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OPTIMIZING RETAILER'S ORDER AND FINANCING DECISIONS ON AN E-COMMERCIAL PLATFORM CONSIDERING CASH FLOW

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Abstract. The mismatch in cash flow often distorts business operational decisions and even lead to bankruptcy for enterprises. This paper investigates the order and financing decisions of a capital-constrained retailer who borrows from an e-commerce platform to fund its business operations. The e-commercial platform, which has full capital, provides three financing schemes: (1) lump-sum repayment (scheme L), (2) average capital plus interest repayment (scheme P), and (3) average capital repayment (scheme A). We first model the financing behaviors of the retailer and determine the circumstances in which the retailer favors a specific financing scheme. Then, we propose a viable cash flow matching strategy in which the retailer retains a portion of its initial capital to address potential repayment shortfalls in each period. To the best of our knowledge, this paper is the first to integrate the capital-constrained retailer's cash flow management into the platform financing scheme. The results show that: In the absence of cash flow considerations, the retailer prefers scheme L since selecting scheme P or A may lead to bankruptcy. In contrast, if the cash flow matching is efficiently realized, the retailer always prefers scheme P to enhance its performance. Numerical examples are used to validate the theoretical results.

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1. Introduction

One of the most pressing concerns for a capital-constrained enterprise is to maximize its expected profit. However, in the real world of business, not every enterprise can successfully achieve this goal by the end of the sales season [17]. In China, over 300,000 online enterprises seek loans each year to facilitate their businesses and maximize their expected profits. For example, online fashion retailers commonly borrow the short-term loans to prepare substantial inventory for peak seasons, such as Double 11 shopping carnival in China. Almost 30% of them, in practice, experience business interruption and bankruptcy in sales midseason [9]. This reflection underscores the insufficiency of working capital in bridging the temporal gap between earnings and financial outlays. The reason behind this failure is that cash inflow from revenue is not synchronized with cash outflow from debt. Consequently, the mismatch between cash inflow and outflow intensifies the retailers' financial costs due to delayed loan repayment [7, 25]. Therefore, the well-scheduled financing schemes are required to match the cash flows generated by financing and trading activities.

Keywords. Capital-constrained retailer, platform financing scheme, time-dependent demand, cash flow management.

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Currently, a type of installment lending service pioneered by the e-commerce platforms, such as Alibaba and JD, is becoming the most prevalent and flexible financial source. The biggest advantage of the installment loan is that borrowers can flexibly choose the repayment schemes, including loan duration and repayment amount in each period. At the same time, the installment repayment schemes also provide borrowers with the opportunity to adjust their cash flows. Typically, there are three repayment schemes available for borrowers, including lump-sum repayment (scheme L), average capital plus interest repayment (scheme P), and average capital repayment (scheme A). The main difference between these three repayment schemes is that the amount of principal and interest payable in each repayment period varies. To be more specific, for the scheme L, the retailer needs to repay the principal and interest in one lump sum on the repayment date; For scheme P, the principal and interest are constant and consistent in each subperiod; For scheme A, the retailer repays a fixed principal amount, but the interest decreases gradually. According to a report from E-commerce News in 2011, approximately 69% of Taobao merchants have applied for such an installment loan with a daily interest rate of 0.06%. However, such financing schemes have also been challenging for e-retailers due to the untimely installment repayments may trigger overdue penalties or even bankrupt.

The financing cases mentioned previously highlight the importance of matching cash inflows and outflows when engaging in platform financing activities [19]. This cash flow mismatch further inspires us to explore the integration of cash flow management into the retailer's platform financing decisions. To ease the concerns of e-retailers, we propose a novel cash flow matching strategy in which the retailer strategically retains part of the initial capital to cover possible repayment shortfalls in each period. In 2015, the Toro Company successfully buffered the time discrepancy between cash inflows and outflows by retaining the operating cash [31]. By contrast, the Hstyle, one of the largest fashion product e-retailer in China, experiences the transaction defaults and platform's penalties, resulting in a loss of over 2.24 million and sales interruption (*Linkshop.com*).

Motivated by these practical observations, several research questions are raised as follows: (1) Are the installment repayment schemes better than the lump-sum repayment scheme? (2) What is the impact of the overdue penalty mechanism on the retailer's cash flow? (3) How to decide a proper order quantity and choose a financing scheme for the capital-constrained retailer? (4) How to match future cash outflows with cash inflows to reduce financing costs? In sharp to previous studies, this paper focuses on two novel concerns: (1) the flexible installment loan provided by the e-commercial platform, and (2) feasible strategy to assist the retailer to match its cash flows.

To reveal these questions, we consider a retailer's order and financing decisions on an e-commercial platform supply chain where such a retailer with limited capital borrows from an e-commercial platform to fund its business. To begin, we develop a time-related newsvendor model to analyze the capital-constrained retailer's order decision and financing scheme selection. The retailer sells the product on the e-commercial platform at a fixed price, and any unmet demand is lost. Three financing schemes are provided by the e-commercial platform as we mentioned before, including scheme L, scheme P and scheme A. Under each financing scheme, we construct a newsvendor model with time-dependent demand to drive the retailer's ordering decision. Then, we characterize the conditions under which the retailer prefers to select a certain financing scheme. We find that only when the overdue penalty interest rate is extremely high and the initial capital level is much low, the scheme L becomes the preferred financing choice. By introducing the cash flow matching, we present a feasible strategy in which the retailer keeps a portion of the initial capital to handle repayment shortfalls. When adopting the cash flow matching, the retailer consistently favors scheme P and scheme A over scheme L. Finally, the numerical examples demonstrate the influence of cash flow matching on the retailer's performance. Remarkably, the retailer's choice to forgo cash flow matching is unlikely to be realized in practice, as the risk of overdue penalties could potentially lead to bankruptcy before generating any sales revenue.

The contributions of this paper can be summarized as threefold. First, we examine the retailer's financing and ordering decisions concerning three types of platform financing schemes. Notably, the installment schemes are subject to a penalty mechanism for overdue repayment, mirroring real-business practices. Second, the paper characterizes the conditions under which the retailer prefers a specific financing scheme, influenced by factors such as the overdue penalty interest rate and initial capital level. Third, the paper proposes a viable strategy to help retailers synchronize their cash flows by retaining a portion of the initial capital. This cash flow matching strategy consistently favors the installment schemes due to their lower financing costs. In summary, this paper uncovers a fundamental relationship between platform financing, ordering decisions, and the implementation of a cash flow matching strategy for retailers.

The remainder of this paper is organized as follows. We review the related literature in the next section, followed by the model setup in Section 3. Sections 4 and 5 formulates the decision models of three financing schemes without and with cash flow considerations, respectively. Section 6 presents numerical examples. Section 7 summarizes the findings and insights of this study. All proofs are relegated to Appendix A.

2. Literature review

This paper contributes to the growing body of research on platform financing and is particularly relevant to the works of Barron [1] and Wang *et al.* [34]. From the perspective of financing source, both this paper and Wang *et al.* [34] share a focus on platform financing. The main difference is that we first introduce two types of installment repayment schemes commonly used in platform financing. This approach enables us to investigate both the retailer's optimal ordering decision and the selection of repayment schemes. Moreover, we also consider an overdue penalty mechanism in that it will be incurred due to delayed repayments and lack in meeting demands for cash flows. From the perspective of the retailer's cash flow management, Barron [1] and this paper both examine the optimal amount of cash that firms should hold to align cash inflows with outflows. However, their work focuses on discrete-time cash balance problems, whereas our study addresses a continuous-time cash flow matching problem that arises when a retailer acts as a newsvendor.

Our paper is related to the literature that studies the platform financing mode [2, 8, 9, 26, 35]. An influential work by Tunca and Zhu [33] investigates how a platform (JD.com) can facilitate loan acquisition for upstream suppliers or downstream retailers. Building on their research, many scholars have shifted their focus towards comparing platform financing with other financing modes [27]. More specifically, Wang *et al.* [34] argue that active platform financing yields greater benefits compared to bank financing. Cai and Yan [3] characterize the financing preferences of two competing retailers by comparing bank financing with platform financing. Their study reveals that both retailers opt for platform financing when the platform charges a high commission rate. Zhen *et al.* [41] provide a capital-constrained manufacturer's financing choice between bank financing, buyer credit financing, and platform financing. Yan *et al.* [36] investigate a retailer's financing channel selection including bank financing, trade credit financing, and platform financing. The results show that the platform is not always willing to provide financing service to the retailer. Gupta and Chen [10] analysis a novel platform financing where platform provides different loan terms to suppliers with vary debt seniority, and the results show that this platform financing weakly improves the expected profits of both parties. Ma *et al.* [20] study a portfolio financing approach where the platform helps its retailer obtain joint loan financing from the platform and the bank. The analytical results unveil that the platform will offer free service for recommending the capital-constrained retailer to bank. Chang *et al.* [5] compare the online retailer's preference between platform financing and bank financing, and find that platform financing is the better choice when the retailer's initial capital is high. Our study differs from previous research by focusing on the retailer's selection of platform financing schemes that offer installment repayment options.

This paper also sets itself apart from traditional cash flow matching studies [12, 16, 18, 23, 25, 28, 39]. Tsai [29] measures the cash flow risk using the standard deviation of net cash flow and demonstrates that early repayment discounts offered by creditors can lower the risk for members of the supply chain. Building on this fundamental work, Tsai [30] and Tsai [31] extend this research by formulating a stochastic optimization model that captures the cash flow risk arising from delayed collection within cash constraints. Barron [1] examines the optimal cash holding strategy that firms should adopt to align cash inflows with outflows, which helps mitigate cash flow risk in the supply chain. In contrast, our study focuses on the interplay between the penalty for cash deficit behavior and cash flow matching in the context of platform financing.

3. Model setup

This paper analyze the financing scheme selection of a capital-constrained retailer who sells a single product on an e-commercial platform. The retailer, as a buyer, decides the order quantity and financing scheme at the beginning of the selling season. Then, the retailer sells the product at a fixed price, and any unmet demand is lost. To simplify our analysis and reduce mathematical complexity, we assume that the residual value of unsold product is zero, which does not affect our main results. The same setting can be found in Huang *et al.* [11] and Panda *et al.* [21]. As a financing service provider, the e-commercial platform offers three financing schemes as we mentioned before, namely scheme L, scheme P and scheme A.

To measure cash flow and prepare financial statements, we assume that the retailer's loan must be repaid in two subperiod under two installment scheme P and A. For scheme P, the principal and interest are constant and consistent in each subperiod; For scheme A, the retailer repays a fixed principal amount, but the interest decreases gradually. Additionally, the repayment period coincides with the sales period of $2T$. The same assumption can be found in Chang and Rhee [4], Yang *et al.* [37] and Zhang *et al.* [38]. The details of the retailer's cash flow changes when adopting the installment schemes are as follows:

(1) In $t \in [0, T]$: The retailer decides to order quantity Q and finances its order from the platform, then pays cQ in full cash to an upstream supplier. Let c be the retailer's unit ordering cost, and B the initial capital level. The retailing price is normalized to 1. The retailer repays the required repayment amount L_i^{Ω} at repayment point i, $\Omega = P, A$. If the retailer's sales income fails to cover the loans, the overdue penalty interest will be charged to the platform daily.
(2) In $t \in [T, 2T]$: The retailer uses the leftover inventory y to meet market demand of the subperiod 2. If the retailer occurs overdue in subperiod 1, the loan interest and overdue fine $L_1^{\Omega}\,(1 + r_c T_c)$ must be paid before time $2T$. Then, the retailer repays the outstanding loan L_2^{Ω} at time $2T$; otherwise, the retailer will declare bankruptcy and transfer all realized sales revenue to the platform. Figures 1a and b illustrate the cash flow diagrams under installment schemes P and scheme A, respectively.

We next introduce the model notations and assumptions. It is worth noting that the cash flows have a direct impact on the retailer's expected profit and financing scheme selection. Therefore, this paper aims to explore the impact of cash flow on the retailer's ordering policy and platform financing scheme selection, and to establish a fundamental relationship between platform financing, ordering and financing decisions, and cash flow matching strategy. To achieve this goal, we develop a time-related newsvendor model that allows us to derive the equilibria of the financing and ordering decisions for three typical platform financing schemes. Table 1 provides a summary of the related notations used in the model. The basic assumptions of the model are listed below.

Assumption 3.1. *The demand rate of successive subperiod i decreases exponentially with time $D_i(t) = ae^{-\delta t} + \varepsilon_i$ ($a > 0, \delta > 0$), where δ and a are constant values. The random noise variables ε_i in subperiod i are independent and identically distributed with continuous distribution function $F(\cdot)$ and probability density function $f(\cdot)$. The consideration of time-dependent demand is common for deteriorating items such as fashion goods, fruits, and vegetables [13, 32, 37, 40, 42].*

Assumption 3.2. *$r_c > r_p$ represents that the overdue penalty interest rate is higher than the platform's loan interest rate. $r_o > r_p$ indicates that the loan interest rate under scheme P and scheme A is lower than that of scheme L. Providing installments schemes is for encouraging the borrower to repay partial debt liabilities in advance, thus the loan interest rate must lower than that of the one-time repayment at the end of the period.*

Assumption 3.3. *$c\,(1 + 2r_oT) < p$ avoids the unreasonable situation in which the retailer is still incapable of repaying full loans even though selling all products at the end of selling season. The same setting can be found in Kouvelis et al. [14, 15].*

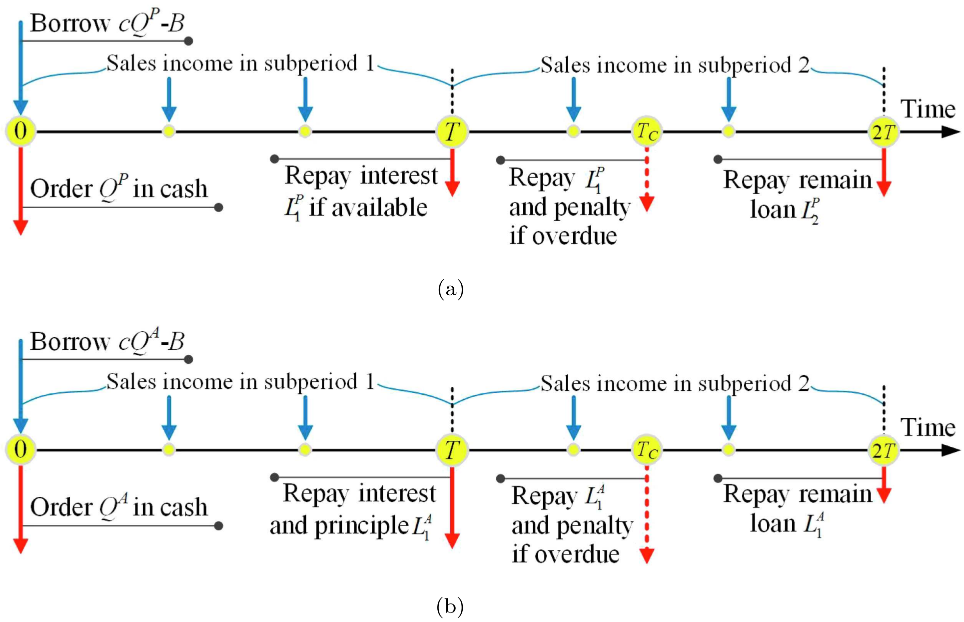


FIGURE 1. Cash flow diagrams under two schemes. (a) Cash ow diagram under scheme P. (b) Cash ow diagram under scheme A.

TABLE 1. Summary of notations.

Notation	Definition
i	Subscript for selling subperiods or repayment points, $i = 1, 2$
Ω	Superscript for two installment financing schemes, $\Omega = P, A$
T	Sales subperiod duration or repayment cycle
c	The retailer's unit ordering cost
B	The retailer's initial capital level
$D_i(t)$	The demand rate in the sales subperiod i
$d_i(t)$	The cumulative deterministic demand in the sales subperiod i
ε_i	The random noise demand in sales subperiod i
r_o	The platform's loan interest rate under scheme L
r_p	The platform's loan interest rate under scheme P and scheme A
r_c	The platform's overdue penalty interest rate
Q	The retailer's order quantity
y	The leftover inventory quantity in the end of subperiod 1
L_i^Ω	The required repayment amount in subperiod i under scheme Ω
T_c	The time point for overdue repayment, where 2T $\geq T_c \geq$ T
K_R	The retailer's expected profit under profit maximization
$\widetilde{K_R}$	The retailer's terminal wealth level under cash flow matching

4. Optimal financing and ordering policy without cash flow considerations

In this section, we formulate the decision-making models under scheme L, P and A respectively. We then derive the optimal order quantity that maximizes the retailer's terminal profit without considering cash flow management. Furthermore, the comparative analyses are conducted to explore the selection of three financing schemes.

4.1. Decision model with scheme L

We now describe scheme L as a benchmark. Following the work of Yang *et al.* [37], we develop the product demand rate as a time dependent function $D^L(t) = ae^{-\delta t} + \varepsilon$. The cumulative deterministic demand is $d^L(t) = \int_0^t ae^{-\delta s}\, ds = \frac{a}{\delta}\left(1 - e^{-\delta t}\right)$ over the period $[0,\ 2T]$. The random noise variable ε follows continuous distribution G$(\cdot)$ with density function g$(\cdot)$. The required repayment amount at time 2T is equal to $(cQ^L - B)(1 + 2r_oT)$. The expected profit of the retailer can be given as:

$$\begin{aligned}\max K_R^L(Q^L) = &\int_0^{2T} \int_0^{Q^L - d^L(t)} (ae^{-\delta t} + \varepsilon) dG(\varepsilon) dt + \int_0^{2T} \int_{Q^L - d^L(t)}^{+\infty} \left[ae^{-\delta t} + Q^L - d^L(t)\right] dG(\varepsilon) dt \\ &- (cQ^L - B)(1 + 2r_oT), \end{aligned} \tag{1}$$

where the first and second terms of K_R^L are the cash inflow (sales revenue) of the retailer when the order quantity is leftover and sold out respectively. The third term of K_R^L is the cash outflow (loan payable) at the end of the selling season.

Proposition 4.1. *Under scheme L, the optimal order quantity Q^{L*} satisfies*

$$\int_0^{2T} \bar{G}[Q^L - d^L(t)]\ dt - c(1 + 2r_oT) = 0.$$

Proposition 4.1 shows that the optimal order quantity under scheme L is decreasing in r_o. Obviously, a high r_o will increase the retailer's financial cost for a product and thus reduces the retailer's order quantity. Moreover, the optimal order quantity under scheme L is independent of B. The reason behind this is the platform's loan is competitively priced. That is, the platform just serves as a money buffer at fixed cost for the retailer. Therefore, we can regard the capital-constrained retailer as the one with sufficient funds but a higher ordering cost. The same result can be found in Yang *et al.* [34]. This outcome further implies that under scheme L, the high B fails to encourage the retailer to order more products. Hence, the retailer's optimal order quantity only depends on the platform's loan interest rate, demand fluctuations and unit ordering cost.

4.2. Decision models with scheme P and A

Under scheme P and A, the selling period is divided into two subperiods considering the nature of the installment repayment schemes, as we stated in Section 3.1. Thus, we formulate a two-subperiod newsvendor model to maximize the retailer's expected profit. Recall our settings in Assumption, the market demand in subperiod i is $D_i^{\Omega}(t) = ae^{-\delta t} + \varepsilon_i$ over the interval $t_1 \in [0, T)$ and $t_2 \in [T, 2T]$. The cumulative deterministic part of demands $d_i^{\Omega}(t_i)$ are shown below:

$$d_1^{\Omega}(t_1) = \int_0^{t_1} ae^{-\delta s} ds = \frac{a}{\delta}\left(1 - e^{-\delta t_1}\right), \qquad t_1 \in [0, T]. \tag{2}$$

$$d_2^{\Omega}(t_2) = \int_{t_2}^{2T} ae^{-\delta s} ds = \frac{a}{\delta}\left(e^{-\delta T} - e^{-\delta t_2}\right), \quad t_2 \in [T, 2T]. \tag{3}$$

Therefore, the retailer's cash inflow (expected sales income) is.

$$
\begin{aligned}
CI^{\Omega} = & \underbrace{\int_0^T \int_0^{Q^{\Omega}-d_1^{\Omega}(T)} \left\{ \int_T^{2T} \int_0^{\left[y-d_2^{\Omega}(2T)\right]^+} \left(ae^{-\delta t_1} + \varepsilon_1 + ae^{-\delta t_2} + \varepsilon_2\right) dF\left(\varepsilon_2\right) dt_2 \right\} dF\left(\varepsilon_1\right) dt_1}_{\text{Leftover}} \\
& + \underbrace{\int_0^T \int_0^{Q^{\Omega}-d_1^{\Omega}(T)} \left\{ \int_T^{2T} \int_{\left[y-d_2^{\Omega}(2T)\right]^+}^{+\infty} \left[ae^{-\delta t_1} + \varepsilon_1 + ae^{-\delta t_2} + y - d_2^{\Omega}(2T)\right] dF\left(\varepsilon_2\right) dt_2 \right\} dF\left(\varepsilon_1\right) dt_1}_{\text{Sold out in subperiod 1}} \\
& + \underbrace{\int_0^T \int_{Q^{\Omega}-d_1^{\Omega}(T)}^{+\infty} \left[ae^{-\delta t_1} + Q^{\Omega} - d_1^{\Omega}(T)\right] dF\left(\varepsilon_1\right) dt_1}_{\text{Sold out in first subperiod 2}} \qquad (4)
\end{aligned}
$$

where the first, second, and third terms of CI^{Ω} are the cash inflow when the order quantity is leftover and used up in subperiod 1 and 2. The stock level at the end of the first subperiod is $y = Q^{\Omega} - d_i^{\Omega}(T) - \varepsilon_i$. The selling process runs into the second subperiod when $y > 0$. Let $x^+ = \max\{0,\ x\}$.

We next model the retailer's cash outflows including loan payable and overdue penalty for retailer. When choosing scheme P, the retailer repays the same loan amount $L_1^P = L_2^P = \frac{r_pT(cQ^P-B)(1+r_pT)^2}{(1+r_pT)^2-1}$ respectively; when selecting scheme A, the retailer first paybacks $L_1^A = (cQ^A - B)(\frac{1}{2} + r_pT)$, then pays $L_2^A = \frac{1}{2}(cQ^A - B)(1 + r_pT)$. Moreover, the random market demand in first subperiod ε_1 determines whether the retailer can avoid the overdue penalty. When $\varepsilon_1 + d_1^{\Omega}(T) < L_1^{\Omega}$, the retailer will trigger the overdue penalty for failure to repay the required loans at time T; otherwise, if $\varepsilon_1 + d_1^{\Omega}(T) > L_1^{\Omega}$, the cash inflow can fully cover the loan payment. Thus, the retailer's cash outflows and total expected profit can be given as:

$$
CO^{\Omega} = \underbrace{\int_0^T \int_{L_1^{\Omega}-d_1^{\Omega}(T)}^{+\infty} \left(L_1^{\Omega} + L_2^{\Omega}\right) dF\left(\varepsilon_1\right) dt_1}_{\text{Non-overdue}} + \underbrace{\int_0^T \int_0^{L_1^{\Omega}-d_1^{\Omega}(T)} \left\{\left[1 + r_c\left(T_c - T\right)\right] L_1^{\Omega} + L_2^{\Omega}\right\} dF\left(\varepsilon_1\right) dt_1}_{\text{overdue}} \qquad (5)
$$

$$
\max K_R^{\Omega}(Q^{\Omega}) = CI_R^{\Omega} - CO_R^{\Omega} \qquad (6)
$$

where the first term of CO^{Ω} is the total loan payable when the retailer successes in repaying the middle loan. The second term represents the loan payable and overdue penalty paid at the time $T_c > T$ when the retailer triggers overdue. If the retailer is still unable to repay the loans and accumulated penalty until 2T, the retailer goes bankrupt and pays all realized profit to the platform.

Proposition 4.2. *(i) Under scheme P and A, the optimal order quantities Q^{P*} and Q^{A*} satisfy:*

$$
\int_0^T \left[\bar{F}\left[Q^{P*} - d_1^P(T)\right] + \int_0^{Q^{P*}-d_1^P(T)} \int_T^{2T} \bar{F}\left[y - d_2^P(2T)\right] dt_2 dF\left(\varepsilon_1\right)\right] dt_1 - R_c\mu\left(Q^{P*}\right) = \frac{2c\left(1 + r_pT\right)^2}{2 + r_pT},
$$

$$
\int_0^T \left[\bar{F}\left[Q^{A*} - d_1^A(T)\right] + \int_0^{Q^{A*}-d_1^A(T)} \int_T^{2T} \bar{F}\left[y - d_2^A(2T)\right] dt_2 dF\left(\varepsilon_1\right)\right] dt_1 - R_c\mu\left(Q^{A*}\right) = c\left(1 + \frac{3}{2}r_pT\right),
$$

where $\mu\left(Q^{\Omega*}\right) = \int_0^T \left[F(L_1^{\Omega*} - d_1^{\Omega}(T)) + L_1^{\Omega*} f(L_1^{\Omega*} - d_1^{\Omega}(T))\right] dt_1$ and $R_c = cr_pr_cT\left(T_c - T\right)$.

(ii) $\frac{\partial Q^{\Omega*}}{\partial r_c} < 0$, $\frac{\partial K_R^{\Omega*}}{\partial r_c} < 0$, $\frac{\partial Q^{\Omega*}}{\partial B} > 0$, $\frac{\partial K_R^{\Omega*}}{\partial B} > 0$.

Proposition 4.2 (i) indicates that the retailer's optimal order quantity under installment schemes (scheme P and A) not only depend on the platform loan interest rate (r_p), but also relies on the initial capital level

(B) and the overdue penalty interest rate (r_c), which differs from the scheme L. Proposition 4.2 (ii) reveals that the retailer's optimal order quantity and expected terminal profits under installment schemes decrease with the r_c. Thus, a high r_c will increase the retailer's penalty of cash deficit, which motivates the retailer to reduce the order quantity. Furthermore, the optimal order quantity and expected terminal profits under installment schemes increase with the B. A high B will decrease the financial cost owing to overdue penalty, which encourage the retailer to order more products. Combining our finding in Proposition 4.1, the optimal order quantity is independent of the initial capital level under scheme L. We conclude that the retailer's initial capital can alleviate the ordering pressure due to installment repayment and overdue penalty. Thereby, the retailer can enjoy better returns on inventory investments.

4.3. Comparative analysis

This subsection compares the three financing schemes without taking cash flow management into consideration. When the retailer does not adopt a cash flow matching strategy, the primary objective of the ordering and financing decisions is to maximize the terminal profit. Finally, we provide a graphical representation to clarify the conditions under which the retailer would prefer the three repayment schemes.

Proposition 4.3. *Only when $r_c \geq \overline{r_c}$ and $B < B_0$, scheme L is the unique financing equilibrium; When $\overline{r_c} > r_c \geq \underline{r_c}$ and $B > B_0$, scheme P is the unique financing equilibrium; Otherwise, when $r_c < \underline{r_c}$, scheme A is the unique financing equilibrium.*

Proposition 4.3 emphasizes that scheme L is the financing equilibrium only when the overdue penalty interest rate is much high and the initial capital is much low. When $r_c \geq \overline{r_c}$ and $B < B_0$, the retailer is highly averse to overdue penalties and therefore prefers scheme L; When $\overline{r_c} > r_c \geq \underline{r_c}$ and $B > B_0$, the retailer is willing to undertake relatively low overdue penalties and select scheme P to reduce financing costs and obtain a greater terminal profit. When $r_c < \underline{r_c}$, the retailer is more willing to accept overdue risk. From Proposition 4.2, the retailer's terminal profit decreases with the overdue penalty interest rate. Therefore, compared with scheme P, the retailer with capital constraints prefers scheme A to enjoy a greater terminal profit.

We assume that the market demands in subperiod i follow an exponential distribution. The mean value is 100, $c = 0.45$, $T = 0.5$, $B = 10$, $\delta = 0.001$, $a = 10$. Figure 2 shows how $\overline{r_c}$ and $\underline{r_c}$ change with B under terminal profit maximization. Note first that only when r_c falls into the region between the blue and red lines, scheme P is the unique financing equilibrium. When r_c and B are above the blue line, scheme L becomes the financing equilibrium. Otherwise, when both values are below the red line, scheme A is the unique financing equilibrium. Figure 2 conveys the message that the threshold of $\overline{r_c}$ and $\underline{r_c}$ increases with B. The scheme A is always the unique equilibrium when r_c is lower than a certain critical value ($r_c > 0.078$). It implies that the retailer can obtain a more profitable financing equilibrium by choosing a radical repayment scheme when the punishment is weak.

5. Optimal financing and ordering policy under cash flow matching

As discussed in the previous section, installment repayment schemes may not be the best option when the overdue penalty is significant. The potential increase in the retailer's financing cost due to overdue penalties could make installment repayment schemes less attractive. To address this issue, we propose a cash flow matching (CFM) mechanism in which the retailer sets aside a portion of the initial capital to avoid any potential overdue penalties. The central research question now becomes whether the retailer can improve its performance by adopting the cash flow matching mechanism.

5.1. Decision models under cash flow matching strategy

Under P-CFM and A-CFM, the retailer borrows the platform's loan $c\widetilde{Q^\Omega} - B + \widetilde{B^\Omega}$ and sets retained capital $\widetilde{B^\Omega}$ to cover possible repayment shortfalls. The probability of overdue repayment at time T changes to

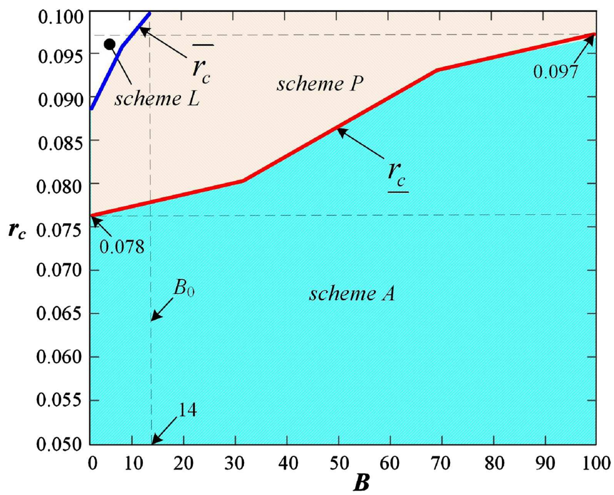


FIGURE 2. $\overline{r_c}$ and $\underline{r_c}$ change with B under terminal profit maximization.

$\left[\widetilde{L_1^\Omega} - d_1^\Omega(T) - \widetilde{B^\Omega} - \varepsilon_1\right]^+$. At the beginning of the selling season, the retailer decides the retained capital and order quantity to maximize the terminal wealth level. The excepted terminal wealth level under P-CFM and A-CFM can be given as:

$$\max \widetilde{K_R^P}\left(\widetilde{Q^P}, \widetilde{B^P}\right) = \widetilde{CI^P} - \frac{2\left(c\widetilde{Q^P} - B + \widetilde{B^P}\right)(1+r_pT)^2}{2+r_pT} + \left[\widetilde{B^P} - \int_0^T \int_0^{\widetilde{L_1^P} - d_1^P(T) - \widetilde{B^P}} R_c \widetilde{L_1^P} dF(\varepsilon_1) dt_1\right] \tag{7}$$

$$\max \widetilde{K_R^A}\left(\widetilde{Q^A}, \widetilde{B^A}\right) = \widetilde{CI^A} - \frac{(2+3r_pT)(c\widetilde{Q^A} - B + \widetilde{B^A})}{2} + \left[\widetilde{B^A} - \int_0^T \int_0^{\widetilde{L_1^A} - d_1^A(T) - \widetilde{B^A}} R_c \widetilde{L_1^A} dF(\varepsilon_1) dt_1\right] \tag{8}$$

where the first term of $\widetilde{K_R^P}$ and $\widetilde{K_R^A}$ is the retailer's sales revenue, which expresses in the same form as CI^Ω in Section 4.2. The second term of $\widetilde{K_R^P}$ and $\widetilde{K_R^A}$ is the loan repayment amount when the retailer occurs no overdue repayment. The last terms of $\widetilde{K_R^P}$ and $\widetilde{K_R^A}$ represent the possible overdue penalty after setting the retained capital.

Theorem 5.1. *(i) Under P-CFM, optimal order quantity $\widetilde{Q^{P*}}$ and retained capital $\widetilde{B^{P*}}$ satisfy:*

$$\begin{cases} \int_0^T \left[\bar{F}\left(\widetilde{Q^{P*}} - d_1^P(T)\right) + \int_0^{\widetilde{Q^{P*}} - d_1^P(T)} \int_T^{2T} \bar{F}\left(y - d_2^P(2T)\right) dt_2 dF(\varepsilon_1)\right] dt_1 + (1-R_C) PY = \frac{2c(1+r_pT)^2}{2+r_pT} \\ R_c \int_0^T \left[\frac{(1+r_pT)^2}{2+r_pT} F\left(\widetilde{L_1^P} - d_1^P(T) - \widetilde{B^{P*}}\right) + \frac{2-(1-r_pT)^2}{2+r_pT} \widetilde{L_1^P} f\left(\widetilde{L_1^P} - d_1^P(T) - \widetilde{B^{P*}}\right)\right] dt_1 = \frac{2(1+r_pT)^2 - (2+r_pT)}{2+r_pT} \end{cases}$$

where $PY = \int_0^T \left[F\left(\widetilde{L_1^P} - d_1^P(T) - \widetilde{B^P}\right) + \widetilde{L_1^P} f\left(\widetilde{L_1^P} - d_1^P(T) - \widetilde{B^P}\right)\right] dt_1$,

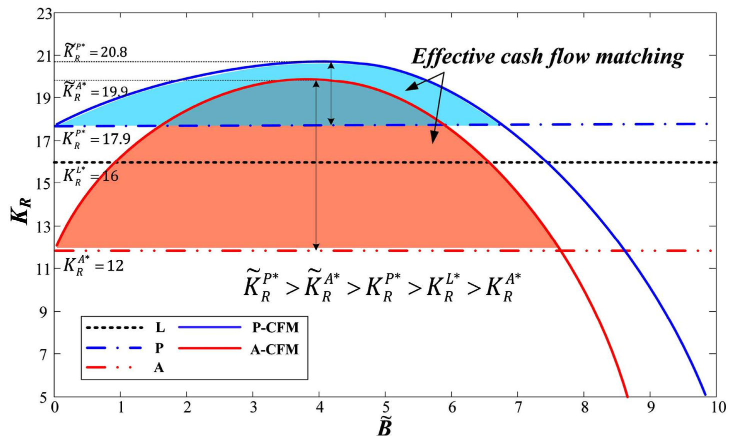


FIGURE 3. K_R changes with $\widetilde{B}$.

(ii) Under A-CFM, the optimal order quantity $\widetilde{Q^{A*}}$ and retained capital $\widetilde{B^{A*}}$ satisfy:

$$\begin{cases} \int_0^T \left[\bar{F}\left(\widetilde{Q^{A*}}-d_1^A(T)\right)+\int_0^{\widetilde{Q^{A*}}-d_1^A(T)}\int_T^{2T}\bar{F}\left(y-d_2^A(2T)\right)dt_2dF(\varepsilon_1)\right]dt_1+(1-R_C)FY=c(1+\frac{3}{2}r_pT) \\ R_c\int_0^T\left[(1-2r_pT)F\left(\widetilde{L_1^A}-d_1^A(T)-\widetilde{B^{A*}}\right)+(3-2r_pT)\widetilde{L_1^A}f\left(\widetilde{L_1^A}-d_1^A(T)-\widetilde{B^{A*}}\right)\right]dt_1=3r_pT \end{cases}$$

where $FY=\int_0^T\left[F\left(\widetilde{L_1^A}-d_1^A(T)-\widetilde{B^{A*}}\right)+\widetilde{L_1^A}f\left(\widetilde{L_1^A}-d_1^A(T)-\widetilde{B^{A*}}\right)\right]dt_1.$

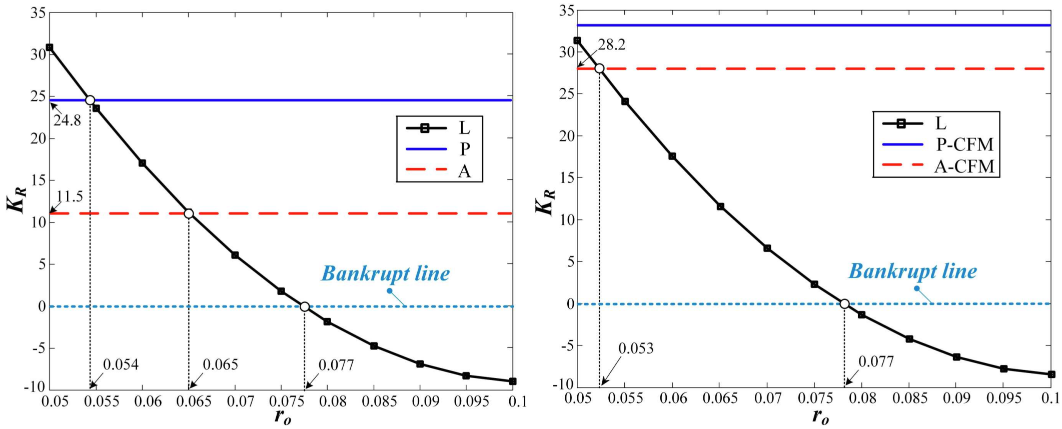
Theorem 5.1 proves that the retailer can retain its partial capital to address potential repayment shortfalls. Since the retailer's excepted terminal wealth level $\widetilde{K_R^{\Omega}}$ is jointly concave in retained capital $\widetilde{B^{\Omega}}$ and order quantity $\widetilde{Q^{\Omega}}$, there exists the unique optimal retained capital under CFM. This is because when possible overdue penalty occurs, the retailer can utilize the retained initial capital to repay, thereby reducing its financing costs. This also implies that the retailer is always willing to retain an appropriate capital to hedge potential cash flow risks when choosing installment schemes.

5.2. Comparative analysis under cash flow matching strategy

In this section, we discuss the retailer's selection of three financing schemes with considering CFM. Specially, we compare two installment scheme P-CFM and A-CFM with scheme L graphically.

Proposition 5.2. *When the cash flow matching is activated, the retailer always prefers scheme P and A to scheme L.*

Due to the complexity of mathematics, we use numerical analysis to compare the retailer's preference of financing schemes under the objective of profit maximization and CFM. Based on the parameter values in Figure 3 and $r_c=0.097$, we drew Figure 3 to examine how the retailer's terminal wealth level $\widetilde{K_R}$ changes with retained capital $\widetilde{B}$. The retailer's expected terminal wealth level first increase, then decrease with the retained capital. Under P-CFM and A-CFM, the retailer's optimal retained capital and terminal wealth level are (4.2,20.8) and (3.9,19.9) respectively. Without retaining the initial capital, the scheme L is the optimal scheme. However, when the retailer adopts CFM and sets the optimal retained capital, the P-CFM scheme is always better than the others. This result demonstrates that the retailer facing capital constraint can mitigate cash flow risks and avoid overdue penalties by implementing a retained capital strategy. By allocating the initial capital effectively, the retailer can efficiently match their cash flows and enhance their performance.



(a) (b)

FIGURE 4. Impacts of r_o on K_R and $\widetilde{K_R}$. (a) K_R changes with r_o without CFM. (b) $\widetilde{K_R}$ changes with r_o under CFM.

6. Numerical analysis

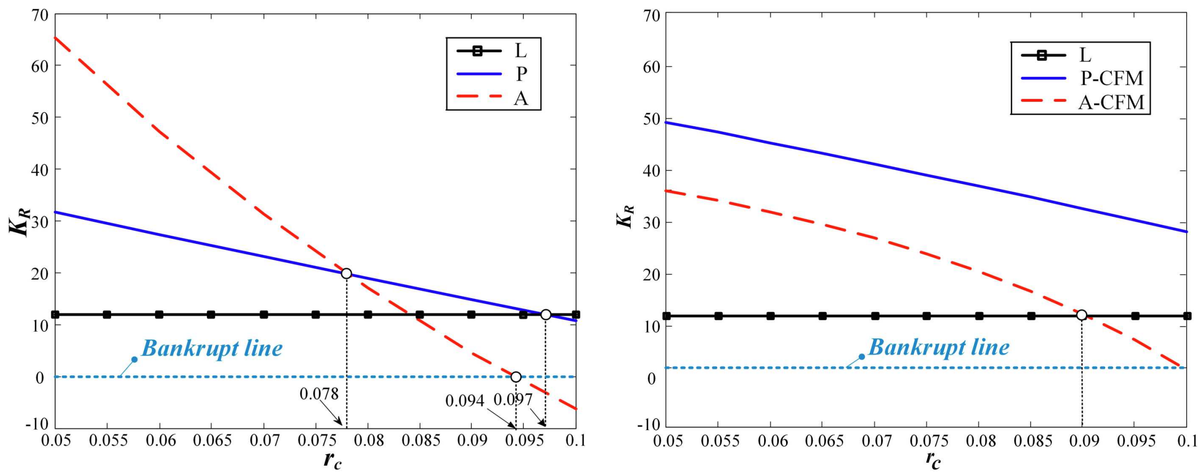
Although we have provided analytical descriptions of the optimal solutions for each scheme, comparing the retailer's financing repayment preference between profit maximization and CFM has proven to be challenging. Therefore, we report numerical examples to glean further managerial insights. We fix the following parameters over the entire numerical studies: (1) the random variables ε_i follow an exponential distribution with mean value 100, $\varepsilon_i \sim E(100)$; (2) $c = 0.45$, $T = 0.5$, $B = 10$, $\delta = 0.001$, $a = 10$, $r_0 = 0.06$, $r_p = 0.05$, $r_c = 0.075$ [2,5,6,38,39].

6.1. Impact of the platform's loan interest rate

We reveal how the retailer's performance changes with the platform's loan interest rate $r_o \in [0.05,\ 0.1]$ with and without the CFM, respectively. In Figure 4a, the expected profits decrease with r_o under scheme L, and keeps unchanged under scheme P and A. When r_o is at a low level ($r_o \leq 0.054$), scheme L is the best choice for the retailer; With an increase in r_o, scheme P always dominates the other two schemes when $0.065 > r_o > 0.054$. When r_o reaches a sufficiently high level $r_o > 0.077$, choosing scheme L will go bankrupt. This outcome indicates that only if the loan interest rate under scheme L is sufficiently low, the retailer prefers scheme L instead of installment schemes. Figure 4b shows that, when the retailer adopts CFM, the scheme P always dominates the others irrespective of the value of loan interest rate. Implementing the CFM can effectively reduce the retailer's financial expenses. The cost-saving effect bring by matching cash flows outweighs the advantage of low interest rate by selecting the scheme L.

6.2. Impact of the overdue penalty interest rate

Figure 5a performs that the retailer's profit decreases with the overdue penalty interest rate r_c under two installment schemes, which proves the correctness of Proposition 4.3. When r_c is at a low level ($r_c < \underline{r_c} = 0.078$), the optimal profit of scheme A is greater than those of scheme P and scheme L. Given a certain value of r_c at a middle range ($\underline{r_c} < r_c < \bar{r_c} = 0.097$), the scheme P dominates the others. Additionally, when setting r_c at a high level ($r_c > \bar{r_c}$), the retailer regards scheme L as the optimal financing scheme. The retailer faces two opposite effects when adopting installment repayment schemes. With the increase of penalty interest rate, the negative effect of overdue penalty accumulation overcomes the positive effect of financial cost-saving, resulting in a sharp decline in profits and even bankrupt (when $r_c > 0.094$).



(a) (b)

FIGURE 5. Impacts of r_c on K_R and $\widetilde{K_R}$. (a) K_R changes with r_c without CFM. (b) $\widetilde{K_R}$ changes with r_c under CFM.

Figure 5b presents that the P-CFM scheme is always optimal. The same findings as before, terminal wealth level decreases penalty interest rate under two installment schemes. Combine with the previous findings in Figure 5a, it sheds lights on the optimal profit of P-CFM scheme may be lower than that of scheme A with profit maximization. However, the cash flow matching makes P-CFM effectively avoid bankruptcy. This discovery further proves that, if firms only focus on profits and ignore cash flow risk, it will lead to the possible bankrupt halfway.

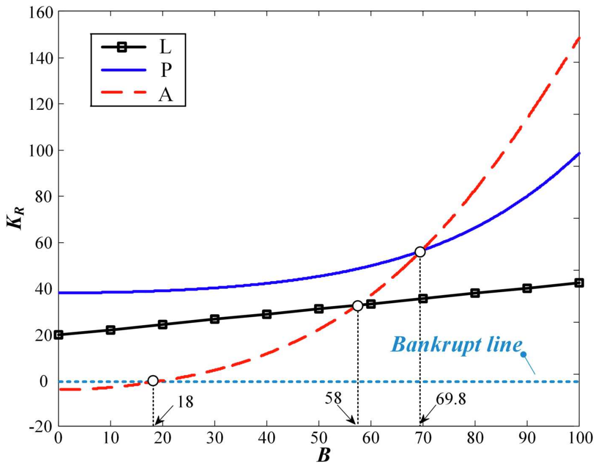
6.3. Impact of the initial capital level

We now research on the impact of the input-output change of initial capital level on scheme selection. We maintain the same settings used in prior subsections and $r_c = 0.075$ (a low level). Figure 6a implies that, when the retailer obtains a sufficiently high initial capital level ($69.8 < B \leq 100$), the retailer is better off selecting scheme A. The relatively abundant initial capital reduces the financing cost and the possibility of overdue repayment caused by insufficient cash flow. In the opposite case, if retailer's the initial capital level is less than a threshold ($B \leq 69.8$), scheme P is a better alternative than scheme A. The retailer obtains inefficient returns owing to loan payables and penalties. Thence, we know that ample initial capital can efficiently avoid overdue financial penalties.

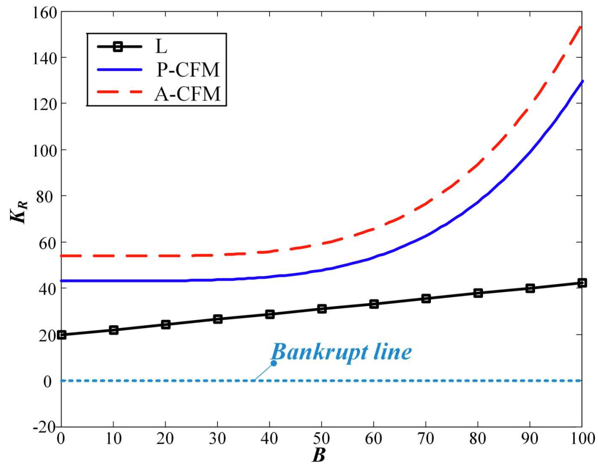
Figure 6b analyzes the retailer's financing schemes selection through the cash flow matching side. Regardless of the variation in initial capital level, adopting A-CFM is preferable to the others. In this case, the edge of financing cost-saving outweighs the negative effect of the overdue penalty. It proves that, implementing perfect cash flow matching will maximize the use of firm's limited initial capital, and tradeoff between chasing profit maximization and staying long-term alive.

7. Conclusions and managerial insights

In this paper, we study the retailer's order and financing decisions in which the capital-constrained retailer resorts to one of three platform financing schemes to fund her business. Our study reveals three main findings: First, we reveal that, for the case without cash flow considerations, two key parameters c can be employed to describe the optimal selection of a retailer's financing scheme. More specifically, only when the overdue penalty interest rate is extremely high and the initial capital level is much low, the scheme L becomes the preferred



(a)



(b)

FIGURE 6. Impacts of r_c on B and $\widetilde{K_R}$. (a) K_R changes with B without CFM. (b) $\widetilde{K_R}$ changes with B under CFM.

financing scheme under the equilibrium. When the overdue penalty interest rate is moderate, the retailer prefers scheme P, otherwise, the retailer selects scheme A when the overdue penalty interest rate is low. Second, the results also reveal that, with the adoption of the cash flow matching strategy, the retailer consistently favors schemes P and A over scheme L. The comparative analysis indicates that opting for the cash flow matching strategy allows the retailer to reserve a portion of their initial capital for handling possible repayment gaps. This proactive approach mitigates the risk of bankruptcy and leads to improved performance. Third, the numerical examples provide additional evidences that, despite the potential for greater expected profit, the retailer's decision to forego cash flow matching is unlikely to be realized in practice due to the risk of overdue penalties leading to bankruptcy before sales revenue can be generated.

Our paper also provides several novel managerial insights. First, implementing cash flow matching may not achieve greater expected profit for the retailer. However, in real business, choosing not to implement cash flow matching may result in the retailer to bankruptcy in the middle of selling season. The capital-constrained retailer should tailor their financing and ordering choices according to their current cash flow circumstances. Second, the retailer who begin with sufficient initial capital can effectively prevent overdue financial penalties and the risk of bankruptcy by retaining a portion of their initial funds. Such a measure empowers retailers to make more flexible adjustments to their cash flows. Lastly, by implementing installment repayment schemes within the framework of cash flow matching, retailers can alleviate the burden of repayments. This, in turn, empowers them to place larger product orders, elevate their terminal wealth, and broaden their market reach.

There are several promising directions for future research in this area. Firstly, investigating the loan interest rate decisions made by platforms would provide valuable insights. Secondly, considering the possibility of retailers borrowing from external lenders alongside platforms would require determining the optimal financing portfolio that aligns with real-world scenarios. This entails the need for retailers to identify the financing mix that is most realistic. Thirdly, exploring the design of mechanisms that accelerate cash flow and mitigate cash flow risks for participants in the supply chain would be highly valuable. Additionally, delving into the competitive behaviors of platforms in the loan services domain presents an intriguing area of study. Lastly, exploring the optimal financing policies for retailers in multi-period decision-making scenarios may introduce complex issues concerning cash flow management.

Appendix A.

Proof of Proposition 4.1. The first-order condition of K_R^L with respect to Q^{L*} yields: $\frac{\partial K_R^L}{\partial Q^L} = \int_0^{2T} g\left(Q^L - d^L(t)\right)\left(ae^{-\delta t} + Q^L - d^L(t)\right)dt + \int_0^{2T}\int_{Q^L-d_o}^{+\infty} g\left(\varepsilon\right)d\varepsilon dt - \int_0^{2T} g\left(Q^L - d^L(t)\right)\left(ae^{-\delta t} + Q^L - d^L(t)\right)dt - c\left(1+2r_oT\right) = \int_0^{2T}\bar{G}\left(Q^L - d^L(t)\right)dt - c\left(1+2r_oT\right)$. Since $\frac{\partial^2 K_R^L}{\partial (Q^L)^2} = -\int_0^{2T} g\left(Q^L - d^L(t)\right)dt < 0, K_R^L(Q^L)$ is a concave function. From $\frac{\partial K_R^L}{\partial Q^{L*}} = 0$, we know that Q^{L*} satisfying $\int_0^{2T}\bar{G}\left[Q^L - d^L(t)\right]dt - c\left(1+2r_oT\right) = 0$. We denote that $H\left(Q^L\right) = \int_0^{2T}\bar{G}\left(Q^L - d^L(t)\right)dt - c\left(1+2r_oT\right)$. Moreover, we obtain $\lim H\left(Q^L\right) = -c\left(1+2r_oT\right) < 0$ and $\lim H\left(Q^L\right) = 2T - c\left(1+2r_oT\right)$. Recall our setting in Assumption , it is clear that $\lim H\left(Q^L\right) > 0$. Thus, we demonstrate the existence of the unique optimal order quantity under scheme L. □

Proof of Proposition 4.2. (i) Similar to the proof of Proposition 4.1, the derivative of K_R^P with respect to Q^P gets: $\frac{\partial K_R^P}{\partial Q^P} = \int_0^T \bar{F}\left(Q^P - d_1\right)dt_1 + \int_0^T\int_0^{Q^P-d_1}\left\{\int_T^{2T}\bar{F}\left(y-d_2\right)dt_2\right\}dF\left(\varepsilon_1\right)dt_1 - \frac{2c(1+r_pT)^2}{2+r_pT} - R_c\int_0^T\left[F\left(D_1^P - d_1\right) + L_1^P f\left(D_1^P - d_1\right)\right]dt_1$, where $R_c = cr_pr_cT\left(T_c - T\right)$. Furthermore, the second-order condition of K_R^P with respect to Q^P is: $\frac{\partial^2 K_R^P}{\partial (Q^P)^2} = -\int_0^T f\left(Q^P - d_1\right)dt_1 - \int_0^T\int_0^{Q^P-d_1}\left\{\int_T^{2T} f\left(y-d_2\right)dt_2\right\}dF\left(\varepsilon_1\right)dt_1 - \int_0^T\int_T^{2T}\bar{F}\left(d_2\right)dt_1dt_2 - \frac{cr_pT(1+r_pT)^2R_c}{2+r_pT}\int_0^T\left[2f\left(D_1^P - d_1\right) + L_1^P f'\left(D_1^P - d_1\right)\right]dt_1 < 0$. Hence, $K_R^{P*}(Q^P)$ is a concave function of Q^P. Similarly, proving that $K_R^{A*}(Q^A)$ is also concave in Q^A.

(ii) We denote the first-order condition of Q^{P*} with respect to r_c as: $-\frac{\partial Q^P}{\partial r_c}Z\left(Q^P\right) - cr_pT\left(T_c - T\right)\int_0^T\left[F\left(L_1^P - d_1\right) + L_1^P f\left(L_1^P - d_1\right)\right]dt_1 = 0$, where $Z\left(Q^P\right) = \int_0^T f\left(Q^P - d_1\right)dt_1 + \int_0^T\int_0^{Q^P-d_1}\left\{\int_T^{2T} f\left(y-d_2\right)dt_2\right\}dF\left(\varepsilon_1\right)dt_1 + \int_0^T\int_T^{2T}\bar{F}\left(d_2\right)dt_1dt_2 + \frac{cr_pT(1+r_pT)^2R_c}{2+r_pT}\int_0^T\left[2f\left(L_1^P - d_1\right) + L_1^P f'\left(L_1^P - d_1\right)\right]dt_1 > 0$. Furthermore, the first derivative of K_R^{P*} with respect to r_c yields $\frac{\partial K_R^{P*}}{\partial r_c} = \frac{\partial Q^{P*}}{\partial r_c}Z(Q^P) - \int_0^T L_1^P\left(T_c - T\right)F\left(L_1^P - d_1\right)dt_1$. Substituting $\frac{\partial Q^{P*}}{\partial r_c}$ into the above expression, we obtain:

$$\frac{\partial K_R^P}{\partial r_c} = -\left[cr_pT\left(T_c - T\right)\int_0^T\left[F\left(D_1^P - d_1\right) + L_1^P f\left(D_1^P - d_1\right)\right]dt_1\right] - \int_0^T L_1^P\left(T_c - T\right)F\left(L_1^P - d_1\right)dt_1 < 0.$$

Similarly, we have $\frac{\partial Q^{A*}}{\partial r_c} < 0$ and $\frac{\partial K_R^{A*}}{\partial r_c} < 0$.

(iii) By driving the first-order of Q^{P*} with respect to B, we obtain: $-\frac{\partial Q^P}{\partial B}Z\left(Q^P\right) + \frac{cr_pT(1+r_pT)^2R_c}{2+r_pT}\int_0^T\left[f\left(L_1^P - d_1\right) + L_1^P f'\left(L_1^P - d_1\right) + f\left(L_1^P - d_1\right)\right]dt_1 = 0$. It is easy to prove that $\frac{\partial Q^{P*}}{\partial B} = \frac{cr_pTR_c(1+r_pT)^2\int_0^T\left[f\left(L_1^P - d_1\right) + L_1^P f'\left(L_1^P - d_1\right) + f\left(L_1^P - d_1\right)\right]dt_1}{(2+r_pT)Z(Q^P)} > 0$. Furthermore, the first derivative of K_R^{P*} with respect to B yields $\frac{\partial K_R^{P*}}{\partial B} = \frac{\partial Q^{P*}}{\partial B}Z\left(Q^P\right) + r_cr_pT\left(T_c - T\right)\left[\int_0^T F\left(L_1^P - d_1\right)dt_1 + L_1^P\int_0^T f\left(L_1^P - d_1\right)dt_1\right] + \frac{2c(1+r_pT)^2}{2+r_pT}$. Substituting $\frac{\partial Q^{P*}}{\partial B}$ into the above expression obtains $\frac{\partial K_R^{P*}}{\partial B} = \frac{cr_pT(1+r_pT)^2R_c}{2+r_pT}\int_0^T\left[f\left(L_1^P - d_1\right) + L_1^P f'\left(L_1^P - d_1\right) + f\left(L_1^P - d_1\right)\right]dt_1 + r_cr_pT\left(T_c - T\right)\left[\int_0^T F\left(L_1^P - d_1\right)dt_1 + L_1^P\int_0^T f\left(L_1^P - d_1\right)dt_1\right] + \frac{2c(1+r_pT)^2}{2+r_pT} > 0$. Similarly, we obtain $\frac{\partial Q^{A*}}{\partial B} > 0$ and $\frac{\partial K_R^{A*}}{\partial B} > 0$.

□

Proof of Proposition 4.3. Since the expression of Q^{P*} and Q^{A*} in Proposition 4.2 (i) can be regarded as $M\left(Q^{P*}\right)-\frac{2c(1+r_pT)^2}{2+r_pT}=0$ and $M\left(Q^{A*}\right)-c\left(1+\frac{3}{2}r_pT\right)=0$, we divide the equations into two parts. One is a function of Q^{Ω}, the other is a constant value. The comparison of constant value is $\frac{2c(1+r_pT)^2}{2+r_pT}>c(1+\frac{3}{2}r_pT)$, which leads to $M\left(Q^{P*}\right)<M\left(Q^{A*}\right)$. Furthermore, we know that $\frac{\partial M(Q^{\Omega})}{\partial Q^{\Omega}}$ equals to $\frac{\partial^2 K_R^{\Omega}}{\partial (Q^{\Omega})^2}$ from Proposition 4.3, so $\frac{\partial M(Q^{\Omega})}{\partial Q^{\Omega}}<0$. Thus, $Q^{A*}>Q^{P*}$ is always established. Furthermore, we compare the expected profits of scheme A with that of scheme P. Let $\underline{Q}$ satisfies that $K_R^{A*}(\underline{Q})=K_R^{P*}\left(Q^{P*}\right)$ and $\underline{r_c}=J^{-1}(\underline{Q})$. From Proposition 4.3 (i), we know that $\frac{\partial Q^{\Omega*}}{\partial r_c}<0$. Thus, when $r_c<\underline{r_c}$, $KR=KRA*-KRP*>0$ is always established.

In addition, we compare the optimal order quantity. First, we rewrite the expression of Q^{P*} by removing the term $cr_pTR_c\int_0^T\left[F\left(D_1^P-d_1\right)+L_1^Pf\left(D_1^P-d_1\right)\right]dt_1$, then we have $\int_0^{2T}\bar{G}\left(Q^P-d_1-d_2\right)dt-\frac{2c(1+r_pT)^2}{2+r_pT}<0$. Second, we compare the above expression yields $\int_0^{2T}\bar{G}\left(Q^P-d_1-d_2\right)dt-\frac{2c(1+r_pT)^2}{2+r_pT}<\int_0^{2T}\bar{G}\left[Q^L-d^L(t)\right]dt-c\left(1+2r_oT\right)=0$. Hence, we have $\int_0^{2T}\bar{G}\left(Q^P-d_1-d_2\right)dt<\int_0^{2T}\bar{G}\left[Q^L-d^L(t)\right]dt$. Furthermore, we know $d_1+d_2=d^L$ and $\frac{\partial^2(K_R^{\Omega})}{\partial(Q)^2}<0$ from Proposition 4.3, thus we further have $Q^{P*}>Q^{L*}$. Furthermore, we compare the terminal wealth level of scheme L and scheme P. Let $\overline{Q}$ satisfies that $K_R^{L*}(\overline{Q})=K_R^{P*}\left(Q^{P*}\right)$. There is a one-to-one mapping between $\overline{Q}$ and r_c, *i.e.*, $\overline{r_c}=J^{-1}(\overline{Q})$. Similarly, we obtain $B_0=K^{-1}(\overline{Q})$. From Proposition 4.3 (i), we know that $\frac{\partial Q^{P*}}{\partial r_c}<0$, $\frac{\partial Q^{P*}}{\partial B}>0$. Thus, when $r_c>\overline{r_c}$ and $B<B_0$, we find that $\Delta K_{LP}=K_R^{L*}-K_R^{P*}>0$ is always established. □

Proof of Theorem 5.1. (i) From equation (7), the first-order condition of $\widetilde{K_R^P}$ with respect to $\widetilde{Q^P}$ yields: $\frac{\partial\widetilde{K_R^P}}{\partial\widetilde{Q^P}}=\int_0^T\left[\bar{F}\left(\widetilde{Q^P}-d_1\right)+\int_0^{\widetilde{Q^P}-d_1}\int_T^{2T}\bar{F}\left(y-d_2\right)dt_2dF\left(\varepsilon_1\right)\right]dt_1+(1-R_C)PY=\frac{2c(1+r_pT)^2}{2+r_pT}$, where $R_c=r_c\left(T_c-T\right)$, where $PY=\int_0^T\left[F\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)+\widetilde{L_1^P}f\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)\right]dt_1$. The first-order condition of $\widetilde{K_R^P}$ with respect to $\widetilde{B^P}$ is:

$$\frac{\partial\widetilde{K_R^P}}{\partial B^P}=R_c\int_0^T\left[\frac{\left(1+r_pT\right)^2}{2+r_pT}F\left(\widetilde{L_1^P}-d_1-\widetilde{B^P*}\right)+\frac{2-\left(1-r_pT\right)^2}{2+r_pT}\widetilde{L_1^P}f\left(\widetilde{L_1^P}-d_1-\widetilde{B^{P*}*}\right)\right]dt_1-\frac{2\left(1+r_pT\right)^2-\left(2+r_pT\right)}{2+r_pT}$$

(ii) To prove $\widetilde{K_R^{P*}}(\widetilde{Q^P},\widetilde{B^P})$ is jointly concave in $(\widetilde{Q^P},\widetilde{B^P})$, we drive the second-order condition of $\widetilde{K_R^P}$ with respect to $\widetilde{Q^P}$ and $\widetilde{B^P}$ as: $\frac{\partial^2\widetilde{K_R^P}}{\partial\left(\widetilde{Q^P}\right)^2}=-\int_0^Tf\left(\widetilde{Q^P}-d_1\right)dt_1-\int_0^T\int_0^{\widetilde{Q^P}-d_1}\left\{\int_T^{2T}f\left(y-d_2\right)dt_2\right\}dF\left(\varepsilon_1\right)dt_1-\int_0^T\int_T^{2T}\bar{F}\left(d_2\right)dt_1dt_2-\frac{2c(1+r_pT)^2}{2+r_pT}R_c\int_0^T\left[2f\left(\widetilde{D_1^P}-d_1\right)+\widetilde{L_1^P}f'\left(\widetilde{D_1^P}-d_1\right)\right]dt_1<0$, $\frac{\partial^2\widetilde{K_R^P}}{\partial\left(\widetilde{B^P}\right)^2}=-\frac{2+r_pT+(1+r_pT)^2}{2+r_pT}$
$R_c\int_0^T\left\{\frac{(1+r_pT)^2}{2+r_pT}f\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)+\frac{2-(1-r_pT)^2}{2+r_pT}\left[f\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)+\widetilde{L_1^P}f'\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)\right]\right\}dt_1<0.$

Moreover, we further drive:

$$\frac{\partial^2\widetilde{K_R^P}}{\partial\left(\widetilde{Q^P}\right)\partial\left(\widetilde{B^P}\right)}=\frac{\partial^2\widetilde{K_R^P}}{\partial\left(\widetilde{B^P}\right)\partial\left(\widetilde{Q^P}\right)}=R_c\int_0^T\left\{\frac{\left(1+r_pT\right)^2}{2+r_pT}F\left(\widetilde{L_1^P}-d_1-\widetilde{B^P}\right)+\frac{2-\left(1-r_pT\right)^2}{2+r_pT}\widetilde{L_1^P}f\left(\widetilde{L_1^P}-d_1-\widetilde{B^P*}\right)\right\}dt_1>0.$$

Since $\frac{2+r_pT+(1+r_pT)^2}{2+r_pT} > 1$, we have $\left|\frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{B^P})^2}\right| > \left|\frac{\partial^2 \widetilde{K_R^P}}{\partial\left(\widetilde{Q^P}\right)\partial\left(\widetilde{B^P}\right)}\right|$. Meanwhile, because of $\frac{2c(1+r_pT)^2}{2+r_pT} > 1$, thus $\left|\frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{Q^P})^2}\right| > R_c \int_0^T \left\{\frac{(1+r_pT)^2}{2+r_pT} F\left(\widetilde{L_1^P} - d_1 - \widetilde{B^P}\right) + \frac{2-(1-r_pT)^2}{2+r_pT} \widetilde{L_1^P} f\left(\widetilde{L_1^P} - d_1 - \widetilde{B^P}\right)\right\} dt_1 = \left|\frac{\partial^2 \widetilde{K_R^P}}{\partial\left(\widetilde{Q^P}\right)\partial\left(\widetilde{B^P}\right)}\right|$. We know the Hessian matrix is negative definite, *i.e.*, $H = \begin{bmatrix} \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{Q^P})^2} & \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{Q^P}) \partial (\widetilde{B^P})} \\ \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{B^P}) \partial (\widetilde{Q^P})} & \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{B^P})^2} \end{bmatrix} = \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{Q^P})^2} \frac{\partial^2 \widetilde{K_R^P}}{\partial (\widetilde{B^P})^2} - \left(\frac{\partial^2 \widetilde{K_R^P}}{\partial\left(\widetilde{Q^P}\right)\partial\left(\widetilde{B^P}\right)}\right)^2 > 0$. Therefore, $\widetilde{K_R^{P*}}(\widetilde{Q^P}, \widetilde{B^P})$ is jointly concave in $(\widetilde{Q^P}, \widetilde{B^P})$. Similarly prove that $\widetilde{K_R^{A*}}(\widetilde{Q^A}, \widetilde{B^A})$ is also concave in $(\widetilde{Q^A}, \widetilde{B^A})$. □

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