RAIRO-Oper. Res. 58 (2024) 803–819
https://doi.org/10.1051/ro/2023073

RAIRO Operations Research
www.rairo-ro.org

OPTIMIZATION OF PRICE, LOT SIZE AND BACKORDERED LEVEL IN AN EPQ INVENTORY MODEL WITH REWORK PROCESS

Ata Allah Taleizadeh[1], Mohabat-Sadat Naghavi-Alhoseiny[1], Leopoldo Eduardo Cárdenas-Barrón[2,*] and Alireza Amjadian[3]

Abstract. In traditional inventory models, the demand rate normally is considered as a constant value, while in marketing and pricing, it is dependent on the selling price. The present study introduces a new type of economic production quantity (EPQ) inventory model. This production-inventory model is among the aspects that make the management of inventories more realistic and practical for managers. The pricing policy, planned backorders, and the rework process are included in the EPQ inventory model. The main contribution is that, in the EPQ inventory model, the price serves as a critical factor that affects the size of demand to maximize total long-term profit. The primarily objective is to determine the optimum selling price, discrete values for the optimal lot size, and the level of optimal backorders so that the total profit is maximized. To accomplish the optimal value for the decision variables an algorithm is developed. The proposed algorithm provides an accurate solution for production managers to jointly decide on lot size, backorder size, and selling price. In addition, a numerical example is solved using real industry data. The results affirm that the total profit obtained using the production-inventory model increases significantly in comparison with the current situation. Furthermore, sensitivity analysis is carried out in order to describe the practical application of the suggested production-inventory model.

Mathematics Subject Classification. 90B05.

Received September 2, 2022. Accepted May 26, 2023.

1. Introduction

Profit optimization is regarded as one of the relevant problems in any organization dedicated to manufacturing and selling products. Usually, one calculates gross profit by the subtraction of the cost of sold goods from the total revenues. Commonly, a company's revenue depends on the sales amount and also the prices of products. From another perspective, the time among production runs, the lot size, and also, the number of backorders for every single production run affect the manufacturing, setup, inventory, and backorder costs.

It is well known that in the economic production quantity (EPQ) inventory model and traditional economic order quantity (EOQ) model, presented by Taft [44] and Harris [26] respectively, the situations of the real world,

Keywords. Pricing, inventory, rework process, shortage, planned backorders.

[1] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.

[2] Tecnológico de Monterrey, School of Engineering and Sciences, Ave. Eugenio Garza Sada 2501, 64849 Monterrey, N.L., Mexico.

[3] Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran.

*Corresponding author: lecarden@tec.mx

where the manufacturing system fabricates defective products or the planned backorders, are not considered. Several studies that have been conducted, including Hayek and Salameh [27], Cárdenas-Barrón [5], and also, Wee *et al.* [11] examine these situations. The contribution of imperfect quality items to the finite production-inventory model was studied by Hayek and Salameh [27]. The defective items, in their production-inventory model, are reworked at a constant rate. In the study of Cárdenas-Barrón [5], an EPQ inventory model is created for a single-stage manufacturing system. This production-inventory model includes planned backorders, rework, and defective items. Wee *et al.* [56] reviewed the production-inventory model from Cárdenas-Barrón [5]. Instead of incorporating the usual decision variables, *i.e.*, backorder level and lot size, they incorporate the time to remove backorders and production time as their suggested decision variables. Their solution produces a new production-inventory strategy for the items characterized by imperfect quality in cases where the ideal manufacturing time is shorter than the optimum time.

Recently, production managers have been interested in the simultaneous optimization of costs and revenues. In this direction, researchers and academicians are integrating pricing techniques into EPQ inventory models. Furthermore, in several manufacturing systems, defective items are the consequence of the low skill level of workers, faulty raw materials, malfunction of production facilities, and poor maintenance policies, among other reasons.

The paper's major objective is to integrate price strategy, rework, and scheduled backorders with the well-known EPQ inventory model. This production-inventory model does not merely utilize the entire expenses of the manufacturing system to determine the ideal lot size and degree of planned backorders. It also considers total income when employing pricing strategies to optimize overall profit and determine the best-selling price for the product. Products are manufactured and sold in discrete numbers in various manufacturing systems, including those for cars, airplanes, electrical devices, business equipment, and casting parts, among others. Therefore, a need exists to develop algorithms to acquire the optimum discrete values of the decision variables. Significant research has been conducted by Zhang and Gerchak [59], Chung [16], Salameh and Jaber [38], and Goyal and Cárdenas-Barrón [25] in the last decades, only to mention a few studies relevant to EPQ inventory models that includes defective product rework. By contrast, an EPQ inventory model with limited manufacturing capacity and scrapped products was developed by Taleizadeh *et al.* [45]. The existence of a single machine limits their inventory model. Glock *et al.* [23] comprehensively analyze literature reviews about lot sizing. When manufactured products are insufficient in order to satisfy demands, then shortages may occur. In these conditions, customers must wait until their demands are satisfied. In the survey conducted by Glock *et al.* [23], readers may achieve better evaluations of inventory models with backorders and imperfect quality items. For instance, deterministic models for the EPQ and EOQ with backordering were investigated by Pentico and Drake [33]. In contrast, certain studies on the EPQ models, such as Cárdenas-Barrón [5], considered the reworking of defective items and planned backorders. His EPQ inventory model, which includes planned backorders, estimates the economic amount of production for an individual product in a single-stage manufacturing system that creates defective products. Likewise, in the inventory model presented by Cárdenas-Barrón [5], all imperfective products undergo reworking within the same cycle. Taleizadeh *et al.* [45] conducted an analysis on a production quantity model characterized by service level restrictions, repair failure, and random defective items. For the purpose of the optimization of the expected total cost, they determine the cycle length, optimal production quantity, and backordered quantity. Taleizadeh *et al.* [47] create and assess an EPQ inventory model, including scraped items, backordering, interruptions in the manufacturing process, and rework. In a reexamination of Cárdenas-Barrón [5]'s EPQ inventory model for a single-stage manufacturing system that included planned backorders, a rework process is considered by Sarkar *et al.* [41]. The basic idea is to create three distinct inventory models for the uniform, triangular, and beta distribution density. They offer closed-form solutions for every inventory model and provide tables that compare optimal distribution function outcomes. The time intervals among orders and the proportion of demands that is procured using the stock are regarded as the decision factors in Pentico *et al.* [36]. They propose two heuristic algorithms for the EOQ/EPQ inventory models with partial backordering. Eventually, they indicate that the performance of the heuristic algorithms is great in cases where the minimum critical value of the backordering rate is 0.50 and is acceptable when it is positive. A line of study in the inventory literature is concerned with pricing. It is essential to note that none of the above research considers the decision of product pricing. In this paper, the selling price (sensitive to demand) is regarded as one of the decision variables used to optimize the profit. Some researchers have added pricing to the basic EOQ inventory model. For example, Within [57] was potentially the first scholar to investigate the integration of pricing into the inventory problem. He includes pricing in the traditional EOQ inventory model, considering a linear demand-sensitive price function. Afterward, numerous studies dealt with the problem of the integration of inventory and pricing. For example, Federgruen and Heching [19] simultaneously derived the strategies of pricing and inventory replenishment for uncertain demand. They focus on finite and infinite horizon models to optimize the total predicted discounted profit or its time-average value, given the assumption that prices remain modifiable. Lau and Lau [31] include demand-curve functions in a pricing and inventory model. They conclude that one may use the common-wisdom implication for single-echelon systems. Still, when distinct demand-curve functions are considered, the obtained results are different in a multi-echelon system. Abad [1] creates a dynamic model for the lot-sizing and pricing purposes and suggests a simple method for addressing the optimization issue. The model of Abad [1] assumes that when it is beneficial to backlog demand, the vendor plans shortage times in which demand seems partly backordered. Abad [2] addresses the lot size and pricing issues for a product bounds by a general deterioration rate and partial backordering. In addition, Chen and Yang [14] examine a periodic review inventory model for a single product, and their inventory model carries out the ordering and pricing decisions across a finite horizon simultaneously. In their inventory model, demands follow a uniform distribution, and it is a function of price. By assuming price-sensitive demand, Hong *et al.* [29] build a novel strategy of transportation and integrated inventory characterized by strategic pricing in order to optimize the overall profit of a worldwide company. The integrated inventory and transportation strategy generally yields the best orders, shipments, and price choices. Taleizadeh *et al.* [48] develop a three-layer supply chain characterized by a distributor, a retailer, and a manufacturer for a single product and generic demand functions subject to three different situations.

In contrast, a deterministic inventory model is suggested by Dye *et al.* [17] for a deteriorating item characterized by a price-dependent demand. In their approach, as the waiting time increases, the unsatisfied demand partly backlogs at a negative exponential pace. Their inventory model allows shortages.

You and Wu [58] simultaneously find the best ordering and price strategy for an inventory system with cancellation. Subsequently, a closed form was developed by Pentico *et al.* [34] in order to decide the best strategy of stocking for an EPQ inventory model with continuous partial backordering. In another study, by modifying their earlier inventory model, Pentico *et al.* [35] accounted for the potential of the percentage of backordered demand to increase after the continuation of the production. They illustrate that the extended inventory model's objective function has a similar mathematical expression as the original model's objective function when the holding, lost sale, and backorder costs are appropriately defined. In a different research project, Ouyang *et al.* [32] evolved an algorithm for resolving an integrated inventory model characterized by a deteriorating price-dependent demand, in which the supplier production lot size and the economic lot size are decided simultaneously so that the overall profit is maximized. In the case of connecting the trade credit to the order quantity in an integrated system of vendor-buyer inventory, Chang *et al.* [13] provide an inventory model to identify the best pricing and order size. They assume that when the retail price increases, the demand rate decreases. Following an analysis of the total channel profit function, they create an algorithm in order to determine the quantity of buyer's orders, the retail price, and the number of vendor-to-buyer shipments for every single manufacturing run. It is noteworthy that the evaluation of pricing inventory systems with defective goods while considering the selling price of goods is another crucial topic. Chan *et al.* [12] include reduced price, reject situations, and rework in model of the EPQ inventory. Their inventory model considers imperfect quality goods. The products characterized by imperfect quality could be purchased by a specific purchaser at lower prices or used in other production processes. They find that as a critical factor, the time required to sell imperfect items affects the inventory costs and the lot size. Kreng and Tan [30] offer an EPQ inventory model. The manufacturing system in the model sorts imperfect quality and scrap items as defective items and maximizes the yearly total projected profit by determining the ideal wholesaler replenishment time decisions for defective items subject to a two-level trade credit policy. Uthayakumar and Palanivel [55] examine the influence of the time value of money and inflation into an inventory model featuring imperfect products in which demand is regarded as a deterministic function of advertising cost and selling price. It should be noted that some of the studies mentioned above have a drawback: they do not account for shortages and backorders. By taking the operational constraints into account, a single-machine multi-product EPQ inventory system under inspection and defective production was presented by Askari *et al.* [4]. An stochastic programming approach was utilized as solution method. Afterwards, an integrated iventory model was provided by Gharaei *et al.* [21] for a four-level supply cahin to optimize the reliability and inventory cost simultaneously. Two algorithms, known as interior point (IP) and sequential quadratic programming (SQP) were utilized for finding the optimum solution. Then, Amjadian and Gharaei [3] designed an inventory model for a closed-loop full-level supply chain and then optimized the model by using a generalised outer approximation with exact penalty method. Later, Gharaei *et al.* [22] provided an integrated lot-sizing policy for the purpose of managing the inventory of constrained multi-level supply chain and optimized their model using null-space method. Eventually, it should be mentioned that Taleizadeh *et al.* [49] carried out a study on the optimum decisions and operational strategies for inventory management of a logistics network by scrutinizing the performances of two capital-constrained manufacturers. The backordering of shortages can repeatedly occur in the real world. Table 1 compares research works according to some features considered in this paper.

Since many products are manufactured and sold in discrete values, some decision variables must take discrete values. In this regard, the lot size must be reported as an integer value; therefore, the technique proposed by García-Laguna *et al.* [20] is considered. They introduce a technique to decide the optimum discrete solution for the standard EOQ and EPQ when the lot size necessarily assumes an integer value. Their suggested technique is quite simple and applies to resolving different inventory models. Afterward, other researchers, such as Teng *et al.* [52], Cárdenas-Barrón *et al.* [6], Cárdenas-Barrón *et al.* [7], Cárdenas-Barrón *et al.* [8], Sarkar [39], Cárdenas-Barrón *et al.* [9], Cárdenas-Barrón *et al.* [10], Teng *et al.* [53], Cárdenas-Barrón *et al.* [11], and Treviño *et al.* [54] use the approach applied in García-Laguna *et al.* [20] in their inventory models for optimizing the integer value of the decision variables. Hong and Lee [28] evolve a single-item inventory system with a time-based consolidation strategy to minimize transportation costs. To optimize overall profit, their developed mathematical model is used to determine the dispatch cycle, replenishment amount, and pricing. In their inventory model, the replenishment quantity is represented as an integer, and their technique of determining the best replenishment quantity is comparable to the technique developed by García-Laguna *et al.* [20].

Considering the environmental challenges in today's world and the need to follow the circular economy, one of the important discussions considered by researchers in the fields of supply chain, production, pricing, and inventory control in recent years, can be sustainability and remanufacturing. Some of these researches are examined here so readers can enrich their knowledge by considering these works in their future investigation. For instance, Sarkar *et al.* [43] conducted a parallel study evaluating environmental risks caused by manufacturing innovative green products through random return and demand rates with insufficient information. Three models were designed for two innovative green products using different production strategies. Further, a distribution-free approach is applied instead of considering any particular distribution for the random variables (return rate and demand). According to the obtained findings, in the case of high uncertainties in demand and supply, extremely innovative green products outperformed less innovative products.

Furthermore, upcoming green products are required to be introduced only when the expected profit of newly manufactured products exceeds the loss incurred by existing products. Compared to traditional innovation policies, the new policy innovation of remanufacturing is more worthwhile. Then, Sarkar *et al.* [42] designed a single-stage smart production model using automated inspection technology. Their study considered different quality products sold through various channels (*i.e.*, online and offline) at different prices. As a result of different qualities and channels, the price may vary.

Consequently, the total demand also changed based on the price of a particular quality product. As a result of the out-of-control situation, imperfect products are produced, leading to backorders. Manufacturing defective items resulted in a waste generation, which has negative economic and environmental effects on the industry. The defective/imperfect items are detected *via* smart, automated inspection technologies, which are then

TABLE 1. Comparison of related research works.

Research work	Features of model					
	Decision variables			Objective function	Manufacturing defective items and/or reworking	Integrating inventory and pricing
	Lot sizing	Backorders	Price			
Abad [1]	✓		✓	Max (TP)		✓
Federgruen and Heching [19]	✓		✓	Max (TP)		✓
Hayek and Salameh [27]	✓	✓		Min (TC)	✓	
Chan *et al.* [12]	✓		✓	Max (TP)	✓	✓
Lau and Lau [31]	✓		✓	Max (TP)		✓
Dye *et al.* [17]	✓		✓	Max (TP)		✓
You and Wu [58]	✓		✓	Max (TP)		✓
Abad [2]	✓		✓	Max (TP)		✓
Cárdenas-Barrón [5]	✓	✓		Min (TC)	✓	
Chang *et al.* [13]	✓		✓	Max (TP)		✓
Ouyang *et al.* [32]	✓		✓	Max (TP)		✓
Chen and Yang [14]	✓		✓	Max (TP)		✓
Taleizadeh *et al.* [45]	✓			Min (TC)	✓	
Taleizadeh *et al.* [46]	✓	✓		Min (TC)	✓	
Pentico *et al.* [35]	✓		✓	Max (TP)		✓
Hong *et al.* [29]	✓		✓	Max (TP)		✓
Wee *et al.* [56]	✓	✓		Min (TC)	✓	
Pentico *et al.* [36]	✓	✓		Min (TC)		
Sarkar *et al.* [41]	✓	✓		Min (TC)	✓	
Taleizadeh *et al.* [47]	✓	✓		Min (TC)	✓	
Taleizadeh *et al.* [48]	✓		✓	Max (TP)		✓
This Paper	✓	✓	✓	Max (TP)	✓	✓

TP: Total profit; TC: Total cost

returned to the same production cycle for remanufacturing. Afterward, Sarkar and Bhuniya [40] designed a novel mathematical model for the same flexible manufacturing–remanufacturing system. In this model, the manufacturer produce its final products by utilizing both the used collected products and new raw materials. The products are then taken to retailers, which are sold there along with their service facilities. Then, such a flexible manufacturing–remanufacturing system is mathematically modeled in order to enhance services and ensure ongoing sustainability. Based on these results, the concept of a service facility enabled customers to select products promptly, while the maximum profits gained from green investments are ensured through the management of supply chain. Ultimately, Tayyab *et al.* [50] investigated a cleaner system of multi-stage production management for resolving the problem of carbon emissions and active involvement in corporate social responsibility activities.

Additionally, they made economic improvements in the system. In their study, they developed three random, imperfect proportion scenarios and implied the multi-stage production system would get cleaned through reworking. To tackle the predicted shortages while improving the service level of systems, a planned backorder policy is recommended.

The study evolves the EPQ inventory model presented by Cárdenas-Barrón [5] into an integrated pricing-inventory model to account for realistic assumptions, including planned backorders, defective items, and rework. This research models demand as a linear function of price, while selling price, lot size, and planned backorders serve as decision variables for maximizing overall profit. The article suggests that all produced items are examined, and defective items are required to be reworked within the same production cycle utilizing the same process of manufacturing. In addition, discrete values are expected for the lot size and planned backorders.

This inventory model, with its assumptions, provides managerial insights for decision-making. The application of these assumptions in the real world is in the manufacturing of those items in integer quantities, which are subject to 100% of screening, and the defective items must be reworked. In some foundry factories, all the manufactured parts are screened, and the defective items must return to the casting furnace as low material.

The following sections of the present article are organized as follows. Section 2 presents a simple description of the integrated pricing-inventory model and related assumptions. The notation and mathematical model for the integrated pricing-inventory model are explained in Section 3. Then, Section 4 outlines the solution technique utilized to concurrently estimate the optimal selling price, lot size, and planned backorders. In Section 5, a numerical problem is solved to exemplify the application of the solution process. A sensitivity analysis is presented in Section 6 to scrutinize the contribution of parameter changes to the optimum solution. Finally, Section 7 concludes the paper with a summary and suggestions recommended for future study directions.

2. Problem definition

In this research, the product pricing strategy is implemented into the EPQ inventory system to simultaneously optimize the lot size, selling price, and backorder level in order to maximize the overall profit function. There are some relevant models associated with the EPQ inventory model, such as Porteus [37], Teng *et al.* [51], Chiu [15], and Eroglu and Ozdemir [18]. Some of them include defective items and planned backorders, but these works do not consider demand sensitive to price. Now, it is briefly described the inventory model that is developed as follows. The manufacturing system of this inventory model produces a proportion of defective items equal to R. Some of these defective items are the imperfect quality ones. In certain businesses, items with imperfect quality can be sold at a discount or shipped as raw materials to another industry.

Sometimes the imperfect items become good quality products with a reworking process at the same manufacturing cycle (*i.e.*, the casting industry). Moreover, in this production-inventory model, 100% of the items are checked, and all products labeled as defective are reworked (reprocessed) within a similar manufacturing system. The yield of defective items leads to the reduction of the production rate from P to $P(1-R)$. Then the defectives are reworked at the rate P. Moreover, all defective items are corrected after just one reworking.

Additionally, this production-inventory model allows shortages while all shortages are backordered. Backorder costs are addressed in two ways: linear backorder costs and fixed backorder costs. Both the demand and production rates are steady and known. Furthermore, the demand rate is regarded as a linear function of the price of the product. Therefore, additional traditional assumptions of the EPQ inventory model are used.

The selling price, the degree of backorders, and the lot size are regarded as the decision variables in the production-inventory model, and they are jointly optimized. The product-pricing plan determines the overall income, and the total cost is obtained from Cárdenas-Barrón [5].

This production-inventory model is created for non-degrading commodities with denumerable quantities or discrete values, such as electronic machines, industrial equipment, cars, etc. In the real world, the lot size of such products generally has an integer value or an integer coefficient of product measurement units. Therefore, the lot size is required to assume an integer value. As a result, the technique described by García-Laguna *et al.* [20] for obtaining the integer value of the lot size is applied to optimize the total profit objective function. Their method is based on the objective function problem, OF, for greater or smaller values than optimum values x^*-1, x^*+1 or has not a better value than the objective function for the optimum value x^* (for maximizing problems: $\text{OF}(x^*+1) \leq \text{OF}(x^*)$ and $\text{OF}(x^*-1) \leq \text{OF}(x^*)$ and for minimizing problems: $\text{OF}(x^*+1) \geq \text{OF}(x^*)$ and $\text{OF}(x^*-1) \geq \text{OF}(x^*)$). The technique introduced by García-Laguna *et al.* [20] is also applied by Teng *et al.* [52], Cárdenas-Barrón *et al.* [6], Cárdenas-Barrón *et al.* [7], Cárdenas-Barrón *et al.* [8], Sarkar [39], Cárdenas-Barrón *et al.* [9], Cárdenas-Barrón *et al.* [10], Teng *et al.* [53], Cárdenas-Barrón *et al.* [11], and Treviño *et al.* [54] for optimizing the integer value of decision variables.

Assumptions of the integrated pricing-production-inventory model

The assumptions presented below are made on the basis of the integrated pricing and the characteristics of the production-inventory model:

- Demand is regarded as a parameter sensitive to the selling price. A linear function as $D = a - b * S$ is considered where S is the selling price, and a and b are demand function coefficients.
- Over horizon planning, the production rate maintains a constant value and is continuously higher than the demand.
- The products are screened completely, and the screening cost is included in the manufacturing costs.
- All the products labeled as defective are reworked and converted into products characterized by good quality.
- During all cycles, no scrap is produced.
- Inventory holding costs are a function of the average quantity of inventory.
- Backorders are permitted, and the whole backorders seem to be satisfied.
- The same manufacturing system is performed for production and reworking at a similar production rate.
- Two backorder costs are included: Fixed and linear backorder costs (backorder cost is applicable to the maximum permitted backorder level and backorder fee is applicable to the average backorders).
- Capital availability and inventory storage space are both unlimited parameters.
- The backorder size and lot size assume integer values.
- The items are non-deteriorating.
- The inventory model depicts a single product.
- An infinite horizon is considered for the planning.

3. The mathematical formulation for the integrated pricing-inventory model

The section gives the notation and mathematical formulation for optimizing pricing, backorder levels, and lot size in an EPQ inventory system. The purpose of determining the selling price, backorder level, and lot size of a product utilizing a one-stage production method is to maximize the overall profit. The discrepancy between revenue and costs is used to calculate the overall profit. Suppose a linear relationship between sales price and demand. Then, the total income is calculated by multiplying the demand by the selling price while also considering that demand decreases as the price increases. Setup, holding, backordering, and production costs are all factors constituting the cost of the manufacturing system. The lot size and backorder levels must have integer values since this inventory model is created for non-degrading products that are denumerable or have discrete values. The notation below is employed when developing the inventory model:

R	Proportion of defective product in every single cycle ($0 < R < 1$)
P	Production rate (units/time unit)
K	Cost of production setup ($/lot)
f	Storage cost ($/unit/time unit)
C	Manufacturing cost of a product, including screening cost ($/unit)
H	Inventory carrying cost determined as $H = iC + f$ ($/unit/time unit)
i	Inventory carrying cost rate in percent (%)
F	Fixed backorder cost ($/unit)
W	Linear backorder cost ($/unit/time unit)
Q	Lot size (units)
B	Backorders level (units)

T	Time interval between production runs, equal to Q/D (time unit)
S	Selling price (\$/unit)
a	Intercept elevation of demand function
b	Slope of demand function
D	Demand rate, a linear function of S, $D = a - bS$ (units/time unit)
TP_c	Total profit per unit of time of the current method (\$/unit time)
$\text{TC}(Q, B, S)$	Total cost per unit of time (\$/unit time)
$\text{TR}(S)$	Total revenue per unit of time (\$/unit time)
$\text{TP}(Q, B, S)$	Total profit per unit of time (\$/unit time)

Below are the four costs that make up the overall cost:

Setup cost: the costs needed to prepare a machine for a production run constitute the setup cost. This cost is regarded as a fixed cost of the corresponding lot/batch; therefore, it is distributed throughout the number of units manufactured.

Holding cost: costs connected with storing unsold inventory constitute holding costs. In addition to ordering and shortage costs, these costs constitute a component of overall inventory costs.

Backordering cost: backorder costs are costs made by a company when it cannot satisfy an order immediately but assure the customer that it is fulfilled with a later delivery date.

Manufacturing cost: the costs incurred throughout a product development are referred to as manufacturing costs, which includes manufacturing overhead, direct labor, and direct material costs.

$$\begin{aligned}
\text{Total cost (TC)} &= \text{Setup cost} + \text{Holding cost} + \text{Manufacturing cost} + \text{Backordering cost}\\
\text{Setup cost} &= \frac{KD}{Q}\\
\text{Holding cost} &= \frac{HQL}{2} + \frac{HB^2A}{2QE} - HB\\
\text{Backordering cost} &= \frac{FBD}{Q} + \frac{WB^2A}{2QE}\\
\text{Manufacturing cost} &= CD(1+R).
\end{aligned}$$

Based on above costs, the total cost is expressed by the sum of these costs as equation (1).

$$\text{TC}(Q,B,S) = \overbrace{\frac{KD}{Q}}^{\text{Setup cost}} + \overbrace{\frac{HQL}{2} + \frac{HB^2A}{2QE} - HB}^{\text{Holding cost}} + \overbrace{\frac{FBD}{Q} + \frac{WB^2A}{2QE}}^{\text{Shortage cost}} + \overbrace{CD(1+R)}^{\text{Manufacturing cost}} \tag{1}$$

In which:

$$\begin{aligned}
A &= 1 - R\\
D &= D(S) = a - bS\\
E &= E(S) = 1 - R - \frac{a - bS}{P}\\
L &= L(S) = 1 - \left(1 + R + R^2\right)\frac{a - bS}{P}\cdot
\end{aligned}$$

It is obvious that the total revenue is calculated as $\text{TR} = S \times (a - bS)$, and the total profit is represented as $\text{TP} = \text{TR} - \text{TC}$, so:

$$\text{TP}(Q,B,S) = SD(S) - \frac{KD(S)}{Q} - \frac{HQL(S)}{2} - \frac{B^2A(H+W)}{2QE(S)} + HB - \frac{FBD(S)}{Q} - CD(S)(1+R). \tag{2}$$

The next challenge is finding the selling price, backorder level, and lot size that maximize the manufacturing system's overall profit. Therefore, the following mathematical formulation is used to explain this optimization problem.

$$\text{Maximize TP}(Q,B,S) = D(S)\left(S - \frac{K}{Q} - C(1+R)\right) - \frac{HQL(S)}{2} - \frac{B^2A(H+W)}{2QE(S)} + HB - \frac{FBD(S)}{Q}$$

$$\text{S.t. } Q \geq 1, B \geq 0, \text{ Integer}$$

$$S \geq 0.$$

4. Solution method

Three lemmas and their proofs are presented in this section. The suggested algorithm is developed utilizing these lemmas.

Lemma 1. *For a given value considered for B, the upper bound of the optimum value of Q, Q_U^* satisfies the following inequality:*

$$Q_U^*(Q_U^*-1) \leq \frac{2KAa(H+W) + \left(A - \frac{a}{P}\right)a^2F^2}{HA(H+W)\left(1 - \frac{a}{P}(1+R+R^2)\right) + \left(A-\frac{a}{P}\right)H^2} \leq Q_U^*(Q_U^*+1). \tag{3}$$

Proof. Assume that S and B are given so the optimum of backorders level (B^*) is calculated by determining the first partial derivative with respect to (B):

$$\frac{\partial \text{TP}(Q,B,S)}{\partial B} = -\frac{BA(H+W)}{QE(S)} + H - \frac{FD(S)}{Q} \tag{4}$$

$$\frac{\partial \text{TP}(Q,B,S)}{\partial B} = 0 \to B = \frac{E(S)(QH - FD(S))}{A(H+W)}. \tag{5}$$

Therefore, the overall profit is demonstrated by:

$$\text{TP}(Q,B,S) = D(S)\left(S - \frac{K}{Q} - C(1+R)\right) + \frac{E(S)(QH-FD(S))^2}{2QA(H+W)} - \frac{HQL}{2}. \tag{6}$$

For a given value of B, the optimum value of Q, *i.e.*, Q^*, satisfies the inequalities presented below:

$$\text{TP}((Q^*+1),B,S) \leq \text{TP}(Q^*,B,S) \tag{7}$$

$$\text{TP}((Q^*-1),B,S) \leq \text{TP}(Q^*,B,S). \tag{8}$$

Equations (7) and (8) are subjected to numerous algebraic operations, obtaining:

$$Q^*(Q^*-1) \leq \frac{2KD(S)A(H+W) - E(S)F^2D^2(S)}{HL(S)A(H+W) + E(S)H^2} \leq Q^*(Q^*+1). \tag{9}$$

$L(S)$ and $E(S)$ are increasing functions of S and they appear in the denominator of equation (9) with positive coefficients. Consequently, for calculating the upper bound of Q^* they must be minimized. In other words, S must be equal to zero. Conversely, $D(S)$ is regarded as a decreasing function of S and it has also a positive coefficient in $2KD(S)A(H+W)$ term. Then for finding the upper bound of Q^*, it must be maximized meaning S has to be equal to zero. In the $-E(S)F^2D^2(S)$ term, $E(S)D^2(S)$ is a cubic function of S with a negative coefficient and for $D \geq \frac{2AP}{3}$ is an increasing function of S. Thus $S=0$ maximize $-E(S)F^2D^2(S)$ if and only if $a \geq \frac{2AP}{3}$. As a result, if one replaces $S=0$ into equation (9) then the lemma is demonstrated. □

Lemma 2. *The upper bound of the optimum value of B, B_U^*, for each Q satisfies the following equation:*

$$B_U^* = \frac{QH}{H+W}\cdot \tag{10}$$

Proof. For a given value of Q, by determining the first partial derivative with respect to B the optimum value of B, B^*, is given by equation (5).

It is simple to demonstrate that equation (2)'s second partial derivative of the total profit function B, $\frac{\partial^2 \mathrm{TP}(Q,B,S)}{\partial B^2} = -\frac{A(H+W)}{QE(S)}$, is less than zero then total profit function $\mathrm{TP}(Q,B,S)$ is a convex function of B, so B^* is calculated by differentiating from $\mathrm{TP}(Q,B,S)$ with respect to B (Eq. (5)).

Since $E(S)$ is regarded as an increasing function of S with a positive coefficient and $D(S)$ is regarded as a decreasing function of S with a negative coefficient, B_U^* is obtained when the sale price, S, has the upper bound value. The upper bound of S is equal to $\frac{a}{b}$ because for $S > \frac{a}{b}$, D takes a negative value. Therefore, if one replaces $S = \frac{a}{b}$ into equation (5) then the lemma is proven. □

Lemma 3. *For given values of Q and B, if $-2b - \frac{b^2B^2A(H+W)}{QP^2}\left(A - \frac{a-bS}{P}\right)^{-3} \leq 0$, the optimum value of S, S^* is equal to the root of the following equation:*

$$a - 2bS - \frac{HQb\left(1+R+R^2\right)}{2P} + bC(1+R) + \frac{b(K+FB)}{Q} + \frac{bB^2A(H+W)}{2PQ}\left(A - \frac{a-bS}{P}\right)^{-2} = 0. \tag{11}$$

Proof. The second partial derivative of the total profit function (Eq. (2)) with respect to S, equals to $\frac{\partial^2 \mathrm{TP}(Q,B,S)}{\partial S^2} = -2b - \frac{b^2B^2A(H+W)}{QP^2}\left(A - \frac{a-bS}{P}\right)^{-3}$. If $\frac{\partial^2 \mathrm{TP}(Q,B,S)}{\partial S^2} \leq 0$, then the total profit (denoted by $\mathrm{TP}(Q,B,S)$) is a convex function of S. In addition, S^* is equal to the root of partial derivative of the total profit with respect to S (see Eq. (11)). □

The findings mentioned above are considered to determine the solution to the production-inventory problem. With these findings an algorithm is created in order to reach the optimum solution.

Algorithm

Step 1. If $a \geq \frac{2AP}{3}$ then compute the upper bound for the optimum value of Q, Q_U^*, with equation (3).
Step 2. For all $Q(Q = 1, \cdots, [Q_U^*])$, calculate the upper bound of optimum value of B, B_U^*, with equation (10).
Step 3. For all $Q(Q = 1, \cdots, [Q_U^*])$ and $B(B = 0, \cdots, [B_U^*])$ compute the real value of optimal selling price, S from equation (11).
Step 4. For each triplet (Q,B,S) determine $k = -2b - \frac{b^2B^2A(H+W)}{QP^2}\left(A - \frac{a-bS^*}{P}\right)^{-3}$. If $k \leq 0$ then compute the related total profit function with equation (2).
Step 5. Select the optimal total profit (denoted by Max $\mathrm{TP}(Q,B,S)$), and Q^*, B^* and S^* corresponding to the maximum total profit as the optimum solution.

5. Numerical example

An example is numerically solved in order to exhibit the applicability of the suggested algorithm. Since there are many mines in Iran, the casting industry is regarded as one of the most important industries and many factories are engaged in this activity. The data source used in the numerical example is from a well-known company in the casting industry called Felzvarzan Asia[1] in one of the big industrial cities of Iran. Consider a casting manufacturing process in which a liquid material is poured into a mold. This mold contains an empty cavity of a specific shape in which the liquid material is solidified. Then the solidified part is ejected from the mold to complete the process. The demand for the casting part is defined as a declining linear function of the

[1] www.felezvarzan.com.

selling price shown by $D = a - b \times S$ in which $a = 450$ and $b = 0.5$, so $D = 450 - 0.5 \times S$. The dimensions of the solidified part must satisfy the specified, designed size, and the surface of the product must have sufficient quality with no blobs. It is important to mention that sometimes it does not occur and some finished items do not follow the designed size and quality features. Therefore, defective items are produced. In this example, the probability of producing imperfect items is $R = 0.1$. All manufactured items are checked before being delivered to consumers. If they do not conform to the specifications, they are returned to the melting pot as low material and the manufacturing process is repeated. The annual output rate is $P = 750$ units. The production setup cost is $K = \$700$ for each lot, and the manufacturing cost incurred by such cast products, comprising the screening costs, is $C = \$30$ per unit. Backorders are permitted in this case, which are completely satisfied. There are two types of backorder costs: fixed backorder cost $F = \$5$ per unit and linear backorder cost: $W = \$7$ per unit annually. Inventory holding cost based on average inventory cost is a function of storage cost and manufacturing cost as $H = iC + f = \$15$ per unit per year. Here $i = 20\%$ are the inventory carrying cost rate and $f = \$9$ per unit per year is storage cost.

It is evident that for the present example, $a \geq \frac{2AP}{3}$. So equation (3) is used for calculating the Q_U^* according to Step 1. Using the equation (3) one obtains $Q_U^* = 290.23$. For $Q = 1, 2, 3, \ldots, 290, B_U^*$ is computed according to Step 2. For all Qs and Bs, the cubic function of S is solved (Eq. (11)) to find the optimum selling price corresponding to Qs and Bs (Step 3). The estimated Ss are the critical points of TP function, so for proving that they are optimum, the convexity of TP function is verified using second derivative test. In Step 4, the second derivative test is used and the results are illustrated in the fourth column of Table 2. The total profit, TP, is calculated for all Qs, Bs and Ss. As Table 2 indicates, the maximum total profit is obtained when $Q^* = 286$, $B^* = 99$ and $S^* = 467.61$ (Step 5). Therefore the maximum of total profit of the optimum solution is $\text{TP}^* = 92\,528.91$. As is observed in Table 2, according to the information received from the total current profit of the company based on the current pricing method, the increase in profit margin is affirmed by the proposed method. On average, 20% increase in the total amount of profit has happened, which confirms the effectiveness of the model and its attractiveness for managers.

6. Sensitivity analysis

The above example is subjected to a sensitivity analysis to determine the impact of various parameters (*i.e.*, costs, the fraction of faulty items, production rate, and demand function coefficient) on the optimal solution. The sensitivity analysis results are illustrated in Tables 3 and 4. In Table 3, the impact of alterations on the optimal solution is illustrated. According to Table 3, the overall profit seems to be highest for $R = 0.001$. It is clear that with lower values for R, total profit is obtained. The bold values are the results when R is not changed, and it has their original value ($R = 0.1$). The backorders level and the lot size (B^*, Q^*) are somewhat sensitive to alterations in R.

Conversely, Q^*, B^* are not sensitive to the changes in R for small R values. Also, the selling price and overall profit (S^*, TP^*) are somewhat sensitive to variations in R. When R increases, Q^*, B^* and S^* increase and TP^* decreases.

Table 4 illustrates the effect of other parameters (costs, demand coefficients and production rate) on the optimal solution.

- The changes in K affects Q^* and B^* moderately. In contrast, S^* and TP^* are only moderately responsive to the changes in K. When K increases (decreases), Q^*, B^* and S^* increase (decreases) and TP^* decreases (increases).
- B^* appears to be insensitive to the changes in this instance. On the contrary, the changes in C slightly affects Q^*, S^* and TP^*. Notice that B^* is almost constant when C changes. When C increases (decreases), B^* and S^* increases (decreases), and TP^* decreases (increases).
- Q^* and B^* are quite sensitive to the changes in f. S^* and TP^* are very slightly sensitive to the changes in f. In the case that f increases (decreases), Q^* and TP^* decreases (increases), S^* increases (decreases), and when f increases (decreases) B^* declines.

TABLE 2. The results of numerical example.

Q (units)	B (units)	S (\$/unit)	Convexity of TP function with respect to S	TP (\$/unit time)	TP_c (\$/unit time)
1	0	816.49	✓	3479.088	2890
2	1	642.75	✓	33 084.84	27 100
2	0	641.49	✓	33 398.99	28 000
3	2	584.84	✓	49 658.63	41 000
3	1	583.99	✓	49 920.97	41 600
3	0	583.15	✓	50 174.46	42 000
⋮	⋮	⋮	⋮	⋮	⋮
285	194	470.16	✓	92 016.28	77 000
285	193	470.13	✓	92 026.97	77 500
285	⋮	⋮	⋮	⋮	⋮
285	1	466.16	✓	91 988.15	77 200
285	0	466.15	✓	91 977.01	78 000
286	194	470.14	✓	92 023.02	77 350
286	193	470.11	✓	92 033.62	76 500
286	⋮	⋮	⋮	⋮	⋮
286	100	467.63	✓	92 528.88	78 000
286	**99**	**467.61**	✓	**92 528.91**	78 300
286	98	467.59	✓	92 528.85	78 300
286	⋮	⋮	⋮	⋮	⋮
286	2	466.15	✓	91 995.96	77 000
286	1	466.15	✓	91 984.93	76 000
286	0	466.14	✓	91 973.78	76 500
287	195	470.15	✓	92 019.06	77 100
287	194	470.12	✓	92 029.68	77 300
287	⋮	⋮	✓	⋮	⋮
287	1	466.14	✓	91 981.70	77 000
287	0	466.13	✓	91 970.54	76 000
⋮	⋮	⋮	⋮	⋮	⋮
290	197	470.14	✓	92 017.77	76 000
290	196	470.11	✓	92 028.35	74 000
290	⋮	⋮	✓	⋮	⋮
290	1	466.11	✓	91 971.94	76 000
290	0	466.1	✓	91 960.73	77 000

- Q^* and B^* are quite sensitive to the changes in i. S^* and TP^* are very slightly sensitive to the changes in i. When i increases (decreases), Q^* and TP^* decrease (increase), S^* increases (decreases), and when i increases (decreases) B^* decreases.
- Q^* and B^* are quite sensitive to the changes in W. S^* and TP^* are very slightly sensitive to the changes in W. When W increases (decreases), Q^*, B^* and TP^* increase (decreases), and S^* is almost constant.
- Q^* and B^* are quite sensitive to the changes in F. S^* and TP^* are very slightly sensitive to the changes in F. When F increases (decreases), S^* increases (decreases) and Q^*, B^* and TP^* decrease (increase).

TABLE 3. Effects of R changes on the optimal solution.

R	Decision variables and objective function				% Changes in			
	Q^*	B^*	S^*	$TP^*(TP_{max})$	Q^*	B^*	S^*	$TP^*(TP_{max})$
0.001	268	95	466.36	93 129.842	−6.29	−4.04	−0.27	+0.65
0.002	268	95	466.37	93 123.707	−6.29	−4.04	−0.26	+0.64
0.005	268	95	466.41	93 105.308	−6.29	−4.04	−0.26	+0.62
0.01	269	95	466.47	93 074.826	−5.94	−4.04	−0.24	+0.59
0.02	271	96	466.61	93 013.889	−5.24	−3.03	−0.21	+0.52
0.05	277	97	466.97	92 831.523	−3.15	−2.02	−0.14	+0.33
0.1	**286**	**99**	**467.61**	**92 528.919**	**0.00**	**0.00**	**0.00**	**0.00**
0.2	294	97	468.91	91 929.191	+2.80	−2.02	+0.28	−0.65
0.3	300	92	470.2	91 334.649	+4.90	−7.07	+0.55	−1.29

Demand function parameters

– Q^*, B^* and S^* are slightly sensitive, while TP^* is very sensitive to changes in a. When a decreases by 20% and 40%, the condition mentioned in Step 1 is not true; consequently, the model is not applicable. When a increases, Q^*, B^*, S^* and TP^* increase.
– S^* and TP^* are almost sensitive to the changes in b. Q^* is slightly sensitive to the changes in b. B^* is not sensitive to the changes in b in this case. When b increases (decreases), Q^*, S^* and TP^* decrease (increase). B^* it will be constant when b changes.

The production rate parameter

– B^* and Q^* are moderately sensitive to the changes in P. S^* and TP^* are slightly sensitive to the changes in P. When P declines by 40%, the problem becomes infeasible because Q_U^* computed from equation (3) has no real value. When P is increased by 20% and 40% the condition mentioned in Step 1 is not true; therefore, the model is not applicable. When P decreases, S^* and Q^* decrease; Q^* and TP^* increase.

7. Conclusions and guidelines for future studies

The present study introduces a new form of the EPQ inventory model, which includes elements that make the production-inventory model more real and applicable to managers. The pricing policy has been considered in an EPQ inventory model in which planned backorders and rework processes have also been included. This study has regarded the demand rate as dependent on the selling price in marketing and pricing. Consequently, this research takes the lot size, the selling price, and the number of backorders as decision variables. It solves a nonlinear optimization problem in which Q and B must take integer values. This paper uses the fact that $TP((Q^*+1), B, S) \leq TP(Q^*, B, S)$ and $TP((Q^*-1), B, S) \leq TP(Q^*, B, S)$ to determine the Q^* and B^*, following which, the convexity of the objective function (TP) concerning S is examined and S^* calculated. The work develops an algorithm for the combined optimization of the lot size, selling price, and backorder level. Furthermore, sensitivity analysis is conducted on the parameter changes to determine the contribution of parameter changes to the objective function and decision variables. In the inventory literature, there are numerous extensions to the EPQ inventory model; the suggested inventory model seems to be a contribution to this line of study. The suggested method offers production managers an accurate solution for simultaneously determining lot size, backorder size, and selling price. Consequently, production managers must have a more precise inventory model for their manufacturing environment. This article also presents a sensitivity analysis so as to examine the contribution of parameter changes to the optimum value of decision variables and overall profit. Price has been included as a critical factor of the EPQ inventory model that affects demand size to

Table 4. Effect of other parameter changes on the optimal solution.

Parameters and % Changes in their values			Decision variables and objective function				% Changes in			
			Q^*	B^*	S^*	$TP^*(TP_{max})$	Q^*	B^*	S^*	$TP^*(TP_{max})$
Costs	K ($/lot)	−40	212	65	467.43	92 772.01	−25.87	−34.34	−0.04	+0.26
		−20	251	83	467.53	92 641.38	−12.24	−16.16	−0.02	+0.12
		+20	314	112	467.69	92 428.60	+9.79	+13.13	+0.02	−0.11
		+40	334	121	467.76	92 349.63	+16.78	+22.22	+0.03	−0.19
	C ($/unit)	−40	290	100	460.98	95 404.59	+1.40	+1.01	−1.42	+3.11
		−20	287	99	464.3	93 961.28	+0.35	0.00	−0.71	+1.55
		+20	285	99	470.92	91 107.49	−0.35	0.00	+0.71	−1.54
		+40	283	99	474.24	89 697.01	−1.05	0.00	+1.42	−3.06
	f ($/unit/time unit)	−40	297	85	467.50	92 630.11	+3.85	−14.14	−0.02	+0.11
		−20	291	93	467.56	92 574.88	+1.75	−6.06	−0.01	+0.05
		+20	271	99	467.71	92 489.18	−5.24	0.00	+0.02	−0.04
		+40	255	97	467.83	92 452.52	−10.84	−2.02	+0.05	−0.08
	i (%)	−40	292	90	467.54	92 592.12	+2.10	−9.09	−0.01	+0.07
		−20	289	95	467.58	92 558.65	+1.05	−4.04	−0.01	+0.03
		+20	277	98	467.68	92 502.04	−3.15	−1.01	+0.02	−0.03
		+40	265	98	467.75	92 476.58	−7.34	−1.01	+0.03	−0.06
	W($/unit/time unit)	−40	290	113	467.84	92 609.07	+1.40	+14.14	+0.05	+0.09
		−20	287	106	467.7	92 566.93	+0.35	+7.07	+0.02	+0.04
		+20	271	87	467.61	92 497.11	−5.24	−12.12	+0.00	−0.03
		+40	259	77	467.61	92 471.24	−9.44	−22.22	+0.00	−0.06
	F ($/unit)	−40	279	109	467.54	92 687.57	−2.45	+10.10	−0.01	+0.17
		−20	284	105	467.58	92 606.19	−0.70	+6.06	−0.01	+0.08
		+20	269	84	467.71	92 456.4	−5.94	−15.15	+0.02	−0.08
		+40	267	77	467.73	92 391.14	−6.64	−22.22	+0.03	−0.15
Demand function coefficients	a	−40	NA	NA	NA	Infeasible	NA	NA	NA	Infeasible
		−20	NA	NA	NA	Infeasible	NA	NA	NA	Infeasible
		+20	329	101	557.18	135 513.95	+15.03	+2.02	+19.16	+46.46
		+40	376	102	646.78	186 636.36	+31.47	+3.03	+38.32	+101.71
	b	−40	289	99	767.57	159 967.04	+1.05	0.00	+64.15	+72.88
		−20	289	99	580.09	117 810.45	+1.05	0.00	+24.05	+27.32
		+20	284	99	392.63	75 684.94	−0.70	0.00	−16.03	−18.20
		+40	283	99	339.07	63 662.47	−1.05	0.00	−27.49	−31.20
Production rate	P (units/time unit)	−40	NA	NA	NA	Infeasible	NA	NA	NA	Infeasible
		−20	307	96	467.14	92 637.36	+7.34	−3.03	−0.10	+0.12
		+20	NA	NA	NA	Infeasible	NA	NA	NA	Infeasible
		+40	NA	NA	NA	Infeasible	NA	NA	NA	Infeasible

Notes. NA: Not Applicable.

maximize long-term profit. Moreover, both the number of backorders and the lot size must assume discrete values.

Like all other research, this research is faced with limiting assumptions, which can be removed to bring this model closer to the real world. Some of them are as follows:

– Considering the environmental conditions in today's world, one of the important areas that can be investigated is the consideration of sustainability in the presented model. For this, you can refer to research such as Sarkar and Bhuniya [40] and Tayyab *et al.* [50].

- Another suggestion that can be made to bring the model closer to the real world conditions is to examine the model in conditions and environments where the amount of demand is uncertain (stochastic, fuzzy, or probabilistic). For further study, you can see Govindan and Cheng [24].
- The use of two-stage and multi-stage planning is a case that can be considered to make the model more practical.
- Considering limitations such as space, available capital, inspection cost, etc., that we face in the real world can help the complexity of the model and application in different industries.
- The use of the neural network, deep learning, and reinforce learning methods, which are of great interest to researchers today to optimize the model and use a large amount of data, will lead to more accurate answers and increase the model's flexibility.

References

[1] P. Abad, Optimal pricing and lot-sizing under conditions of perishability and partial backordering. *Manage. Sci.* **42** (1996) 1093–1104.

[2] P.L. Abad, Optimal price and order size under partial backordering incorporating shortage, backorder and lost sale costs. *Int. J. Prod. Econ.* **114** (2008) 179–186.

[3] A. Amjadian and A. Gharaei, An integrated reliable five-level closed-loop supply chain with multi-stage products under quality control and green policies: generalised outer approximation with exact penalty. *Int. J. Syst. Sci. Oper. Logistics* **9** (2022) 429–449.

[4] R. Askari, M.V. Sebt and A. Amjadian, A multi-product EPQ model for defective production and inspection with single machine, and operational constraints: stochastic programming approach, in International Conference on Logistics and Supply Chain Management. Springer, Cham (2020) 161–193.

[5] L.E. Cárdenas-Barrón, Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Comput. Ind. Eng.* **57** (2009) 1105–1113.

[6] L.E. Cárdenas-Barrón, H.M. Wee and M.F. Blos, Solving the vendor–buyer integrated inventory system with arithmetic–geometric inequality. *Math. Comput. Modell.* **53** (2011) 991–997.

[7] L.E. Cárdenas-Barrón, A.A. Taleizadeh and G. Treviño-Garza, An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework. *Expert Syst. App.* **39** (2012) 13540–13546.

[8] L.E. Cárdenas-Barrón, J.T. Teng, G. Treviño-Garza, H.M. Wee and K.R. Lou, An improved algorithm and solution on an integrated production-inventory model in a three-layer supply chain. *Int. J. Prod. Econ.* **136** (2012) 384–388.

[9] L.E. Cárdenas-Barrón, B. Sarkar and G. Treviño-Garza, Easy and improved algorithms to joint determination of the replenishment lot size and number of shipments for an EPQ model with rework. *Math. Comput. App.* **18** (2013) 132–138.

[10] L.E. Cárdenas-Barrón, B. Sarkar and G. Treviño-Garza, An improved solution to the replenishment policy for the EMQ model with rework and multiple shipments. *Appl. Math. Modell.* **37** (2013) 5549–5554.

[11] L.E. Cárdenas-Barrón, G. Treviño-Garza, G.A. Widyadana and H.M. Wee, A constrained multi-products EPQ inventory model with discrete delivery order and lot size. *Appl. Math. Comput.* **230** (2014) 359–370.

[12] W. Chan, R. Ibrahim and P. Lochert, A new EPQ model: integrating lower pricing, rework and reject situations. *Prod. Plann. Control* **14** (2003) 588–595.

[13] H.-C. Chang, C.-H. Ho, L.-Y. Ouyang and C.-H. Su, The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity. *Appl. Math. Modell.* **33** (2009) 2978–2991.

[14] Z. Chen and Y. Yang, Optimality of (s, S, p) policy in a general inventory-pricing model with uniform demands. *Oper. Res. Lett.* **38** (2010) 256–260.

[15] Y.P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Eng. Optim.* **35** (2003) 427–437.

[16] K.-J. Chung, Bounds for production lot sizing with machine breakdowns. *Comput. Ind. Eng.* **32** (1997) 139–144.

[17] C.-Y. Dye, T.-P. Hsieh and L.-Y. Ouyang, Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. *Eur. J. Oper. Res.* **181** (2007) 668–678.

[18] A. Eroglu and G. Ozdemir, An economic order quantity model with defective items and shortages, *Int. J. Prod. Econ.* **106** (2007) 544–549.

[19] A. Federgruen and A. Heching, Combined pricing and inventory control under uncertainty. *Oper. Res.* **47** (1999) 454–475.

[20] J. García-Laguna, L.A. San-José, L.E. Cárdenas-Barrón and J. Sicilia, The integrality of the lot size in the basic EOQ and EPQ models: applications to other production-inventory models. *Appl. Math. Comput.* **216** (2010) 1660–1672.

[21] A. Gharaei, A. Amjadian and A. Shavandi, An integrated reliable four-level supply chain with multi-stage products under shortage and stochastic constraints. *Int. J. Syst. Sci. Oper. Logistics* (2021) 1–22.

[22] A. Gharaei, A. Amjadian, A. Amjadian, A. Shavandi, A. Hashemi, M. Taher and N. Mohamadi, An integrated lot-sizing policy for the inventory management of constrained multi-level supply chains: null-space method. *Int. J. Syst. Sci. Oper. Logistics* (2022) 1–14.

[23] C.H. Glock, E.H. Grosse and J.M. Ries, The lot sizing problem: a tertiary study. *Int. J. Prod. Econ.* **155** (2014) 39–51.
[24] K. Govindan and T.C.E. Cheng, Advances in stochastic programming and robust optimization for supply chain planning. *Comput. Oper. Res.* **100** (2018) 262–269.
[25] S.K. Goyal and L.E. Cárdenas-Barrón, Note on: economic production quantity model for items with imperfect quality: a practical approach. *Int. J. Prod. Econ.* **77** (2002) 85–87.
[26] F. Harris, How many parts to make at once. *Factory Mag. Manage.* **10** (1913) 135–136, and 152.
[27] P.A. Hayek and M.K. Salameh, Production lot sizing with the reworking of imperfect quality items produced. *Prod. Planning Control* **12** (2001) 584–590.
[28] K.S. Hong and C. Lee, Optimal time-based consolidation policy with price sensitive demand. *Int. J. Prod. Econ.* **143** (2013) 275–284.
[29] K.S. Hong, S.S. Yeo, H.J. Kim, E.P. Chew and C. Lee, Integrated inventory and transportation decision for ubiquitous supply chain management. *J. Intell. Manuf.* **23** (2012) 977–988.
[30] V.B. Kreng and S.J. Tan, Optimal replenishment decision in an EPQ model with defective items under supply chain trade credit policy. *Expert Syst. App.* **38** (2011) 9888–9899.
[31] A.H.L. Lau and H.S. Lau, Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model. *Eur. J. Oper. Res.* **147** (2003) 530–548.
[32] L.-Y. Ouyang, C.-H. Ho and C.-H. Su, An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. *Comput. Ind. Eng.* **57** (2009) 920–930.
[33] D.W. Pentico and M.J. Drake, A survey of deterministic models for the EOQ and EPQ with partial backordering. *Eur. J. Oper. Res.* **214** (2011) 179–198.
[34] D.W. Pentico, M.J. Drake and C. Toews, The deterministic EPQ with partial backordering: a new approach. *Omega* **37** (2009) 624–636.
[35] D.W. Pentico, M.J. Drake and C. Toews, The EPQ with partial backordering and phase-dependent backordering rate. *Omega* **39** (2011) 574–577.
[36] D.W. Pentico, C. Toews and M.J. Drake, Two heuristics for the basic EOQ and EPQ with partial ackordering. *Int. J. Inf. Syst. Supply Chain Manage.* **7** (2014) 31–49.
[37] E.L. Porteus, Optimal lot sizing, process quality improvement and setup cost reduction. *Oper. Res.* **34** (1986) 137–144.
[38] M. Salameh and M. Jaber, Economic production quantity model for items with imperfect quality. *Int. J. Prod. Econ.* **64** (2000) 59–64.
[39] B. Sarkar, A production-inventory model with probabilistic deterioration in two-echelon supply chain management. *Appl. Math. Modell.* **37** (2013) 3138–3151.
[40] B. Sarkar and S. Bhuniya, A sustainable flexible manufacturing–remanufacturing model with improved service and green investment under variable demand. *Expert Syst. App.* **202** (2022) 117154.
[41] B. Sarkar, L.E. Cárdenas-Barrón, M. Sarkar and M.L. Singgih, An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. *J. Manuf. Syst.* **33** (2014) 423–435.
[42] B. Sarkar, B.K. Dey, M. Sarkar and S.J. Kim, A smart production system with an autonomation technology and dual channel retailing. *Comput. Ind. Eng.* **173** (2022) 108607.
[43] B. Sarkar, M. Ullah and M. Sarkar, Environmental and economic sustainability through innovative green products by remanufacturing. *J. Cleaner Prod.* **332** (2022) 129813.
[44] E.W. Taft, The most economical production lot. *Iron Age* **101** (1918) 1410–1412..
[45] A.A. Taleizadeh, A. Najafi and N.S. Akhavan, Economic production quantity model with scrapped items and limited production capacity. *Sci. Iran. Trans. E: Ind. Eng.* **17** (2010) 58–69.
[46] A.A. Taleizadeh, H.-M. Wee and S.J. Sadjadi, Multi-product production quantity model with repair failure and partial backordering. *Comput. Ind. Eng.* **59** (2010) 45–54.
[47] A.A. Taleizadeh, L.E. Cárdenas-Barrón and B. Mohammadi, A deterministic multi product single machine EPQ model with backordering, scraped products, rework and interruption in manufacturing process. *Int. J. Prod. Econ.* **150** (2014) 9–27.
[48] A.A. Taleizadeh, M. Noori-daryan and R. Tavakkoli-Moghaddam, Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items. *Int. J. Prod. Res.* **53** (2015) 4553–4582.
[49] A.A. Taleizadeh, A.Z. Safaei, A. Bhattacharya and A. Amjadian, Online peer-to-peer lending platform and supply chain finance decisions and strategies. *Ann. Oper. Res.* **315** (2022) 397–427.
[50] M. Tayyab, M.S. Habib, M.S.S. Jajja and B. Sarkar, Economic assessment of a serial production system with random imperfection and shortages: a step towards sustainability. *Comput. Ind. Eng.* **171** (2022) 108398.
[51] J.-T. Teng, M.-S. Chern, H.-L. Yang and Y.J. Wang, Deterministic lot-size inventory models with shortages and deterioration for fluctuating demand. *Oper. Res. Lett.* **24** (1999) 65–72.
[52] J.-T. Teng, L.E. Cárdenas-Barrón and K.-R. Lou, The economic lot size of the integrated vendor–buyer inventory system derived without derivatives: a simple derivation. *Appl. Math. Comput.* **217** (2011) 5972–5977.
[53] J.T. Teng, L.E. Cárdenas-Barrón, K.R. Lou and H.M. Wee, Optimal economic order quantity for buyer–distributor–vendor supply chain with backlogging derived without derivatives. *Int. J. Syst. Sci.* **44** (2013) 986–994.
[54] G. Treviño-Garza, K.K. Castillo-Villar and L.E. Cárdenas-Barrón, Joint determination of the lot size and number of shipments for a family of integrated vendor–buyer systems considering defective products. *Int. J. Syst. Sci.* **46** (2015) 1705–1716.
[55] R. Uthayakumar and M. Palanivel, An inventory model for defective items with trade credit and inflation. *Prod. Manuf. Res.* **2** (2014) 355–379.

[56] H.M. Wee, W.-T. Wang and L.E. Cárdenas-Barrón, An alternative analysis and solution procedure for the EPQ model with rework process at a single-stage manufacturing system with planned backorders. *Comput. Ind. Eng.* **64** (2013) 748–755.

[57] T.M. Whitin, Inventory control and price theory. *Manage. Sci.* **2** (1955) 61–68.

[58] P.-S. You and M.-T. Wu, Optimal ordering and pricing policy for an inventory system with order cancellations. *OR Spectrum* **29** (2007) 661–679.

[59] X. Zhang and Y. Gerchak, Joint lot sizing and inspection policy in an EOQ model with random yield. *IIE Trans.* **22** (1990) 41–47.