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PAYMENT POLICY FOR A THREE-ECHELON SUPPLY CHAIN MANAGEMENT UNDER ADVERTISEMENT-DRIVEN DEMAND

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Abstract. Payment and selling are two important policies for a supply chain management. All participating supply chain players can earn profit based on the successful implementation of these two policies. The payment policy provides buyers some extra time to pay for the product. This research introduces a single-supplier, a single-manufacturer, and multi-retailer-based three-echelon supply chain management under advertising and payment policies. Product delivery among supply chain players is made with the help of a transportation policy. The transportation policy is environment-friendly and helps retailers with their cost management. It is demonstrated that advertising positively influences sales through an advertisement-driven market demand for the product. The model aims to reduce supply chain cost and maximizes profit by considering a single-setup multiple-delivery policy, variable transportation cost, variable carbon emissions costs, and trade-credit policy. The objective function is optimized for cases: Case 1 and Case 2, based on the payment period. A classical optimization method is employed to obtain the solution of the model. A numerical example, sensitivity analysis, and graphical representations are given to illustrate the model. Results show that Case 2, where cycle time is greater than the payment period, is 45.36% more profitable than Case 1.

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1. Introduction

To increase sales, members of the supply chain use some policies such that the profit becomes always maximum. It is proven that advertising creates more sales in the marketing system [44], which is profitable for all the supply chain players. Based on the recommendation of Xie and Neyret [44], market demand for the product includes the effect of advertising policy to ensure the effectiveness of the advertisement. This study considers that the demand is dependent on advertising expenditure along with the retail price of the product. For such a retail-based supply chain environment, a centralized supply chain management (SCM) policy [30] is more profitable than a decentralized policy. Therefore, the centralized supply chain with retail price and advertising

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demand is considered within a credit-based SCM. A credit-based SCM provides an opportunity for the supply chain players to pay the purchasing costs within a provided credit period. This credit period helps to generate revenue and then repay the purchasing cost using the revenue. But, there is always a risk of opportunity loss if a delay-in-payment situation occurs. All players have the opportunity to earn an interest from their buyer. The goal of this study is to find a strategic payment policy that helps the supply chain players to earn profit utilizing the advertisement policy.

Two stages of a single type of product are involved within the SCM: one is semi-finished products and another one is finished products. Transportation of these products is maintained by a single-setup-multiple-delivery (SSMD) policy. There are multiple deliveries for a single order for an SSMD policy. It may seem that a single delivery can reduce the supply chain costs along with carbon emissions from transportation, but it exists in the literature [45] that an SSMD policy provides more profit than a single-setup-single-delivery (SSSD) policy. This SSMD policy not only supports SCM profit but also supports supply chain players who have higher holding costs. For example, the product is from the textile industry. The brand image is high and the main showcase of the finished clothes is the retail stores. Semi-finished products are less expensive than finished products. Finally, when finished products appear in retail stores, they become valuable with higher holding costs than the manufacturer. Then for the supply chain players supplier, manufacturer, and retailers, holding costs form an ascending order per unit per unit time. Delivery in multiple shipments helps to save holding costs. In contrast, to reduce the emissions costs, variable carbon emissions costs are used by the supplier and the manufacturer based on the number of shipments. A supply chain with variable transportation costs has the goal to increase profit with optimized shipment numbers. The current model allows supply chain members to increase profit by reducing the total opportunity and interest income.

Research gaps and the corresponding contributions are given in the following sections.

1.1. Research gaps

This study finds the following research gaps.

- Two managerial policies are used: trade-credit for payment policy and advertisement for retail policy. The study finds the effect of two managerial policies on the three-echelon SCM.
- Two trade-credit periods are discussed in comparison with the cycle time. Which credit policy will be more profitable for the supply chain?
- Do the selling price and credit period affect the advertisement policy of the supply chain?
- Does the SSMD policy profitable for the three-echelon supply chain when a payment policy is involved within the system?

1.2. Contributions of the study

This study has the following contribution for a three-echelon SCM.

- This research describes a three-echelon supply chain model including a single-supplier, a single-manufacturer, and multiple-retailer. The product is a single-type and every downstream supply chain player have a credit period to their upstream player.
- The supplier and manufacturer use SSMD policy for product transportation. Besides, the manufacturer and retailers use a trade-credit policy for payment. This study investigates the effect of the combination of an SSMD and trade-credit policy. This study finds which supply chain player earns more profit from these policies.
- Market demand for the product is advertisement dependent, *i.e.*, the more advertisement earns more profit. This study examines the scenario of using a credit period for an advertisement-driven demand policy. Does the credit period help to earn more profit from the advertisement-driven demand policy?

1.3. Structure of the study

The representation of this research is as follows: Section 2 expresses literature analysis. Section 3 contains problem definitions, notation, and assumptions for the model. The mathematical model is presented in the next section, *i.e.*, in Section 4. Section 5 consists of the solution methodology of the model and Section 6 contains the numerical analysis. Section 7 gives a detail discussions about results and sensitivity analysis whereas Section 8 provides managerial insights. Lastly, Section 9 gives conclusions of the proposed research and its possible future expansions.

2. Literature review

The analysis of the literature investigates advertisement policies, payment policies, transportation policies, and carbon emissions issues within the SCM.

2.1. Supply chain management

Recently, many research work are designed for production lot-size problem and they focus on a two-stage and three-stage supply chain. Hanh *et al.* [14] investigated a supply chain coordination framework to minimize the double marginalization effect in the case of price-sensitive demand. They investigated the impact of pricing strategy and ordering/production policy on the performance of a three-layer SCM by using the Salop spatial model. Seo [36] described a multi-echelon SCM by upgrading reorder decision policy along real-time shared stock information. Sana [27] calculated imperfect products from a three-echelon SCM model which included a single-supplier, a single- manufacturer, and a single-retailer. Roy *et al.* [25] proposed as a three-echelon SCM depending on a uncertain environment (i) when the demand is uncertain and (ii) when a uncertain demand spread uniformly over horizon. Sebatjanea and Adetuji [35] presented a model for inventory management for perishable food products whereas Wang and Wan [43] introduced a bi-objective mixed-integer multi-objective model with profit maximization and carbon emissions minimization. Ben-Daya *et al.* [4] considered an environment-friendly remanufacturing process within a two-stage closed-loop supply chain management (CLSCM) for a single-vendor and multi-buyers. All supply chain players were involved in a centralized consignment-stock agreement. Sana [28] presented a dynamical stock-dependent model where the demand was dependent on team effort along with the stock. Chen *et al.* [9] investigated a manufacturing system which emit low carbon in environment. They investigated a two-opponent firms under a game policy to maintain production efficiency.

2.2. Payment and advertisement policies within a supply chain management

A lot of SCM studies used payment and advertisement policies to address the profit maximization problem [13, 17]. Lou [18] discussed a coordination policy between the manufacturer and the distributor for credit incentives. Duan *et al.* [13] studied a coordination model for fixed lifetime products with quantity discount. They found that both the players can accomplish the winning result after following the amount discount strategy. Sarkar *et al.* [33] described a two-echelon SCM model for perishable products with credit facility. Credit period and delay payment were established as a useful policy for maximizing supply chain profit. Cárdenas-Barrón and Sana [7] developed a multi-product-based two-echelon supply chain model with delayed payments. They found that the order was influenced by the promotion policy. Further, manufacturers give advertisement to sell more products. Over time, many manufacturers find that customers choose similar products from other manufacturers too. Manufacturers become interested in advertising policy to sell more products than other manufacturers. A manufacturer then needs extra information on the market. Chatha and Jalil [8] proposed a structural complexity of the supply chain, represented by the supply chain members. Suppliers were associated with the main manufacturing firm, whereas the operational complexity was represented by demand variability experienced by the manufacturing firm. They found that increasing the number of customers reduces capacity utilization, demand fulfillment, and increases inventory level. Increasing number of suppliers increases the capacity utilization, demand fulfillment, and inventory level of the main manufacturing firm. Bhatnagar *et al.* [5] presented a mathematical model to maximize the inventory and transportation costs under inflation. Yue *et al.* [46] studied the co-operative

TABLE 1. Author(s) contributions table based on literature.

Author(s)	Three-echelon SCM	SSMD policy	Variable transportation	Carbon emissions	Policy
Park *et al.* [23]	√	–	√	–	–
Roy *et al.* [24]	–	√	√	–	–
Sana [27]	√	–	–	–	–
Roy *et al.* [25]	√	–	–	–	–
Ullah and Sarkar [40]	–	–	√	–	–
Ben-Daya *et al.* [3]	√	–	–	–	–
Sana *et al.* [29]	√	√	–	–	–
Salas-Navarro *et al.* [26]	√	–	–	–	VMI
Cárdenas-Barrón and Sana [6]	√	–	–	–	–
Davoudi *et al.* [10]	–	√	–	√	Bargaining
Yadav et al. [45]	–	–	√	√	–
Aljazzar *et al.* [1]	√	√	–	–	–
Aljazzar *et al.* [2]	√	√	–	–	Trade-credit
Sepheri *et al.* [37]	–	–	–	√	–
Chen *et al.* [9]	–	–	–	√	–
This paper	√	√	√	√	Trade-credit

Notes. "–" – Not applicable; SCM – Supply chain management; SSMD – Single-setup multi-delivery; VMI – Vendor-managed inventory.

advertising policy under coordination of the manufacturer and retailer. The manufacturer used discount policy for products with advertisement.

Xie and Neyret [44] described another cooperative advertising policy for promotion of products. They considered that the demand was affected by both the retail promotion and advertisement. Seyed *et al.* [38] investigated a vertical-cooperative advertising and pricing strategy and built of demand-price combination as linear, convex or concave. They solved the model by four game theoretic models. Karray and Amin [16] created a twain advertisement-dependent retail strategy and solved the model using game theory. Besides, there were supply chain studies which did not consider payment an advertisement policies. Lee [17] created a manufacturer–distributor supply chain model for product distribution. Ullah *et al.* [41] enhanced the supply chain possibilities with remanufacturing without any payment policy. Sarkar *et al.* [19] considered a closed-loop SCM with bargaining without any payment and advertisement policies. Wang and Sarker [42] designed a assembly system for just-in-time production and kanban system.

2.3. Transportation policy and carbon emissions within a supply chain management

Roy *et al.* [24] developed the optimal transportation policy for defective products for out of stock situation. Park *et al.* [23] described the supply chain networking problem through a three-echelon network. They considered risk-pooling and lead time within the networking. But did not describe any carbon policy for networking. Mardanya and Roy [20] presented a transportation model for soild products and solved with fuzzy approach. Hosseini *et al.* [15] proposed a healthcare supply chain network where transportation plays an important role irrespective of the supply chain cost. Zhao *et al.* [47] addressed a problem of optimal ordering quantity and frequency for a supplier–retailer logistic system in which transportation cost as well as the multiple uses of the vehicle were considered. Mridha *et al.* [21] discussed an SCM model for green product manufacturing and transportation facility. They considered carbon emissions from both the manufacturing system and transportation. Debnath and Sarkar [11] discussed a CLSCM for waste nullification in a two-stage supply chain. They discussed circular economy along with carbon emissions from the system. Sarkar *et al.* [31] proposed a supply chain model that aimed to improve the delivery policy for supplier selection as a supply chain member. They transported products along with used

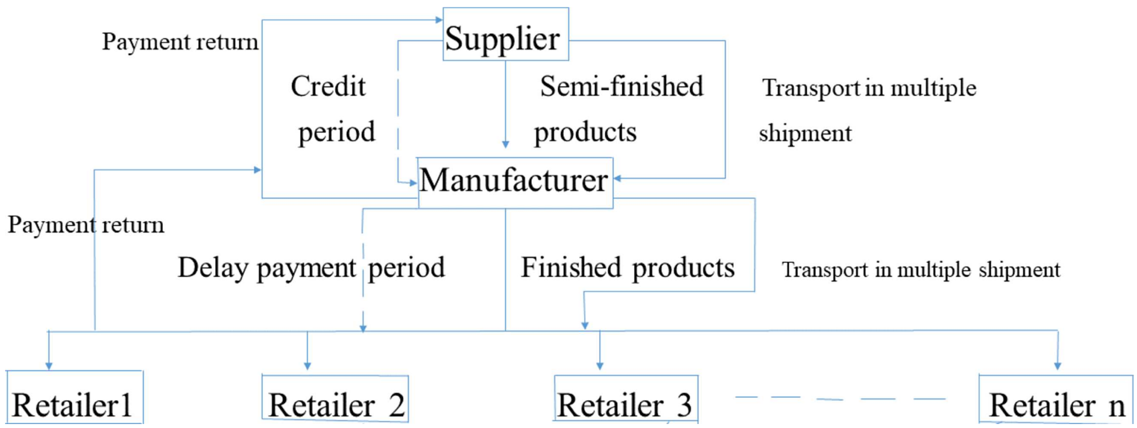

FIGURE 1. Graphical representation of the three-echelon SCM with semi-finished and finished product flow between the single-supplier, single-manufacturer, and multi-retailer.

packaging products. Singh *et al.* [39] described an advertisement-driven supply chain model with transportation policy that reduces waste from the system. Saxena *et al.* [34] proposed a model that simultaneously considered the production of eco-designed products through a reverse logistics facility and solved the model using game policy. They considered carbon emissions from the system. There are other studies as Ben-Daya *et al.* [3] and Sana *et al.* [29], who described a three-echelon SCM without considering multiple delivery shipment and carbon emissions from the system. Table 1 appears the improvement and insert of authors.

3. Problem definition, notation, and assumptions

This section consists of problem definition of the model (Fig. 1), associative notation, and assumptions.

3.1. Problem definition

This research examines a three-echelon supply chain with a single-supplier, a single-manufacturer, and multi-retailer. Supplier is the upstream player and manufacturer & retailers are downstream players compared with the supplier. This study discusses three policies as payment, advertisement, and transportation policies within a three-echelon SCM. The study reduces carbon emissions by increasing consumer awareness through advertising. In this course, SSMD policy is used as transportation policy to deliver products to retailers. The aim is to diminish the total cost for the SCM by applying proposed policies. Advertising policy creates more demand opportunity for retailers than no-advertisement. The order of product is influenced by advertising. The supplier offers credit period to the manufacturer. The manufacturer offers a delay payment period to all retailers. Time limit of the return repayment affects the interest earn and opportunity value of the supply chain players. If the payment are made before the allowable time, the manufacturer earns interest through the sales return. The SSMD policy is used by the supplier and the manufacturer to save holding costs of retailers. In the case of multiple deliveries, transportation cost depends on distance and quantity. Carbon emissions cost associated with transportation is variable. All three players work together and the total cost of the supply chain is minimized for cooperative case. Based on the credit period, two cases are discussed.

3.2. Notation

Index

j	Retailer, $i = 1, 2, \dots, n$

Decision variables

T	Cycle time of all retailers (year)
p	Selling price of retailer ($/unit)
b	Advertisement percentage

Parameters

Supplier's parameters

v_1	Shipment number of the supplier (integer)
p_{11}	Selling price of semi-finished product to the manufacturer ($/unit)
p_s	Production rate of semi-finished products (units/year)
s_D	Demand rate of semi-finished products (units/year)
h_{TS}	Holding cost for raw materials ($ /unit/year)
h_{sm}	Holding cost for semi-finished products ($/unit/year)
A_c	Setup cost ($/setup)
O_c	Ordering cost for raw material ($/shipment)
Q_c	Lot size (units/cycle)
S_c	Shipment size of supplier, $S_c = \frac{Q_c}{v_1}$ (units)
F	Cost of fixed transportation ($/shipment)
V	Cost of variable transportation ($/unit)
C_{cfe}	Fixed carbon emissions cost ($/shipment/year)
C_{cve}	Variable carbon emissions cost ($/unit)
C_p	Purchasing cost of raw materials ($/unit)
I_{ps}	Capital opportunity cost ($/$/year)
I_{es}	Interest earned ($/$/year)
$\mathrm{TS}_{c1}, \mathrm{TS}_{c2}$	Supplier's total cost in Cases 1 and 2, respectively ($/cycle)
$\mathrm{TP}_{c1}, \mathrm{TP}_{c2}$	Supplier's total profit in Cases 1 and 2, respectively ($/cycle)

Manufacturer's parameters

v_2	Shipment number for unfinished item received by manufacturer (integer number)
p_{22}	Selling price of finished product to retailers ($/unit)
p_1	Integer of the manufacturer's cycle time (integer number)
T_z	$p_1 T_k (T_z = p_1 p_2 T)$ (integer number)
p_p	Production rate (unit/year)
D_r	Demand rate of finished product (unit/year)
h_p	Finished product's holding cost ($/unit/year)
A_s	Setup cost ($/setup)
O_s	Ordering cost for semi-finished product ($/shipment)
Q_s	Lot size (unit/cycle)
S_s	Shipment size, $S_s = \frac{Q_s}{v_2}$ (unit/shipment)
F	Fixed transportation cost ($/shipment)
V	Variable transportation cost ($/unit)
C_{emf}	Fixed carbon emissions cost ($/shipment/year)
C_{evm}	Variable carbon emissions cost ($/unit)
C_c	Purchasing cost of semi-finished product ($/unit)
I_{pm}	Cost of capital opportunity ($/year)
I_{em}	Manufacturer's interest earned ($/$/year)
Y	Manufacturer's payment time (year)
$\mathrm{TC}_{p1}, \mathrm{TC}_{p2}$	Manufacturer's total cost in Cases 1 and 2, respectively ($/cycle)
$\mathrm{TP}_{p1}, \mathrm{TP}_{p2}$	Manufacturer's total profit in Cases 1 and 2, respectively ($/cycle)

Retailers' parameters

p_2	Integer of the retailers' cycle period (integer number)
T_k	p_2T ($T_z = p_1p_2T$) (integer number)
D_{rj}	Retailer j's demand (unit/year)
ρ	Advertising number (point scale, *i.e.*, one-point, two-points) (integer number)
a, α	Scaling parameters of demand
S_{rj}	Ordered quantity of retailer j (units/cycle)
$p_{\max}$	Selling price (maximum) (\$/unit)
$p_{\min}$	Selling price (minimum) (\$/unit)
β	Investment parameter (\$/cycle)
γ	Shape parameter of demand
k	Efficiency of advertisement (1–10 scale value)
d	Advertising parameter
h_c	Holding cost per unit per year (\$/unit/year)
R_o	Ordering cost to manufacturer (\$/order)
n	Number of retailer (integer number)
C_s	Purchasing cost of finished product (\$/unit)
I_{pr}	Cost of capital opportunity (\$/year)
I_{er}	Interest earned of retailer (\$/\$/year)
X	Retailer's permissible payment time (year)
$\mathrm{TC}_{s1}, \mathrm{TC}_{s2}$	Retailers' total cost in Cases 1 and 2, respectively (\$/cycle)
$\mathrm{TP}_{s1}, \mathrm{TP}_{s2}$	Retailers' total profit in Cases 1 and 2, respectively (\$/cycle)

Other parameters

$\mathrm{ETC}_1, \mathrm{ETC}_2$	Total cost of the SCM in Cases 1 and 2, respectively (\$/cycle)
$\mathrm{ETP}_1, \mathrm{ETP}_2$	Total profit of the SCM in Cases 1 and 2, respectively (\$/cycle)

3.3. Assumptions

The following hypotheses are proposed to construct a mathematical model.

(1) The three-echelon SCM consists of a single-supplier, a single-manufacturer, and multi-retailer for a single type of product. The supplier sends the semi-finished products to the manufacturer. The manufacturer finishes the product and sends finished products to retailers.

(2) Demand for the supplier (s_D) and the manufacturer (D_r) is the same as the market demand (D), *i.e.*, $s_D = D_r = D$. That is, both the supplier and manufacturer produce the product based on the market demand. The production rate of semi-finished products (p_s) and finished products (p_p) are greater than the market demand (D). Then, the supply chain does not allow any shortage.

(3) Market demand of retailer i depends on selling price p and advertising expenditure b as follows: $\sum_{j=1}^{n} D_{rj} = D = a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{\rho k\sqrt{d}}{1+\rho}$, where α is a scaling parameter of advertisement, α is the scaling parameter of selling price, γ is a shape parameter for advertising, $\frac{\rho}{1+\rho}$ is action of the advertising on demand, and $k\sqrt{d}$ is the effect of advertising expenditure on demand. $\frac{\rho k\sqrt{d}}{1+\rho}$ is treated as constant. $p_{\max}$ is the maximum selling price and $p_{\min}$ is the minimum selling price of the product. If $p = p_{\min}$, then D is infinity [22].

(4) Holding and ordering costs are considered equal for all retailers. Retailers spend the advertising cost for the product. Holding costs for supply chain players are increasing toward the downstream players [39]. Besides, the advertisement cost for each retailer is the same.

(5) The cycle time T of the SCM is the retailer's cycle time. The cycle time of the supplier (p_1p_2T) is as an integer multiple (p_1) of the manufacturer's cycle time. The cycle time of the manufacturer (p_2T) is considered as an integer multiple (p_2) of the retailer's cycle time [12].

(6) The supplier allows a delay payment period Y for the manufacturer. The supplier earns interest for the delay period and loses an opportunity cost. Meanwhile, the manufacturer provides a permissible delay period to retailers for payment and earns opportunity cost as well as interest from retailers based on the delay period. The supplier earns interest at a rate of I_{es} and the manufacturer earns interest at the rate of I_{em}. If the manufacturer and retailers return the debt to the supplier and the manufacturer, respectively within the approved period, they earn an opportunity cost at a rate of I_{pm} and I_{pr}, respectively [12].

(7) Both semi-finished and finished product delivery by the supplier and the manufacturer are completed by an SSMD policy, respectively. According to the SSMD policy, both types of products are delivered in multiple shipments. Therefore, a variable carbon emissions cost is considered for transportation policy [45].

4. Mathematical modelling

The section develops a mathematical modelling on a three-echelon SCM with a single-supplier, single-manufacturer, and multi-retailer. The supplier uses raw materials and produces semi-finished items at a production rate of p_s. These semi-finished items are subsequently transferred to the manufacturer at a fixed batch size through SSMD policy.

4.1. Supplier's model

The supplier buys raw materials to produce products for the entire period. Raw materials are delivered to a supplier within the p_1p_2T cycle time. Total amount of produced semi-finished products within the cycle time is

$$Q_c = p_1p_2T\left(a\frac{p_{\max} - p}{p - p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)\cdot \tag{1}$$

The supplier's cost and total profit formulation are given below.

4.1.1. Setup cost

In each cycle, the supplier requires a fixed cost for a production setup. The setup cost for the supplier per cycle is

$$S_t = \frac{A_c}{p_1p_2T}\cdot \tag{2}$$

4.1.2. Ordering cost of raw materials

The supplier satisfies the raw material requirement through sub-suppliers in each cycle p_1p_2T. The number of shipments used by the supplier within the cycle time is v_1. O_c is the unit ordering cost per shipment. Therefore, the total ordering cost for raw material per cycle is

$$\frac{O_cv_1}{p_1p_2T}\cdot \tag{3}$$

4.1.3. Holding cost of raw materials

The supplier receives total required raw materials in v_1 shipments, triangles that split the cycle time (Fig. 2). Then, using equation (1), raw materials in each shipment becomes

$$S_c = \frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)}{v_1}\cdot \tag{4}$$

Total holding cost of raw materials in each cycle is

$$\mathrm{HC}_p = \frac{h_{\mathrm{TS}}Q_c\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)}{2p_s} = \frac{h_{\mathrm{TS}}p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_1p_s}\cdot \tag{5}$$

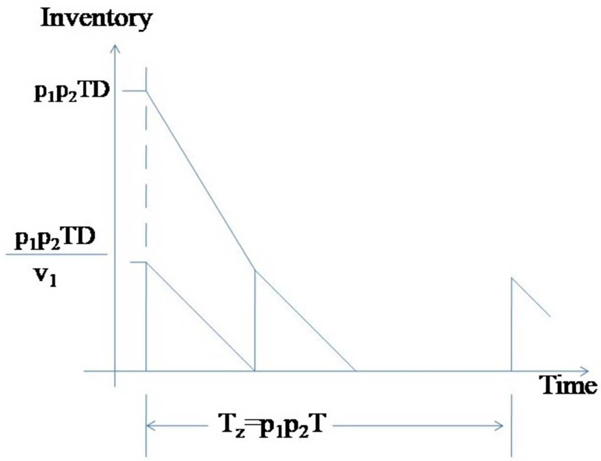

FIGURE 2. Inventory position of the supplier for raw materials products in two shipments.

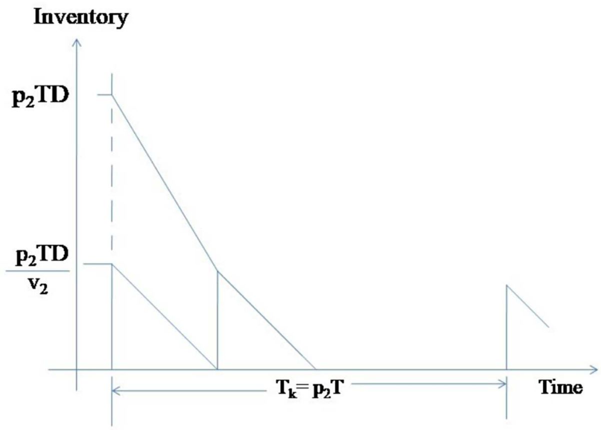

FIGURE 3. Inventory position of the manufacturer for raw materials (semi-finished products) in two shipments.

4.1.4. Holding cost of semi-finished products

Semi-finished products are the final product of the supplier. The supplier sends semi-finished products to the manufacturer in v_2 substantial shipments in each cycle p_2T. This circumstance is represented in Figure 3. Therefore, the batch size of semi-finished product in each shipment is

$$A = \frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)}{v_2}. \tag{6}$$

Let the corresponding accumulative production inventory of the supplier is denoted by B_1 (Figure 4). Then, B_1 is

$$B_1 = \frac{p_2T}{2}\left(2p_1^2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\sqrt{d}\rho}{1+\rho}\right) - \frac{p_1^2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\sqrt{d}\rho}{1+\rho}\right)^2}{p_s}\right). \tag{7}$$

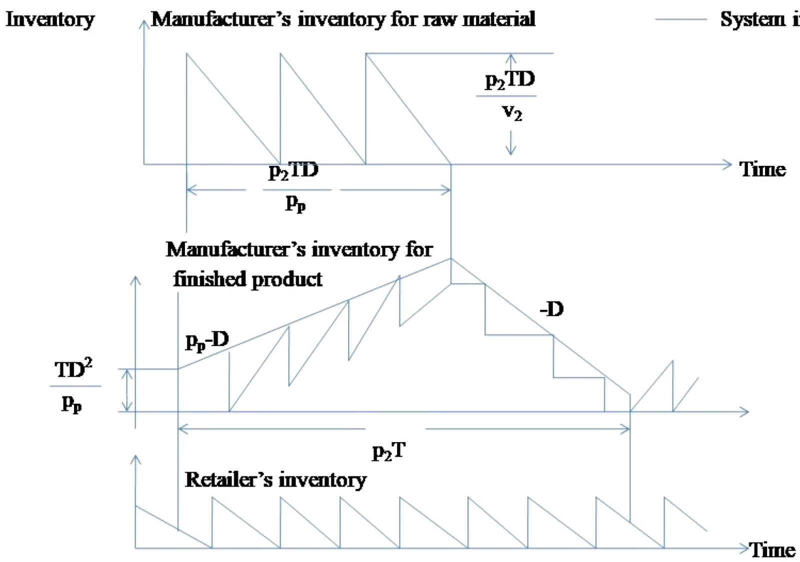

Figure 4. Manufacturer's raw material (semi-finished products), finished products, and retailers' on-hand inventory.

However, B_2 represents the total demand of the manufacturer and is divided into three rectangles for simplification.

$$A_1 = \frac{p_2 T}{2}\left(\frac{p_1 p_2 T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2 (v_2+1)}{v_2 p_p}\right) \tag{8}$$

$$A_2 = \frac{p_2 T}{2}\left(p_1 p_2 T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)(p_1+1) - \frac{p_1 p_2 T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2 (p_1+1)}{p_p}\right.$$
$$\left. - \frac{2 p_1 p_2 T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{v_2 p_s}\right) \tag{9}$$

$$A_3 = \frac{p_2 T}{2}\left(\frac{p_1 p_2 T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2 (p_1-1)}{p_p}\right). \tag{10}$$

Area B_2 is the sum of three areas A_1, A_2, and A_3. Thus, the total inventory holding cost of semi-finished products is given by the $B_1 - B_2$, cumulative semi-finished products after replenish demand. Semi-finished product's holding cost per cycle is (Figure 5)

$$f_p \mathrm{HC}_p = \frac{h_{sm} p_2 T}{2}\left(\left(\frac{2}{v_2} - p_1\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_s} + \left(1 - \frac{1}{v_2}\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.$$

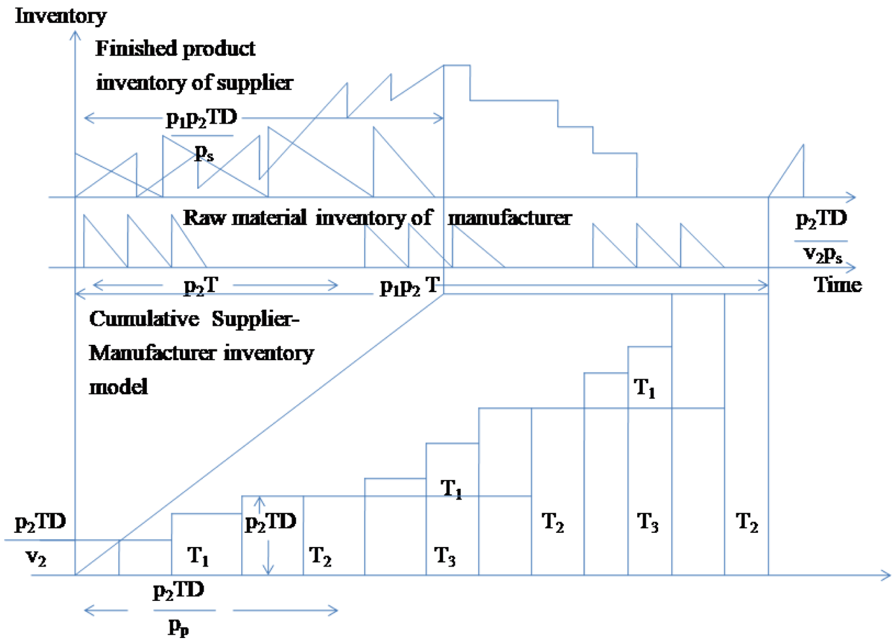

FIGURE 5. Common inventory representation for the supplier and manufacturer ($p_1 = 3, v_1 = 4$, and $v_2 = 3$).

$$+\,(p_1-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\Bigg). \tag{11}$$

4.1.5. Cost of transportation of semi-finished products

The variable transportation cost for quantity is VQ_c and the fixed transportation cost for v_1 shipments is v_1F. Thus, the total transportation cost of the supplier for v_1 number of shipments is

$$T_P = \frac{v_1F + \mathrm{VQ}_c}{p_1p_2T}. \tag{12}$$

4.1.6. Cost of carbon emissions for transportation

The carbon emissions cost is two types: fixed and variable. C_{cve} is the variable carbon emissions cost per unit product and C_{cfe} is the fixed carbon emissions cost shipment per cycle. Then, the total carbon emissions cost for transportation is

$$C_{dc} = C_{cfe}v_1 + C_{cve}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right). \tag{13}$$

4.1.7. Cost of opportunity and interest earned

Based on the business strategy, if the manufacturer's cycle time is shorter than the approved credit period Y to the supplier, the suppler pays an equal opportunity cost. It bears the interest earned by the supplier. The opportunity cost is more and the interest income is small when the permissible credit period Y is shorter than the cycle time p_2T. Consequently, the supplier earns a return by paying a low opportunity cost. In such cases, the supplier pays a low opportunity cost and earns interest profit. The model of the supplier includes two cases.

Case 1. If $p_2T < Y$, then the cost of transfer opportunity for purchasing raw materials with unit cost C_p is

$$Sc_p = C_p I_{ps}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y-\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right). \tag{14}$$

Case 2. If $p_2T \geq Y$, then the cost of transfer opportunity for purchasing raw materials is

$$S_{tp} = C_p I_{ps}\left(\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}\right). \tag{15}$$

The income of the transfer cost for selling semi-finished products with unit price C_c is

$$S_{it} = \frac{C_c I_{es}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2(p_1p_2T-Y)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{16}$$

4.1.8. Total cost of the supplier

Total cost of the supplier is given below:

Case 1. $p_2T < Y$

$$\begin{aligned}
\mathrm{TS}_{c1} = {} & \frac{A_c+O_cv_1}{p_1p_2T}+h_{\mathrm{TS}}\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_1p_s}+\frac{h_{sm}p_2T}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_s}\right.\\
&\left.+\left(1-\frac{1}{v_2}\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}+(p_1-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_1F+\mathrm{VQ}_c}{p_1p_2T}\\
&+\left(C_{cfe}v_1+C_{cve}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+C_pI_{ps}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right.\\
&\left.-\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right).
\end{aligned} \tag{17}$$

Unit selling price of semi-finished products is p_{11}. Then, the total profit of the supplier in Case 1 is

$$\mathrm{TP}_{c1} = p_{11}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\mathrm{TS}_{c1}. \tag{18}$$

Case 2. $p_2T \geq Y$

$$\begin{aligned}
\mathrm{TS}_{c2} = {} & \frac{h_{sm}v_2T}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.\\
&\left.+(p_1-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_1F+\mathrm{VQ}_c}{p_1p_2T}
\end{aligned}$$

$$
\begin{aligned}
&+\left(C_{cfe}v_1+C_{cve}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+C_pI_{ps}\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}\\
&-C_cI_{es}\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}(p_1p_2T-Y)^2+\frac{A_c+O_cv_1}{p_1p_2T}\\
&+h_{\mathrm{TS}}\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_1p_s}\cdot
\end{aligned}
\tag{19}
$$

Total profit of the supplier in Case 2 is

$$
\mathrm{TP}_{c2}=p_{11}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\mathrm{TS}_{c2}. \tag{20}
$$

4.2. Manufacturer's model

Costs of the manufacturer include setup cost, ordering cost, holding cost of semi-finished products & finished products, variable & fixed shipments costs, carbon emissions cost, and interest received including opportunity cost. Inventory representation appears in Figure 4. Cycle length of the manufacturer is p_2T. Total lot size of the manufacturer is

$$
Q_s=p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\cdot \tag{21}
$$

4.2.1. Setup cost of the manufacturer

Setup cost is the cost for configuring equipment for production. This expenditure is considered as fixed. The setup cost per setup is A_s and per cycle is

$$
S_m=\frac{A_s}{p_2T}\cdot \tag{22}
$$

4.2.2. Ordering cost of raw materials (semi-finished products)

The manufacturer orders raw materials to the supplier within a cycle p_2T. Raw materials of the manufacturer is the semi-finished products. The manufacturer uses v_2 number of shipment in each cycle. If the ordering cost per shipment is O_s, then total ordering cost of the manufacturer per cycle is

$$
\frac{O_sv_2}{p_2T}\cdot \tag{23}
$$

4.2.3. Holding cost of raw materials (semi-finished products)

The manufacturer uses v_2 number of shipments to deliver products total product. The cycle length of the manufacturer is p_2T. Then, the shipment size of semi-finished products is

$$
S_s=\frac{Q_s}{v_2}=\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{v_2}\cdot \tag{24}
$$

The area of semi-finished products is triangle over the cycle time p_2T, as appears in Figure 3. Hence, the holding cost of semi-finished products is

$$
H_t=h_{sm}\frac{Q_s\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2p_p}\cdot \tag{25}
$$

4.2.4. Holding cost of finished products

The manufacturer delivers finished products to retailer's in v_2 shipment with equal shipment size. Initially, the manufacturer does not have any finished products and the lowest inventory of retailers is $\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}$. After starting the production, the inventory increases at a rate of $\left[p_p-\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right]$. Consequently, record of the inventory appears in Figure 3. Maximum inventory of finished products of the manufacturer is

$$M_{hc}=\frac{\left(p_p-\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{p_p}+\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\cdot \tag{26}$$

At the end of the cycle time, the finished product consumption rate is $-\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)$, and the average inventory of finished products is

$$M_{ev}=\frac{\left(p_p-\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2p_p}+\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\cdot \tag{27}$$

The average inventory of finished product for the manufacturer is

$$\begin{aligned}R_{iv}=&\frac{\left(p_p-\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2p_p}+\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\\&-\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}.\end{aligned} \tag{28}$$

The above amount of products are sent to retailers in v_2 number of shipments through SSMD policy. The holding cost the manufacturer under the SSMD strategy is given by

$$H_{ha}=\frac{h_pT}{2}\left((2-p_2)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}+(p_2-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)\cdot \tag{29}$$

4.2.5. Transportation cost of finished products

The manufacturer pays the transportation cost when finished products are shipped to the retailers. Thus, V is the unit variable transportation cost per product, and F is fixed transportation cost per shipment. If v_2 is the shipment number, then the transportation cost of the manufacturer is

$$T_t=\frac{v_2F+\mathrm{VQ}_s}{p_2T}\cdot \tag{30}$$

4.2.6. Carbon emissions cost from transportation

For manufacturer, C_{emf} is the unit fixed carbon emissions cost per shipment per cycle, and C_{evm} is the variable carbon emissions cost per unit of product. Therefore, the cost of carbon emissions from the transportation is

$$C_{emf}v_2+C_{evm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\cdot \tag{31}$$

4.2.7. Cost of opportunity and interest earned

Two cases based on the acceptable payment periods of the manufacturer Y, retailers X, and the cycle time of the manufacturer are described below.

Case 1. $p_2T < Y$ If cycle time of the manufacturer is less than the payment period of the manufacturer Y, then the interest earned for selling finished products with selling price C_s is provided as follows:
Interest income is

$$C_sI_{em}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right). \tag{32}$$

Transfer opportunity cost for purchasing raw materials with unit cost C_c is

$$C_cI_{pm}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)X-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right). \tag{33}$$

Case 2. $p_2T \geq Y$ Similar to Case 1, the opportunity cost for purchasing raw materials is as follows:

$$\text{Transfer opportunity} = C_cI_{pm}\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)X\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{34}$$

The cost of opportunity for purchasing raw materials is

$$C_{co} = \frac{C_cI_{pm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)(p_2T-Y)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{35}$$

The interest earned for selling finished products is follows:

$$\text{Interest} = C_sI_{em}\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{36}$$

Interest earned of the manufacturer is

$$C_{iit} = \frac{C_sI_{em}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)(p_2T-X)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{37}$$

4.2.8. Total cost of the manufacturer

The manufacturer's total cost is given below.

Case 1. If $p_2T < Y$

$$\begin{aligned}\mathrm{TC}_{p1} = {} & \frac{A_s+O_sv_2}{p_2T}+h_{sm}\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_2p_p}+\frac{h_pT}{2}\left((2-p_2)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.\\ & \left.+(p_2-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_2F+\mathrm{VQ}_s}{p_2T}\end{aligned}$$

$$+\left(C_{emf}v_2+C_{evm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)-C_sI_{em}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right.$$
$$\left.-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right)+C_cI_{pm}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)X\right.$$
$$\left.-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right). \tag{38}$$

Unit selling price of finished products to retailers is p_{22}. Manufacturer's total profit in Case 1 is

$$\mathrm{TP}_{p1}=p_{22}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\mathrm{TC}_{p1}. \tag{39}$$

Case 2. $p_2T\geq Y$

$$\mathrm{TC}_{p2}=\frac{A_s+O_sv_2}{p_2T}+h_{sm}\frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2p_2p_p}+\frac{h_pT}{2}\left((2-p_2)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.$$
$$\left.+(p_2-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_2F+\mathrm{VQ}_s}{p_2T}+C_{emf}v_2$$
$$+C_{evm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)+\frac{C_cI_{pm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+k\sqrt{d}(\rho/(1+\rho))\right)^2(p_2T-Y)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+k\sqrt{d}(\rho/(1+\rho))\right)}$$
$$+C_cI_{pm}\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)X\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)}-C_sIem\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)}$$
$$-\frac{C_sI_{em}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2(p_2T-X)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)}. \tag{40}$$

Total profit of the manufacturer in Case 2 is

$$\mathrm{TP}_{p2}=p_{22}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^\gamma+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\mathrm{TC}_{p2}. \tag{41}$$

4.3. Retailers' model

This section focuses on four categories of costs incurred by retailers: (i) ordering cost, (ii) cost of holding finished products, (iii) interest earned & opportunity cost, and (iv) investment for advertising cost.

4.3.1. Ordering cost

Retailers order finished products to the manufacturer. The ordering cost of all the retailers is

$$R_t=\frac{R_o}{T}\cdot \tag{42}$$

4.3.2. Finished product holding cost

Total finished product of retailer j over cycle time T is

$$S_{rj} = TD_{rj}. \tag{43}$$

The holding cost of average inventory of retailer j is

$$R_h = h_c \frac{TD_{rj}}{2}. \tag{44}$$

4.3.3. Investment for advertising

Retailer j uses investment for marketing of products. Advertisement is denoted by b, and the corresponding investment parameter is denoted by β, then the total investment of all retailers is

$$\frac{\beta b^2}{2}. \tag{45}$$

4.3.4. Earned interest and cost of opportunity

Two cases of allowable settlement period X over cycle time T are explained below.

Case 1. If $T < X$, then interest income becomes

$$pI_{er}\left(D_{rj}X - \frac{TD_{rj}}{2}\right). \tag{46}$$

Case 2. If $T \geq X$, then interest income is

$$\frac{pI_{er}(D_{rj}X)^2}{2TD_{rj}} \tag{47}$$

and the corresponding opportunity cost is

$$\frac{C_s I_{pr}(D_{rj}T - D_{rj}X)^2}{2TD_{rj}}. \tag{48}$$

4.3.5. Total cost of retailers

The total cost of all retailer are given below.

Case 1. $T < X$

$$\mathrm{TC}_{s1} = \sum_{j}^{n}\left(\frac{R_o}{T} + h_c\frac{TD_{rj}}{2} - pI_{er}\left(D_{rj}X - \frac{TD_{rj}}{2}\right)\right) + \frac{\beta b^2}{2}. \tag{49}$$

The profit of retailers in Case 1 is

$$\mathrm{TP}_{s1} = p\left(a\frac{p_{\max} - p}{p - p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right) - \mathrm{TC}_{s1}. \tag{50}$$

Case 2. $T \geq X$

$$\mathrm{TC}_{s2} = \sum_{j}^{n}\left(\frac{R_o}{T} + h_c\frac{TD_{rj}}{2} - \frac{pI_{er}(D_{rj}X)^2}{2TD_{rj}} + \frac{C_s I_{pr}(D_{rj}T - D_{rj}X)^2}{2TD_{rj}}\right) + \frac{\beta b^2}{2}. \tag{51}$$

Corresponding total profit of retailer in Case 2 is

$$\mathrm{TP}_{s2} = p\left(a\frac{p_{\max} - p}{p - p_{\min}} + \alpha b^{\gamma} + \frac{k\rho\sqrt{d}}{1+\rho}\right) - \mathrm{TC}_{s2}. \tag{52}$$

4.4. Total profit of the supply chain

Total profit of the supply chain is found for two cases: Cases 1 and 2.

Case 1. Total profit of the supply chain in Case 1 is derived by adding the supplier's profit, manufacturer's profit, and retailers' profit for Case 1.

$$
\begin{aligned}
\mathrm{ETP}_1 ={}& (p_{11}+p_{22}+p)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\left[\frac{A_c+O_c v_1}{p_1p_2T}+h_{\mathrm{TS}}\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_1p_s}\right.\\
&+\frac{h_{sm}p_2T}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.\\
&\left.+(p_1-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_1F+\mathrm{VQ}_c}{p_1p_2T}+C_{cfe}v_1+C_{cve}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\\
&+C_pI_{ps}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y-\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right)+\frac{A_s+O_sv_2}{p_2T}\\
&+h_{sm}\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_2p_p}+\frac{h_pT}{2}\left((2-p_2)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.\\
&\left.+(p_2-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_2F+\mathrm{VQ}_s}{p_2T}+C_{emf}v_2+C_{evm}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\\
&-C_sI_{em}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)Y-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right)\\
&+C_cI_{pm}\left(\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)X-\frac{p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)}{2}\right)\\
&\left.+\sum_{j}^{n}\left(\frac{R_o}{T}+h_c\frac{TD_{rj}}{2}-pI_{er}(D_{rj}X-\frac{TD_{rj}}{2})\right)+\beta\frac{b^2}{2}\right].
\end{aligned}
\tag{53}
$$

Case 2. Similarly, the total profit of the supply chain is found in Case 2 by adding profits of supplier, manufacturer, and retailers.

$$
\begin{aligned}
\mathrm{ETP}_2 ={}& (p_{11}+p_{22}+p)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)-\left[\frac{A_c+O_c v_1}{p_1p_2T}+h_{\mathrm{TS}}\frac{p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2v_1p_s}\right.\\
&+\frac{h_{sm}v_2T}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}\right.\\
&\left.+(p_1-1)\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)\right)+\frac{v_1F+\mathrm{VQ}_c}{p_1p_2T}+C_{cfe}v_1+C_{cve}\left(a\frac{p_{\max}-p}{p-p_{\min}}+\alpha b^{\gamma}+\frac{k\rho\sqrt{d}}{1+\rho}\right)
\end{aligned}
$$

$$+ C_p I_{ps} \frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)} - C_c I_{es} \frac{\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2p_1p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)}(p_1p_2T - Y)^2$$

$$+ \frac{A_s + O_s v_2}{p_2 T} + h_{sm} \frac{T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{2p_2p_p} + \frac{h_pT}{2}\Bigg((2-p_2)\frac{\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2}{p_p}$$

$$+ (p_2-1)\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)\Bigg) + \frac{v_2F + \mathrm{VQ}_s}{p_2T} + C_{emf}v_2$$

$$+ C_{evm}\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right) + \frac{C_cI_{pm}\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2(p_2T-Y)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)}$$

$$+ C_cI_{pm}\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)X\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)} - C_sIem\frac{\left(\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)Y\right)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)}$$

$$- \frac{C_sI_{em}\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)^2(p_2T-X)^2}{2p_2T\left(a\frac{p_{\max}-p}{p-p_{\min}} + \alpha b^\gamma + \frac{k\rho\sqrt{d}}{1+\rho}\right)} + \sum_j^n\Bigg(\frac{R_o}{T} + h_c\frac{TD_{rj}}{2} - \frac{pI_{er}(D_{rj}X)^2}{2TD_{rj}}$$

$$+ \frac{C_sI_{pr}(D_{rj}T - D_{rj}X)^2}{2TD_{rj}}\Bigg) + \beta\frac{b^2}{2}\Bigg]. \tag{54}$$

Equations (53) and (54) give the total profit functions of Cases 1 and 2, respectively. There are three continuous decision variables T, b, and p. Solution procedures are given in the next section.

5. Solution methodology

Optimum solutions of found separately for two cases.

5.1. Optimum solutions of Case 1

Optimum values of continuous decision variables are found by equating the first order derivatives of Equation (53) with respect to decision variables T, b, and p with zero, respectively. The unique values of T, b, and p are

$$T^* = \frac{\sqrt{A_c + O_cv_1 + v_1F + \mathrm{VQ}_c + v_2F + \mathrm{VQ}_s + \sum j^n R_o}}{\sqrt{A}}. \text{ (See Appendices A and B)} \tag{55}$$

$$b^* = \left(\frac{\beta}{2\alpha\gamma}\right)^{\frac{1}{\gamma-2}}. \text{ (See Appendices A and C)} \tag{56}$$

$$p^* = \frac{\sqrt{(p_{\min} - p_{\max})C}}{\sqrt{\sum_j^n I_{er}(D_{rj}X - D_{rj}T) - Z}} + p_{\min}. \text{ (See Appendices A and B)} \tag{57}$$

From equations (55) to (57) the unique values of decision variables are found. This following Proposition 1 proves the global maximum profit at T^*, b^*, and p^*.

Proposition 1. *Global maximum profit of the supply chain (equation (53)) is achieved at T^*, b^*, and p^* if* $U^2 + UA_{66}\frac{2}{p-p_{\min}} + \left(\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2 > U + A_{55}\frac{2Z}{p-p_{\min}}$ *and* $\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s} + \frac{2h_{sm}p_2T}{v_2p_s} + \frac{h_{sm}p_2T}{p_p} + \frac{h_{sm}p_2T}{v_2p_p} + \frac{2h_pT}{p_p} > \frac{h_{sm}p_1p_2T}{p_s} + \frac{h_{sm}p_2T}{v_2p_p} + \frac{h_pTp_2}{p_p}$.

Proof. See the Appendix D. □

5.2. Optimum solutions of Case 2

Optimum solutions of the equation (54) with respect to T, b, and p are found by the necessary conditions of classical optimization. Unique solutions are as follows:

$$T^* = \frac{\sqrt{A_{12}}}{\sqrt{A_{21}}}. \text{ (See Appendices A and B)} \tag{58}$$

$$b^* = \left(\frac{\beta}{B_{11}}\right)^{\frac{1}{\gamma-2}}. \text{ (See Appendices A and C)} \tag{59}$$

$$p^* = \frac{\sqrt{2Ta(p_{\min} - p_{\max})(C_{11} - zC_{111})}}{\sqrt{\sum_j^n I_{er}D_{rj}X^2}} + p_{\min}. \text{ (See Appendices A and B)} \tag{60}$$

The unique values of the decision variables are given in equations (58)–(60). This following Proposition 2 proves the global maximum profit at T^*, b^*, and p^*.

Proposition 2. *Global maximum profit of the supply chain (equation (54)) is achieved at T^*, b^*, and p^* if* $U^2 + UA_{66}\frac{2}{p-p_{\min}} + \left(\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2 > U + A_{55}\frac{2Z}{p-p_{\min}}$ *and* $\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s} + \frac{2h_{sm}p_2T}{v_2p_s} + \frac{h_{sm}p_2T}{p_p} + \frac{h_{sm}p_2T}{v_2p_p} + \frac{2h_pT}{p_p} > \frac{h_{sm}p_1p_2T}{p_s} + \frac{h_{sm}p_2T}{v_2p_p} + \frac{h_pTp_2}{p_p}$.

Proof. See the Appendix E. □

6. Numerical analysis

The numeral example is provided to demonstrate the model numerically. Input values are summarized in Table 2. Data is taken from Dey *et al.* [12] to obtain the global optimum profit, T, p, and b. Data is modified for the convergence of the objective functions. Case 2 provides more profit than Case 1. This implies that SSMD policy is fruitful with the second credit policy, where the cycle time of the SCM players is greater or equal to the payment period. It is seen that the advertisement percentage is more in Case 2 than Case 1, *i.e.*, retailers give more effort for advertisement. As a result, the profit is higher than Case 1. Case 2 requires more cycle length than Case 1. This implies that more cycle time provides more time to SCM player for payments. As cycle time > payment period provides maximum profit, a large cycle time provides them little bit more time for payment. In that mean time, they can sell products and earn revenue.

The optimum solutions are provided in Tables 3 and 4. The number of shipments for SSMD policy is 5. The supplier and the manufacturer sends their ordered products in 5 times.

Besides the propositions, some graphical representation of the concavity of the total profit functions for Cases 1 and 2 are given in Figures 6–10, respectively. Figures 6–8 show that the objective function in Case 1 obtains global optimum profit. Similarly, Figures 9 and 10 show that the objective function in Case 2 obtains its global optimum profit at T^*, p^*, and b^*.

7. Discussions

Few discussions and a detail comparison of the model with existing literature are provided here.

TABLE 2. Parametric values.

Parameter	Supplier	Manufacturer	Retailers
Ordering cost	$O_c = \$200$/shipment	$O_s = \$150$/shipment	$R_o = \$100$/order
Setup cost	$A_c = \$600$/setup	$A_s = \$300$/setup	–
Holding cost of final products	$h_{sm} = \$0.8$/unit/year	$h_p = \$0.5$/unit/year	$h_c = \$0.8$/unit/year
Holding cost of raw material	$h_{\mathrm{TS}} = \$.4/$	$h_{sm} = \$0.8$/unit/year	–
Production rate	$p_s = \$5500$ unit/year	$p_p = \$4300$ unit/year	–
Fixed carbon emissions cost	$C_{cfe} = \$0.3$/shipment/year	$C_{emf} = \$0.3$/shipment/year	–
Variable carbon emissions cost	$C_{cve} = \$0.2$/unit	$C_{evm} = \$0.3$/unit	–
Fixed transportation cost	$F = \$0.7$/shipment	$F = \$0.7$/shipment	–
Variable transportation cost	$V = \$0.2$/unit	$V = \$0.2$/unit	–
Purchasing cost	$C_p = \$10$/unit	$C_c = \$25$/unit	$C_s = \$40$/unit
Selling price	$p_{11} = \$25$/unit	$p_{22} = \$40$/unit	–
Opportunity cost	$I_{ps} = \$0.10$/year	$I_{pm} = \$0.40$/year	$I_{pr} = \$0.05$/year
Interest earned	$I_{es} = \$0.25$/year	$I_{em} = \$0.15$/year	$I_{er} = \$0.28$/year
Integer for cycle length	–	$p_1 = 10$	$p_2 = 10$
Permissible delay period	–	$Y = 0.26$ years	$X = 0.26$ years
Scaling parameter	–	–	$(a, \alpha, \beta, \gamma) = (50, 10, 100, 1.1)$
Maximum selling price	–	–	$P_{\max} = \$3000$/unit
Minimum selling price	–	–	$P_{\min} = \$80$/unit
Advertisement parameter	–	–	$(k, d, \rho) = (1, 9, 5)$

Notes. "–" denotes not applicable.

TABLE 3. Optimum solutions of Case 1: Cycle time is less than payment period.

T (year)	p (\$/unit)	b	ETP$_1$ (\$/year)
0.53	250.97	52.62	396 521

TABLE 4. Optimum solutions of Case 2: Cycle time is greater or equal to payment period.

T (year)	p (\$/unit)	b	ETP$_2$ (\$/year)
0.95	314.42	62.48	576 371

7.1. Comparison with existing literature

The result of comparison are given below:

- Sebatjanea and Adetuji [35] presented a three-echelon SCM for living beings & perishable products and discussed inventory management procedures for three supply chain players. The present study is applicable to non-perishable and physical products. Besides, if the present model is considered with the following relaxations, then it can conceptually converge to the study of Sebatjanea and Adetuji [35]: SSMD policy for the supplier, advertisement policy for demand, and credit policy for three SCM players.
- Wang and Wan [43] introduced a three-echelon supply chain and found solutions through metaheuristics the model for a greenness-dependent product with uncertainty. In contrast, the present study finds global solution for non-deteriorated and non-perishable products. Thus, the comparison is based on the solution methodology of a three-echelon SCM.
- Ben-Daya *et al.* [4] discussed a two-echelon closed-loop SCM under a consignment stock agreement, which is an inventory management policy. The present study discusses a three-echelon SCM with advertisement and payment policies. The differences between the two models are dimension and policies-based.

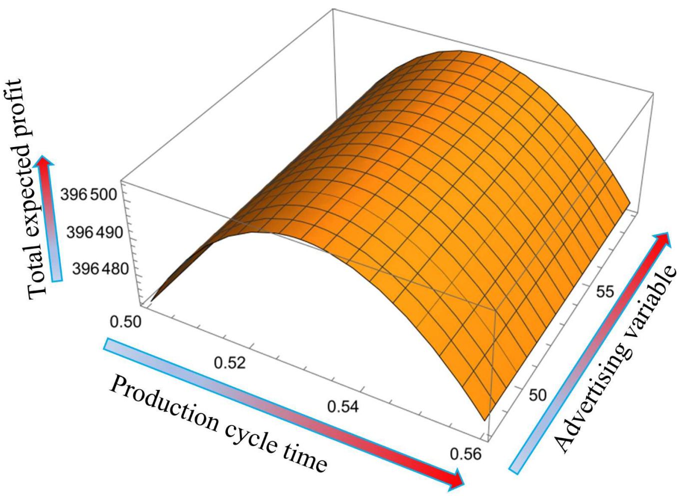

FIGURE 6. Concavity of the total profit *versus* advertising cost and production cycle time.

- Chatha and Jalil [8] proposed a structural model of supply chain network. Whereas the present study formulates supply chain management with inventory management and developed policies for transportation, payment, and marketing. Thus, the purpose and aim of these two models are different.
- Hanh *et al.* [14] investigated a three-echelon supply chain non-coordination model with a price-sensitive linear demand to choose the most efficient players through game theory. In contrast, the present paper proposes the best policies for a three-echelon supply chain coordination model with price and advertisement-sensitive non-linear demand.

7.2. Sensitivity analysis

This section presents sensitiveness of cost parameters with respect to the profit functions. Analysis for two cases are given below.

7.3. Case 1. When cycle time is less than the payment period

The result of sensitivity of the total profit are observed for changes in −50%, −25%, +25%, and +50%. This section provides results for several key parameter changes O_s, A_s, h_p, h_c, C_{cve}, C_{evm}, I_{er}, and h_{TS} of the total profit ETP_1. Changes of cost parameters are expressed in the following Table 5.

The following results are found:

(1) Changes in supplier's ordering cost O_s are inversely proportional to the total profit of Case 1. If the value of O_s raise by 50% and 25%, than value of ETP_1 declines. Likewise, if the value of O_s decreases by 50% and 25%, total profit raises. Changes are identical for the both negative and positive changes.
(2) Changes in the supplier's setup cost A_s results a slight variation in the total profit of Case 1. If the cost of A_s raises to 50% and 25%, total profit decreases. Likewise, if the parameter A_s decreases by 50% and 25%, total profit increases.
(3) If the holding cost of the manufacturer h_p raises, then the total profit ETP_1 of Case 1 declines. Similarly, if holding costs decreases, then the total profit increases. The variation in parameters causes an inverse relation with the total profit, but the changes are similar for both positive and negative changes. Changes for retailers' holding costs are similar.

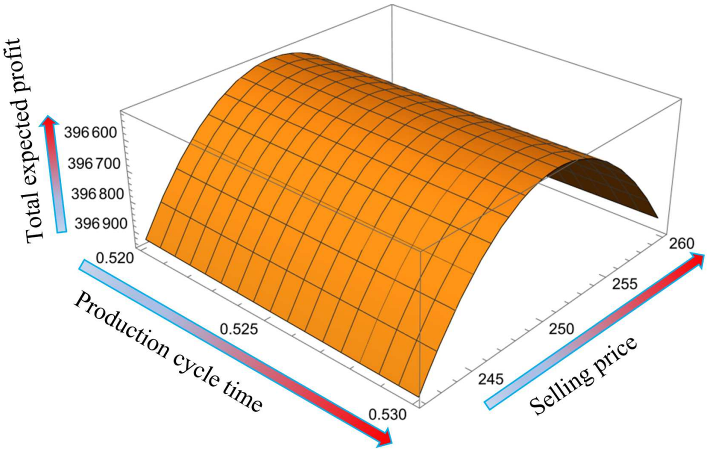

FIGURE 7. Concavity of the total profit *versus* selling price and production cycle time.

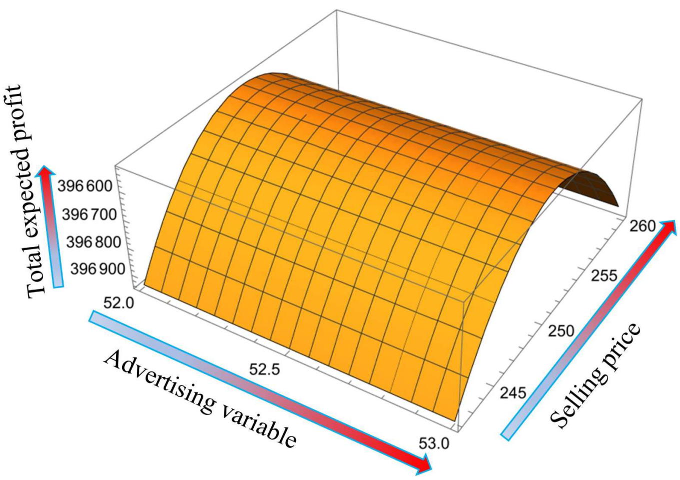

FIGURE 8. Concavity of the total profit *versus* selling price and advertising cost.

(4) Capital opportunity cost I_{pm} of the manufacturer is the most sensitive parameter for the total profit of Case 1. If the opportunity cost raises by 25% and 50%, then ETP_1 decreases. Likewise, if the opportunity cost declines by 25% and 50%, total profit increases by 0.93% and 1.86%, respectively.
(5) Interest earned I_{er} of the retailer is the second most sensitive parameter. If amount of I_{er} raises by 25% and 50%, then ETP_1 of Case 1 raises by 0.485% and 0.970%, respectively. Likewise, if interest earned decreases by 25% and 50%, total profit declines.
(6) For a single product, the supplier's variable carbon emissions cost C_{cvc} is inversely proportional to the total profit and changes for both negative and positive percentages are similar. Changes for supplier's fixed holding cost h_{TS} is similar to the variable carbon emissions cost. Decreasing parametric value of h_{TS} increases total profit of Case 1.

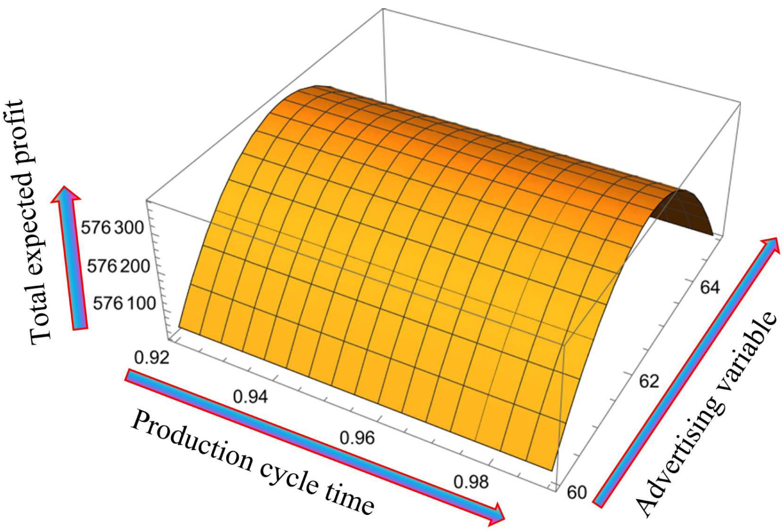

FIGURE 9. Concavity of the total profit *versus* advertising cost and production cycle time.

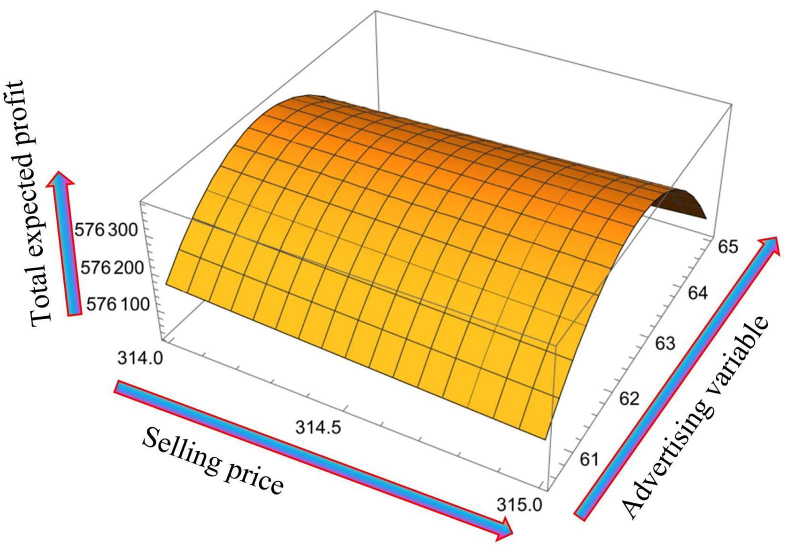

FIGURE 10. Concavity of the total profit *versus* selling price and advertising cost.

7.4. Case 2: When cycle time is greater or equal to the payment period

The results of sensitivity of the total profit are observed for changes in −50%, −25%, +25%, and +50%. This section provides results for several key parameter changes O_s, A_s, h_p, h_c, C_{cve}, C_{evm}, I_{er}, and h_{TS} of the total profit ETP_1. Changes of cost parameters are expressed in the following Table 6. Changes for all above-mentioned parameters are very less, *i.e.*, cost parameters are very less sensitive for the total cost in Case 2. Discussions are given below.

(1) Supplier's ordering cost O_s and retailers' holding cost h_c are the least sensitive parameters. Changes in cost parameters are directly proportional to the total profit of Case 2.

TABLE 5. Result of changes in essential cost parameters for Case 1.

Parameters	Changes (in %)	ETP_1 (%)	Parameters	Changes (in %)	ETP_1 (%)
O_s	−50	+0.024	I_{pm}	−50	+1.860
	−25	+0.012		−25	+0.930
	+25	−0.012		+25	−0.930
	+50	−0.024		+50	−1.860
A_s	−50	+0.050	I_{er}	−50	−0.970
	−25	+0.025		−25	−0.485
	+25	−0.025		+25	+0.485
	+50	−0.050		+50	+0.970
h_p	−50	+0.148	C_{cve}	−50	+0.076
	−25	+0.074		−25	+0.038
	+25	−0.074		+25	−0.038
	+50	−0.148		+50	−0.076
h_c	−50	+0.027	h_{TS}	−50	−0.087
	−25	+0.013		−25	−0.044
	+25	−0.013		+25	+0.044
	+50	−0.027		+50	+0.087

TABLE 6. Result of changes in essential cost parameters for Case 2.

Parameters	Changes (in %)	ETP_2 (%)	Parameters	Changes (in %)	ETP_2 (%)
O_s	−50	−0.0069	I_{pm}	−50	−0.1476
	−25	−0.0035		−25	−0.0739
	+25	+0.0033		+25	+0.0737
	+50	+0.068		+50	+0.1478
A_s	−50	+0.028	I_{er}	−50	+0.4122
	−25	+0.014		−25	+0.2060
	+25	−0.012		+25	−0.2060
	+50	−0.026		+50	−0.4122
h_p	−50	−0.1942	C_{cve}	−50	−0.0527
	−25	−0.0972		−25	−0.264
	+25	+0.0972		+25	+0.264
	+50	+0.1944		+50	+0.0528
h_c	−50	−0.0190	h_{TS}	−50	−0.1393
	−25	−0.0095		−25	−0.0698
	+25	+0.0094		+25	+0.0698
	+50	+0.0189		+50	+0.1394

(2) Manufacturer's holding cost for finished products h_s & capital opportunity cost I_{pm} and supplier's variable emissions cost C_{cve} & and raw material holding costs are the next least sensitive cost parameters for the total profit of Case 2. Changes are directly proportional to the profit function, *i.e.*, increasing cost causes increasing profit.

(3) manufacturer's setup cost A_s and retailers' interest earned I_{er} are two less sensitive parameters but the changes are inversely proportional to the total profit, that is, increasing cost causes decreasing profit and vice-versa.

8. Managerial insights

(1) The results presented in this paper clarifies three advanced policies as payment, advertisement, and transportation within a three-echelon SCM. An important observation is that cycle time and payment period are two important drivers of a three-echelon SCM for profit maximization.
(2) The SSMD transportation strategy is profitable with the combination of credit-period and advertisement policy. As products are delivered into multiple shipments, holding costs of retailers reduce along with the profitable payment period.
(3) A longer cycle time provides more profit when trade-credit involves as a payment method of supply chain players. The more payment period to the supply chain players earns more profit for the coordination case among players.
(4) Industry managers determines opportunity cost, holding cost, earned interest, paying interest, transportation cost, and carbon emissions cost. Variable cost structure for emissions from transportation restrict the emissions cost rather than a lumpsum amount investment.

9. Conclusions

This research described a three-echelon SCM with a single supplier, a single manufacturer, and, multiple retailers. Supply chain players used a trade-credit payment policy to pay later, specifically after generating revenue. The used advertisement policy helps to gain more revenue for the supply chain than a constant or advertisement-free demand policy. The SSMD transportation policy was found to be the best-fit transportation policy with the other two policies. Besides, the SSMD policy saved holding cost of retailers. Fixed and variable type of emissions from transportation was used by all players. Results found that the proposed policies earned a profit for the coordination policy among three types of supply chain players. Through numerical research, it was identified that fixed transport and carbon emissions cost were lesser sensitive to the total profit. Therefore, industry managers can use the SSMD policy for transportation, save holding cost, and generates more revenue. This study did not consider any effect of transportation policy on the market demand. This study can be expanded by considering the effect of transportation policy on market demand. For an effective study, an international supply chain can be considered with variable fuel charges and carbon emissions from fuel, whereas the present study discussed a domestic supply chain. Then, the dollar exchange rate, international taxes, and other tariffs should be considered within the supply chain. Future research can draw other multiple aspects too. This study considered a single type of products without imperfect products, which can be extend for multi-product-based three-echelon SCM with lead time. The effect of imperfect items and waste from those imperfect items can be studied further. Effect of energy (renewable and fossil fuels) is another promising area of research to extend the present work. The application of uncertainty [32] with lead time within a three-echelon will be another interesting extension of this study.

Appendix A.

$$U = a\frac{(p_{\min} - p_{\max})}{(p - p_{\min})^2},$$

$$W = v_1 C_{cfe} + ZC_{cve} + ZYC_p I_{ps} + v_2 C_{cfe} + ZC_{cve} - C_s I_{em} ZY + XZC_c I_{pm} - \sum_j^n pI_{er} D_{rj} X + \beta\frac{b^2}{2},$$

$$Z = \left(a\frac{p_{\max} - p}{p - p_{\min}} + \alpha b^\gamma + k\sqrt{d}(\rho/(1+\rho))\right),$$

$$A = \Bigg[h_{\mathrm{TS}}\frac{p_1 p_2 Z^2}{2v_1 p_s} + \frac{h_{sm} p_2}{2}\left(\left(\frac{2}{v_2} - p_1\right)\frac{Z^2}{p_s} + \left(1 - \frac{1}{v_2}\right)\frac{Z^2}{p_p} + (p_1 - 1)Z\right) - C_p I_{ps}\frac{p_1 p_2 Z}{2} + h_{sm}\frac{p_2 Z^2}{2v_2 p_p}$$

$$+\frac{h_p}{2}\left((2-p_2)\frac{Z^2}{p_p}+(p_1-1)Z\right)+C_sI_{em}\frac{p_2Z}{2}-C_cI_{pm}\frac{p_2Z}{2}+\sum_j^n h_c\frac{D_{rj}}{2}+\sum_j^n pI_{er}\frac{D_{rj}}{2}\Bigg],$$

$$B=-\Bigg[p+\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s}+\frac{h_{sm}p_2T}{p_s}\left(\frac{2}{v_2}-p_1\right)+\frac{h_{sm}p_2T}{p_p}\left(1-\frac{1}{v_2}\right)+(p_1-1)+2C_{cve}+C_pI_{ps}Y-C_pI_{ps}\frac{p_1p_2T}{2}$$

$$+\frac{h_{sm}p_2T}{v_2p_p}+\frac{h_pT}{p_p}(2-p_2)+\frac{h_2T}{2}(p_2-1)-C_sI_{em}Y+\frac{p_2T}{2}C_sI_{em}+C_cI_{pm}X-\frac{p_2T}{2}C_cI_{pm}\Bigg],$$

$$C=p+\frac{h_{\mathrm{TS}}}{v_1p_s}Zp_1p_2T+h_{sm}p_2T\left(\frac{2}{v_2}-p_1\right)\frac{Z}{p_s}+\frac{h_{sm}p_2T}{p_p}\left(1-\frac{1}{v_2}\right)Z+(p_1-1)+C_{cve}+C_pI_{ps}Y-\frac{C_pI_{ps}p_1p_2T}{2}$$

$$+\frac{h_{sm}p_2T}{v_2p_p}Z+\frac{h_pT}{p_p}(2-p_2)Z+\frac{h_pT}{2}(p_2-1)+C_{cve}-C_sI_{em}Y+\frac{C_sI_{em}p_2T}{2}+C_cI_{pm}X-\frac{p_2T}{2}C_cI_{pm},$$

$$G=\frac{A_c+O_cv_1}{Tp_2},L=\frac{v_1F+\mathrm{VQ}_c}{p_2T},I*=2F-2E-2A^*-\frac{2FZ}{p_s},E=h_{\mathrm{TS}}T\frac{Z^2}{2v_1p_s}p_2,F=\frac{h_{sm}p_2TZ}{2},$$

$$A^*=\frac{p_2TZ}{2},B^*=\frac{A_c+O_cv_1}{p_1T}+\frac{v_1F+\mathrm{VQ}_c}{p_1T}+\frac{A_s+O_sv_2}{T}+\frac{v_2F+\mathrm{VQ}_s}{T},$$

$$M=\frac{h_{\mathrm{TS}}p_1TZ^2}{v_1p_s}+h_{sm}TN*+C_pI_{ps}TZ+\frac{h_{sm}TZ^2}{v_2p_p}+\frac{h_pTZ^2}{p_p}+2h_pTZ+C_{cve}TZ+C_cI_{pm}TZ,$$

$$N^*=\left(\frac{2}{v_2}-p_1\right)\frac{Z^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{Z^2}{p_p}+(p_1-1)Z,K=\frac{h_{\mathrm{TS}}p_1p_2TZ^2}{2p_s},S=\frac{h_{sm}p_2TZ^2}{2p_p}$$

$$N=\frac{h_{sm}p_2TZ^2}{p_s}-\frac{h_{sm}p_2TZ^2}{p_p}+\frac{h_{sm}p_2TZ^2}{2p_p}\cdot$$

Appendix B.

$$\mathrm{ETP}_1=pZ-\frac{A_c+O_cv_1+v_1F+\mathrm{VQ}_c+p_1A_s+p_1O_sv_2+p_1v_2F+p_1\mathrm{VQ}_s+p_1p_2\sum_j^n R_o}{p_1p_2T}-T\Bigg[h_{\mathrm{TS}}\frac{p_1p_2Z^2}{2v_1p_s}$$

$$+\frac{h_{sm}p_2}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{Z^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{Z^2}{p_p}+(p_1-1)Z\right)-C_pI_{ps}\frac{p_1p_2Z}{2}+h_{sm}\frac{p_2Z^2}{2v_2p_p}$$

$$+\frac{h_p}{2}\left((2-p_2)\frac{Z^2}{p_p}+(p_1-1)Z\right)+C_sI_{em}\frac{p_2Z}{2}-C_cI_{pm}\frac{p_2Z}{2}+\sum_j^n h_c\frac{D_{rj}}{2}+\sum_j^n pI_{er}\frac{D_{rj}}{2}\Bigg]-W$$

$$\frac{\partial\mathrm{ETP}_1}{\partial T}=\frac{A_c+O_cv_1+v_1F+\mathrm{VQ}_c+p_1A_s+p_1O_sv_2+p_1v_2F+p_1\mathrm{VQ}_s+p_1p_2\sum_j^n R_o}{p_1p_2T^2}-\Bigg[h_{\mathrm{TS}}\frac{p_1p_2Z^2}{2v_1p_s}$$

$$+\frac{h_{sm}p_2}{2}\left(\left(\frac{2}{v_2}-p_1\right)\frac{Z^2}{p_s}+\left(1-\frac{1}{v_2}\right)\frac{Z^2}{p_p}+(p_1-1)Z\right)-C_pI_{ps}\frac{p_1p_2Z}{2}+h_{sm}\frac{p_2Z^2}{2v_2p_p}$$

$$+\frac{h_p}{2}\left((2-p_2)\frac{Z^2}{p_p}+(p_1-1)Z\right)+C_sI_{em}\frac{p_2Z}{2}-C_cI_{pm}\frac{p_2Z}{2}+\sum_j^n h_c\frac{D_{rj}}{2}+\sum_j^n pI_{er}\frac{D_{rj}}{2}\Bigg]$$

$$\frac{\partial\mathrm{ETP}_1}{\partial b}=p\alpha\gamma b^{(\gamma-1)}-\Bigg[\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s}Z\alpha\gamma b^{(\gamma-1)}+h_{sm}p_2T\left(\frac{2}{v_2}-p_1\right)\frac{Z}{p_s}\alpha\gamma b^{(\gamma-1)}+\frac{h_{sm}p_2T}{p_p}\left(1-\frac{1}{v_2}\right)Z\alpha\gamma b^{(\gamma-1)}$$

$$+\frac{h_{sm}p_2T}{2}(p_1-1)\alpha\gamma b^{(\gamma-1)}+C_{cve}\alpha\gamma b^{(\gamma-1)}+C_pI_{ps}Y\alpha\gamma b^{(\gamma-1)}-C_pI_{ps}\frac{p_1p_2T}{2}\alpha\gamma b^{(\gamma-1)}+\frac{h_{sm}p_2T}{v_2p_p}$$

$$\times Z\alpha\gamma b^{(\gamma-1)} + \frac{h_pT}{p_p}(2-p_2)Z\alpha\gamma b^{(\gamma-1)} + \frac{h_pT}{2}(p_2-1)\alpha\gamma b^{(\gamma-1)} + C_{cve}\alpha\gamma b^{(\gamma-1)} - C_sI_{em}Y\alpha\gamma b^{(\gamma-1)}$$
$$+ \frac{p_2T}{2}C_sI_{em}\alpha\gamma b^{(\gamma-1)} + C_cI_{pm}\alpha\gamma b^{(\gamma-1)} - \frac{p_2T}{2}C_cI_{pm}\alpha\gamma b^{(\gamma-1)} + b\beta\Bigg]$$

$$\frac{\partial \mathrm{ETP}_1}{\partial p} = Z - \left[\frac{h_{\mathrm{TS}}}{v_1p_s}ZUp_1p_2T + h_{sm}p_2T\left(\frac{2}{v_2}-p_1\right)\frac{ZU}{p_s} + \frac{h_{sm}p_2T}{p_p}\left(1-\frac{1}{v_2}\right)ZU + \frac{h_{sm}p_2T}{2}(p_1-1)U\right.$$
$$+ C_{cve}U + C_pI_{ps}YU - \frac{C_pI_{ps}p_1p_2T}{2}U + \frac{h_{sm}p_2T}{v_2p_p}ZU + \frac{h_pT}{p_p}(2-p_2)ZU + \frac{h_pT}{2}(p_2-1)U$$
$$\left.+ C_{cve}U - C_sI_{em}YU + \frac{C_sI_{em}p_2T}{2}U + C_cI_{pm}XU - \frac{p_2T}{2}C_cI_{pm}U - \sum_j^n I_{er}\left(D_{rj}X - \frac{TD_{rj}}{2}\right)\right]$$

$$\frac{\partial^2 \mathrm{ETP}_1}{\partial T^2} = -\frac{A1}{T^3}, \frac{\partial^2 \mathrm{ETP}_1}{\partial T\partial b} = -2ZUB1 - UC1, \frac{\partial^2 \mathrm{ETP}_1}{\partial T\partial p} = -\sum_{j=1}^n \frac{I_{er}D_{rj}}{2}$$
$$\frac{\partial^2 \mathrm{ETP}_1}{\partial b^2} = b^{(\gamma-1)}\left[p\alpha\gamma(\gamma-1) - \alpha^2\gamma^2 b(\gamma-1)A_{11} - \frac{\alpha\gamma ZA_{11}}{b} - \frac{\alpha\gamma(\gamma-1)A_{22}}{b}\right] + \beta,$$
$$\frac{\partial^2 \mathrm{ETP}_1}{\partial b\partial T} = -Z\alpha\gamma b(\gamma-1)A_{33} - \alpha\gamma b(\gamma-1)A_{44}, \frac{\partial^2 \mathrm{ETP}_1}{\partial b\partial p} = \alpha\gamma b^{(\gamma-1)}(1-A_{11}U)$$
$$\frac{\partial^2 \mathrm{ETP}_1}{\partial p^2} = -U^2 - UA_{66}\frac{2}{p-p_{\min}} + U + A_{55}\frac{2Z}{p-p_{\min}}, \frac{\partial^2 \mathrm{ETP}_1}{\partial p\partial T} = -ZUA_{77} - UA_{88},$$
$$\frac{\partial^2 \mathrm{ETP}_1}{\partial p\partial T} = \alpha\gamma b(\gamma-)(1-UA_{55}).$$

Appendix C.

$$A_1 = \frac{2(A_c + O_cv_1 + v_1F + \mathrm{VQ}_c + p_1A_s + p_1O_sv_2 + p_1v_2F + p_1\mathrm{VQ}_s + p_1p_2Q_s + p_1p_2\sum_{j=1}^n R_o)}{p_1p_2}$$
$$B_1 = \frac{h_{\mathrm{TS}}p_1p_2}{2v_1p_s} + \left(\frac{2}{v_2}-p_1\right)\frac{h_{sm}p_2}{2p_s} + \left(1-\frac{1}{v_2}\right)\frac{h_{sm}p_2}{2p_p} + \frac{h_{sm}p_2}{2v_2p_p} + (2-p_2)\frac{h_p}{2p_p}$$
$$C_1 = (p_1-1)\frac{h_{sm}p_2}{2} - \frac{C_pI_{ps}p_1p_2}{2} + (p_1-1)\frac{h_p}{2} + \frac{C_sI_{em}p_2}{2} - \frac{C_cI_{pm}p_2}{2}$$
$$A_{11} = \frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s} + \frac{h_{sm}p_2T}{p_s}\left(\frac{2}{v_2}-p_1\right) + \frac{h_{sm}p_2T}{p_p}\left(1-\frac{1}{v_2}\right) + \frac{h_{sm}p_2T}{v_2p_p} + \frac{h_pT}{p_p}(2-p_2)$$
$$A_{22} = \frac{h_{sm}p_2T}{2}(p_1-1) + C_{cve} + C_pI_{ps}Y - C_pI_{ps}\frac{p_1p_2T}{2} + \frac{h_pT}{2}(p_2-1) + C_{cve} - C_sI_{em}Y + \frac{p_2T}{2}C_sI_{em}$$
$$+ C_cI_{pm} - \frac{p_2T}{2}C_cI_{pm}$$
$$A_{33} = \frac{h_{\mathrm{TS}}p_1p_2}{v_1p_s} + \frac{h_{sm}p_2}{p_s}\left(\frac{2}{v_2}-p_1\right) + \frac{h_{sm}p_2}{p_p}\left(1-\frac{1}{v_2}\right) + \frac{h_{sm}p_2}{v_2p_p} + \frac{h_p}{p_p}(2-p_2)$$
$$A_{44} = \frac{h_{sm}p_2}{2}(p_1-1) - \frac{C_pI_{ps}p_1p_2}{2} + \frac{h_p}{2}(p_2-1) + \frac{p_2}{2}C_sI_{em} - \frac{p_2}{2}C_cI_{pm}$$
$$A_{55} = \frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s} + \frac{h_{sm}p_2T}{p_s}\left(\frac{2}{v_2}-p_1\right) + \left(1-\frac{1}{v_2}\right)\frac{h_{sm}p_2T}{p_p} + \frac{h_{sm}p_2T}{v_2p_p} + \frac{h_pT}{p_p}(2-p_2)$$

$$A_{66} = (p_1-1)\frac{h_{sm}p_2T}{2} + C_{cve} + C_pI_{ps}Y - \frac{C_pI_{ps}p_1p_2T}{2} + \frac{h_pT}{2}(p_2-1) + C_{cve} - C_sI_{em}Y + \frac{C_sI_{em}p_2T}{2} + C_cI_{pm}X - \frac{p_2T}{2}C_cI_{pm} - \sum_j^n I_{er}\left(D_{rj}X - \frac{TD_{rj}}{2}\right)$$

$$A_{77} = \frac{h_{\mathrm{TS}}p_1p_2}{v_1p_s} + \left(\frac{2}{v_2}-p_1\right)\frac{h_{sm}p_2}{p_s} + \left(1-\frac{1}{v_2}\right)\frac{h_{sm}p_2}{p_p} + \frac{h_{sm}p_2}{v_2p_p} + \frac{h_p}{p_p}(2-p_2)$$

$$A_{88} = (p_1-1)\frac{h_{sm}p_2}{2} - C_sI_{em}Y + \frac{C_sI_{em}p_2}{2} + \frac{p_2}{2}C_cI_{pm} + \sum_j^n \frac{D_{rj}}{2}$$

$$A_{12} = \frac{A_c+O_cv_1}{p_1p_2} + \frac{v_1F+\mathrm{VQ}_c}{p_1p_2} + \frac{C_pI_{ps}ZY^2}{2p_1p_2} - \frac{C_cI_{es}ZY^2}{2p_1p_2} + \frac{A_s+O_sv_2}{p_2} + \frac{v_2F+\mathrm{VQ}_s}{p_2} + \frac{C_cI_{pm}ZY^2}{2p_2} + \frac{C_cI_{pm}ZX^2}{2p_2} - \frac{C_sI_{em}ZY^2}{2p_2} - \frac{C_sI_{em}ZX^2}{2p_2} + \sum_j^n R_0 - \sum_j^n \frac{pI_{er}X^2}{2} + \sum_j^n \frac{C_sI_{pr}D_{rj}X^2}{2}$$

$$A_{21} = \frac{h_{\mathrm{TS}}p_1p_2Z^2}{2v_1p_s} + \frac{h_{sm}v_2}{2}\left[\left(\frac{2}{v_2}-p_1\right)\frac{Z^2}{p_s} + \left(1-\frac{1}{v_2}\right)\frac{Z^2}{p_p} + (p_1-1)Z\right] + \frac{h_{sm}Z^2}{2p_pp_2} + \frac{h_p}{2}\left[(2-p_2)\frac{Z^2}{p_p} + (p_2-1)Z\right] + \sum_j^n \frac{h_cD_{rj}}{2} + \sum_j^n \frac{C_sI_{pr}D_{rj}}{2} + \frac{C_cI_{pm}Zp_2}{2} - \frac{C_sI_{pm}Zp_2}{2}$$

$$B_{11} = (p_{11}+p_{22}+p)\alpha\gamma - \frac{Zh_{\mathrm{TS}}p_1p_2T\alpha\gamma}{v_1p_s} - \frac{h_{sm}v_2T}{2}\left[\left(\frac{2}{v_2}-p_1\right)\frac{2Z\alpha\gamma}{p_s} + \left(1-\frac{1}{v_2}\right)\frac{2Z\alpha\gamma}{p_p} + (p_1-1)b^{\gamma-1}\right] + \frac{C_cI_{es}\alpha\gamma(p_1p_2T-Y)^2}{2p_1p_2T} - C_{cve}\alpha\gamma - \frac{C_pI_{ps}Y\alpha\gamma}{2p_1p_2T} - \frac{h_{sm}Z\alpha\gamma}{p_2p_p} - \frac{h_pT}{2}\left[\frac{(2-p_2)2Z\alpha\gamma}{p_p} + (p_2-1)\alpha\gamma\right] + \frac{C_cI_{pm}\alpha\gamma}{2p_2T}(p_2T-Y)^2 + \frac{C_sI_{em}\alpha\gamma}{2p_2T}(p_2T-X)^2 - \frac{C_cI_{pm}\alpha\gamma X^2}{2p_2T} + \frac{C_sI_{em}\alpha\gamma Y^2}{2p_2T}$$

$$C_{11} = (p_{11}+p_{22}+p) - \frac{(p_1-1)h_{sm}v_2T}{2} - C_{cve} - \frac{C_pI_{ps}Y}{2p_1p_2T} + \frac{C_s(p_1p_2T-Y)^2}{2p_1p_2} - \frac{(p_2-1)h_pT}{2} - C_{emf} - \frac{C_cI_{pm}X^2}{2p_2T} + \frac{C_cI_{em}Y^2}{2p_2T} - \frac{C_cI_{pm}(p_2T-Y)^2}{2p_2T} + \frac{C_sI_{em}(p_2T-X)^2}{2p_2T}$$

$$C_{111} = \frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s} + \frac{h_{sm}v_2T}{p_s}\left(\frac{2}{v_2}-p_1\right) + \left(1-\frac{1}{v_2}\right)\frac{2h_{sm}v_2T}{p_p} + \frac{h_{sm}T}{p_2p_P} + \frac{h_pT}{p_p}(2-p_2) + \sum_j^n \frac{I_{er}D_{rj}X^2}{2T}$$

$$A_{10} = \frac{2\left(A_c+O_cv_1+v_1F+\mathrm{VQ}_c+p_1A_s+p_1O_sv_2+p_1v_2F+p_1\mathrm{VQ}_s+p_1p_2Q_s+p_1p_2\sum_{j=1}^n R_o\right)}{p_1p_2} + \frac{C_pI_{ps}Y^2}{2p_1p_2} - \frac{C_cI_{es}ZY^2}{2p_1p_2} + \frac{C_cI_{pm}ZY^2}{2p_2} + \frac{C_cI_{pm}X^2}{2p_2} - \frac{C_sI_{em}ZX^2}{2p_2} - \frac{C_sI_{em}ZY^2}{2p_2} - \sum_j^n \frac{pI_{er}X^2}{2} + \sum_j^n \frac{C_sI_{pr}D_{rj}X^2}{2}.$$

Appendix D.

Proof of Proposition 1. Total profit function in equation (53) can be written as a Hessian matrix.

$$H=\begin{vmatrix}\frac{\partial^2\mathrm{ETP}_1}{\partial T^{*2}} & \frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial P^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_1}{\partial p^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial p^{*2}} & \frac{\partial^2\mathrm{ETP}_1}{\partial p^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_1}{\partial b^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial b^*\partial p^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial b^{*2}}\end{vmatrix}.$$

The first principal minor is $H_{11}=\frac{\partial^2\mathrm{ETP}_1}{\partial T^{*2}}=-\frac{A_1}{T^3}<0$.

The second principal minor is

$$\begin{aligned}H_{22}&=\frac{\partial^2\mathrm{ETP}_1}{\partial T^{*2}}\frac{\partial^2\mathrm{ETP}_1}{\partial p^{*2}}-\left(\frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial p^*}\right)^2\\ &=\left(-\frac{A_1}{T^3}\right)\left(-\left(U^2+UA_{66}\frac{2}{p-p_{\min}}-U-A_{55}\frac{2Z}{p-p_{\min}}\right)\right)-\left(-\sum_j^n\frac{I_{er}D_{rj}}{2}\right)^2\\ &=\left(-\frac{A_1}{T^3}\right)(-)\left[\left(U^2+UA_{66}\frac{2}{p-p_{\min}}-U-A_{55}\frac{2Z}{p-p_{\min}}\right)+\left(\sum_j^n\frac{I_{er}D_{rj}}{2}\right)^2\right]\\ &=\frac{A_1}{T^3}\left[\left(U^2+UA_{66}\frac{2}{p-p_{\min}}-U-A_{55}\frac{2Z}{p-p_{\min}}\right)+\left(\sum_j^n\frac{I_{er}D_{rj}}{2}\right)^2\right]>0,\end{aligned}$$

if $U^2+UA_{66}\frac{2}{p-p_{\min}}+\left(\sum_j^n\frac{I_{er}D_{rj}}{2}\right)^2>U+A_{55}\frac{2Z}{p-p_{\min}}$.

The third principal minor is

$$H_{33}=\begin{vmatrix}\frac{\partial^2\mathrm{ETP}_1}{\partial T^{*2}} & \frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial P^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_1}{\partial p^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial p^{*2}} & \frac{\partial^2\mathrm{ETP}_1}{\partial p^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_1}{\partial b^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial b^*\partial p^*} & \frac{\partial^2\mathrm{ETP}_1}{\partial b^{*2}}\end{vmatrix}.$$

$H_{33}=\frac{\partial^2\mathrm{ETP}_1}{\partial b^{*2}}det(H_{22})-(\frac{\partial^2\mathrm{ETP}_1}{\partial T^*\partial b^*})^2(\frac{\partial^2\mathrm{ETP}_1}{\partial b^{*2}})=-a\frac{(p_{\max}-p_{\min})}{(p-p_{\min})^2}\frac{\beta}{2T^3p_1p_2}A_{11}<0$, if $A11>0$. That is, $\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s}+\frac{h_{sm}p_2T}{p_s}(\frac{2}{v_2}-p_1)+\frac{h_{sm}p_2T}{p_p}(1-\frac{1}{v_2})+\frac{h_{sm}p_2T}{v_2p_p}+\frac{h_pT}{p_p}(2-p_2)>0$, *i.e.*, $\frac{h_{\mathrm{TS}}p_1p_2T}{v_1p_s}+\frac{2h_{sm}p_2T}{v_2p_s}+\frac{h_{sm}p_2T}{p_p}+\frac{h_{sm}p_2T}{v_2p_p}+\frac{2h_pT}{p_p}>\frac{h_{sm}p_1p_2T}{p_s}+\frac{h_{sm}p_2T}{v_2p_p}+\frac{h_pTp_2}{p_p}$.

(See Appendix B for all second order derivatives and calculations) Thus, the first and third order principal minors have negative values and the second order principal minor has positive value. Thus, it is concluded that the maximum total profit in Case 1 is global maximum. □

Appendix E.

Proof of Proposition 2. Total profit function in equation (54) can be written as a Hessian matrix.

$$H=\begin{vmatrix}\frac{\partial^2\mathrm{ETP}_2}{\partial T^{*2}} & \frac{\partial^2\mathrm{ETP}_2}{\partial T^*\partial P^*} & \frac{\partial^2\mathrm{ETP}_2}{\partial T^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_2}{\partial p^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_2}{\partial p^{*2}} & \frac{\partial^2\mathrm{ETP}_2}{\partial p^*\partial b^*}\\ \frac{\partial^2\mathrm{ETP}_2}{\partial b^*\partial T^*} & \frac{\partial^2\mathrm{ETP}_2}{\partial b^*\partial p^*} & \frac{\partial^2\mathrm{ETP}_2}{\partial b^{*2}}\end{vmatrix}.$$

The first order principal minor is $H_{11} = \frac{\partial^2 \mathrm{ETP}_2}{\partial T^{*2}} = -\frac{A_{10}}{T^3} < 0$. The second order principal minor is

$$
\begin{aligned}
H_{22} &= \frac{\partial^2 \mathrm{ETP}_2}{\partial T^{*2}} \frac{\partial^2 \mathrm{ETP}_2}{\partial p^{*2}} - \left(\frac{\partial^2 \mathrm{ETP}_2}{\partial T^* \partial p^*}\right)^2 \\
&= \left(-\frac{A_1}{T^3}\right)\left(-\left(U^2 + UA_{66}\frac{2}{p - p_{\min}} - U - A_{55}\frac{2Z}{p - p_{\min}}\right)\right) - \left(-\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2 \\
&= \left(-\frac{A_1}{T^3}\right)(-).\left[\left(U^2 + UA_{66}\frac{2}{p - p_{\min}} - U - A_{55}\frac{2Z}{p - p_{\min}}\right) + \left(\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2\right] \\
&= \frac{A_1}{T^3}\left[\left(U^2 + UA_{66}\frac{2}{p - p_{\min}} - U - A_{55}\frac{2Z}{p - p_{\min}}\right) + \left(\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2\right] > 0,
\end{aligned}
$$

if $U^2 + UA_{66}\frac{2}{p-p_{\min}} + \left(\sum_j^n \frac{I_{er}D_{rj}}{2}\right)^2 > U + A_{55}\frac{2Z}{p-p_{\min}}$.

The third order principal minor is

$$
H_{33} = \begin{vmatrix}
\frac{\partial^2 \mathrm{ETP}_2}{\partial T^{*2}} & \frac{\partial^2 \mathrm{ETP}_2}{\partial T^* \partial P^*} & \frac{\partial^2 \mathrm{ETP}_2}{\partial T^* \partial b^*} \\
\frac{\partial^2 \mathrm{ETP}_2}{\partial p^* \partial T^*} & \frac{\partial^2 \mathrm{ETP}_2}{\partial p^{*2}} & \frac{\partial^2 \mathrm{ETP}_2}{\partial p^* \partial b^*} \\
\frac{\partial^2 \mathrm{ETP}_2}{\partial b^* \partial T^*} & \frac{\partial^2 \mathrm{ETP}_2}{\partial b^* \partial p^*} & \frac{\partial^2 \mathrm{ETP}_2}{\partial b^{*2}}
\end{vmatrix}.
$$

$H_{33} = \frac{\partial^2 \mathrm{ETP}_2}{\partial b^{*2}} det(H_{22}) - (\frac{\partial^2 \mathrm{ETP}_2}{\partial T^* \partial b^*})^2 (\frac{\partial^2 \mathrm{ETP}_2}{\partial b^{*2}}) = -\left(a\frac{(p_{\max}-p_{\min})}{(p-p_{\min})^2}\right)\frac{\beta}{2T^3 p_1 p_2} A_{11} < 0$, if $A11 > 0$. That is, $\frac{h_{\mathrm{TS}} p_1 p_2 T}{v_1 p_s} + \frac{h_{sm} p_2 T}{p_s}(\frac{2}{v_2} - p_1) + \frac{h_{sm} p_2 T}{p_p}(1 - \frac{1}{v_2}) + \frac{h_{sm} p_2 T}{v_2 p_p} + \frac{h_p T}{p_p}(2 - p_2) > 0$, *i.e.*, $\frac{h_{\mathrm{TS}} p_1 p_2 T}{v_1 p_s} + \frac{2h_{sm} p_2 T}{v_2 p_s} + \frac{h_{sm} p_2 T}{p_p} + \frac{h_{sm} p_2 T}{v_2 p_p} + \frac{2h_p T}{p_p} > \frac{h_{sm} p_1 p_2 T}{p_s} + \frac{h_{sm} p_2 T}{v_2 p_p} + \frac{h_p T p_2}{p_p}$.

(See Appendix B for all second order derivatives and calculations) Thus, the first and third order principal minors are negative definite and the second order principal is positive definite. Thus, it is concluded that the total profit obtained by the SCM is global maximum. □

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