RESEARCH ON THE BIG DATA INFORMATION SHARING IN CLOSED-LOOP SUPPLY CHAIN WITH TRIPLE-CHANNEL RECYCLING

HAN SONG, YANMING CAO, YI ZHANG AND YING DAI*

Abstract. Based on big data techniques to improve recycling efficiency and uncertain market information on whether manufacturers share, we construct a closed-loop supply chain where a manufacturer, a retailer, and a third-party collector compete for recycling at the same time. From the perspectives of manufacturer monopoly information market (Model-M), manufacturer and retailer share information (Model-MR), manufacturer and third-party collector share information (Model-MT), and supply chain tripartite shared information (Model-MRT), we build four types of Stackelberg game models dominated by the manufacturer to analyze the optimal strategies of the manufacturer in the four models and conduct numerical analysis to verify the effectiveness of the models. Research shows that as competition intensifies, the negative impact of big data technology costs on manufacturer decision-making and profitability diminishes. Furthermore, when the competitive intensity of recycling is wild, the optimal decision for the manufacturer is to share information only with the retailer. While competition is intense, the optimal strategy for the manufacturer is information monopoly. However, it is not always optimal for the manufacturer to share information with the third-party collector.

Mathematics Subject Classification. 90B06.

1. Introduction

With the development of the economy, people pay more attention to the environment and resources. More and more scholars are studying closed-loop supply chain (CLSC). A CLSC involves not only the forward flow process of new products but also the recycling process of used products. On the one hand, recycling and remanufacturing used products is conducive to the development of enterprises, enabling them to save costs and improve profits. For example, Xerox saved more than $200 million through remanufacturing in five years [21]. On the other hand, sustainable development can help consumers save money [37]. Hence, recycling is beneficial for both enterprises and consumers.

Generally, manufacturers, retailers, and third-party collectors may recycle used products [22]. In the past, most studies have focused on a single recycling channel. There are three main recycling channels, namely manufacturer recycling, retailer recycling, or independent third-party collector recycling. However, due to development needs, the traditional single recycling channel has yet to meet the market demand. Therefore, it is important for the company to design an appropriate recycling channel. Then, there has been an increasing number of studies...
on dual-channel recovery. However, triple-channel recycling, in which manufacturers, retailers and third-party collectors recycle jointly, has been less studied. In fact, triple-channel recycling exists in real life. For example, Apple's recycling program: users can mail their no-longer-used Apple devices, such as iPhone, iPad, Mac, Apple Watch, etc. directly to Apple or bring them to Apple retail stores for recycling. In addition, Apple partners with a number of third-party recyclers to provide additional recycling options, such as Apple’s partnership with Sims Lifecycle Services, a specialized global e-waste recycling and resource management company. Another example is Dell’s own direct recycling service, which allows users to visit Dell’s recycling website, submit a recycling request on that site, and then send the no-longer-needed device back to Dell. Dell has also partnered with retailer Staples to launch the “Dell Reconnect” program. Under this program, consumers can bring electronic devices and cartridges they no longer use to Staples stores. In addition, Dell has partnered with Electronic Recyclers International (ERI), a specialized electronic equipment recycling company, to recycle equipment. However, in the process of triple-channel recycling, there will inevitably be competition for recycling. It is therefore meaningful to study competition between recycling channels.

In addition, manufacturers struggle to collect enough used products to meet set targets due to uncertain demand, i.e., uncertainty in the amount of recycling, recycling location, and recycling time. With the development of information technology, the application of big data technology (BDT) can effectively solve problems such as demand uncertainty in the recycling process and help enterprises make better decisions. For example, Mani et al. [18] believed that companies can predict various social issues through big data analysis, thereby reducing supply chain risks. Addo-Tenkorang and Helo [1] believed that large amounts of data can create new opportunities to capture more value. Mageto [17] argued that big data can detect those supply chain components and members whose activities are unsustainable and take corrective action. Nowadays, BDT is widely used in reality, such as P&G, one of the world’s largest manufacturers, uses BDT to analyze consumer buying and usage behavior. This data helps them to understand the life cycle of products and when consumers typically discard or need to replace products, which helps to develop sustainability strategies, including recycling and remanufacturing.

Moreover, by sharing data information, resources can be allocated more rationally, social costs can be saved and more wealth can be created. Typically, information is shared by retailers. However, manufacturers can also share information. For example, IBM shares supply chain information with suppliers and distributors, which helps to improve supply chain visibility and collaboration.

Therefore, based on the above limitations, our study considers the competition between recycling channels in a CLSC consisting of a manufacturer, a retailer, and a third-party collector. The manufacturer invests in BDT to gain information about the uncertain demand of used products and is allowed to share information with other members. We construct four Stackelberg models to study the manufacturer’s optimal information sharing strategy.

The research questions of our study are given as follows:

(1) How does BDT in triple-channel recycling affect the manufacturer’s decisions and CLSC’s members’ profits?
(2) How does recycling competition among the manufacturer, the retailer, and the third-party collector affect the CLSC members’ performance?
(3) What is the optimal information sharing strategy?

The innovation and contribution are as follows:

(1) The triple-channel recycling model, where the manufacturer, the retailer, and the third-party collector all participate in recycling.
(2) The manufacturer invests in BDT for market information.
(3) The manufacturer considers information sharing.
(4) Competition among collectors was considered.

The rest of this paper is as follows: Section 2 reviews the literature related to this paper and addresses the research gap. Section 3 describes the research problem and solves the models. Section 4 discusses the optimal
strategy of manufacturer information sharing and makes a comparative analysis of the equilibrium decision and performs the sensitivity analysis. In Sections 5 and 6 managerial implications, practical insights, and conclusions are drawn.

2. Literature review

We study the information sharing problem of manufacturers in a triple recycling channel with channel competition. The existing studies on CLSC, recycling channel, BDT and information sharing, are most relevant to our work.

Closed-loop supply chain (CLSC)

As for CLSC, scholars have studied it from different perspectives. For example, Zheng et al. [35] studied two recycling cooperation models, the recycling alliance model and the cost-sharing model. An analytical approach was developed to determine whether and how manufacturers and retailers in a CLSC enter into a recycling cooperative agreement. Ma and Huang [16] considered the life-cycle attributes of products and explored the situation where manufacturers tend to invest in green innovation and establish alliances with other supply chain members. In a two-cycle CLSC consisting of a manufacturer, a retailer and a third-party collector, the Stackelberg game method is used to analyze and compare the optimal decisions and benefits of CLSC members. Song et al. [24] analyzed a CLSC in which the operator sets the price and recovery effort, and the supplier determines the wholesale price. The rent and recovery strategies of integrated and decentralized channels are analyzed. Liu and Wang [12] studied the different government subsidy policies of the government and members of the CLSC and their impact on profits and social welfare. Wang et al. [26] considered a CLSC with a competitive recycling market (competing collectors) and a product market (new products and recycled products) and studied three competitive schemes selected by manufacturers. Georgiadis and Besiou [4] studied the operational characteristics of both environmental sustainability strategies and CLSC, as well as their interactions and types of impacts on the environment and economic sustainability. Long et al. [13] established a multiple remanufacturing mode model considering WTP heterogeneity or different remanufactured products in one-period and two-period CLSC scenarios to determine the best recycling and remanufacturing strategies of manufacturers. Ranjbar et al. [19] considered a three-level CLSC consisting of manufacturers, retailers, and third-party collectors. Evaluated optimal pricing and collection decisions through two competitive recycling channels: retailer collection and third-party collection. Zhao et al. [33] used the evolutionary game model to analyze the effectiveness of reward and punishment mechanism on the implementation of EPR system and established the producer-led reverse CLSC model under the effective conditions. Then, they compared and analyzed the channel choices of producers in implementing reverse supply chain under different reward and punishment mechanisms. Zhou et al. [36] considered multi-channel recycling, the conflict between sales channels and recycling channels, and dual trust in complex real-world environments. The corresponding pricing decision model is constructed for analysis and research, and the coordination and incentive mechanism of each node enterprise in the CLSC system is further discussed, so as to realize the overall optimization of the CLSC. Zu-Jun et al. [38] studied the interaction between different parties in a three-level CLSC consisting of a manufacturer, a retailer, and two collectors, and focused on how cooperative strategies affect CLSC decisions. Goli [5] proposed a comprehensive framework to design a blockchain-enabled CLSC. Lotfi et al. [14] considered a CLSC by taking into account sustainability, resilience, robustness, and risk aversion for the first time. The above literatures have studied CLSC from different perspectives. Considering the competitive relationship in the CLSC recycling channel, this paper studies the influence of competition intensity on CLSC.

Recycling channel

Most research about recovery channel focus on the single-channel recovery and the dual-channel recovery. First of all, research on single-channel recycling are as follows: Savaskan et al. [22] built three kinds of recycling modes i.e., the manufacturer direct recovery and the retailer recovery and the third-party collector recovery. Hong and Yeh [6] proposed the model of retailer recycling and non-retailer recycling and drew the conclusion that
retailer recycling was the optimal recycling mode. Wu et al. [28] considered a CLSC consisting of a manufacturer and a retailer. Each member as channel leader has three different channels to collect the used products, which are manufacturer (M channel), retailer (R channel) and third party (T channel). A mathematical model was developed to study the performance of CLSC under different combinations of channel leadership and recycling channels. Through comparative analysis, they founded that M channel is the most effective recycling channel. Zheng et al. [34] considered three recycling channel structures: manufacturer collection (Model M), retailer collection (Model R), and third-party collection (Model C) structures. They found that manufacturers and retailers can get more profits in Models M and R.

Secondly, research on dual-channel recycling are as follows: Hong et al. [7] respectively compared CLSC decision-making problems under three modes: co-recycling between manufacturer and retailer, co-recycling between retailer and third-party collector, and co-recycling between manufacturer and third-party collector. De Giovanni et al. [3] explored an incentive strategy model for both manufacturer and retailer to recycle waste products. Zhang et al. [30] researched a WEEE CLSC composed of manufacturer, retailer and third-party collector, established four dynamic game models combined with the dual-channel recycling model and government fund policies, and analyzed the impact of government fund policies on CLSC decisions.

Finally, there are also some literatures compared the single channel recycling and dual-channel recycling. For example, Li et al. [11] explored three recycling models, i.e., the single offline recycling channel (Model A), the single online recycling channel (Model B) and the mixed recycling channel of online and offline recycling (Model C). They obtained the optimal pricing and recycling quantity decision, as well as the corresponding profit. Zhang et al. [31] proposed a reward and punishment mechanism and made a comparative analysis of six recycling modes by using Stackelberg game theory. Including manufacturer recycling (Model-M), manufacturer recycling through retailer (Model-R), manufacturer recycling through third-party collector (Model-TP), manufacturer and retailer recycling together (Model-M&R), manufacturer and third-party collector recycling together (Model-M&TP) and retailer and third-party collector recycling together (Model-R&TP). Suvadarshini et al. [25] studied different CLSC return channel structures under the influence of collection efficiency, individual rationality and information asymmetry.

Through the review of the above literature, it is found that much research on the recycling channel selection of CLSC focus on the single channel recycling. However, with the requirements of the environment and a large number of waste products in urgent need of recycling market, the traditional single channel recycling cannot meet the requirements of need. Hence, achievements have been made in the research on dual-channel recycling. In addition, under the attraction of considerable profits in the recycling market and the difficulties of effective recycling of all waste products, the triple-channel recycling mode is common in the market. However, there are few research on the triple-channel recycling. Therefore, based on the deficiency of research, our paper explores the triple-channel recycling mode in which a manufacturer, a retailer and a third-party collector all participate in the recycling activities.

**Big Data Technology (BDT) and information sharing**

With the development of BDT, more and more enterprises use BDT to obtain consumers’ or products’ information. Therefore, there are much research on BDT in supply chain. Ma and Hu [15] constructed the differential game among manufacturers, retailers and Internet service platforms. By using Behrman’s continuous dynamic programming theory, the optimal feedback strategies of price and big data marketing efforts, second-hand product return rate are obtained under three business models. Mageto [17] indicated new developments in information and communication technologies, especially big data analytics (BDA), can help create new insights that can detect parts and members of a supply chain whose activities are unsustainable and take corrective actions. Mani et al. [18] had studied the application of BDA to identify and mitigate social issues in supply chains. Hu et al. [8] applied game theory to study the decision-making and coordination of green supply chains for big data targeted advertising by online retailers. In addition, research on BDT in the field of product recycling has also made some progress. For example, Kong and Lu [10] studied the status quo and strategies of packaging recycling for online shopping in China under the environment of big data. Yu et al. [29] built a recycling platform
for used batteries of new energy vehicles based on BDT. Combined with the comprehensive characteristics of big data information, the operation mechanism of big data battery recycling platform is analyzed. In a CLSC, Jiao et al. [9] indicated that a data-driven approach is proposed to generate reliable CLSC designs that reduce the burden of uncertainty and greenhouse gas emissions.

The above research mostly focused on the role of BDT in the supply chain. However, there is a huge value in the information obtained by using the BDT. Whether this information is shared among supply chain enterprises is crucial to the performance of supply chain. Shin and Zeevi [23] investigated the problem of reviews information sharing between a retail platform and a manufacturer in a supply chain. Bian et al. [2] studied manufacturer-retailer bilateral information sharing in two competing supply chains, where both the manufacturer and the retailer have partial demand information. Raweewan and Ferrell [20] investigated information sharing in supply chain collaboration. Wang et al. [27] developed a game theory model of the dual-channel CLSC to determine whether the retailer has incentive to share private demand information with the manufacturer and/or the collector, and how different information sharing modes affect the operational performance of chain members under different channel power structures. Similarly, Zhang et al. [32] regarded a retailer as the holder of demand information and study information sharing models under different channel power structures. However, the information holders in the above studies are all retailers. Therefore, different from the above studies, our paper considers the manufacturer invest BDT to be the information holder and study his optimal information sharing decision.

**Research gap**

A summary of the relevant works is presented in Table 1. This research tries to design a novel triple-channel recycling CLSC by applying BDT to improve recycling efficiency and by considering information sharing and recycling competition among collectors. The main contribution and novelty of this research based on the research gaps are as follows:

1. The triple-channel recycling model.
2. The manufacturer invests in BDT for market information.
3. The manufacturer considers information sharing.
4. Considering competition among the three recycling channels.

### 3. Problem statement

**Problem statement**

In this paper, we consider a CLSC with a triple recycling channel, that is, consisting of a manufacturer, a retailer, and a third-party collector. The manufacturer sells new products to the consumer through the retailer. Then, the manufacturer works with the retailer and the third-party collector to recycle the used products. In addition, the manufacturer invests in BDT to improve the recovery efficiency of used products. The manufacturer relies on BDT to obtain information about used products and can share the information with other collectors in reverse logistics. However, the manufacturer’s information-sharing decision is critical to improve his profits. Hence, there are four cooperation strategies as follows (Figs. 1 and 2):

1. The manufacturer monopolizes the information market after the BDT expansion (Model-M).
2. The manufacturer shares the recycling market information only with the retailer (Model-MR).
3. To promote professional recycling activities, the manufacturer allows the third-party collector to have the recycling market information driven by BDT (Model-MT).
4. To stimulate overall recycling activities, all collectors participate in the expanded recycling market of BDT (Model-MRT).
Table 1. Comparison between the related literature with our article.

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<th>Related paper</th>
<th>*SC</th>
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Assumptions

Assumption 1. In this paper, we use the Stackelberg game theory and assume that the manufacturer is the leader, both the retailer and the third-party collector are followers. In the forward logistics process, it is assumed that the manufacturer produces new products at the cost $c_m$ per unit and sells it to the retailer at the wholesale price $w$ per unit, while the retailer sells it directly to the consumer at the retail price $p$ per unit. For reverse logistics, we assume that the manufacturer, the retailer, and the third-party collector all collect the used products and bring unit residual value $A$ to the manufacturer. Then, the manufacturer pays the retailer and third-party collector $A$ unit repurchase price.
Assumption 2. According to Savaskan et al. [22], we assume that the market demand is linear, i.e., \( D = \alpha - \beta p \). Where \( \alpha \) represents the potential market demand, \( \beta \) represents the elasticity coefficient of consumers to the retail price.

Assumption 3. In reality, there are cases of the triple-channel recycling. For example, Apple collects old devices directly from consumers or allows consumers to bring used devices to Apple retail stores for recycling. Additionally, Apple turns over recycling activities to the third-party collector such as Sims Lifecycle Services. In the process of recycling used products, however, there is inevitable competition between recycling channels. Hence, we use \( \varepsilon \) to reflect the intensity of competition between recycling channels.

Assumption 4. Referring to Zhang et al. [30], we assume that the recycling costs for the manufacturer, the retailer, and the third-party collector are

\[
\Pi_m = (w - c_m)D + A(D_m + D_r + D_t) - \mu(\tau_m^2 + \varepsilon\tau_r^2 + \varepsilon\tau_t^2) - ke^2 - AD_r - AD_t,
\]

\[
\Pi_r = (p - w)D + AD_r - \mu(\tau_r^2 + \varepsilon\tau_m^2 + \varepsilon\tau_t^2),
\]

\[
\Pi_t = AD_t - \mu(\tau_t^2 + \varepsilon\tau_m^2 + \varepsilon\tau_r^2).
\]

Assumption 5. In reality, Samsung, Dell and others are actively utilizing BDT in the field of electronics recycling. They collect data such as usage data of devices and return information of recycled devices in order to analyze consumer recycling behavior and trends. Automobile manufacturers such as Tesla, Ford, and General Motors also utilize BDT to understand recycling trends for abandoned vehicles. Therefore, we assume that the BDT investment level \( e \) is determined by the manufacturer. The more BDT is invested in, the more quantity will be collected. As the manufacturer invests in BDT, he has to incur additional costs \( ke^2 \).

Assumption 6. According to Hu et al. [8], the recycling amount of the manufacturer, the retailer and the third-party collector in the information sharing model based on BDT is respectively \( D_m = \tau_m(\theta D + \lambda c) \), \( D_r = \tau_r(\theta D + \lambda c) \), \( D_t = \tau_t(\theta D + \lambda c) \). If the manufacturer does not share the information, then the retailer and the third-party collector recycle volume becomes \( D_r = \tau_r(\theta D + \lambda c), D_t = \tau_t(\theta D + \lambda c) \).

Assumption 7. Profits function of the manufacture, the retailer and the third-party collector are

\[
\Pi_m = (w - c_m)D + A(D_m + D_r + D_t) - \mu(\tau_m^2 + \varepsilon\tau_r^2 + \varepsilon\tau_t^2) - ke^2 - AD_r - AD_t,
\]

\[
\Pi_r = (p - w)D + AD_r - \mu(\tau_r^2 + \varepsilon\tau_m^2 + \varepsilon\tau_t^2),
\]

\[
\Pi_t = AD_t - \mu(\tau_t^2 + \varepsilon\tau_m^2 + \varepsilon\tau_r^2).
\]
### Notation list

**Sets**

- $i$: Sets of CLSC' member $i = I \in \{m, r, t\}$.
- $j$: Sets of model $j = J \in \{M, MR, MT, MRT\}$.

**Parameters**

- $\alpha$: Potential market demand.
- $\beta$: The sensitivity of demand of the retail price.
- $c_m$: The unit manufacturing costs of the product.
- $\epsilon$: The competition intensity between the recycling channels. $0 < \epsilon < 1$.
- $\mu$: The cost coefficient of the recycling rate.
- $k$: The cost coefficient of BDT investment.
- $\theta$: Effective recycling coefficient based on market demand.
- $\lambda$: The effective coefficient of BDT for the recyclable market.
- $A$: The unit value of recycled products.
- $D$: The market demands.
- $D_i$: The recycling quantity of member $i$.
- $\Pi_j^i$: The profit of channel member $i$ in model $j$.

**Decision variables**

- $w_j$: The unit wholesale price.
- $p_j$: The unit retail price.
- $e_j$: The investment level of BDT.
- $\tau_j^i$: The recycling rate of channel member $i$ in model $j$.

### Mathematical model

There are four information-sharing models for the manufacture in the CLSC. In a Stackelberg game led by the manufacture, the sequence of moves is as follows: firstly, the manufacturer decides the investment level of BDT $e$, the recycling rates of used products $\tau_m$ and the wholesale prices $w$ to maximize his profits $\Pi_m$. Secondly, the retailer makes decisions on the retail prices $p$ and the recycling rates of used products $\tau_r$ to maximize his profits $\Pi_r$. Thirdly, the third-party collector updates the used product recycling rates $\tau_t$ to maximize his profits $\Pi_t$. All proofs are listed in Appendix A.

**Model-M**

In this model, the manufacture withholding the information market expanded by BDT from other collectors. The profits functions of the manufacturer, the retailer and the third-party collector are:

\[
\begin{align*}
\text{Max}(e, w, \tau_m)\Pi_m^M &= (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda \epsilon) - \mu(\tau_m^2 + \epsilon\tau_r^2 + \epsilon\tau_t^2) - ke^2 \\
\text{Max}(p, \tau_r)\Pi_r^M &= (p - w)(\alpha - \beta p) + A\tau_r\theta(\alpha - \beta p) - \mu(\tau_r^2 + \epsilon\tau_m^2 + \epsilon\tau_t^2) \\
\text{Max}(\tau_t)\Pi_t^M &= A\tau_t\theta(\alpha - \beta p) - \mu(\tau_t^2 + \epsilon\tau_m^2 + \epsilon\tau_r^2).
\end{align*}
\]

**Proposition 1.** If $k > \frac{1}{6} \frac{A^2 \lambda^2 (\alpha^2 \beta^2 - 4\mu)}{\lambda^2 (\beta^2 - \frac{4}{3} \mu) \mu}$ and $\mu > \frac{3}{8} A^2 \beta^2$ are satisfied, then we can get the optimal values when there is no information sharing in the CLSC:

\[
\begin{align*}
\tau_m^{M^*} &= \frac{A\theta(\alpha - \beta c_m)(A^2 \lambda^2 - 4k\mu)}{4\lambda^2 \beta(\epsilon - \frac{1}{2})^2 \theta^2 A^4 - 16(k\theta^2(\epsilon - \frac{3}{4})\beta - \frac{1}{2} \lambda^2)\mu A^2 - 32k\mu^2} \\
\tau_r^{M^*} &= \frac{A\theta(A^2 \lambda^2 - 4k\mu)(\alpha - \beta c_m)}{4A^4 \beta^2 \lambda^2 (\epsilon - \frac{1}{2}) - 16\mu(k\theta^2(\epsilon - \frac{3}{4})\beta - \frac{1}{2} \lambda^2) A^2 - 32k\mu^2}.
\end{align*}
\]
Figure 2. Four triple-channel recycling models. (a) Model-M. (b) Model-MR. (c) Model-MT. (d) Model-MRT.

\[ P^* = \frac{\alpha \theta^2 \lambda^2 \left( \epsilon - \frac{1}{2} \right) A^4 - 4\mu(k\alpha(\epsilon - \frac{3}{4}) \theta^2 - \frac{1}{8}c_m \lambda^2)A^2 - 2kc_m \mu^2}{\beta(\lambda^2(\epsilon - \frac{1}{2})A^2 - 4\mu k(\epsilon - \frac{3}{4}))A^2\theta^2 \beta + 2A^2 \lambda^2 \mu - 8k\mu^2} \]  

\[ r_m^* = \frac{2\beta^2 \lambda^2(\epsilon - \frac{1}{2})A^4 - 8\mu(k\beta(\epsilon - \frac{3}{4}) \theta^2 - \frac{1}{2} \lambda^2)A^2 - 16k\mu^2}{2\beta^2 \lambda^2(\epsilon - \frac{1}{2})A^4 - 8\mu(k\beta(\epsilon - \frac{3}{4}) \theta^2 - \frac{1}{2} \lambda^2)A^2 - 16k\mu^2} \]  

\[ e^* = \frac{A^2 \theta^2 \lambda^2(\epsilon - 1)(A^2 \lambda^2 - 4k\mu) \beta^2 + (3\theta^2(\epsilon - \frac{3}{4}) \lambda^2 \alpha_A^4 - 12\mu(k\alpha(\epsilon - \frac{3}{4}) \theta^2 - \frac{1}{4}c_m \lambda^2)A^2}{\beta(\lambda^2(\epsilon - \frac{1}{2})A^2 - 4k\mu(\epsilon - \frac{3}{4}))\theta^2 \beta + 2A^2 \lambda^2 \mu - 8k\mu^2} \]  

\[ w^* = \frac{1}{4} \frac{1}{\beta(\lambda^2(\epsilon - \frac{1}{2})A^2 - 4k\mu(\epsilon - \frac{3}{4}))\theta^2 \beta + 2A^2 \lambda^2 \mu - 8k\mu^2} \]
Proposition 2. If \( \frac{3A^2}{4} \mu (A^2 \theta^2 + 8k\mu) \) and \( \mu > \frac{3\alpha^2}{8} A^2 \beta \theta^2 \) are satisfied, we can derive the optimal values of the Model-MR:

\[
\Pi^*_m = \frac{1}{4} \mu (\beta c_m - \alpha)^2 (A^2 \lambda^2 - 4k\mu)
\]
\[
\Pi^*_r = \frac{1}{16} \mu \left( \theta^2 (A^2 \lambda^2 (\epsilon - \frac{1}{2}) A^2 - 4k (\epsilon - \frac{1}{2}) \mu) + 2A^2 \lambda^2 \mu - 8k\mu^2 \right) (\beta c_m - \alpha)^2
\]
\[
\Pi^*_t = \frac{1}{16} \theta^2 \mu A^2 \left( -\lambda^2 (\epsilon - 1) A^4 + 8k\lambda^2 (\epsilon - 1) A^2 - 32\mu^2 (\epsilon - \frac{1}{2}) k^2 \right) (\beta c_m - \alpha)^2.
\]

Model-MR

Typically, manufacturers sell their products to consumers through retailers. In this model, considering the trading relationship, the manufacturer shares the recycling market information with the retailer. Therefore, the profits of the manufacturer, the retailer, and the third-party collector are formulated as follows:

\[
\begin{align*}
\text{Max}(c, w, \tau_m) \Pi^*_{M} & = (w - c_m)(\alpha - \beta p) + A\tau_m(\alpha - \beta p) \theta + \lambda e - \mu \left( \tau_m^2 + \epsilon \tau_m^2 + \epsilon \tau_m^2 \right) - ke^2 \\
\text{Max}(p, \tau_r) \Pi^*_{R} & = (p - w)(\alpha - \beta p) + A\tau_r(\alpha - \beta p) \theta + \lambda e - \mu \left( \tau_r^2 + \epsilon \tau_r^2 + \epsilon \tau_r^2 \right) \\
\text{Max}(\tau_t) \Pi^*_{T} & = A\tau_t(\alpha - \beta p) - \mu \left( \tau_t^2 + \epsilon \tau_t^2 + \epsilon \tau_t^2 \right).
\end{align*}
\]

Proposition 2. If \( k > \frac{1}{4} \mu (\frac{3A^2}{4} \theta^2 + 8k\mu) \) and \( \mu > \frac{3\alpha^2}{8} A^2 \beta \theta^2 \) are satisfied, we can derive the optimal values of the Model-MR:

\[
\begin{align*}
\tau^*_m & = A\theta ((\epsilon - 1) A^2 + 4k\mu)(\alpha - \beta c_m) \\
\tau^*_r & = 3\theta^2 A^2 ((\epsilon - 1) A^2 + 16((\frac{1}{2} \epsilon - \frac{1}{2}) \lambda^2 + k\beta^2 (\epsilon - \frac{1}{2})) A^2 + 32k\mu^2 \\
\tau^*_t & = A\theta ((\alpha - \beta c_m)(A^2 \lambda^2 + 4k\mu) A^2 + 3\theta^2 A^2 ((\epsilon - \frac{1}{2}) A^2 + 16((\frac{1}{2} \epsilon - \frac{1}{2}) \lambda^2 + k\beta^2 (\epsilon - \frac{1}{2})) A^2 + 32k\mu^2 \\
p^*_m & = 2A^2 \lambda^2 (\epsilon - \frac{1}{2}) A^2 + 16((\frac{1}{2} \epsilon - \frac{1}{2}) \lambda^2 + k\beta^2 (\epsilon - \frac{1}{2}) \lambda^2 A^2 + 32k\mu^2 \\
p^*_r & = (\alpha - \beta c_m)(A^2 \lambda^2 + 4k\mu) A^2 + 2A^2 \lambda^2 (\epsilon - \frac{1}{2}) A^2 + 16((\frac{1}{2} \epsilon - \frac{1}{2}) \lambda^2 + k\beta^2 (\epsilon - \frac{1}{2}) \lambda^2 A^2 + 32k\mu^2 \\
p^*_t & = 1 + (c_m (\epsilon - 1) A^2 + 3\alpha (\epsilon - \frac{1}{2}) \theta^2 k) \beta + \alpha \lambda^2 (\epsilon - 1) \mu A^2 + 16k\mu^2 (\beta c_m + \alpha) \\
p^*_r & = 3 \beta (\theta^2 A^2 ((\epsilon - \frac{1}{2}) A^2 + 16((\frac{1}{2} \epsilon - \frac{1}{2}) \lambda^2 + k\beta^2 (\epsilon - \frac{1}{2}) \lambda^2 A^2 + 32k\mu^2 \\
p^*_t & = \frac{1}{3} \left( (\beta c_m - \alpha)^2 \mu (\epsilon - 1) \lambda^2 A^2 + 4k\mu) \right) \\
P^*_m & = \frac{1}{3} \left( (\beta c_m - \alpha)^2 \mu (\epsilon - 1) \lambda^2 A^2 + 4k\mu) \right) \\
P^*_r & = \frac{1}{9} \left( (\beta c_m - \alpha)^2 \mu (\epsilon - 1) \lambda^2 A^2 + 4k\mu) \right) \\
P^*_t & = \frac{1}{9} \left( (\beta c_m - \alpha)^2 \mu (\epsilon - 1) \lambda^2 A^2 + 4k\mu) \right)
\end{align*}
\]
Model-MT

In order to promote professional recycling activities, the manufacturer allows the third-party collector to have the recycling market information driven by BDT. Therefore, the manufacturer's, the retailer's, and the third-party collector's problem are given, respectively:

\[
\begin{align*}
\max (e, w, \tau_m) \Pi^\text{MT}_m &= (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_m + \varepsilon \tau^2_r + \varepsilon \tau^2_t) - k e^2 \\
\max (p, \tau_r) \Pi^\text{MT}_r &= (p - w)(\alpha - \beta p) + A\tau_r(\alpha - \beta p) - \mu(\tau^2_r + \varepsilon \tau^2_m + \varepsilon \tau^2_t) \\
\max (\tau_t) \Pi^\text{MT}_t &= A\tau_t((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_t + \varepsilon \tau^2_m + \varepsilon \tau^2_r).
\end{align*}
\]

Proposition 3. If \( k > \frac{1}{6} A^2 \lambda^2 (A^2 \beta^2 - 4\mu) \) and \( \mu > \frac{3}{8} A^2 \beta^2 \) are satisfied, the optimal solutions of values of are derived as follows:

\[
\begin{align*}
\tau^\text{MT}_t &= \frac{A\theta(\alpha - \beta c_m)(A^2 \varepsilon \lambda^2 - 4k\mu)}{3(\varepsilon - \frac{\lambda}{3})\beta \lambda^2 \beta^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\frac{\varepsilon}{2} - \frac{1}{2})\lambda^2}{4})A^2 - 32k\mu^2} \\
\tau^\text{MT}_r &= \frac{A\theta(\beta c_m - \alpha)((\varepsilon - 1)\lambda^2 A^2 + 4k\mu)}{3(\varepsilon - \frac{\lambda}{3})\beta \lambda^2 \beta^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\frac{\varepsilon}{2} - \frac{1}{2})\lambda^2}{4})A^2 - 32k\mu^2} \\
p^\text{MT}_r &= \frac{\beta((\varepsilon - \frac{\lambda}{3})\beta \lambda^2 \beta^2 A^4 + \frac{(\beta \lambda^2 \beta^2 A^4)}{2}+ \frac{(\beta \lambda^2 \beta^2 A^4)}{2}(\beta c_m + 3\alpha))}{\beta((\varepsilon - \frac{\lambda}{3})\beta \lambda^2 \beta^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2} \\
\tau^\text{MT}_m &= \frac{3\beta \lambda^2 \lambda^2 \lambda^2 (\varepsilon - \frac{\lambda}{3})A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2}{3(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2} \\
e^\text{MT}_r &= \frac{-A^2 \lambda \mu (\varepsilon - \frac{\lambda}{2}))((\beta c_m - \alpha))}{3(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2} \\
u^\text{MT}_r &= \frac{1}{3}(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2 \\
\Pi^\text{MT}_m &= \frac{1}{3}(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2 \\
\Pi^\text{MT}_r &= \frac{4}{9}(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2 \\
\Pi^\text{MT}_t &= \frac{2}{9}(\beta \lambda^2 \lambda^2 \lambda^2 A^4 - 16\mu(k\beta \theta^2(\varepsilon - \frac{\lambda}{4}) + \frac{(\beta c_m + 3\alpha)}{2}))A^2 - 32k\mu^2.
\end{align*}
\]

Model-MRT

To stimulate overall recycling activity, all recyclers participate in the expanded recycling market of BDT. Thus, the problem of Model-MRT can be stated as follows:

\[
\begin{align*}
\max (e, w, \tau_m) \Pi^\text{MRT}_m &= (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_m + \varepsilon \tau^2_r + \varepsilon \tau^2_t) - k e^2 \\
\max (p, \tau_r) \Pi^\text{MRT}_r &= (p - w)(\alpha - \beta p) + A\tau_r(\alpha - \beta p) + \lambda e) - \mu(\tau^2_r + \varepsilon \tau^2_m + \varepsilon \tau^2_t) \\
\max (\tau_t) \Pi^\text{MRT}_t &= A\tau_t((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_t + \varepsilon \tau^2_m + \varepsilon \tau^2_r).
\end{align*}
\]
Proposition 4. If \( k > \frac{1}{4} A^2 \theta^2 (2 + 8 \mu) \) and \( \mu > \frac{3}{4} A^2 \theta^2 \) are satisfied, the model-MRT obtains the maximum values, and the optimal solution values are derived as follows:

\[
\begin{align*}
\tau_{t}^{\text{MRT}} & = \frac{A \theta (\alpha - \beta c_m) (\lambda^2 (e - 1) A^2 - 4k \mu)}{\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2} \\
\tau_{r}^{\text{MRT}} & = \frac{A \theta (\alpha - \beta c_m) (\lambda^2 (e - 1) A^2 - 4k \mu)}{\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2} \\
p^{\text{MRT}} & = \frac{(\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2) \beta}{(\lambda^2 (e - 1) A^2 - 4k \mu) (\alpha - \beta c_m) A \theta} \\
e^{\text{MRT}} & = \frac{6A^2 \lambda \mu \theta (e - \frac{3}{4}) (\alpha - \beta c_m)}{\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2} \\
w^{\text{MRT}} & = \frac{(\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2) \beta}{(\alpha - \beta c_m) (\lambda^2 (e - 1) A^2 - 4k \mu) A \theta} \\
\Pi_{m}^{\text{MRT}} & = -\frac{2 ((e - \frac{1}{2}) \lambda^2 A^2 + 2k \mu) (\beta c_m - \alpha)^2 \mu}{(\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2) \beta} \\
\Pi_{r}^{\text{MRT}} & = -\frac{6 \mu (\beta c_m - \alpha)^2 ((e - \frac{1}{2}) (e - 1)^2 \theta^2 \lambda^2 \beta^2 A^2 - 4 \mu \lambda^2 ((e - 1) \theta^2 k (e - \frac{1}{2}) \beta + \frac{3}{4} (e - \frac{1}{2})^2 \lambda^2) A^4 + \frac{3}{4} k^2 \mu^2) \beta}{(\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2)^2} \\
\Pi_{t}^{\text{MRT}} & = \frac{2 (\frac{1}{2} - e) \mu \theta^2 A^2 ((\alpha - \beta c_m) A^2 - 4k \mu) (\beta c_m - \alpha)^2}{(\beta \theta^2 \lambda^2 (e - 1)^2 A^4 - 16 ((e - \frac{1}{2}) \lambda^2 + k \beta \theta^2 (e - \frac{3}{4})) \mu A^2 - 32k \mu^2)^2}.
\end{align*}
\]

Solution approach

We build four Stackelberg game models, which are solved using backward induction. In addition, we solved the models and performed sensitivity analysis with the help of the mathematical software Maple 2021. In these models, the manufacturer, as a leader, makes decisions first. Then, the retailer and the third-party collector, as followers, make their own decisions based on the manufacturer’s decisions.

4. RESULTS

Description

In Section 3, the optimal profits of the manufacturer, the retailer, and the third-party collector under the four models are calculated respectively. As the manufacturer invests in BDT and decides to share information, we only compare the decisions and profits of the manufacturer in this part. We find that competition among recycling channels always harms the manufacturer. However, the impact of BDT inputs on the manufacturer varies with the intensity of competition. Moreover, for the manufacturer, his optimal information sharing decision in most cases is to share information only with the retailer. It is instructive in that the retailer is closer to consumers, and therefore, it is more beneficial for the retailer to have more effective market information. All proofs are listed in Appendix B.
Computational results

**Proposition 5.** The optimal recycling rates in the four models satisfy:

1. \( \frac{\partial r_{m}^{*}}{\partial \varepsilon} < 0; \frac{\partial r_{m}^{MT\ast}}{\partial \varepsilon} < 0; \frac{\partial r_{m}^{MRT\ast}}{\partial \varepsilon} < 0. \)

2. \( \frac{\partial r_{m}^{*}}{\partial \kappa} < 0; \frac{\partial r_{m}^{MT\ast}}{\partial \kappa} < 0; \) if \( 0 < \varepsilon < \frac{1}{2}, \frac{\partial r_{m}^{MT\ast}}{\partial \kappa} < 0, \) if \( \frac{1}{2} < \varepsilon < 1, \frac{\partial r_{m}^{MT\ast}}{\partial \kappa} > 0; \) if \( 0 < \varepsilon < \frac{2}{3}, \frac{\partial r_{m}^{MRT\ast}}{\partial \kappa} < 0, \) if \( \frac{2}{3} < \varepsilon < 1, \frac{\partial r_{m}^{MRT\ast}}{\partial \kappa} > 0. \)

**Proposition 5(1)** shows that the recycling rates of the manufacturer in the four models are all decreasing with the increase of the competition intensity between recycling channels. That is to say, the competition will reduce the manufacturer’s enthusiasm for recycling used products regardless of whom he shares information with.

**Proposition 5(2)** indicates that the recycling rates of the manufacturer in the Models-M and -MR are decreasing with the BDT investment costs coefficient without conditions. Because when the manufacturer monopolizes the BDT information, he has a recycling advantage and competitiveness over other recyclers. At this point, excessive BDT costs are not necessary for the manufacturer. As a result, the higher the costs of the BDT, the less effort the manufacturer puts into the BDT, leading to a decrease in his recycling rate. When the manufacturer shares information with the retailer that has a channel advantage, the retailer becomes more competitive and undermines the unique advantage that BDT gives to the manufacturer, leading to a threat to the manufacturer’s recycling activities. Therefore, regardless of the amount of BDT investment, the impact on the manufacturer’s recycling volume is negative. However, monotonically decreasing among the optimal decisions in Models-MT and -MRT requires that the competition intensity satisfies some conditions. When the manufacturer shares information with the third-party collector and the competition intensity is less than \( 1/2, \) the manufacturer shares the acquired BDT information, which will weaken his competitiveness and bring benefits to the third-party collector. Therefore, the greater the costs of BDT, the more reluctant the manufacturer is to invest in BDT, which further leads to a lower recycling rate for the manufacturer. However, when the competitive intensity exceeds \( 1/2, \) the retailer who does not have the information advantage is less motivated to recycle, and the overall competitiveness of the third-party collector is weaker than that of the retailer. At this point, even if the costs of BDT increase, the manufacturer is willing to invest in BDT to recycle larger quantities of used products. When both the retailer and the third-party collector have access to BDT information, and the intensity of competition is less than two-thirds, the manufacturer’s advantage weakens while the retailer’s and the third-party collector’s advantages increase. Hence, the manufacturer’s recycling rate decreases as the costs of BDT increase. When the competition intensity is higher than \( 2/3, \) the competition is fierce at this time, and the recycling enthusiasm of recyclers decreases. At this point, the manufacturer is willing to invest in BDT to recycle more used products, even if the costs of BDT increase. However, as both the retailer and the third-party collector gain an information advantage, the scope for increased recycling rates is narrowed.

**Proposition 6.** The optimal BDT level in the four models satisfy:

1. \( \frac{\partial c_{m}^{*}}{\partial \varepsilon} < 0; \frac{\partial c_{m}^{MT\ast}}{\partial \varepsilon} < 0; \frac{\partial c_{m}^{MRT\ast}}{\partial \varepsilon} < 0. \)

2. \( \frac{\partial c_{m}^{*}}{\partial \kappa} < 0; \frac{\partial c_{m}^{MT\ast}}{\partial \kappa} < 0; \) if \( 0 < \varepsilon < \frac{1}{2}, \frac{\partial c_{m}^{MT\ast}}{\partial \kappa} < 0, \) if \( \frac{1}{2} < \varepsilon < 1, \frac{\partial c_{m}^{MT\ast}}{\partial \kappa} > 0; \) if \( 0 < \varepsilon < \frac{2}{3}, \frac{\partial c_{m}^{MRT\ast}}{\partial \kappa} < 0, \) if \( \frac{2}{3} < \varepsilon < 1, \frac{\partial c_{m}^{MRT\ast}}{\partial \kappa} > 0. \)

**Proposition 6(1)** shows that the BDT investment levels in the four models are all decreasing with the increase of the competition intensity between recycling channels. In other words, the intense competition for recycling used products will lead to a decline in the recycling rates of the manufacturer, and the revenue generated from recycling used products will not be sufficient to offset the BDT investment costs. As a result, the manufacturer will reduce the effort they invest in BDT.

From **Proposition 6(2)**, we can find that the investment BDT levels in Models-M and -MR decrease with the BDT investment cost coefficient. However, the BDT effort level decreases only when the competitive intensity...
satisfies certain conditions in Models-MT and -MRT. The unique advantage provided to the manufacturer by the information obtained through BDT technology already makes him competitive with other recyclers when the manufacturer keeps the information private, so the increased costs of BDT lead the manufacturer to lower the effort invested in BDT. When the manufacturer shares information with the retailer that has advantages in both the forward and reverse channels, it further enhances the retailer’s advantages and weakens the manufacturer’s advantages. Thus, regardless of the intensity of competition, increasing BDT costs lead the manufacturer to invest less in BDT. When the manufacturer and the third-party collector share information and the competitive intensity is less than 1/2, the manufacturer’s sharing of BDT information will benefit the third-party collector and reduce its benefits. So, the larger k is, the less the manufacturer is willing to invest in BDT. However, when the competitive intensity exceeds 1/2, the competitive environment is at the middle to upper level, and the retailer who does not gain the information advantage is less motivated to recycle. At this point, the manufacturer is willing to invest in BDT even if the costs of BDT increase. When the manufacturer shares BDT information with the retailer and the third-party collector, and when the intensity of competition is less than 2/3, the advantage of the manufacturer diminishes while the advantage of the retailer and the third-party collector increases. Hence, the level of the manufacturer’s efforts to invest in BDT decreases as the costs of BDT increase. When the competitive intensity is greater than 2/3, the competition is fierce, and the recycling enthusiasm of recyclers decreases. At this point, the manufacturer is willing to invest in BDT to recycle more used products, even if the costs of BDT increase. As the retailer and the third-party collector gain an information advantage, the scope of competition for the manufacturer willing to invest in BDT narrows.

**Proposition 7.** *The optimal wholesale prices in the four models satisfy:*

1. $\frac{\partial w^{MT}}{\partial \varepsilon} > 0$; $\frac{\partial w^{MR}}{\partial \varepsilon} > 0$; $\frac{\partial w^{MRT}}{\partial \varepsilon} > 0$; $\frac{\partial w^{MRT*}}{\partial \varepsilon} > 0$.

2. $\frac{\partial w^{MT}}{\partial k} > 0$; if $0 < \varepsilon < \varepsilon'$, $\frac{\partial w^{MR}}{\partial k} > 0$, if $\varepsilon' < \varepsilon < 1$, $\frac{\partial w^{MR}}{\partial k} < 0$; if $0 < \varepsilon < \frac{1}{2}$, $\frac{\partial w^{MRT}}{\partial k} > 0$, if $\frac{2}{3} < \varepsilon < 1$, $\frac{\partial w^{MRT}}{\partial k} < 0$.

$$\varepsilon' = -\frac{1}{2} \frac{A^2 \beta \theta^2 - \sqrt{5A^4 \beta^2 \theta^4 + 8A^2 \beta \theta^2 + 16\mu^2 + 4\mu}}{A^2 \beta \theta^2} < \frac{1}{2}.$$
Proposition 8. The optimal profits of the manufacture in the four models satisfy:

(1) \( \frac{\partial \Pi_m^{M^*}}{\partial \varepsilon} < 0; \frac{\partial \Pi_m^{MR^*}}{\partial \varepsilon} < 0; \frac{\partial \Pi_m^{MRT^*}}{\partial \varepsilon} < 0; \frac{\partial \Pi_m^{MRRT^*}}{\partial \varepsilon} < 0. \)

(2) \( \frac{\partial \Pi_m^{M^*}}{\partial k} < 0; \frac{\partial \Pi_m^{MR^*}}{\partial k} < 0; \frac{\partial \Pi_m^{MRT^*}}{\partial k} < 0; \frac{\partial \Pi_m^{MRRT^*}}{\partial k} < 0. \)

From the perspective of the manufacturer’s profits, Proposition 8(1) and (2) show that the manufacturer’s profits all decrease with increasing \( \varepsilon \) and \( k \) in four models. Intensified competition will lead to a reduction in the level of effort put into BDT and less recycling of used products by the manufacturer, who will have to raise wholesale prices in order to make higher profits. However, a price increase will further lead to a decrease in demand, affecting the quantity of products used.

Based on the previous analysis, it is clear that the impact of BDT costs on \( \varepsilon \) and \( \tau_m \) is not always negative. Similarly, wholesale prices do not always increase with BDT costs. The manufacturer adapts his strategies to different information-sharing decisions and different competitive environments. However, from a profit point of view, the impact of BDT costs on the manufacturer’s profits is always negative in the four models. It suggests that under a particular competitive environment and information-sharing model, even if the manufacturer is willing to invest more BDT regardless of costs to recycle more used products and lower wholesale prices to expand market demand, the benefits from the additional number of used products and expanded market demand cannot compensate for the excessive costs.

Comparing model

Proposition 9. The optimal profits of the manufacture in the four models satisfy:

(1) If \( \varepsilon \in (0, \varepsilon_1) \), \( \Pi_m^{M^*} < \Pi_m^{MR^*}; \) if \( \varepsilon \in (\varepsilon_1, 1) \), \( \Pi_m^{M^*} > \Pi_m^{MR^*}. \)

\( \varepsilon_1 = \frac{1}{9} \frac{5A^2\lambda^2-24k\mu+\sqrt{-24A^4\lambda^4-2(24A^2\lambda k\lambda^2\mu+144k^2\mu^2)}}{A^2\lambda^2-4k\mu}. \)

(2) If \( \varepsilon \in (0, \varepsilon_2) \), \( \Pi_m^{M^*} < \Pi_m^{MR^*}; \) if \( \varepsilon \in (\varepsilon_2, 1) \), \( \Pi_m^{M^*} > \Pi_m^{MR^*}. \)

\( \varepsilon_2 = \frac{1}{2} \frac{3A^2\lambda^2-16k\mu+\sqrt{-3A^4\lambda^4+64k^2\mu^2}}{A^2\lambda^2-4k\mu}. \)

(3) \( \Pi_m^{M^*} > \Pi_m^{MT^*}, \Pi_m^{MR^*} > \Pi_m^{MRT^*}, \Pi_m^{MR^*} > \Pi_m^{MRT^*}. \)

Proposition 9(1) states that when the intensity of competition between channels is relatively low (i.e., \( \varepsilon < \varepsilon_1 \)), the manufacturer can consider sharing information with both the retailer and the third-party collector to stimulate the whole recycling activities. However, when the competition among the three members of the CLSC gradually becomes fierce (i.e., \( \varepsilon < \varepsilon_1 \)), it leads to the reduction of the recycling quantity of the manufacturer and the decline of his profits. At this time, the manufacturer keeps the information about the used product private.

Similar to Proposition 9(1). Proposition 9(2) shows that if the manufacturer considers whether to share information with the retailer. To a certain extent (i.e., \( \varepsilon < \varepsilon_2 \)), the manufacturer is willing to allow the retailer to participate in the recycling market expanded by BDT. On the contrary, when the competition intensity is greater than \( \varepsilon_2 \), the manufacturer will own the recycling market information after the expansion of BDT.

Proposition 9(3) shows that the manufacturer is consistently unwilling to share information with the third-party collector. Because the manufacturer, as the producer, decides what kind of products to produce and has some knowledge of his own products. The retailer directly faces consumers and controls the consumer market, giving him advantages not only in the forward channel but also in the reverse channel. However, as an independent outsourcing company, the third-party collector has insufficient advantages in the forward channel.
Table 2. Parameter-values.

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<td>20</td>
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Corollary 1. The recycling competition intensity satisfy:

1. \(\varepsilon_1 < \frac{1}{2}\);
2. If \(k \in (k_0, k')\), \(\varepsilon_2 < \frac{1}{2}\); if \(k \in (k', +\infty)\), \(\varepsilon_2 > \frac{1}{2}\). \(k_0 = -\frac{A^2\lambda^2(A^2\beta\theta^2 + 8\mu)}{4\mu(3A^2\beta\theta^2 - 8\mu)}\), \(k' = \frac{7}{20}A^2\lambda^2\).

Corollary 1(1) indicates that \(\varepsilon_1\) is always low. This suggests that in most cases, the manufacturer prefers to have exclusive access to information rather than share it with both the retailer and the third-party collector. The significant advantages of sharing information for one party are diminished when all recyclers have BDT information.

Corollary 1(2) shows that \(\varepsilon_2\) is uncertain. The \(\varepsilon_2\) becomes intense as the \(k\) increases. Unlike Corollary 1, the manufacturer has more options when determining to share information with the retailer, which means he can consider increasing or reducing the costs of investing in BDT. However, Corollary 1 shows that no matter how much the manufacturer spends on acquiring used product information, the overall recycling activities cannot be activated. It suggests that it is sub-optimal for the manufacturer to share information with the retailer and the third-party collector.

Corollary 2. The optimal profits of the manufacturer in the four models satisfy:

1. When \(\varepsilon \in (0, \varepsilon_1)\), \(\Pi_{m}^{MR^*} > \Pi_{m}^{MRT^*} > \Pi_{m}^M > \Pi_{m}^{MT^*}\);
2. When \(\varepsilon \in (\varepsilon_1, \varepsilon_2)\), \(\Pi_{m}^{MR^*} > \Pi_{m}^M > (\Pi_{m}^{MRT^*}, \Pi_{m}^{MT^*})\);
3. When \(\varepsilon \in (\varepsilon_2, 1)\), \(\Pi_{m}^M > \Pi_{m}^{MR^*} > (\Pi_{m}^{MT^*}, \Pi_{m}^{MRT^*})\).

It indicates that it is optimal for the manufacturer to share information with the retailer when the intensity of competition between channels is at a low or medium level. When the competition for recycling activities is at a high level, the optimal option for the manufacturer is to withhold the information. However, sharing information with the third-party collector is always a disadvantage.

In reality, proper competition can promote the enthusiasm of CLSC members to recycle products to a certain extent, thereby increasing profits. In addition, based on the previous analysis, it is clear that a more competitive retailer will have a more significant influence on the manufacturer's decision. Therefore, it is more important for the manufacturer to cooperate with the retailer.

Sensitivity analysis

In the previous chapters, we solved the decision variables of the four models and studied the manufacturer’s optimal information-sharing decision. Due to the complexity of the profits function, the impact of some key parameters on profits cannot be directly determined by comparative analysis. Therefore, this part aims to illustrate the impact of the parameters change on the manufacturer profits more intuitively and precisely through the design of numerical experiments. In order to meet \(\mu > \frac{3}{8} A^2 \beta \theta^2\) and \(k > -\frac{A^2\lambda^2(A^2\beta\theta^2 + 8\mu)}{4\mu(3A^2\beta\theta^2 - 8\mu)}\), and ensure that all optimal results are meaningful, \(\varepsilon\) varied within the range of \((0, 1)\), and \(k\) varied within the range of \((1.8, 2)\). This range can clearly describe the relationship of the optimal results. The parameters’ values used are listed in Table 2.
Sensitivity analysis 1

Impacts of BDT and recycling competition on the manufacturer’s decisions.

Figure 3 indicates that, with the increase of $k$, the BDT input levels in the four models are decreasing, and the effect of $k$ on the levels of the manufacturer effort devoted to BDT diminishes as competition increases. Excessive costs will reduce the enthusiasm of the manufacturer to invest in BDT. However, when the competition becomes fierce, the enthusiasm of other collectors has weakened. At this time, even if the costs of BDT are high, the manufacturer is willing to invest in BDT to a certain extent to obtain effective information.

When the recycling competition intensifies, the manufacturer will decrease the effort levels of investing BDT. When the recycling competition is moderate, the Model-MR has the highest BDT level. When recycling competition becomes fierce, the manufacturer is more willing to invest in BDT without sharing information. This suggests that in a milder competitive environment, the manufacturer is more willing to invest in BDT when he shares information with the retailer. When the competition becomes intense, the manufacturer chooses not to share information.

Figure 4 shows that the wholesale prices increase with the increase of $k$, but as the competition increases, the impact of $k$ on $w$ is increasingly not obvious. As the costs of BDT increase, the manufacturer makes up for the expense by raising wholesale prices. However, when recycling competition becomes fierce, other collectors are less motivated to recycle. Even if the costs of BDT increase, the manufacturer is willing to invest in BDT to recycle more used products. So, the manufacturer can set lower wholesale prices. The lower wholesale price expands the market demand, and the costs of investing in BDT are offset by the benefit from increased demand, so the effect of $k$ on $w$ is diminished.

In addition, with the increase of $\varepsilon$, the wholesale prices of products are all increasing regardless of whether the manufacturer shares information and with whom. The manufacturer will raise wholesale prices to compensate for the decline in recycled products as competition intensifies. When the competition intensity is high, the manufacturer will set the highest wholesale prices when the retailer obtains BDT information.

Sensitivity analysis 2

Impacts of BDT and recycling competition on performance of CLSC.

As shown in Figure 5, the increased costs of BDT lead to lower manufacturer profits. In addition, the impact of BDT costs on the manufacturer’s profits is gradually weakened as competition intensifies. Because of increased recycling competition, the third-party collector and the retailer are less enthusiastic about recycling. At this
point, even if the costs of BDT increase, the manufacturer is willing to invest in BDT as well as set lower wholesale prices to get more used products. The revenue from increased used products compensates to some extent for the expense of BDT costs, and therefore, the impact of $k$ on the manufacturer’s profits is diminished.

According to Figure 5, we can find that the manufacturer’s profits decrease with the increase in competition intensity. Firstly, although the intensity of competition will lead to increase in wholesale prices, it will eventually lead to the promotion of retail prices, further leading lead to the decline of consumer demand. Secondly, the increased intensity of competition will lead to a decrease in the recycling rates of the manufacturer and the level of effort invested in BDT, further leading to a decrease in the amount of recycled used products. The combination of factors ultimately leads to a decline in the manufacturer’s profits.

In addition, under a moderate competitive environment, the manufacturer’s profits in Model-MR are obviously better than those of other models. If the recycling competition intensifies, the manufacturer will keep the BDT information private. Moreover, the profit gap of the manufacturer in different models gradually narrows as competition intensifies. Therefore, to make the function of information sharing more effective and obvious,
enterprises should participate in recycling activities within a reasonable competition and then choose to share information with the retailer.

Models-MR and -MRT mean that information is shared with the retailer. Figure 5 shows that when the manufacturer shares information with the retailer and the recycling competition is wild, as \( k \) increases, the retailer’s profits gradually decrease. When the manufacturer shares information with the retailer, the costs of BDT indirectly affect the retailer’s profits. That is, increased BDT costs can lead to the manufacturer’s reluctance to invest in BDT, resulting in the effective information being shared with the retailer becoming limited, further leading to a decrease in the number of recalls by the retailer and, ultimately, a decrease in the retailer’s profits.

In addition, the optimal profits of the retailer decrease with the increase of channel competition intensity. Figure 6 indicates that in a reasonable competitive intensity, the retailer benefits significantly from information sharing with the manufacturer. As competition increases, the retailer’s profits will not only shrink, but the benefits of information sharing will also diminish. So, for the retailer to reap the benefits of information sharing, he needs to be in a more moderate competitive environment.
Figure 8. Impact of $\varepsilon$ and $k$ on the $\Pi_{sc}$.

Models-MT and -MRT mean that the third-party collector obtains information. Figure 7 shows that the larger $k$ is, the less profitable the third-party collector is. Similarly, when the manufacturer shares information with the third-party collector, the costs of BDT affect the profits of the third-party collector, i.e., the larger the costs of BDT, the more reluctant the manufacturer is to invest in BDT, resulting in limited information shared to the third-party collector, further leading to a decline in the number of the third-party collector, and ultimately, a decline in the profits of the third-party collector.

Similar to the previous result, Figure 7 states that the optimal profits of the third-party collector decrease with the increase of channel competition. In addition, Figure 7 indicates that the third-party collector can always benefit from information sharing. When the third-party collector has access to effective recycling information, his profits increase significantly. This is because the third-party collector, as an enterprise specializing in recycling activities, faces greater risks from the uncertain demand market. Once the manufacturer is willing to share the valid information with him, it will significantly improve his recycling efficiency and profits.

According to Figure 8, when the competition intensity is small, the larger $k$ is, the smaller the profits of CLSC; when the competition becomes intense, the negative impact of BDT costs on the profits of CLSC diminishes. When the competitive environment is relatively mild, high BDT costs lead the manufacturer to reduce the level of effort put into BDT and to set higher wholesale prices, resulting in less used products recycled and less market demand, which ultimately hurts the profits of the manufacturer and CLSC. When competition is intense, and collectors’ recycling effort levels are reduced, the profits of CLSC increase as the manufacturer is willing to invest in BDT and set lower wholesale prices to obtain more used products, even though BDT costs increase.

From a horizontal perspective, Figure 8 indicates that the entire CLSC’s profits decrease as the channel competition increases. From the vertical point of view, information sharing by the manufacturer will increase the profits of the CLSC. Therefore, no matter with whom the manufacturer decides to share BDT information, it can benefit the whole CLSC if he decides to share the information. However, a more moderate competitive environment and the manufacturer’s decision to share information only with the retailer should be maintained if the benefits to the CLSC as a whole are to be maximized.

Discussion

As can be seen, we show a CLSC with triple-channel recycling that integrates recycling channel competition, BDT, and information sharing. In this paper, we consider competition between recycling channels, examine the manufacturer’s investment in BDT to obtain more efficient information about the recycling market for used products, and consider information sharing in the context of triple-channel recycling. We obtain optimal
information-sharing strategies for the manufacturer and understand the impact of recycling competition, level of investment in BDT, and different information-sharing strategies on CLSC members.

This suggests that competition between recycling channels, BDT costs, and information are important sources of profit for the manufacturer and CLSC. When competition is low or medium, the manufacturer shares information with the retailer. When recycling competition is high, the manufacturer keeps information confidential. However, sharing information between the manufacturer and the third-party collector is always to the detriment of the manufacturer. In the recycling process of used products, there is bound to be competition, and reasonable competition can promote recycling activities. If the recycling competition is too fierce, the manufacturer will choose to keep information private from any member not to jeopardize his interests. In addition, the impact of BDT costs on the manufacturer’s decisions is related to differences in the intensity of competition and information-sharing decisions: the intensity of the manufacturer’s investment in BDT and the rate of recycling does not always decrease as the costs of BDT increase. Similarly, wholesale prices do not always increase as BDT costs increase. When competition reaches a certain level of intensity, the manufacturer is willing to increase his investment in BDT and lower his wholesale prices even as the costs of BDT increase to obtain more used products and expand market demand. However, even if the manufacturer invests in BDT at any cost and lowers wholesale prices, the increased volume of used products and market demand will not make up for the high costs of BDT, leading to a decrease in the manufacturer’s overall profitability. Finally, more competitive retailers are in a better position to influence the manufacturer’s decisions, so the manufacturer should choose to share information with retailers in order to achieve a stronger alliance.

To sum up, the impacts of information sharing on the whole CLSC and its members’ profits are related to the intensity of channel competition and BDT costs. Although the negative impact of BDT costs on the manufacturer’s decisions and profits has diminished as the intensity of competition has increased, in general, intense competition has had a more significant negative impact on the manufacturer and other collectors. In addition, compared with the no information-sharing model, sharing information can benefit the whole supply chain when the competition intensity is reasonable. When competition among collectors is too intense, the benefits of information sharing for the CLSC will be offset and eroded by fierce competition.

5. Managerial insights and practical implications

The model can solve the problem of information sharing in a multi-channel CLSC. In practice, the results of the model can provide a better basis for managers’ decisions. Further, it can help managers develop more appropriate recycling strategies and information-sharing strategies to meet the requirements of sustainable development. Fierce competition in recycling can be detrimental to the interests of collectors. Therefore, managers should think about how to avoid vicious competition between industries. In addition, the manufacturer has no incentive to share demand information voluntarily with the third-party collector. So, the third-party collector can negotiate with the manufacturer or take other steps to incentivize the manufacturer to share information. Moreover, in most cases, the manufacturer is willing to share information with the retailer, so the retailer should maintain a close relationship with the manufacturer.

6. Conclusions and outlook

Conclusions

Under the growing pressure of demand uncertainty, BDT has played an increasingly important role in the CLSC. In this paper, we consider the competition between recycling channels and investigate the problem of a manufacturer investing in BDT to obtain more effective recycling market information for used products under the triple-channel recycling (i.e., the manufacturer, the retailer, and the third-party collector all participating in recycling activities) and consider manufacturer information sharing. We examine the manufacturer monopolizes the recycling information obtained through BDT and the other three information-sharing cases. We then inves-
tigate the effects of recycling competition and the level of investment in BDT and different information-sharing strategies on CLSC members.

The findings are as follows:

(1) Competition among recycling channels is an important factor in manufacturer’s decisions and the profitability of CLSC members. When the competition is at a low level or a medium level, the manufacturer chooses to share information with the retailer. When the competition in recycling is high, the manufacturer will keep the information private.

(2) It shows that information shared between the manufacturer and the third-party collector is always disadvantageous to the manufacturer.

(3) The manufacturer’s level of effort for investing in BDT and recycling rates does not always decrease as the costs of BDT increase. Similarly, wholesale prices do not always increase as the costs of BDT increase.

(4) When the intensity of competition reaches a certain level, the manufacturer is willing to invest more in BDT and lower wholesale prices to obtain more used products and expand market demand, even if the costs of BDT increase. However, even if the manufacturer invests in BDT at any cost and reduces wholesale prices, the increased number of used products and market demand cannot compensate for the high BDT costs, resulting in lower overall profits for the manufacturer.

(5) More competitive, the retailer is better able to influence the manufacturer’s decisions, so the manufacturer should choose to share information with the retailer in order to achieve a strong alliance.

(6) The impacts of the information sharing on the whole CLSC, and its members’ profits are related to the intensity of channel competition and BDT costs. Although the negative impact of BDT costs on the manufacturer’s decisions and profits has diminished as the intensity of competition has increased, in general, intense competition has had a greater negative impact on the manufacturer as well as other collectors.

(7) Compared to the model with no information sharing, information sharing can benefit the entire supply chain if the intensity of competition is at a reasonable level. If competition among collectors is too intense, the benefits of information sharing for the CLSC will be offset and eroded by fierce competition.

Limitation and future research

Our research only considers the used products to be homogeneous. However, the reality is that used products are often heterogeneous. In addition, we consider that the residual value obtained by the three collectors is the same. Therefore, different transfer prices for the manufacturer, the retailer, and the third-party collector can be considered. We only consider forward sales and reverse recycling activities and can then combine them to study remanufacturing activities. This paper assumes that the market demand is certain, so it is also possible to consider the case where the demand is uncertain.

APPENDIX A.

Proof of Model-M

The profit functions for the manufacturer, retailer and the third-party collector are respectively

\[ \max(e, w, \tau_m) \Pi^M_m = (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_m^2 + \varepsilon \tau_r^2 + \varepsilon \tau_t^2) - ke^2 \]  
\[ \max(p, \tau_r) \Pi^M_r = (p - w)((\alpha - \beta p) + A\tau_r(\alpha - \beta p) - \mu(\tau_r^2 + \varepsilon \tau_m^2 + \varepsilon \tau_t^2)) \]  
\[ \max(\tau_t) \Pi^M_t = A\tau_t(\alpha - \beta p) - \mu(\tau_t^2 + \varepsilon \tau_m^2 + \varepsilon \tau_r^2). \]

The manufacturer, the retailer, and the third-party collector engage in a Stackelberg game, with the manufacturer as the leader. According to the backward induction method, first, it is easy to verify that \( \frac{\partial \Pi^M_M}{\partial \tau_t} = -2\mu < 0 \). Therefore, based on the first-order condition: \( \frac{\partial \Pi^M_M}{\partial \tau_t} = 0 \), we can obtain the optimal recycling rate of the third-party collector \( \tau_t^M^* \). Then, substitute it in equation (A.2).
We can obtain the second-order Hessian matrix of the retailer’s profit concerning $\tau^r_M$ and $p^M$. The Hesse matrix $H^r_M = \begin{pmatrix} -2\beta - \frac{1}{2}eA^2\theta^2 & -A\beta \mu \\ -A\beta \mu & -2\mu \end{pmatrix}$. We can easily find $|M_1| < 0$. To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to $\tau^r_M$, $p^M$, and the existence of extreme value points, we let $|H^r_M| = \beta(A^2\beta e^2 - A^2\theta^2 + 4\mu) > 0$. Then, we find the constraint $\mu > \frac{1}{4}A^2\theta^2\beta$. Based on the first-order condition: $\frac{\partial M^r}{\partial \tau_r} = 0$ and $\frac{\partial M^r}{\partial p} = 0$, we can obtain $\tau^r_M$ and $p^M$. Then substitute them in equation (A.1).

We can obtain the third-order Hessian matrix of the manufacturer’s profit with respect to $\tau^m_M$, $e^M$, and $w^M$.

The Hesse matrix $H^M_m = \begin{pmatrix} -\frac{8\mu(A^2\theta^2(e-\frac{1}{2})\beta + 2\mu)\beta}{(A^2\theta^2\beta(e-1)+4\mu)^2} & 0 & -\frac{2A\beta\mu}{A^2\theta^2\beta(e-1)+4\mu} \\ 0 & -2k & A\lambda \\ -\frac{2A\beta\mu}{A^2\theta^2\beta(e-1)+4\mu} & A\lambda & -2\mu \end{pmatrix}$. Because $\mu > \frac{1}{4}A^2\theta^2\beta$, $|M_1| < 0$.

To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to $\tau^m_M$, $e^M$, and $w^M$, we let $|M_2| > 0$, i.e., $\frac{-8\mu(A^2\theta^2(e-\frac{1}{2})\beta + 2\mu)\beta}{(A^2\theta^2\beta(e-1)+4\mu)^2} > 0$. So, if $\frac{\partial M^m}{\partial \tau_m} = 0, \frac{\partial M^m}{\partial e} = 0, \frac{\partial M^m}{\partial w} = 0$, we can obtain: $\tau^m_M$, $e^M$, $w^M$.

Finally, substituting the above variables into the profit functions, we can get $\Pi^m_M$, $\Pi^r_M$ and $\Pi^t_M$.

**Proof of Model-MR**

The profit functions for the manufacturer, retailer, and the third-party collector are respectively

\[
\begin{align*}
\text{Max}(e, w, \tau_m)\Pi^M_m &= (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_m + e\tau^2 + \varepsilon e^2) - ke^2 \\
\text{Max}(p, \tau_r)\Pi^M_r &= (p - w)(\alpha - \beta p) + A\tau_r((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2_r + e\tau^2 + \varepsilon e^2) \\
\text{Max}(\tau)\Pi^t_M &= A\tau_r(\alpha - \beta p) - \mu(\tau^2_r + e\tau^2_r + \varepsilon e^2).
\end{align*}
\] (A.4)

The manufacturer, the retailer, and the third-party collector engage in a Stackelberg game, with the manufacturer as the leader. According to the backward induction method, first, it is easy to verify that $\frac{\partial^2\Pi^M_M}{\partial \tau^2_m} = -2\mu < 0$.

Therefore, based on the first-order condition: $\frac{\partial M^M}{\partial \tau_m} = 0$, we can obtain the optimal recycling rate of the third-party collector $\tau^t_M$. Then, substitute it in equation (A.5).

We can obtain the second-order Hessian matrix of the retailer’s profit concerning $\tau^r_M$ and $p^M$. The Hesse matrix $H^r_M = \begin{pmatrix} -2\beta - \frac{1}{2}eA^2\theta^2 & -A\beta \mu \\ -A\beta \mu & -2\mu \end{pmatrix}$. We can easily find $|M_1| < 0$. To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to $\tau^r_M$, $p^M$, and the existence of extreme value points, we let $|H^r_M| = \beta(A^2\beta e^2 - A^2\theta^2 + 4\mu) > 0$. Then, we can obtain $\mu > \frac{1}{4}A^2\theta^2\beta$. Based on the first-order condition $\frac{\partial M^r}{\partial \tau_r} = 0$ and $\frac{\partial M^r}{\partial p} = 0$, we can obtain $\tau^r_M$ and $p^M$. Then substitute them in equation (A.4).

We can obtain the third-order Hessian matrix of the manufacturer’s profit with respect to $\tau^m_M$, $e^M$, and $w^M$. The Hesse matrix $H^M_m = \begin{pmatrix} -\frac{8\mu(A^2\theta^2(e-\frac{1}{2})\beta + 2\mu)\beta}{(A^2\theta^2\beta(e-1)+4\mu)^2} & 0 & -\frac{2A\beta\mu}{A^2\theta^2\beta(e-1)+4\mu} \\ 0 & -2k & A\lambda \\ -\frac{2A\beta\mu}{A^2\theta^2\beta(e-1)+4\mu} & A\lambda & -2\mu \end{pmatrix}$. Because $\mu > \frac{1}{4}A^2\theta^2\beta$, $|M_1| < 0$.

To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to $\tau^m_M$, $e^M$, and $w^M$, we let $|M_2| > 0$, i.e., $\frac{-8\mu(A^2\theta^2(e-\frac{1}{2})\beta + 2\mu)\beta}{(A^2\theta^2\beta(e-1)+4\mu)^2} > 0$. So, if $\frac{\partial M^m}{\partial \tau_m} = 0, \frac{\partial M^m}{\partial e} = 0, \frac{\partial M^m}{\partial w} = 0$, we can obtain: $\tau^m_M$, $e^M$, $w^M$.

Finally, substituting the above variables into the profit functions, we can get $\Pi^m_M$, $\Pi^r_M$ and $\Pi^t_M$.
The Hesse matrix

\[ H_{m}^{MR} = \begin{pmatrix}
-8\beta(A^2\theta^2(\varepsilon - 1/2)\varepsilon + 2)\mu & \lambda A^2\beta(A^2\theta^2(\varepsilon + 2)\theta^2 + 4\mu(\varepsilon + 1))
\end{pmatrix}
\]

Because \( \mu > \frac{1}{4} A^2\theta^2 \beta \), we can get \( |M_1| < 0 \). To ensure that the Hesse matrix is a negative definite matrix, we let \( |M_2| > 0 \), i.e.,

\[
-8\beta(A^2\theta^2(\varepsilon - 1/2)\varepsilon + 2)\mu - 2A^2\beta(A^2\theta^2(\varepsilon + 2)\theta^2 + 4\mu(\varepsilon + 1)) = 0
\]

\[
-2A^2\beta(A^2\theta^2(\varepsilon + 2)\theta^2 + 4\mu(\varepsilon + 1)) = 0
\]

\[
\lambda A^2\beta(A^2\theta^2(\varepsilon + 2)\theta^2 + 4\mu(\varepsilon + 1)) = 0
\]

Finally, substituting the above variables into the profits function, we can get \( \Pi_{m}^{MR} \).

\[
\Pi_{m}^{MR} = (w - c_m)(\alpha - \beta p) + A r_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_m^2 + \varepsilon \tau_m^2) - k e^2
\]

\[
\Pi_{m}^{MR} = (p - w)(\alpha - \beta p) + A r_\tau((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_\tau^2 + \varepsilon \tau_\tau^2)
\]

\[
\Pi_{m}^{MR} = A r_\tau((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_\tau^2 + \varepsilon \tau_\tau^2)
\]

**Proof of Model-MT**

The profit functions for the manufacturer, retailer, and the third-party collector are respectively

\[
\max(e, w, \tau_m) \Pi_{m}^{MT} = (w - c_m)(\alpha - \beta p) + A r_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_m^2 + \varepsilon \tau_m^2) - k e^2
\]

\[
\max(p, \tau_\tau) \Pi_{m}^{MT} = (p - w)(\alpha - \beta p) + A r_\tau((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_\tau^2 + \varepsilon \tau_\tau^2)
\]

The manufacturer, the retailer, and the third-party collector engage in a Stackelberg game, with the manufacturer as the leader. According to the backward induction method, first, it is easy to verify that \( \frac{\partial^2 \Pi_{m}^{MT}}{\partial \tau_m} = -2\mu < 0 \). Therefore, based on the first-order condition \( \frac{\partial \Pi_{m}^{MT}}{\partial \tau_m} = 0 \), we can obtain \( \tau_m^{MT*} \). Then substitute it in equation (A.8).
We can obtain the second-order Hessian matrix of the retailer’s profit concerning \( \tau_r^\text{MR} \) and \( p^\text{MR} \). The Hesse matrix \( H_r^\text{MT} = \begin{pmatrix} -2\lambda & -A_\lambda \beta \\ -A_\lambda \beta & -2\beta - \frac{1}{2} \varepsilon A^2 \beta \varepsilon^2 \end{pmatrix} \). We can find \( |M_1| < 0 \). To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to \( \tau_r^\text{MT} \) and \( p^\text{MT} \), and the existence of extreme value points, let \( |H_r^\text{MT}| = \beta (A^2 \beta \varepsilon^2 - A^2 \beta \theta^2 + 4\mu) > 0 \). Then, we can obtain \( \mu > \frac{1}{4} A^2 \beta \varepsilon^2 \beta \theta^2 \). According to \( \frac{\partial H_r^\text{MT}}{\partial \tau_r} = 0 \) and \( \frac{\partial H_r^\text{MT}}{\partial p} = 0 \), we can obtain \( \tau_r^\text{MT}^*, p^\text{MT}^* \). Then substitute them in equation (A.7).

We can obtain the third-order Hessian matrix of the manufacturer’s profit with respect to \( \tau_m^\text{MT} \), \( e^\text{MT} \), and \( w^\text{MT} \). The Hesse matrix

\[
H_m^\text{MT} = \begin{pmatrix}
8(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 2\mu) \mu \beta \\
-2(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu) \\
-2(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu)
\end{pmatrix} - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{\mu A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - 2\mu
\]

Because \( \mu > \frac{1}{4} A^2 \beta \varepsilon^2 \beta \theta^2 \), we can get \( |M_1| < 0 \). To ensure that the Hesse matrix is a negative definite matrix, we let \( |M_2| > 0 \), i.e.,

\[
\begin{vmatrix}
8(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 2\mu) \mu \beta \\
-2(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu) \\
-2(A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu)
\end{vmatrix} - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{\mu A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - 2\mu
\]

We can obtain \( \mu > \frac{3}{8} A^2 \beta \varepsilon^2 \beta \theta^2 \) and \( k > \frac{1}{4} A^2 \beta \varepsilon^2 \beta \theta^2 \). Let

\[
H_m^\text{MT} = \begin{pmatrix}
2(3A^4 \beta \varepsilon \beta \theta^2 - 2A^4 \beta \lambda^2 \theta^2 - 16A^2 \beta \varepsilon \varepsilon \beta \mu \theta^2 + 12A^2 \beta \varepsilon \beta \mu \theta^2 - 8A^2 \varepsilon \theta^2 \theta^2 + 8A^2 \lambda^2 \mu + 32(\beta \theta^2 + \frac{1}{2} \beta \varepsilon^2) (\varepsilon - 1) \beta \varepsilon^2 \mu - 32k \mu^2) \beta \mu
\end{pmatrix} - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - \frac{2A^4 \beta \varepsilon^2 \beta \theta^2}{A^2 \beta \varepsilon^2 (e - \frac{1}{2}) \beta + 4\mu})^2 - 2\mu
\]

We can obtain \( k > \frac{1}{6} A^2 \lambda^2 (A^2 \beta \varepsilon^2 - 4\mu) \). We find \( \frac{4}{6} A^2 \lambda^2 \mu < \frac{1}{6} A^2 \lambda^2 (A^2 \beta \varepsilon^2 - 4\mu) \). So, if \( \mu > \frac{3}{8} A^2 \beta \varepsilon^2 \beta \theta^2 \) and \( k > \frac{1}{6} A^2 \lambda^2 (A^2 \beta \varepsilon^2 - 4\mu) \), the matrix is negative definite and there exists a unique optimal solution. Let \( \frac{\partial H_m^\text{MT}}{\partial \tau_m} = 0 \), \( \frac{\partial H_m^\text{MT}}{\partial e} = 0 \), \( \frac{\partial H_m^\text{MT}}{\partial w} = 0 \), we can obtain \( \tau_m^\text{MT}^*, e^\text{MT}^*, w^\text{MT}^* \).

Finally, substituting the above variables into the profits function, we can get \( \Pi_m^\text{MT}^*, \Pi_r^\text{MT}^* \) and \( \Pi_t^\text{MT}^* \).

**Proof of Model-MRT**

The profit functions for the manufacturer, retailer and the third-party collector are respectively

\[
\text{Max}(e, w, \tau_m) = (w - c_m)(\alpha - \beta p) + A\tau_m((\alpha - \beta p)\theta + \lambda e) - \mu(\tau_m^2 + e\tau_m + e\tau_m^2) - ke^2
\]
\[ \text{Max}(p, \tau) \Pi^\text{MRT}_r = (p - w)(\alpha - \beta p) + A\tau_r(\theta(\alpha - \beta p) + \lambda e) - \mu(\tau^2 + \varepsilon \tau^2_m + \varepsilon \tau^2) \] (A.11)

\[ \text{Max}(\tau) \Pi^\text{MRT}_l = A\tau_l((\alpha - \beta p)\theta + \lambda e) - \mu(\tau^2 + \varepsilon \tau^2_m + \varepsilon \tau^2) \] (A.12)

The manufacturer, the retailer, and the third-party collector engage in a Stackelberg game, with the manufacturer as the leader. According to the backward induction method, first, it is easy to verify that \( \frac{\partial^2 \Pi^\text{MRT}_r}{\partial \tau^2} = -2\mu < 0 \). Therefore, based on the first-order condition \( \frac{\partial \Pi^\text{MRT}_r}{\partial \tau} = 0 \), we can obtain \( \tau^*_r \), then substitute it in equation (A.11).

We can obtain the second-order Hessian matrix of the retailer’s profit concerning \( \tau^*_r \) and \( p^\text{MRT} \). The Hesse matrix \( H^\text{MRT}_r = \begin{pmatrix} -2\beta - \frac{1}{2} \varepsilon A^2 \beta^2 \mu^2 & A\theta \beta \\ -A\theta \beta & -2\mu \end{pmatrix} \). We can find \( |M_1| < 0 \). To ensure that the Hesse matrix is a negative definite matrix, a joint concave function with respect to \( \tau^*_r \) and \( p^\text{MRT} \), and the existence of extreme value points, we let \( |H^\text{MRT}_r| = \beta(A^2\beta\varepsilon\theta^2 - A^2\beta\theta^2 + 4\mu) > 0 \). Then, we can obtain \( \mu > \frac{1}{2} A^2 \theta^2 \beta \). Let \( \frac{\partial \Pi^\text{MRT}_r}{\partial \tau} = 0 \) and \( \frac{\partial \Pi^\text{MRT}_r}{\partial p} = 0 \), we can obtain \( \tau^*_r \) and \( p^\text{MRT} \). Then substitute them in equation (A.10).

We can obtain the third-order Hessian matrix of the manufacturer’s profit with respect to \( \tau^*_m \), \( e^\text{MRT} \), and \( w^\text{MRT} \). The Hesse matrix

\[
H^\text{MRT}_m = \begin{pmatrix}
\frac{-8(A^2\beta^2(e^{-\frac{1}{2}})\beta + 2\mu)\mu\beta}{(A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu)^2} & \frac{-\theta A^2\lambda\beta(A^2\beta(e^{-\frac{1}{2}})\beta^2 - 4\mu(e + 1))}{(A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu)^2} & \frac{-2A\beta^2\mu}{A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu} \\
\frac{-\theta A^2\lambda\beta(A^2\beta(e^{-\frac{1}{2}})\beta^2 - 4\mu(e + 1))}{(A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu)^2} & \frac{-2k\beta^2\theta^2(e^{-\frac{1}{2}})A^2 - 16(k\beta(e^{-\frac{1}{2}})\beta^2 + \lambda e)\mu A^2 - 32k\mu^2}{(A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu)^2} & \frac{4A\lambda\mu}{A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu} \\
\frac{2\mu\beta(A^4\beta^2\varepsilon^2\theta^2 - 2A^4\beta\varepsilon^2\theta^2 + A^4\beta^2\varepsilon^2\theta^2 - 16A^2\beta e\varepsilon^2\theta^2 + 12A^2\beta^2\varepsilon^2\theta^2 - 16A^2\varepsilon^2\theta^2 + 8A^2\varepsilon^2\theta^2 - 32k\mu^2)}{(A^2\beta^2\theta^2 - A^2\beta^2\theta^2 + 4\mu)^2} & \frac{4A\lambda\mu}{A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu} & -2\mu
\end{pmatrix}
\]

\[> 0,\]

\[
\frac{-8(A^2\beta^2(e^{-\frac{1}{2}})\beta + 2\mu)\mu\beta}{(A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu)^2} - \frac{2A\beta^2\mu}{A^2\beta^2(e^{-\frac{1}{2}})\beta + 4\mu} = \frac{4\mu^2\beta(4A^2\beta^2\theta^2 - 3A^2\beta^2\theta^2 + 8\mu)}{(2A^2\beta^2\theta^2 - A^2\beta^2\theta^2 + 4\mu)^2} > 0,
\]

\[= 4\mu(A^4\beta^2\varepsilon^2\theta^2 - 2A^4\beta^2\varepsilon^2\theta^4 + A^4\beta^2\varepsilon^2\theta^4 + 8A^2\beta e\varepsilon^2\theta^2 + 8A^2\beta^2\varepsilon^2\theta^2 + 8A^2\varepsilon^2\theta^2 - 8A^2\varepsilon^2\theta^2 + 8A^2\varepsilon^2\theta^2 - 4A^2\varepsilon^2\theta^2 + 16k^2\mu^2) > 0.
\]

We can obtain \( \mu > \frac{3}{8} A^2 \theta^2 \beta \) and \( k > \frac{4A^2\mu\lambda^2}{(A^2\beta^2\theta^2 - A^2\beta^2\theta^2 + 4\mu)^2} \). Let

\[
H^\text{MRT}_m = \frac{2\mu\beta(A^4\beta^2\varepsilon^2\theta^2 - 2A^4\beta^2\varepsilon^2\theta^2 + A^4\beta^2\varepsilon^2\theta^2 - 16A^2\beta e\varepsilon^2\theta^2 + 12A^2\beta^2 e\varepsilon^2\theta^2 - 16A^2\varepsilon^2\theta^2 + 8A^2\varepsilon^2\theta^2 - 32k\mu^2)}{(A^2\beta^2\theta^2 - A^2\beta^2\theta^2 + 4\mu)^2} < 0,
\]
we can obtain \( k > -\frac{1}{4} \frac{A^2 \lambda^2 (A^2 \beta^2 + 8\mu)}{\mu (3A^2 \beta^2 - 8\mu)} \). We find \( \frac{A^2 \lambda^2}{(A^2 \beta^2 - 4\mu)^2} < -\frac{1}{4} \frac{A^2 \lambda^2 (A^2 \beta^2 + 8\mu)}{\mu (3A^2 \beta^2 - 8\mu)} \). So, if \( \mu > \frac{3}{8} A^2 \beta^2 \beta \) and \( k > -\frac{1}{4} \frac{A^2 \lambda^2 (A^2 \beta^2 + 8\mu)}{\mu (3A^2 \beta^2 - 8\mu)} \), the matrix is negative definite and there exists a unique optimal solution. Let \( \frac{\partial M^{R_{MRT^*}}}{\partial \tau_m} = 0, \frac{\partial M^{R_{MRT^*}}}{\partial \tau_e} = 0 \), we can obtain \( \tau_{m}^{R_{MRT^*}}, \nu^{R_{MRT^*}}, \psi^{R_{MRT^*}} \).

Finally, substituting the above variables into the profits function, we can get \( \Pi_{m}^{R_{MRT^*}}, \Pi_{r}^{R_{MRT^*}} \) and \( \Pi_{t}^{R_{MRT^*}} \).

**APPENDIX B.**

Based on the judgment of Hesse matrix in Appendix A, we obtained the constraint conditions:

\[
\mu > \frac{3}{8} A^2 \beta^2 \beta \\
k > -\frac{1}{4} A^2 \lambda^2 \left( A^2 \beta^2 + 8\mu \right)
\]

**Proof of Proposition 5.**

\[
\frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} = -\frac{2A k \mu \theta (\beta c_m - \alpha) (2 \beta^2 \lambda^2 A^4 - 8\mu A^2 k \beta \theta^2)}{2 \left( \beta \theta^2 \lambda^2 (\varepsilon - \frac{1}{2}) A^4 - 8\mu (k \beta (\varepsilon - \frac{1}{2}) \theta^2 - \frac{\lambda^2}{2}) A^2 - 16k \mu^2 \right)^2} \\
\]

\[
2 \left( \beta \theta^2 \lambda^2 \left( \varepsilon - \frac{1}{2} \right) A^4 - 8\mu \left( k \beta \left( \varepsilon - \frac{3}{4} \right) \theta^2 - \frac{\lambda^2}{2} \right) A^2 - 16k \mu^2 \right)^2 > 0, \quad \left( \beta c_m - \alpha \right) < 0,
\]

\[
(2 \beta^2 \lambda^2 A^4 - 8\mu A^2 k \beta \theta^2) < 0.
\]

So, \( \frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} < 0. \)

\[
\frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} = \left( \beta c_m - \alpha \right) \left( A^2 \lambda^2 + 4k \mu \right) A \theta \left( 3 \beta^2 \lambda^2 (2\varepsilon - 1) A^4 + 16 \mu \left( \frac{\lambda^2}{2} + k \beta \theta^2 \right) A^2 \right) \\
\left( 3 \beta^2 \lambda^2 \left( \varepsilon^2 - \varepsilon - \frac{1}{3} \right) A^4 + 16 \mu \left( \frac{1}{2} \varepsilon - \frac{1}{2} \right) \lambda^2 + k \beta \left( \varepsilon - \frac{3}{4} \right) \theta^2 \right) A^2 + 32k \mu^2 \right)^2 > 0, \quad \left( \beta c_m - \alpha \right) < 0, \quad \text{and} \quad \left( 3 \beta^2 \lambda^2 (2\varepsilon - 1) A^4 + 16 \mu \left( \frac{\lambda^2}{2} + k \beta \theta^2 \right) A^2 \right) > 0.
\]

So, \( \frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} < 0. \)

\[
\frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} = 2 \left( \beta \theta^2 \lambda^2 A^2 - 4\mu (k \beta \theta^2 + \lambda^2) \right) \left( A^2 \lambda^2 - 8k \mu \right) A^3 \theta (\beta c_m - \alpha) \\
9 \left( \varepsilon - \frac{1}{2} \right) \beta \lambda^2 \theta^2 A^4 - \frac{16 \mu \left( \frac{1}{2} \varepsilon - \frac{1}{2} \right) \lambda^2 + k \beta (\varepsilon - \frac{3}{4} \theta^2) A^2 - 32k \mu^2 \right)^2 > 0, \quad \left( A^2 \lambda^2 - 8k \mu \right) < 0, \quad \left( \beta \theta^2 \lambda^2 A^2 - 4\mu (k \beta \theta^2 + \lambda^2) \right) < 0.
\]

So, \( \frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} < 0. \)

\[
\frac{\partial \tau_{m}^{R_{MRT^*}}}{\partial \varepsilon} = A^3 \left( \beta \theta^2 \lambda^4 (\varepsilon - 1)^2 A^4 - 8\lambda^2 \mu (-\lambda^2 + k \beta \theta^2 (\varepsilon - \frac{3}{2})) A^2 + 64 \left( k \beta \theta^2 + \frac{\lambda^2}{2} \right) k \mu^2 \theta (\beta c_m - \alpha) \\
(\beta \theta^2 \lambda^2 (\varepsilon - 1)^2 A^4 - 16 \left( \varepsilon - \frac{1}{2} \right) \lambda^2 + k \beta (\varepsilon - \frac{3}{4} \theta^2) \mu A^2 - 32k \mu^2 \right)^2
\]
\[(\beta \theta^2 \lambda^2(\varepsilon - 1)^2 A^4 - 16 \left( (\varepsilon - \frac{1}{2}) \lambda^2 + k\beta (\varepsilon - \frac{3}{4}) \theta^2 \right) \mu A^2 - 32 k \mu^2 \right)^2 > 0,\]

\[(\beta \theta^2 \lambda^4(\varepsilon - 1)^2 A^4 - 8 \lambda^2 \mu \left( -\lambda^2 + k\beta \theta^2 (\varepsilon - \frac{3}{2}) \right) A^2 + 64 \left( k\beta \theta^2 + \frac{3 \lambda^2}{2} \right) k \mu^2 \right)^2 > 0, \]

So, \(\frac{\partial \tau_{MRT}^{*}}{\partial \varepsilon} < 0\).

\[
\frac{\partial \tau_{MRT}^{*}}{\partial k} = \frac{\theta A^3 \mu(\beta \varepsilon - \alpha) \lambda^2 (\beta \theta^2 (\varepsilon - \frac{1}{2}) A^2 + 2 \mu)}{\left( \beta \theta^2 \lambda^2 (\varepsilon - \varepsilon - \frac{1}{2}) A^4 - 4 \mu (k\beta (\varepsilon - \frac{3}{4}) \theta^2 + \frac{1}{2} (\varepsilon - 1) \lambda^2) \mu A^2 + \frac{32}{3} k \mu^2 \right)^2}
\]

\[
\left( \beta \theta^2 \lambda^2 \left( \varepsilon - \frac{1}{2} \right) A^4 - 4 \mu \left( k\beta (\varepsilon - \frac{3}{4}) \theta^2 - \frac{1}{2} (\varepsilon - 1) \lambda^2 \right) A^2 - 8 k \mu^2 \right)^2 > 0,
\]

So, \(\frac{\partial \tau_{MT}^{*}}{\partial k} < 0\).

\[
\frac{\partial \tau_{MRT}^{*}}{\partial k} = -\frac{4}{3} \frac{\theta A^3 \mu(\beta \theta^2 (\varepsilon - \frac{1}{2}) A^2 + \frac{8}{3} \mu)(\varepsilon - 2) \lambda^2}{\left( \beta \theta^2 \lambda^2 (\varepsilon - \varepsilon - \frac{1}{2}) A^4 + \frac{1}{2} (k\beta (\varepsilon - \frac{3}{4}) \theta^2 + \frac{1}{2} (\varepsilon - 1) \lambda^2) \mu A^2 + \frac{32}{3} k \mu^2 \right)^2}
\]

\[
\left( \beta \theta^2 \lambda^2 \left( \varepsilon - \frac{1}{3} \right) A^4 + \frac{16}{3} \left( k\beta (\varepsilon - \frac{3}{4}) \theta^2 + \frac{1}{2} (\varepsilon - 1) \lambda^2 \right) \mu A^2 + \frac{32}{3} k \mu^2 \right)^2 > 0,
\]

So, \(\frac{\partial \tau_{MT}^{*}}{\partial k} < 0\).

\[
\frac{\partial \tau_{MRT}^{*}}{\partial k} = -\frac{16}{9} \frac{\theta A^3 \mu(\beta \theta^2 (\varepsilon - 1) A^2 + 4 \mu)(\varepsilon - \frac{1}{2}) \lambda^2}{\left( \beta \theta^2 \lambda^2 (\varepsilon - \varepsilon - \frac{2}{3}) A^4 - 16 \left( k\beta (\varepsilon - \frac{3}{4}) \theta^2 + \frac{1}{2} (\varepsilon - 1) \lambda^2 \right) \mu A^2 - \frac{32}{3} k \mu^2 \right)^2}
\]

\[
\left( \beta \theta^2 \lambda^2 (\varepsilon - 1) A^4 - 16 \left( k\beta (\varepsilon - \frac{3}{4}) \theta^2 + \left( \varepsilon - \frac{1}{2} \right) \lambda^2 \right) \mu A^2 - 32 k \mu^2 \right)^2 > 0.
\]

If \(\varepsilon \in (0, \frac{1}{2})\), \(\beta \theta^2 (\varepsilon - 1) A^2 + 4 \mu)(\varepsilon - \frac{1}{2}) > 0, \frac{\partial \tau_{MT}^{*}}{\partial k} > 0\).

If \(\varepsilon \in (\frac{1}{2}, 1)\), \(\beta \theta^2 (\varepsilon - 1) A^2 + 4 \mu)(\varepsilon - \frac{1}{2}) < 0, \frac{\partial \tau_{MT}^{*}}{\partial k} < 0\).

If \(\varepsilon \in (0, \frac{2}{3})\), \(\beta \theta^2 (\varepsilon - 1) A^2 + 8 \mu)(\varepsilon - \frac{2}{3}) < 0, \frac{\partial \tau_{MRT}^{*}}{\partial k} < 0\).

If \(\varepsilon \in (\frac{2}{3}, 1)\), \(\beta \theta^2 (\varepsilon - 1) A^2 + 8 \mu)(\varepsilon - \frac{2}{3}) > 0, \frac{\partial \tau_{MRT}^{*}}{\partial k} > 0\).

Proofs of Propositions 6–8 are the same.
Proof of Proposition 9.

\[ \Pi_m^{\mathrm{M^*}} - \Pi_m^{\mathrm{MRT^*}} = \frac{9}{4} \left( \frac{\lambda^2(e^2 - \frac{10}{9}e + \frac{1}{3})}{\beta^2} A^2 - 4(e - 1)k\mu(e - \frac{1}{3}) \right) \mu \lambda^2 A^4 (3c_m - \alpha)^2 \theta^2 \]

After proving that the Hesse matrix is negative definite, we can get

\[ \left( \beta \theta^2 \lambda^2 (e - 1)^2 A^4 + 16 \left( \lambda^2 \left( e - \frac{1}{2} \right) - k\beta \theta^2 \left( e - \frac{3}{4} \right) \right) \mu A^2 + 32k \mu^2 \right) A^2 - 8k \mu^2 > 0, \]

It can be easily found that \( \mu \lambda^2 A^4 (3c_m - \alpha)^2 \theta^2 > 0. \)

So, we only need to discuss the sign (positive or negative) of \( \lambda^2 (e^2 - \frac{10}{9}e + \frac{1}{3}) A^2 - 4(e - 1)k\mu(e - \frac{1}{3}) \).

If we combine the same terms, we get \( (A^2 \lambda^2 - 4k \mu) e^2 + (-\frac{10}{9} A^2 \lambda^2 + \frac{16}{3} k \mu) e + \frac{1}{3} A^2 \lambda^2 - \frac{4}{3} k \mu > 0, \) so \( \Pi_m^{\mathrm{M^*}} < \Pi_m^{\mathrm{MRT^*}}. \)

The same way, we can prove \( \Pi_m^{\mathrm{M^*}} < \Pi_m^{\mathrm{MRT^*}}. \)

\[ \Pi_m^{\mathrm{M^*}} - \Pi_m^{\mathrm{MRT^*}} = \frac{1}{3} \left( \frac{\lambda^2(e^2 - \frac{2}{3})}{\beta^2} A^2 - 4k\mu(e - 1) \right) \lambda^2 A^4 - 4k \mu^2 A^2 - 8k \mu^2 \]

After proving that the Hesse matrix is negative definite, we can get

\[ \left( \beta \theta^2 \lambda^2 (e - \frac{2}{3}) A^4 + 16 \left( \lambda^2 \left( e - \frac{1}{2} \right) - k\beta \theta^2 \left( e - \frac{3}{4} \right) \right) \mu A^2 + 32k \mu^2 \right) A^2 - 8k \mu^2 > 0. \]

It can be easily found that \( \mu \theta^2 A^2 (3c_m - \alpha)^2 > 0. \)

So, we only need to discuss the sign (positive or negative) of \( \lambda^2 (e^2 - 3e + 3) A^2 - 4k \mu (e - 1)(e - 3) \).

If we combine the same terms, we get \( (A^2 \lambda^2 - 4k \mu) e^2 + (-3A^2 \lambda^2 + 16k \mu) e + 3A^2 \lambda^2 - 12k \mu > 0, \) so \( \Pi_m^{\mathrm{M^*}} > \Pi_m^{\mathrm{MRT^*}}. \)

The same way, we can prove \( \Pi_m^{\mathrm{M^*}} > \Pi_m^{\mathrm{MRT^*}}. \)

And we can find \( \mu \theta^2 A^2 (3c_m - \alpha)^2 > 0. \) easily.

So, we only need to discuss the sign (positive or negative) of \( \lambda^2 (e - \frac{2}{3}) A^2 - 4k \mu (e - 1) \).

From the proof in Appendix A, we can obtain \( \lambda^2 (e - \frac{2}{3}) A^2 - 4k \mu (e - 1) > 0, \) so \( \Pi_m^{\mathrm{M^*}} > \Pi_m^{\mathrm{MRT^*}}. \)

In the same way, we can prove \( \Pi_m^{\mathrm{M^*}} > \Pi_m^{\mathrm{MRT^*}} \) and \( \Pi_m^{\mathrm{MRT^*}} > \Pi_m^{\mathrm{M^*}}. \)
Proof of Corollary 1.

\[ \epsilon_1 - \frac{1}{2} = \frac{A^2 \lambda^2 - 12k\mu + 2\sqrt{-24A^4\lambda^4 - 24A^2k\lambda^2\mu + 144k^2\mu^2}}{18A^2\lambda^2 - 72k\mu}. \]

Because \( k > \frac{1}{4} \frac{A^2\lambda^2}{\mu} \)

\[ 18A^2\lambda^2 - 72k\mu < 0 \]
\[ A^2\lambda^2 - 12k\mu + 2\sqrt{-24A^4\lambda^4 - 24A^2k\lambda^2\mu + 144k^2\mu^2} > 0 \]
\[ \frac{A^2\lambda^2 - 12k\mu + 2\sqrt{-24A^4\lambda^4 - 24A^2k\lambda^2\mu + 144k^2\mu^2}}{18A^2\lambda^2 - 72k\mu} < 0. \]

So, \( \epsilon_1 < \frac{1}{2} \)

\[ \epsilon_2 - \frac{1}{2} = \frac{2A^2 \lambda^2 - 12k\mu + \sqrt{-3A^4\lambda^4 + 64k^2\mu^2}}{2A^2\lambda^2 - 8k\mu}. \]

Let \( \epsilon_2 - \frac{1}{2} = 0. \)

We can get \( k' = \frac{7}{20} \frac{A^2\lambda^2}{\mu} \)

\[ k' - k_0 = \frac{1}{10} \frac{A^2\lambda^2(13A^2\beta\theta^2 - 8\mu)}{\mu(3A^2\beta\theta^2 - 8\mu)}. \]

Let \( k' - k_0 = 0. \)

We can get \( \mu' = \frac{13}{8} A^2\beta\theta^2 \), and we can judge \( \mu' > \mu_0 \).

So, when \( \mu \in (\mu_0, \mu') \), \( k' < k_0 \); when \( \mu \in (\mu', +\infty) \), \( k' > k_0 \).

Hence, Corollary 1 is proved.

\[ \epsilon_1 - \epsilon_2 = \frac{-17\lambda^2 A^2 + 96k\mu - 9\sqrt{-3A^4\lambda^4 + 64k^2\mu^2} + 2\sqrt{-24A^4\lambda^4 - 24A^2k\lambda^2\mu + 144k^2\mu^2}}{18A^2\lambda^2 - 72k\mu}. \]

Let \( \epsilon_1 - \epsilon_2 = 0. \)

We can obtain \( k'' = \frac{73}{320} \frac{A^2\lambda^2}{\mu} . \)

When \( k \in (0, k'') \), \( \epsilon_1 - \epsilon_2 > 0. \)

When \( k \in (k'', +\infty) \), \( \epsilon_1 - \epsilon_2 < 0. \)

Because \( k > \frac{1}{4} \frac{A^2\lambda^2}{\mu} \)

So \( \epsilon_1 - \epsilon_2 < 0. \)

\[ \Box \]

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