RAIRO-Oper. Res. 58 (2024) 1–18
https://doi.org/10.1051/ro/2023179

RAIRO Operations Research
www.rairo-ro.org

COOPERATIVE ADVERTISING AND COORDINATION IN A SUPPLY CHAIN: THE ROLE OF NASH BARGAINING FAIRNESS CONCERNS

Xianjin Du*, Shan Jiang, Shaokun Tao* and Shulei Wang

Abstract. This study constructs a game-theoretic model of cooperative advertising in a supply chain with Nash bargaining solutions as the fairness reference point. We use a square root response function to describe the saturation effect of advertising. We find that the retailer's Nash bargaining fairness concerns (NBFC) improve the local advertising investment even more than the level of the centralized case. The effect of NBFC on the retailer's profit is inverted U-shaped, rising first and then falling, and vice versa for the manufacturer. As the fairness-concerned coefficient increases from small to large, the efficiency of the supply chain changes from improvement to decline. Moreover, we find that a two-way subsidy mechanism in cooperative advertising still works on coordination although the retailer possesses NBFC. The study offers practical management insights into the operational strategies of supply chain members.

Mathematics Subject Classification. 91-10, 91A40, 90B60.

Received August 20, 2023. Accepted November 19, 2023.

1. Introduction

Within the enterprise business operation, many products need to be advertised jointly by manufacturers and retailers to enhance the brand and achieve sales [21, 46]. Many well-known companies, such as Apple, invest heavily in Internet and television (TV) advertising and gift-giving campaigns at retail terminals within the territory [58]. As retailers are familiar with the local market and advertising resources, manufacturers widely adopt cooperative advertising to increase retailers' incentive to advertise their products [8, 15, 61]. Examples of such collaborations are everywhere: billboards to catalogs, TV commercials to digital platforms. In the US, the expenditures on cooperative advertising in 2015 and 2018 are very significant commercial operating expenses, with a sizeable amount of approximately \$50 billion and \$70 billion, respectively[1]. Therefore, making cooperative advertising investments highly efficient is an important management decision for supply chain members.

A cooperative advertising program usually has two stakeholders: the manufacturer (*he*) and the retailer (*she*). The manufacturer invests in national advertisement and offers the retailer a part of her local advertising investment, called a subsidy rate or advertising allowance [35, 45]. As an efficient cost-sharing mechanism, the subsidy program affects stakeholders' advertising investments and incomes. Given the program's importance in business operations, game theory has been employed to examine its decision process [18, 30].

Keywords. Cooperative advertising, Nash bargaining fairness concerns, subsidy rate, two-way subsidy, game theory.

School of Management, Hefei University of Technology, Hefei 230009, China.
*Corresponding authors: xianjindu@hfut.edu.cn, shaokuntao@mail.hfut.edu.cn

[1]https://martech.org/product-listing-ads-digital-co-op-and-the-13-billion-opportunity/ (accessed on July 2, 2022).

Despite the potential benefits on sales, the retailer has other appeals in engaging in supply chain cooperation, such as fairness concerns that are thought to be widespread in the decision-making process [27, 40, 49, 65]. For instance, Galanz Group, a microwave oven maker, announced that it had suffered a massive loss of profits because of the unfair treatment it received from Alibaba. During the cooperation, Galanz was asked to choose only one platform to sell their products between Alibaba and Pinduoduo, a famous platform in China[2]. Another practice case is the Langsha Group (a well-known sock manufacturer) which believes that the profits generated by the cooperation with Wal-Mart have not been relatively allocated and decides to terminate it [10, 13, 37].

To the best of our knowledge, only a small amount of literature has introduced fairness concerns into cooperative advertising studies through exogenous other-regarding fairness concern parameters [32,60,65]. Nevertheless, [10] and [37] pointed out that the existing exogenous fairness concerns cannot describe the impact of supply chain members' bargaining power. To reflect the endogenous contributions of supply chain members and the bargaining power, many studies adopt the Nash bargaining solutions as the fairness reference point, which impacts the stakeholders' decision-making [7, 10, 37]. Motivated by this new perspective, we introduce the Nash bargaining fairness concerns (NBFC) into the cooperative advertising decisions and get the following research questions:

– Considering the NBFC, how about the cooperative advertising decisions?
– Can NBFC improve advertising subsidy rates, supply chain enterprises' advertising investments, and profits?
– Can the supply chain benefit from NBFC?

To address the above questions, we construct a game-theoretic model of cooperative advertising. Furthermore, as commonly assumed in the literature, the retailer has NBFC, but the manufacturer does not. To maximize his profit, the upstream manufacturer optimizes the national advertising level and advertising subsidy rate. Upon receiving the counterpart's decisions, the retailer maximizes her utility by calculating the optimal local advertising level with NBFC. We investigate how NBFC affect stakeholders' cooperative advertising decisions. Moreover, the two-way subsidy (TWS) policy is examined as the coordination mechanism for the supply chain.

By determining and contrasting the equilibrium strategies under different cases, we obtain the following conclusions: (1) We find that local advertising improves with an increase in the NBFC, but national advertising is unaffected. The manufacturer's advertising subsidy rate initially increases in NBFC and then decreases; (2) In the presence of NBFC, the retailer's profit is inverted U-shaped, rising first and then falling, and vice versa for the manufacturer. That said, the NBFC sometimes reward and sometimes negatively affect supply chain partners according to the intensity of fairness concerns. Interestingly, when the NBFC coefficient falls within a particular range, the overall supply chain efficiency is improved. Therefore, the double marginalization effect can be mitigated; (3) Our analytical results show that cooperative advertising with the retailer's NBFC cannot coordinate the supply chain. However, we verify that the TWS mechanism in cooperative advertising still works on coordination although the retailer possesses NBFC.

The main motivations and contributions we find are as follows:

– Integrating NBFC in supply chain cooperative advertising program.
– Revealing the role of NBFC on channel members' cooperative advertising strategies.
– Developing the supply chain coordination mechanism in the cooperative advertising with NBFC.

The paper is further structured as follows. Section 2 briefly reviews the literature. Section 3 sets up the cooperative advertising framework and benchmark model without considering NBFC and then introduces the Nash bargaining solutions to fairness concerns. Section 4 compares and discusses supply chain enterprises' cooperative advertising decisions under different scenarios. Section 5 shows the managerial insights and practical implications. Section 6 concludes the study, and Appendix A provides proof.

[2] https://www.ft.com/content/b55d0e0a-33a1-11ea-9703-eea0cae3f0de/

2. Literature review

Our study is related to two streams of literature: cooperative advertising and NBFC in the supply chain.

The first stream is cooperative advertising. Considerable research exists on cooperative advertising, and the theoretical model is one of the most important research methods. Among these studies, many scholars focused on the static model of cooperative advertising or marketing efforts where participants make decisions in one period [18, 34, 50, 54, 61]. The other stream of research adopts dynamic cooperative advertising models, which calculate time-dependent effects [4, 8, 12, 15–17, 22, 28, 30, 45, 62]. We refer to [26] and [1] for details as they comprehensively reviewed relevant research. In the present study, we take a closer look at a few cooperative advertising studies that consider stakeholders' behaviors. [60] took the lead in incorporating the participants' fairness concerns when investigating cooperative advertising in dyadic channels. Their findings show that cooperative advertising improves the channel members' utilities, which depends on the fairness concern coefficient. [32] examined supply chain cooperative advertising with the manufacturer's fairness concerns under dual-channel settings. In addition to the fairness concerns, some other managerial behaviors, such as reference price, risk, or inequality aversion, have been investigated with a cooperative advertising program. [62] and [46] analyzed the reference price impact on cooperative advertising strategies with differential equations and a two-period Stackelberg game, respectively. [66] examined the cooperative advertising and ordering policies when both supply chain members are risk-averse. This finding shows that cooperative advertising reduces the retailer's risk and encourages her to increase advertising investment. [45] studied how price stickiness affects cooperative advertising investment while participants are myopia or farsighted. [36] employed a laboratory experiment and found that the manufacturer's inequality-averse behavior is beneficial to the cooperative supply chain.

The second stream focuses on NBFC in the supply chain. Various academic studies investigated fairness concerns about profit allocation in the supply chain [5,9,11,19,24,39,40,47–49,57]. Different from taking the profit distributed by the partner as the reference point, NBFC apply the Nash bargaining solution as a standpoint, which leads to a new perspective on fairness concerns research and is closer to reality [6, 7, 10, 13, 37, 53, 55]. [10] addressed the newsvendor problem where the channel members have NBFC, which reduces channel efficiency. [6, 7] investigated the pricing and ordering decisions with buy-back guarantee financing and stochastic demands, respectively, in the context of a retailer with NBFC. [53] studied the equilibrium strategies with the option contracts when the channel partners have NBFC. [52] investigated the pricing decisions of distribution channels and found that fairness-concerned retailer performs great. [13] introduced consumer goodwill and two types of arrangements to investigate how NBFC affect enterprise decisions within the supply chain and found that the channel leader is susceptible to fairness concerns, changing its pricing investment strategy. [37] examined the effect of NBFC on different contract mechanisms and concluded that NBFC benefit the supply chain's performance. [55] found that channel members' carbon emission reduction and green marketing efforts decrease, whereas the manufacturer's utility benefits with NBFC. Expect for the research on how fairness preferences and other behaviors affect operational management, the global pandemic has sparked extensive studies on supply chain resilience and viability [25, 41, 44].

Table 1 presents the location of the typical papers and literature of this study. As described in the literature, cooperative advertising has been studied in many ways that consider fairness issues [32, 60, 64]. The critical difference between them and our study is the fairness reference point, in which we use Nash bargaining solutions. With this extension, we investigate the impact of the endogenous power and contribution within the distributional channels on cooperative advertising decisions [7, 10, 13, 37]. Moreover, we gain insight into the management of the NBFC regarding cooperative advertising decisions, profits earned by supply chain members, and channel efficiency.

3. Problem statement

This section considers a two-echelon monopoly supply chain consisting of a manufacturer and a retailer, where they develop cooperative advertising. As in the literature [21, 33, 35], the manufacturer advertises nationally,

Table 1. Comparison of related studies.

	Type	SC member	Theme	Advertising effect	FC reference point	FC member
[51]	SC	One M,One R	Co-op advertising	Square root	–	–
[60]	SC	One M,One R	Co-op advertising	Linear	Π_m	R
[62]	SC	One M,One R	Co-op advertising	Linear	–	–
[2]	SC	One M,Two R	Co-op advertising	Square root	–	–
[10]	SC	One S,One R	Channel efficiency	–	NBS	S,R
[32]	DSC	One M,One R	Co-op advertising	Linear	Π_r	M
[29]	SC	Two M,One R	Co-op advertising	Linear	–	–
[66]	SC	One M,One R	Co-op advertising	Power	–	–
[64]	CLSC	One M,One D,One R	Surplus profit allocation	–	Π_d, Π_{md}	R
[53]	SC	One M,One R	Option contract	–	NBS	M,R
[13]	SC	One M,One R	Channel coordination	Linear	NBS	M,R
[52]	DSC	One M,One R	Pricing	–	NBS	R
[49]	SC	Two M,One R	Pricing	–	Π_{m1} ,Π_{m2}	R
[57]	SSC	One M,One R	Contract	–	Π_m	R
[37]	SC	One M,One R	Contract	–	NBS	M,R
[55]	LCSC	One M,One R	Carbon reduction	Linear	NBS	R
[46]	SC	One M,One R	Co-op advertising	Linear	–	–
[11]	SC	Two M,One R	Pricing	–	Π_{m1} ,Π_{m2}	R
This study	SC	One M,One R	Co-op advertising	Square root	NBS	R

Notes. SC: supply chain; CLSC: Closed-loop SC; DSC: Dual-channel SC; SSC: Sustainable SC; LCSC: Low-carbon SC; M: manufacturer; R: retailer; S:supplier; D: distributor; NBS: Nash bargaining solutions; FC: fairness concerns.

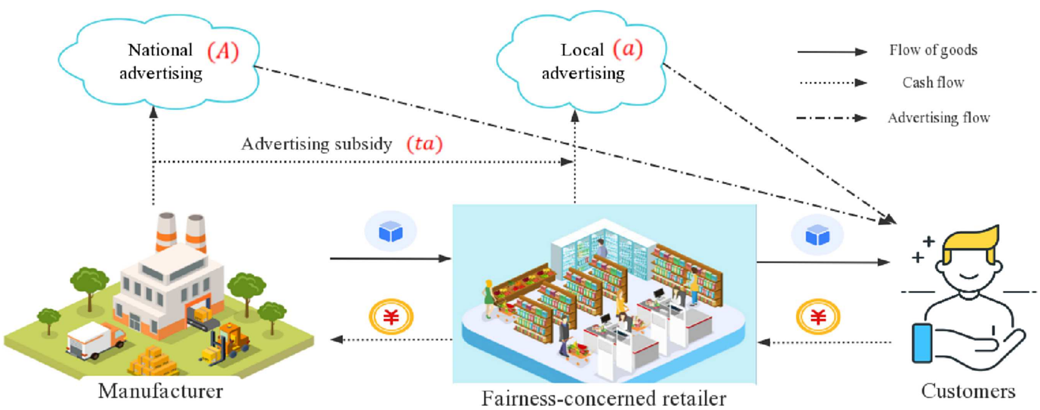

Figure 1. Two-echelon supply chain.

whereas the retailer invests in local advertisement. The manufacturer reimburses a portion of the retailer's advertising expenditures to incentivize local advertising investment. The Stackelberg game is used in modeling the cooperative advertising decision process between supply chain members [31, 32, 36]. Figure 1 shows the framework of the supply chain. The sequence of decisions is that the manufacturer first determines the national advertising expenditures A and the subsidy rate t to the retailer. Then, the retailer receives the solutions and determines her optimal local advertising expenditures a to maximize her utility. In the decision-making process, we consider the retailer with or without NBFC and introduce a decision utility function that considers the fairness reference point with the Nash bargaining solutions.

TABLE 2. Notation list.

Indices	
i	Index of supply chain member $i \in \{m, r\}$
j	Index of member's advertisement effectiveness $j \in \{1, 2\}$
Parameters	
α	Retailer's bargaining power
λ_i	Nash bargaining fairness concerns of firm i
ρ_i	Marginal profit of firm i
k_j	Advertisement effectiveness
Π_i	Firm i's profit
Π_c	Supply chain's profit
$\overline{\Pi_i}$	Firm i's Nash bargaining solution which is the fairness reference point
U_i	Firm i's utility
Decision variables	
A	National advertising expenditures
a	Local advertising expenditures
t	Manufacturer's local advertising subsidy rate for the retailer, where $t \in [0, 1)$
φ	Retailer's national advertising subsidy rate for the manufacturer, where $\varphi \in [0, 1)$

3.1. Notations and assumptions

For ease of reference, Table 2 summarizes notations' descriptions. In addition, the subscripts "m", "r", and "c" indicate the manufacturer, retailer, and supply chain system, respectively.

For constructing the decision model, we have some key assumptions.

Assumption 3.1. *The sales response to advertising $S(a, A) = k_1\sqrt{a} + k_2\sqrt{A} + \epsilon$.*

We assume that the sales response function with national and local advertising efforts is $S(a, A) = k_1\sqrt{a} + k_2\sqrt{A} + \epsilon$ [14, 63]. The saturation effect of advertising decreases marginal utility for increased advertising expenditures. Owing to the uncertainty of the impact of advertising on demand, we use ϵ to denote the random quantity. In addition, the value range of ϵ is $(\underline{\epsilon}, \overline{\epsilon})$ with mean μ and variance σ^2. Without losing generality, we assume that $\mu = 0$. The cumulative distribution function and probability density function are $F(\epsilon)$ and $f(\epsilon)$, respectively [56, 59].

Assumption 3.2. *The product is sold in one season.*

We assume that the product is sold in one season to focus on how NBFC influence cooperative advertising decisions. This assumption has been used in many studies [34, 50, 61]. Furthermore, the retail price remains unchanged, consistent with the enterprise operations. For example, the iPhone 13 has been priced at $799 since its release in September 2021 until now. Moreover, this assumption of exogenous retail price is commonly used by other research [18, 23, 32].

Assumption 3.3. *The supply chain members' marginal profits are ρ_m and ρ_r.*

As the retail price is exogenous, we assume that the manufacturer's marginal profit is ρ_m, and the retailer's is ρ_r. This result is in line with several studies [22, 38, 60]. In addition, we assume that $\rho_m > \frac{\rho_r}{2}$. As ρ_m is less than $\frac{\rho_r}{2}$, and the manufacturer will not make a profit by using the cooperative advertising strategy.

3.2. Benchmark model

Before analyzing the effect of NBFC, we start with the case that the members are rational. Moreover, we have two cases, centralized and decentralized.

Under the centralized case, the optimization aims at the overall channel. The manufacturer and the retailer work together to determine the optimal national and local advertisements. We set market demand: $D = (\Theta -$

TABLE 3. Equilibrium results in the benchmark model.

Case	Centralized case	Decentralized case
Subsidy rate	–	$t^{sn} = \frac{2\rho_m-\rho_r}{2\rho_m+\rho_r}$
National advertisement expenditures	$A^{co} = \frac{{k_2}^2(\rho_m+\rho_r)^2}{4}$	$A^{sn} = \frac{{k_2}^2{\rho_m}^2}{4}$
Local advertisement expenditures	$a^{co} = \frac{{k_1}^2(\rho_m+\rho_r)^2}{4}$	$a^{sn} = \frac{{k_1}^2(2\rho_m+\rho_r)^2}{16}$
Manufacturer's profit	–	$\Pi_m^{sn} = \frac{\left(4{k_1}^2+4{k_2}^2\right){\rho_m}^2+4\rho_m{k_1}^2\rho_r+{\rho_r}^2{k_1}^2}{16}$
Retailer's profit	–	$\Pi_r^{sn} = \frac{(2\rho_m+\rho_r){k_1}^2\rho_r+4{k_2}^2\rho_m\rho_r}{8}$
Supply chain's profit	$\Pi_c^{co} = \frac{(\rho_m+\rho_r)^2\left({k_1}^2+{k_2}^2\right)}{4}$	$\Pi_c^{sn} = \frac{\left(4{k_1}^2+4{k_2}^2\right){\rho_m}^2+8\left({k_1}^2+{k_2}^2\right)\rho_m\rho_r+3{k_1}^2{\rho_r}^2}{16}$

$p)(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon)$, which combines the action of price and advertisement. When the retail price is constant, we normalize the price effect $\Theta - p$ as 1. The utility function can be shown as follows.

$$\max_{A,a} E[\Pi_c(A,a)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} (\rho_m+\rho_r)\left(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon\right) f(\epsilon)\, \mathrm{d}\epsilon - A - a. \tag{1}$$

Under a decentralized case, the following expected profit functions can be derived:

$$\max_{A,t} E[\Pi_m(A,a,t)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \rho_m\left(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon\right) f(\epsilon)\, \mathrm{d}\epsilon - A - ta, \tag{2}$$

$$\max_{a} E[\Pi_r(A,a,t)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \rho_r\left(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon\right) f(\epsilon)\, \mathrm{d}\epsilon - (1-t)\,a. \tag{3}$$

The superscripts co and sn indicate the optimal strategy for centralized and decentralized cases without NBFC, respectively. Moreover, the equilibrium strategies can be obtained using the backward induction method for the decentralized case. Table 3 lists the equilibrium results, including the national and local advertisement expenditures, the subsidy rate, and the stakeholders' profits.

3.3. Model with NBFC

This subsection analyzes the advertising strategies with NBFC. The Nash bargaining process is not a real game but a psychological process to get the decisions' reference point. Therefore, we first introduce the decision utility functions that consider fairness reference points with the Nash bargaining model. Then, members' decision-making processes and the equilibrium strategies are analyzed based on the Stackelberg game.

When the decision-makers have NBFC, their utility depends on the actual profits and the gap between profits and the Nash bargaining solution. If their real yields are larger than Nash bargaining solutions, then the utilities increase, and vice versa. According to the literature [7,10,40], the utility functions are represented by the following:

$$U_i = \Pi_i^{sn} + \lambda_i\left(\Pi_i^{sn} - \overline{\Pi_i}\right), i = m, r. \tag{4}$$

λ_i indicates the decision maker's fairness concern coefficient, which is a positive constant. $\overline{\Pi_i}$ denotes their Nash bargaining solution, which is his (her) fairness reference point. Notably, in equation (4), we do not assume $\Pi_i^{sn} \geq \overline{\Pi_i}$. That means the utility increases or decreases as the comparative relationship between Π_i^{sn} and $\overline{\Pi_i}$ changes. In line with Nash's axiomatic definition [7,10,13,37], the Nash bargaining solutions of the supply chain partners can be obtained by maximizing the following model:

$$\max_{\Pi_m^{sn},\Pi_r^{sn}} (U_r)^{\alpha}(U_m)^{1-\alpha}$$

$$s.t. \begin{cases} \Pi_m^{sn} + \Pi_r^{sn} = \Pi_c^{sn} \\ \Pi_m^{sn}, \Pi_r^{sn} \in (0, \Pi_c^{sn}), \end{cases}$$

where α $(0 < \alpha < 1)$ is the retailer's bargaining power, which is her expected profit share.

Following the solving process [10], we can get the Nash bargaining solutions for the supply chain partners as follows:

$$\overline{\Pi_m} = \frac{(1-\alpha)(1+\lambda_m)}{\alpha(1+\lambda_r)+(1-\alpha)(1+\lambda_m)}\Pi_c^{sn},$$
$$\overline{\Pi_r} = \frac{\alpha(1+\lambda_r)}{\alpha(1+\lambda_r)+(1-\alpha)(1+\lambda_m)}\Pi_c^{sn}.$$

Inserting the Nash bargaining solution $\overline{\Pi_r}$ and $\overline{\Pi_m}$ into equation (4), the utility functions of the supply chain partners are as follows:

$$\max_{A,t} E[U_m(A,a,t)] = (1+\lambda_m)\Pi_m^{sn} - \frac{(1-\alpha)(1+\lambda_m)\lambda_m}{\alpha(1+\lambda_r)+(1-\alpha)(1+\lambda_m)}\Pi_c^{sn},$$

$$\max_{a} E[U_r(A,a,t)] = (1+\lambda_r)\Pi_r^{sn} - \frac{\alpha(1+\lambda_r)\lambda_r}{\alpha(1+\lambda_r)+(1-\alpha)(1+\lambda_m)}\Pi_c^{sn}.$$

From a general logical analysis, both supply chain partners can be fairness-concerned. However, here, we set the retailer to possess NBFC but not for the manufacturer. In other words, the retailer has a social comparison with its profit from the view of endogenous power and contribution within the channel. Many studies have quoted this assumption [6,7,20,60] , but some argue that both members are fairness-concerned. We follow the former because in our research setting, the manufacturer dominates the game and allocates the profit, whereas the retailer is the follower and receiver. Consequently, she is concerned about the equality of the allocating process [6,7]. With $\lambda_m = 0$, we obtain the following retailer's utility function:

$$\begin{aligned}\max_{a} E[U_r(A,a,t)] &= (1+\lambda_r)\Pi_r^{sn} - \frac{\alpha\lambda_r(1+\lambda_r)}{1+\alpha\lambda_r}\Pi_c^{sn} \\ &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} \Bigg[(1+\lambda_r)\left(\rho_r\left(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon\right)-(1-t)\,a\right) - \\ &\quad \frac{\alpha\,\lambda_r\,(1+\lambda_r)\left((\rho_m+\rho_r)\left(k_1\sqrt{a}+k_2\sqrt{A}+\epsilon\right)-A-a\right)}{\alpha\,\lambda_r+1}\Bigg] f(\epsilon)\,\mathrm{d}\epsilon.\end{aligned} \tag{5}$$

We remove the subscript r from λ_r for simplicity. The Stackelberg game can be solved by backward induction. We have a detailed process for solving the equilibrium in Appendix A.

4. Results and discussion

The purpose of this section is to analyze the decision-makers' optimal advertising strategies by comparing the equilibrium solutions for three scenarios described in Section 3. We also investigate whether and how the cooperative advertising program with NBFC coordinates the supply chain.

4.1. Equilibrium solutions

By solving the game described in Section 3.3, we get equilibrium solutions in Proposition 4.1. The superscript sf denotes the optimal strategy when the retailer possesses NBFC.

Proposition 4.1. *When the retailer possesses NBFC, we obtain equilibrium results for the Stackelberg game as follows:*

(a) *The decision variables' solutions are given by*

$$a^{sf} = \frac{{k_1}^2\left(2\,\rho_m + \rho_r + \alpha\,\lambda\,\rho_m\right)^2}{16}, \left(A^{sf}, t^{sf}\right) = \left(\frac{{k_2}^2{\rho_m}^2}{4}, \frac{2\,\rho_m - \rho_r + 3\,\alpha\,\lambda\,\rho_m}{\left(2\,\rho_m + \rho_r + \alpha\,\lambda\,\rho_m\right)\left(\alpha\,\lambda + 1\right)}\right).$$

(b) *The members' profits are as follows:*

$$\Pi_m^{sf} = \frac{{k_2}^2{\rho_m}^2}{4} + \frac{{k_1}^2\left(2\,\rho_m + \rho_r + \alpha\,\lambda\,\rho_m\right)^2}{16\left(1 + \alpha\lambda\right)},$$

$$\Pi_r^{sf} = \frac{{k_2}^2\rho_m\,\rho_r}{2} + \frac{{k_1}^2\left(2\,\rho_m + \rho_r + \alpha\,\lambda\,\rho_m\right)\left(2\rho_r + \alpha\lambda\left(3\rho_r - \alpha\lambda\rho_m\right)\right)}{16\left(1 + \alpha\lambda\right)},$$

$$\Pi_c^{sf} = \frac{{k_2}^2\rho_m\left(\rho_m + 2\rho_r\right)}{4} + \frac{{k_1}^2\left(2\,\rho_m + \rho_r + \alpha\,\lambda\,\rho_m\right)\left(2\rho_m + 3\rho_r - \alpha\lambda\rho_m\right)}{16}.$$

4.2. Comparative analysis

We compare the equilibrium results for three cases to obtain the following propositions.

Proposition 4.2. *The following are the analyses of the optimal advertising decisions:*

(a) *The comparative result of local advertisement: when* $0 < \lambda < \lambda_1$, $a^{co} > a^{sf} > a^{sn}$; *and when* $\lambda \geq \lambda_1$, $a^{sf} > a^{co} > a^{sn}$;
(b) *For subsidy rate in the local advertisement: when* $0 < \lambda < \lambda_2$, $t^{sf} > t^{sn}$; *and when* $\lambda \geq \lambda_2$, $t^{sf} \leq t^{sn}$;
(c) *The comparative result of national advertisement:* $A^{sf} = A^{sn} < A^{co}$;

where $\lambda_1 = \frac{\rho_r}{\alpha\rho_m}, \lambda_2 = \frac{\rho_r(4\rho_m+\rho_r)}{\alpha\rho_m(2\rho_m-\rho_r)}$ and $\lambda_1 < \lambda_2$.

As shown in Proposition 4.2a, the retailer's local advertisement benefits when she has NBFC as $a^{sf} > a^{sn}$. Furthermore, if $\lambda \geq \lambda_1$, then the local advertisement is higher under the NBFC case than the centralized one. To address his counterpart's fairness concerns, the rational manufacturer can allocate profit by enhancing his local advertising subsidy rate shown in Proposition 4.2b. Moreover, if $\lambda \geq \lambda_2$, then the manufacturer would reduce the advertising subsidy rate to protect his benefit. According to Proposition 4.2c, the manufacturer distributes the supply chain's profit via his subsidy rate while maintaining his national advertising investment.

Proposition 4.3. *The coefficient of NBFC has variable effects on the profits comparison can be expressed as:*

(a) *For the retailer's profit, when* $0 < \lambda < \lambda_3, \Pi_r^{sf} > \Pi_r^{sn}$; *when* $\lambda \geq \lambda_3, \Pi_r^{sf} \leq \Pi_r^{sn}$;
(b) *For the manufacturer's profit, when* $0 < \lambda < \lambda_4, \Pi_m^{sf} < \Pi_m^{sn}$; *when* $\lambda \geq \lambda_4, \Pi_m^{sf} \geq \Pi_m^{sn}$;

where $\lambda_3 = \frac{\sqrt{(\rho_m+\rho_r)^2+\rho_r^2}-\rho_m+\rho_r}{\alpha\rho_m}, \lambda_4 = \frac{\rho_r(2\rho_m+\rho_r)}{\alpha\rho_m^2}$ and $\lambda_1 < \lambda_3 < \lambda_4 < \lambda_2$.

As shown in Proposition 4.3a, the retailer's NBFC positively affect her profit, and the converse is true for the manufacturer if $0 < \lambda < \lambda_3$. However, if $\lambda_3 < \lambda < \lambda_4$, both the supply chain partners' profits are lower than that without NBFC. Moreover Proposition 4.3b shows that when $\lambda \geq \lambda_4$, the manufacturer's profit benefits from the NBFC coefficient.

Proposition 4.4. *As to the supply chain's profit, we have the following comparative analysis: when* $0 < \lambda < \lambda_5$, $\Pi_c^{sn} < \Pi_c^{sf} < \Pi_c^{co}$; *when* $\lambda \geq \lambda_5$, $\Pi_c^{sf} \leq \Pi_c^{sn} < \Pi_c^{co}$, *where* $\lambda_5 = \frac{2\rho_r}{\alpha\rho_m}$ *and* $\lambda_1 < \lambda_5 < \lambda_3$.

Proposition 4.4 shows that whether λ is greater or less than λ_5, the profit of the supply chain is lower than that of the centralized case. However, when λ is smaller than λ_5, we get $\Pi_c^{sf} > \Pi_c^{sn}$. Nevertheless, if the NBFC coefficient still grows, $\Pi_c^{sf} < \Pi_c^{sn}$.

4.3. Sensitivity analysis

Although several parameters exist, such as the advertisement effectiveness and the participant's marginal profit, we focus on the NBFC's impact on cooperative advertising as the other parameters have been studied in related literature. The fairness concern coefficient and bargaining power are completely symmetrical in the equilibrium solutions. That is, the analysis applicable to one parameter is also applicable to the other parameter when exchanged. Thus, we only need to pay attention to the analysis of one of them. In this section, all analyses will be conducted through the fairness concern coefficient, which is our focal point.

Observation 4.5. *When the Nash bargaining fairness-concerned coefficient λ increases:*

(a) The retailer increases the local advertising expenditures a;
(b) The manufacturer's national advertising expenditures A remain unchanged. The local advertising subsidy rate t increases if $0 < \lambda < \lambda_1$ and decreases if $\lambda > \lambda_1$.

From Proposition 4.1, we can know that $a^{sf} = \frac{{k_1}^2(\alpha\lambda\rho_m+2\rho_m+\rho_r)^2}{16}$. Evidently, λ positively impacts the local advertising investment a. Furthermore, we also find an interesting result that the manufacturer does not change his decision on national advertising. Instead, he may set a local advertising subsidy rate to respond to the retailer's appeal. When $0 < \lambda < \lambda_1$, the subsidy rate on the local advertisement expenditures increases in the retailer's NBFC. However, if $\lambda > \lambda_1$, then the subsidy rate decreases. In this way, the manufacturer can simplify his decision-making without modifying national advertising investment.

Observation 4.6. *When the Nash bargaining fairness-concerned coefficient λ increases:*

(a) The manufacturer's profit decreases if $0 < \lambda < \lambda_1$, and increases if $\lambda > \lambda_1$;
(b) The profits of the retailer and the whole supply chain increase if $0 < \lambda < \lambda_1$, and decrease if $\lambda > \lambda_1$.

Observation 4.6 shows that the retailer's NBFC have different influences on the stakeholders' profits. When the fairness concern coefficient is less than λ_1, the profits of the retailer and supply chain increase, whereas those of the manufacturer decrease. However, when the fairness-concerned coefficient exceeds a certain threshold, the trends reverse.

4.4. Coordination with a TWS contract

According to Proposition 4.4, we find when the retailer has NBFC, cooperative advertising cannot coordinate the supply chain. Therefore, we propose in this section a TWS contract for coordinating the decentralized supply chain with NBFC, similar to [62] and [18]. In the TWS contract, the supply chain members share each other's advertising investments with subsidy rates t and φ. Thus, we have their profit functions as follows:

$$\max_{A,t} E[\Pi_m^{tw}(A,a,t,\varphi)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \rho_m \left(k_1\sqrt{a} + k_2\sqrt{A} + \epsilon\right) f(\epsilon)\, \mathrm{d}\epsilon - (1-\varphi)\,A - ta, \tag{6}$$

$$\max_{a,\varphi} E[\Pi_r^{tw}(A,a,t,\varphi)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \rho_r \left(k_1\sqrt{a} + k_2\sqrt{A} + \epsilon\right) f(\epsilon)\, \mathrm{d}\epsilon - \varphi A - (1-t)\,a, \tag{7}$$

where the subscript tw denotes the scenario of a TWS contract.

As the retailer has NBFC, her utility changes to:

$$\begin{aligned}\max_{a,\varphi} E[U_r^{tw}(A,a,t,\varphi)] = \int_{\underline{\epsilon}}^{\overline{\epsilon}} \bigg[&(1+\lambda)\left(\rho_r\left(k_1\sqrt{a} + k_2\sqrt{A} + \epsilon\right) - (1-t)\,a - \varphi\,A\right) \\ &-\frac{\alpha\,\lambda\,(1+\lambda)}{\alpha\,\lambda+1}\left((\rho_m+\rho_r)\left(k_1\sqrt{a} + k_2\sqrt{A} + \epsilon\right) - A - a\right)\bigg] f(\epsilon)\, \mathrm{d}\epsilon.\end{aligned} \tag{8}$$

Moreover, we obtain the optimal national and local advertising investments with backward induction if the subsidy rates φ and t are fixed.

$$A^{tw} = \frac{{\rho_m}^2 {k_2}^2}{4\,(1-\varphi)^2}, \tag{9}$$

$$a^{tw} = \frac{{k_1}^2\,(\alpha\,\lambda\,\rho_m - \rho_r)^2}{4\,(\alpha\,\lambda\,t + t - 1)^2}. \tag{10}$$

For the TWS contract, the coordination happens only if parameters φ and t have specific values such that $A^{tw} = A^{co}$ and $a^{tw} = a^{co}$, respectively. Based on the solutions, we arrive at the following conclusion.

Proposition 4.7. *When the retailer has NBFC, a cooperative advertising program is capable of coordinating the supply chain through a TWS policy, but only if the subsidy rates satisfy*

$$\varphi^{tw} = \frac{\rho_r}{\rho_m + \rho_r}, \tag{11}$$

$$t^{tw} = \begin{cases} \frac{\rho_m}{\rho_m+\rho_r}, & 0 < \lambda \le \lambda_1 \\ \frac{\rho_m + 2\,\rho_r - \alpha\,\lambda\,\rho_m}{(\alpha\,\lambda+1)(\rho_m+\rho_r)}, & \lambda > \lambda_1. \end{cases} \tag{12}$$

We set $t_1^{tw} = \frac{\rho_m}{\rho_m+\rho_r}$ and $t_2^{tw} = \frac{\rho_m + 2\,\rho_r - \alpha\,\lambda\,\rho_m}{(\alpha\,\lambda+1)(\rho_m+\rho_r)}$. Among them, t_1^{tw} is independent of the fairness-concerned coefficient λ, and t_2^{tw} decreases in λ. Compared with the participating rate in the decentralized Stackelberg game in Proposition 4.1, $t_1^{tw} > t^{sf}$, $t_2^{tw} > t^{sf}$, and $\varphi^{tw} > 0$. Therefore, we obtain the following Observation 4.8.

Observation 4.8. *To coordinate the supply chain with a fairness-concerned retailer through cooperative advertising, the supply chain members share further each other's advertising investment in the TWS contract.*

4.5. Numerical analysis

To greatly present the relevant analysis results, we carry out a numerical analysis as shown in this section. The parameters and values are chosen based on relevant literature: $k_1 = 1, k_2 = 1, \rho_m = 5, \rho_r = 5, \alpha = 0.5$ [2, 10, 18].

4.5.1. Variation of advertising expenditures

Figures 2a and b show the variation of the local and national advertising expenditures based on the NBFC coefficient. The numerical analysis results validate the relevant conclusions in Proposition 4.2.

4.5.2. Variation of subsidy rate

Figure 2c depicts the variation of the local advertising subsidy rate based on the NBFC coefficient. In the NBFC case, Figure 2c clearly shows an inverted U-shaped relation between the retailer's NBFC coefficient and the manufacturer's local advertising subsidy rate. In the TWS case, we find that when the NBFC coefficient is small, the manufacturer will not change the local advertising subsidy rate. However, when the NBFC coefficient is large, the local advertising subsidy rate decreases as the retailer's NBFC coefficient increases.

4.5.3. Variation of profits

Figures 2d–f present the variation of the retailer's, manufacturer's, and supply chain's profits based on the NBFC coefficient. The relevant conclusions of Proposition 4.3, Proposition 4.4, and Observation 4.6 are visually presented in the figures. The supply chain members' profits in the TWS case are noteworthy. With Figure 2f, the TWS contract can coordinate the supply chain. However, in Figures 2d and e, when the NBFC coefficient is large, the retailer's profit decreases in the NBFC coefficient, and the manufacturer's profit is the opposite.

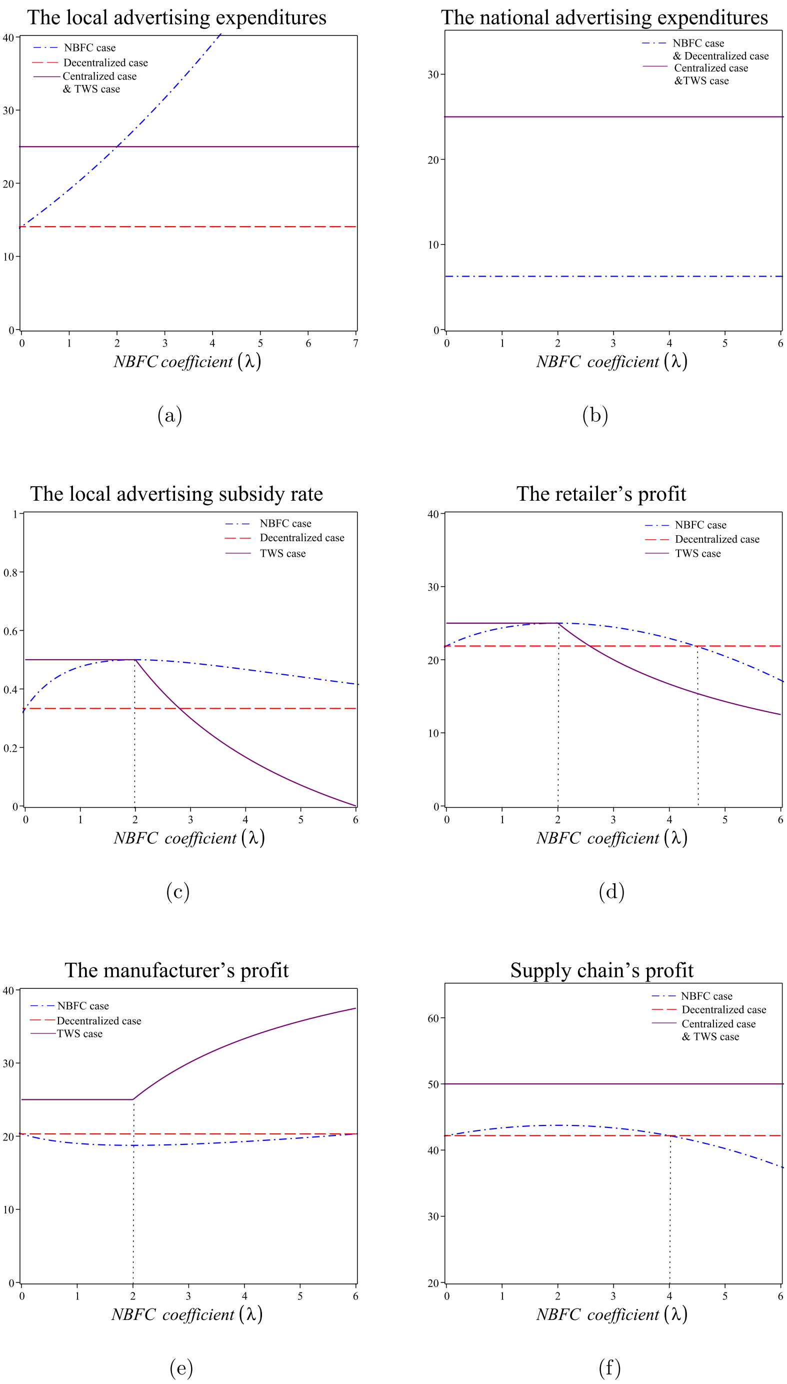

FIGURE 2. Numerical analysis. (a) Variation of the local advertising expenditures. (b) Variation of the national advertising expenditures. (c) Variation of the local advertising subsidy rate. (d) Variation of the retailer's profit. (e) Variation of the manufacturer's profit. (f) Variation of the supply chain's profit.

4.6. Discussion

4.6.1. Cooperative advertising decisions

Proposition 4.2 and the numerical analysis show that local advertising is higher than the case without NBFC. If the NBFC coefficient exceeds a certain threshold ($\lambda > \lambda_1$), then the local advertising expenditures become the highest as $a^{sf} > a^{co} > a^{sn}$, even more than under the centralized case. This result is different from [60] and [55]. As discussed in Observation 4.5, the fairness-concerned retailer has a solid incentive to enhance local advertising to increase sales and gain profit with the participating mechanism. For the manufacturer, the retailer's NBFC will not change his national advertising investment, but relatively low fairness will make the manufacturer increase his subsidy to the retailer's local advertising. The reason is that the manufacturer is responding to the retailer's NBFC by adjusting the subsidy rate. The result means that the NBFC can be used as a market penetration tool in the supply chain.

4.6.2. Supply chain members' profits

Proposition 4.3 shows that the retailer's profit with $\lambda < \lambda_3$ is better than the case without NBFC, whereas the manufacturer is worse. The reason is that the manufacturer is willing to comprise and support local advertising investment by enhancing the subsidy rate. This also incentivizes the retailer to increase her advertising efforts. Because of the increasing shared costs, the manufacturer's profit is less than the case without fairness concerns. However, the additional local advertising promotes the growth of the retailer's profit by bringing additional sales.

However, as the fairness concern coefficient increases, the retailer's profit gradually decreases, finally below the case without NBFC. The reason is that increased revenue can not offset the increased advertising costs, whereas local advertising investment has a declining marginal effect. For the manufacturer, the fairness concern coefficient over a certain threshold ($\lambda \geq \lambda_4$) can contribute to his profit growth. Notably, local advertising investment continues to increase in λ. The improvement of his profit means that the sales brought by local advertising exceed the cost the manufacturer undertakes. Different from their conclusions [55, 60], the impact of the retailer's fairness concerns on her profits is not purely unfavorable or favorable but is influenced by the degree of fairness.

4.6.3. Supply chain efficiency and coordination

Proposition 4.4 and the numerical study note that the supply chain efficiency with a small fairness concern coefficient is relatively better than the situation without NBFC. The NBFC could improve efficiency when $0 < \lambda < \lambda_5$. Nevertheless, if the NBFC coefficient still grows, then the "double marginalization effect" becomes more heavily as $\Pi_c^{sf} < \Pi_c^{sn}$. As analysis from Proposition 4.3, when the fairness concern coefficient takes a small value, the manufacturer loses part of the profit to subsidize the retailer. However, the income brought by the increased advertising investment of the retailer is conducive to the channel's profit. Therefore, the profit lost by the manufacturer is less than the additional income of the retailer. From another aspect, if this coefficient continues to increase, then the channel's profit declines. Therefore, the NBFC cannot improve channel members' profit simultaneously. That is, the NBFC cannot produce a Pareto improvement. Coupled with Proposition 4.1, we find a very interesting interval. When $\lambda_1 < \lambda < \lambda_5$, the manufacturer can gain great market share and improve channel efficiency by transferring profits to the retailer.

Considering that the cooperative advertising with NBFC is not capable of coordinating the supply chain, the TWS and transfer payment mechanisms help. According to the coordination definition [3], the supply chain members' advertising expenditures with NBFC should be identical to the solutions of the centralized case. However, Proposition 4.1 gives different conclusions that $A^{sf} < A^{co}$ and $a^{sf} \neq a^{co}$. Moreover, in Proposition 4.4, we find that $\Pi_c^{sf} < \Pi_c^{co}$ always holds. The conclusion is consistent with [65]. With the equilibrium solutions in Proposition 4.7, we design the bilateral subsidy strategy with the TWS contract to coordinate the supply chain, which is also supported by Figure 2f. However, as shown in Figure 2d, the retailer's profit remains unchanged when the NBFC coefficient is small. Nevertheless, with the increase of λ, her profit decreases. As the retailer's profit may be lower than the unilateral subsidies scenario, she would not like to use the TWS contract. To solve this problem, we can introduce the transfer payment mechanism used in literature, such as [62] and [18]. Let $T = \frac{1}{2}[(\pi_m^{tw} - \pi_m^{sf}) - (\pi_r^{tw} - \pi_r^{sf})]$ and transfer it from the manufacturer to the retailer if $T > 0$ or vice versa to ensure that their profits are improved.

5. Managerial insights and practical implications

This research investigates the cooperative advertising decisions with NBFC. With the Stackelberg game, we model the supply chain members decision processes and offer the equilibrium strategies. Furthermore, we construct the TWS contract to coordinate the supply chain. The findings shed some managerial insights for the manufacturer, the retailer, and the whole supply chain.

With the previous analysis, the retailer should hold a relatively small fairness concern coefficient, which can increase her profit. Otherwise, she may fall into a lose-lose situation with the manufacturer where $\lambda_3 < \lambda < \lambda_4$ showed in Proposition 4.3. At this time, the NBFC negatively affect both members. Thus, the retailer should control her NBFC in a suitable range. The obsessive pursuit of fairness may lead to a decrease in the profit and even the whole supply chain. However, if the NBFC coefficient is big enough, that is, $\lambda > \lambda_4$, it could benefit the manufacturer. In conclusion, NBFC do not always negatively affect the manufacturer.

The effect of NBFC on the total profit of the supply chain shows an inverted U-shaped curve. In other words, NBFC may be helpful for supply chain efficiency but cannot achieve supply chain coordination. This result is in line with [10] and [13] on the impact of NBFC. To solve the issue, we present the TWS contract with a transfer arrangement to achieve supply chain coordination. In summary, the supply chain partners must manage NBFC wisely. The manufacturer and the retailer can strengthen the cooperation and sharing of advertisements by implementing TWS to achieve a win-win situation.

6. Conclusion

Considering its importance in the enterprise's marketing strategy, cooperative advertising has attracted extensive studies from marketing and operation management scholars. Meanwhile, many studies showed that fairness concerns can significantly impact supply chain decisions, such as cooperative advertising strategy. In addition, few scholars systematically examined the relationship between NBFC and cooperative advertising decisions. In doing so, we establish a two-tiered monopoly supply chain. The leading manufacturer determines the national advertising expenditures and subsidy rate, whereas the following retailer sets the local advertising investment. In light of the optimal Nash bargaining solutions, we can draw the equilibrium strategies of a Nash bargaining fairness-concerned retailer with a rational manufacturer. Some meaningful conclusions are obtained by comparing the equilibrium strategies under different cases.

– By solving the model with NBFC, the equilibrium solutions of the supply chain members are derived in Proposition 4.1.
– With NBFC, the local advertising effort increases, even more than the investment in centralized decision-making (*cf.* Obs. 4.5, Prop. 4.2, and Fig. 2a).
– Regardless of whether the retailer has NBFC, the manufacturer's advertising investment remains unchanged (*cf.* Obs. 4.5, Prop. 4.2, and Fig. 2b).
– The NBFC coefficient influences the subsidy rate and retailer's profit showing an inverted U-shaped curve, and vice versa for the manufacturer (*cf.* Obs. 4.5 and 4.6, Prop. 4.3, and Figs. 2c and d and e).
– The NBFC can improve the channel efficiency with a small NBFC coefficient but cannot coordinate the supply chain (*cf.* Prop. 4.4 and Fig. 2f).
– The TWS contract can coordinate the supply chain in the cooperative advertising program even if the retailer has NBFC (*cf.* Prop. 4.7 and Fig. 2f).

Despite the importance of the NBFC on the supply chain's cooperative advertising program, this study has some limitations. In the actual operation, several manufacturers and retailers may be involved in the complex supply chain. The competing supply chain should be introduced into cooperative advertising under the NBFC framework [8, 30]. In addition, the increase in advertising investment may lead to changes in retail prices. Therefore, researching retail price as a decision variable while considering NBFC in cooperative advertising may be meaningful [11, 46].

Furthermore, we focus on the impact of NBFC on the cooperative advertising program. Introducing other behaviors, such as reference effect and overconfidence, would be interesting [11, 22]. Moreover, as supply chain viability is a relatively new research area, combining it with fairness preferences for research is also a worthwhile direction to explore [25, 42, 43].

Appendix A. Proofs of Propositions and Observations

Proof of Proposition 4.1. With the model considering the retailer's NBFC, the following utility functions are derived:

$$\begin{aligned}\max_{A,t} E[U_m(A,a,t)] &= \max_{A,t} E[\Pi_m(A,a,t)] \\ &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} \rho_m \left(k_1\sqrt{a} + k_2\sqrt{A} + \epsilon\right) f(\epsilon)\ \mathrm{d}\epsilon - A - ta,\end{aligned} \tag{A.1}$$

$$\max_{a} E[U_r(A,a,t)] = \max_{a} E[(1+\lambda)\Pi_r - \frac{\alpha\lambda(1+\lambda)}{1+\alpha\lambda}\Pi_c]. \tag{A.2}$$

With the assumption in Section 3.1, we know the above utility functions are strictly concave in a. According to equations (A.1) and (A.2), we apply the first-order condition and get the optimality conditions:

$$\frac{\partial U_r}{\partial a} = 0 \Leftrightarrow a = \frac{{k_1}^2 (\alpha\lambda\rho_m - \rho_r)^2}{4(\alpha\lambda t + t - 1)^2}. \tag{A.3}$$

Substituting equation (A.3) into equation (A.1) and applying game theory, we can obtain:

$$\frac{\partial U_m}{\partial A} = 0 \Leftrightarrow A = \frac{{k_2}^2 {\rho_m}^2}{4}, \tag{A.4}$$

$$\frac{\partial U_m}{\partial t} = 0 \Leftrightarrow t = \frac{3\alpha\lambda\rho_m + 2\rho_m - \rho_r}{(\alpha\lambda\rho_m + 2\rho_m + \rho_r)(\alpha\lambda + 1)}. \tag{A.5}$$

Combining equations (A.3), (A.4), and (A.5), we can get:

$$a^{sf} = \frac{{k_1}^2 (2\rho_m + \rho_r + \alpha\lambda\rho_m)^2}{16}, \left(A^{sf}, t^{sf}\right) = \left(\frac{{k_2}^2{\rho_m}^2}{4}, \frac{2\rho_m - \rho_r + 3\alpha\lambda\rho_m}{(2\rho_m + \rho_r + \alpha\lambda\rho_m)(\alpha\lambda + 1)}\right).$$

Hence, the stakeholders' profits are obtained:

$$\Pi_m^{sf} = \frac{{k_2}^2{\rho_m}^2}{4} + \frac{{k_1}^2 (2\rho_m + \rho_r + \alpha\lambda\rho_m)^2}{16(1+\alpha\lambda)},$$

$$\Pi_r^{sf} = \frac{{k_2}^2\rho_m\rho_r}{2} + \frac{{k_1}^2 (2\rho_m + \rho_r + \alpha\lambda\rho_m)(2\rho_r + \alpha\lambda(3\rho_r - \alpha\lambda\rho_m))}{16(1+\alpha\lambda)},$$

$$\Pi_c^{sf} = \frac{{k_2}^2\rho_m(\rho_m + 2\rho_r)}{4} + \frac{{k_1}^2 (2\rho_m + \rho_r + \alpha\lambda\rho_m)(2\rho_m + 3\rho_r - \alpha\lambda\rho_m)}{16}.$$

Proof of Proposition 4.2. According to Proposition 4.1 and the equilibrium solutions without fairness concerns given in Table 3, we can get the following:

$$A^{co} - A^{sf} = \frac{{k_2}^2 \rho_r \, (2\,\rho_m + \rho_r)}{4} > 0 \Leftrightarrow A^{co} > A^{sf}, \tag{A.6}$$

$$a^{sf} - a^{sn} = \frac{\rho_m {k_1}^2 \lambda \left((\alpha\lambda + 4)\,\rho_m + 2\rho_r \right) \alpha}{16} > 0 \Leftrightarrow a^{sf} > a^{sn}, \tag{A.7}$$

$$a^{co} - a^{sf} = \frac{{k_1}^2 \left(\rho_r - \alpha\lambda\rho_m \right) \left(\alpha\lambda\rho_m + 4\,\rho_m + 3\,\rho_r \right)}{16}, \tag{A.8}$$

$$t^{sf} - t^{sn} = \frac{\alpha\lambda \left(\alpha\lambda\rho_m\rho_r + 4\rho_m\rho_r + {\rho_r}^2 - 2\alpha\lambda{\rho_m}^2 \right)}{\left(\alpha\lambda\rho_m + 2\rho_m + \rho_r \right) \left(\alpha\lambda + 1 \right) \left(2\rho_m + \rho_r \right)}. \tag{A.9}$$

For equation (A.8), it is easy to get that if $0 < \lambda < \lambda_1$, then $a^{co} > a^{sf}$; if $\lambda \geq \lambda_1$, then $a^{co} \leq a^{sf}$, where $\lambda_1 = \frac{\rho_r}{\alpha\rho_m}$. And $\Delta_1 = \alpha\lambda\rho_m\rho_r + 4\rho_m\rho_r + {\rho_r}^2 - 2\alpha\lambda{\rho_m}^2$ determines the sign of equation (A.9). Δ_1 is continuous with λ and $\Delta_1 = 0$ when $\lambda_2 = \frac{\rho_r(4\rho_m + \rho_r)}{\alpha\rho_m(2\rho_m - \rho_r)}$. Thus, we have:

$$0 < \lambda < \lambda_2 \Leftrightarrow \Delta_1 > 0 \Leftrightarrow t^{sf} > t^{sn}, \tag{A.10}$$

$$\lambda \geq \lambda_2 \Leftrightarrow \Delta_1 \leq 0 \Leftrightarrow t^{sf} \leq t^{sn}. \tag{A.11}$$

Also, we can have $\lambda_1 < \lambda_2$. Thus, Proposition 4.2 is proved.

Proof of Proposition 4.3. With the equilibrium solutions of different cases, we get:

$$\Pi_r^{sf} - \Pi_r^{sn} = -\frac{\left(\alpha^2\lambda^2{\rho_m}^2 + 2\alpha\rho_m \left(\rho_m - \rho_r \right) \lambda - 4\rho_m\rho_r - {\rho_r}^2 \right) {k_1}^2 \lambda\alpha}{16\alpha\lambda + 16}, \tag{A.12}$$

$$\Pi_m^{sf} - \Pi_m^{sn} = \frac{\alpha{k_1}^2\lambda \left(\alpha\lambda{\rho_m}^2 - 2\rho_m\rho_r - {\rho_r}^2 \right)}{16\alpha\lambda + 16}. \tag{A.13}$$

Consistent with the proof process in Proposition 4.2, we can get:

$$\lambda_3 = \frac{\sqrt{\left(\rho_m + \rho_r \right)^2 + {\rho_r}^2} - \rho_m + \rho_r}{\alpha\rho_m}, \lambda_4 = \frac{\rho_r \left(2\rho_m + \rho_r \right)}{\alpha{\rho_m}^2}.$$

And we have:

$$0 < \lambda < \lambda_3 \Leftrightarrow \Pi_r^{sf} > \Pi_r^{sn}; \lambda \geq \lambda_3 \Leftrightarrow \Pi_r^{sf} \leq \Pi_r^{sn}, \tag{A.14}$$

$$0 < \lambda < \lambda_4 \Leftrightarrow \Pi_m^{sf} < \Pi_m^{sn}; \lambda \geq \lambda_4 \Leftrightarrow \Pi_m^{sf} \geq \Pi_m^{sn}. \tag{A.15}$$

By comparison, we can get $\lambda_1 < \lambda_3 < \lambda_4 < \lambda_2$.

Therefore, Proposition 4.3 is proved.

Proof of Proposition 4.4. Comparing the supply chain's profit under different scenarios, we obtain:

$$\Pi_c^{sf} - \Pi_c^{sn} = \frac{\alpha\lambda{k_1}^2\rho_m \left(\alpha\lambda\rho_m - 2\rho_r \right)}{16}, \tag{A.16}$$

$$\Pi_c^{co} - \Pi_c^{sf} = \frac{\left(\alpha\lambda\rho_m - \rho_r \right)^2 {k_1}^2 + 4{\rho_r}^2{k_2}^2}{16} > 0 \Leftrightarrow \Pi_c^{co} > \Pi_c^{sf}. \tag{A.17}$$

With equations (A.16) and (A.17), we can get that if $0 < \lambda < \lambda_5, \Pi_c^{sn} < \Pi_c^{sf} < \Pi_c^{co}$. And if $\lambda \geq \lambda_5, \Pi_c^{sf} < \Pi_c^{sn} < \Pi_c^{co}$, where $\lambda_5 = \frac{2\rho_r}{\alpha\rho_m}$ and $\lambda_5 < \lambda_3$.

Proof of Observation 4.5. According to Proposition 4.1, it is easy to get: $\frac{\partial a^{sf}}{\partial \lambda} = \frac{{k_1}^2 \alpha\rho_m(\alpha\lambda\rho_m+2\rho_m+\rho_r)^2}{8} > 0$. $\frac{\partial t^{sf}}{\partial \lambda} = -\frac{(3\alpha\lambda\rho_m+4\rho_m+\rho_r)\alpha(\alpha\lambda\rho_m-\rho_r)}{(\alpha\lambda\rho_m+2\rho_m+\rho_r)^2(\alpha\lambda+1)^2}$, which is easily compared to $\frac{\partial t^{sf}}{\partial \lambda} > 0$ if $0 < \lambda < \frac{\rho_r}{\alpha\rho_m}$, and vice versa. And the national advertising expenditures A have no bearing on the NBFC coefficient.

Proof of Observation 4.6. Based on Proposition 4.1, we can obtain that:

$$\frac{\partial \Pi_m^{sf}}{\partial \lambda} = \frac{{k_1}^2\left(\alpha\lambda\rho_m + 2\rho_m + \rho_r\right)\alpha\left(\alpha\lambda\rho_m - \rho_r\right)}{16\left(\alpha\lambda+1\right)^2},$$

$$\frac{\partial \Pi_r^{sf}}{\partial \lambda} = -\frac{{k_1}^2\left(2\alpha^2\lambda^2\rho_m + 5\alpha\lambda\rho_m + 4\rho_m + \rho_r\right)\alpha\left(\alpha\lambda\rho_m - \rho_r\right)}{16\left(\alpha\lambda+1\right)^2},$$

$$\frac{\partial \Pi_c^{sf}}{\partial \lambda} = -\frac{\alpha\rho_m{k_1}^2\left(\alpha\lambda\rho_m - \rho_r\right)}{8}.$$

Hence, we can get that if $0 < \lambda < \frac{\rho_r}{\alpha\rho_m}$, then $\frac{\partial \Pi_m^{sf}}{\partial \lambda} < 0$, $\frac{\partial \Pi_r^{sf}}{\partial \lambda} > 0$, $\frac{\partial \Pi_c^{sf}}{\partial \lambda} > 0$, and vice versa.

Proof of Proposition 4.7 and Observation 4.8. The processes are explained in Section 4, we will not repeat them here.

Acknowledgements. The authors thank the editor-in-chief, associate editor and the anonymous reviewers for their insightful comments on the earlier version of this manuscript. They also acknowledge the financial support provided by the National Natural Science Foundation of China (Nos.72071065).

References

[1] G. Aust and U. Buscher, Cooperative advertising models in supply chain management: A review. *Eur. J. Oper. Res.* **234** (2014) 1–14.

[2] G. Aust and U. Buscher, Vertical cooperative advertising in a retailer duopoly. *Comput. Ind. Eng.* **72** (2014) 247–254.

[3] G.P. Cachon, Supply chain coordination with contracts. In: *Supply Chain Management: Design, Coordination and Operation. Vol. 11 of Handbooks in Operations Research and Management Science*. Elsevier (2003) 227–339.

[4] D. Cao, X. Zhang, L. Yang and J. Xiao, Dynamic cooperative advertising strategy in oao supply chain with customer return. *RAIRO:RO* **54** (2020) 1537–1553.

[5] Y. Chen and T.H. Cui, The benefit of uniform price for branded variants. *Mark. Sci.* **32** (2013) 36–50.

[6] J. Chen, Y. Zhou and Y. Zhong, A pricing ordering model for a dyadic supply chain with buyback guarantee financing and fairness concerns. *Int. J. Prod. Res.* **55** (2017) 5287–5304.

[7] J. Chen, T. Zhang, Y. Zhou and Y. Zhong, Joint decision of pricing and ordering in stochastic demand with nash bargaining fairness. *Comput. Oper. Res.* **123** (2020) 105037.

[8] A. Chutani and S.P. Sethi, Dynamic cooperative advertising under manufacturer and retailer level competition. *Eur. J. Oper. Res.* **268** (2018) 635–652.

[9] H.T. Cui, J.S. Raju and Z.J. Zhang, Fairness and channel coordination. *Manage. Sci.* **53** (2007) 1303–1314.

[10] S. Du, T. Nie, C. Chu and Y. Yu, Newsvendor model for a dyadic supply chain with Nash bargaining fairness concerns. *Int. J. Prod. Res.* **52** (2014) 5070–5085.

[11] S. Ganguly, M.R. Kuiti, P. Das, and M. Maiti, Effect of fairness and overconfidence on pricing strategy of substitute bundles in a two-echelon supply chain. *RAIRO:RO* **57** (2023) 401–425.

[12] Q. Gou, J. Zhang, L. Liang, Z. Huang and A. Ashley, Horizontal cooperative programmes and cooperative advertising. *Int. J. Prod. Res.* **52** (2014) 691–712.

[13] Z. Guan, T. Ye and R. Yin, Channel coordination under Nash bargaining fairness concerns in differential games of goodwill accumulation. *Eur. J. Oper. Res.* **285** (2020) 916–930.

[14] A. Hafezalkotob, M. Khodabakhsh, A. Saghaei and M. Eshghipour, Cooperation of advertising companies in social networks: A graph and game theory approaches. *Comput. Ind. Eng.* **125** (2018) 212–220.

[15] J. Han, S.P. Sethi, C.C. Siu and S.C.P. Yam, Co-op advertising in randomly fluctuating markets. *Prod. Oper. Manag.* **32** (2023) 1617–1635.

[16] X. He, A. Prasad and S.P. Sethi, Cooperative advertising and pricing in a dynamic stochastic supply chain: Feedback Stackelberg strategies. *Prod. Oper. Manag.* **18** (2009) 78–95.

[17] Y. He, Q. Gou, C. Wu and X. Yue, Cooperative advertising in a supply chain with horizontal competition. *Math. Probl. Eng.* **2013** (2013) 1–16.

[18] Y. He, H. Wang, Q. Guo and Q. Xu, Coordination through cooperative advertising in a two-period consumer electronics supply chain. *J. Retail. Consum. Serv.* **50** (2019) 179–188.

[19] T.-H. Ho and X. Su, Peer-induced fairness in games. *Am. Econ. Rev.* **99** (2009) 2022–2049.

[20] T.-H. Ho, X. Su and Y. Wu, Distributional and peer-induced fairness in supply chain contract design. *Prod. Oper. Manag.* **23** (2014) 161–175.

[21] Z. Huang and S.X. Li, Co-op advertising models in manufacturer–retailer supply chains: A game theory approach. *Eur. J. Oper. Res.* **135** (2001) 527–544.

[22] Z. Huang and P. Bai, Dynamic cooperative promotion in the presence of consumer reference effect with competing retailers. *J. Retail. Consum. Serv.* **60** (2021) 102441.

[23] Z. Huang, S.X. Li and V. Mahajan, An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. *Decis. Sci.* **33** (2002) 469–494.

[24] M. Huang, Y. Zhang, and H. Fan, Research on retail channel decisions considering consumer's fairness concern. *RAIRO:RO* **56** (2022) 23–47.

[25] D. Ivanov, A. Dolgui, J.V. Blackhurst and T.-M. Choi, Toward supply chain viability theory: from lessons learned through covid-19 pandemic to viable ecosystems. *Int. J. Prod. Res.* **61** (2023) 2402–2415.

[26] S. Jørgensen and G. Zaccour, A survey of game-theoretic models of cooperative advertising. *Eur. J. Oper. Res.* **237** (2014) 1–14.

[27] D.Kahneman, J.L. Knetsch and R.H. Thaler, Fairness and the assumptions of economics. *J. Bus.* **59** (1986) 285–300.

[28] S. Karray, G. Martín-Herrán and S.P. Sigué, Cooperative advertising for competing manufacturers: The impact of long-term promotional effects. *Int. J. Prod. Econ.* **184** (2017) 21–32.

[29] S. Karray, G. Martín-Herrán and G. Zaccour, Assessing the profitability of cooperative advertising programs in competing channels. *Int. J. Prod. Econ.* **187** (2017) 142–158.

[30] S. Karray, G. Martín-Herrán and S.P. Sigué, Cooperative advertising in competing supply chains and the long-term effects of retail advertising. *J. Oper. Res. Soc.* **73** (2022) 2242–2260.

[31] A.P. Kennedy, S.P. Sethi, C.C. Siu, S.C.P. Yam, Cooperative advertising in a dynamic three-echelon supply chain. *Prod. Oper. Manag.* **30** (2021) 3881–3905.

[32] B. Li, P.-W. Hou and Q.-H. Li, Cooperative advertising in a dual-channel supply chain with a fairness concern of the manufacturer. *IMA J. Manag. Math.* **28** (2017) 259–277.

[33] X. Li, Y. Li and W. Cao, Cooperative advertising models in O2O supply chains. *Int. J. Prod. Econ.* **215** (2019) 144–152.

[34] J. Li, X. Ji,, Z. Chen, and J. Wu, How cooperative advertising interacts with distributional contracts in a dual-channel system. *RAIRO:RO* **56** (2022) 1655–1684.

[35] M. Li, X. Zhang and B. Dan, Cooperative advertising contract design in a supply chain with an offline showroom under asymmetric information. *J. Oper. Res. Soc.* **73** (2022) 261–272.

[36] Y. Li, X. Zhao, J. Xie and W. Zhu, Inequality aversion in cooperative advertising in supply chain: an experimental study. *Int. J. Prod. Res.* **60** (2022) 7113–7135.

[37] Z.P. Li, J.J. Wang, S. Perera and J.J. Shi, Coordination of a supply chain with nash bargaining fairness concerns. *Transp. Res. Part E: Logist. Transp. Rev.* **159** (2022) 102627.

[38] D. Liang, J. Xie, W. Zhu and X. Zhao, Cooperative advertising in social networks with positive externalities. *Nav. Res. Logist.* **69** (2022) 702–714.

[39] Z. Liu, X.-X. Zheng, D.-F. Li, C.-N. Liao and J.-B. Sheu, A novel cooperative game-based method to coordinate a sustainable supply chain under psychological uncertainty in fairness concerns. *Transp. Res. Part E: Logist. Transp. Rev.* **147** (2021) 102237.

[40] C.H. Loch and Y. Wu, Social preferences and supply chain performance: An experimental study. *Manag. Sci.* **54** (2008) 1835–1849.

[41] R. Lotfi, Z. Sheikhi, M. Amra, M. AliBakhshi and G.W. Weber, Robust optimization of risk-aware, resilient and sustainable closed-loop supply chain network design with lagrange relaxation and fix-and-optimize. *Int. J. Logist. Res. Appl.* (2021) 1–41.

[42] R. Lotfi, B. Kargar and M. Rajabzadeh, Hybrid fuzzy and data-driven robust optimization for resilience and sustainable health care supply chain with vendor-managed inventory approach. *Int. J. Fuzzy Syst.* **24** (2022) 1216–1231.

[43] R. Lotfi, H. Nazarpour and A. Gharehbaghi, Viable closed-loop supply chain network by considering robustness and risk as a circular economy. *Environ. Sci. Pollut. Res.* **29** (2022) 70285–70304.

[44] R. Lotfi, M. Rajabzadeh and A. Zamani, Viable supply chain with vendor-managed inventory approach by considering blockchain, risk and robustness. *Ann. Oper. Res.* (2022) 1–20.

[45] F. Lu, W. Tang, G. Liu and J. Zhang, Cooperative advertising: A way escaping from the prisoner's dilemma in a supply chain with sticky price. *Omega* **86** (2019) 87–106.

[46] G. Martín-Herrán and S.P. Sigué, An integrative framework of cooperative advertising with reference price effects. *J. Retail. Consum. Serv.* **70** (2023) 103166.

[47] T. Nie and S. Du, Dual-fairness supply chain with quantity discount contracts. *Eur. J. Oper. Res.* **258** (2017) 491–500.

[48] J.A. Niederhoff and P. Kouvelis, Effective and necessary: Individual supplier behavior in revenue sharing and wholesale contracts. *Eur. J. Oper. Res.* **277** (2019) 1060–1071.

[49] K. Pan, Z. Cui, A. Xing and Q. Lu, Impact of fairness concern on retailer-dominated supply chain. *Comput. Ind. Eng.* **139** (2020) 106209.

[50] S.J. Sadjadi and A. Alirezaee, Impact of pricing structure on supply chain coordination with cooperative advertising. *RAIRO:RO* **54** (2020) 1613–1629.

[51] M.M. SeyedEsfahani, M. Biazaran and M. Gharakhani, A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains. *Eur. J. Oper. Res.* **211** (2011) 263–273.

[52] A. Sharma and D. Jain, A game theoretic analysis of dual-channel supply chain with nash bargaining fairness concern. *J. Bus. Ind. Mark.* **35** (2020) 244–259.

[53] A. Sharma, G. Dwivedi and A. Singh, Game-theoretic analysis of a two-echelon supply chain with option contract under fairness concerns. *Comput. Ind. Eng.* **137** (2019) 106096.

[54] R. Shoeleh, M. Seifbarghy and D. Pishva, Modelling pricing, vertical co-op advertising and quality improvement in a non-cooperative three-echelon supply chain using game theory approach. *RAIRO:RO* **53** (2019) 1937–1965.

[55] H. Sun and Y. Zhong, Carbon emission reduction and green marketing decisions in a two-echelon low-carbon supply chain considering fairness concern. *J. Bus. Ind. Mark.* **38** (2022) 905–929.

[56] J. Xia, W. Niu, X. Chen and L. Zhang, Investing in a shared supplier to encourage environmental responsibility under spillovers and demand uncertainty. *Transp. Res. Part E: Logist. Transp. Rev.* **174** (2023) 103134.

[57] Q. Xiao, L. Chen, M. Xie and C. Wang, Optimal contract design in sustainable supply chain: Interactive impacts of fairness concern and overconfidence. *J. Oper. Res. Soc.* **72** (2021) 1505–1524.

[58] R. Yan, Cooperative advertising, pricing strategy and firm performance in the e-marketing age. *J. Acad. Mark. Sci.* **38** (2010) 510–519.

[59] R. Yan, Z. Cao and Z. Pei, Manufacturer's cooperative advertising, demand uncertainty, and information sharing. *J. Bus. Res.* **69** (2016) 709–717.

[60] J. Yang, J. Xie, X. Deng and H. Xiong, Cooperative advertising in a distribution channel with fairness concerns. *Eur. J. Oper. Res.* **227** (2013) 401–407.

[61] L. Yu, X. He, J. Zhang and C. Xu, Horizontal cooperative advertising with advertising threshold effects. *Omega* **98** (2021) 102104.

[62] J. Zhang, Q. Gou, L. Liang and Z. Huang, Supply chain coordination through cooperative advertising with reference price effect. *Omega* **41** (2013) 345–353.

[63] T. Zhang, X. Guo and J. Hu, Cooperative advertising models under different channel power structure. *Ann. Oper. Res.* **291** (2020) 1103–1125.

[64] X.X. Zheng, Z. Liu, K.W. Li, J. Huang and J. Chen, Cooperative game approaches to coordinating a three-echelon closed-loop supply chain with fairness concerns. *Int. J. Prod. Econ.* **212** (2019) 92–110.

[65] Y. Zhou, M. Bao, X. Chen and X. Xu, Co-op advertising and emission reduction cost sharing contracts and coordination in low-carbon supply chain based on fairness concerns. *J. Clean. Prod.* **133** (2016) 402–413.

[66] Y.W. Zhou, J. Li and Y. Zhong, Cooperative advertising and ordering policies in a two-echelon supply chain with risk-averse agents. *Omega* **75** 201897–117.