government-enterprise guarantee financing model. We mainly investigate full coordination conditions and Pareto improvement areas. Lastly, we compare the two financing models under different scenarios and contracts, and present financing model selection strategies for the farmer and the enterprise. The results indicate that the price commitment contract cannot achieve Pareto improvement in any scenarios and financing models. Nonetheless, when the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, in the traditional bank financing model, the revenue-sharing contract can achieve full coordination and Pareto improvement, but they cannot coexist. In the government-enterprise guarantee financing model, the revenue-sharing contract can achieve full coordination and Pareto improvement at the same time.

The contributions of this paper are summarized as follows. First, to the best of our knowledge, this paper is among the first to study financing models of price commitment and revenue-sharing contracts in an agricultural supply chain with a capital-constrained farmer. As is known, the price commitment and revenue-sharing contracts, which are commonly used contract forms, are simple to administer. Moreover, bankruptcy risks arising from yield uncertainty may have substantial impacts on the operation of a supply chain. Therefore, it is important to understand the interplay of operational and financing decisions under the price commitment and revenue-sharing contracts. Second, with the price commitment and revenue-sharing contracts, we explore the risk-sharing between the enterprise and the farmer under two financing modes. Specifically, we characterize the conditions under which supply chain members achieve financing equilibrium and which supply chain achieves full coordination and Pareto improvement, which has not been considered in the literature. Third, we are the first to incorporate the government and enterprises jointly guarantee into the agricultural supply chain finance, the choice of traditional and innovative financing model and the choice of two contracts are studied.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the agricultural supply chain finance (ASCF) model. Section 4 analyzes financing and coordination strategies in traditional bank financing. Section 5 presents the analysis of financing and coordination strategies in government-enterprise guarantee financing. Section 6 compares traditional bank financing and government-enterprise guarantee financing. Section 7 summarizes our research results and future research suggestions. All proofs are provided in Appendix B.

2. Literature review

Literature related to this paper includes the operation and finance decision on supply chain, ASCF, and supply chain contract.

In recent years, numerous scholars have studied the intersection of finance and operational decisions. Buzacott and Zhang [7] first incorporated asset-based financing into production decisions, they found that banks are better off using asset-based financing with appropriately chosen parameters. Ding and Wan [14] considered financing (bank loan or advance payment) and coordinating issues in a supply chain with one supplier and one manufacturer, in which the supplier is capital-constrained and her production yield is random. Zhu and Ou [43] first attempt to take banks into account and endogenises their interest rates in the modelling of reverse

[5] https://www.sohu.com/a/397778652_120214189

factoring. Traditional financing methods could be costly or unavailable to small enterprises [35]. Therefore, other financing models have emerged. For example, some scholars have studied trade credit [9, 12, 31]). Jing *et al.* [17] indicated that when both bank and trade credits are viable, the unique equilibrium is trade credit financing if the production cost is relatively low, whereas it is bank credit financing otherwise. Trade credit is an important form of short-term financing for capital-constrained small firms [13]. However, Yan *et al.* [38] analyzed the equilibrium financing strategies and the coordination conditions for the PCG contract. Xu *et al.* [37] derived the equilibrium strategies of the supply chain members under PCG and a combination of trade credit and PCG. We find that most of the research has aimed to solve the problem of shortage of funds for small- and medium-sized enterprises (SMEs). On the contrary, our research is on capital-constrained farmers with high loan risks, providing farmers with financing strategies.

In terms of ASCF, Liu and Zhan [20] indicated that agricultural enterprises are facing serious financial problems and that financing difficulty is essentially a question of financing efficiency. Zhuo *et al.* [44] revealed the hidden factor that restricts the agricultural loan efficiency of Chinese banks. Bergen *et al.* [5] placed upstream intervention schemes in the theoretical context of supply chain finance (SCF) and modeled their application to a three-echelon agricultural supply chain. Yan *et al.* [39] analyzed the financing strategies adopted by SMEs on the basis of the characteristics of a fresh agricultural supply chain. They showed that the optimal level of financing by SMEs is negatively related to the freshness effort cost coefficient and positively related to the sensitivity coefficient of market freshness. Liang *et al.* [19] proposed a co-decision method to classify the credit ratings of agricultural SMEs and provided a reference for commercial banks to determine the credit of agricultural SMEs with a low decision risk. Yi *et al.* [41] considered an agricultural supply chain consisting of a capital-constrained smallholder farmer and an intermediary platform and showed that under guarantor and direct financing, the smallholder farmer's production level can even be higher than that in a centralized system. Considering the limited studies on ASCF, we first incorporate government-enterprise guarantee financing into a farmer's production decision. In accordance with whether a scenario with bankrupt risk for the farmer exists, we mainly study the ASCF with bank and government-enterprise guarantee financing models under the yield uncertainty of agricultural products.

Our work is also related to studies on supply chain contract. Enterprises can achieve optimal performance *via* a supply chain contract [8]. Our paper mainly discusses the price commitment and revenue-sharing contracts in ASCF. In terms of a price commitment contract, Liu *et al.* [21] studied the operational decisions and resulting profits for a supply chain facing price-dependent demand tinder, a policy in which an ex-ante commitment is made on the retail price markup. Liu *et al.* [22] also examined the use of price commitment policies in dynamic contracting in multiple-period, finite-time horizons. Xiao *et al.* [36] showed that a pure price commitment contract can neither improve suppliers' sustainable investment level nor coordinate the supply chain. Likely, Kabul and Parlaktrk [18] showed that price or quantity commitments can harm not only a firm itself but also the profitability of other firms in the supply chain and the entire supply chain because such commitments aggravate double-marginalization inefficiency in the supply chain. We also found that a price commitment contract will seriously damage an enterprise's profits and cannot make supply chain achieve Pareto improvement. Thus, we considered a revenue-sharing contract. Revenue-sharing contracts have gained considerable popularity over the past two decades and have attracted great research attention in the domain of supply chain management [4]. Giri *et al.* [15] indicated that a contingent buyback with target sales rebate and penalty between the retailer and the manufacturer, and a revenue sharing contract between the manufacturer and the supplier can coordinate supply chain. Moon *et al.* [25] proposed revenue sharing coupled with investment cost sharing to coordinate and achieve a win-win outcome and maintain fairness for each member. Ma *et al.* [24] studied the coordination problem of a three-echelon fresh agricultural product supply chain, cost and revenue sharing contracts are designed for the two transaction processes to achieve Pareto improvement. Chen [10] compared bank and trade credit under a classical channel contract, revenue-sharing contract. Unlike existing studies, we compare bank and government-enterprise guarantee financing models under a price commitment or revenue-sharing contract. We also analyze the conditions for full coordination of the supply chain, the region of Pareto development, and the financing models' selection preferences for farmers and the enterprises under different contracts.

TABLE 1. Comparisons of our paper and the related literature.

Researches	Yield uncertainty	Agricultural supply chain	Coordination mechanism	Traditional bank financing	PCG
Chen [10]			√	√	
Yan *et al.* [38]			√	√	√
Ye *et al.* [40]	√	√	√		
Ma *et al.* [24]		√	√		
Ding and Wan [14]	√		√	√	
Xu *et al.* [37]				√	√
Yan *et al.* [39]		√		√	
Yuan *et al.* [42]	√			√	
Yi *et al.* [41]		√		√	√
This study	√	√	√	√	√

We summarize the research gap between our research and the closely related literature and further highlight our contributions. Table 1 provides a summary. To the best of our knowledge, this is the first research to incorporate simultaneously yield uncertainty, government-enterprise guarantee financing (a type of PCG), and coordination mechanisms into the ASCF. We intend to examine how the supply chain members' cooperative mechanisms and financing models affect their strategies and profits in an agricultural supply chain. In addition, in accordance with whether a scenario with bankrupt risk for the farmer exists, we compare traditional bank and government-enterprise guarantee financing models under a price commitment or revenue-sharing contract. The capital-constrained farmer obtains funds by choosing a suitable financing model, and the enterprise signs appropriate contracts with the farmer to achieve full coordination and Pareto improvement of the entire supply chain. Additionally, we model that the wholesale price is determined endogenously by the enterprise. Several studies propose that wholesale prices are determined through negotiations between both parties [2, 26, 45]. Meanwhile, some argue that wholesale prices are exogenous parameters [23]. This is also a point of differentiation in our analysis.

3. Agricultural supply chain financing model

A capital-constrained farmer who plants one kind of agricultural product is considered. The farmer has no direct access to the consumer market and relies on one enterprise to distribute his products. In this study, we consider this "enterprise + farmer" contract-farming supply chain with one farmer and one enterprise. Owing to the particularity of agricultural products, the farmer's output is susceptible to natural disasters and abnormal weather, leading to high uncertainty on yield [30]. We assume that the planting seasons of agricultural products are divided into "high-yield season" and "low-yield season". Without loss of generality, we assume that the probability of a high-yield season is k, the probability of a low-yield season is $1-k$, and $k \in (0,1)$. In the high-yield season, the farmer can obtain a higher input-output rate of agricultural products, denoted as x, in the low-yield season, the input-output rate is θx, θ is the low-yield discount factor and $\theta \in (0,1)$. Before the planting season, the farmer should decide his planting input Q, such as planting acreage and culture area. The output q in the end of the season is Qx or $Q\theta x$. The enterprise purchases all agricultural products produced by the farmer with a wholesale price w and sells it to the end market with a price $p = a - bq$ [6]. Furthermore, we suppose that all selling price is greater than the wholesale price, *i.e.*, $a - bQx > w$. We denote $\mu = kx + (1-k)\theta x$ as the mathematical expectation of output rate and $\delta = kx^2 + (1-k)\theta^2 x^2$ as the second moment of the uncertain input-output rate.

Given scale diseconomy in agriculture and the limited production capacity of the farmer, when the production input increases, its marginal cost would increase [34]. Therefore, many studies regard agricultural production cost as a quadratic cost function $C(Q) = c_1 Q + cQ^2$ [3, 26], where c_1 represents the cost on planting one unit input, such as seeds and other production materials, and c is the coefficient of the effort cost of the farmer's planting. For ease of calculation and without loss of generality, we assume that the farmer's planting cost is $C(Q) = cQ^2$, *i.e.*, $c_1 = 0$, $c > 0$.

In this study, we suppose the farmer can borrow from a bank without guarantee or with government-enterprise guarantee. So there exist two kinds of financing models: a traditional bank financing model and a government-enterprise guarantee financing model. In the traditional bank financing model, the bank bears the financing risk and the farmer gets loans from the bank. We suppose that loan interest rate r_b is exogenous. In the government-enterprise guarantee financing model, the government and the enterprise jointly bear the financing risk, and the government guarantee fee rate is s, then the farmer gets loans from the bank. We suppose that loan interest rate r_g is also exogenous. If the farmer could not fully repay the bank, then the policy guarantee bears a proportion λ_g of the default, the enterprise bears a proportion λ_e, and the bank bears the remaining part $\lambda_0 = 1 - \lambda_g - \lambda_e$. To eliminate trivial cases, we assume that $\frac{1+r_g}{1-s} < 1 + r_b$. With some extended research, we find that the initial capital level does not affect the equilibrium results and has little impact on the selection of different financing models. So, following [17, 32, 41], we assume that the farmer has no initial capital.

Given yield uncertainty, in accordance with whether bankruptcy risk exists for the farmer, three scenarios in the end of the selling season are considered, where bankruptcy represents that the farmer could not fully repay the bank. First, the farmer has no bankruptcy, even if he encounters a low-yield season. Second, a bankruptcy risk occurs in a low-yield season, whereas no bankrupt risk occurs in a high-yield season. Third, the farmer will always go bankrupt, even in a high-yield season. Considering that engaging in the operation in the third scenario is not rational for the farmer, we will only assess the first two scenarios. We summarized all notations of the paper in Table A.1 in Appendix A.

Furthermore, to establish a performance benchmark, we first analyze the problem of a centralized supply chain, we assume that the enterprise, farmer, and bank belong to the same virtual economic entity [40]. The total cost of the supply chain is cQ^2 and the revenue is kp_1Q_0x or $(1-k)p_2Q_0\theta x$, where $p_1 = a - bQ_0x$ and $p_2 = a - bQ_0\theta x$. Accordingly, the total expected profit of the supply chain is

$$\begin{aligned}\Pi_0(Q_0) &= k(p_1Q_0x - cQ_0^2) + (1-k)(p_2Q_0\theta x - cQ_0^2)\\ &= -(b\delta + c)Q_0^2 + a\mu Q_0.\end{aligned}$$

$\Pi_0(Q_0)$ is strictly concave in Q_0. Thus, the optimal production input is $Q_0^* = \frac{a\mu}{2(c+b\delta)}$, and the total expected profit is $\Pi_0^* = \frac{a^2\mu^2}{4(c+b\delta)}$.

4. Financing and coordination strategies in traditional bank financing

4.1. Traditional bank financing model

In the traditional bank financing model, first, the core enterprise decides the wholesale price w and offers a purchase contract to the farmer. Second, the farmer decides his production input Q and acquires a loan cQ^2 from the bank. Third, the farmer plants and delivers product to the enterprise. Lastly, the enterprise sells products to the market and pays the farmer. The farmer repays the loan and interest to the bank. If the farmer goes bankrupt, the bank will bear all loss.

In the following, we consider a decentralized supply chain. The farmer's revenues in high- and low-yield seasons are w_bQ_bx and $w_bQ_b\theta x$, respectively. The sum of loan and interest is $cQ_b^2(1+r_b)$. Thus, if the farmer will go bankrupt in the low-yield season and will not go bankrupt in the high-yield season, the farmer's production input Q_b must meet a condition: $w_bQ_b\theta x \le cQ_b^2(1+r_b) \le w_bQ_bx$. The farmer's expected profit is

$$\Pi_b(Q_b) = k[w_bQ_bx - cQ_b^2(1+r_b)].$$

We obtain the optimal production input $Q_b^* = w_b\eta_b$, where $\eta_b = \frac{1}{(1+r_b)c}\max(\frac{x}{2}, \theta x)$.

TABLE 2. Optimal values for Scenario 1.

$\Pi_{bp}^* = [\mu\eta_{bp} - c(1+r_b)\eta_{bp}^2]w_{bp}^2$
$\Theta_{bp}^* = a\mu\eta_{bp}w_{bp} - (\mu + b\delta\eta_{bp})\eta_{bp}w_{bp}^2$
$\Omega_{bp}^* = cr_b\eta_{bp}^2 w_{bp}^2$
$SW_{bp} = a\mu\eta_{bp}w_{bp} + (\frac{b\delta}{2} - c)\eta_{bp}^2 w_{bp}^2$

The enterprise's expected profit is

$$\begin{aligned}\Theta_b(Q_b) &= k(a - bQ_bx - w_b)Q_bx + (1-k)(a - bQ_b\theta x - w_b)Q_b\theta x\\ &= (a - w_b)\mu Q_b - b\delta Q_b^2.\end{aligned}$$

To be consistent with reality, the enterprise's decision on w_b must be lower than sales prices, *i.e.*, $a - bQ_bx > w_b$. Accordingly, we suppose that $k > \hat{k}$, where $\hat{k} = \frac{\theta[b\eta_b x(1-2\theta)-1]}{(1-\theta)[1+b\eta_b x(1+2\theta)]}$. Then, we obtain the optimal decisions in the decentralized supply chain.

Lemma 4.1. *In the decentralized supply chain with traditional bank financing model, the enterprise's optimal wholesale price is* $w_b^* = \frac{a\mu}{2(\mu+b\delta\eta_b)}$*, and the farmer's optimal production input is* $Q_b^* = \frac{a\mu\eta_b}{2(\mu+b\delta\eta_b)}$.

Correspondingly, the farmer's profit is $\Pi_b^* = k[x - c\eta_b(1+r_b)]\frac{a^2\mu^2\eta_b}{4(\mu+b\delta\eta_b)^2}$, the enterprise's profit is $\Theta_b^* = \frac{a^2\mu^2\eta_b}{4(\mu+b\delta\eta_b)}$, and the bank's profit is $\Omega_b^* = kcQ_b^2r_b + (1-k)(w_bQ_b\theta x - cQ_b^2) = [(kr_b+k-1)c\eta_b^2 + (1-k)\theta x\eta_b]\frac{a^2\mu^2}{4(\mu+b\delta\eta_b)^2}$. Consumers measure their interests by consumer surplus, which is equal to the difference between the highest willing price and the actual price paid. Consequently, $CS_b = E[\int_0^q (a - bq)dx] = E(\frac{bq^2}{2}) = \frac{b}{2}\delta Q_b^2$. Then, the social welfare is $SW_b = \Pi_b^* + \Theta_b^* + \Omega_b^* + CS_b = (kx + \theta x - k\theta x + \frac{1}{2}b\delta\eta_b - c\eta_b)\frac{a^2\mu^2\eta_b^2}{4(\mu+b\delta\eta_b)^2} + \frac{a^2\mu^2\eta_b}{4(\mu+b\delta\eta_b)}$.

Corollary 4.2. *In decentralized supply chain with traditional bank financing,* $\frac{dw_b^*}{dr_b} > 0$, $\frac{dQ_b^*}{dr_b} < 0$.

Corollary 4.2 indicates that when the bank's loan interest rate increases, the cost of the farmer will be increased and the production input will be reduced. At this time, the enterprise will increase the wholesale price to encourage the farmer to increase the production input.

4.2. Supply chain coordination with price commitment contract

4.2.1. Scenario 1: no bankrupt risk

If the farmer will not go bankrupt even in a low-yield season, then the farmer's production input Q_{bp} must satisfy a condition: $w_{bp}Q_{bp}\theta x \geq cQ_{bp}^2(1+r_b)$. The farmer's expected profit is $\Pi_{bp}(Q_{bp}) = k[w_{bp}Q_{bp}x - cQ_{bp}^2(1+r_b)] + (1-k)[w_{bp}Q_{bp}\theta x - cQ_{bp}^2(1+r_b)]$. We determine the optimal decision $Q_{bp}^* = w_{bp}\eta_{bp}$, where $\eta_{bp} = \frac{1}{(1+r_b)c}\min(\frac{\mu}{2}, \theta x)$. Q_{bp}^* can ensure that the farmer has no bankrupt risk. To ensure that $a - bQ_{bp}^*x > w_{bp}$, we suppose that $w_{bp} < \frac{a}{1+bx\eta_{bp}}$. In addition, $\Theta_{bp}(Q_{bp}) = (a - w_{bp})\mu Q_{bp} - b\delta Q_{bp}^2$, $\Omega_{bp} = kcQ_{bp}^2r_b + (1-k)(w_{bp}Q_{bp}\theta x - cQ_{bp}^2)$, and $SW_{bp} = \Pi_{bp}^* + \Theta_{bp}^* + \Omega_{bp}^* + CS_{bp}$. Therefore, we obtain the supply chain members' optimal expected profits, which are listed in Table 2. In the following, we consider the conditions of full coordination in the supply chain.

Proposition 4.3. *In a supply chain with traditional bank financing model in Scenario 1, for a price commitment contract, there are:*

1. *If* $k_1 < k < \frac{\theta}{1-\theta}$ *and* $w_{bp} = \frac{ac(1+r_b)}{b\delta+c}$*, then* $Q_{bp}^* = Q_0^*$.
2. *If* ($c < c_2$ *and* $k > \max(\frac{\theta}{1-\theta}, k_2)$) *or* ($c > c_2$ *and* $\frac{\theta}{1-\theta} < k \leq k_2$)*, and* $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$*, then* $Q_{bp}^* = Q_0^*$.

3. *In the following situations: (i)* $k < k_1$*; (ii)* $c < c_2$ *and* $k < \max(\frac{\theta}{1-\theta}, k_2)$*; (iii)* $c > c_2$ *and* $k < \frac{\theta}{1-\theta}$*; (iv)* $c > c_2$ *and* $k > k_2$*, for any* w_{bp}*, there is* $Q^*_{bp} < Q^*_0$*.*

Where $c_2 = \frac{(1+2\theta)\theta b x^2}{1+r_b}$, $k_1 = \frac{b\theta x^2+2cr_b-2b\theta^2x^2}{bx^2+b\theta x^2-2b\theta^2x^2}$*, and* $k_2 = \frac{\theta}{1-\theta} * \frac{c(r_b-1)+b\theta x^2-2b\theta^2x^2}{b\theta x^2+2b\theta^2x^2-c(1+r_b)}$*.*

Proposition 4.3 shows that in a supply chain with traditional bank financing model and no bankrupt risk, if the enterprise wants to maximize the total profit of the supply chain when he provides a price commitment contract to the farmer, then he needs to pay attention to the probability of a high-yield season k, the effort cost of the farmer's planting c, and the wholesale price w_{bp} simultaneously. Specifically, when k is low, the enterprise should provide a wholesale price $w_{bp} = \frac{ac(1+r_b)}{b\delta+c}$ to the farmer. When k is high, the enterprise needs to consider the size of c. At this time, the enterprise should provide a wholesale price $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$ to the farmer. In short, the enterprise should provide the farmer with different wholesale prices based on specific conditions, otherwise he will not be able to maximize supply chain profits.

From Proposition 4.3, two cases in which the supply chain can achieve full coordination exist. First, if $k_1 < k < \frac{\theta}{1-\theta}$ and $w_{bp} = \frac{ac(1+r_b)}{b\delta+c}$, then the supply chain can achieve full coordination. The farmer's profit is $\Pi^*_{bp} = \frac{a^2\mu^2c(1+r_b)}{4(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{bp} = (b\delta - 2cr_b)\frac{a^2\mu^2}{4(b\delta+c)^2}$, and the bank's profit is $\Omega^*_{bp} = \frac{a^2\mu^2cr_b}{4(b\delta+c)^2}$. Second, if $k \geq \frac{\theta}{1-\theta}$ and $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$, then the supply chain achieves full coordination. The farmer's profit is $\Pi^*_{bp} = \frac{a^2\mu^2c(\mu-\theta x)(1+r_b)}{4\theta x(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{bp} = \frac{a^2\mu^2[\theta x b\delta+2\theta x c-\mu c(1+r_b)]}{4\theta x(b\delta+c)^2}$, and the bank's profit is $\Omega^*_{bp} = \frac{a^2\mu^2cr_b}{4(b\delta+c)^2}$.

Corollary 4.4. *In a supply chain with traditional bank financing model in Scenario 1, for the price commitment contract, there are:*

(1) If $w_{bp} \geq \sqrt{\frac{a^2\mu^2\eta_b k[x-c\eta_b(1+r_b)]}{\eta_{bp}(2\mu+2b\delta\eta_b)^2[\mu-\min(\theta x,\frac{\mu}{2})]}}$*, then* $\Pi^*_{bp} \geq \Pi^*_b$*, otherwise,* $\Pi^*_{bp} < \Pi^*_b$*;*

(2) For any given w_{bp}*,* $\Theta^*_{bp} < \Theta^*_b$*.*

From Proposition 4.3 and Corollary 4.4, we find that, although the price commitment contract can achieve full supply chain coordination, it cannot make the profits both of the farmer and core enterprise be greater than the profits under a decentralized decision. If the promised price given by the enterprise is low, then the farmer will reduce the amount of production input. Although the sale price will increase with the decrease in the quantity of agricultural products, the decrease in the quantity of agricultural products will reduce the total income of the core enterprise. If the promised price given by the enterprise is high, then the farmer will increase the amount of production input, and the corresponding sale price will decrease. The increase in cost will reduce the total revenue of the core enterprise. Therefore, the profits of the farmer and the enterprise cannot be increased at the same time. In a supply chain with a traditional bank financing model and no bankruptcy risk, the price commitment contract can be advantageous to the farmer, but it is consistently detrimental to the enterprise. As a result, in practical scenarios, the enterprise cannot apply this contract unless they intend to provide support to the farmer.

4.2.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

If the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, then the production input Q_{bp} must meet a constraint: $w_{bp}Q_{bp}\theta x \leq cQ^2_{bp}(1+r_b) \leq w_{bp}Q_{bp}x$. His profit function is $\Pi_{bp}(Q_{bp}) = k[w_{bp}Q_{bp}x - cQ^2_{bp}(1+r_b)]$. $\Pi_{bp}(Q_{bp})$ is strictly concave in Q_{bp}. $Q^*_{bp} = w_{bp}\eta_b$ can ensure that the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season. To ensure that $a - bQ^*_{bp}x > w_{bp}$, we suppose that $w_{bp} < \frac{a}{1+bx\eta_b}$. In addition, $\Theta_{bp}(Q_{bp}) = (a - w_{bp})\mu Q_{bp} - b\delta Q^2_{bp}$, $\Omega_{bp} = kcQ^2_{bp}r_b + (1-k)(w_{bp}Q_{bp}\theta x - cQ^2_{bp})$, and $SW_{bp} = \Pi^*_{bp} + \Theta^*_{bp} + \Omega^*_{bp} + CS_{bp}$. Therefore, Table 3 lists the corresponding optimal expected profits in the supply chain.

Proposition 4.5. *In a supply chain with traditional bank financing model in Scenario 2, for a price commitment contract, there are:*

TABLE 3. Optimal values for Scenario 2.

$\Pi^*_{bp} = k[x - c(1 + r_b)\eta_b]\eta_b w^2_{bp}$
$\Theta^*_{bp} = a\mu\eta_b w_{bp} - (\mu\eta_b + b\delta\eta^2_b)w^2_{bp}$
$\Omega^*_{bp} = [kcr_b - (1 - k)c]\eta^2_b w^2_{bp} + (1 - k)\theta x\eta_b w^2_{bp}$
$SW_{bp} = a\mu\eta_b w_{bp} - (\frac{b\delta}{2} + c)\eta^2_b w^2_{bp}$

1. *Case $\theta > \frac{1}{2}$. If ($c < c_2$ and $k > k_2$) or ($c \geq c_2$ and $k \leq k_2$), and $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$, then $Q^*_{bp} = Q^*_0$; if ($c < c_2$ and $k \leq k_2$) or ($c \geq c_2$ and $k > k_2$), for any w_{bp}, then there is $Q^*_{bp} < Q^*_0$.*
2. *Case $\theta \leq \frac{1}{2}$. If ($c < c_3$ and $k > k_3$) or ($c > c_3$ and $k < k_3$), and $w_{bp} = \frac{a\mu c(1+r_b)}{x(b\delta+c)}$, then $Q^*_{bp} = Q^*_0$; if ($c < c_3$ and $k \leq k_3$) or ($c > c_3$ and $k \geq k_3$), for any w_{bp}, then there is $Q^*_{bp} < Q^*_0$.*

Where $c_3 = \frac{(1+2\theta)bx^2}{2(1+r_b)}$, $k_3 = \frac{2c\theta(r_b+1)+b\theta x^2-2b\theta^2x^2-2c}{(1-\theta)[bx^2+2b\theta x^2-2c(1+r_b)]}$.

Proposition 4.5 demonstrates that in a supply chain with traditional bank financing and bankruptcy risk, the enterprise must pay attention to the low-yield discount factor θ, the probability of a high-yield season k, the coefficient of the farmer's planting effort cost c, and the wholesale price w_{bp} simultaneously. This is essential if the enterprise wants to maximize the total profit of the supply chain by providing a price commitment contract to the farmer. Specifically, when θ is high, the enterprise should provide a wholesale price $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$ to the farmer. Conversely, when θ is low, the enterprise should provide a wholesale price $w_{bp} = \frac{a\mu c(1+r_b)}{x(b\delta+c)}$ to the farmer. At this juncture, the enterprise must also consider the magnitude of c and k. Unlike Proposition 4.3, when the supply chain entails the risk of bankruptcy, it is necessary to take into account the level of the low-yield discount factor.

From Proposition 4.5, in this scenario, two cases in which the supply chain can achieve full coordination exist. First, if $\theta > \frac{1}{2}$ and the supply chain achieves full coordination, the farmer's profit is $\Pi^*_{bp} = \frac{a^2\mu^2ckx(1+r_b)(1-\theta)}{4\theta x(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{bp} = \frac{a^2\mu^2[\theta xb\delta+2\theta xc-\mu c(1+r_b)]}{4\theta x(b\delta+c)^2}$, and the bank's profit is $\Omega^*_{bp} = \frac{a^2\mu^2cr_b}{4(b\delta+c)^2}$. Second, if $\theta \leq \frac{1}{2}$ and the supply chain achieves full coordination, the farmer's profit is $\Pi^*_{bp} = \frac{a^2\mu^2ck(1+r_b)}{4(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{bp} = \frac{a^2\mu^2[xb\delta+2xc-2\mu c(1+r_b)]}{4x(b\delta+c)^2}$, and the bank's profit is $\Omega^*_{bp} = \frac{a^2\mu^2[kcr_b+c(1-k)(2\theta+2\theta r_b-1)]}{4(b\delta+c)^2}$.

Corollary 4.6. *In a supply chain with traditional bank financing model in Scenario 2, for the price commitment contract, there are:*
(1) If $w_{bp} \geq \frac{a\mu}{2\mu+2b\delta\eta_b}$, then $\Pi^*_{bp} \geq \Pi^*_b$, otherwise, $\Pi^*_{bp} < \Pi^*_b$;
(2) For any given w_{bp}, $\Theta^*_{bp} < \Theta^*_b$.

From Corollary 4.6, in this scenario, we also cannot use a price commitment contract to achieve Pareto improvement. The reason is similar to that in Scenario 1, the profits of the farmer and enterprise cannot be increased at the same time. Therefore, combining Corollary 4.4, we find that in reality, regardless of whether the supply chain has bankruptcy risk, the enterprise cannot provide a price commitment contract to the farmer.

4.3. Supply chain coordination with revenue-sharing contract

In a revenue-sharing contract, the enterprise and the farmer share the revenue pQx (or $pQ\theta x$) from sales. The enterprise retains a ratio φ, and the farmer retains a ratio $\bar{\varphi} = 1 - \varphi$. The following is a study on whether a revenue-sharing contract can achieve supply chain coordination, and the benefits of all parties are compared under the traditional wholesale price contract to find the range of Pareto improvement.

TABLE 4. Optimal values for scenario 1.

$$\begin{aligned}
&\Pi_{br}^* = (w_{br} + a\bar{\varphi})^2\gamma_1[\mu - (1-\varphi)b\delta\gamma_1 - c\gamma_1(1+r_b)] \\
&\Theta_{br}^* = -b\varphi\delta(w_{br} + a\bar{\varphi})^2\gamma_1^2 + (\varphi a - w_{br})(w_{br} + a\bar{\varphi})\mu\gamma_1 \\
&\Omega_{br}^* = cr_b\gamma_1^2(w_{br} + a\bar{\varphi})^2 \\
&CS_{br} = E[\textstyle\int_0^q (a - bq)dx] \\
&\quad\;\; = \tfrac{1}{2}b\delta\gamma_1^2(w_{br} + a\bar{\varphi})^2 \\
&SW_{br} = (w_{br} + a\bar{\varphi})^2\gamma_1(\mu - \tfrac{1}{2}b\delta\gamma_1 - c\gamma_1) + (\varphi a - w_{br})(w_{br} + a\bar{\varphi})\mu\gamma_1
\end{aligned}$$

4.3.1. Scenario 1: no bankrupt risk

In a revenue-sharing contract, the farmer's total revenues in high- and low-yield seasons are $w_{br}Q_{br}x + (1-\varphi)p_{br1}Q_{br}x$ and $w_{br}Q_{br}\theta x + (1-\varphi)p_{br2}Q_{br}\theta x$, respectively, where $p_{br1} = a - bQ_{br}x$ and $p_{br2} = a - bQ_{br}\theta x$. The sum of loan and interest is $cQ_{br}^2(1+r_b)$. Thus, if the farmer will not go bankrupt, then the farmer's production input Q_{br} must satisfy a condition: $w_{br}Q_{br}\theta x + (1-\varphi)p_{br2}Q_{br}\theta x \geq cQ_{br}^2(1+r_b)$. The farmer's expected profit is

$$\begin{aligned}
\Pi_{br}(Q_{br}) &= k[w_{br}Q_{br}x + (1-\varphi)p_{br1}Q_{br}x] + (1-k)[w_{br}Q_{br}\theta x + (1-\varphi)p_{br2}Q_{br}\theta x] - cQ_{br}^2(1+r_b) \\
&= -[(1-\varphi)b\delta + c(1+r_b)]Q_{br}^2 + w_{br}Q_{br}\mu + (1-\varphi)aQ_{br}\mu.
\end{aligned}$$

In the traditional bank financing model with revenue-sharing contract, $Q_{br}^* = (w_{br} + a\bar{\varphi})\gamma_1$ can ensure that the farmer has no bankrupt risk. To ensure that $a - bQ_{br}^*x > w_{br}$, we suppose that $w_{br} < \frac{a(1-bx\gamma_1\bar{\varphi})}{1+bx\gamma_1}$, where $\gamma_1 = \min(\frac{\mu}{2[b\delta\bar{\varphi}+c(1+r_b)]}, \frac{\theta x}{(1+r_b)c+\bar{\varphi}b\theta^2x^2})$. In addition, $\Theta_{br}(Q_{br}) = k(\varphi(a - bQ_{br}x) - w_{br})Q_{br}x + (1-k)(\varphi(a - bQ_{br}\theta x) - w_{br})Q_{br}\theta x$, $\Omega_{br} = kcQ_{br}^2r_b + (1-k)(w_{br}Q_{br}\theta x - cQ_{br}^2)$, and $SW_{br} = \Pi_{br}^* + \Theta_{br}^* + \Omega_{br}^* + CS_{br}$. Table 4 presents the members' optimal expected profits.

Proposition 4.7. *In a supply chain with traditional bank financing model in Scenario 1, for a revenue-sharing contract, there are:*

1. *If* $\bar{\varphi} < \nu_1$, $k > k_4$, *and* $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - a\bar{\varphi}$, *then* $Q_{br}^* = Q_0^*$;
2. *If* $\bar{\varphi} > \nu_1$ *(or* $\bar{\varphi} < \nu_1, k < k_4$*), and* $w_{br} = \frac{ac(\varphi+r_b)}{b\delta+c}$, *then* $Q_{br}^* = Q_0^*$.

Where $\nu_1 = \frac{c(1+r_b)}{b\theta x^2(2+\theta)}$, $k_4 = \frac{\theta}{1-\theta} * \frac{c(1+r_b)+b\theta^2x^2\bar{\varphi}}{c(1+r_b)-2b\theta x^2\bar{\varphi}-b\theta^2x^2\bar{\varphi}}$.

Proposition 4.7 illustrates that in a supply chain with a traditional bank financing model and no bankruptcy risk, in order to maximize total supply chain profit while providing a revenue-sharing contract to the farmer, the enterprise must consider both the probability of a high-yield season k and the farmer's revenue-sharing $\bar{\varphi}$. Specifically, when $\bar{\varphi}$ is low and k is high, the optimal wholesale price w_{br} for the farmer is given by $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - a\bar{\varphi}$. When either $\bar{\varphi}$ is high, or both $\bar{\varphi}$ and k are low, the optimal wholesale price is $w_{br} = \frac{ac(\varphi+r_b)}{b\delta+c}$. If these conditions are not met, the enterprise will not be able to maximize the supply chain's profit.

To explore the area of Pareto improvement, we conduct a numerical experiment. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.6$, $k = 0.7$, $r_b = 0.15$, $x_0 = 3$, and $\varphi = 0.8$. In the traditional bank financing model, numerical analysis compares the expected profits of the farmer or the enterprise in the revenue-sharing contract and decentralized supply chain. As shown in Figure 1, the expected profit of the farmer under the revenue-sharing contract is always greater than the expected profit under the decentralized decision and increases with the wholesale price. On the contrary, the expected profit of the core enterprise is always lower than the expected profit under the decentralized decision and decreases with the wholesale price. Consequently, finding the area of Pareto improvement in Scenario 1 is impossible.

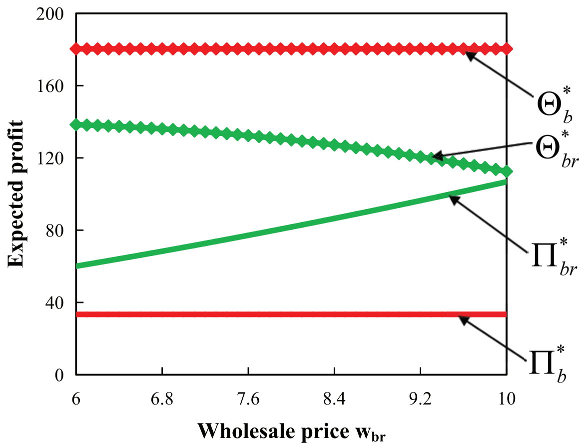

FIGURE 1. Pareto improvement analysis.

TABLE 5. Optimal values for scenario 2.

$\Pi^*_{br} = k\gamma_2(w_{br} + a\bar{\varphi})^2[x - (1-\varphi)b\gamma_2 x^2 - c\gamma_2(1+r_b)]$
$\Theta^*_{br} = -b\varphi\delta(w_{br} + a\bar{\varphi})^2\gamma_2^2 + (a\varphi - w_{br})(w_{br} + a\bar{\varphi})\mu\gamma_2$
$\Omega^*_{br} = \gamma_2(w_{br} + a\bar{\varphi})^2[ckr_b\gamma_2 + (1-k)\theta x - (1-k)(1-\varphi)b\theta^2 x^2\gamma_2 - (1-k)c\gamma_2]$
$CS_{br} = \frac{1}{2}b\delta\gamma_2^2(w_{br} + a\bar{\varphi})^2$
$SW_{br} = (w_{br} + a\bar{\varphi})^2\gamma_2(\mu - \frac{1}{2}b\delta\gamma_2 - c\gamma_2) + (\varphi a - w_{br})(w_{br} + a\bar{\varphi})\mu\gamma_2$

4.3.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

If the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, then the production input Q_{br} must meet a constraint: $w_{br}Q_{br}\theta x + (1-\varphi)p_{br2}Q_{br}\theta x \le cQ_{br}^2(1+r_b) \le w_{br}Q_{br}x + (1-\varphi)p_{br1}Q_{br}x$. His profit function is

$$\begin{aligned}\Pi_{br}(Q_{br}) &= k[w_{br}Q_{br}x + (1-\varphi)p_{br1}Q_{br}x - cQ_{br}^2(1+r_b)] \\ &= k[(a\bar{\varphi} + w_{br})Q_{br}x - ((1-\varphi)bx^2 + c(1+r_b))Q_{br}^2].\end{aligned}$$

In the traditional bank financing model, $Q^*_{br} = (w_{br} + a\bar{\varphi})\gamma_2$ can ensure that the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season. To ensure that $a - bQ^*_{br}x > w_{br}$, we suppose that $w_{br} < \frac{a(1-bx\gamma_2\bar{\varphi})}{1+bx\gamma_2}$, where $\gamma_2 = \max(\frac{x}{2[bx^2\bar{\varphi}+c(1+r_b)]}, \frac{\theta x}{(1+r_b)c+\bar{\varphi}b\theta^2x^2})$. In addition, $\Theta_{br}(Q_{br}) = k(\varphi(a - bQ_{br}x) - w_{br})Q_{br}x + (1-k)(\varphi(a - bQ_{br}\theta x) - w_{br})Q_{br}\theta x$, $\Omega_{br} = kcQ_{br}^2 r_b + (1-k)(w_{br}Q_{br}\theta x - cQ_{br}^2)$, and $SW_{br} = \Pi^*_{br} + \Theta^*_{br} + \Omega^*_{br} + CS_{br}$. Table 5 demonstrates the members' optimal expected profits.

Proposition 4.8. *In a supply chain with traditional bank financing model in Scenario 2, for a revenue-sharing contract, there are:*

1. *If* $\theta < \frac{1}{2}$, $\bar{\varphi} < \nu_2$, *and* $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}bx^2]}{x(b\delta+c)} - a\bar{\varphi}$, *then* $Q^*_{br} = Q^*_0$.
2. *If* $\theta > \frac{1}{2}$ *(or* $\theta < \frac{1}{2}$, $\bar{\varphi} > \nu_2$*), and* $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - a\bar{\varphi}$, *then* $Q^*_{br} = Q^*_0$.

Where $\nu_2 = \frac{c(1+r_b)(1-2\theta)}{b\theta x^2(2-\theta)}$.

Proposition 4.8 demonstrates that in a supply chain with a traditional bank financing model and bankruptcy risk, in order to maximize total supply chain profit while providing a revenue-sharing contract to the farmer, the

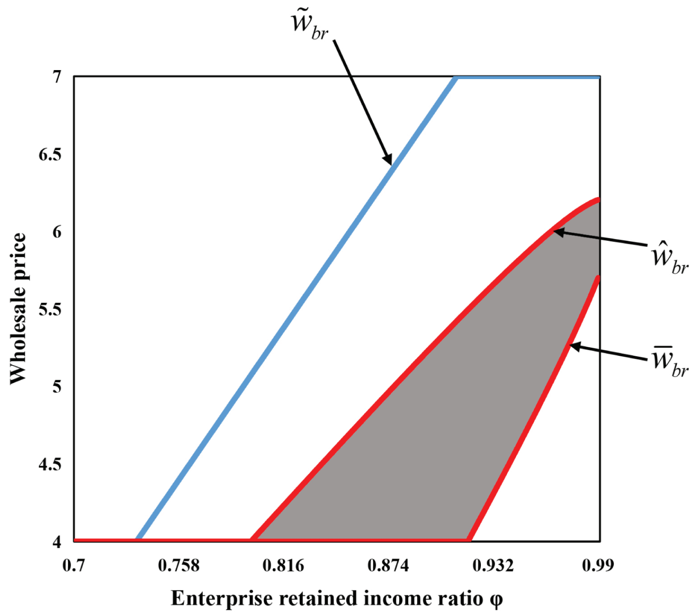

FIGURE 2. Pareto improvement analysis.

enterprise must consider both the low-yield discount factor θ and the farmer's revenue-sharing $\bar{\varphi}$. Specifically, when both θ and $\bar{\varphi}$ are low, the optimal wholesale price w_{br} for the farmer is given by $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}bx^2]}{x(b\delta+c)} - a\bar{\varphi}$. When θ is high or when θ is low and $\bar{\varphi}$ is high, the optimal wholesale price is $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - a\bar{\varphi}$. If these conditions are not met, the supply chain will not be able to maximize its profits. Combining Propositions 4.3, 4.5, and 4.7, we can conclude that both price commitment contracts and revenue-sharing contracts can achieve full coordination under certain conditions, regardless of whether the supply chain has bankruptcy risk.

To explore the area of Pareto improvement, we perform a numerical experiment. The parameters are set as follows: $c = 3$, $a = 20$, $b = 0.2$, $\theta = 0.5$, $k = 0.6$, $r_b = 0.15$, and $x_0 = 5$. In the numerical study, we compare the supply chain with a revenue-sharing contract and the decentralized supply chain. As shown in Figure 2, line $\widetilde{w}_{br}$ represents $\Pi^*_{br} + \Theta^*_{br} = \Pi^*_0$, the supply chain can achieve full coordination in this line; Lines $\overline{w}_{br}$ and $\widehat{w}_{br}$ represent $\Theta^*_{br} = \Theta^*_b$. When $\overline{w}_{br} \leq w_{br} \leq \widehat{w}_{br}$, the core enterprise can obtain more profits in the supply chain with a revenue-sharing contract. The shaded region indicates the area where Pareto improvement can be achieved. We can conclude that, when the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, the core enterprise can adjust the wholesale price and the revenue-sharing ratio to realize Pareto improvement of the supply chain. However, when the supply chain members achieve Pareto improvement, the supply chain cannot achieve full coordination.

5. FINANCING AND COORDINATION STRATEGIES IN GOVERNMENT-ENTERPRISE GUARANTEE FINANCING

5.1. Government-enterprise guarantee financing model

In government-enterprise guarantee financing model, first, the core enterprise decides the wholesale price w and offers a purchase contract to the farmer. Second, the farmer decides his production input Q and acquires a loan $\frac{cQ^2}{1-s}$ from the bank. Third, the farmer plants and delivers products to the enterprise. Lastly, the enterprise sells the products to the market and pays the farmer. The farmer repays the loan and interest to the bank. If the farmer goes bankrupt, the government, core enterprise, and bank will share the loss. In the following, we consider a decentralized supply chain.

5.1.1. Scenario 1: no bankrupt risk

The farmer's revenues in high- and low-yield seasons are w_gQ_gx and $w_gbQ_g\theta x$, respectively. The sum of loan and interest is $\frac{cQ_g^2(1+r_g)}{1-s}$. Thus, if the farmer will not go bankrupt, then the farmer's production input Q_g must satisfy a condition: $w_gQ_g\theta x \geq \frac{cQ_g^2(1+r_g)}{1-s}$. The farmer's expected profit is $\Pi_g(Q_g) = k[w_gQ_gx - \frac{cQ_g^2(1+r_g)}{1-s}] + (1-k)[w_gQ_g\theta x - \frac{cQ_g^2(1+r_g)}{1-s}]$, and the enterprise's expected profit is $\Theta_g(Q_g) = k(a - bQ_gx - w_g)Q_gx + (1-k)(a - bQ_g\theta x - w_g)Q_g\theta x = (a - w_g)\mu Q_g - b\delta Q_g^2$. Then, we determine the optimal decisions.

Lemma 5.1. *In the government-enterprise guarantee financing model, the enterprise's optimal wholesale price is* $w_g^* = \frac{a\mu}{2(\mu+b\delta\eta_{g1})}$, *the farmer's optimal production input is* $Q_g^* = \frac{a\mu\eta_{g1}}{2(\mu+b\delta\eta_{g1})}$, *where* $\eta_{g1} = \frac{1-s}{(1+r_g)c}\min(\frac{\mu}{2}, \theta x)$.

Correspondingly, the farmer's profit is $\Pi_g^* = \frac{a^2\mu^2}{4(\mu+b\delta\eta_{g1})^2}[\mu\eta_{g1} - \frac{c\eta_{g1}^2(1+r_g)}{1-s}]$, the enterprise's profit is $\Theta_g^* = \frac{a^2\mu^2\eta_{g1}}{4(\mu+b\delta\eta_{g1})}$, and the bank's profit is $\Omega_g^* = k\frac{cQ_g^2r_g}{1-s} + (1-k)\frac{cQ_g^2r_g}{1-s} = \frac{a^2\mu^2\eta_{g1}^2cr_g}{4(1-s)(\mu+b\delta\eta_{g1})^2}$, the consumer surplus is $CS_g = E[\int_0^q (a - bq)dx] = E(\frac{bq^2}{2}) = \frac{a^2\mu^2\eta_{g1}^2b\delta}{8(\mu+b\delta\eta_{g1})^2}$. Accordingly, the social welfare is $SW_g = \Pi_g^* + \Theta_g^* + \Omega_g^* + CS_g + \frac{scQ_g^2}{1-s} = \frac{a^2\mu^2\eta_{g1}}{4(\mu+b\delta\eta_{g1})^2}(-c\eta_{g1} + \frac{3}{2}b\delta\eta_{g1} + 2\mu)$.

Corollary 5.2. *In decentralized supply chain with government-enterprise guarantee financing, for Scenario 1,* $\frac{\partial w_g^*}{\partial r_g} > 0$, $\frac{\partial Q_g^*}{\partial r_g} < 0$; $\frac{\partial w_g^*}{\partial s} > 0$, $\frac{\partial Q_g^*}{\partial s} < 0$.

Corollary 5.2 indicates that when the guarantee fee ratio increases, the bank's loan interest rate will increase, then the cost of the farmer will be increased and the production input will be reduced. At this time, the enterprise will increase the wholesale price to encourage the farmer to increase the production input.

5.1.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

If the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, then the production input Q_g must meet a constraint: $w_gQ_g\theta x \leq \frac{cQ_g^2(1+r_g)}{1-s} \leq w_gQ_gx$. His profit function is $\Pi_g(Q_g) = k[w_gQ_gx - \frac{cQ_g^2(1+r_g)}{1-s}]$. The enterprise's revenues in high- and low-yield seasons are $(p_{g1} - w_g)Q_gx$ and $(p_{g2} - w_g)Q_g\theta x - \lambda_e[\frac{cQ_g^2(1+r_g)}{1-s} - w_gQ_g\theta x]$, respectively, where $p_{g1} = a - bQ_gx$ and $p_{g2} = a - bQ_gx$. Consequently, the enterprise's expected profit is $\Theta_g(Q_g) = k(a - bQ_gx - w_g)Q_gx + (1-k)[(a - bQ_g\theta x - w_g)Q_g\theta x - \lambda_e[\frac{cQ_g^2(1+r_g)}{1-s} - w_gQ_g\theta x]]$. Lemma 5.3 shows the optimal decisions.

Lemma 5.3. *In the government-enterprise guarantee financing model, the enterprise's optimal wholesale price is* $w_g^* = \frac{a\mu(1-s)}{2\zeta_g}$, *the farmer's optimal production input is* $Q_g^* = \frac{a\mu(1-s)\eta_{g2}}{2\zeta_g}$, *where* $\eta_{g2} = \frac{1-s}{(1+r_g)c}\max(\frac{x}{2}, \theta x)$, $\zeta_g = \eta_{g2}[(1-s)(\mu + b\delta\eta_{g2} - \lambda_e(1-k)\theta x) + c\lambda_e(1-k)(1+r_g)]$.

Correspondingly, the farmer's profit is $\Pi_g^* = \frac{a^2\mu^2k\eta_{g2}(1-s)[(1-s)x - c(1+r_g)\eta_{g2}]}{4\zeta_g^2}$, the enterprise's profit is $\Theta_g^* = \frac{a^2\mu^2\eta_{g2}(1-s)}{4\zeta_g}$, the bank's profit is $\Omega_g^* = \frac{cQ_g^2r_g}{1-s} - \lambda_0(1-k)(\frac{cQ_g^2(1+r_g)}{1-s} - w_gQ_g\theta x) = \frac{a^2\mu^2\eta_{g2}(1-s)}{4\zeta_g^2}[c\eta_{g2}(r_g - \lambda_0(1-k)(1+r_g)) + \lambda_0\theta x(1-k)(1-s)]$, the consumer surplus is $CS_g = E[\int_0^q (a - bq)dx] = E(\frac{bq^2}{2}) = \frac{a^2\mu^2\eta_{g2}^2b\delta(1-s)^2}{8\zeta_g^2}$, and the social welfare is $SW_g = \Pi_g^* + \Theta_g^* + \Omega_g^* + CS_g + \frac{scQ_g^2}{1-s} - (1-k)\lambda_g[\frac{cQ_g^2(1+r_g)}{1-s} - w_gQ_g\theta x] = \frac{a^2\mu^2\eta_{g2}(1-s)}{8\zeta_g^2}\{-2(1+r_g)c\eta_{g2}[k + (1-k)(\lambda_0 + \lambda_g)] + (1-s)[2xk + 2x\theta(1-k)(\lambda_0 + \lambda_g) + b\delta\eta_{g2}] + 2c\eta_{g2}r_g + 2\eta_{g2}sc + 2\zeta_g\}$.

TABLE 6. Optimal values for scenario 1.

$\Pi^*_{gp} = [\mu - \frac{c(1+r_g)\eta_{g1}}{1-s}]\eta_{g1} w^2_{gp}$
$\Theta^*_{gp} = a\mu\eta_{g1} w_{gp} - (\mu\eta_{g1} + b\delta\eta^2_{g1})w^2_{gp}$
$\Omega^*_{gp} = \frac{cr_g\eta^2_{g1}w^2_{gp}}{1-s}$
$SW_{gp} = a\mu\eta_{g1}.w_{gp} - (\frac{b\delta}{2} + c)\eta^2_{g1}w^2_{gp}$

5.2. Supply chain coordination with price commitment contract

5.2.1. Scenario 1: no bankrupt risk

If the farmer will not go bankrupt, then the farmer's production input Q_{gp} must satisfy a condition: $w_{gp}Q_{gp}\theta x \geq \frac{cQ^2_{gp}(1+r_g)}{1-s}$. The farmer's expected profit is $\Pi_{gp}(Q_{gp}) = k[w_{gp}Q_{gp}x - \frac{cQ^2_{gp}(1+r_g)}{1-s}] + (1-k)[w_{gp}Q_{gp}\theta x - \frac{cQ^2_{gp}(1+r_g)}{1-s}]$. $Q^*_{gp} = w_{gp}\eta_{g1}$ can ensure that the farmer has no bankrupt risk. To ensure that $a - bQ^*_{gp}x > w_{gp}$, we suppose that $w_{gp} < \frac{a}{1+bx\eta_{g1}}$. In addition, $\Theta_{gp}(Q_{gp}) = k(a - bQ_{gp}x - w_{gp})Q_{gp}x + (1-k)(a - bQ_{gp}\theta x - w_{gp})Q_{gp}\theta x$, $\Omega_{gp} = k\frac{cQ^2_{gp}r_g}{1-s} + (1-k)\frac{cQ^2_{gp}r_g}{1-s}$, and $SW_{gp} = \Pi^*_{gp} + \Theta^*_{gp} + \Omega^*_{gp} + CS_{gp} + \frac{scQ^2_{gp}}{1-s}$. Table 6 presents the members' optimal expected profits.

Proposition 5.4. *In a supply chain with government-enterprise guarantee financing model in Scenario 1, for a price commitment contract, there are:*

1. *If* $k_5 < k < \frac{\theta}{1-\theta}$ *and* $w_{gp} = \frac{ac(1+r_g)}{(b\delta+c)(1-s)}$, *then* $Q^*_{gp} = Q^*_0$.
2. *If (* $c > c_4$ *and* $k > \max(\frac{\theta}{1-\theta}, k_6)$ *) or (* $c < c_4$ *and* $\frac{\theta}{1-\theta} < k \leq k_6$ *), and* $w_{gp} = \frac{a\mu c(1+r_g)}{2\theta x(b\delta+c)(1-s)}$, *then* $Q^*_{gp} = Q^*_0$.
3. *In the following situations, for any* w_{gp}, *there is* $Q^*_{gp} < Q^*_0$*: (i)* $c > c_4$ *and* $k < \max(\frac{\theta}{1-\theta}, k_6)$*; (ii)* $c < c_4$ *and* $k < \frac{\theta}{1-\theta}$*; (iii)* $c < c_4$ *and* $k > k_6$.

Where $c_4 = \frac{(1+2\theta)(1-s)b\theta x^2}{1+r_g}$, $k_5 = \frac{2c(r_g+s)+\theta bx^2(1-2\theta)(1-s)}{(1+2\theta)(1-\theta)(1-s)bx^2}$, $k_6 = \frac{\theta}{1-\theta} * \frac{c(r_g+2s-1)+b\theta x^2(1-s)(1-2\theta)}{b\theta x^2(1-s)(1+2\theta)-c(r_g+1)}$.

Proposition 5.4 states that in a supply chain with a government-enterprise guarantee financing model and no bankruptcy risk, in order to maximize total supply chain profit while providing a price commitment contract to the farmer, the enterprise must consider the probability of a high-yield season k, the coefficient of the farmer's effort cost of planting c, and the wholesale price w_{gp} simultaneously. Specifically, when k is low, the optimal wholesale price w_{gp} for the farmer is given by $w_{gp} = \frac{ac(1+r_g)}{(b\delta+c)(1-s)}$. When k is high, the optimal wholesale price is $w_{gp} = \frac{a\mu c(1+r_g)}{2\theta x(b\delta+c)(1-s)}$. If these conditions are not met, the supply chain will not be able to maximize its profits. This is similar to Proposition 4.3.

From Proposition 5.4, two cases in which the supply chain can achieve full coordination exist. First, if $k_5 < k < \frac{\theta}{1-\theta}$ and $w_{gp} = \frac{ac(1+r_g)}{(b\delta+c)(1-s)}$, the supply chain can achieve full coordination. The farmer's profit is $\Pi^*_{gp} = \frac{a^2\mu^2 c(1+r_g)}{4(1-s)(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{gp} = \frac{a^2\mu^2[b\delta(1-s)-2c(s+r_g)]}{4(1-s)(b\delta+c)^2}$, the bank's profit is $\Omega^*_{gp} = \frac{a^2\mu^2 cr_g}{4(1-s)(b\delta+c)^2}$, and the total revenue of the government is $\Psi_{gp} = \frac{a^2\mu^2 sc}{4(1-s)(b\delta+c)^2}$. Second, if $k \geq \frac{\theta}{1-\theta}$ and $w_{gp} = \frac{a\mu c(1+r_g)}{2\theta x(b\delta+c)(1-s)}$, the supply chain can achieve full coordination. The farmer's profit is $\Pi^*_{gp} = \frac{a^2\mu^2 c(1+r_g)(\mu-\theta x)}{4\theta x(1-s)(b\delta+c)^2}$, the enterprise's profit is $\Theta^*_{gp} = \frac{a^2\mu^2[\theta x(b\delta+2c)(1-s)-\mu c(1+r_g)]}{4\theta x(1-s)(b\delta+c)^2}$, the bank's profit is $\Omega^*_{gp} = \frac{a^2\mu^2 cr_g}{4(1-s)(b\delta+c)^2}$, and the total revenue of the government is $\Psi_{gp} = \frac{a^2\mu^2 sc}{4(1-s)(b\delta+c)^2}$.

Corollary 5.5. *In a supply chain with government-enterprise guarantee financing model in Scenario 1, for the price commitment contract, there are:*

TABLE 7. Optimal values for scenario 2.

$\Pi^*_{gp} = [x - \frac{c(1+r_g)\eta_{g2}}{1-s}]k\eta_{g2}w^2_{gp}$
$\Theta^*_{gp} = a\mu\eta_{g2}w_{gp} - [\mu\eta_{g2} + b\delta\eta^2_{g2} + (1-k)\lambda_e(\frac{c(1+r_g)\eta^2_{g2}}{1-s} - \eta_{g2}\theta x)]w^2_{gp}$
$\Omega^*_{gp} = [r_g - \lambda_0(1-k)(1+r_g)]\frac{c\eta^2_{g2}w^2_{gp}}{1-s} + \lambda_0\eta_{g2}w^2_{gp}\theta x(1-k)$
$\Psi_{gp} = [s - \lambda_g(1-k)(1+r_g)]\frac{c\eta^2_{g2}w^2_{gp}}{1-s} + \lambda_g\eta_{g2}w^2_{gp}\theta x(1-k)$
$SW_{gp} = a\mu\eta_{g2}w_{gp} - (\frac{b\delta}{2} + c)\eta^2_{g2}w^2_{gp}$

(1) If $w_{gp} \geq \frac{a\mu}{2\mu+2b\delta\eta_{g1}}$, then $\Pi^*_{gp} \geq \Pi^*_g$, otherwise, $\Pi^*_{gp} < \Pi^*_g$;
(2) For any given w_{gp}, $\Theta^*_{gp} < \Theta^*_g$.

From Corollary 5.5, in the government-enterprise guarantee financing model, when the farmer has no bankrupt risk, the enterprise cannot adjust the promised price to make the profits of the farmer and core enterprise greater than the profits under a decentralized decision. The reason is similar to that in the traditional bank financing model. Government-enterprise guarantee financing only reduces the financing costs of the farmer, and still cannot change the profit loss of the core enterprise. Thus, we cannot use a price commitment contract to achieve Pareto improvement. This is similar to Corollary 4.4.

5.2.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

If the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, then the production input Q_{gp} must meet a constraint: $w_{gp}Q_{gp}\theta x \leq \frac{cQ^2_{gp}(1+r_g)}{1-s} \leq w_{gp}Q_{gp}x$. His profit function is $\Pi_{gp}(Q_{gp}) = k[w_{gp}Q_{gp}x - \frac{cQ^2_{gp}(1+r_g)}{1-s}]$. $\Pi_{gp}(Q_{gp})$ is strictly concave in Q_{gp}. Thus, we determine the farmer's optimal decision: $Q^*_{gp} = w_{gp}\eta_{g2}$ can ensure that the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season. To ensure that $a - bQ^*_{gp}x > w_{gp}$, we suppose that $w_{gp} < \frac{a}{1+bx\eta_{g2}}$. In addition, $\Theta_{gp}(Q_{gp}) = k(a - bQ_{gp}x - w_{gp})Q_{gp}x + (1-k)[(a - bQ_{gp}\theta x - w_{gp})Q_{gp}\theta x - \lambda_e[\frac{cQ^2_{gp}(1+r_g)}{1-s} - w_{gp}Q_{gp}\theta x]]$, $\Omega_{gp} = \frac{cQ^2_{gp}r_g}{1-s} - \lambda_0(1-k)(\frac{cQ^2_{gp}(1+r_g)}{1-s} - w_{gp}Q_{gp}\theta x)$, and $SW_{gp} = \Pi^*_{gp} + \Theta^*_{gp} + \Omega^*_{gp} + CS_{gp} + \frac{scQ^2_{gp}}{1-s} - (1-k)\lambda_g[\frac{cQ^2_{gp}(1+r_g)}{1-s} - w_{gp}Q_{gp}\theta x]$. Table 7 presents the members' optimal expected profits.

Proposition 5.6. *In a supply chain with government-enterprise guarantee financing model in Scenario 2, for a price commitment contract, there are:*

1. *Case $\theta > \frac{1}{2}$. If ($c > c_4$ and $k > \max(\frac{\theta}{1-\theta}, k_6)$) or ($c < c_4$ and $\frac{\theta}{1-\theta} < k \leq k_6$), and $w_{gp} = \frac{a\mu c(1+r_g)}{2\theta x(b\delta+c)(1-s)}$, then $Q^*_{gp} = Q^*_0$; if (i) $c > c_4$ and $k < \max(\frac{\theta}{1-\theta}, k_6)$; (ii)$c < c_4$ and $k < \frac{\theta}{1-\theta}$; (iii) $c < c_4$ and $k > k_6$, for any w_{gp}, then there is $Q^*_{gp} < Q^*_0$.*
2. *Case $\theta \leq \frac{1}{2}$. If ($c < c_5$ and $k > k_7$) or ($c > c_5$ and $k < k_7$), and $w_{gp} = \frac{a\mu c(1+r_g)}{x(b\delta+c)(1-s)}$, then $Q^*_{gp} = Q^*_0$; if ($c < c_5$ and $k < k_7$) or ($c > c_5$ and $k > k_7$), for any w_{gp}, then there is $Q^*_{gp} < Q^*_0$.*

Where $c_5 = \frac{(1+2\theta)(1-s)bx^2}{2c(1+r_g)}$, $k_7 = \frac{2c(1-s)+2\theta c(r_g+1)+b\theta x^2(1-s)(1-2\theta)}{(1-\theta)[bx^2(1-s)(1+2\theta)-2c(r_g+1)]}$.

Proposition 5.6 is similar to Proposition 4.5. When θ is high, the optimal wholesale price is $w_{gp} = \frac{a\mu c(1+r_g)}{2\theta x(b\delta+c)(1-s)}$. When θ is low, the optimal wholesale price is $w_{gp} = \frac{a\mu c(1+r_g)}{x(b\delta+c)(1-s)}$. It is important for the enterprise to choose the appropriate wholesale price based on the value of θ, c and k in order to maximize the total profit of the supply chain.

From Proposition 5.6, in this scenario, two cases in which the supply chain can achieve full coordination exist. First, if $\theta > \frac{1}{2}$ and the supply chain achieves full coordination, the farmer's profit is $\Pi^*_{gp} = ckx(1 +$

TABLE 8. Optimal values for scenario 1.

$\Pi_{gr}^* = (w_{gr} + a\bar{\varphi})^2\gamma_3(1-s)[\mu - \bar{\varphi}b\delta\gamma_3(1-s) - c\gamma_3(1+r_g)]$
$\Theta_{gr}^* = (w_{gr} + a\bar{\varphi})\gamma_3(1-s)[(a\varphi - w_{gr})\mu - b\delta\varphi\gamma_3(1-s)(w_{gr} + a\bar{\varphi})]$
$\Omega_{gr}^* = cr_g\gamma_3^2(1-s)(w_{gr} + a\bar{\varphi})^2$
$CS_{gr} = \frac{1}{2}b\delta\gamma_3^2(1-s)^2(w_{gr} + a\bar{\varphi})^2$
$\Psi_{gr} = sc(1-s)\gamma_3^2(w_{gr} + a\bar{\varphi})^2$
$SW_{gr} = (w_{gr} + a\bar{\varphi})\gamma_3(1-s)[a\mu - \gamma_3(\frac{1}{2}b\delta(1-s) + c - cs)(w_{gr} + a\bar{\varphi})]$

$r_g)(1-\theta)\frac{a^2\mu^2}{4\theta x(1-s)(b\delta+c)^2}$, the enterprise's profit is $\Theta_{gp}^* = \frac{a^2\mu^2[b\theta x\delta(1-s)+2\theta xc(1-s)-\mu c(1+r_g)]}{4\theta x(1-s)(b\delta+c)^2}$, the bank's profit is $\Omega_{gp}^* = \frac{a^2\mu^2 cr_g}{4(1-s)(b\delta+c)^2}$, and the total revenue of the government is $\Psi_{gp} = \frac{a^2\mu^2 sc}{4(1-s)(b\delta+c)^2}$. Second, if $\theta \le \frac{1}{2}$ and the supply chain achieves full coordination, the farmer's profit is $\Pi_{gp}^* = \frac{a^2\mu^2 ck(1+r_g)}{4(1-s)(b\delta+c)^2}$, the enterprise's profit is $\Theta_{gp}^* = \frac{a^2\mu^2[bx\delta(1-s)+2xc(1-s)-2\mu c(1+r_g)-cx\lambda_e(1-k)(1+r_g)(1-2\theta)]}{4x(1-s)(b\delta+c)^2}$, the bank's profit is $\Omega_{gp}^* = \frac{a^2\mu^2 c[r_g+\lambda_0(2\theta-1)(1+r_g)(1-k)]}{4(1-s)(b\delta+c)^2}$, and the total government expenditure is $\Psi_{gp} = \frac{a^2\mu^2 c[\lambda_g(1-2\theta)(1+r_g)(1-k)-s]}{4(1-s)(b\delta+c)^2}$.

Corollary 5.7. *In a supply chain with government-enterprise guarantee financing model in Scenario 2, for the price commitment contract, there are:*
(1) If $w_{gp} \ge \frac{a\mu(1-s)}{2\zeta_g}$*, then,* $\Pi_{gp}^* \ge \Pi_g^*$*, otherwise,* $\Pi_{gp}^* < \Pi_g^*$*;*
(2) For any given w_{gp}*,* $\Theta_{gp}^* < \Theta_g^*$*.*

From Corollary 5.7, in this scenario, we also cannot use a price commitment contract to achieve Pareto improvement. The reason is similar to that in Scenario 1, the profits of the farmer and enterprise cannot be increased at the same time. By combining Corollaries 4.4, 4.6, and 5.5, we conclude that in the real world, the enterprise cannot use a price commitment contract to incentivize the farmer, regardless of whether the supply chain has bankruptcy risk. This limitation applies to both the traditional bank financing model and the government-enterprise guarantee financing model.

5.3. Supply chain coordination with revenue-sharing contract

5.3.1. Scenario 1: no bankrupt risk

In a revenue-sharing contract, the farmer's revenues in high- and low-yield seasons are $w_{gr}Q_{gr}x + (1-\varphi)p_{gr1}Q_{gr}x$ and $w_{gr}Q_{gr}\theta x + (1-\varphi)p_{gr2}Q_{gr}\theta x$, respectively, where $p_{gr1} = a - bQ_{gr}x$ and $p_{gr2} = a - bQ_{gr}\theta x$. The sum of loan and interest is $\frac{cQ_{gr}^2(1+r_g)}{1-s}$. Thus, if the farmer will not go bankrupt, the farmer's production input Q_{gr} must satisfy a condition: $w_{gr}Q_{gr}\theta x + (1-\varphi)p_{gr2}Q_{gr}\theta x \ge \frac{cQ_{gr}^2(1+r_g)}{1-s}$. The farmer's expected profit is

$$\begin{aligned}\Pi_{gr}(Q_{gr}) &= k[w_{gr}Q_{gr}x + (1-\varphi)p_{gr1}Q_{gr}x] + (1-k)[w_{gr}Q_{gr}\theta x + (1-\varphi)p_{gr2}Q_{gr}\theta x] - \frac{cQ_{gr}^2(1+r_g)}{1-s}\\ &= -[\bar{\varphi}b\delta + \frac{c(1+r_g)}{1-s}]Q_{gr}^2 + (w_{gr} + \bar{\varphi}a)Q_{gr}\mu.\end{aligned}$$

We determine the optimal decision: $Q_{gr}^* = (w_{gr} + a\bar{\varphi})(1-s)\gamma_3$ can ensure that the farmer has no bankrupt risk. To ensure that $a - bQ_{gr}^*x > w_{gr}$, we suppose that $w_{gr} < \frac{a-bx\gamma_3 a\bar{\varphi}(1-s)}{1+bx\gamma_3(1-s)}$, where $\gamma_3 = \min(\frac{\mu}{2[b\delta\bar{\varphi}(1-s)+c(1+r_g)]}, \frac{\theta x}{(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2})$. In addition, $\Theta_{gr}(Q_{gr}) = k(\varphi(a-bQ_{gr}x) - w_{gr})Q_{gr}x + (1-k)(\varphi(a-bQ_{gr}\theta x) - w_{gr})Q_{gr}\theta x$, $\Omega_{gr} = k\frac{cQ_{gr}^2 r_g}{1-s} + (1-k)\frac{cQ_{gr}^2 r_g}{1-s}$, and $SW_{gr} = \Pi_{gr}^* + \Theta_{gr}^* + \Omega_{gr}^* + CS_{gr} + \frac{scQ_{gr}^2}{1-s}$. In Table 8, we indicate the members' optimal expected profits.

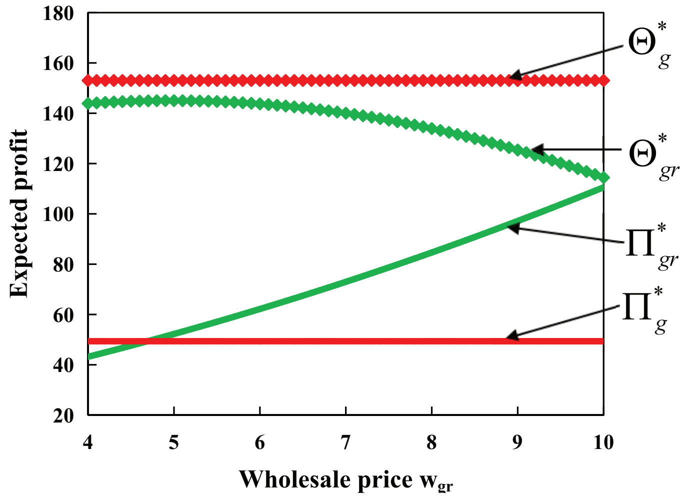

FIGURE 3. Pareto improvement analysis.

Proposition 5.8. *In a supply chain with government-enterprise guarantee financing model in Scenario 1, for a revenue-sharing contract, there are:*

1. *If* $c > c_6$, $k > k_8$, *and* $w_{gr} = \frac{a\mu[(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)(1-s)} - a\bar{\varphi}$, *then* $Q^*_{gr} = Q^*_0$.
2. *If (* $c > c_6$, $k < k_8$ *) or (* $c < c_6$ *), and* $w_{gr} = \frac{ac(1+r_g)-ac\bar{\varphi}(1-s)}{(1-s)(b\delta+c)}$, *then* $Q^*_{gr} = Q^*_0$.

Where $c_6 = \frac{(2+\theta)(1-s)\bar{\varphi}b\theta x^2}{1+r_g}$, $k_8 = \frac{\theta}{1-\theta} * \frac{c(r_g+1)+b\bar{\varphi}\theta^2x^2(1-s)}{c(r_g+1)-b\bar{\varphi}\theta x^2(1-s)(2+\theta)}$.

Proposition 5.8 demonstrates that in a supply chain with a government-enterprise guarantee financing model and no bankruptcy risk, the enterprise should set a wholesale price of $w_{gr} = \frac{a\mu[(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)(1-s)} - a\bar{\varphi}$ when both c and k are high. In cases where c is low or when c is high and k is low, the enterprise should provide a wholesale price of $w_{gr} = \frac{ac(1+r_g)-ac\bar{\varphi}(1-s)}{(1-s)(b\delta+c)}$ to the farmer. Failing to follow these pricing strategies will prevent the enterprise from maximizing supply chain profits.

To explore the area of Pareto improvement, we conduct a numerical experiment. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.6$, $k = 0.7$, $s = 0.02$, $r_g = 0.08$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, $\lambda_g = 0.5$, and $\varphi = 0.8$. The results are similar to those in the traditional bank financing model. As shown in Figure 3, in the government-enterprise guarantee financing model, the expected profit of the farmer under the revenue-sharing contract increases with the wholesale price and gradually exceeds the expected profit under a decentralized decision. On the contrary, the expected profit of the core enterprise is always lower than the expected profit under a decentralized decision. That is, the gap in the enterprise's profit is not as large as that under traditional bank financing, such that the area of Pareto improvement in Scenario 1 is also impossible to find.

5.3.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

If the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, then the production input Q_{gr} must meet a constraint: $w_{gr}Q_{gr}\theta x + (1-\varphi)p_{gr2}Q_{gr}\theta x \le \frac{cQ^2_{gr}(1+r_g)}{1-s} \le w_{gr}Q_{gr}x + (1-\varphi)p_{gr1}Q_{gr}x$. His profit function is

$$\begin{aligned}\Pi_{gr}(Q_{gr}) &= k[w_{gr}Q_{gr}x + (1-\varphi)p_{gr1}Q_{gr}x - \frac{cQ^2_{gr}(1+r_g)}{1-s}] \\ &= k\{[-\bar{\varphi}bx^2 - \frac{c(1+r_g)}{1-s}]Q^2_{gr} + (a\bar{\varphi} + w_{gr})Q_{gr}x\}.\end{aligned}$$

TABLE 9. Optimal values for scenario 2.

$$\begin{aligned}
\Pi^*_{gr} &= k\gamma_4(w_{gr}+a\bar{\varphi})^2(1-s)[x-\bar{\varphi}(1-s)b\gamma_4 x^2 - c\gamma_4(1+r_g)]\\
\Theta^*_{gr} &= \gamma_4(w_{gr}+a\bar{\varphi})(1-s)[(a\varphi - w_{gr})\mu - b\delta\varphi\gamma_4(1-s)(w_{gr}+a\bar{\varphi})]\\
&\quad +(1-k)\gamma_4(w_{gr}+a\bar{\varphi})^2(1-s)\lambda_e[\theta x-\bar{\varphi}(1-s)b\gamma_4\theta^2x^2 - c\gamma_4(1+r_g)]\\
\Omega^*_{gr} &= cr_g\gamma_4^2(w_{gr}+a\bar{\varphi})^2(1-s)\\
&\quad +(1-k)\gamma_4(w_{gr}+a\bar{\varphi})^2(1-s)\lambda_0[\theta x-\bar{\varphi}(1-s)b\gamma_4\theta^2x^2 - c\gamma_4(1+r_g)]\\
CS_{gr} &= \tfrac{1}{2}b\delta\gamma_4^2(w_{gr}+a\bar{\varphi})^2(1-s)^2\\
SW_{gr} &= \gamma_4(w_{gr}+a\bar{\varphi})^2(1-s)[\mu-\tfrac{1}{2}b\gamma_4\delta(1-s)-c\gamma_4(1-s)]\\
&\quad +\mu\gamma_4(w_{gr}+a\bar{\varphi})(1-s)(a\varphi-w_{gr})
\end{aligned}$$

We obtain the farmer's optimal decision: $Q^*_{gr}=(w_{gr}+a\bar{\varphi})(1-s)\gamma_4$ can ensure that the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season. To ensure that $a-bQ^*_{gr}x>w_{gr}$, we suppose $w_{gr}<\frac{a-bx\gamma_4 a\bar{\varphi}(1-s)}{1+bx\gamma_4(1-s)}$, where $\gamma_4=\max(\frac{x}{2[b\bar{\varphi}x^2(1-s)+c(1+r_g)]},\frac{\theta x}{(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2})$. In addition, $\Theta_{gr}(Q_{gr})=k(\varphi(a-bQ_{gr}x)-w_{gr})Q_{gr}x+(1-k)[(\varphi(a-bQ_{gr}\theta x)-w_{gr})Q_{gr}\theta x-\lambda_e[\frac{cQ^2_{gr}(1+r_g)}{1-s}-w_{gr}Q_{gr}\theta x-((1-\varphi)p_{gr1}-w_{gr})Q_{gr}\theta x]]$, $\Omega_{gr}=\frac{cQ^2_{gr}r_g}{1-s}-\lambda_0(1-k)[\frac{cQ^2_{gr}(1+r_g)}{1-s}-w_{gr}Q_{gr}\theta x-((1-\varphi)p_{gr1}-w_{gr})Q_{gr}\theta x]$, and $SW_{gr}=\Pi^*_{gr}+\Theta^*_{gr}+\Omega^*_{gr}+CS_{gr}+\frac{scQ^2_{gr}}{1-s}-(1-k)\lambda_g[\frac{cQ^2_{gr}(1+r_g)}{1-s}-w_{gr}Q_{gr}\theta x-((1-\varphi)p_{gr1}-w_{gr})Q_{gr}\theta x]$. In Table 9, we present the members' optimal expected profits.

Proposition 5.9. *In a supply chain with government-enterprise guarantee financing model in Scenario 2, for a revenue-sharing contract, there are:*

1. *If* $(\theta>\frac{1}{2})$ *or* $(\theta<\frac{1}{2}$ *and* $\bar{\varphi}>\frac{(1-2\theta)(1+r_g)c}{(2-\theta)(1-s)b\theta x^2})$, *and* $w_{gr}=\frac{a\mu[(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)(1-s)}-a\bar{\varphi}$, *then* $Q^*_{gr}=Q^*_0$.
2. *If* $\theta<\frac{1}{2}$, $\bar{\varphi}<\frac{(1-2\theta)(1+r_g)c}{(2-\theta)(1-s)b\theta x^2}$, *and* $w_{gr}=\frac{(1-s)a\mu bx^2\bar{\varphi}+a\mu c(1+r_g)}{x(1-s)(b\delta+c)}-a\bar{\varphi}$, *then* $Q^*_{gr}=Q^*_0$.

Proposition 5.9 demonstrates that in a supply chain with government-enterprise guarantee financing model and bankruptcy risk, the enterprise should offer a wholesale price of $w_{gr}=\frac{a\mu[(1+r_g)c+(1-s)\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)(1-s)}-a\bar{\varphi}$ to the farmer when θ is high or when θ is low and $\bar{\varphi}$ is high. When both θ and $\bar{\varphi}$ are low, the enterprise should provide a wholesale price of $w_{gr}=\frac{(1-s)a\mu bx^2\bar{\varphi}+a\mu c(1+r_g)}{x(1-s)(b\delta+c)}-a\bar{\varphi}$ to the farmer. Not following these pricing strategies will prevent the enterprise from maximizing supply chain profits. Combining Propositions 5.4, 5.6, and 5.8, it can be concluded that price commitment contracts and revenue sharing contracts can achieve full coordination under certain conditions in a government-enterprise guarantee financing model, regardless of whether the supply chain has bankruptcy risk or not.

To explore the area of Pareto improvement, we conduct a numerical simulation. The parameters are set as follows: $c=3$, $a=20$, $b=0.6$, $\theta=0.6$, $k=0.7$, $s=0.02$, $r_g=0.0584$, and $x_0=6$. As shown in Figure 4, in the government-enterprise guarantee financing model, line $\widetilde{w}_{gr}$ represents $\Pi^*_{gr}+\Theta^*_{gr}=\Pi^*_0$, the supply chain can achieve full coordination in this line; and lines $\overline{w}_{gr}$ and $\widehat{w}_{gr}$ represent $\Theta^*_{gr}=\Theta^*_g$. When $\overline{w}_{gr}\le w_{gr}\le\widehat{w}_{gr}$, the core enterprise can obtain more profits in the supply chain with a revenue-sharing contract. The shaded region indicates the area where Pareto improvement can be achieved. When the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, the wholesale price and revenue-sharing ratio can be adjusted to increase the profits of the farmer and core enterprise at the same time. Therefore, the revenue-sharing contract can achieve Pareto improvement.

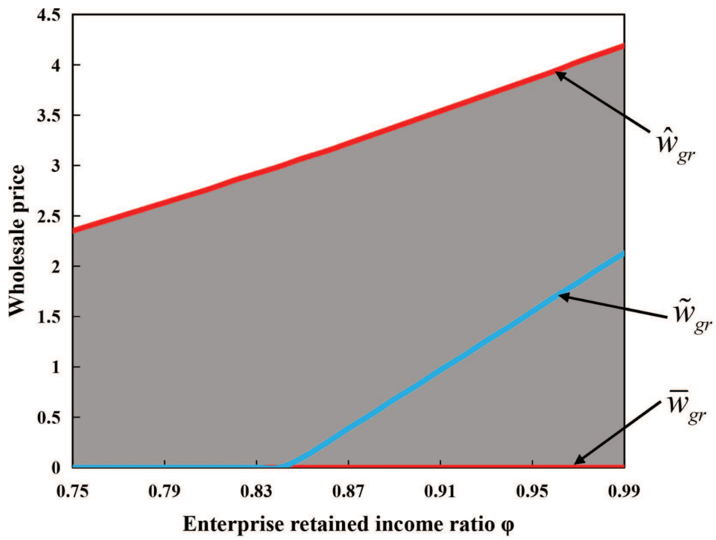

FIGURE 4. Pareto improvement analysis.

6. Financing models comparison

6.1. Financing models comparison under price commitment contract

6.1.1. Scenario 1: no bankrupt risk

Proposition 6.1. *Under the price commitment contract, in Scenario 1, there are:*

1. *If* $\frac{w_{gp}}{w_{bp}} > \frac{1+r_g}{(1-s)(1+r_b)}$*, then* $Q^*_{gp} > Q^*_{bp}$*; otherwise,* $Q^*_{gp} \leq Q^*_{bp}$.
2. *If* $\frac{w^2_{gp}}{w^2_{bp}} > \frac{1+r_g}{(1-s)(1+r_b)}$*, then* $\Pi^*_{gp} > \Pi^*_{bp}$*; otherwise,* $\Pi^*_{gp} \leq \Pi^*_{bp}$.
3. *If* $\frac{w^2_{gp}}{w^2_{bp}} > \frac{r_b(1+r_g)^2}{r_g(1-s)(1+r_b)^2}$*, then* $\Omega^*_{gp} > \Omega^*_{bp}$*; otherwise,* $\Omega^*_{gp} \leq \Omega^*_{bp}$.

Proposition 6.1(1) indicates that when the ratio of the promised price of government-enterprise guarantee financing to the promised price of traditional bank financing is greater than a threshold, the input production volume of the farmer under the government-enterprise guarantee mode is higher, otherwise, the input production volume of the farmer under the traditional bank financing mode is higher. Proposition 6.1(2) indicates that if the ratio squared of the promised price of government-enterprise guarantee financing to the promised price of traditional bank financing is greater than a threshold, then the expected profit of the farmer in the government-enterprise guarantee financing mode is higher, otherwise, it is less. Proposition 6.1(3) represents the difference between the promised price of government-enterprise guarantee financing and the promised price of traditional bank financing when the ratio is greater than a threshold, in which the bank's expected return under the government-enterprise guarantee financing mode is higher; otherwise, it is less.

To explore the selection of financing mode for a supply chain with a price commitment contract in Scenario 1, we conduct a numerical experiment. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.2$, $\theta = 0.8$, $k = 0.7$, $w_{bp} = 10$, $r_b = 0.15$, $r_g = 0.06$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, and $\lambda_g = 0.5$. In Figure 5, the commitment price ratio means that $\frac{w_{gp}}{w_{bp}}$; ρ_1 means that $\Pi_{bp} = \Pi_{gp}$; ρ_2 means that $\Theta_{bp} = \Theta_{gp}$. Region A means that the farmer tends to government-enterprise guarantee financing and the enterprise prefers traditional bank financing. Region B means that the farmer and enterprise prefer traditional bank financing. Region C means that the farmer tends to traditional bank financing and the enterprise prefers government-enterprise guarantee financing.

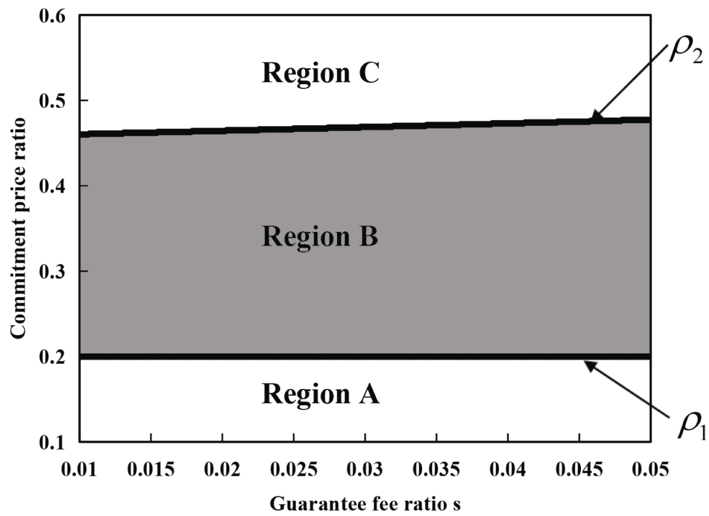

FIGURE 5. The selection of financing model under the price commitment contract in Scenario 1.

For the farmer, when the committed price of the core enterprise under traditional bank financing is higher, the input and production volume of the farmer in traditional bank financing are higher than those in government-enterprise guarantee financing. The income from agricultural products under traditional bank financing is higher, but the production and financing costs also increase rapidly, making the increase in costs greater than the increase in income. Therefore, the total income of the farmer under traditional bank financing is lower than that under government-enterprise guarantee financing, and the farmer will tend to government-enterprise guarantee financing. Otherwise, the farmer will tend to traditional bank financing. For the enterprise, although the market price of agricultural products is lower in the traditional bank financing model than that in government-enterprise guarantee financing, increasing the quantity of agricultural products makes the increase in revenues greater than the increase in costs, and the final revenue of the core enterprise under traditional bank financing is higher. Hence, the core enterprise tends to traditional bank financing. Otherwise, the core enterprise will prefer government-enterprise guarantee financing. Thus, we can find a win-win situation for the farmer and core enterprise.

6.1.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

Proposition 6.2. *Under the price commitment contract, in Scenario 2, there are*

1. *If* $\frac{w_{gp}}{w_{bp}} > \frac{1+r_g}{(1-s)(1+r_b)}$, *then* $Q^*_{gp} > Q^*_{bp}$, *otherwise,* $Q^*_{gp} \le Q^*_{bp}$.
2. *If* $\frac{w^2_{gp}}{w^2_{bp}} > \frac{1+r_g}{(1-s)(1+r_b)}$, *then* $\Pi^*_{gp} > \Pi^*_{bp}$, *otherwise,* $\Pi^*_{gp} \le \Pi^*_{bp}$.

The conclusion from Proposition 6.2 is the same as that from Proposition 5.6, which shows that changes in the farmer's scenarios have no effect on the threshold of the commitment price ratio.

Similar to Scenario 1, we perform a numerical simulation. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.8$, $k = 0.7$, $w_{bp} = 6$, $r_b = 0.15$, $r_g = 0.08$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, and $\lambda_g = 0.5$. In Figure 6, the commitment price ratio represents that $\frac{w_{gp}}{w_{bp}}$; ρ_4 means that $\Pi_{bp} = \Pi_{gp}$; ρ_3 and ρ_5 mean that $\Theta_{bp} = \Theta_{gp}$. Region A means that the farmer and enterprise prefer traditional bank financing. Region B means that the farmer tends to traditional bank financing and the enterprise prefers government-enterprise guarantee financing. Region C means that the farmer and enterprise tend to government-enterprise guarantee financing. Region D means that the farmer tends to government-enterprise guarantee financing and the enterprise prefers traditional bank financing.

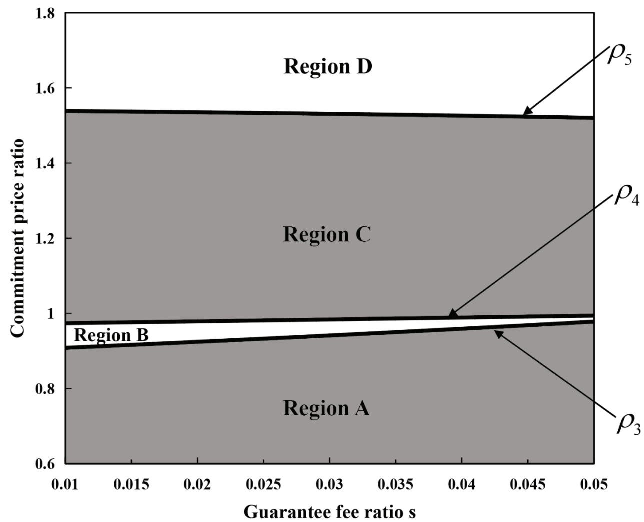

FIGURE 6. The selection of financing model under the price commitment contract in Scenario 2.

The analysis process is similar to that in Scenario 1. With the increase in the proportion of wholesale prices, the expected income of the farmer continues to increase, and the farmer will tend to government-enterprise guarantee financing. The enterprise will tend to government-enterprise guarantee financing in the area from ρ_3 to ρ_5. When the wholesale price ratio is excessively high, the procurement costs of the enterprise under the government-enterprise guarantee continue to increase; therefore, the enterprise will prefer traditional bank financing.

6.2. Financing models comparison under revenue-sharing contract

6.2.1. Scenario 1: no bankrupt risk

Proposition 6.3. *Under the revenue-sharing contract, in Scenario 1, if* $\frac{w_{br}\gamma_1+a\bar{\varphi}[\gamma_1-(1-s)\gamma_3]}{(1-s)\gamma_3} < w_{gr} < \frac{a-bx\gamma_3 a\bar{\varphi}(1-s)}{1+bx\gamma_3(1-s)}$*, then* $Q^*_{gr} > Q^*_{br}$*; if* $w_{gr} \le \frac{w_{br}\gamma_1+a\bar{\varphi}[\gamma_1-(1-s)\gamma_3]}{(1-s)\gamma_3}$*, then* $Q^*_{gr} \le Q^*_{br}$.

Proposition 6.3 shows that under the revenue-sharing contract and no bankruptcy risk, when the wholesale price given by the enterprise is relatively high in the government-enterprise guarantee financing, the farmer's enthusiasm for farming agricultural products will be stimulated, resulting in larger production inputs compared to traditional bank financing.

We perform a numerical simulation to explore the selection of financing mode for supply chain members under the revenue-sharing contract in Scenario 1. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.6$, $k = 0.6$, $w_{br} = 8$, $s = 0.02$, $r_b = 0.15$, $r_g = 0.1$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, and $\lambda_g = 0.5$. In Figure 7, the wholesale price ratio represents that $\frac{w_{gr}}{w_{br}}$; ρ_7 means that $\Pi_{br} = \Pi_{gr}$; ρ_6 and ρ_8 mean that $\Theta_{br} = \Theta_{gr}$. Region A means that the farmer and enterprise prefer traditional bank financing. Region B means that the farmer tends to traditional bank financing and the enterprise prefers government-enterprise guarantee financing. Region C means that the farmer and enterprise tend to government-enterprise guarantee financing. Region D means that the farmer tends to government-enterprise guarantee financing and the enterprise prefers traditional bank financing.

For the farmer, under the revenue-sharing contract, when the committed price of the enterprise under traditional bank financing is higher, the input and production volume of the farmer are higher than those under government-enterprise guarantee financing, and the farmer's income under traditional bank financing is higher,

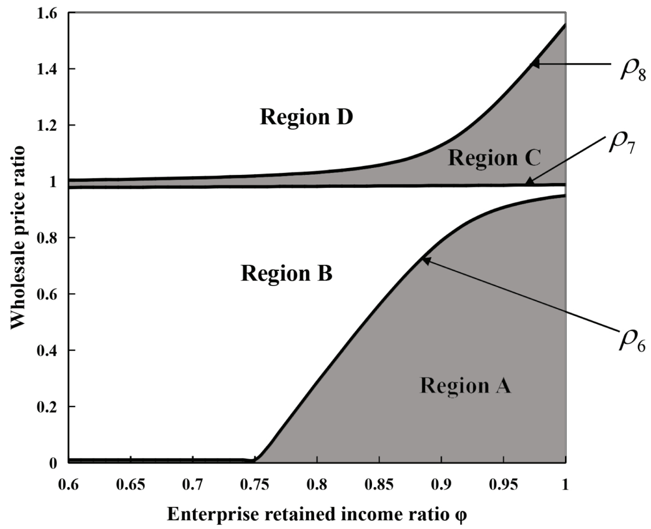

FIGURE 7. The selection of financing model under the revenue-sharing contract in Scenario 1.

thus, the farmer tends to traditional bank financing. Otherwise, the farmer tends to government-enterprise guarantee financing.

For the enterprise, under the revenue-sharing contract, the enterprise prefers government-enterprise guarantee financing in area $\rho_6 \leq \frac{w_{gr}}{w_{br}} \leq \rho_8$. When the promised price of the enterprise under the government-enterprise guarantee financing is low, *i.e.*, the wholesale price ratio is extremely low, and the proportion of retained earnings is high, the farmer will reduce the amount of production input due to the decrease in income. Although the sale price will rise due to the reduction in products, the reduced benefits of the enterprise are greater than the reduced costs. Consequently, the total income of the enterprise under government-enterprise guarantee financing is lower than that under traditional bank financing, and the enterprise prefers traditional bank financing. When the promised price under government-enterprise guarantee financing is high, *i.e.*, the wholesale price ratio is excessively high, and the proportion of retained earnings of the core enterprise is low, the farmer will continue to increase the amount of production input due to the increase in income, resulting in a decline in the sale price of agricultural products. The enterprise's increased revenues are less than the increased costs, which in turn makes the total income of the enterprise under the government-enterprise guarantee financing lower than that under traditional bank financing. Thus, the enterprise prefers traditional bank financing. Only when the promised price of the enterprise is set within a reasonable range and the corresponding revenue-sharing ratio is set under government-enterprise guarantee financing, this model can encourage the farmer to increase the production input reasonably and raise the total income of the enterprise.

6.2.2. Scenario 2: bankrupt in a low-yield season and not bankrupt in a high-yield season

Proposition 6.4. *Under the revenue-sharing contract, in Scenario 2, if* $\frac{w_{br}\gamma_2 + a\bar{\varphi}[\gamma_2 - (1-s)\gamma_4]}{(1-s)\gamma_4} < w_{gr} < \frac{a - bx\gamma_4 a\bar{\varphi}(1-s)}{1+bx\gamma_4(1-s)}$*, then* $Q^*_{gr} > Q^*_{br}$*; if* $w_{gr} \leq \frac{w_{br}\gamma_2 + a\bar{\varphi}[\gamma_2 - (1-s)\gamma_4]}{(1-s)\gamma_4}$*, then* $Q^*_{gr} \leq Q^*_{br}$.

This Proposition is similar to Proposition 6.3. The result demonstrates that in a revenue-sharing contract with bankruptcy risk and in a government-enterprise guarantee financing model, a high wholesale price offered by the enterprise will stimulate the farmer's enthusiasm for farming agricultural products, resulting in larger production inputs compared to traditional bank financing.

Similar to Scenario 1, we perform a numerical simulation. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.6$, $k = 0.6$, $w_{br} = 8$, $s = 0.02$, $r_b = 0.15$, $r_g = 0.08$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, and $\lambda_g = 0.5$.

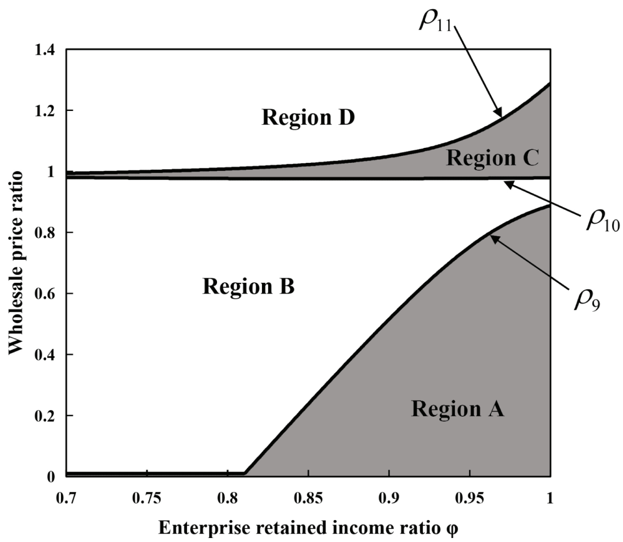

FIGURE 8. The selection of financing model under the revenue-sharing contract in Scenario 2.

In Figure 8, the wholesale price ratio represents that $\frac{w_{gr}}{w_{br}}$; ρ_{10} means that $\Pi_{br} = \Pi_{gr}$; ρ_9 and ρ_{11} mean that $\Theta_{br} = \Theta_{gr}$. Region A means that the farmer and enterprise prefer traditional bank financing. Region B means that the farmer tends to traditional bank financing and the enterprise prefers government-enterprise guarantee financing. Region C means that the farmer and enterprise tend to government-enterprise guarantee financing. Region D means that the farmer tends to government-enterprise guarantee financing and the enterprise prefers traditional bank financing.

The analysis process is similar to that in Scenario 1. With the increase in the proportion of wholesale prices, the farmer's expected income continues to increase, and the farmer will tend to government-enterprise guarantee financing. The enterprise will tend to government-enterprise guarantee financing in the area from ρ_9 to ρ_{11}. When the wholesale price ratio is exceedingly high, the procurement costs of the enterprise continue to increase under the government-enterprise guarantee financing model, hence, the enterprise will prefer traditional bank financing. When the wholesale price ratio is low and the proportion of the enterprise's retained earnings is high, the enterprise will prefer traditional bank financing.

To explore the changes in social welfare, we conduct a numerical simulation. The parameters are set as follows: $c = 3$, $a = 30$, $b = 0.5$, $\theta = 0.6$, $k = 0.7$, $s = 0.02$, $r_b = 0.15$, $r_g = 0.06$, $x_0 = 3$, $\lambda_e = 0.4$, $\lambda_0 = 0.1$, $\lambda_g = 0.5$, and $\varphi = 0.8$. As shown in Figure 9, under the revenue-sharing contract, when the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season, the social welfare will increase with the wholesale price under the two financing models. The social welfare is more sensitive to the change in wholesale price under the traditional bank financing model, which makes the social welfare gap smaller. Furthermore, the social welfare under the government-enterprise guarantee financing model is always greater than the social welfare under the traditional bank financing model.

7. CONCLUSIONS

Most of the farmers have insufficient funds. The lack of creditworthiness and valuable collateral means that the bank treats the farmer as a high-risk candidate for a loan. At the same time, affected by uncontrollable factors such as natural disasters, the farmer's production has yield uncertainty, which exacerbates the difficulty to obtain loans and the probability of default for the farmer. The capital constraints of agriculture not only prevent the farmer from normal planting but also considerably affect the supply of agricultural products and the

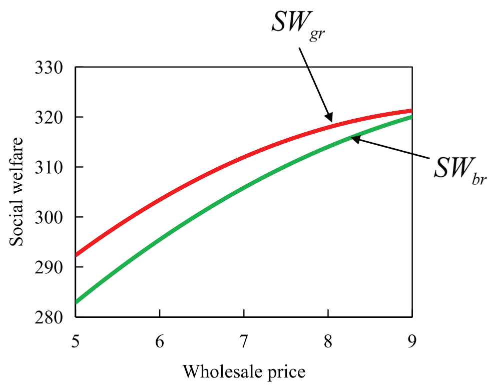

FIGURE 9. Comparison of the social welfare under the two models.

stability of the entire supply chain. Thus, we quantitatively investigate an agricultural supply chain consisting of a capital-constrained farmer and a core enterprise. To help the farmer solve the financing issues and improve the performance of the supply chain and increase the benefits of all parties, we not only compare traditional bank financing and government-enterprise guarantee financing models, but also compare the two commonly used contracts in contract farming: price commitment and revenue-sharing contracts. We summarized key findings as follows.

First, in the traditional bank financing model, the price commitment and revenue-sharing contract can achieve full coordination of the supply chain. Due to the serious reduction in the enterprise's revenue, the price commitment contract cannot make the supply chain achieve Pareto improvement. Although the revenue-sharing contract still cannot achieve Pareto improvement when the farmer has no default risk, it can achieve Pareto improvement when the farmer has default risk in a low-yield season. However, the increase in the profits of the enterprise and farmer cannot be achieved at the same time as the maximization of the total profits of the supply chain in this time. In addition, the production input decreases with the bank's loan interest rate and the wholesale price increases with the bank's loan interest rate.

Second, in the government-enterprise guarantee financing model, the price commitment and revenue-sharing contract can also achieve full coordination. The price commitment and revenue-sharing contracts also cannot achieve Pareto improvement in Scenario 1. Nonetheless, under certain conditions, the revenue-sharing contract can simultaneously achieve full coordination and Pareto improvement in Scenario 2, in this time, the social welfare in the government-enterprise guarantee financing is larger than that under the traditional bank financing. In addition, the production input decreases with guarantee fee ratio and the wholesale price increases with guarantee fee ratio.

Third, comparing traditional bank financing and government-enterprise guarantee financing models, under the price commitment contract, we find that in Scenario 1, when the promised price of traditional bank financing is higher, the farmer and enterprise prefer traditional bank financing. In Scenario 2, when the promised price of government-enterprise guarantees financing is higher, the farmer and enterprise prefer government-enterprise guarantee financing. Under the revenue-sharing contract, we find that in Scenarios 1 and 2, when the promised price and enterprise retains ratio of traditional bank financing are higher, the farmer and enterprise prefer traditional bank financing. When the promised price and enterprise retains ratio of government-enterprise guarantees financing are higher, the farmer and enterprise prefer government-enterprise guarantee financing. In a word, the farmer and enterprise prefer to choose financing models with higher promised prices and enterprise retains ratio, and the supply chain can achieve financing equilibrium.

The model designed in this paper provides some suggestions for the management of supply chains for ASCF. First, in the agricultural supply chain with a capital-constrained farmer, if the government actively provides financing guarantee services to the farmer, it cannot only increase the profits of the farmer and the enterprise, but also remarkably improve social welfare. Specifically, the production enthusiasm of the farmer increases with the government's guarantee ratio, which in turn increases the production input of agricultural products, but the guarantee fee ratio should be decreased. As a result, the enterprise obtains increased returns, and the entire society will benefit eventually. Second, for the enterprise, a price commitment contract should be carefully considered, given that it encourages the farmer to increase the amount of planting and improve his income, but it would reduce the enterprise's profit. On the contrary, the enterprise can choose a revenue-sharing contract and set a reasonable revenue retention ratio, such that all parties can benefit. Third, for the farmer, choosing the government-enterprise guarantee financing model can extremely free himself from capital troubles with the joint guarantee from the government and enterprise. Regardless of which contract the farmer chooses, the contract can improve the farmer's benefit. However, to make the supply chain operation stable, the farmer should prioritize the revenue-sharing contract.

Although the findings and managerial insights are important, there are still some limitations and valuable extensions should be addressed. First, this study only considers the situation of a strong market demand, *i.e.*, all agricultural products can be sold in the market. Nevertheless, the market demand for agricultural products is random, and an imbalance may exist between supply and demand in reality. Follow-up research can consider demand uncertainty. Second, this work only studies a two-echelon supply chain consisting of a farmer and an enterprise. In reality, an agricultural supply chain may include multiple farmers or multiple companies, which is also a valuable research direction. Additionally, a critical assumption in the current model is that the wholesale price is determined endogenously by the enterprise. In the future, it may be worthwhile to consider scenarios where the wholesale price is negotiated and dependent on the bargaining power of the enterprise and the farmer. Given that these explorations are beyond the scope of our research, we leave them to future research.

Appendix A. Notations

Table A.1. Notations and descriptions.

Decision variables	
Q	The production input of the farmer
w	The wholesale price
Parameters	
p	Market selling price
a	The suffocation price, which means that if the market selling price is higher than it, then consumers will not buy this agricultural product, $a > 0$.
b	The price sensitivity coefficient, indicating the sensitivity to market selling price, $b > 0$.
c	The coefficient of the effort cost of the farmer's planting
k	The probability of a high-yield season. The probability of a low-yield season is $1 - k(0 < k < 1)$.
x	The higher input-output rate of agricultural products. The input-output rate is $\theta x(0 < \theta < 1)$ in the low-yield season. The mean value is μ.
r_i	Loan interest rate, $i = b, g$
φ	Enterprise retains ratio, $\varphi \in [0, 1)$
s	Government-enterprise guarantee fee ratio, $s \in (0, 1)$
$\lambda_g \setminus \lambda_e \setminus \lambda_0$	Government \ enterprise \ bank bears a proportion of the default when the farmer could not fully repay the bank in government-enterprise guarantee financing model.
$\Pi \setminus \Theta \setminus \Omega \setminus \Psi$	The expected profits of the farmer \ enterprise \ bank \ government
SW	Social welfare
Superscript	
$*$	Optimal expected profit or decision
Subscript	
0	Centralized supply chain
$b \setminus g$	Traditional bank financing \ government-enterprise guarantee financing model
$bp \setminus br$	Traditional bank financing with price commitment \ revenue-sharing contract
$gp \setminus gr$	Government-enterprise guarantee financing with price commitment \ revenue-sharing contract

Appendix B. Proofs

Proof to Lemma 4.1. From $w_bQ_b\theta x \le cQ_b^2(1+r_b) \le w_bQ_bx$, we get $\frac{w_b\theta x}{c(1+r_b)} \le Q_b \le \frac{w_bx}{c(1+r_b)}$, at this time, the farmer has bankrupt risk in a low-yield season and no bankrupt risk in a high-yield season.

Obviously, $\frac{\partial^2\Pi_b}{\partial Q_b^2} < 0$, thus Π_b is strictly concave on Q_b. Let $\frac{\partial\Pi_b}{\partial Q_b} = 0$, we obtain that the optimal production input of farmer is $Q_b = \frac{w_bx}{2(1+r_b)c}$.

From $\frac{w_b\theta x}{c(1+r_b)} \le Q_b \le \frac{w_bx}{c(1+r_b)}$, we can rewrite the optimal Q_b^* as $w_b\eta_b$, where $\eta_b = \frac{1}{c(1+r_b)}\max(\frac{x}{2}, \theta x)$.

After substituting Q_b^* into the expected profit function of the enterprise, we get $\frac{\partial^2\Theta_b}{\partial w_b^2} < 0$, and the expected profit function of the enterprise is the quadratic function on w_b. So, let $\frac{\partial\Theta_b}{\partial w_b} = 0$, we obtain the optimal wholesale price $w_b^* = \frac{a\mu}{2\mu+2b\delta\eta_b}$.

Finally, we substitute w_b^* into Q_b^*, we obtain $Q_b^* = \frac{a\mu\eta_b}{2\mu+2b\delta\eta_b}$. In addition, from $w_b < p_1 = a - bQ_bx$, we get $k > \frac{\theta[b\eta_bx(1-2\theta)-1]}{(1-\theta)[1+b\eta_bx(1+2\theta)]}$. □

Proof to Corollary 4.2. From Lemma 4.1, we know that $w_b^* = \frac{a\mu}{2\mu+2b\delta\eta_b}$ and $Q_b^* = \frac{a\mu\eta_b}{2\mu+2b\delta\eta_b}$.

Then, we can obtain that $\frac{dw_b^*}{dr_b} = \frac{ab\mu\delta\max(\frac{x}{2},\theta x)}{2c(\mu+b\delta\eta_b)^2(1+r_b)^2} > 0$, $\frac{dQ_b^*}{dr_b} = -\frac{2ac\mu^2\max(\frac{x}{2},\theta x)}{[2c\mu(1+r_b)+2b\delta\max(\frac{x}{2},\theta x)]^2} < 0$. □

Proof to Proposition 4.3. In order to the supply chain can achieve full coordination, we need to get $Q_{bp}^* = Q_0^* = \frac{a\mu}{2(b\delta+c)}$, *i.e.*, $w_{bp} = \frac{a\mu}{2\eta_{bp}(b\delta+c)}$. According to the size of $\eta_{bp} = \frac{1}{c(1+r_b)}\min(\frac{\mu}{2}, \theta x)$, there exist two cases.

(1) If $\eta_{bp} = \frac{\mu}{2c(1+r_b)}$, then it shows that $\frac{\mu}{2} < \theta x$, where $\mu = kx + (1-k)\theta x$. From this, we get $k < \frac{\theta}{1-\theta}$. At this time, $w_{bp} = \frac{ac(1+r_b)}{b\delta+c}$.

From $w_{bp} < p_1 = a - bQ_{bp}^*x$, we need to get $w_{bp} < \frac{2ac(1+r_b)}{bx\mu+2(1+r_b)c}$. Thus, from this, we get $k > k_1 = \frac{b\theta x^2+2cr_b-2b\theta^2x^2}{bx^2+b\theta x^2-2b\theta^2x^2}$.

Therefore, when $k_1 < k < \frac{\theta}{1-\theta}$ and $w_{bp} = \frac{ac(1+r_b)}{b\delta+c}$, the supply chain can achieve full coordination, *i.e.*, $Q_{bp}^* = Q_0^*$.

(2) If $\eta_{bp} = \frac{\theta x}{c(1+r_b)}$, then it shows that $\frac{\mu}{2} \ge \theta x$, where $\mu = kx + (1-k)\theta x$. From this, we get $k \ge \frac{\theta}{1-\theta}$. At this time, $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$.

From $w_{bp} < p_1 = a - bQ_{bp}^*x$, we need to get $w_{bp} < \frac{ac(1+r_b)}{b\theta x^2+(1+r_b)c}$. Thus, from this, we get if $c < c_2$, then $k > \max(\frac{\theta}{1-\theta}, k_2)$; if $c > c_2$, then $\frac{\theta}{1-\theta} < k \le k_2$.

Therefore, when ($c < c_2$ and $k > \max(\frac{\theta}{1-\theta}, k_2)$) or ($c > c_2$ and $\frac{\theta}{1-\theta} < k \le k_2$), and $w_{bp} = \frac{a\mu c(1+r_b)}{2\theta x(b\delta+c)}$, the supply chain can achieve full coordination, *i.e.*, $Q_{bp}^* = Q_0^*$.

Otherwise, we get (3), *i.e.*, $Q_{bp}^* < Q_0^*$. □

Proof to Corollary 4.4. First, the difference of farmer's expected profit between in the price commitment contract and in the decentralized decision is

$\Delta\Pi_{bp} = \Pi_{bp}^* - \Pi_b^* = w_{bp}^2\eta_{bp}[\mu - c(1+r_b)\eta_{bp}] - k[x - c\eta_b(1+r_b)]\frac{a^2\mu^2\eta_b}{(2\mu+2b\delta\eta_b)^2}$.

From this, we obtain that: if $w_{bp} \ge \sqrt{\frac{a^2\mu^2\eta_bk[x-c\eta_b(1+r_b)]}{\eta_{bp}(2\mu+2b\delta\eta_b)^2[\mu-\min(\theta x,\frac{\mu}{2})]}}$, then, $\Delta\Pi_{bp} \ge 0$, otherwise, $\Delta\Pi_{bp} < 0$.

Second, the difference of enterprise's expected profit between in the price commitment contract and in the decentralized decision is

$\Delta\Theta_{bp} = \Theta_{bp}^* - \Theta_b^* = -(\mu + b\delta\eta_{bp})\eta_{bp}w_{bp}^2 + a\mu\eta_{bp}w_{bp} - \frac{a^2\mu^2\eta_b}{4(\mu+b\delta\eta_b)}$,

which is the quadratic function of w_{bp}.

From this, we know discriminant of quadratic form $\Delta\Theta_{bp}$ is $a^2\mu^3\eta_{bp}\frac{\eta_{bp}-\eta_b}{\mu+b\delta\eta_b} < 0$. Thus, for any w_{bp}, $\Delta\Theta_{bp} < 0$. □

Proof to Proposition 4.5. The proof process is similar to Proposition 4.3, so we omit it here. □

Proof to Corollary 4.6. First, the difference of the farmer's expected profit between in the price commitment contract and in the decentralized decision is

$\Delta\Pi_{bp} = \Pi_{bp}^* - \Pi_b^* = k\eta_b[x - \max(\frac{x}{2}, \theta x)][w_{bp}^2 - \frac{a^2\mu^2}{(2\mu+2b\delta\eta_b)^2}]$.

From this, we can obtain that: if $w_{bp} \geq \frac{a\mu}{2\mu+2b\delta\eta_b}$, then, $\Delta\Pi_{bp} \geq 0$, otherwise, $\Delta\Pi_{bp} < 0$.

Second, the difference of the enterprise's expected profit between in the price commitment contract and in the decentralized decision is

$\Delta\Theta_{bp} = \Theta_{bp}^* - \Theta_b^* = -(\mu + b\delta\eta_{bp})\eta_b w_{bp}^2 + a\mu\eta_b w_{bp} - \frac{a^2\mu^2\eta_b}{4(\mu+b\delta\eta_b)} = -(\mu + b\delta\eta_{bp})\eta_b(w_{bp} - \frac{a\mu}{2\mu+2b\delta\eta_b})^2$.

From this, we find that there is a unique solution $w_{bp} = \frac{a\mu}{2\mu+2b\delta\eta_b}$ that can meet the conditions of Pareto improvement. When $w_{bp} = \frac{a\mu}{2\mu+2b\delta\eta_b}$, the profits of both parties of the supply chain are the same as under decentralized decision. However, there is no $\Delta\Theta_{bp} > 0$. □

Proof to Proposition 4.7. In centralized decision, $Q_0^* = \frac{a\mu}{2(b\delta+c)}$. In the traditional bank financing model with revenue-sharing contract, $Q_{br}^* = (w_{br} + a\bar{\varphi})\gamma_1$. In order to the supply chain can achieve full coordination, we need to obtain $Q_{br}^* = Q_0^*$. Therefore, according to the size of γ_1, there exist two cases.

(1) When $\frac{\theta x}{\bar{\varphi}b\theta^2x^2+(1+r_b)c} < \frac{\mu}{2[b\delta\bar{\varphi}+(1+r_b)c]}$, $Q_{br}^* = \frac{(w_{br}+\bar{\varphi}a)\theta x}{\bar{\varphi}b\theta^2x^2+(1+r_b)c}$, then making $Q_{br}^* = Q_0^*$, we have that $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - \bar{\varphi}a$.

From $\frac{\theta x}{\bar{\varphi}b\theta^2x^2+(1+r_b)c} < \frac{\mu}{2[b\delta\bar{\varphi}+(1+r_b)c]}$, we get $\bar{\varphi} < \nu_1$ and $k > k_4$. So when $\bar{\varphi} < \nu_1$, $k > k_4$ and $w_{br} = \frac{a\mu[(1+r_b)c+\bar{\varphi}b\theta^2x^2]}{2\theta x(b\delta+c)} - \bar{\varphi}a$, the supply chain can achieve full coordination, *i.e.*, $Q_{br}^* = Q_0^*$.

(2) When $\frac{\theta x}{\bar{\varphi}b\theta^2x^2+(1+r_b)c} > \frac{\mu}{2[b\delta\bar{\varphi}+(1+r_b)c]}$, $Q_{br}^* = \frac{(w_{br}+\bar{\varphi}a)\mu}{2[b\delta\bar{\varphi}+(1+r_b)c]}$, then making $Q_{br}^* = Q_0^*$, we have that $w_{br} = \frac{ac(\varphi+r_b)}{b\delta+c}$.

From $\frac{\theta x}{\bar{\varphi}b\theta^2x^2+(1+r_b)c} > \frac{\mu}{2[b\delta\bar{\varphi}+(1+r_b)c]}$, we get ($\bar{\varphi} < \nu_1$ and $k < k_4$) or ($\bar{\varphi} > \nu_1$). So when $\bar{\varphi} > \nu_1$ (or $\bar{\varphi} < \nu_1, k < k_4$) and $w_{br} = \frac{ac(\varphi+r_b)}{b\delta+c}$, the supply chain can achieve full coordination, i,e., $Q_{br}^* = Q_0^*$.

In summary, we obtain this proposition. □

Proof to Proposition 4.8. The proof process is similar to Proposition 4.7, so we omit it here. □

Proof to Lemma 5.1. The proof process is similar to Lemma 4.1, so we omit it here. □

Proof to Corollary 5.2. The proof process is similar to Corollary 4.2, so we omit it here. □

Proof to Lemma 5.3. The proof process is similar to Lemma 4.1, so we omit it here. □

Proof to Proposition 5.4. The proof process is similar to Proposition 4.3, so we omit it here. □

Proof to Corollary 5.5. The proof process is similar to Corollary 4.6, so we omit it here. □

Proof to Corollary 5.7. The proof process is similar to Corollary 4.6, so we omit it here. □

Proof to Proposition 5.6. The proof process is similar to Proposition 4.3, so we omit it here. □

Proof to Proposition 5.8. The proof process is similar to Proposition 4.7, so we omit it here. □

Proof to Proposition 5.9. The proof process is similar to Proposition 4.7, so we omit it here. □

Proof to Proposition 6.1. Under the price commitment contract, in Scenario 1, there are:

(1) From $Q_{bp}^* = w_{bp}\eta_{bp}$ and $Q_{gp}^* = w_{gp}\eta_{g1} = \frac{w_{gp}\eta_{bp}(1-s)(1+r_b)}{1+r_g}$, we have that $\frac{Q_{gp}^*}{Q_{bp}^*} = \frac{w_{gp}(1-s)(1+r_b)}{w_{bp}(1+r_g)}$.

From this, we can know that when $\frac{w_{gp}(1-s)(1+r_b)}{w_{bp}(1+r_g)} > 1$, $Q_{gp}^* > Q_{bp}^*$, *i.e.*, when $\frac{w_{gp}}{w_{bp}} > \frac{1+r_g}{(1-s)(1+r_b)}$, $Q_{gp}^* > Q_{bp}^*$; otherwise, when $\frac{w_{gp}}{w_{bp}} \leq \frac{1+r_g}{(1-s)(1+r_b)}$, $Q_{gp}^* \leq Q_{bp}^*$.

(2) From $\Pi_{gp}^* = w_{gp}^2\eta_{g1}\mu - \frac{cw_{gp}^2\eta_{g1}^2(1+r_g)}{1-s}$ and $\Pi_{bp}^* = w_{bp}^2\eta_{bp}[\mu - c(1+r_b)\eta_{bp}]$, we have that $\Pi_{gp}^* - \Pi_{bp}^* = \eta_{bp}[\frac{(1-s)(1+r_b)}{1+r_g}w_{gp}^2 - w_{bp}^2][\mu - \min(\theta x, \frac{\mu}{2})]$.

From this, we find that $\mu > \min(\theta x, \frac{\mu}{2})$. Thus, when $\frac{(1-s)(1+r_b)}{1+r_g}w_{gp}^2 - w_{bp}^2 > 0$, $\Pi_{gp}^* > \Pi_{bp}^*$, *i.e.*, when $\frac{w_{gp}^2}{w_{bp}^2} > \frac{1+r_g}{(1-s)(1+r_b)}$, $\Pi_{gp}^* > \Pi_{bp}^*$; otherwise, when $\frac{w_{gp}^2}{w_{bp}^2} \leq \frac{1+r_g}{(1-s)(1+r_b)}$, $\Pi_{gp}^* \leq \Pi_{bp}^*$.

(3) Similarly, from $\Omega_{gp}^* - \Omega_{bp}^* = c\eta_{bp}^2[\frac{r_g(1-s)(1+r_b)^2}{(1+r_g)^2}w_{gp}^2 - r_b w_{bp}^2]$, we can obtain that when $\frac{w_{gp}^2}{w_{bp}^2} > \frac{r_b(1+r_g)^2}{r_g(1-s)(1+r_b)^2}$, $\Omega_{gp}^* > \Omega_{bp}^*$; when $\frac{w_{gp}^2}{w_{bp}^2} \leq \frac{r_b(1+r_g)^2}{r_g(1-s)(1+r_b)^2}$, $\Omega_{gp}^* \leq \Omega_{bp}^*$.

In summary, we obtain this proposition. □

Proof to Proposition 6.2. The proof process is similar to Proposition 6.1, so we omit it here. □

Proof to Proposition 6.3. The proof process is similar to Proposition 6.1, so we omit it here. □

Proof to Proposition 6.4. The proof process is similar to Proposition 6.1, so we omit it here. □

Acknowledgements. This research is supported by the National Natural Science Foundation of China (Nos. 72271219, 72232002), Natural Science Foundation of Zhejiang Province (No. LY22G020007) and National Social Science Fund of China (No. 21BGL109).

References

[1] E. Anderson and M. Monjardino, Contract design in agriculture supply chains with random yield. *Eur. J. Oper. Res.* **277** (2019) 1072–1082.

[2] I. Arizono and Y. Takemoto, Collaborative bargaining solution in tandem supply chain through cooperative game theoretical approach. *RAIRO: OR* **56** (2022) 2581–2593.

[3] S. Alizamir, F. Iravani and H. Mamani, An analysis of price vs. revenue protection: Government subsidies in the agriculture industry. *Manage. Sci.* **65** (2018) 32–49.

[4] N. Bart, T. Chernonog and T. Avinadav, Revenue-sharing contracts in supply chains: a comprehensive literature review. *Int. J. Prod. Res.* **59** (2020) 6633–6658.

[5] M. Bergen, M. Steeman, M. Reindorp and L. Gelsomino, Supply chain finance schemes in the procurement of agricultural products. *J. Purch. Supply Manag.* **25** (2018) 172–184.

[6] O. Boyabatli, P. Kleindorfer and S. Koontz, Integrating long-term and short-term contracting in beef supply chains. *Manage. Sci.* **57** (2011) 1771–1787.

[7] J. Buzacott and R.Q. Zhang, Inventory management with asset-based financing. *Manage. Sci.* **50** (2004) 1274–1292.

[8] G. Cachon, Supply chain coordination with contracts. *Handbooks Oper. Res. Manage. Sci.* **11** (2003) 227–339.

[9] G. Cai, X. Chen and Z.Xiao, The roles of bank and trade credits: theoretical analysis and empirical evidence. *Prod. Oper. Manag.* **23** (2014) 583–598.

[10] X. Chen, A model of trade credit in a capital-constrained distribution channel. *Int. J. Prod. Econ.* **159** (2015) 347–357.

[11] K. Chen and T. Xiao, Pricing and replenishment policies in a supply chain with competing retailers under different retail behaviors. *Comput. Ind. Eng.* **103** (2017) 145–157.

[12] J. Chod, Inventory, risk shifting, and trade credit. *Manage. Sci.* **63** (2016) 3207–3225.

[13] S. Deng, K. Fu, J. Xu and K. Zhu, The supply chain effects of trade credit under uncertain demands. *Omega* **98** (2021) 102113.

[14] W. Ding and G. Wan, Financing and coordinating the supply chain with a capital-constrained supplier under yield uncertainty. *Int. J. Prod. Econ.* **230** (2020) 107813.

[15] B. Giri, J. Majhi, S. Bardhan and K. Chaudhuri, Coordinating a three-level supply chain with effort and price dependent stochastic demand under random yield. *Ann. Oper. Res.* **307** (2021) 1–32.

[16] IFC, Access to finance for smallholder farmers: Learning from the experiences of microfinance institutions in Latin America. International Finance Corporation (2014).

[17] B. Jing, X. Chen and G. Cai, Equilibrium financing in a distribution channel with capital constraint. *Prod. Oper. Manag.* **21** (2012) 1090–1101.

[18] M. Kabul and A. Parlaktrk, The value of commitments when selling to strategic consumers: A supply chain perspective. *Manage. Sci.* **65** (2019) 4754–4770.

[19] D. Liang, W. Cao and M. Wang, Credit rating of sustainable agricultural supply chain finance by integrating heterogeneous evaluation information and misclassification risk. *Ann. Oper. Res.* (2021) DOI: 10.1007/s10479-021-04453-x.

[20] L. Liu and X. Zhan, Analysis of financing efficiency of chinese agricultural listed companies based on machine learning. *Complexity* **2019** (2019) 9190273.

[21] Y. Liu, F. Qin, M. Fry and A.S.Ruturi, Retail price markup commitment in decentralized supply chains. *Eur. J. Oper. Res.* **192** (2009) 277–292.

[22] Y. Liu, F. Qin, M. Fry and A.S. Ruturi, Multi-period modeling of two-way price commitment under price-dependent demand. *Eur. J. Oper. Res.* **221** (2012) 546–556.

[23] Y. Liu, D. Ma, J. Hu and Z. Zhang, Sales mode selection of fresh food supply chain based on blockchain technology under different channel competition. *Comput. Ind. Eng.* **162** (2021) 107730.

[24] X. Ma, S. Wang, S. Islam and X. Liu, Coordinating a three-echelon fresh agricultural products supply chain considering freshness-keeping effort with asymmetric information. *Appl. Math. Model.* **67** (2019) 337–356.

[25] I. Moon, Y. Jeong and S. Saha, Investment and coordination decisions in a supply chain of fresh agricultural products. *Oper. Res-ger.* **20** (2020) 2307–2331.

[26] B. Niu, D. Jin and X. Pu, Coordination of channel members' efforts and utilities in contract farming operations. *Eur. J. Oper. Res.* **255** (2016) 869–883.

[27] OECD, Financing SMEs and entrepreneurs. 4 (2015) 1–519.

[28] D. Roettger, Agricultural finance for smallholder farmers: Rethinking traditional microfinance risk and cost management approaches. Columbia University Press, New York (2015).

[29] J. Ruan, Y. Wang, F.T.S. Chan, X. Hu, M. Zhao, F. Zhu, B. Shi, Y. Shi and F. Lin, A life cycle framework of green IoT-based agriculture and its finance, operation, and management issues. *IEEE. Commun. Mag.* **57** (2019) 90–96.

[30] Z. Shi and E. Cao, Contract farming problems and games under yield uncertainty. *Aust. J. Agric. Resour. Econ.* **64** (2020) 1210–1238.

[31] J. Shi, Q. Li, L. Chu and Y. Shi, Effects of demand uncertainty reduction on the selection of financing approach in a capital-constrained supply chain. *Transp. Res. E: Logist. Transp. Rev.* **148** (2021) 102266.

[32] C. Tang, S. Yang and J. Wu, Sourcing from suppliers with financial constraints and performance risk. *Manuf. Serv. Oper. Manag.* **20** (2017) 70–84.

[33] M. Thierry, K. Omer and Mylene, Financing smallholder farmers and rural entrepreneurs through IFAD-supported operations in the Near East and North Africa. New Directions for Smallholder Agriculture (2011) 24–25.

[34] J. Tirole, The theory of industrial organization. MIT Press Books (1988) 1.

[35] T.I. Tunca and W. Zhu, Buyer intermediation in supplier finance. *Manage. Sci.* **64** (2017) 5631–5650.

[36] D. Xiao, J. Wang and Q.Lu, Stimulating sustainability investment level of suppliers with strategic commitment to price and cost sharing in supply chain. *J. Clean. Prod.* **252** (2020) 119732.

[37] S. Xu, L. Fang and W. Talley, Partial credit guarantee and trade credit in an emission-dependent supply chain with capital constraint. *Transp. Res. E: Logist. Transp. Rev.* **135** (2020) 101859.

[38] N. Yan, B. Sun, H. Zhang and C. Liu, A partial credit guarantee contract in a capital-constrained supply chain: Financing equilibrium and coordinating strategy. *Int. J. Prod. Econ.* **173** (2016) 122–133.

[39] B. Yan, G. Liu, Z. Zhang and C. Yan, Optimal financing and operation strategy of fresh agricultural supply chain. *Aust. J. Agric. Resour. Econ.* **64** (2020) 776–794.

[40] F. Ye, Q. Lin and Y. Li, Coordination for contract farming supply chain with stochastic yield and demand under cvar criterion. *Oper. Res.-Ger.* **20** (2020) 369–397.

[41] Z. Yi, Y. Wang and Y. Chen, Financing an agricultural supply chain with a capital-constrained smallholder farmer in developing economies. *Prod. Oper. Manag.* **30** (2021) 2102–2121.

[42] X. Yuan, G. Bi, Y. Fei and L. Liu, Supply chain with random yield and financing. *Omega* **102** (2020) 102334.

[43] L. Zhu and Y. Ou, Enhance financing for small- and medium-sized suppliers with reverse factoring: a game theoretical analysis. *Ann. Oper. Res.* (2021) DOI: 10.1007/s10479-021-04361-0.

[44] J. Zhuo, Y. Hu and M. Kang, Agricultural loan efficiency in centralized bank supply chains with fairness concern: a DEA-based analysis. *Ind. Manage. Data Syst.* **121** (2020) 839–855.

[45] F. Zhong, Z. Zhou and M. Leng, Negotiation-sequence, pricing, and ordering decisions in a three-echelon supply chain: A coopetitive-game analysis. *Eur. J. Oper. Res.* **294** (2021) 1096–1107.

[46] W. Zhou, T. Lin and G.Cai, uarantor financing in a four-party supply chain with leadership influence. *Prod. Oper. Manag.* **29** (2020) 2035–2056.