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2-POWER DOMINATION NUMBER FOR KNÖDEL GRAPHS AND ITS APPLICATION IN COMMUNICATION NETWORKS

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Abstract. In a graph G, if each node $v \in V(G) \setminus S$ is connected to some node in S, then the set S of nodes is referred to as a dominating set. The domination number of G is the minimum cardinality of all dominating sets of G and is represented by $\gamma(G)$. If a dominating set S monitors every node in the system under a set of guidelines for power systems monitoring, then the set S is referred to as a power-dominating set of G. The power domination number of G is the least number of vertices of a power dominating set of G. A generalization of power domination is the k-power domination in a graph G. The k-power domination number of G is the minimum cardinality of all k-power dominating sets of G and is represented by $\gamma_{p,k}(G)$. In this paper, we have obtained the 2-power domination number represented by $\gamma_{p,2}(G)$ for 4-regular Knödel graphs and given the lower bound for 5-regular Knödel graphs.

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1. Introduction

The challenge of monitoring electrical systems gives birth to the theory of power domination in networks. By locating Phase Measurement Units, termed PMUs, at certain places throughout the system, electric systems seek to explore the nature of the system continuously. Due to the high price of such synchronized devices, obtaining the minimum number of possible devices is necessary while overseeing the complete system. This problem was referred to as the power domination problem in networks, which is a variation of the domination problem, by Hayens *et al.* in 2002 [16]. A graph that represents the nodes as the electrical nodes and the lines as the communication links connecting those nodes can be used to simulate an electric power network. Finding the least number of nodes from which the complete network can be perceived while adhering to specific criteria is the goal of the power domination problem in this model. Such a minor set of nodes will offer the actual network places where the PMUs should be located to observe the complete network at the lowest possible price [5]. The following findings were achieved for a connected graph G by Paul Dorbec *et al.* in 2012 using the idea of the k-power domination problem, which is a generalization of the power domination problem in graphs. If G

Keywords. Domination, power domination, 2-power domination, Knödel graph.

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is linked and $\delta(G) \leq k+1$, then $\gamma_{p,k}(G) = 1$. An higher bound for the k-power domination value of any linked network G of nodes n, $\gamma_{p,k}(G) \leq \frac{n}{k+2}$, is given in the same article [7].

For an undirected simple network G, the nodes and lines of the network are named as $V(G)$ and $E(G)$, respectively. For a node $a \in V(G)$, $N(a) = \{b \in V(G) | ab \in E(G)\}$ is the open neighbourhood of node a and $N[a] = N(a) \cup \{a\}$ is the closed neighbourhood of the node a. Let $N_i(a) = \{b | d(a,b) = i\}$, where $d(a,b)$ is the shortest distance between a and b in G. Clearly $N_1(a) = N(a)$. Let $\delta(G) = min\{deg(a) | a \in V(G)\}$ and $\triangle(G) = max\{deg(a) | a \in V(G)\}$, where $deg(a)$ is the degree of node a. If a set S of nodes in a graph G is such that every node v in $V(G) \setminus S$ is adjacent to some node in S, then S is called a dominating set of G. The least cardinality of all the dominating sets of G is the domination number of G and is symbolized by $\gamma(G)$. For more results on domination, see [2,24]. Haynes *et al.* presented propagation laws regarding nodes and lines in a network in 2002, which served as the basis for introducing power domination in graphs. Considering a network $G(V,E)$ and $S \subseteq V(G)$, we define $p^i(S)$ of nodes observed by S inductively at stage $i, i \geq 0$, as follows:

1. $p^0(S) = N[S]$.
2. $p^{i+1}(S) = p^i(S) \cup \{w : \exists v \in p^i(S), N(v) \cap (V(G) \setminus p^i(S)) = \{w\}\}$.

The set S is referred to as a power dominating set of G, if $p^\infty(S) = V(G)$. The power domination number of G, indicated by $\gamma_p(G)$, is the smallest number of elements in a power dominating set of G. We note that $p^i(S) \subseteq p^{i+1}(S) \subseteq V(G)$ for any i. Furthermore, integer $k \geq 0$. We define the k-power domination by $p^i(S)$ of nodes observed by S inductively at stage $i, i \geq 0$, as follows:

1. $p^0(S) = N[S]$.
2. $p^{i+1}(S) = \cup\{N[v] : v \in p^i(S) \text{ such that } |N[v] \setminus p^i(S)| \leq k\}$.

Every stage a node of the set $p^i(S)$ has maximum k neighbours in $V(G) \setminus p^i(S)$, we include its neighbours to the next $p^{i+1}(S)$. We thus describe $p^\infty(S) = V(G)$. The k-power domination number of G, defined by $\gamma_{p,k}(G)$ is the k-power dominating set with the smallest number of nodes of S. Put $k = 0$, then the k-power domination is normal domination. Put $k = 1$, the k-power domination is normal power domination. If $k = 2$, then the k-power domination comes under our case 2-power domination.

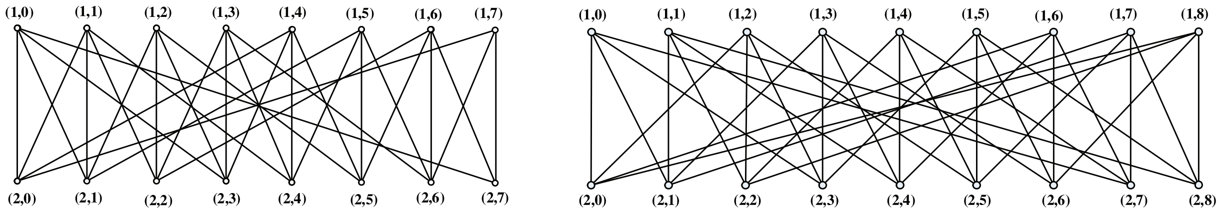
The k-power domination has been examined, and proved it for chordal graphs and bipartite graphs is NP-complete [7], regular graphs [10], Sierpinski graph [9], split graphs and chordal graphs [20], grid graphs [11], claw-free graphs [31], block graphs [30], cylinders, torus and generalized Petersen graph [4], hypertrees, hexagonal mesh, honeycomb mesh, Christmas tree [26], Cartesian product of graphs [19], hyper-graphs [6], and r-uniform hypergraphs [1], octahedral structures [25]. In this paper, we have discussed the 2-power domination number for Knödel graphs of regularity 4 and 5.

2. Knödel graph

Walter Knödel first proposed the Knödel graph in 1975 [18]. It is a unique network because it is the smallest gossip graph. In gossiping, given a group of n people, each of them has a piece of information and is attempting to communicate it to another using binary calls, with each call lasting a set amount of time, and finally finding how much time must elapse before everyone is aware of everything. Similarly, in broadcasting, at time $t = 0$, only one person knows the information, and at time $t = 1$, two persons know the information, and at time $t = 2$, 2^2-persons know the information; finally, if there are n members in the system, the required time is at least $\lceil log_2 n \rceil$. In this case, the persons are referred to as vertices. Knödel graph is Δ-regular, $1 \leq \Delta \leq \lfloor log_2 n \rfloor$ and bipartite.

Knödel graph is defined as follows [12]: $W_{\Delta,n}$ denotes the Knödel graph having $2n$ vertices, $n \geq 2$, and $1 \leq \Delta \leq \lfloor log_2 n \rfloor$ [14]. The pairs of vertex set of Knödel graph are i and j, where $i = 1$ and 2, and j lies between 0 and $n/2 - 1$. For all j, there exist edges between node $(1,j)$ and each of the nodes $(2, j + 2^k - 1 \ mod\ (n/2))$, for $k = 0, 1, \ldots, \Delta - 1$. The Knödel graph $W_{4,16}$ and $W_{4,18}$ are shown in Figure 1.

The Knödel graph is a suitable competitor for hypercubes and circulant graphs in broadcasting and gossiping. It is a good option for solving the challenge of information dissemination. Moreover, any Knödel graph can be



(a) (b)

FIGURE 1. (a) Knödel graph $W_{4,16}$ (b) Knödel graph $W_{4,18}$.

TABLE 1. Power domination and, 2-power domination numbers for Knödel graphs.

Δ	$\gamma_p(G)$	$\gamma_{p,2}(G)$
3	$=2$, $n \geq 8$ [29]	1, $n \geq 8$ [Theorem 3.1]
4	≤ 2, $n = 16$ [29]	$=2$, $n \geq 16$ [Theorem 3.2]
5	≤ 4, $n = 32$ [29]	≥ 3, if $n \geq 46$ [Theorem 3.3] $= 2$, if $n \leq 44$ [Remark 3.4]

described as a Cayley graph [17]. For topological properties and for more information on Knödel graph, see [13]. Mojdeh *et al.* discussed the value of total domination for cubic Knödel graph in [23]. Domination number for 4-regular Knödel graph [21], and domination number for critical Knödel graph [22] have also been determined. Varghese *et al.* discussed the power domination number for Knödel graphs in [29], and the details along with the results obtained in this paper are listed in Table 1.

3. MAIN RESULTS

In this section, we find the 2-power domination number for 4-regular and obtain the lower bound for 5-regular Knödel graphs. Since Knödel graphs are bipartite, in the sequel, we take X, Y as the bipartition of the node-set.

Theorem 3.1. *Let G be a graph with maximum degree Δ, $\Delta \geq 2$. Then $\gamma_{p,k}(G) = 1$, if $k = \Delta - 1$.*

Proof. Select a node x in $V(G)$ such that $\deg(x) = \Delta$. Then the degree of each node in $N(x)$ is at most $\Delta - 1$, hence x propagates to the next stage. Proceeding along this way, all the nodes will be supervised. Thus $\gamma_{p,\Delta-1}(G) = 1$. □

Theorem 3.2. *Let G be a 4-regular Knödel graph $W_{4,n}$, $n \geq 18$. Then $\gamma_{p,2}(G) = 2$.*

Proof. Suppose we assume that $\gamma_{p,2}(G) = 1$. Since G is bipartite, and the vertex representation in X and Y as $(1, j)$ and $(2, j)$, $0 \leq j \leq \frac{n}{2} - 1$. Without loss of generality we take a node x in X and put it in S, $p^0(S) = N_1[S]$, then by the definition of Knödel graph, for any vertex $x = (1, j)$ in X, the adjacent vertices are $(2, j + 2^k - 1\ (mod\ \frac{n}{2}))$, for $k = 0, 1, \ldots, \Delta - 1$, and for any vertex $y = (2, j)$ in Y, the adjacent vertices are $(1, \frac{n}{2} - 2^k + j + 1\ (mod\ \frac{n}{2}))$ for $k = 0, 1, \ldots, \Delta - 1$. Then $|N(y) \cap N_2(x)| \geq 3$, for any $y \in N(x)$, a contradiction. Hence $\gamma_{p,2}(G) \geq 2$.

To prove the reverse inequality, we exhibit the dominating set S having two vertices namely $(1, 0), (2, 8)$, which will be the 2-power dominating set for $W_{4,n}, n \geq 18$. Clearly $p^0(S) = \{(i, j) : i \in [2], j \in [1, 7]\} \cup \{(2, 0), (2, 3), (1, 5), (1, 8)\}$. Then there exists a vertex $x \in N_2(x)$ such that $|N[x] \cap N_3[S]| \leq 2$, then x will start

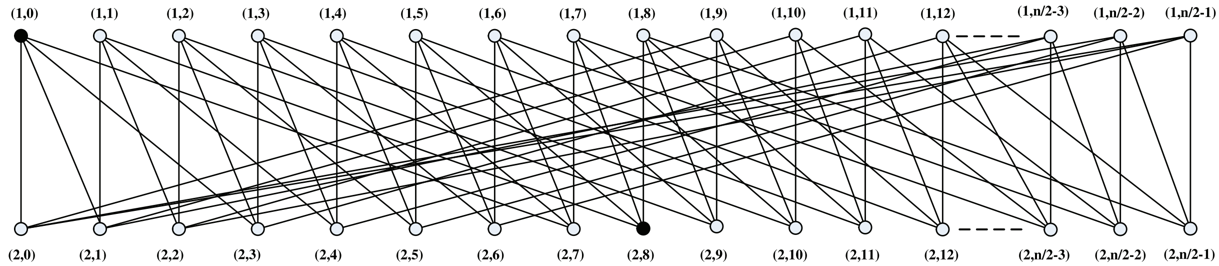


FIGURE 2. 2-power dominating vertices in $W_{4,n}$.

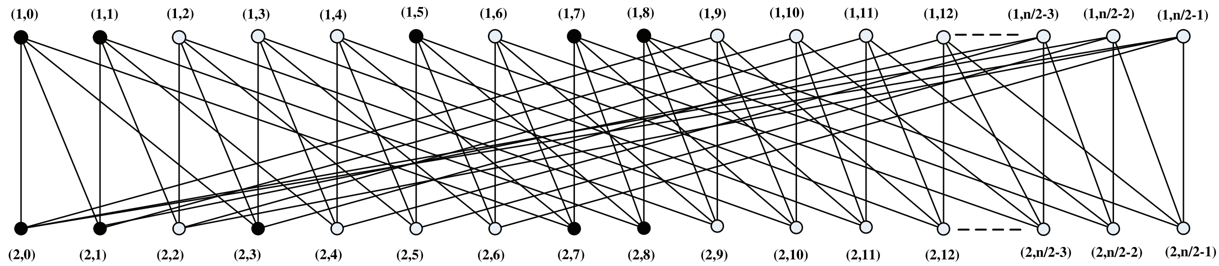


FIGURE 3. Neighbourhood is monitored.

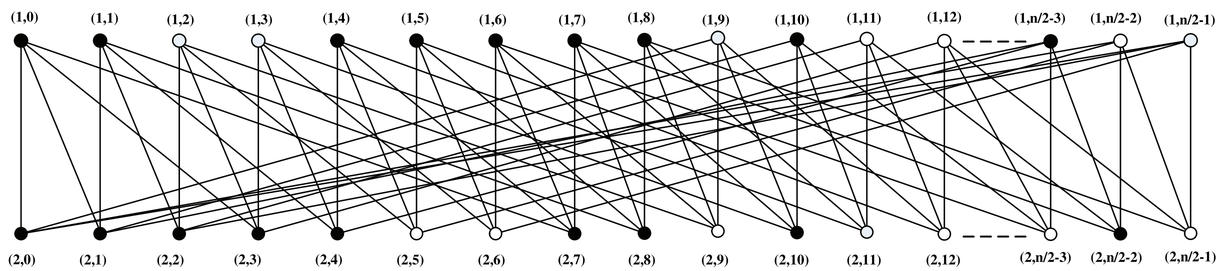


FIGURE 4. Propagation occurs.

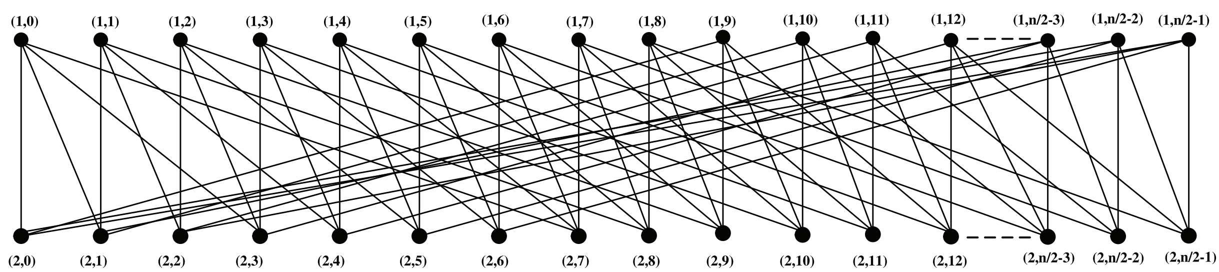


FIGURE 5. End of propagation.

the propagation. Now, $p^1(S) = p^0(S) \cup \{(1,4),(1,6),(1,10),\ (1,13),(2,2),(2,4),(2,10),(2,14)\}$. Proceeding along the same line all the nodes in the graph G are supervised. Thus $\gamma_{p,2}(G) = 2$. $\square$

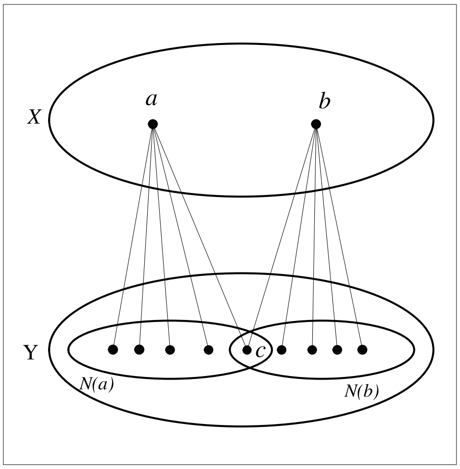


FIGURE 6. The neighbourhood of a and b, and the connection pattern with respect to Case 1.

For illustration, the propagation of each stage in $W_{4,n}$ is represented pictorially step-by-step in Figures 2 to 5.

In Figure 2, the dominated nodes are highlighted in black color and the other nodes are white in color.

In Figure 3, the dominated nodes and their neighbors $p^0(S)$ are highlighted in black color and the other nodes are white in color.

In Figure 4, vertices in $S \cup p^0(S) \cup p^1(S)$ is highlighted in black color and the other nodes are white in color.

In Figure 5, the stage is reached in which all the vertices are supervised.

Theorem 3.3. *Let G be a 5-regular Knödel graph $W_{5,n}$, $n \geq 46$. Then $\gamma_{p,2}(G) \geq 3$.*

Proof. Suppose $\gamma_{p,2}(G) = 2$, let $S = \{a, b\} \subset V(G)$ be a 2-power dominating set of G. Now we have the following cases.

Case 1. (**Both a and b in X**): Then for each node c in $N(a)$, $N(c) \cap N(b) = \emptyset$. Similarly, for each node d in $N(b)$, $N(d) \cap N(a) = \emptyset$. See Figure 6. Hence $|N(x) \setminus \{a, b\}| \geq 3$, for any x in $N(a) \cup N(b)$. Thus further propagation is not possible. Therefore $\gamma_{p,2}(G) \geq 3$.

Case 2. (a **in** X **and** b **in** Y): We have the following subcases.

Subcase($\boldsymbol{i}$). If $ab \in E(G)$, then by the definition of Knödel graph, for any vertex $(1, j)$ in X, the adjacent vertices are $(2, j + 2^k - 1\ (mod\ \frac{n}{2}))$ for $k = 0, 1, \ldots, \Delta - 1$, and for any vertex $(2, j)$ in Y, the adjacent vertices are $(1, \frac{n}{2} - 2^k + j + 1\ (mod\ \frac{n}{2}))$ for $k = 0, 1, \ldots, \Delta - 1$. Then $|N(x) \cap N(b)| \leq 1$ for any $x \in N(a)$. See Figure 7. Thus x will not propagate to the next level. The same arguments hold for any vertex y in $N(b)$. Hence $\gamma_{p,2}(G) \geq 3$.

Subcase($\boldsymbol{ii}$). If $ab \notin E(G)$, then by the definition of Knödel graph, for any vertex $(1, i)$ in X, for a fixed k, the vertices in the second neighbourhood are $(2, \frac{n}{2} - 2^k - 1 + (2^i - 1) \pmod{\frac{n}{2}})$ for $i = 0, 1, \ldots, \Delta - 1$, and for any vertex $(2, j)$ in Y, for a fixed k, the vertices in the second neighbourhood are $(1, j + 2^k - 1 - (2^i - 1) \pmod{\frac{n}{2}})$ for $i = 0, 1, \ldots, \Delta - 1$. Then $|N(x) \cap N_2(b)| \leq 1$ for any $x \in N_2(a)$. See Figure 8. Thus x will not propagate to the next level. A similar argument holds for any vertex y in $N(b)$. Hence $\gamma_{p,2}(G) \geq 3$.

□

For illustration, the propagation of each stage in $W_{5,46}$ is represented pictorially step-by-step in Figures 9 to 12. In Figure 9, the dominated nodes are highlighted in black and the other nodes are white in color.

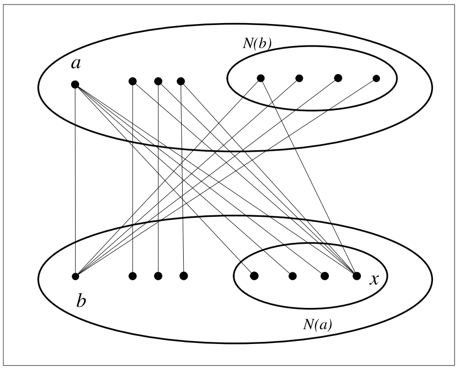


FIGURE 7. The neighbourhood of a and b, and the connection pattern with respect to Subcase(i).

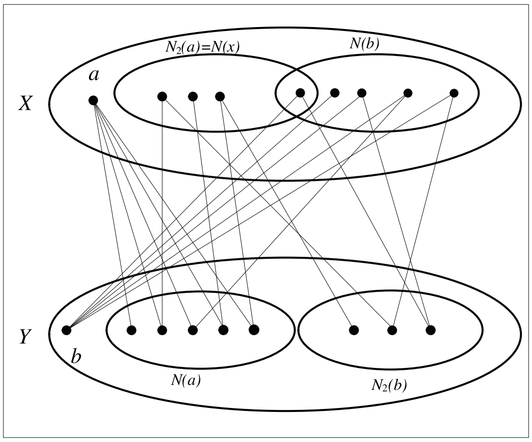


FIGURE 8. The neighbourhood of a and b, and the connection pattern with respect to Subcase(ii).

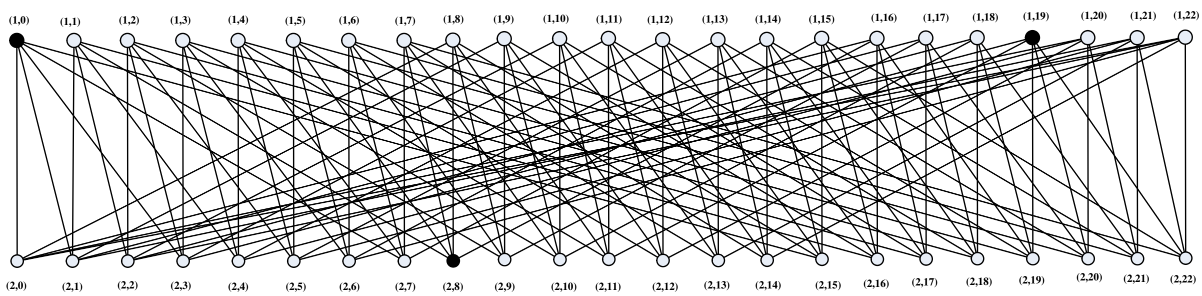


FIGURE 9. 2-power dominating vertices in $W_{5,46}$.

In Figure 10, the dominated nodes and their neighbors $p^0(S)$ are highlighted in black color and the other nodes are white in color.

In Figure 11, vertices in $S \cup p^0(S) \cup p^1(S)$ is highlighted in black color and the other nodes are white in color.

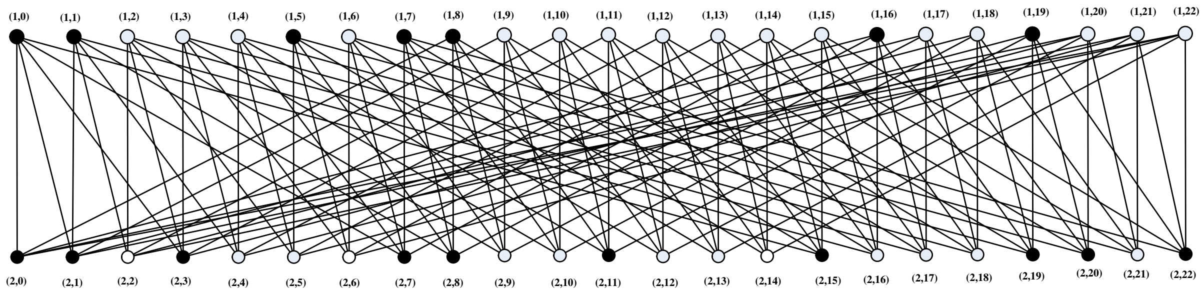


FIGURE 10. Neighbourhood is monitored.

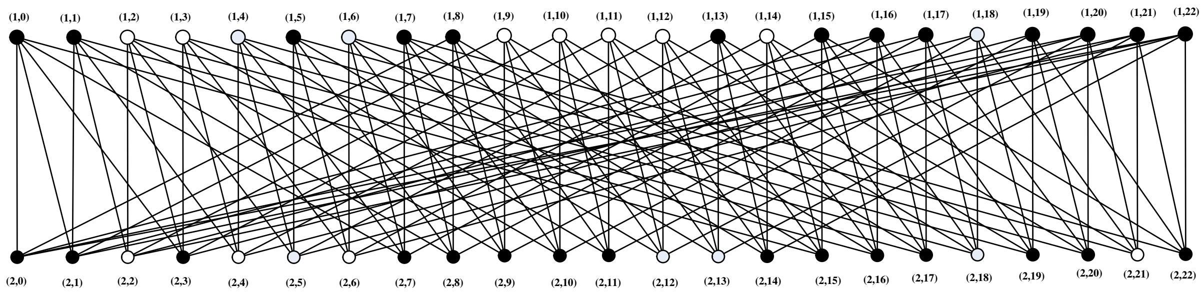


FIGURE 11. Propagation occurs.

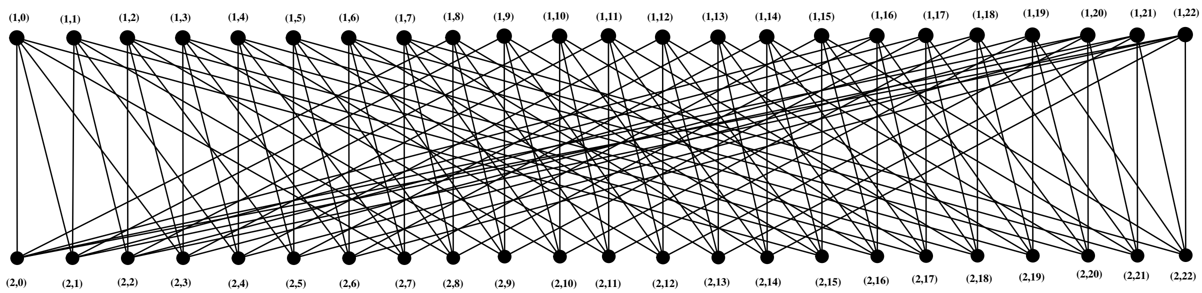


FIGURE 12. End of propagation.

In Figure 12, the stage is reached in which all the vertices are supervised. Thus we conclude that $\gamma_{p,2}(W_{5,46}) = 3$.

Remark 3.4. Let G be a 5-regular Knödel graph $W_{5,n}$, $n \leq 44$. Then $\gamma_{p,2}(G) = 2$.

Proof. Suppose we assume that $\gamma_{p,2}(G) = 1$. Since G is bipartite, and the vertex representation in X and Y as $(1, j)$ and $(2, j)$, $0 \leq j \leq \frac{n}{2} - 1$. Without loss of generality we take a node x in X and put it in S, $p^0(S) = N_1[S]$, then by the definition of Knödel graph, for any vertex $x = (1, j)$ in X, the adjacent vertices are $(2, j + 2^k - 1 \ (mod\ \frac{n}{2}))$, for $k = 0, 1, \ldots, \Delta - 1$, and for any vertex $y = (2, j)$ in Y, the adjacent vertices are $(1, \frac{n}{2} - 2^k + j + 1 \ (mod\ \frac{n}{2}))$ for $k = 0, 1, \ldots, \Delta - 1$. Then $|N(y) \cap N_2(x)| \geq 3$, for any $y \in N(x)$, a contradiction. Hence $\gamma_{p,2}(G) \geq 2$.

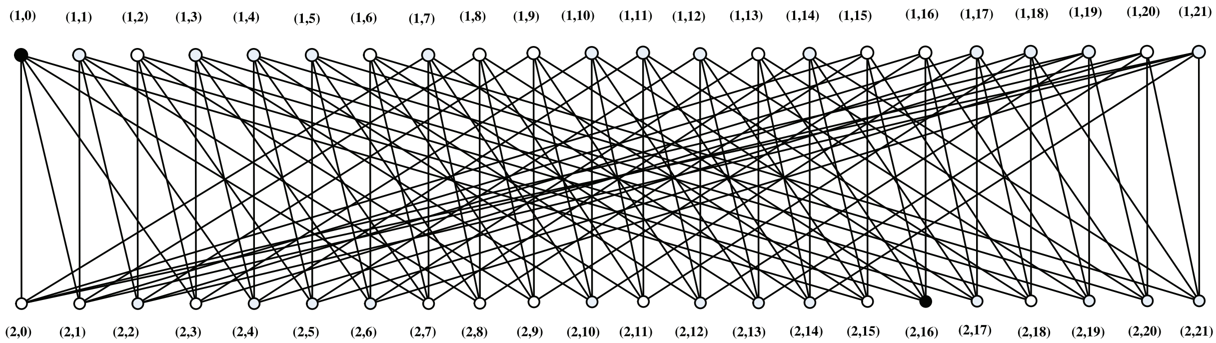


FIGURE 13. 2-power dominating vertices in $W_{5,44}$.

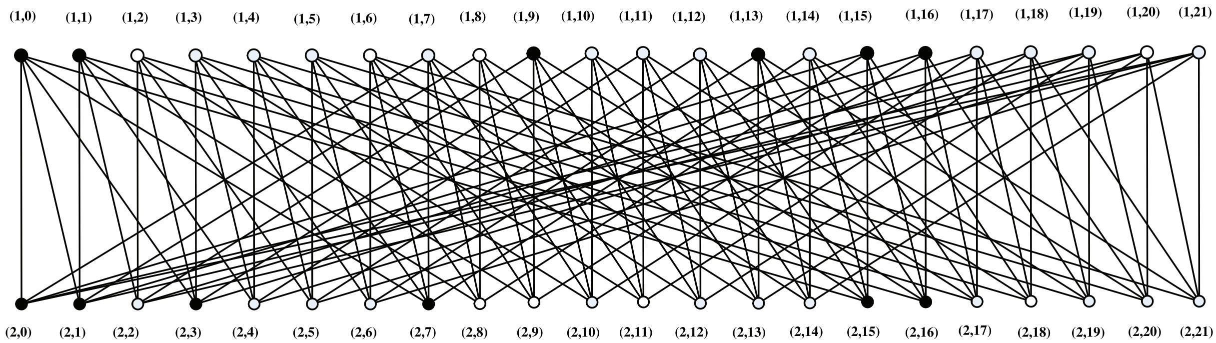


FIGURE 14. Neighbourhood is monitored.

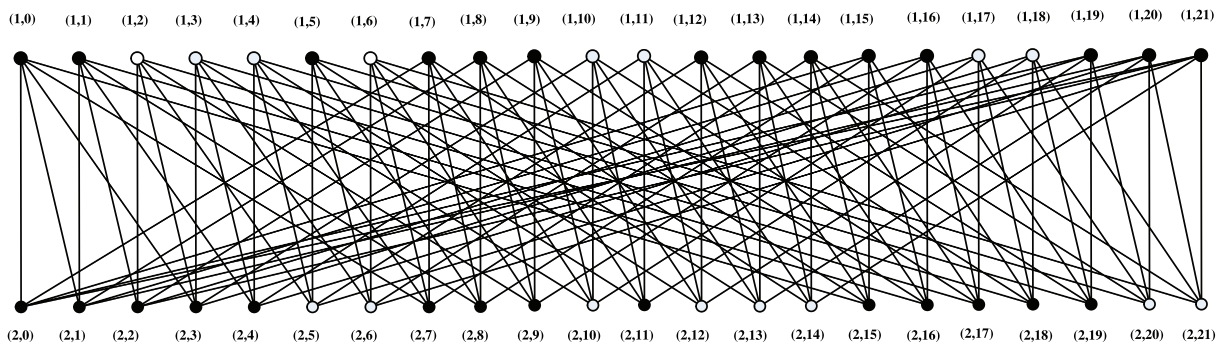


FIGURE 15. Propagation occurs.

To prove the reverse inequality, we exhibit the 2-power dominating set S having two vertices namely,

$$\gamma_{p,2}(W_{5,n}) = \begin{cases} (1,0),(2,16), \text{ for } n = 34,36,42,44 \\ (1,0),(2,8), \text{ for } n = 32,38 \\ (1,0),(2,19), \text{ for } n = 40. \end{cases}$$

Proceeding along the same lines as Theorem 3.2, the nodes in the graph G are supervised. Thus $\gamma_{p,2}(G) = 2$. □

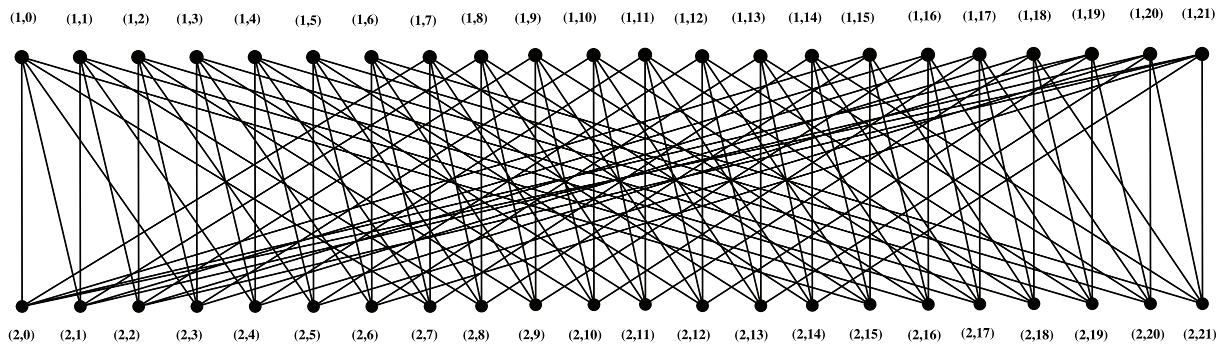


FIGURE 16. End of propagation.

For illustration, the propagation of each stage in $W_{5,44}$ is represented pictorially step-by-step in Figures 13 to 16.

In Figure 13, the dominated nodes are highlighted in black and the other nodes are white in color.

In Figure 14, the dominated nodes and their neighbors $p^0(S)$ are highlighted in black color and the other nodes are white in color.

In Figure 15, vertices in $S \cup p^0(S) \cup p^1(S)$ is highlighted in black color and the other nodes are white in color.

In Figure 16, the stage is reached in which all the vertices are supervised. Thus we conclude that $\gamma_{p,2}(W_{5,44}) = 2$.

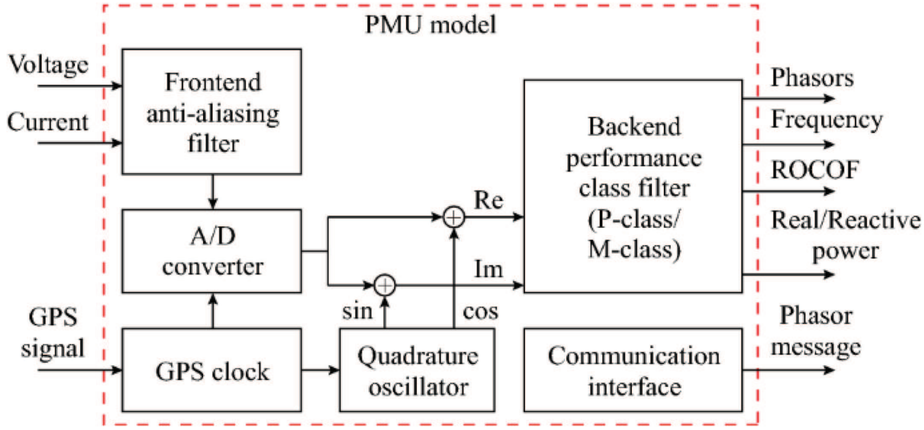
Conjecture. Let G be a 5-regular Knödel graph $W_{5,n}, n \geq 46$. Then $\gamma_{p,2}(G) = 3$.

4. Execution and trials

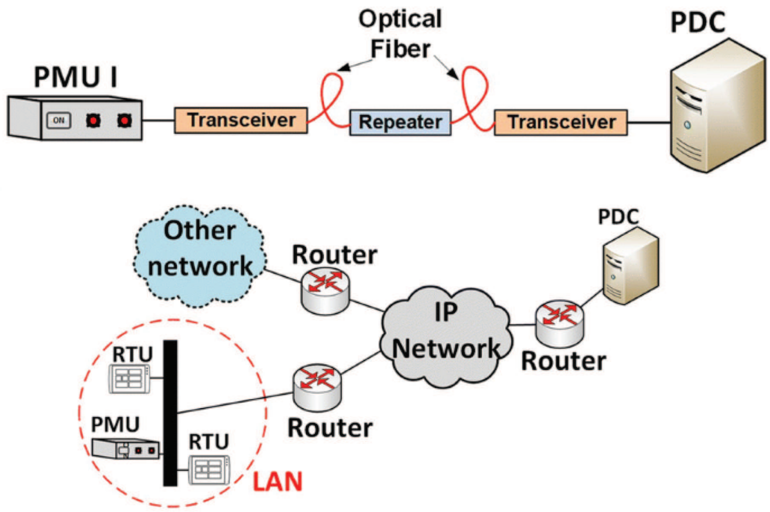
In this section, we look at some of the applications of our proposed problem 2-power domination number and several of our network Knödel graph in practical settings. In this instance, the vertices or subsets of the 2-power dominant set can be completely detached. To achieve energy-saving routing and structuring, disjoint sets are used. Different disjoint sets can ensure various message-carrying pathways and node architectures in mobile wireless networks, each with no nodes in common. To maintain effective and efficient operation, power utilities should monitor their power networks and adapt to changing conditions of claim and obtainability. Several power industries endlessly use PMUs to monitor their power networks. The node covering network-theoretic problem, or else, a variant of the dominating set problem, is precisely how to supervise an electric power system using the limited available measurement devices.

Production, sub-broadcast, broadcast, loads, and supply comprise the existing power system (PS), a sophisticated multicomponent interconnection network (Graph structures). To prevent an unexpected incident, PS's multidimensional ecosystem requires constant supervision and defense of its components. The notion of monitoring the power industry in original-life situations has become more and more relevant in the modern day due to the numerous previous power system events [27].

Supervisory control and data acquisition (SCADA) centered examining is typically spent for present days; however, it is ineffective in portraying original-time dynamics of the outline due to its asynchronous valuations of network factors, low determination, and lack of data on structure-activity. Phasor is nothing more than a mathematical representation of electrical signals or waveforms. An electrical device known as a PMU delivers synchrophasors and recurrence estimates for 3-step AC voltage and current waveforms. PMU-based synchrophasor technology (ST) is employed instead of SCADA due to the restrictions they have overcome [28]. Using the dynamic state of measurement, these PMUs are widely employed for grid security and safety monitoring. See Figure 17.



(a)



(b)

FIGURE 17. (a) Optimal PMU placement model; (b) Synchrophasor communication system.

Electrical hubs and broadcasting wires (linking loads and generators) make up a power generation and delivery network. Electric power industries must regularly monitor their power networks. It's critical to limit the voltage size at loads and the device stage at generators. Machine and voltage are variables. Positioning PMUs in the scheme's defined locations must be monitored. Due to the high charge of PMUs, it is vital to reduce the quantity of PMUs used and also retain the aptitude to pursue the entire scheme. By Haynes [16], this summary is described as a mathematical example of problems in graph theory and is referred to as a type of domination problem called power domination.

A variation of the k-power domination is the 2-power domination problem. When there is just one unobserved vertex in a power domination situation, a vertex from the dominating set initiates propagation activity. If a vertex has two or more unobserved vertices, it cannot propagate. A vertex can propagate to the next level in the situation of 2-power domination even if it contains at most two unobserved vertices in its neighbor. It is possible to shorten the time needed to complete the operation for the entire network and the number of nominators in the dominant set.

The ability of an interconnection network to transfer messages is one of its essential characteristics. The communication delay between nodes determines a network's quality in large part. The nodes could be processors, computers, or other kinds of terminals. Let's say a node knows certain information and needs to send it to each network node. Typically, this process is known as broadcasting. When every node is aware of a set of data that needs to be sent to each other node, we are talking about gossiping.

In the year 1975, Walter Knödel [18] introduced an interconnection network called Knödel graph where gossiping can be done in the shortest period in the assumption of 1-port telephone model. To accomplish gossiping in the shortest amount of time, Knödel provided a method in a complete graph with the nodes of an even number. Fraigniaud and Peters officially described the Knödel graph, which is the network that underlies Knödel's communication design [14].

The Knödel graphs have many interesting properties. The general Knödel graph's diameter $W_{\Delta,n}$, is unknown, the diameter of W_k is known; a sharp inferior and superior bounds on the diameter of Knödel graph is obtained by Grigoryan and Harutyuanyan in [15]. Harutyuanyan and Morosan have examined the spectrum of W_k and have used the spectrum to estimate the maximum number of spanning trees for W_k. For more properties of Knödel graphs, see[3].

The numerous properties and exciting applications in broadcasting and gossiping of Knödel graphs motivate us to study the 2-power domination on it. Comparing our results with the results found in [29], the power domination and 2-power domination in PMUs, the 2-power domination reduced the number of PMUs needed to supervise the power networks.

5. Concluding remarks

In this paper, we have proved that the 2-power domination number for 4-regular Knödel graph is 2, and the lower bound for 5-regular Knödel graph is 3. The 2-power domination number for other regular Knödel graphs are under investigation.

Open problem. Obtain the k-power domination number for Δ-regular Knödel graph $W_{\Delta,n}$, $n \geq 2$.

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