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A NOVEL PROFIT ALLOCATION RULE OF COMPLEX PRODUCT DEVELOPMENT NETWORKS UNDER THE POSITION VALUE

Mingzhen Zhang, Naiding Yang*, Xianglin Zhu and Yan Wang

Abstract. A fair and reasonable profit allocation mechanism is the key to ensuring the stability of complex product development networks. Considering the disconnectedness of development networks, this paper takes the Position value to allocate profits. Initially, this paper constructs the profit function of complex product development networks, which serves as the characteristic function in graph cooperative games. Subsequently, a real-world case is presented to demonstrate the profit allocation process using the Position value, allowing for an examination of the relationship between profits and firms' investments. Moreover, various factors are investigated to assess their influence on the profit allocation result, and a comparison is made between the Position value and the Myerson value. By adjusting the parameters and observing the numerical simulation, the research delves into the impact of key parameters on firms' profit allocation. The findings indicate that the network position and investment are directly proportional to firms' allocated profits. Additionally, the synergistic coefficient and benefit coefficient positively moderate firms' profits, while the cost coefficient of investment negatively moderates them. Notably, the Position value proves to be more suitable for complex product development networks than the Myerson value.

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1. Introduction

Different from mass-produced goods, complex products such as aircraft [23] and gas turbines [39] possess distinctive characteristics including high development cost, technology intensity, customization, and small-batch production [42]. The development of complex products involves a wide range of knowledge and fields that are often beyond the capabilities of individual firms. Participation in development networks enables firms to attain complementary resources, share development costs, mitigate development risks, and expedite development cycles, thus emerging as an essential approach for firms to pursue complex product development [10]. For example, Boeing has formed a tight collaborative network with GE Aviation, Spirit Aero Systems, Honeywell Aerospace, Rockwell Collins, and others to finish the research and manufacturing of the Boeing 747. In the cooperation process, an important issue is to distribute the total profit to enterprises fairly and reasonably [22]. When the distribution of profits among enterprises is unfair, it can lead to negative consequences, including member dissatisfaction, instability in cooperative relationships, disadvantages in competitive advantage, loss of

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market share, increased costs, and decreased revenue [30]. Therefore, it is imperative to establish a fair profit distribution rule to maintain the stability and competitiveness of complex product development networks.

Regarding the content of profit allocation, the existing research mostly presumes that the profit produced by all sub-networks is predicted in advance without considering the process of profit created. However, studies have shown that changes in factors in the profit function can have a significant impact on profit distribution. As stated by Jiang *et al.* [20], variables such as research and development input, contribution coefficient, synergy effect, and cost coefficient play a significant role in formulating the profit function and influencing the profit distribution within technology innovation cooperative alliances. Similarly, Feess and Thun [11] highlighted that factors such as network innovation profit, input resources, and technology spillover are crucial in determining the profit distribution within the supply chain. Hence, when allocating profits to complex product development networks, it is essential to consider the factors that impact network profits and incorporate them into the allocation rules.

In terms of methods for profit allocation, the Shapley value, which is a solution concept for cooperative games, is widely used due to its properties of validity, additivity, and uniqueness. However, the Shapley value assumes that all participants can successfully collaborate, which is contrary to the cooperation condition of firms in actual development networks. In actual collaborative processes, there are often limitations imposed by factors such as cultural differences, technological disparities, and geographical distances, which create constraints on cooperation. To address this shortcoming, Myerson [41] proposed a graph cooperative game, which is a cooperative game with an undirected graph structure. The Myerson value is a point solution for cooperative graph games, similar to the Shapley value, as it allocates benefits based on the marginal contributions of the participants. Later, Meessen [40] introduced the Position value based on the Myerson value, which considers links as participants and allocates benefits to links under the Myerson value and then evenly distributes them to the two nodes at both ends. Given that the benefits in the process of developing complex products are generated through collaborative cooperation, this aligns well with the principle of the Position value. Therefore, this paper employs the Position value, which is the solution of cooperative graph games, to allocate benefits in complex product development networks.

From the aforementioned analysis, we can derive the drawbacks of the previous studies:

1. Prior studies focus on the outcomes or results of profit allocation but do not fully consider the underlying mechanisms and dynamics that lead to the generation of profits.
2. A complex product development network has the scale-free feature, however, prior studies in profit allocation have predominantly relied on traditional cooperative game theory methods and overlooked the cooperative restrictions and limitations that exist between firms.
3. The Position value effectively captures the process of profit generation within development networks, but existing research on Position value has primarily focused on theoretical extensions and conceptual frameworks. Its application to the study of network profit allocation has been relatively limited.

To address these challenges, this study initially formulates the profit function of the network by considering investments and the synergistic coefficient. This function is then adopted as the characteristic function of the graph cooperative game. Then we take the local network of high-speed train traction and braking system project as the research objective and allocates profits to firms with the Position value. After thoroughly analyzing firms' profit, several meaningful propositions are derived. Finally, numerical analysis is employed to quantitatively and visually illustrate the examined relationships, thereby facilitating a comprehensive understanding of the profit distribution process within complex product development networks.

The main contribution and novelty of this research are as follows:

1. Constructing the profit function of complex product development networks to analyze the process of profit generation.
2. Considering the non-connectivity of the complex product development network, adopting the Position value, which is the solution of the graph cooperative game, as the profit allocation rule.

3. After a comparative analysis with the Myerson value, the superiority and applicability of using the Position value is concluded.
4. Putting forward the recommendations for improving the profit value of firms from the aspects of synergistic coefficient, benefit coefficient, and cost coefficient.

The remainder is structured as follows: Section 1 provides a comprehensive literature review of relevant research in the field. Section 2 introduces the problem and the solution approach. Section 3 presents the results and discussion. Section 4 demonstrates managerial insights and practical implications. Finally, Section 5 presents the conclusion of the study.

2. Literature review

2.1. Profit allocation

Profit in the development network should be allocated reasonably and fairly to motivate cooperation and thus avoid internal conflict [1]. Currently, research on network profit allocation can be mainly divided into two categories. One method is to allocate profit using non-cooperative game theory. Such as the symmetric Nash bargaining solution [31], maximum and minimum and the Nash bargaining model [32], the modified asymmetric Nash bargaining model[15], Stackelberg game theory [58], and the distribution-robust Stackelberg game model [13]. This type of approach concentrates on obtaining the optimal distribution proportion to maximize individual profits. The other category is to allocate profit networks using cooperative game theory. Since the solution of the cooperative game satisfies the properties of validity, additivity, and symmetry, which can provide an equitable and satisfactory scheme, it has been frequently used to handle the profit allocation issue in recent years [17]. As the unique solution in a cooperative game, the Shapley value is the most widely used profit allocation rule in the cooperative game. For example, applying the Shapley value to address pollution responsibility and cost-benefit allocation within multi-layer supply chains [7], as well as addressing the benefit distribution issue in supply chains [26, 57], enterprise technological innovation cooperation alliances [20] and high-tech enterprise modular research and development networks [50].

Furthermore, the Shapley value has been modified from two aspects. One aspect is incorporating other factors to construct profit allocation rules. Zhang *et al.* [55] integrated factors such as knowledge sharing level, knowledge absorption level, investment amount, and risk resistance ability to redefine the Shapley value for profit allocation in industry-university-research collaboration networks. Zhu *et al.* [59] incorporated a penalty factor into the profit allocation process and modified the Shapley value model by integrating the ratio of firms' contribution to their risk level. This modified model was then applied to a dynamic multimodal transportation alliance. The other is to improve the Shapley value with uncertain payoffs. Han and Li [49] proposed the Shapley value method of intuitionistic fuzzy coalition cooperative game, investigating the profit distribution mechanism among collaborating enterprises. Gao *et al.* [14] introduced the definition and characteristics of uncertain Shapley value, including expected Shapley value and α-optimistic Shapley value. The method subsequently was applied to address profit allocation issues within supply chain alliances. Asrol [2] enhanced the fuzzy Shapley value method by considering the uncertain risks and value-added in the agro-industry supply chain alliance and applied it to the sugarcane agro-industry supply chain. Wang and Yin [51] improved the interval Shapley value method by introducing correction factors such as risk assumption, cooperative effort, market competition, innovation contribution, and investments. Subsequently, a solution for fuzzy cooperative games was developed, which was applied to profit allocation in alliances within the manufacturing and logistics industries.

Indeed, while existing studies have employed various methods for profit allocation, few have thoroughly explored the process of profit generation and considered the cooperation constraints that enterprises face within networks.

2.2. Graph cooperative game

The Shapley value assumes that all participants can successfully collaborate, which is contrary to the cooperation condition of firms in actual research and development networks. Due to the constraints of various cultural backgrounds, technology, and location, the cooperative game among firms is restricted in the network structure. To address this shortcoming, Myerson [41] proposed the graph cooperative game and the solution of graph cooperative gameMyerson value. It has introduced cooperative game theory into more complex situations. For instance, utilizing the Myerson value to allocate profits in HSR express networks [38], agricultural supply chain networks [44], and networked microgrids [48], respectively. Additionally, some scholars have extended the Myerson value from different perspectives [18, 21, 24, 40, 46]. Meessen [40], for example, proposed the Position value based on the Myerson value. Unlike the Myerson value, which is a solution concept for point games, the Position value is a solution concept for link games. By considering the links as the players in the game, the Position value allocates benefits to the links utilizing the Myerson value, which reflects the structure of feasible networks. The Position value suggests that members who control more resources and occupy more advantageous positions in the network should receive a larger share of the profits. The process of profit generation in complex product development networks aligns seamlessly with the principle of benefit allocation based on the link game in the Position value. Therefore, the Position value is more suitable for profit allocation in complex product development networks.

Existing research on the Position value primarily focuses on its extension. Such as, extending link games and the Position value to local structures [25], proposing a generalized Position value, called the "v-Position" value, in the hypernetwork game [45], extending the Position value of the hypernetwork game by considering the number of participants included in each hyperlink [28], introducing the Position value on a probabilistic network [5], and investigating the Position value on fuzzy communication structures [12, 27]. Further research is warranted to explore the applications of the Position value to profit allocation in greater depth.

2.3. Research gap

Inspired by [33–37], we summarize the different types of research work conducted in this field in Table 1. Different from previous research, this paper not only constructs a profit function to describe the fundamental mechanism of profit creation in complex product development networks. In addition, considering the heterogeneous network characteristics of such networks, employ the Position value which is the solution of graph cooperative games to allocate profits. This approach provides a reference for enhancing the stability of complex product development networks and extends the applicability of graph cooperative game theory. The algorithm of this study is shown in Figure 1.

3. Problem statement

Following a thorough analysis of the relevant literature, we conclude that the variables of investment and the synergistic coefficient hold paramount significance in the research and development stage of complex product development. Specifically, investment pertains to the diverse array of resources allocated by organizations toward the advancement of complex products, encompassing investment capital, personnel, technological reserves, and infrastructure. In the process of collaboration, firms create values by investing in complementary resources, technologies, and knowledge, as expounded by [16]. Consequently, investment emerges as a decisive determinant impacting the overall profit of the network. Greater levels of investment by firms yield higher network profits and bolster their contributions to the product's success.

The synergistic coefficient denotes the impact of a firm's harmonization and amalgamation of internal and external resources by the strategic objectives of the network. This leads to a total system efficacy that exceeds the outcome of individual firm's endeavors [3], which in turn enhances the overall performance of the network [43, 53]. Consequently, the synergistic coefficient is a crucial determinant of the network's overall profit [4, 52]. The greater the synergistic coefficient among firms, the greater the overall profit of the network.

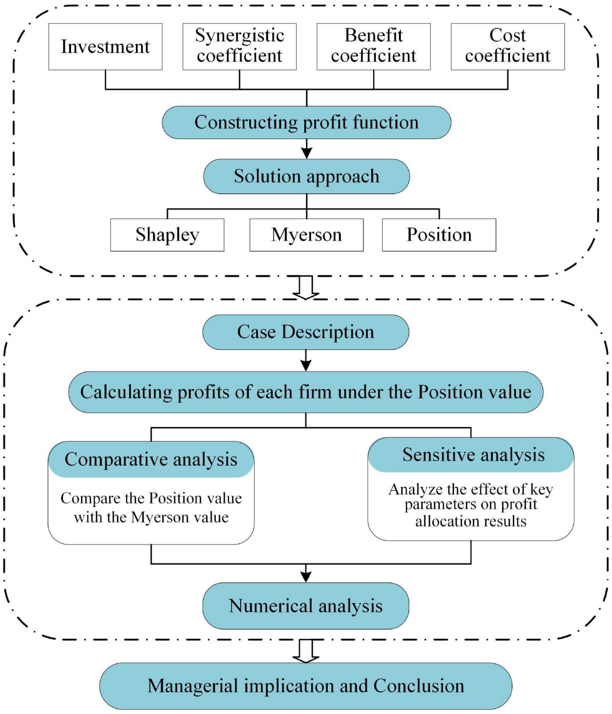

FIGURE 1. The algorithm of this study.

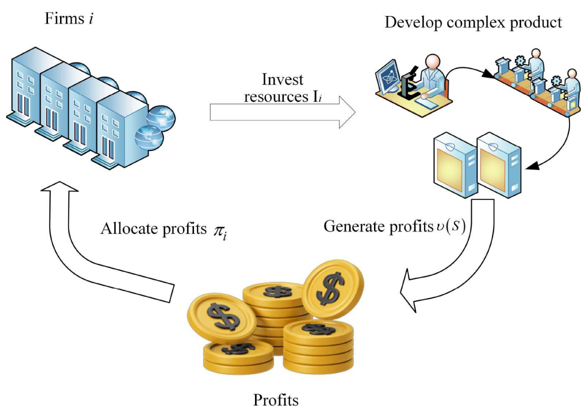

FIGURE 2. The process of profit allocation.

The profit allocation process can be depicted in Figure 2.

3.1. Notations and assumptions

The indices, parameters, and decision variables are defined as follows.

Indices (Sets).
i Index of firms in the network $i \in S \in N = \{3, 6, 15, 16, 18, 19\}$,

TABLE 1. Study of the profit allocation.

Classification	Research	Method	Context Considering	Restricted structure	Considering the profit creates process
Non-cooperative game	Liu *et al.* [31]	Symmetric Nash bargaining solution Urban renewal project	×	×	
	Liu *et al.* [32]	Nash bargaining model	Logistics enterprise coalitions	×	×
	Ge *et al.* [15]	Modified asymmetric Nash bargaining model	Industry-university-research cooperative innovation strategy alliance	×	×
	Zhong *et al.* [58]	Stackelberg game theory	E-commerce logistics service supply chain	×	✓
	Fu *et al.* [13]	Distribution-robust Stackelberg game	Decentralized supply chain	×	✓
	Hu [17]	Clique solution	Social cooperation	×	×
Cooperative game	Ciardiello and Genovese [7]	Shapley value	Supply chain	×	×
	Zheng *et al.* [57]	Shapley value	Three-echelon closed-loop supply chain	×	✓
	Li and Chen [26]	Equal allocation, propositional allocation and the Shapley value	Assembly supply chain	×	✓
	Jiang *et al.* [20]	Shapley value	Enterprises' technological innovation cooperation alliance	×	✓
	Wang *et al.* [50]	Shapley value	Modular R&D network of high-tech enterprises	×	×
	Zhang *et al.* [55]	Shapley value	University-industry cooperation network	×	×
	Zhu *et al.* [59]	Modified Shapley value	Dynamic multimodal transportation alliance	×	×
	Huang and Li [49]	Intuitionistic fuzzy Shapley value	Collaborating enterprises	×	×
	Gao *et al.* [14]	Uncertain Shapley value	Supply chain alliances	×	×
	Asrol *et al.* [2]	Fuzzy Shapley value	Sugarcane agro-industry supply chain	×	✓
	Wang and Yin[51]	Interval Shapley value	The alliance within the manufacturing and logistics industries	×	×
Graph Cooperative game	Lv *et al.* [38]	Myerson value	HSR express	✓	×
	Li *et al.* [44]	Myerson value	Multiple agricultural communes and multiple supermarkets	✓	×
	Suh and Yoon [48]	Myerson value	Networked micrograids	✓	×
	This paper	Position value	Complex product development network	✓	✓

l Index of connection links in the network $l \in A \in L = \{a, b, c, d, \ldots\}$.

Parameters.

$I_i\,(I_i > 1)$ Investment of firm i,

$I_i\,(I_i > 1)$ The synergistic coefficient of cooperation,

$\delta\,(0 < \delta < 1)$ The benefit coefficient of investment,

$\gamma\,(0 < \gamma < 1)$ The cost coefficient of investment.

Variables.

$\upsilon\,(S)$ The profit of the subnetwork S,

Π The profit of the complex product development network,

π_i The profit allocated to firm i under the Position value,

$\bar{\pi}_i$ The profit allocated to firm i under the Position value when investment is I,

μ_i The profit allocated to firm i under the Myerson value,
$\bar{\mu}_i$
The profit allocated to firm i under the Myerson value when investment is I.

Assumption 3.1. *The benefit distribution procedure is static. That is, the value of a firm's investment and the cooperative connection between firms do not alter once they have been decided.*

Assumption 3.2. *The study focuses on the research and development stage of complex product development, excluding the subsequent marketization phase following successful product development. Thus, we do not consider the issues about price and production decisions of the product.*

Assumption 3.3. *To demonstrate that the Position value considers not only the contribution made by input but also the contribution made by the network position, this study assumes that firms are homogeneous in terms of capabilities, knowledge, and resources. This means that the synergistic coefficient, benefit coefficient, and cost coefficient are equal for all firms.*

Assumption 3.4. *Considering the specific process of development, we suppose that $r\delta^2 \le \gamma$, which is consistent with Jiang et al. [20].*

Assumption 3.5. *To prevent the network profit value from being negative, we assume $r\delta^2 I_i + \delta - \gamma I_i \ge 0\,(i = 3, 6, 15, 16, 18, 19)$.*

3.2. Mathematical model

The overall profit of the complex product development network consists of the foundational benefits converted from investments, the benefits generated by synergistic effects, and the costs associated with investments [8].

For a sub-network S, the foundational benefit converted from investments is $\delta \sum_{i\in S} I_i$. Synergistic effects imply that as one party increases its input, the marginal benefits of the other party's input also increase. A higher coefficient of synergistic effects signifies stronger collaborative innovation capabilities among firms. Dai and Fan [8] pointed out that the synergistic effect is closely associated with a firm's investments. Thus, we suppose that the benefits generated by synergistic effects are $\frac{1}{2}r\left(\delta \sum_{i\in S} I_i\right)^2$. According to D'Aspremont and Jacquemin [9], there is a quadratic relationship between the investment and the development cost. Here, we suppose that $C = \frac{1}{2}\gamma \sum_{i\in S} I_i^2$. Then the profit function of the complex product development network is established as follows.

$$\upsilon\left(S\right) = \frac{1}{2} r \left(\delta \sum_{i\in S} I_i\right)^2 + \delta \sum_{i\in S} I_i - \frac{1}{2}\gamma \sum_{i\in S} I_i^2 \tag{1}$$

The profit of the whole network is $\Pi = \frac{1}{2}r\left(\delta \sum_{i\in N} I_i\right)^2 + \delta \sum_{i\in N} I_i - \frac{1}{2}\gamma \sum_{i\in N} I_i^2$. Then we can get that: $\frac{\partial \Pi}{\partial I_i} = r\delta^2 \left(\sum_{i\in N} I_i\right) + \delta - \gamma I_i$ and $\frac{\partial^2 \Pi}{\partial {I_i}^2} = r\delta^2 - \gamma$. When $r\delta^2 - \gamma \ge 0$, then $\frac{\partial \Pi}{\partial I_i} > 0$ and $\frac{\partial^2 \Pi}{\partial {I_i}^2} > 0$. This means that as a firm invests more in the research and development process, the benefits of the development network will grow exponentially. If this conclusion holds, then with enough investment of resources and capital, exponential gains can be achieved. This is not consistent with reality. Therefore, we assume that $r\delta^2 \le \gamma$, this indicates that the marginal effect of the firm's investment is diminishing, and the benefits generated by the development network will gradually saturate as the investment increases. Moreover, to avoid the investment has a negative impact on profits, we need to make $\frac{\partial \Pi}{\partial I_i} > 0$. Thus we set $r\delta^2 I_i + \delta - \gamma I_i \ge 0\,(i \in N)$. These two conditions have been mentioned in Assumptions 3.4 and 3.5.

Theorem 3.6. *The profit function $\upsilon\left(S\right)$ of the complex product development network satisfies non-negativity and super-additivity. That is:*

(1) Non-negativity: In the graph game (N, υ, L), for any$S \subseteq N$, there is $\upsilon(S) \geq 0$.

(2) Super-additivity: In the graph game (N, υ, g), for any $S \subseteq N$ and $T \subseteq N$, if $S \cap T = \varnothing$, then $\upsilon(S \cup T) \geq \upsilon(S) + \upsilon(T)$.

The proof is provided in the Appendix.

According to Theorem 3.6, it is known that the profit function proposed in this paper is feasible. In the subsequent sections, we will consider the profit function as the characteristic function of the graph cooperative game and employ the Position value, which is the solution of the graph cooperative game, to distribute the profits of complex product development networks.

3.3. Solution approach

The Position value is an extended form of the Shapley value, which is suitable for the allocation of profits in restricted networks. This paper introduces the notion of the Position value through the concepts of Shapely and Myerson values, which provide the foundation for the Position value.

3.3.1. Shapley value

The cooperative n-person game with transferable utility is a pair (N, υ), wherein the set of players and $\upsilon : 2^N \rightarrow \mathbb{R}$, the characteristic function, is a map. For any $S \subseteq N$, $\upsilon(S)$ is the utility of the subset S. The point solution, or allocation rule, of the cooperative game, is a map. And Shapley value is one of the best-known allocation rules, which can be expressed below.

$$Sh_i(N, \upsilon) = \sum_{S \subseteq N \backslash \{i\}} \frac{|S|!\,(|N| - |S| - 1)!}{|N|!} \left[\upsilon\left(S \bigcup \{i\}\right) - \upsilon(S)\right] \tag{2}$$

where $Sh_i(N, \upsilon)$ is the outcome of the player i in the game, $N \backslash \{i\}$ is the set of players where the player i removed from the network N, $|S|$ and $|N|$ are the cardinality of S and N, $\upsilon(S \bigcup \{i\}) - \upsilon(S)$ is the marginal contributions that player i to the network N.

3.3.2. Myerson value

The cooperative game with restrictions in the communication is a triple (N, υ, L), where N is the set of limited players, υ is the characteristic function, L is the set of links, and the pair (N, L) represents a graph or a network. If there exists a path between i and j in the graph, it is said that i and j are connected. And the graph (N, L) is connected if all $i, j \in N$ are connected. As any non-empty set $T \subseteq N$, (T, L_T) is the subgraph of the graph T, and $L_T = \{ij \in L : i, j \in T\}$. If the subgraph (T, L_T) is connected, then the graph T can be said as a connected subset. In the graph (N, L), every maximal connected subset is regarded as a partition, then we denote the set of partitions as N/L. And the L_i represents the partition containing the player i, for any $i \in N$. In 1977, Myerson [41] proposed the solution of the graph-restricted games, Myerson value. The allocation rule supposes that only connected networks can cooperate and that the utility of a non-connected network is considered to be the sum of the utilities of all partitions.

The characteristic function υ^L of the graph-restricted game $\left(N, \upsilon^L\right)$ is denoted as follows:

$$\upsilon^L(S) = \sum_{C \in S/L} \upsilon(C), S \subseteq N. \tag{3}$$

The $\upsilon^L(S)$ is the utility that the subgraph, S can generate under the restrictions.

Then the Myerson value μ, the allocation rule in the communication (N, υ, L), can be denoted below.

$$\begin{aligned} \mu_i(N, \upsilon, L) &= Sh_i\left(N, \upsilon^L\right) \\ &= \sum_{S \subseteq N \backslash \{i\}} \frac{|S|\,(|N| - |S| - 1)!}{|N|!} \left[\upsilon^L(S \cup \{i\}) - \upsilon^L(S)\right], \forall i \in N \end{aligned} \tag{4}$$

where $\mu_i(N, \upsilon, L)$ is the value that the player i can allocate in the communication (N, υ, L), and $Sh_i\left(N, \upsilon^L\right)$ is the Shapley value of the graph-restricted game.

3.3.3. Position value

Based on the Myerson value, Meessen, and Borm [6] introduced a new allocation rule on the communication situation (N, υ, L), the Position value, following the concept of link game. In the link game, the link $l \in L$ is regarded as the player and the focus is on the marginal contribution of the link in the cooperation.

The characteristic function r^υ of the link game (L, r^υ) can be expressed as follows:

$$r^\upsilon(A) = \sum_{C \in N/A} \upsilon(C), A \subseteq L \tag{5}$$

where N/A represents the set of partitions in the graph (N, A) and $\upsilon(C)$ is the utility that the subgraph C can generate.

Then the Position value is another allocation rule on communication situations (N, υ, L), which can be denoted below.

$$\begin{aligned}\pi_i(N, \upsilon, L) &= \sum_{l \in L_i} \frac{1}{2} Sh_l(L, r^\upsilon) \\ &= \sum_{l \in L_i} \frac{1}{2} \sum_{A \subseteq L \setminus \{l\}} \frac{|A|(|L| - |A| - 1)!}{|L|!}\left[r^\upsilon(A \cup \{l\}) - r^\upsilon(A)\right], \forall i \in N\end{aligned} \tag{6}$$

where $\pi_i(N, \upsilon, L)$ is the profit that allocates to the player i in the communication (N, υ, L), $L_i = \{l \in L | i \in l\}$ is the set of links that contain the player i, $Sh_l(L, r^\upsilon)$ is the Shapley value of the link l, $L \setminus \{l\}$ is the set of links that removed the link l, $|L|$ and $|A|$ are the number of links in the network N and A, and $r^\upsilon(A \cup \{l\}) - r^\upsilon(A))$ is the marginal contribution of the link l in the network.

Furthermore, Slikker [47] proved that the Position value is the unique solution of the cooperative game and satisfies component efficiency and balanced link contributions. The component efficiency property means that the value of any alliance of A in N/A is divided into the links of the alliance. The balanced link contribution refers to a situation in which each player equally contributes to the profits of the other.

4. Results and discussion

4.1. Description

Complex product development networks, such as the co-development networks of projects like Boeing 787 and Airbus A350 XWB [19], manifest scale-free network characteristics [29,56] and serve as paradigmatic instances of heterogeneous networks. To elucidate the applicability of the Position value in the allocation of profits within complex product development networks, we draw inspiration from an authentic real-world complex product development endeavor, specifically, the high-speed train traction and braking system project [54], which is visually depicted in Figure 3. We focus our investigation on the local network composed of nodes 3, 6, 15, 16, 18, and 19, as portrayed in Figure 4.

4.2. Profit allocation process with the Position value

In this section, we initially compute the value generated by the coalition of cooperative relationships using equation (5), as depicted in Table 2. Subsequently, we ascertain the profits assigned to each firm according to equation (6), illustrated in Table 3.

Through a systematic analysis of firms' profits, we have summarized the following three propositions.

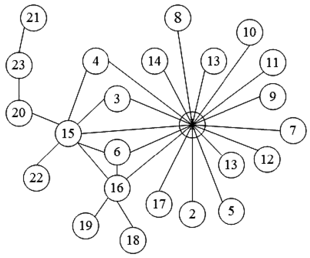

FIGURE 3. The network structure of the 2009BAG12A05 project.

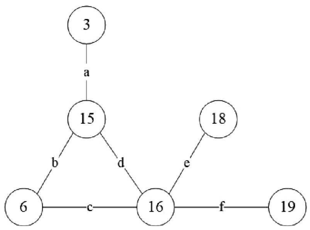

FIGURE 4. The structure of the local network.jpg.

TABLE 2. The value created by each coalition of cooperative relationships under the Position value.

A	$r^{\upsilon}(A)$	A	$r^{\upsilon}(A)$	A	$r^{\upsilon}(A)$
$\{a\}$	$\upsilon(3,15)$	$\{a,b,d\}$	$\upsilon(3,6,15,16)$	$\{a,b,d,f\}$	$\upsilon(3,6,15,16,19)$.
$\{b\}$	$\upsilon(6,15)$	$\{a,c,d\}$	$\upsilon(3,6,15,16)$	$\{a,c,d,e\}$	$\upsilon(3,6,15,16,18)$
$\{c\}$	$\upsilon(6,16)$	$\{a,d,e\}$	$\upsilon(3,15,16,18)$	$\{a,c,d,f\}$	$\upsilon(3,6,15,16,19)$
$\{d\}$	$\upsilon(15,16)$	$\{a,d,f\}$	$\upsilon(3,15,16,19)$	$\{a,d,e,f\}$	$\upsilon(3,15,16,18,19)$
$\{e\}$	$\upsilon(16,18)$	$\{b,c,d\}$	$\upsilon(6,15,16)$	$\{b,c,d,e\}$	$\upsilon(6,15,16,18)$
$\{f\}$	$\upsilon(16,19)$	$\{b,c,e\}$	$\upsilon(6,15,16,18)$	$\{b,c,d,f\}$	$\upsilon(6,15,16,19)$
$\{f\}$	$\upsilon(16,19)$	$\{b,c,e\}$	$\upsilon(6,15,16,18)$	$\{b,c,d,f\}$	$\upsilon(6,15,16,19)$
$\{a,d\}$	$\upsilon(3,15,16)$	$\{b,d,e\}$	$\upsilon(6,15,16,18)$	$\{b,d,e,f\}$	$\upsilon(6,15,16,18,19)$
$\{b,c\}$	$\upsilon(6,15,16)$	$\{b,d,f\}$	$\upsilon(6,15,16,19)$	$\{c,d,e,f\}$	$\upsilon(6,15,16,18,19)$
$\{b,d\}$	$\upsilon(6,15,16)$	$\{c,d,e\}$	$\upsilon(6,15,16,18)$	$\{a,b,c,d,e\}$	$\upsilon(3,6,15,16,18)$
$\{c,d\}$	$\upsilon(6,15,16)$	$\{c,d,f\}$	$\upsilon(6,15,16,19)$	$\{a,b,c,d,f\}$	$\upsilon(3,6,15,16,19)$
$\{c,e\}$	$\upsilon(6,16,18)$	$\{c,e,f\}$	$\upsilon(6,16,18,19)$	$\{a,b,c,e,f\}$	$\upsilon(3,6,15,16,18,19)$
$\{c,f\}$	$\upsilon(6,16,19)$	$\{d,e,f\}$	$\upsilon(15,16,18,19)$	$\{a,b,d,e,f\}$	$\upsilon(3,6,15,16,18,19)$
$\{d,e\}$	$\upsilon(15,16,18)$	$\{a,b,c,d\}$	$\upsilon(3,6,15,16)$	$\{a,c,d,e,f\}$	$\upsilon(3,6,15,16,18,19)$
$\{d,f\}$	$\upsilon(15,16,19)$	$\{a,b,c,e\}$	$\upsilon(3,6,15,16,18)$	$\{b,c,d,e,f\}$	$\upsilon(6,15,16,18,19)$
$\{e,f\}$	$\upsilon(16,18,19)$	$\{a,b,c,f\}$	$\upsilon(3,6,15,16,19)$	$\{a,b,c,d,e,f\}$	$\upsilon(3,6,15,16,18,19)$
$\{a,b,c\}$	$\upsilon(3,6,15,16)$	$\{a,b,d,e\}$	$\upsilon(3,6,15,16,18)$		

Proposition 4.1. *Under the Position value, let the investment values of all firms be equal to I. Then the ranking results of profits is $\bar{\pi}_{16} > \bar{\pi}_{15} > \bar{\pi}_6 > \bar{\pi}_{18} = \bar{\pi}_{19} > \bar{\pi}_3$ when $I > \frac{10\delta}{9r\delta^2+5\gamma}$ and the ranking results of profits is $\bar{\pi}_{16} > \bar{\pi}_{15} > \bar{\pi}_6 > \bar{\pi}_3 > \bar{\pi}_{18} = \bar{\pi}_{19}$ when $I < \frac{10\delta}{9r\delta^2+5\gamma}$.*

The proof is provided in the Appendix.

As shown in Figure 2, the ranking result of degree is $d_{16} > d_{15} > d_6 = d_{18} = d_{19} = d_3$. The ranking of profits corresponds closely to the ranking of degree centrality. The only difference is that firms 3, 18, and 19

TABLE 3. The profit allocation results under the Position value.

Firm	π_i
3	$\frac{1}{240}[r\delta^2 I_3 (60I_3 + 70I_6 + 120I_{15} + 70I_{16} + 46I_{18} + 46I_{19}) + 20r\delta^2 {I_{15}}^2 + 120\delta I_3 + 40\delta I_{15} - 60\gamma {I_3}^2 - 20\gamma {I_{15}}^2]$
6	$\frac{1}{240} r\delta^2 I_3 (40I_6 + 20I_{16} + 12I_{18} + 12I_{19}) + 15r\delta^2 {I_{16}}^2 + 120\delta I_6 + r\delta^2 I_6 (60I_6 + 100I_{15} + 100I_{16} + 40I_{18} + 40I_{19}) + 40\delta I_{15} + 30\delta I_{16} + r\delta^2 I_{15} (20I_{15} + 40I_{16} + 20I_{18} + 20I_{19}) - 60\gamma {I_6}^2 - 20\gamma {I_{15}}^2 - 15\gamma {I_{16}}^2]$
15	$\frac{1}{240}[r\delta^2 I_3 (60I_3 + 110I_6 + 120I_{15} + 110I_{16} + 68I_{18} + 68I_{19}) + r\delta^2 I_6 (30I_6 + 100I_{15} + 40I_{16} + 20I_{18} + 20I_{19}) + r\delta^2 I_{15} (60I_{15} + 100I_{16} + 40I_{18} + 40I_{19}) + 15r\delta^2 {I_{16}}^2 + 120\delta I_3 + 60\delta I_6 + 120\delta I_{15} + 30\delta I_{16} - 60\gamma {I_3}^2 - 30\gamma {I_6}^2 - 60\gamma {I_{15}}^2 - 15\gamma {I_{16}}^2]$
16	$\frac{1}{240}[r\delta^2 I_3 (20I_6 + 40I_{16} + 68I_{18} + 68I_{19}) + 60\delta I_6 + 40\delta I_{15} + r\delta^2 I_6 (30I_6 + 40I_{15} + 100I_{16} + 110I_{18} + 110I_{19}) + 60r\delta^2 (I_{18} + I_{19})^2 + 120\delta I_{16} + 120\delta I_{18} + 120\delta I_{19} - 30\gamma {I_6}^2 - 20\gamma {I_{15}}^2 - 60\gamma {I_{16}}^2 - 60\gamma {I_{18}}^2 - 60\gamma {I_{19}}^2]$
18	$\frac{1}{240}[r\delta^2 I_{18} (46I_3 + 70I_6 + 70I_{15} + 120I_{16} + 60I_{18} + 60I_{19}) + 15r\delta^2 {I_{16}}^2 + 30\delta I_{16} + 120\delta I_{18} - 15\gamma {I_{16}}^2 - 60\gamma {I_{18}}^2]$
19	$\frac{1}{240}[r\delta^2 I_{19} (46I_3 + 70I_6 + 70I_{15} + 120I_{16} + 60I_{18} + 60I_{19}) + 15r\delta^2 {I_{16}}^2 + 30\delta I_{16} + 120\delta I_{19} - 15\gamma {I_{16}}^2 - 60\gamma {I_{19}}^2]$

have the same degree values, but their profit rankings differ. When the level of investment is low, firm 3 has higher profits compared to firms 18 and 19, but when the level of investment is high, firms 18 and 19 have higher profits than firm 3. Through our analysis of the mechanism of profit allocation, we find that when the level of investment is low, firms 15 and 16 have similar profit values. Firm 15 shares its profits with firms 3, 6, and 16, while firm 16 shares its profits with firms 6, 15, 18, and 19. As a result, firms 18 and 19 receive lower profits compared to firm 3. However, when the level of investment is high, firm 16 has higher profits than firm 15, and correspondingly, firms 18 and 19 receive higher profits than firm 3. Therefore, the allocation of profits is closely related to the firm's level of investment and its position in the network.

Proposition 4.2. *The amount of firms' investment has a positive effect on their profits. That is:* $\frac{\partial \pi_i}{\partial I_i} > 0\,(i = 3, 6, 15, 16, 18, 19)$.

The proof is provided in the Appendix.

During the process of research and development collaboration, firms invest resources and share information to jointly create profits, which are then distributed among the participating firms. However, due to the nature of collaboration, there is a risk of opportunistic behavior, where some firms may exploit the cooperative process for their gain. Proposition 4.2 addresses this issue by suggesting that when firms increase their investments in development actively, their profits also increase. Profit growth deters opportunistic behavior, as firms have more to gain from the cooperative process and are less likely to engage in actions that may harm the overall collaboration. In essence, Proposition 4.2 highlights the positive relationship between investment and the mitigation of opportunistic behavior, emphasizing the importance of fostering a cooperative and mutually beneficial environment in development collaborations.

Propositions 4.1 and 4.2 indicate that the profit allocation results based on the Position value incorporate both the contribution of firms' investment and the role of their network positions.

4.3. Comparative analysis

A complex product development network is a disconnected network, the Shapley value is not applicable. Then we calculate the allocated profits with the Myerson value which is shown in Table 4, and compare the results calculated with the Position value.

Similar to Proposition 4.1, we set firms' investments to be I. Then the profit of firms can be expressed below.

$\bar{\mu}_3 = \frac{1}{12}\left(26r\delta^2 I^2 + 12\delta I - 6\gamma I^2\right)$, $\bar{\mu}_6 = \frac{1}{12}\left(30r\delta^2 I^2 + 12\delta I - 6\gamma I^2\right)$

TABLE 4. The profit allocation results under the Myerson value.

Firm	μ_i
3	$\frac{1}{12}\left[r\delta^2 I_3\left(6I_3+4I_6+6I_{15}+4I_{16}+3I_{18}+3I_{19}\right)+12\delta I_3-6\gamma {I_3}^2\right]$
6	$\frac{1}{12}\left[r\delta^2 I_6\left(4I_3+6I_6+6I_{15}+6I_{16}+4I_{18}+4I_{19}\right)+12\delta I_6-6\gamma {I_6}^2\right]$
15	$\frac{1}{12}[r\delta^2 I_3\left(4I_6+6I_{15}+4I_{16}+3I_{18}+3I_{19}\right)+12\delta I_{15}+r\delta^2 I_{15}\left(6I_6+6I_{15}+6I_{16}+4I_{18}+4I_{19}\right)-6\gamma {I_{15}}^2]$
16	$\frac{1}{12}[r\delta^2 I_3\left(4I_{16}+3I_{18}+3I_{19}\right)+r\delta^2 I_6\left(6I_{16}+4I_{18}+4I_{19}\right)+r\delta^2 I_{15}(6I_{16}+4I_{18}+4I_{19})+r\delta^2 I_{16}\left(6I_{16}+6I_{18}+6I_{19}\right)+4r\delta^2 I_{18}I_{19}+12\delta I_{16}-6\gamma {I_{16}}^2]$
18	$\frac{1}{12}\left[r\delta^2 I_{18}\left(3I_3+4I_6+4I_{15}+6I_{16}+6I_{18}+4I_{19}\right)+12\delta I_{18}-6\gamma {I_{18}}^2\right]$
19	$\frac{1}{12}\left[r\delta^2 I_{19}\left(3I_3+4I_6+4I_{15}+6I_{16}+4I_{18}+6I_{19}\right)+12\delta I_{19}-6\gamma {I_{19}}^2\right]$

$\bar{\mu}_{15}=\frac{1}{12}\left(46r\delta^2 I^2+12\delta I-6\gamma I^2\right)$, $\bar{\mu}_{16}=\frac{1}{12}\left(60r\delta^2 I^2+12\delta I-6\gamma I^2\right)$
$\bar{\mu}_{18}=\frac{1}{12}\left(27r\delta^2 I^2+12\delta I-6\gamma I^2\right)$, and $\bar{\mu}_{19}=\frac{1}{12}\left(27r\delta^2 I^2+12\delta I-6\gamma I^2\right)$.

To get the ranking results of profits among firms, we calculate their differences in the following.
$\bar{\mu}_{19}-\bar{\mu}_{18}=0$, $\bar{\mu}_{19}-\bar{\mu}_3=\frac{r\delta^2 I^2}{12}>0$, $\bar{\mu}_6-\bar{\mu}_{19}=\frac{r\delta^2 I^2}{4}>0$, $\bar{\mu}_{15}-\bar{\mu}_6=\frac{4r\delta^2 I^2}{3}>0$, and $\bar{\mu}_{16}-\bar{\mu}_{15}=\frac{7r\delta^2 I^2}{6}>0$.

Then we can get the ranking results of profits is $\bar{\mu}_{16}>\bar{\mu}_{15}>\bar{\mu}_6>\bar{\mu}_{19}=\bar{\mu}_{18}>\bar{\mu}_3$.

To analyze the change of results under different profit allocation rules, we calculate the difference between $\bar{\mu}_i$ and $\bar{\pi}_i\,(i=3,6,15,16,18,19)$.

$\bar{\mu}_3-\bar{\pi}_3=\frac{1}{240}\left[\left(88r\delta^2-40\gamma\right)I^2+80\delta I\right]$,
$\bar{\mu}_6-\bar{\pi}_6=\frac{1}{240}\left[\left(61r\delta^2-25\gamma\right)I^2+50\delta I\right]$,
$\bar{\mu}_{15}-\bar{\pi}_{15}=\frac{1}{240}\left[\left(-81r\delta^2+45\gamma\right)I^2-90\delta I\right]$,

$\bar{\mu}_{16}-\bar{\pi}_{16}=\frac{1}{240}\left[\left(-266r\delta^2+110\gamma\right)I^2-220\delta I\right]$,

$\bar{\mu}_{18}-\bar{\pi}_{18}=\frac{1}{240}\left[\left(99r\delta^2-45\gamma\right)I^2+90\delta I\right]$

and $\bar{\mu}_{19}-\bar{\pi}_{19}=\frac{1}{240}\left[\left(99r\delta^2-45\gamma\right)I^2+90\delta I\right]$.

As Assumption 3.5 shows, $r\delta^2 I_i+\delta-\gamma I_i\geq 0\,(i=3,6,15,16,18,19)$. Then we can get that $\bar{\mu}_{15}<\bar{\pi}_{15}$, $\bar{\mu}_{16}<\bar{\pi}_{16}$, $\bar{\mu}_3>\bar{\pi}_3$, $\bar{\mu}_6>\bar{\pi}_6$, $\bar{\mu}_{18}>\bar{\pi}_{18}$, and $\bar{\mu}_{19}>\bar{\pi}_{19}$.

As demonstrated above, under the Position value, the ranking of profits among firms depends on both the centrality degree and the level of investment. In contrast, under the Myerson value, the ranking of profits is solely based on the centrality degree. This indicates that compared to the Myerson value, the Position value can more objectively reflect the level of contribution of firms. Additionally, we can observe that under the Position value, firms with a higher centrality degree can obtain higher profits than under the Myerson value, which indicates that the Position value recognizes and acknowledges the power of firms in the network. This aligns well with the situation in complex product development networks where a few firms hold the majority of resources. In summary, the Position value is more suitable for complex product development networks and it effectively captures the contribution of firms in terms of both investment and network position.

4.4. Sensitive analysis

This section will systematically analyze the effect of the synergistic coefficient, benefit coefficient, and cost coefficient on firms profits. The relationship between the above parameters and firms profits is summarized in Proposition 4.3.

Proposition 4.3. *The synergistic coefficient r has a positive effect on firms' profits, the benefit coefficient δ has a positive effect on firms' profits, and the cost coefficient γ has a negative effect on firms' profits. That is:*

(1) $\frac{\partial \pi_i}{\partial r} > 0\,(i = 3, 6, 15, 16, 18, 19)$;
(2) $\frac{\partial \pi_i}{\partial \delta} > 0\,(i = 3, 6, 15, 16, 18, 19)$;
(3) $\frac{\partial \pi_i}{\partial \gamma} < 0\,(i = 3, 6, 15, 16, 18, 19)$.

The proof is provided in the Appendix.

The strengthening of the synergistic coefficient r results in greater value creation of the network and consequently higher profits for firms. The rationale behind this conclusion lies in the notion that collaboration and synergy among firms allow for the optimization of research processes, faster development cycles, reduced costs, and better utilization of available resources. This collaborative environment fosters innovation, reduces duplication of efforts, and promotes the exchange of ideas, increasing the value generated by the collaborative efforts of these firms, ultimately leading to higher profits that can be distributed among them. It is important to note that this conclusion emphasizes the positive relationship between synergy and profit allocation in the context of firms. It highlights the potential benefits of fostering collaboration and cooperative relationships within the research and development stage, as it can lead to mutually advantageous outcomes for the participating firms.

As the benefit coefficient δ for firms increases, the amount of profit that can be allocated to these firms also increases. The benefit coefficient reflects the degree to which a firm's efforts drive the success and value creation of the development network. The rationale behind this conclusion lies in the notion that the coefficient of benefit serves as a measure of the value or significance of each firm's contributions within a development network. When a firm's benefit coefficient increases, it signifies that its contributions, whether in terms of expertise or resources, hold greater weight or importance in the overall network performance. The increase in the allocated profit is a recognition of the enhanced value or influence of the firm within the development network. It signifies that the firm's efforts are considered more instrumental in achieving the project's objectives, leading to a fairer distribution of the generated profits based on their respective contributions. It is important to note that this conclusion emphasizes the significance of the benefit coefficient as a determining factor in profit allocation for firms. By acknowledging and rewarding firms with higher coefficients, it aims to incentivize and promote the continuous improvement of contributions, fostering a collaborative environment that encourages innovative and impactful outcomes.

As the cost coefficient γ of a firm increases, the profit allocated to the firm diminishes. The fundamental principle underlying this conclusion is that the cost coefficient reflects the relative costs or expenses incurred by a firm in the development activity. A higher cost coefficient indicates that the firm's expenditures in the research and development process are relatively high. This may be due to lower resource utilization efficiency, suboptimal technological utilization, or complex requirements, among other reasons. Consequently, the proportion of profit allocated to these firms decreases, reflecting the higher costs incurred during the development process. The negative correlation between the cost coefficient and allocated profit encourages firms to maximize cost-effectiveness in their development activities. This impels firms to seek ways to reduce costs while maintaining the desired level of product quality.

4.5. Numerical analysis

In this section, we engage in parameter adjustments and observe simulation processes to discuss the impact of key parameters in this study on the profit allocation to firms, as well as the differences between the Myerson value and Position value allocation rules.

We assume that the investments of firms are $I_3 = 5$, $I_6 = 2$, $I_{15} = 2$, $I_{16} = 5$, $I_{18} = 4$, $I_{19} = 5$, the synergistic coefficient is $r = 0.24$, the benefit coefficient is $\delta = 0.8$, and the cost coefficient is $\gamma = 0.2$. We can see that the parameters satisfy the conditions $r\delta^2 \leq \gamma$ and $r\delta^2\left(\sum_{i\in N} I_i\right) + \delta - \gamma I_i \geq 0$. Based on the profit allocation results under the Position value in Table 4, we can get that $\pi_3 = 5.90$, $\pi_6 = 5.65$, $\pi_{15} = 11.43$, $\pi_{16} = 15.98$, $\pi_{18} = 4.83$, and $\pi_{19} = 5.74$. And $\sum_{i\in N} \pi_i = \Pi = 49.53$, it indicates that the profit allocation rule satisfies validity.

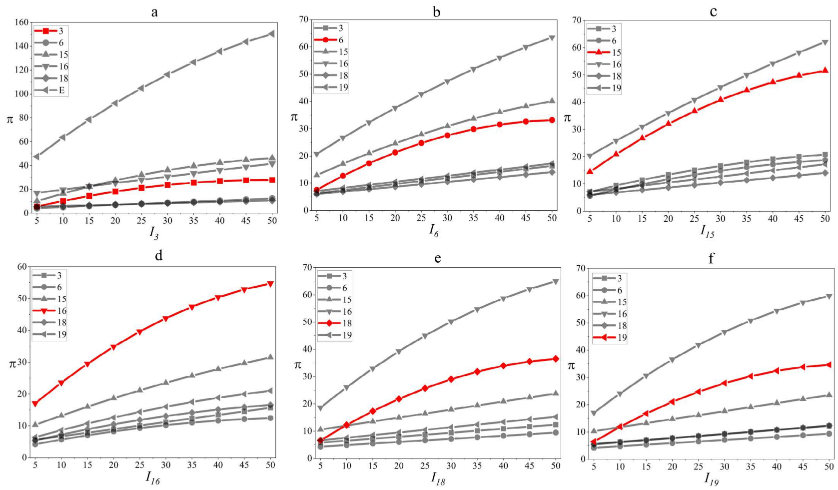

FIGURE 5. The relationship between the investments and the allocated profits of firms under the Position value.

The ranking result of firms' profits is $\pi_{16} > \pi_{15} > \pi_3 > \pi_{19} > \pi_6 > \pi_{18}$. The degree of firms in the network are $d_3 = 1$, $d_6 = 2$, $d_{15} = 3$, $d_{16} = 4$, $d_{18} = 1$, and $d_{19} = 1$. By comparing the investment values, degree centrality values, and achievable profit values of firms, several conclusions can be made:

(1) When the degree centrality values are the same, firms with higher investment values can achieve higher profit values. For example, $d_{18} = d_{19}$ and $I_{18} < I_{19}$, then $\pi_{18} < \pi_{19}$.
(2) When the investment values are equal, firms with higher centrality degree values obtain higher profit values. For example, $\pi_{18} < \pi_{19}$ and I_6=I_{15}, then $\pi_6 < \pi_{15}$.
(3) Even if a firm has a relatively low centrality degree, it can still achieve substantial profits if its investment value is sufficiently high. For example, $d_3 < d_6$ and $I_3 > I_6$, then $\pi_3 > \pi_6$.
(4) When the investment value of a firm is low, being in an advantageous position within the network can still result in higher profits. For example, $d_3 < d_{15}$ and $I_3 > I_{15}$, then $\pi_3 < \pi_{15}$.

These comparisons indicate that the profit allocated to a firm is not solely determined by its investment value or network position. Instead, it is influenced by the combination of these two factors. The relationship between allocated profit, investment value, and network position is a complex and interdependent one.

Next, we will observe the changes in profit allocation results when we vary the investment value, synergistic coefficient, benefit coefficient, and cost coefficient of the firms separately

4.5.1. The effect of the investment on profit allocation

To facilitate analysis the impact of individual firm's investment on the distribution of profits, we set $r = 0.24$, $\delta = 0.8$, and $\gamma = 0.2$. We also assume that the investment values of all other firms remain constant, with $I_3 = 5$, $I_6 = 2$, $I_{15} = 2$, $I_{16} = 5$, $I_{18} = 4$, and $I_{19} = 5$. The simulated environment satisfies the conditions of Assumptions 3.4 and 3.5. The final simulation results are depicted in Figure 5. From Figures 5a to f, we sequentially modify the investment values of firms 3, 6, 15, 16, 18, and 19, and observe the changes in the profit allocated to firms. As the investment values of the firm increased, its allocated profit values also increased. This observation aligns with the conclusion drawn in Proposition 4.2. This is due to the parameter setting in Assumption 3.4. Additionally, we notice that the ranking of firms' profit values is not constant but varies based on changes in their investment values. This indicates that the Position value considers a combination of investment values and network position when distributing profits.

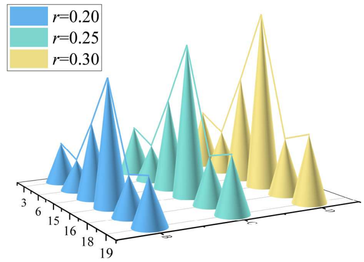

FIGURE 6. The relationship between the synergistic coefficient r and the allocated profits of firms under the Position value.

4.5.2. The effect of the synergistic coefficient on profit allocation

To set $\delta = 0.8$, $\gamma = 0.2$, $I_3 = 5$, $I_6 = 2$, $I_{15} = 2$, $I_{16} = 5$, $I_{18} = 4$, and $I_{19} = 5$, we study the allocation of network profits by adjusting the synergistic coefficient r, the results are shown in Figure 6. As the synergistic coefficient r increases, the profits of firms increase significantly. Specifically, when the synergistic coefficient r increases from 0.2 to 0.3, which satisfies conditions in Assumptions 3.4 and 3.5, the profit of firm 3 increases from 4.84 to 6.79, the profit of firm 6 increases from 3.66 to 5.00, the profit of firm 15 increases from 8.89 to 12.42, the profit of firm 16 increases from 14.71 to 20.67, the profit of firm 18 increases from 4.71 to 6.57, and the profit of firm 19 increases from 5.54 to 7.84. At the same time, we can see that the ranking of the profits to the participating firms remains $\pi_{16} > \pi_{15} > \pi_{19} > \pi_3 > \pi_{18} > \pi_6$, regardless of the synergistic coefficient r. The result is consistent with Proposition 4.3(1).

As the synergistic coefficient r increases, the profits of the network increase exponentially with the same number of resources invested, so the overall profits of the network increase significantly. In the case of a synergistic coefficient without differentiating between firms, the profit that can be allocated to each firm also increases exponentially.

4.5.3. The effect of the benefit coefficient on profit allocation results

To set $r = 0.24$, $\gamma = 0.2$, $I_3 = 5$, $I_6 = 2$, $I_{15} = 2$, $I_{16} = 5$, $I_{18} = 4$, and $I_{19} = 5$, we study the allocation of network profits by adjusting the synergistic coefficient δ, the results are shown in Figure 7. As the synergistic coefficient δ increases, the profits of firms increase significantly. Specifically, when the synergistic coefficient δ increases from 0.6 to 0.8, which satisfies conditions in Assumptions 3.4 and 3.5, the profit of firm 3 increases from 3.01 to 6.46, the profit of firm 6 increases from 2.40 to 5.45, the profit of firm 15 increases from 5.67 to 17.10, the profit of firm 16 increases from 9.28 to 10.30, the profit of firm 18 increases from 2.97 to 4.20, and the profit of firm 19 increases from 3.42 to 5.62. At the same time, we can see that the ranking of the profits to the participating firms remains $\pi_{16} > \pi_{15} > \pi_{19} > \pi_3 > \pi_{18} > \pi_6$, regardless of the synergistic coefficient δ. The result is consistent with Proposition 4.3(2).

As the benefit coefficient δ increases, the proportion of investment that can be converted into profit increases as well, leading to an increase in network profit. When the profit coefficient is not differentiated among enterprises, the allocated profit to each enterprise also increases proportionally.

4.5.4. The effect of the cost coefficient on profit allocation

To set $r = 0.24$, $\delta = 0.8$, $I_3 = 5$, $I_6 = 2$, $I_{15} = 2$, $I_{16} = 5$, $I_{18} = 4$, and $I_{19} = 5$, we study the allocation of network profits by adjusting the investment cost coefficients γ, and the results are shown in Figure 8. With the increase of the cost coefficient γ, the profits of firms all decrease proportionally. Specifically, when the cost coefficient γincreases from 0.2 to 0.3, which satisfies conditions in Assumptions 3.4 and 3.5, the profit of firm 3 decreases from 5.62 to 4.96, the profit of firm 3 decreases from 4.20 to 3.91, the profit of firm 15 decreases from 10.30 to 9.37, the profit of firm 16 decreases from 17.10 to 15.26, the profit of firm 18 decreases from 5.45 to

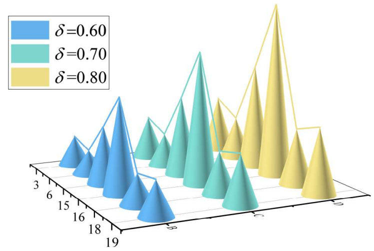

FIGURE 7. The relationship between the benefit coefficient δ and the allocated profits of firms under the Position value.

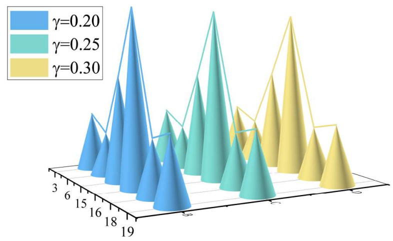

FIGURE 8. The relationship between the benefit coefficient γ and the allocated profits of firms under the Position value.

4.90, and the profit of firm 19 decreases from 6.46 to 5.68. At the same time, we can see that the ranking of the profits to the participating firms does not change as the cost coefficient changes, *i.e.*, the ranking results of the participating firms remain $\pi_{16} > \pi_{15} > \pi_{19} > \pi_3 > \pi_{18} > \pi_6$. The result is consistent with Proposition 4.3(3).

With the increase of the cost coefficient γ, the cost of collaborative development has increased to a certain extent, and the cost of the development network has increased with the same output, so the overall profit of the network has decreased. Since the investment cost coefficient of each firm is equal, the profit that each firm can allocate to the network also decreases proportionally.

4.5.5. Comparison with the Myerson value

As shown in Figure 9, firms 15 and 16, which have higher degrees, can obtain higher profits under the Position value. This is consistent with the conclusion in Section 4.3. Complex product development networks often exhibit scale-free network characteristics, where a few core enterprises control the majority of resources, and other enterprises rely on these core enterprises for collaboration. Core enterprises typically play the role of intermediaries, facilitating cooperation and information flow among different enterprises. Thus, in addition to their explicit contributions such as input, core enterprises also have implicit contributions, such as their role as intermediaries and connectors. Allocating profits based on the Position value can better reflect the power and contributions of enterprises within the network. Therefore, the Position value is more suitable for complex product development networks as it can accurately evaluate the contributions of enterprises and determine their deserved share of profits.

4.6. Discussion

Considering the restricted communication among enterprises in complex product development networks, this paper adopts the Position value method as its profit allocation rule. In this section, taking a local network of the "2009BAG12A05" project as an example, we calculate the profit functions for each enterprise under the Position value method through mathematical derivation. The results indicate that the profit values allocated

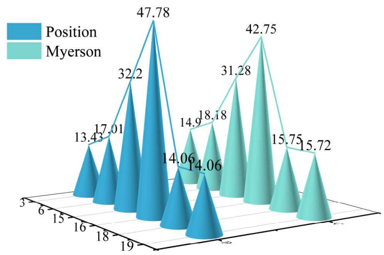

FIGURE 9. The allocation results under the Position value and Myerson value when $I_i = 4$, $r = 0.24$, $\delta = 0.8$, and $\gamma = 0.2$.

by the Position value are directly proportional to the centrality degree and investment of the enterprises. To demonstrate the superiority of this method, we compare the profit distribution results between the Position value and the Myerson value methods. The outcomes reveal that enterprises with higher centrality degrees under the Position value method attain higher profit values, highlighting the importance of enterprises as intermediary roles more distinctly. Furthermore, we conduct sensitivity analysis on the investment, synergistic coefficient, benefit coefficient, and cost coefficient to dissect the impact of key parameters on enterprises' profit values. Jiang *et al.* [20] considered the profit generation process during the collaboration among enterprises but did not account for the restricted communication among them–a scientific issue overlooked in many studies on profit allocation in collaborative networks. On the other hand, Lv *et al.* [38], Li *et al.* [44], Suh and Yoon [48] utilized the solution of graph cooperative games–the Myerson value–as the profit allocation rule, considering restricted communication among enterprises, but they did not analyze the profit generation process. Prior research has not comprehensively studied the aforementioned aspects and failed to recognize the superiority of the solution of graph cooperative games–the Position value–as a profit allocation rule. Therefore, this research attempts to fill the research gap, demonstrating the superiority of the Position value as a profit allocation rule in complex product development networks and providing practical measures to enhance enterprise profit values.

5. Managerial insights and practical implications

This study suggests that the Position value, a solution of graph cooperative games, is effective in allocating profits within complex product development networks by considering the network position and marginal contributions of enterprises. Given the scale-free characteristic of complex product development networks, the centrality degree of enterprises reflects their roles as bridges and intermediaries in collaboration. Compared with the Myerson value, the Position value recognizes and acknowledges the role of brokers. Consequently, the Position value serves as a more suitable rule for profit allocation in complex product development networks.

For managers, ensuring the stability of the network is a top priority. A fair and appropriate benefit distribution rule is critical to network stability. We recommend that managers adopt the Position value as the foundation for benefit distribution. This is because the Position value not only allocates earnings based on the firm's marginal contribution but also treats the firm's contribution as an intermediary. It is more applicable to unconnected networks such as complex product development networks than other cooperative game solutions.

Furthermore, during the research and development process, managers should establish connections with a greater number of enterprises, enhancing their intermediary roles and consequently increasing their marginal contributions. Simultaneously, the findings reveal that the synergistic coefficient has a positive impact on firms' profits, while the benefit and cost coefficients have a positive and negative impact on firms' profits, respectively. Therefore, efforts should be directed at improving communication and coordination with partners during the development phase to enhance the synergistic coefficient among enterprises. Additionally, a focus on reinforcing technological innovation and capabilities, along with optimizing resource allocation, will boost the benefit coefficient and minimize the cost coefficient.

6. Conclusions and outlook

6.1. Conclusions

Existing studies have often overlooked the process of profit generation and assumed that the profits of individual enterprises or the entire network are known in advance. Additionally, many of these studies have not taken into account the structural constraints among enterprises during the profit allocation process. To address these gaps, this paper constructs a profit function for complex product development networks to reveal the dynamics and mechanism of profit generation. Subsequently, the Position value, which is the solution of graph cooperative game theory, is employed for profit allocation in complex product development networks. Through theoretical derivation and numerical analysis, the paper draws conclusions regarding profit allocation and conducts sensitivity analysis on the impact of various factors on the allocation results. Furthermore, a comparative analysis with the Myerson value is conducted to further validate the applicability and superiority of the Position value in complex product development networks. Our work points to several meaningful findings as follows:

1. The Position value takes into account both the investment made by firms and their network position, resulting in a fair and reflective distribution of profits among the participating firms. Compared with the Myerson value, the Position value tends to allocate higher profits to firms located at the center of the network. This aligns well with the characteristics of complex product development networks, where a few hub firms possess the majority of the resources. Therefore, the Position value is indeed more suitable as a profit allocation rule for complex product development networks.
2. When firms increase their investments in research and development, the overall profits of the development network tend to increase and the focus firm receives a larger portion of the profits distributed. By investing more resources in research and development activities, firms demonstrate their commitment and dedication to the collaborative process. This not only contributes to the overall success of the network but also enhances the trust and cooperation among participating firms. Therefore, it is crucial for firms to allocate a significant portion of their resources to research and development if they aim to maximize both the general profits of the development network and their share of those profits.
3. When the synergistic coefficient among firms increases, the overall profit of the development network also rises, leading to an increase in the individual profits of each firm. Strengthening the synergistic coefficient implies enhanced resource sharing, information exchange, and collaboration, which contribute to improving the cooperative and coordination capabilities of the enterprises. This, in turn, facilitates effective utilization and optimization of resources, resulting in an overall boost in the network's profit and subsequently increasing the individual profits of the participating firms. Therefore, during the research and development stage, firms can foster a collaborative environment by enhancing communication and coordination, establishing shared objectives, and implementing incentivization mechanisms to enhance the synergistic coefficient among enterprises.
4. The benefit coefficient of research and development has a positive impact on both the overall profit of the network and the profits allocated to individual firms. The benefit coefficient reflects the value that a firm can create about the resources it invests in development activities. A high benefit coefficient means that a firm can achieve higher innovation output or technological advancement with relatively fewer investments. Therefore, firms can enhance their benefit coefficient and maximize the overall profit of the network and the profits allocated to each participating firm by strengthening technological innovation and improving their capabilities.
5. When the cost coefficient of research and development increases, the overall profit of the network decreases, leading to a decrease in the allocated profits for each firm. A heightened cost coefficient leads to a comparatively suboptimal efficiency in the utilization of resources and technology in endeavors, accompanied by escalated costs associated with such activities. Therefore, during the research and development process, firms can mitigate the burden of investments and reduce the cost coefficient by optimizing resource allocation,

adopting more efficient development processes, and strengthening cooperation and sharing with partners. This will maximize the overall profit of the network and the profits allocated to each participating firm.

6.2. Limitations and future research directions

This study addresses the profit allocation issue in complex product development networks with the Position value. In further study, there are two potential directions for development.

1. The model in this paper assumes that firms are homogeneous in terms of technology, resources, and capabilities. However, individuals have differences in knowledge-sharing and risk-undertaking, etc. The Position value can be optimized through incorporating firms' investments, knowledge sharing degree, and risk-resistant ability in future studies to design a more reasonable profit allocation mechanism.
2. This study does not consider the information uncertainty in the profit allocation process, but the player may lack full information about the other players' payoffs (or even their own) in real game situations. Studying the Position value with uncertain information deserves further investigation.

Appendix A. Proof of Theorem 3.6

(1) $\upsilon(S) = \frac{1}{2}r\left(\delta\sum_{i\in S} I_i\right)^2 + \delta\sum_{i\in S} I_i - \frac{1}{2}\gamma\sum_{i\in S} I_i^2$, $\frac{\partial \upsilon(S)}{\partial I_i} = r\delta^2\left(\sum_{i\in S} I_i\right) + \delta - \gamma I_i$. As Assumption 3.5 shows, $r\delta^2 I_i + \delta - \frac{1}{2}\gamma I_i \geq 0$. This indicates there is a positive correlation between $\upsilon(S)$ and I_i. Moreover, $\upsilon(\varnothing) = 0$. It can be obtained $\upsilon(S) \geq 0$. Then, the profit function satisfies non-negativity is proved.

(2) Based on the profit function,

$$\upsilon(g|_{S\cup T}) = \frac{1}{2}r\delta^2\left(\sum_{i\in S\cup T} I_i\right)^2 + \delta\sum_{i\in S\cup T} I_i - \frac{1}{2}\gamma\sum_{i\in S\cup T} {I_i}^2$$

and

$$\begin{aligned}
&\upsilon(g|_S) + \upsilon(g|_T)\\
&= \frac{1}{2}r\left(\delta\sum_{i\in S} I_i\right)^2 + \delta\sum_{i\in S} I_i - \frac{1}{2}\gamma\sum_{i\in S} {I_i}^2 + \frac{1}{2}r\left(\delta\sum_{i\in T} I_i\right)^2 + \delta\sum_{i\in T} I_i - \frac{1}{2}\gamma\sum_{i\in T} {I_i}^2\\
&= \frac{1}{2}r\delta^2\left[\left(\sum_{i\in S} I_i\right)^2 + \left(\sum_{i\in T} I_i\right)^2\right] + \delta\sum_{i\in S\cup T} I_i - \frac{1}{2}\gamma\sum_{i\in S\cup T} {I_i}^2
\end{aligned}$$

Then,

$$\begin{aligned}
\upsilon(g|_{S\cup T}) - \upsilon(g|_S) - \upsilon(g|_T) &= \frac{1}{2}r\delta^2\left(\sum_{i\in S\cup T} I_i\right)^2 - \frac{1}{2}r\delta^2\left[\left(\sum_{i\in S} I_i\right)^2 + \left(\sum_{i\in T} I_i\right)^2\right]\\
&= r\delta^2\left(\sum_{i\in S} I_i\cdot\sum_{i\in T} I_i\right) \geq 0.
\end{aligned}$$

The profit function satisfies super-additivity is proved.

Thus, Theorem 3.6 is proved.

Appendix B. Proof of Proposition 4.1

We set firms investment to be I. Then the profits of firms $\bar{\pi}_i$ $(i = 3, 6, 15, 16, 18, 19)$ can be expressed below.
$\bar{\pi}_3 = \frac{1}{240}\left(432r\delta^2 I^2 + 160\delta I - 80\gamma I^2\right)$, $\bar{\pi}_6 = \frac{1}{240}\left(539r\delta^2 I^2 + 190\delta I - 95\gamma I^2\right)$,
$\bar{\pi}_{15} = \frac{1}{240}\left(1001r\delta^2 I^2 + 330\delta I - 165\gamma I^2\right)$, $\bar{\pi}_{16} = \frac{1}{240}\left(1466r\delta^2 I^2 + 460\delta I - 230\gamma I^2\right)$,
$\bar{\pi}_{18} = \frac{1}{240}\left(441r\delta^2 I^2 + 150\delta I - 75\gamma I^2\right)$, and $\bar{\pi}_{19} = \frac{1}{240}\left(441r\delta^2 I^2 + 150\delta I - 75\gamma I^2\right)$.

To get the ranking results of profits among firms, we calculate their differences in the following. $\bar{\pi}_{19} - \bar{\pi}_{18} = 0$, $\bar{\pi}_{19} - \bar{\pi}_3 = \frac{1}{240}\left(9r\delta^2 I^2 + 5\gamma I^2 - 10\delta I\right)$,$\bar{\pi}_6 - \bar{\pi}_{19} = \frac{1}{240}\left(98r\delta^2 I^2 - 20\gamma I^2 + 40\delta I\right)$, $\bar{\pi}_{15} - \bar{\pi}_6 = \frac{1}{240}\left(462r\delta^2 I^2 - 70\gamma I^2 + 140\delta I\right)$, and $\bar{\pi}_{16} - \bar{\pi}_{15} = \frac{1}{240}\left(465r\delta^2 I^2 - 65\gamma I^2 + 130\delta I\right)$.

When $I > \frac{10\delta}{9r\delta^2+5\gamma}$, $\bar{\pi}_{19} > \bar{\pi}_3$; when $I < \frac{10\delta}{9r\delta^2+5\gamma}$, $\bar{\pi}_{19} < \bar{\pi}_3$.

Then we can get the ranking results of profits is $\bar{\pi}_{16} > \bar{\pi}_{15} > \bar{\pi}_6 > \bar{\pi}_{18} = \bar{\pi}_{19} > \bar{\pi}_3$ when $I > \frac{10\delta}{9r\delta^2+5\gamma}$ and the ranking results of profits is $\bar{\pi}_{16} > \bar{\pi}_{15} > \bar{\pi}_6 > \bar{\pi}_3 > \bar{\pi}_{18} = \bar{\pi}_{19}$ when $I < \frac{10\delta}{9r\delta^2+5\gamma}$.

Thus, Proposition 4.1 is proved.

Appendix C. Proof of Proposition 4.2

To prove this proposition, we use $\pi_i\,(i = 3, 6, 15, 16, 18, 19)$ to calculate the first order derivative of $I_i\,(i = 3, 6, 15, 16, 18, 19)$, respectively.

$$\begin{aligned}
\frac{\partial \pi_3}{\partial I_3} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_3 + r\delta^2\left(70I_6 + 120I_{15} + 70I_{16} + 46I_{18} + 46I_{19}\right) + 120\delta\right],\\
\frac{\partial \pi_6}{\partial I_6} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_6 + r\delta^2\left(40I_3 + 100I_{15} + 100I_{16} + 40I_{18} + 40I_{19}\right) + 120\delta\right],\\
\frac{\partial \pi_{15}}{\partial I_{15}} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_{15} + r\delta^2\left(120I_3 + 100I_6 + 100I_{16} + 40I_{18} + 40I_{19}\right) + 120\delta\right],\\
\frac{\partial \pi_{16}}{\partial I_{16}} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_{16} + r\delta^2\left(60I_3 + 80I_6 + 100I_{15} + 120I_{18} + 120I_{19}\right) + 120\delta\right],\\
\frac{\partial \pi_{18}}{\partial I_{18}} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_{18} + r\delta^2\left(46I_3 + 70I_6 + 70I_{15} + 120I_{16} + 90I_{19}\right) + 120\delta\right],\\
\frac{\partial \pi_{19}}{\partial I_{19}} &= \frac{1}{240}\left[120\left(r\delta^2-\gamma\right)I_{19} + r\delta^2\left(46I_3 + 70I_6 + 70I_{15} + 120I_{16} + 90I_{18}\right) + 120\delta\right].
\end{aligned}$$

As shown in Assumption 3.5, $r\delta^2 I_i + \delta - \gamma I_i \geq 0\,(i = 3, 6, 15, 16, 18, 19)$. Then we can find that $\frac{\partial \pi_i}{\partial I_i} > 0\,(i = 3, 6, 15, 16, 18, 19)$.

Thus, Proposition 4.2 is proved.

Appendix D. Proof of Proposition 4.3

We use $\pi_i\,(i = 3, 6, 15, 16, 18, 19)$ to calculate the first order derivative of r, δ, and γ, respectively.

(1) Using $\pi_i\,(i = 3, 6, 15, 16, 18, 19)$ to calculate the first order derivative of r.

$$\begin{aligned}
\frac{\partial \pi_3}{\partial r} &= \frac{1}{240}\Big[\delta^2 I_3\left(60I_3 + 70I_6 + 120I_{15} + 70I_{16} + 46I_{18} + 46I_{19}\right) + 20\delta^2 {I_{15}}^2\Big] > 0,\\
\frac{\partial \pi_6}{\partial r} &= \frac{1}{240}\Big[\delta^2 I_3\left(40I_6 + 20I_{16} + 12I_{18} + 12I_{19}\right) +\\
&\quad \delta^2 I_6\left(60I_6 + 100I_{15} + 100I_{16} + 40I_{18} + 40I_{19}\right) +\\
&\quad \delta^2 I_{15}\left(20I_{15} + 40I_{16} + 20I_{18} + 20I_{19}\right) + 15\delta^2 {I_{16}}^2\Big] > 0,\\
\frac{\partial \pi_{15}}{\partial r} &= \frac{1}{240}\Big[\delta^2 I_3\left(60I_3 + 110I_6 + 120I_{15} + 110I_{16} + 68I_{18} + 68I_{19}\right) +\\
&\quad \delta^2 I_6\left(30I_6 + 100I_{15} + 40I_{16} + 20I_{18} + 20I_{19}\right) +\\
&\quad \delta^2 I_{15}\left(60I_{15} + 100I_{16} + 40I_{18} + 40I_{19}\right) + 15\delta^2 {I_{16}}^2\Big] > 0,\\
\frac{\partial \pi_{16}}{\partial r} &= \frac{1}{240}\Big[\delta^2 I_3\left(20I_6 + 40I_{16} + 68I_{18} + 68I_{19}\right) +\\
&\quad \delta^2 I_6\left(30I_6 + 40I_{15} + 100I_{16} + 110I_{18} + 110I_{19}\right) +\\
&\quad \delta^2 I_{15}\left(20I_{15} + 100I_{16} + 110I_{18} + 110I_{19}\right) +
\end{aligned}$$

$$\delta^2 I_{16}\left(60I_{16}+120I_{18}+120I_{19}\right)+60\delta^2(I_{18}+I_{19})^2\Big]>0,$$

$$\frac{\partial\pi_{18}}{\partial r}=\frac{1}{240}\Big[\delta^2 I_{18}\left(46I_3+70I_6+70I_{15}+120I_{16}+60I_{18}+60I_{19}\right)+15\delta^2{I_{16}}^2\Big]>0,$$

$$\frac{\partial\pi_{19}}{\partial r}=\frac{1}{240}\Big[\delta^2 I_{19}\left(46I_3+70I_6+70I_{15}+120I_{16}+60I_{18}+60I_{19}\right)+15\delta^2{I_{16}}^2\Big]>0.$$

(2) Using $\pi_i\,(i=3,6,15,16,18,19)$ to calculate the first order derivative of δ

$$\frac{\partial\pi_3}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_3\left(60I_3+70I_6+120I_{15}+70I_{16}+46I_{18}+46I_{19}\right)$$
$$+40r\delta{I_{15}}^2+120I_3+40I_{15}\Big]>0,$$

$$\frac{\partial\pi_6}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_3\left(40I_6+20I_{16}+12I_{18}+12I_{19}\right)+$$
$$+2r\delta I_6\left(60I_6+100I_{15}+100I_{16}+40I_{18}+40I_{19}\right)+30r\delta{I_{16}}^2+120I_6$$
$$+2r\delta I_{15}\left(20I_{15}+40I_{16}+20I_{18}+20I_{19}\right)+40I_{15}+30I_{16}\Big]>0,$$

$$\frac{\partial\pi_{15}}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_3\left(60I_3+110I_6+120I_{15}+110I_{16}+68I_{18}+68I_{19}\right)+120I_3+60I_6$$
$$+2r\delta I_6\left(30I_6+100I_{15}+40I_{16}+20I_{18}+20I_{19}\right)+30r\delta{I_{16}}^2$$
$$+2r\delta I_{15}\left(60I_{15}+100I_{16}+40I_{18}+40I_{19}\right)+120I_{15}+30I_{16}\Big]>0,$$

$$\frac{\partial\pi_{16}}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_3\left(20I_6+40I_{16}+68I_{18}+68I_{19}\right)+120r\delta(I_{18}+I_{19})^2$$
$$+2r\delta I_6\left(30I_6+40I_{15}+100I_{16}+110I_{18}+110I_{19}\right)+60I_6+40I_{15}$$
$$+2r\delta I_{15}\left(20I_{15}+100I_{16}+110I_{18}+110I_{19}\right)+120I_{16}$$
$$+2r\delta I_{16}\left(60I_{16}+120I_{18}+120I_{19}\right)+120I_{18}+120I_{19}\Big]>0,$$

$$\frac{\partial\pi_{18}}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_{18}\left(46I_3+70I_6+70I_{15}+120I_{16}+60I_{18}+60I_{19}\right)$$
$$+30r\delta{I_{16}}^2+30I_{16}+120I_{18}\Big]>0,$$

$$\frac{\partial\pi_{19}}{\partial\delta}=\frac{1}{240}\Big[2r\delta I_{19}\left(46I_3+70I_6+70I_{15}+120I_{16}+60I_{18}+60I_{19}\right)$$
$$+30r\delta{I_{16}}^2+30I_{16}+120I_{19}\Big]>0.$$

(3) Using $\pi_i\,(i=3,6,15,16,18,19)$ to calculate the first order derivative of γ.

$$\frac{\partial\pi_3}{\partial\gamma}=\frac{1}{12}\left(-3{I_3}^2-{I_{15}}^2\right)<0,\ \frac{\partial\pi_6}{\partial\gamma}=\frac{1}{48}\left(-12{I_6}^2-4{I_{15}}^2-3{I_{16}}^2\right)<0,$$

$$\frac{\partial\pi_{15}}{\partial\gamma}=\frac{1}{16}\left(-4{I_3}^2-2{I_6}^2-4{I_{15}}^2-{I_{16}}^2\right)<0,$$

$$\frac{\partial\pi_{16}}{\partial\gamma}=\frac{1}{24}\left(-3{I_6}^2-2{I_{15}}^2-6{I_{16}}^2-6{I_{18}}^2-6{I_{19}}^2\right)<0,$$

$$\frac{\partial\pi_{18}}{\partial\gamma}=\frac{1}{16}\left(-{I_{16}}^2-4{I_{18}}^2\right)<0,$$

$$\frac{\partial\pi_{19}}{\partial\gamma}=\frac{1}{16}\left(-{I_{16}}^2-4{I_{19}}^2\right)<0.$$

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