RAIRO-Oper. Res. 57 (2023) 2735–2755
https://doi.org/10.1051/ro/2023130

RAIRO Operations Research
www.rairo-ro.org

STRATEGIC INVESTMENT MODELLING FOR RETAIL SECTOR POST COVID-19

Aditi Khanna, Priyamvada, Shikha Yadav and Chandra K. Jaggi*

Abstract. Amidst the unprecedented COVID-19 pandemic, the online grocery retail industry has faced significant obstacles. To overcome these challenges and adapt to shifting customer attitudes, retailers must embrace innovative strategies. These include implementing a home delivery service with rigorous sanitization measures, leveraging social media advertising to enhance consumer awareness, and utilizing preservation technology to uphold grocery items' quality and freshness. In such a dynamic setting, it is only rational to acknowledge that the demand for products relies heavily upon the delivery firm's service performance and the awareness it generates. The present study explores these vital investments within the online grocery retail store, comparing them with models lacking such investments. By optimizing investments in preservation technology, service, and advertisement, the model seeks to maximize the retailer's overall profit. The findings unequivocally demonstrate that despite incurring additional costs, these investments wield financial dominance, boosting the total profit by an impressive 32%. The study concludes by presenting valuable insights derived from numerical and sensitivity analysis, offering invaluable guidance for the effective management of grocery items in the current post-pandemic era.

Mathematics Subject Classification. 90B05, 90B30.

Received January 31, 2023. Accepted August 23, 2023.

1. Introduction

The COVID-19 pandemic is having an unprecedented impact on societies all over the world, as the situation is rapidly evolving. To prevent the spread of the virus, governments have introduced various measures such as travel restrictions, social distancing, closing educational institutes, public places, and nearly all businesses. In a time of such extreme uncertainty, the majority of industries have suspended their operations which has paralyzed the global economies. Although the lockdown ensured free movement of grocery items, whereas food and grocery supply chains have faced overwhelming challenges and huddles. According to James Stock notes: "*For the week ended March 14, there were two countervailing effects. Consumer confidence plummeted and new claims for unemployment jumped sharply, but same-store sales surged as a result of the run-on groceries and supplies*". As per the article by Liam O'Connell, "*Consumers intended to increase their spending on grocery and non-food products for children. Vegetarian and vegan items have seen a substantial sales increase during the*

Keywords. Inventory, retail sector, grocery, preservation technology, service facility, advertisement, COVID-19.

Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi 110007, India.
*Corresponding author: ckjaggi@yahoo.com

pandemic, with a reported increase in sales of oat milk of over 300% and sales of meat substitutes growing by over 200%". This data clearly suggests that the demand for grocery goods (food items and grocery) has drastically increased during the pandemic. Moreover, due to the lockdown, there has been a supply chain disruption, which has put extra pressure on retailers to maintain enough inventories to avoid shortages. People in many countries have been found to engage in panic purchasing, which has adversely affected the supply system for the retail market [30]. As per the research conducted by Dumitras *et al.* [15], consumers' food consumption patterns have been noticeably changed due to the COVID-19 pandemic, and the new changes are being reflected in the post-pandemic period as well.

The grocery food items are perishable in nature and could spoil if not stored properly; thus, efficient preservation technology is required to pacify the loss due to spoilage/deterioration. Although investment in preservation techniques contributes to the total cost component, at the same time, it helps to preserve the perishable product for a longer time. Further, in this pandemic, there has been great concern about the health and safety of individuals; therefore, retailers need to adopt several safety measures such as social distancing, cleanliness, proper sanitization, and hygiene, as well as educating their staff to reassure the customers. Also, there has been a tremendous increase in the online purchase of grocery goods, which makes the home delivery service a mandate for retailers. The home delivery service plays an important role in today's crisis, as the product has to be delivered from door to door, taking into account all the safety measures. In order to reach out to the maximum number of customers and to maintain the government's protocol for a hygienic and contamination-free environment, extra investment in the service facility is required by the retailers to ensure safe delivery to the customers. Due to the various travel restrictions, many agitated local delivery systems significantly promote home delivery during the pandemic. To survive these difficult times of COVID-19, any running business would not mind going the extra mile to sustain itself in the competition.

Another major challenge is to create awareness among the masses about the delivery service and safety measures initiated by retailers to serve their customers. Advertisement on different social media platforms is helpful in creating awareness and to reach out to the maximum number of people. Advertising a product is essential to inventory management in order to promote brand awareness and assess the brand's availability in various markets. Typically, producers and retailers inform consumers about their products and services through advertisements, especially when introducing new products or modifying existing ones [7, 23]. The efforts of the advertising media and sales team encourage clients to purchase limited quantities of necessities in order to prevent shortages in the pandemic environment. Because marketing efforts influence supply chain market demand, they are regarded as decisive factors [44]. Most recent study in this area is about the "service and advertisement investment strategies affect the inventory selling price and stocking policy" by Mahajan and Tomar [25]. The efforts of the advertising media and sales team encourage the customers to buy the daily utility in a limited amount and prefer the home delivery service which is a safer option in the current scenario.

2. Literature survey

The present section puts forward the existing research in the area of the retail grocery sector and different strategic investments in the related field. It also provides a summary of related research in Table 1 which highlights the gap and contribution of this model.

Impact of COVID-19 on retail grocery. In accordance with current consumer surveys conducted by Numerator in the US, "90% of customers accepted that they had experienced a transform in their shopping activities due to the pandemic and 32% of them continue to build up stocks goods that they usually don't. Due to this stockpiling, shortage of products has developed as one of the most challenging issues, with 65% of consumers experiencing scarcity in the market. There is an amazing boom in online grocery shopping, and the survey states that 11% of online consumers made their first purchase in the last 6 months". Consequently, the supply chain and logistics management need to constantly adapt to the new crisis and look out for different ways to handle grocery items efficiently during the pandemic. In this regard, Ivanov and Dolgui [21] studied the impact of the COVID-19 pandemic on the supply and demand of grocery items. Singh and Rakshit [41] critically analyzed

the panic-buying behavior of Mumbaikar's in the COVID-19 era. Roggeveen and Sethuraman [36] gave some theoretical insights on the impact of supply chain disruption caused by COVID-19 on retailers. Chesbrough [13] threw light on how innovation will be acting as a recovery method from the economic havoc caused by COVID-19 in many industries. Sarkis *et al.* [38] have given a few ways for production managers to survive in times of coronavirus spread. Baker *et al.* [3] investigated the changes in the spending of households that have drifted to grocery items, particularly during the COVID-19 pandemic. A temporary framework for the supply and distribution of grocery goods during the COVID-19 pandemic has been proposed by Borocci *et al.* [9].

Preservation technology investment. The COVID-19 pandemic has adversely impacted consumer food stockpiling behavior, which forces the retail systems to store large inventory. The grocery food items being perishable are prone to deterioration/spoilage and thus can result in financial loss if not managed properly. Such monetary loss due to decay is a major challenge for the retailers of grocery items, and consequently, an investment in preservation technology is imperative to deal with it. In this direction, Hsu *et al.* [20] obtained the optimal inventory policy incorporating preservation technology investment in the model. Recently, Bardhan *et al.* [4], Tiwari *et al.* [42], and Saha *et al.* [37] developed different inventory models by considering preservation technology investment for items that are deteriorating in nature.

Service facility investment. Further, researchers have been putting immense effort into exploring the critical aspects of this spread and its harsh impact on businesses. To overcome the loss due to the lockdown, a home delivery service facility is a viable option to sustain in this pandemic situation. The success factor of e-grocery home delivery has been initially studied by Punakivi and Saranen [35]. Kämäräinen and Punakivi [22] proposed different home delivery models with e-grocery supply chains. While there has been a massive shift to online shopping platforms, grocery e-commerce growth penetration is expected to continue, to reach 14–18% in the next three to five years [27]. Laato *et al.* [24] presented the linking mechanism for the quarantine period and making unusual purchases during the COVID-19 pandemic. Hall *et al.* [18] proposed a study that grocery spending in New Zealand increased appreciably in mid-March compared to the same dates of the last year which demonstrated panic-buying and over-stocking by consumers. There is a continuous rise in online orders and the need for home delivery services is ever-expanding.

Investment in advertisement. The impact of traditional advertising on the demand for the product is initially studied by Bhunia and Maiti [8], and Goyal and Gunasekaran [17]. Later, how the demand is influenced by advertising media and the salesman's effort is discussed by Batarfi *et al.* [5], Chernonog and Avinadav [12], etc. In today's scenario, advertisements on social media can surely prove to be an effective approach to make people aware of the new services and measures which are taken by retail outlets to contain the COVID-19 outbreak. The fluctuation of the COVID-19 pandemic and the ways it influences shopping habits will likely continue into the foreseeable future. Online shopping will continue to pervade the retail world, particularly aligned to concerned shoppers with a reduced-contact mindset. Hence, digital marketing or advertisements on social media platforms is a new way to accelerate demand and promote sales in multiple places at the same time.

Although scientists and researchers worldwide are working together to accelerate the research and development process to overcome the challenges imposed by the COVID-19 crisis, most of the existing literature is theoretical and analytical. This paper attempts to develop a mathematical framework for online retail grocery stores, which would encourage retailers to incorporate different investment strategies in their system so as to increase their profit in this newly changed post-COVID world. The distinctiveness of the research lies in the fact that it offers three different models for the retail outlet which they can adopt depending on their usability and applicability. The mathematical models are supported by numerical investigations, and the comparative analysis has also been presented for the decision-makers to ease their managerial impediments. Table 1 provides a summary of related research which clearly highlights the gap and contribution of this model.

2.1. Research gap, objectives, and our contribution

In the wake of recent events, it has become evident through various research articles that the demand for grocery goods has become increasingly unpredictable, posing significant challenges for supply chains to meet

TABLE 1. Summary of grocery-supply chain-related research.

Author(s)	Problem addressed	Solution approach	Investment strategies		
			Service facility	Advertisement	Preservation
Perdana *et al.* [31]	Impact of COVID-19 on the food supply network	Location-routing method	No	No	No
Ćirić *et al.* [14]	Consumer behavior in online shopping of organic food during the Covid-19 pandemic	Survey-based approach	No	No	No
Alaimo *et al.* [2]	COVID-19 challenge on the online food buying behavior	Survey-based approach	Yes	No	No
Hao *et al.* [19]	Chinese grocery stockpile behavior	Bivariate profit model	Yes	No	No
Chenarides *et al.* [11]	Food consumption behavior during COVID-19 with grocery delivery service	Survey-based approach	Yes	No	No
Schmitt *et al.* [39]	Consumption and food waste pattern	Survey-based approach	No	No	No
Sharma *et al.* [40]	Supply chain resilience strategies in the grocery industry	Survey-based approach	Yes	No	No
Pérez Vergara *et al.* [32]	Variety of preservation strategies for service facilities during the pandemic days	Survey-based approach	Yes	No	Yes
Priyamvada and Kumar [33]	Production model with the investment in service, advertisement, and Variable selling price	Production Model	Yes	Yes	No
This model	Retailer model with the investment in service, advertisement, and preservation	Optimization model for Ordering Quantity	Yes	Yes	Yes

these fluctuating demands. Burgos and Ivanov [10] explored the resilience of the food retail supply chain in response to a pandemic, highlighting the substantial impact of demand surges and supplier shutdowns, whereas transportation interruptions had a minor influence. They found that utilizing online sales channels led to higher revenues. Sharma *et al.* [40] investigated the factors influencing food supply chains amid a disruptive pandemic, emphasizing the contribution of Blockchain Technology (BC-T) in managing disruptions and risks. During a pandemic, Pérez Vergara *et al.* [32] examined inventory management solutions for businesses dealing with different items with volatile demand, with a focus on ensuring customer satisfaction.

Indisputably, the COVID-19 pandemic has significantly altered consumer purchasing patterns, necessitating the retail industry's prompt adoption of new initiatives to cope with demand and supply limitations. In the case of groceries, retailers must invest in preservation techniques to prolong product shelf life and allocate resources to establishing efficient home delivery services to meet high demand. Consequently, increasing awareness of these services becomes essential, requiring advertising sector investments. Although, in the past three years, researchers have proposed various strategies to address disruptions in food supply chains during a pandemic; however, there has been limited focus on developing a mathematical model that combines different investment strategies (preservation technology, service facility, and advertising) for retailers dealing with grocery items in order to address this situation promptly. In this scenario, the retailer's investments in service and advertisement have a substantial impact on demand. This paper seeks to address this gap in the literature by presenting

three distinct scenarios for the post-pandemic period, comparing and analyzing these models based on different investment strategies. The objective is to boost demand and maximize the retailer's total profit by optimizing investments in preservation technology, service facility, and advertisement. In addition, this study provides valuable managerial insights to stakeholders for determining appropriate post-pandemic actions. Therefore, the research objectives of this study are as follows:

- Address the challenges posed by the COVID-19 pandemic to a grocery store retail establishment.
- Develop a mathematical framework for retailers considering investments in preservation technology, service facility, and advertisement.
- Emphasize the significance of the proposed strategies by presenting a comparative mathematical and analytical analysis of the three models.

The proposed study contributes to the existing literature by developing an optimization model for a practical real-world challenge imposed by the pandemic, which has been adopted by the retail industry post-pandemic. The study presents three investment models with different investment approaches, emphasizing the potential for profit in each case. The objective is to suggest a more resilient retail strategy to enhance customer service. The developed models are validated through numerical examples and sensitivity analysis, offering a range of policy decisions for stakeholders to consider based on their specific circumstances.

3. Notations and assumptions

The proposed article presents a mathematical model for retail grocery stores to deal with the consumer's buying behavior which is changed during the COVID-19 pandemic and continues to be the same till date. Investment in service facilities and advertisement along with preservation is a powerful tool for retailers to meet the challenges imposed by the pandemic and post-pandemic scenario. This paper develops three models to compare their performance in the present – post-pandemic period. The models are validated with different numerical examples so as to deliver valuable managerial insights. The following notations and assumptions are used to develop the three models presented by this study:

3.1. Notations

The notations used in the model are given below:

Decision variables

I	Service investment per unit time (\$/time)
τ	Investment in preservation technology per unit time (\$/time)
A_d	Advertisement cost per unit time (\$/time)

Constant parameters

C_o	Cost of ordering (per order) (\$/order)
$D(A_d, S(I))$	Demand rate
P	Unit selling price (\$/item)
T	Cycle time (Days)
h	Holding cost per unit per unit time (\$/time)
C_p	Purchase cost per unit (\$/item)
b	Demand sensitivity parameter related to the advertisement investment
q	Demand sensitivity parameter related to the service performance
I_0	Initial investment to start home delivery service per unit time (\$/time)
$S(I)$	Service performance which influences the demand
a	Demand function intercept
β	Sensitivity parameter for advertisement cost
γ	Sensitivity parameter for service investment performance

y_0	Deterioration rate without investment in preservation technology
α	Sensitivity parameter of preservation technology investment where $0 < \alpha < 1$
$y(\tau)$	Deterioration rate with investment in preservation technology
Q	Order quantity (units)

3.2. Assumptions

(i) The model considers online grocery retail stores for the items which are deteriorating in nature. The mathematical model is developed for a single type of product at a time.

(ii) The rate of deterioration can be controlled by investment in preservation technology. Deterioration rate can be expressed as a function that satisfies the following conditions $\partial y(\tau)/\partial\tau < 0$, $\partial^2 y(\tau)/\partial\tau^2 > 0$. Therefore, this model considers that $y(\tau) = y_0 e^{-\alpha\tau}$ where α is the sensitivity parameter of investment in preservation where $0 < \alpha < 1$ and τ is investment in preservation technology [16].

(iii) In order to contain the virus transmission, the retailers have switched to home delivery service of goods, following government guidelines and adopting the necessary safety measures (like sanitization before the next delivery, regular medical check-ups of the working staff, etc.). The home delivery service continues to be part of the present-day business.

(iv) The rate of demand is assumed to be directly proportional to the effectiveness of the investment in advertisement and service performance. Thus, the demand is positively and linearly dependent on the effectiveness of the advertisement expense and service performance [26]. The optimal impact of service parameters with the ordering decision model for deteriorating inventory is studied by Al Hamadi *et al.* [1], and Priyamvada and Kumar [33].

(v) The model assumes negligible lead time with no shortages, so as to maintain a steady supply of goods to consumers in the time of crisis [34, 43].

4. Problem description and mathematical modeling

4.1. Problem description

The pandemic has changed the consumers' perception as well as the way how businesses work. The retail sector has seen a major shift from offline to online purchases, making online shopping the most preferred and comfortable way for the majority of the population. Online grocery shopping is the "next normal" for most consumers worldwide. This transition has put up a challenge for the management of retail grocery stores. Retailers are in a tough situation, since they have to administer the consumer demand and delivery the items with an adequate level of service and safety conditions [6, 30]. Moreover, for grocery items, deterioration is also a major challenge that needs to be addressed. Grocers are confronting this situation by investing in different categories, *viz.*, preservation technology to control the deterioration of food products; service facilities so as to ensure a smooth customer experience; and digital marketing to promote their sales. With this motivation, the present paper instigates the real-life inventory management problem of an online grocery retail store for items that are deteriorating in nature. The objective is to develop optimal investment strategies in preservation technology, service facility, and promotional efforts for maximizing the retailer's total profit.

4.2. Mathematical model

In this section, the above-discussed retail grocery problem has been investigated by developing mathematical models under three different conditions. A model for pandemic and post-pandemic scenarios has been formulated in Section 4.2.1, followed by two more models to compare the usability and applicability of the proposed model. In the present study, it has been considered that retailers are offering service facilities with some additional investment in advertisement (digital marketing) to achieve maximum profit for the post-pandemic period. The rest of the two models are given to compare the strategy without investment in advertisement and service facility. Thus, the proposed three models are (a) the Retailer's model for the post-pandemic scenario with

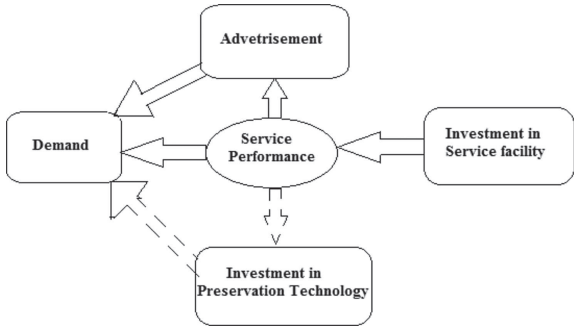

FIGURE 1. Demand and their influencing factors.

investment in preservation technology, service facility, and advertisement; (b) the Retailer's model for the post-pandemic scenario with investment in preservation technology and service facility; and (c) Retailer's model for the traditional scenario with investment in preservation technology.

4.2.1. Retailer's model for the post-pandemic scenario with investment in preservation technology, service facility, and advertisement

During the pandemic period, many efforts have been made by the authorities to satisfy the demand for grocery goods and to protect the health of society. The proposed mathematical model takes into account the three important aspects in such a scenario *i.e.*, investment in preservation technology, service facility, and awareness created by advertisement.

Figure 1 represents the demand function which is assumed to be dependent on two dominant factors *i.e.*, investment in service facility and advertisement. Also, the investment in preservation technology influences the demand indirectly. Similarly, the service performance affects the investment indirectly. Here, the advertisement factor is taken similarly to the existing paper by Mashud *et al.* [26].

The service investment influences the demand through service performance, where the performance includes different components like customer satisfaction, sanitization, regular health check-ups of the delivery boys, feedback, etc. Thus, the demand function can be written as:

$$D(A_d, S(I)) = a + bA_d^{\beta} + qS(I), \quad a, b, q > 0 \text{ and } 0 < \beta < 1. \tag{1}$$

Service performance $S(I) = \gamma \ln(I)$, $0 < \gamma < 1$ where I is the investment to maintain home delivery service.

b and q are the parameters that denote the effect of advertisement cost and service performance on the demand pattern. β is the sensitivity parameter for the efficiency of the advertisement cost. Since advertisement cost enhances the total cost linearly, it is quite obvious to control that component and at the same time, its impact on demand can be regulated by the parameter b. γ denotes the performance of the investment into the service facilities. Here, the performance includes all the attributes which are mentioned above.

$S(I)$ is the curve that represents the influencing behavior of that investment in terms of demand. Figure 2 reflects the curve of the investment in terms of the resultant effect on the other axis as $S(I)$.

Now, consider an inventory system where the inventory level varies with demand and deterioration. The behavior of the inventory with time is presented in Figure 3.

The items in inventory are deteriorating in nature, but the rate of deterioration has been controlled by an investment in preservation technology. Thus, the reduced deterioration rate is given by $y(\tau)$. Further, inventory depletes due to demand and deterioration as well, thus the equation governing the inventory level at any time t can be represented as:

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} + y(\tau)I(t) = -D(A_d, S(I)), \qquad 0 \le t \le T. \tag{2}$$

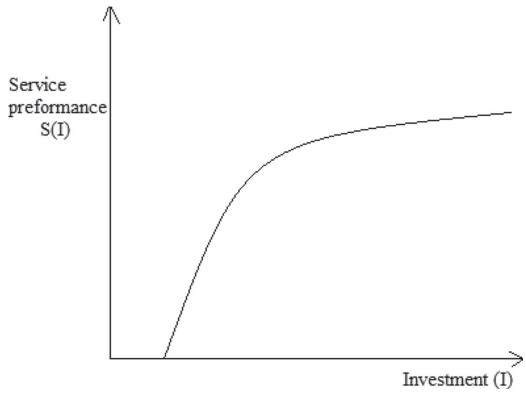

FIGURE 2. Graph of the Investment and their effect ($S(I)$).

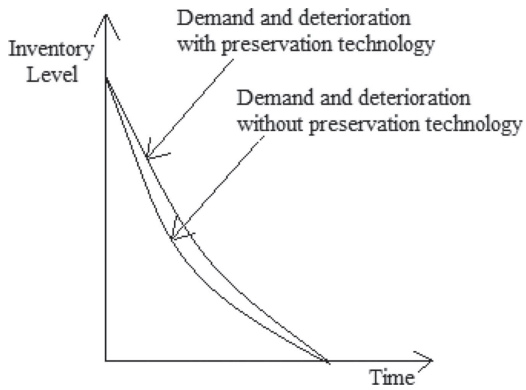

FIGURE 3. Graph of the inventory depletion with and without preservation technology.

The solution of equation (1) using the boundary condition $I_1(T) = 0$, is given as:

$$I_1(t) = \frac{D(A_d, S(I))}{y(\tau)} \left[e^{y(\tau)(T-t)} - 1 \right] \tag{3}$$

and the initial inventory level is:

$$Q = I_1(0) = \frac{D(A_d, S(I))T}{y(\tau)} \left[e^{y(\tau)T} - 1 \right]. \tag{4}$$

The total cost per cycle of the present model (Model 1) comprises the ordering cost, purchase cost, holding cost, preservation technology cost, total service cost, and advertisement cost.

The cost of ordering a lot is C_o which is considered constant for every cycle.

Since C_p is the unit purchase cost thus the total purchase cost is C_pQ.

The holding cost of the average inventory is h per unit per time and hence the total holding cost for one cycle time is $h \int_0^T I_1(t)\,\mathrm{d}t$ where $I_1(t)$ is taken from equation (2). Thus the total holding cost is given by the following equation

$$\text{holding cost} = h\int_0^T I_1(t)\,\mathrm{d}t = \frac{hD(p,G)}{(y(\tau))^2}\Big[e^{y(\tau)T} - T(y(\tau)) - 1\Big]. \tag{5}$$

For perishable items, the rate of deterioration can be controlled with the help of investment in preservation technology. Although it is an additional cost component, it helps to reduce the loss due to deterioration and hence overall maximize the total profit. Preservation technology investment is expressed as τT.

Home delivery (service delivery) bridges the gap between the retailer and the customer during this pandemic. As per government guidelines, maintaining a hygienic and contamination-free environment puts an extra burden on the retailer in terms of sanitization costs, regular medical check-ups of staff, etc. To provide service of home delivery there must be an initial cost to start with this service which is I_0. The other component of the service cost is variable and depends upon the total demand so as to maintain customer satisfaction as well as minimize the service cost. Thus, the service cost is given as follows:

$$\text{Total Service cost} = I_0 + \frac{I}{D(A_d, S(I))}. \tag{6}$$

The service facility and safety measures adopted by the retailers are worthless if the public is not aware of the same. In order to create awareness, advertisements on different social media platforms are an effective strategy to reach out to the maximum number of customers. The advertisement cost is given as A_dT.

Summing all the cost components, we get the total cost function per unit of time as:

$$= \frac{C_o}{T} + \frac{hD(A_d,S(I))}{T(y(\tau))^2}\Big[e^{y(\tau)T} - T(y(\tau)) - 1\Big] + \frac{\tau T}{T} + \left(I_0 + \frac{I}{D(A_d,S(I))}\right)\frac{1}{T} + \frac{C_pQ}{T} + \frac{A_dT}{T}. \tag{7}$$

Total revenue per unit time (TR)

$$= pD(A_d, S(I)). \tag{8}$$

Hence the total profit per unit time of the system is (TP)

$$= pD(A_d,S(I)) - \frac{C_o}{T} - \frac{hD(A_d,S(I))}{T(y(\tau))^2}\Big[e^{y(\tau)T} - T(y(\tau)) - 1\Big] - \tau - \left(I_0 + \frac{I}{D(A_d,S(I))}\right)\frac{1}{T} - \frac{C_pQ}{T} - A_d. \tag{9}$$

After substituting the value of Q from equation (4) and expanding the $e^{y(\tau)T}$ with ignoring the second and higher powers of $e^{y(\tau)T}$, as $y(\tau)T \ll 1$ [28] in equation (9) reduces to

$$\begin{aligned} &= pD(A_d,S(I)) - \frac{hD(A_d,S(I))T}{2} - \tau - \frac{C_o}{T} - I - A_d \\ &\quad - \frac{C_p}{T}\left[D(A_d,S(I))T + \frac{D(A_d,S(I))y(\tau)T^2}{2}\right] - \left(I_0 + \frac{I}{D(A_d,S(I))}\right)\frac{1}{T}. \end{aligned} \tag{10}$$

4.2.2. Retailer's model for the post-pandemic scenario with investment in preservation technology and service facility

The second scenario for a retailer could be where he has a good consumer base, which is loyal to him and places regular orders. Accordingly, the investment in advertisement or promotional efforts is assumed to be zero. This is the case which represents the retailer's model with investment in service facility; without any additional investment being done for the advertisement. However, the model does consider an investment in preservation technology in order to reduce the rate of deterioration.

In this case, the demand is considered as a function of investment in service facilities only. The total cost per cycle of this model (Model 2) comprises the ordering cost, purchase cost, holding cost, preservation technology cost, and total service facilities cost which are same as in the previous section. Hence, the total profit of the system for this model is

$$= pD(S(I)) - \frac{hD(S(I))T}{2} - \tau - \frac{C_o}{T} - I - \frac{C_p}{T}\left[D(S(I))T + \frac{D(S(I))y(\tau)T^2}{2}\right] - \left(I_0 + \frac{I}{D(S(I))}\right)\frac{1}{T}. \tag{11}$$

4.2.3. Retailer's model for traditional scenario with investment in preservation technology

This scenario considers both the investments in promotional efforts and service facilities to be zero. It is the very basic model with investment in preservation technology only. This model is not suitable for the pandemic situation. In this case, demand is considered to be constant, without any additional investment being done in the service facility and advertisement. The total cost per cycle of this model (Model 3) comprises the ordering cost, purchase cost, holding cost, and preservation technology cost, which are same as in the previous section. Hence, the total profit of the system for this model is

$$= pD - \frac{hDT}{2} - \tau - \frac{C_o}{T} - \frac{C_p}{T}\left[DT + \frac{Dy(\tau)T^2}{2}\right]. \tag{12}$$

5. Optimal solution

The main objective of the present study is to maximize the total profit of the model by jointly optimizing the investment in service cost (I), the total investment in preservation technology (τ), and the investment in advertisement (A_d) in the case of Model 1. For Model 2, the objective function should be optimized with respect to the total investment in preservation technology (τ) and investment in service cost (I). Lastly for Model 3, the objective function is optimized with respect to the total investment in preservation technology (τ).

5.1. Optimality conditions for Model 1

To establish optimality, the necessary conditions for Model 1 (First order partial derivatives of TP given in Eq. (10)) are as follows:

$$\frac{\partial \mathrm{TP}}{\partial \tau} = \frac{C_p \alpha T y(\tau) D(A_d, S(I))}{2} - 1 = 0, \tag{13}$$

$$\frac{\partial \mathrm{TP}}{\partial I} = \frac{1}{T}\left(\frac{q\gamma}{(D(A_d, S(I)))^2} - \frac{1}{D(A_d, S(I))}\right) + \frac{q\gamma}{I}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau) T}{2}\right) - 1 = 0, \tag{14}$$

$$\frac{\partial \mathrm{TP}}{\partial A_d} = \frac{I b \beta A_d^{\beta-1}}{T(D(A_d, S(I)))^2} + b\beta A_d^{\beta-1}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau) T}{2}\right) - 1 = 0. \tag{15}$$

For the sufficiency of the optimality condition, the propositions have been used by Mishra *et al.* [29]. Please refer to Appendix B.

Further, for more rigorous sufficiency conditions, the Hessian matrix is needed which is given below

$$H = \begin{bmatrix} \frac{\partial^2 \mathrm{TP}}{\partial \tau^2} & \frac{\partial^2 \mathrm{TP}}{\partial \tau \partial I} & \frac{\partial^2 \mathrm{TP}}{\partial \tau \partial A_d} \\ \frac{\partial^2 \mathrm{TP}}{\partial I \partial \tau} & \frac{\partial^2 \mathrm{TP}}{\partial I^2} & \frac{\partial^2 \mathrm{TP}}{\partial I \partial A_d} \\ \frac{\partial^2 \mathrm{TP}}{\partial A_d \partial \tau} & \frac{\partial^2 \mathrm{TP}}{\partial A_d \partial I} & \frac{\partial^2 \mathrm{TP}}{\partial A_d^2} \end{bmatrix} \text{ and conditions are } \frac{\partial^2 \mathrm{TP}}{\partial \tau^2} < 0,$$

$$\begin{vmatrix} \frac{\partial^2 \mathrm{TP}}{\partial \tau^2} & \frac{\partial^2 \mathrm{TP}}{\partial \tau \partial I} \\ \frac{\partial^2 \mathrm{TP}}{\partial I \partial T} & \frac{\partial^2 \mathrm{TP}}{\partial I^2} \end{vmatrix} > 0 \text{ and } H < 0. \tag{16}$$

The second-order derivatives for the calculations of the Hessian matrix and sufficient conditions to maximize the total profit function have been mentioned in Appendix A. Further, it is very difficult to establish sufficiency criteria mathematically, and thus the graphical method is used to establish the concavity. Thus, the optimality graphs are generated using the software Maple (see Figs. 4–6).

Finally, to determine the optimal solution for Model 1, the following solution procedure is proposed: Using the numerical parametric values, derive the optimal values of I, τ and A_d by solving the equations (13)–(15) simultaneously. Check the sufficient conditions for optimality which are given in (16). Hence, the optimal value of the total profit is obtained by substituting the parameters and optimal values of decision variables into the total profit function given in equation (10).

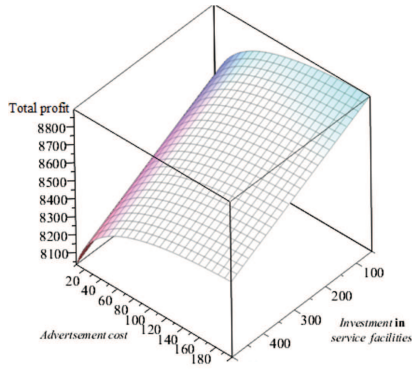

FIGURE 4. Graphical representation of total profit *versus* investment in service and advertisement.

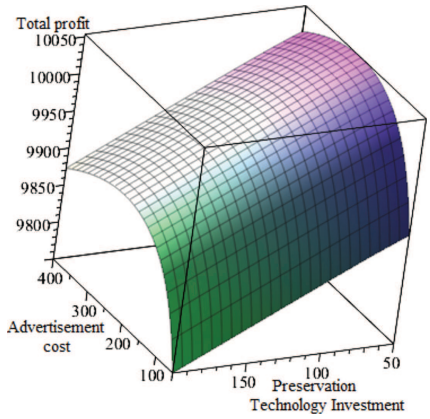

FIGURE 5. Graphical representation of total profit *versus* investment in preservation and advertisement.

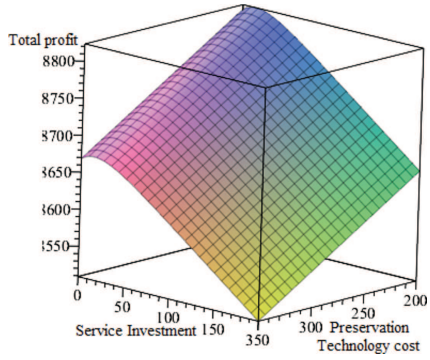

FIGURE 6. Graphical representation of total profit *versus* investment in service and advertisement.

5.2. Optimality conditions for Model 2

To establish optimality, the necessary conditions for Model 2 (First order partial derivatives of TP given in Eq. (11)) are as follows:

$$\frac{\partial \mathrm{TP}}{\partial \tau} = \frac{C_p \alpha T y(\tau) D(S(I))}{2} - 1 = 0, \tag{17}$$

$$\tau^* = \frac{2}{i} Ln(C_p \alpha T y_0 D(S(I))), \tag{18}$$

$$\frac{\partial \mathrm{TP}}{\partial I} = \frac{1}{T}\left(\frac{q\gamma}{(D(S(I)))^2} - \frac{1}{D(S(I))}\right) + \frac{q\gamma}{I}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau) T}{2}\right) - 1 = 0. \tag{19}$$

Due to complex mathematical expression, a closed-form solution for the value of I cannot be obtained. However, the optimal value of I can be obtained by solving equation (19). Further, the sufficiency criteria for the optimality of equation (11) has been established using the propositions given in Appendix C.

Finally, to determine the optimal solution for Model 2, the following solution procedure is proposed: Using the numerical parametric values, derive the optimal values of τ and I by solving the equations (17) and (19) simultaneously. Check the sufficient conditions for the optimality of the profit function. Hence, the optimal value of the total profit is obtained by substituting the parameters and optimal values of decision variables into the total profit function given in equation (11).

5.3. Optimality conditions for Model 3

To establish optimality, the necessary condition for Model 3 (First order derivative of TP given in Eq. (12)) is as follows:

$$\frac{\partial \mathrm{TP}}{\partial \tau} = \frac{C_p \alpha T y(\tau) D}{2} - 1 = 0, \tag{20}$$

$$\tau^* = \frac{2}{i} Ln(C_p \alpha T y_0 D). \tag{21}$$

The sufficiency conditions needed to be satisfied for optimality are:

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau^2} < 0 \quad i.e., \quad -\frac{C_p \alpha^2 T y(\tau) D}{2} < 0. \tag{22}$$

Now, to determine the optimal solution for Model 3, the following solution procedure is proposed: Using the numerical parametric values, derive the optimal value of τ by using equation (21). Check the sufficient conditions for the optimality of the profit function. Hence, the optimal value of the total profit is obtained by substituting the parameters and optimal value of decision variables into the total profit function given in equation (12).

6. Numerical and comparative analysis

The parametric values for the given models are taken from existing literature [33,44], with some modifications as per our assumptions which are given as follows.

6.1. Numerical examples

Example 6.1. The parametric values for Model 1: $T = 30$ days, $C_0 = \$500$/order, $a = 100$, $b = 20$, $C_p = \$10$/unit, $h = \$0.05$/unit/time, $y_0 = 0.2$, $\gamma = 0.02$, $\alpha = 0.05$, $\beta = 0.1$, $p = \$70$/unit, $I_0 = \$150$, $q = 2$.

This is an example of the present model with different investment strategies to meet the challenges imposes by the pandemic. The optimal values are given in Table 2.

TABLE 2. Optimal results for Model 1.

Parameters	Optimal values
Total Profit (TP)	\$7455.261
Service investment (I)	\$317.401
Investment in preservation (τ)	\$107.442
Investment in advertisement (A_d)	\$200.942
Demand	134 units (approx.)

TABLE 3. Optimal results for Model 2.

Parameters	Optimal values
Total Profit (TP)	\$5647.450
Service investment (I)	\$236.827
Investment in preservation (τ)	\$101.722
Investment in advertisement (A_d)	\$00
Demand	100 units (approx.)

TABLE 4. Optimal results for Model 3.

Parameters	Optimal values
Total Profit (TP)	\$5636.898
Service investment (I)	\$00
Investment in preservation (τ)	\$101.679
Investment in advertisement (A_d)	\$00
Demand	100 units (approx.)

Example 6.2. The parametric values for Model 2 are given as follows: $T = 30$ days, $C_0 = \$500/\text{order}$, $a = 100$, $C_p = \$10/\text{unit}$, $h = \$0.05/\text{unit/time}$, $y_0 = 0.2$, $\gamma = 0.02$, $\alpha = 0.05$, $p = \$70/\text{unit}$, $I_0 = \$150$, $q = 2$.

This scenario considers the investment in promotional efforts to be zero which is the case for deteriorating items during the pandemic days with service facility investment. The optimal values are recorded in Table 3. This model is not suitable for the pandemic situation.

Example 6.3. The parametric values for Model 3 are given as follows: $T = 30$ days, $C_0 = \$500/\text{order}$, $D = 100$, $C_p = \$10/\text{unit}$, $h = \$0.05/\text{unit/time}$, $y_0 = 0.2$.

This scenario considers the investment in promotional efforts and service facilities to be zero which is the traditional case for deteriorating items in normal days. This is the case which represents the model without any investment in service facilities and promotional efforts. This is the very basic model with investment in preservation technology only. The optimal values are recorded in Table 4. This model is not suitable for the pandemic situation.

6.2. Comparative analysis

To overcome the heavy loss due to the pandemic COVID-19, the home delivery facility is indeed a good alternative for retailers to suffice the demand. At the same time, there should be an investment in advertising media to create awareness among consumers about their services and safety measures. During the lockdown

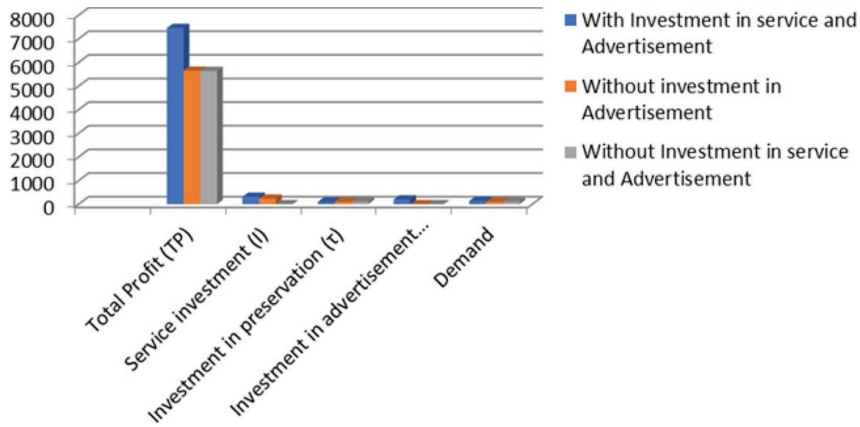

FIGURE 7. Comparison among all three models.

TABLE 5. Comparative results for different models.

Optimal values	With Investment in service and advertisement	Without investment in advertisement	Without investment in service and advertisement
Total Profit (TP)	\$7455.261	\$5647.450	\$5636.898
Service investment (I)	\$317.401	\$236.827	\$00
Investment in preservation (τ)	\$107.442	\$101.722	\$101.679
Investment in advertisement (A_d)	\$200.942	\$00	\$00
Demand	134 units (approx.)	101 units (approx.)	100 units (approx.)

and also post-pandemic, people preferred to get the grocery items in home delivery mode. So, investing in a service facility with promotional investment is quite pertinent. Investment in a service facility is only beneficial when people are aware of it. To make them aware of this facility, investment in promotional efforts (through advertisement on social media platforms) is quite beneficial for retailers; so as to convert their investment into higher demand, which ultimately turns into much higher profit. In order to check the financial capability of such a set-up, a comparative study of different settings has been done which is analyzed through different cases. The optimal results of the three cases are summarized in Table 5. It is directed to a retailer on how profit margin can be maximized under different investment strategies. Figure 7, represents the difference in total profit and other decision variables in all three cases. It is evident from the findings of Table 5 that the investment in a new service facility and advertisement helps to boost the demand and also results in a considerable increase in total profit for the retailer. Thus, it can be stated that controlling deterioration by implementing preservation technology, investing in the service facility, and creating awareness through advertisement is an effective strategy to sustain this economic and health crisis.

The comparative optimal results for different cases are recorded in Table 5.

7. SENSITIVITY ANALYSIS

To analyze the impact of key parameters on the optimal solution of this model, sensitivity analysis is performed for the numerical values which are taken in the numerical section. The consequent results are given in Table 6, in order to establish the robustness of the given model.

TABLE 6. Sensitivity analysis for the key parameters with decision variables.

a	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
80	200.857	104.276	269.983	3482.881	114.211	6273.074
90	200.903	105.922	293.693	3783.052	124.215	6864.111
100	200.942	107.442	317.351	4083.206	134.219	7455.260
110	200.975	108.854	341.109	4383.344	144.222	8046.506
120	201.005	110.173	364.816	4683.471	154.225	8637.835
q	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
1.0	200.908	107.423	158.523	4079.316	134.089	7448.813
1.5	200.927	107.432	237.921	4081.211	134.152	7451.936
2.0	200.942	107.442	317.351	4083.206	134.219	7455.260
2.5	200.959	107.452	396.986	4085.278	134.288	7458.734
3.0	200.975	107.462	476.670	4087.412	134.359	7462.328
γ	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
0.010	200.908	107.423	158.523	4079.316	134.089	7448.814
0.015	200.927	107.432	237.925	4081.153	134.153	7451.937
0.020	200.942	107.442	317.351	4083.206	134.219	7455.260
0.025	200.957	107.432	337.925	4084.542	134.262	7458.406
0.030	200.989	107.432	398.925	4086.205	134.316	7461.641
β	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
0.08	140.486	106.805	307.242	3954.656	129.934	7262.506
0.09	168.834	107.109	312.052	4015.479	131.962	7353.960
0.10	200.942	107.442	317.351	4083.206	134.219	7455.260
0.11	237.397	107.806	323.363	4158.670	136.734	7567.460
0.12	278.890	108.205	330.012	4242.848	139.540	7691.796
b	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
10	092.988	104.574	274.125	3535.308	115.958	6484.169
15	145.939	106.032	295.355	3804.093	124.916	6960.514
20	200.942	107.442	317.351	4083.206	134.219	7455.260
25	257.524	108.795	340.075	4370.257	143.786	7964.172
30	315.399	110.090	363.260	4663.796	153.570	8484.672
α	A_d	τ	I	Q	$D(A_d, S(I))$	Total profit
0.01	198.804	383.025	317.348	4299.356	134.182	7103.675
0.03	200.575	162.438	317.35	4120.219	134.213	7387.247
0.05	200.942	107.442	317.351	4083.206	134.219	7455.260
0.07	201.100	081.477	317.353	4067.211	134.221	7486.851
0.09	201.188	066.131	317.356	4034.565	134.221	7505.335

7.1. Observations and managerial insights

The present paper proposes a benchmark study related to the problem imposed by the pandemic and provides a solution to overcome the challenges faced by the retail business. The objective is an inventory optimization problem to maximize the retail store's profit with the optimum investment in preservation technology, promotional activities, and service facility to provide a better consumer experience. From Table 6, the following operational and decision-making implications can be drawn:

– When the scaling factor of demand (a) increases, investment in preservation technology, service facility, and total profit increases; whereas investment in the advertisement decreases. Amplified demand increases the order size which affects the service cost significantly, and has a mild effect on preservation technology investment, but results in higher profit. This follows due to a high surge in demand, which further suggests investing less in advertisement.

- An increase in the demand sensitivity parameter of service performance (q), elevates the service cost notably and this also influences the demand positively. There is a negligible increase in other costs like investment in advertisement and preservation. The total profit goes up for the retailer as demand increases due to better service performance.
- Increase in service investment performance parameter (γ) increases the service performance, which influences the demand and thus leads to higher profit. Order size also increases in order to meet the growing demand. Since improved service performance can be achieved by putting more resources into the service facility, hence the service cost rises. However, greater service performance boosts the demand and results in higher profit for the retailer.
- With a rise in the effectiveness parameter of advertisement cost (β), there is a steep rise in demand which leads to higher profits. Order size also increases in order to satisfy the rising demand. Since the effectiveness of the advertisement can be enhanced by increasing the investment, thus the advertisement cost escalates. Also, there is a slight increase in service costs due to high demand.
- Also, an increase in demand sensitivity parameter related to advertisement cost (b), gives a boost to demand. High demand can be pulled by investing more in the advertisement sector, which increases the cost significantly. Moreover, service cost increases to satisfy the surge in demand, but at the same time, the total profit of the system increases. Thus, it indicates that although investment in the advertisement is an additional expense for the retailer, it helps to increase the demand and fetch a higher profit.
- An increase α leads to an increase in the effectiveness of the investment in preservation technology. With efficient preservation technology, the investment amount decreases. Because of the lower preservation technology cost, the profit will increase rationally. All this has a negligible impact on other investments like the investment in service and advertisement.

8. Conclusion

As the COVID-19 pandemic hits the world, retailers are retooling every aspect of their operations to satisfy the need for grocery goods and to protect the health and well-being of society. Many efforts *viz.*, adopting better preservation techniques for the food items, initiating home-delivery service, undertaking proper sanitation measures, advertising in media to create awareness, etc., have been taken up by the retailers to overcome this health and economic crisis. To achieve this, the present paper developed an inventory model taking into account different investments – preservation technology, service facility, and awareness strategies for a retailer supplying grocery goods (food and grocery items). Being perishable goods, an investment in preservation technology is inevitable to minimize the financial loss due to deterioration/spoilage. Home delivery service has become a more appealing alternative for consumers during the pandemic, with service performance affecting the demand competently. Moreover, investment in advertisement is dynamic which also influences the demand. The proposed model targets the realistic challenges that are encountered by the grocery retail industry, especially with the supply chain of grocery items. Extra investment to provide the delivery service to attract more customers has been considered to enhance the total profit. Practically, promotional efforts through advertisement on different social media platforms are also required to elevate sales. Thus, the demand is considered to be a function of service performance and investment in the advertisement. Accordingly, three mathematical models have been developed to analyze the impact of investment strategies for the current post-pandemic period. Model 1 optimizes the investment in preservation technology, service facility, and advertisement which helps to fetch more demand and results in maximum profit. Model 2 represents the case where investment in a service facility has been considered without any promotional effort. The third model discusses the traditional case with basic assumptions for deteriorating items. The results of the numerical section (Table 5) validate that although the advertisement strategy introduces an additional cost, it has a financial dominance to increase the total profit by 32% approximately. The major contribution of this paper is the assembly of strategies with different investments to overcome the challenges imposed by the COVID-19 pandemic on the retail grocery establishment. To conclude, the study establishes more resilient retail strategies to improve consumer service.

9. Limitations and future research directions

The presented research is limited to items having fixed deterioration rates. This model can be extended in several ways. For instance, the model can allow shortages for complete and partial backlogging cases. Other possible extensions may include analyzing the effect of non-linear and time-dependent holding costs due to the pandemic scenario. The presence of imperfect items, and their screening is another challenge during the crisis. Advance payments, trade credit, and inflation on economic policies can also be considered because of income shock due to COVID-19. Moreover, the effect of the pandemic on production firms under the scenario of multi-echelon supply chains and multiple inspections can be studied. Also, the data set is taken from existing literature. It would be more effective if the whole scenario were considered for any industry data set in today's scenario. Fuzzy data set consideration is a better option for more practical managerial implications.

Appendix A.

$$\begin{aligned}\mathrm{TP} &= pD(A_d,S(I)) - \frac{C_o}{T} - \frac{hD(A_d,S(I))}{T(y(\tau))^2}\left[e^{y(\tau)T} - T(y(\tau)) - 1\right] - \tau - \left(I_0 + \frac{I}{D(A_d,S(I))}\right)\frac{1}{T} \\ &\quad - \frac{C_pQ}{T} - A_d, \end{aligned} \tag{A.1}$$

$$\frac{\partial \mathrm{TP}}{\partial \tau} = \frac{C_p\alpha T y(\tau) D(A_d,S(I))}{2} - 1 = 0, \tag{A.2}$$

$$\frac{\partial \mathrm{TP}}{\partial I} = \frac{1}{T}\left(\frac{q\gamma}{(D(A_d,S(I)))^2} - \frac{1}{D(A_d,S(I))}\right) + \frac{q\gamma}{I}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right) - 1 = 0, \tag{A.3}$$

$$\frac{\partial \mathrm{TP}}{\partial A_d} = \frac{Ib\beta A_d^{\beta-1}}{T(D(A_d,S(I)))^2} + b\beta A_d^{\beta-1}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right) - 1 = 0, \tag{A.4}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau^2} = -\frac{C_p\alpha^2 T y(\tau) D(A_d,S(I))}{2}, \tag{A.5}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial I^2} = -\frac{1}{T}\left(\frac{2q^2\gamma^2}{I(D(A_d,S(I)))^3} - \frac{q\gamma}{I(D(A_d,S(I)))^2}\right) - \frac{q\gamma}{I^2}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right), \tag{A.6}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial A_d^2} = \frac{Ib\beta(\beta-1)A_d^{\beta-2}}{T(D(A_d,S(I)))^2} - \frac{2Ib^2\beta^2 A_d^{2\beta-2}}{T(D(A_d,S(I)))^3} + b\beta(\beta-1)A_d^{\beta-1}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right), \tag{A.7}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau \partial I} = \frac{\partial^2 \mathrm{TP}}{\partial I \partial \tau} = \frac{C_p q\gamma\alpha y(\tau)T}{2}, \tag{A.8}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau \partial A_d} = \frac{\partial^2 \mathrm{TP}}{\partial A_d \partial \tau} = \frac{bC_p\alpha T\beta A_d^{\beta-1} y(\tau)}{2}, \tag{A.9}$$

$$\frac{\partial^2 \mathrm{TP}}{\partial I \partial A_d} = \frac{\partial^2 \mathrm{TP}}{\partial A_d \partial I} = \frac{b\beta A_d^{\beta-1}(a + bA_d^{\beta} + (\ln I - 2)q\gamma)}{T(a + bA_d^{\beta} + q\gamma \ln I)^3}. \tag{A.10}$$

Appendix B.

Proposition B.1. *When the investment in service facility and advertisement is given, the profit function* (A.1) *is concave with respect to the investment in preservation technology.*

Proof. Without loss of generality, suppose I and A_d are continuous variables, and hence, expressions (A.3) and (A.4) are the first order derivatives of expression (A.1) with respect to I and A_d, for any $\tau \gg 0$. Also, the expression (A.1) is differentiable with respect to τ and the first derivative is given by expression (A.2).

The second derivative with respect to τ is:

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau^2} = -\frac{C_p \alpha^2 T y(\tau) D(A_d, S(I))}{2} < 0. \tag{B.1}$$

Since all the terms are positive, the total expression is negative. Hence, equation (B.1) is a concave function with respect to τ when I and A_d are given. □

Proposition B.2. *When the investment in preservation technology and advertisement is given, the profit function* (A.1) *is concave with respect to the service facility for all* $(2p - h - 2C_p - C_p y(\tau)T) > 0$.

Proof. Without loss of generality, suppose τ and A_d are continuous variables, and hence, expression (A.2) and (A.4) are the first order derivatives of expression (A.1) with respect to τ and A_d, for any $I > 0$. Also, the expression (A.1) is differentiable with respect to I and the first derivative is given by expression (A.3).

The second derivative with respect to I is:

$$\frac{\partial^2 \mathrm{TP}}{\partial I^2} = -\frac{1}{T}\left(\frac{2q^2\gamma^2}{I(D(A_d, S(I)))^3} - \frac{q\gamma}{I(D(A_d, S(I)))^2}\right) - \frac{q\gamma}{I^2}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right). \tag{B.2}$$

In this expression every term is negative and hence, for all $(2p - h - 2C_p - C_p y(\tau)T) > 0$, equation (B.2) is a concave function with respect to I when τ and A_d are given. □

Proposition B.3. *When the investment in preservation technology and service facility is given, the profit function* (A.1) *is concave with respect to the investment in the advertisement for all* $(2p - h - 2C_p - C_p y(\tau)T) > 0$.

Proof. Without loss of generality, suppose τ and I are continuous variables, and hence, expressions (A.2) and (A.3) are the first order derivatives of expression (A.1) with respect to τ and I, for any $A_d > 0$. Also, the expression (A.1) is differentiable with respect to A_d and the first derivative is given by expression (A.4).

The second derivative with respect to A_d is:

$$\frac{\partial^2 \mathrm{TP}}{\partial A_d^2} = -\frac{Ib\beta(1-\beta)A_d^{\beta-2}}{T(D(A_d, S(I)))^2} - \frac{2Ib^2\beta^2 A_d^{2\beta-2}}{T(D(A_d, S(I)))^3} - b\beta(1-\beta)A_d^{\beta-1}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right). \tag{B.3}$$

Since, all the terms are negative and hence, equation (B.3) is a concave function with respect to A_d when τ and I are given. □

Appendix C.

Proposition C.1. *When the investment in a service facility is given, the profit function* (A.1) *is concave with respect to the investment in preservation technology.*

Proof. Without loss of generality, suppose I is a continuous variable, and hence, expression (A.3) is the first order derivative of expression (A.1) with respect to I, for any $\tau > 0$. Also, the expression (A.1) is differentiable with respect to τ and the first order derivative is given by expression (A.2).

The second derivative with respect to τ is:

$$\frac{\partial^2 \mathrm{TP}}{\partial \tau^2} = -\frac{C_p \alpha^2 T y(\tau) D(S(I))}{2} < 0. \tag{C.1}$$

Since all the terms are positive, the total expression is negative. Hence, equation (C.1) is a concave function with respect to τ when I is given. □

Proposition C.2. *When the investment in preservation technology is given, the profit function* (A.1) *is concave with respect to the service facility for all* $(2p - h - 2C_p - C_p y(\tau)T) > 0$.

Proof. Without loss of generality, suppose τ is continuous variables, and hence, expression (A.2) is the first order derivative of expression (A.1) with respect to τ, for any $I > 0$. Also, the expression (A.1) is differentiable with respect to I and the first derivative is given by expression (A.3).

The second derivative with respect to I is:

$$\frac{\partial^2 \mathrm{TP}}{\partial I^2} = -\frac{1}{T}\left(\frac{2q^2\gamma^2}{I(D(S(I)))^3} - \frac{q\gamma}{I(D(S(I)))^2}\right) - \frac{q\gamma}{I^2}\left(p - \frac{h}{2} - C_p - \frac{C_p y(\tau)T}{2}\right). \tag{C.2}$$

In this expression every term is negative and hence, for all $(2p - h - 2C_p - C_p y(\tau)T) > 0$, equation (C.2) is a concave function with respect to I when τ is given. □

Acknowledgements. The authors would like to thank the Editors-in-chief and the anonymous referees for their valuable comments and suggestions for improving the research paper. The first and corresponding author would like to thank the University of Delhi, Delhi, for funding this work through the Faculty Research Programme (FRP) of the IoE scheme under the grant number IoE/2021/12/FRP.

Conflict of Interest. On behalf of all the authors, the corresponding author declares that there is no conflict of interest.

References

[1] H.M. Al Hamadi, N. Sangeetha and B. Sivakumar, Optimal control of service parameter for a perishable inventory system maintained at service facility with impatient customers. *Ann. Oper. Res.* **233** (2015) 3–23.

[2] L.S. Alaimo, M. Fiore and A. Galati, How the COVID-19 pandemic is changing online food shopping human behaviour in Italy. *Sustainability* **12** (2020) 9594.

[3] S.R. Baker, R.A. Farrokhnia, S. Meyer, M. Pagel and C. Yannelis, How does household spending respond to an epidemic? Consumption during the 2020 COVID-19 pandemic. *Rev. Asset Pricing Stud.* **10** (2020) 834–862.

[4] S. Bardhan, H. Pal and B.C. Giri, Optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand. *Oper. Res.* **19** (2019) 347–368.

[5] R. Batarfi, M.Y. Jaber and S. Zanoni, Dual-channel supply chain: a strategy to maximize profit. *Appl. Math. Modell.* **40** (2016) 9454–9473.

[6] A. Baveja, A. Kapoor and B. Melamed, Stopping COVID-19: a pandemic-management service value chain approach. *Ann. Oper. Res.* **289** (2020) 173–184.

[7] U.K. Bhattacharya, A chance constraints goal programming model for the advertising planning problem. *Eur. J. Oper. Res.* **192** (2009) 382–395.

[8] A.K. Bhunia and M. Maiti, A two warehouse inventory model for deteriorating items with a linear trend in demand and shortages. *J. Oper. Res. Soc.* **49** (1998) 287–292.

[9] S. Borocci, F. Brunet, S. Cisnal De Ugarte, M. L. Yuen and M. Tagara, The EU commission publishes a temporary framework to provide guidance to companies that are cooperating to ensure the supply and distribution of grocery products during the COVID-19 outbreak. e-Competitions Bulletin (2020) (Preview).

[10] D. Burgos and D. Ivanov, Food retail supply chain resilience and the COVID-19 pandemic: a digital twin-based impact analysis and improvement directions. *Transp. Res. Part E: Logistics Transp. Rev.* **152** (2021) 102412.

[11] L. Chenarides, C. Grebitus, J.L. Lusk and I. Printezis, Food consumption behavior during the COVID-19 Pandemic. *Agribusiness* **37** (2021) 44–81.

[12] T. Chernonog and T. Avinadav, Pricing and advertising in a supply chain of perishable products under asymmetric information. *Int. J. Prod. Econ.* **209** (2019) 249–264.

[13] H. Chesbrough, To recover faster from COVID-19, open up: managerial implications from an open innovation perspective. *Ind. Marketing Manage.* **88** (2020) 410–413.

[14] M.R. Ćirić, D.S. Ilić, S.D. Ignjatijević and S.D. Brkanlić, Consumer behaviour in online shopping organic food during the COVID-19 pandemic in Serbia. *Food Feed Res.* **47** (2020) 149–158.

[15] D.E. Dumitras, R. Harun, F.H. Arion, D.I. Chiciudean, E. Kovacs, C.F. Oroian, A. Porutiu and I.C. Muresan, Food consumption patterns in Romania during the COVID-19 pandemic. *Foods* **10** (2021) 2712.

[16] C.Y. Dye and T.P. Hsieh, An optimal replenishment policy for deteriorating items with effective investment in preservation technology. *Eur. J. Oper. Res.* **218** (2012) 106–112.

[17] S.K. Goyal and A. Gunasekaran, An integrated production-inventory-marketing model for deteriorating items. *Comput. Ind. Eng.* **28** (1995) 755–762.

[18] M.C. Hall, G. Prayag, P. Fieger and D. Dyason, Beyond panic buying: consumption displacement and COVID-19. *J. Serv. Manage.* **32** (2020) 113–128.

[19] N. Hao, H.H. Wang and Q. Zhou, The impact of online grocery shopping on stockpile behavior in COVID-19. *China Agric. Econ. Rev.* **12** (2020) 459–470.

[20] P.H. Hsu, H.M. Wee and H.M. Teng, Preservation technology investment for deteriorating inventory. *Int. J. Prod. Econ.* **124** (2010) 388–394.

[21] D. Ivanov and A. Dolgui, Viability of intertwined supply networks: extending the supply chain resilience angles towards survivability. A position paper motivated by COVID-19 outbreak. *Int. J. Prod. Res.* **58** (2020) 2904–2915.

[22] V. Kämäräinen and M. Punakivi, Developing cost-effective operations for the e-grocery supply chain. *Int. J. Logistics* **5** (2002) 285–298.

[23] M.A.A. Khan, A.A. Shaikh, I. Konstantaras, A.K. Bhunia and L.E. Cárdenas-Barrón, Inventory models for perishable items with advanced payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price. *Int. J. Prod. Econ.* **230** (2020) 107804.

[24] S. Laato, A.N. Islam, A. Farooq and A. Dhir, Unusual purchasing behavior during the early stages of the COVID-19 pandemic: the stimulus-organism-response approach. *J. Retail. Consum. Serv.* **57** (2020) 102224.

[25] K. Mahajan and S. Tomar, COVID-19 and supply chain disruption: evidence from food markets in India. *Am. J. Agric. Econ.* **103** (2021) 35–52.

[26] A.H.M. Mashud, M.R. Hasan, H.M. Wee and Y. Daryanto, Non-instantaneous deteriorating inventory model under the joined effect of trade-credit, preservation technology and advertisement policy. *Kybernetes* **49** (2020) 1645–1674.

[27] S. Meyer, Understanding the COVID-19 effect on online shopping behavior. The BigCommerce Blog (2020).

[28] U. Mishra, L.E. Cárdenas-Barrón, S. Tiwari, A.A. Shaikh and G. Treviño-Garza, An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. *Ann. Oper. Res.* **254** (2017) 165–190.

[29] U. Mishra, J.Z. Wu, Y.C. Tsao and M.L. Tseng, Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates. *J. Cleaner Prod.* **244** (2020) 118807.

[30] N.A. Omar, M.A. Nazri, M.H. Ali and S.S. Alam, The panic buying behavior of consumers during the COVID-19 pandemic: examining the influences of uncertainty, perceptions of severity, perceptions of scarcity, and anxiety. *J. Retail. Consum. Serv.* **62** (2021) 102600.

[31] T. Perdana, D. Chaerani, A.L.H. Achmad and F.R. Hermiatin, Scenarios for handling the impact of COVID-19 based on food supply network through regional food hubs under uncertainty. *Heliyon* **6** (2020) e05128.

[32] I.G. Pérez Vergara, M.C. López Gómez, I. Lopes Martínez and J. Vargas Hernández, Strategies for the preservation of service levels in the inventory management during COVID-19: a case study in a company of biosafety products. *Global J. Flexible Syst. Manage.* **22** (2021) 65–80.

[33] Priyamvada and A. Kumar, Modelling retail inventory pricing policies under service level and promotional efforts during COVID-19. *J. Cleaner Prod.* **381** (2022) 134784.

[34] Priyamvada, S. Yadav, A. Khanna and C.K. Jaggi, Sustainable preservation strategies with deterioration management and environment sensitive demand. *Int. J. Math. Eng. Manage. Sci.* **6** (2021) 1089.

[35] M. Punakivi and J. Saranen, Identifying the success factors in e-grocery home delivery. *Int. J. Retail Distrib. Manage.* **29** (2001) 156–163.

[36] A.L. Roggeveen and R. Sethuraman, How the COVID pandemic may change the world of retailing. *J. Retail.* **96** (2020) 169.

[37] S. Saha, D. Chatterjee and D. Sarkar, The ramification of dynamic investment on the promotion and preservation technology for inventory management through a modified flower pollination algorithm. *J. Retail. Consum. Serv. 58* (2021) 102326.

[38] J. Sarkis, M.J. Cohen, P. Dewick and P. Schröder, A brave new world: lessons from the COVID-19 pandemic for transitioning to sustainable supply and production. *Res. Conserv. Recycl.* **159** (2020) 104894.

[39] V.G.H. Schmitt, M.M. Cequea, J.M.V. Neyra and M. Ferasso, Consumption behavior and residential food waste during the COVID-19 pandemic outbreak in Brazil. *Sustainability* **13** (2021) 3702.

[40] M. Sharma, S. Joshi, S. Luthra and A. Kumar, Managing disruptions and risks amidst COVID-19 outbreaks: role of blockchain technology in developing resilient food supply chains. *Oper. Manage. Res.* **15** (2021) 268–281.

[41] C.K. Singh and P. Rakshit, A critical analysis to comprehend panic buying behaviour of Mumbaikar's in COVID-19 era. *Stud. Indian Place Names* **40** (2020) 44–51.

[42] S. Tiwari, C.K. Jaggi, M. Gupta and L.E. Cárdenas-Barrón, Optimal pricing and lot-sizing policy for supply chain system with deteriorating items under limited storage capacity. *Int. J. Prod. Econ.* **200** (2018) 278–290.

[43] S. Yadav, F. Siddiqui and A. Khanna, Sustainable inventory model with carbon emission dependent demand under different carbon emission policies, in Soft Computing in Inventory Management. Springer Singapore, Singapore (2021) 163–175.

[44] S. Yadav, P. Priyamvada and A. Khanna, COVID-19 impact on a sustainable production model with volume agility and advertisement dependent demand. *Int. J. Supply Oper. Manage.* **10** (2023) 136–150.