GREEN INITIATIVE IN A TWO-ECHELON SUPPLY CHAIN WITH CO-ORDINATION AND CONTRACT

SANTANU KUMAR GHOSH¹,²* AND PALASH GOSWAMI²

Abstract. In recent times, environmental responsibility is an important factor that determines the success of a Supply Chain. In this study, we have considered green production in the light of various co-ordinations and contracts. This is a two-echelon Supply chain consisting of one manufacturer who designs and develops a green product and the retailer sells it to the environmentally aware customers and the awareness is converted to actual purchasing behaviour by the retailers marketing strategy and the manufacturer's product design and development which includes technology usage to develop the greenness, packaging and several other factors which were not studied earlier. All these factors are involved in our demand function which is distinct from the existing literature. The model is developed under three contracts, Price-only, green marketing cost sharing and two-part tariff contracts. This is an well-established fact that co-ordination enhances the economic benefits to every tier member of a chain. Our findings also establish that co-ordination and co-operation among members will enhance their environmental sustainability. In this way they can carry out their social responsibilities towards our environment. It is also noticed that as the environmental consciousness of the consumers increases, the cost sharing contract is more profitable for manufacturer than that for retailer whereas the price only contract is profitable for the retailer.

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1. Introduction

Sustainable green supply chain management has been one of the growing interest among the researchers nowadays. The increasing attention of green production of the manufacturer and green marketing and selling of the product to the consumers are very important to the environment-conscious customers. Green initiatives include green product design, green manufacturing, green supplying of the products to the retailers, green marketing, green by-product generation and recycling of the used products. So, to maintain sustainability of the supply chain, the supply chain members have to be more careful as customers are now more environment conscious. As per a survey on 2009 by Accenture, more than 80 percent consumers prefer greenness while buying a product. Another survey by Carbon Trust in 2011 reveals that more than 20 percent of customers prefer to buy green products even though they are costlier than the regular products. Environmental consciousness and green

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1 Department of Mathematics, Kazi Nazrul University, Asansol, West Bengal 713340, India.
2 Department of Mathematics, Michael Madhusudan Memorial College, Durgapur, West Bengal 713216, India.
*Corresponding author: santanukumar.ghosh@knu.ac.in

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product purchasing behavior of the consumers have forced the manufacturing companies to manufacture green products. This behaviour of the customers also forced the retailers to use green marketing. The social responsibility of the manufacturer and the retailer is another drive-force for green production and green marketing. Therefore the social responsibilities of the manufacturing companies and the retailers and the consciousness of the benefits of using green products have led the production of environment-friendly green products which are less harmful than the usual goods. These green products are recyclable and environment friendly. It is true that in order to produce green products a certain cost is incurred which is called technology improvement cost. For example Walmart prepares a green marketing strategy for its suppliers. Manufacturers and retailers can execute environmental responsibilities in their production and related activities enjoying economic benefits as well [16].

It is an established fact that green marketing has been highly successful to increase consumers’ environmental awareness to such an extent so that they actually are inclined to consume such products [24]. Great retailers like Walmart and Carrefour applied green marketing in their strategies and were very much successful. Co-operation and co-ordination among tier members is very important for the betterment of the performance of a sustainable supply chain [25]. Green product design [26, 29], green marketing [27, 33] and recycling [11] play a very crucial role to meet environmental responsibilities. Xu and Wang [30] studied a closed loop supply chain model with price and emission reduction dependent demand and observed the effect of remanufacturing and recycling on demand and the optimal decisions. They considered the Nash bargaining profit distribution co-ordination in the decentralized scenario. Heydari et al. [12] studied the problem of financing a two-stage sustainable supply chain using green bonds. The model uses discounts to motivate the customers to buy again and recycle the products.

In order to execute the environmental responsibility, the manufacturer designs and develops a green product for the environmentally aware customers and for that some investment should be done for technology development [32] to enhance the product greenness. The manufacturer incurs other costs in developing, packaging, transporting the green product in an environmental friendly way. The degree of greenness of a product will be evaluated by the profit of the manufacturer in selling the green product to the environmentally aware customers. The retailer then sells the product to the customers after purchasing them from the manufacturer. In order to enhance the purchasing behavior of the consumers, the retailer uses several green marketing strategies for instance environmental advertisements or educating the consumers regarding environmental improvements to project the green product [1] and the level of marketing is reflected in the profit of the retailers.

Supply chain co-ordination has a positive effect on environment. This is verified in our work in the context of some contracts where the degree of co-operation is ascending. The contracts are price-only, cost sharing and two-part tariff contracts. In the price-only contract, the wholesale price is the sole connection between the manufacturer and the retailer. In the cost sharing contract, the green marketing cost which was beared by the retailer is being shared by the manufacturer. This contract is being practically used by many companies such as Intel, who shared 60% of the retailer’s cost in marketing in 2001 [7]. In the two-part tariff contract, every decision of the tier members is equivalent to those in the fully co-operative or centralized scenario. Thus it is the most co-operative among all other contracts.

But Channel Co-ordination in a Sustainable Supply Chain under contracts were not treated in any of the above literature. Toktas-Palut [28] considered an integrated three-stage forward and reverse supply chain, which provides new and remanufactured green products to a green concerned market. Xu et al. [31] studied co ordination of the supply chain considering the green technology in the block chain era with an online platform. Block chain technology helps the products become greener and bring more profits for the manufacturer and the platform. Jian et al. [17] attempted to find the impact of the manufacturer’s fairness concern on a green close-loop supply chain decisions and profits. Hong and Guo [13] analyzed and modeled a green product supply chain under three co-ordination contracts namely price-only, cost sharing and two-part tariff. Their demand function involved the green awareness of the consumers and the green marketing effort of the retailer. Our work is an extension of this work in the sense that we involve several other criteria like green packaging, green transport, green awareness, green technology, green marketing etc in the demand function. In our work, we study green initiatives of the Chain members in the form of green product design, packaging, development, marketing and many other factors in the context of co-ordination contracts which was not studied so far.
We compare the decisions of the manufacturer and the retailer in the above mentioned contracts. This analysis will help us to know as to how the co-operation level effects the degree of environmental improvements. It is found that the environmental attributes of the product increases with the the increase in the co-operation levels and also with the increase in the environmental awareness of the consumers. Thus co-ordination from the two-part tariff contract is very much beneficial for the tier members not only economically but also for environmental improvements. It is also noteworthy that the manufacturer gains more from the cost sharing contract than in the price-only contract whereas for the retailer, it is the opposite case.

The rest of the Paper is organized as follows. The Review of previous research is discussed in Section 2. In Section 3, the notations and assumptions are given The Mathematical formulation of the Problem is discussed in Section 4. The full Channel co-ordination is discussed under the two part tariff contract in Section 5. The comparison among the decisions of all the contracts are discussed and the necessary managerial insights were given in Section 6. The conclusion and the further research that can be done to extend this study are given in Section 7.

2. LITERATURE REVIEW

Two areas of Research are very much related to our study. One is Sustainable Supply Chain Management and the other is Chain Co-ordination. Sustainable supply chain management deals with matters related to environment such as the design and development of green product and its by-products, by-products that are produced during the usage of the product [8, 9]. Such product design keeps in mind both the functional and environmental attributes. Another product design problem in which the environmental regulations are imposed to control the emissions as a result of production [14]. Gouda et al. [10] studied the design of the product with a composite regulatory constraint to treat emission related quality levels. Hong et al. [15] studied a supply chain configuration problem for a green product family in consideration of guaranteed service time and emission constraints. Xu and Wang [30] studied a closed loop supply chain model with price and emission reduction dependent demand. Yalabik and Fairchild [32] examined the effects of consumers’ green awareness, competitive pressure on company investments in green production. Various studies [5, 24] disclose that increasing green awareness and green knowledge increases the green production and the awareness increases with the green marketing initiatives.

Co-ordination is also very much crucial in a sustainable supply chain [3]. In this paper, we investigate the role of co-operation in achieving economic goals and also environmental developments. In another study, it has been shown that for both simultaneous moves and leader-follower games, the supply chain co-ordination can always be achieved by a pair of properly designed contracts [19]. Swami and Shah [27] addressed the questions such as the extent of effort in greening of operations by manufacturer or the retailer, level of co-operation between them and how to co-ordinate their operations. Xu and Wang [30] in their work used Nash bargaining profit distribution co-ordination contract which is better than the traditional cost sharing or revenue sharing contracts considering consumer’s low carbon and remanufacturing preferences. Lan and Yu [18] introduced Promotion effort and service level into system decision-making and tried to coordinate the CGB Supply chain comprising of a Supplier, a group buying platform and a group leader. Li et al. [20] investigated the effects of two types of Government subsidies on the green decisions of a two-echelon under cap and trade mechanism. Meng et al. [23] studied a dual-channel green supply chain considering consumers’ green preference and channel preference and explored the pricing policy of green product and the manufacturer’s collaborative pricing of common products and green products with and without Government subsidies. Heydari et al. [12] studied the problem of financing a two-stage sustainable supply chain using green bonds. The recent pandemic has greatly affected Green Supply Chain Management. This has been extensively reviewed by Chowdhury et al. [6]. A part of their discussion was Supply chain sustainability in the light of the Pandemic. The following table shows the contributions of various authors in the related literature.
Table 1. Comparative study of contributions made by different authors.

<table>
<thead>
<tr>
<th>Author</th>
<th>G/CSR/S effort</th>
<th>No of stages</th>
<th>No of members</th>
<th>Contract</th>
<th>Demand</th>
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<tr>
<td>Swami and Shah [27]</td>
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<td>Xu et al. [29]</td>
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<td>Subramanian [26]</td>
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<td>Heydari et al. [12]</td>
<td>S</td>
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<td>Govindan and Popiuc [11]</td>
<td>None</td>
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<td>Proposed model</td>
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The abbreviations used in Table 1 are given below:

3. Notations and Assumptions

The following notations are used throughout this paper

3.1. Notations

$p_d$ : Unit retail price of the product under the price only contract.

$p_{cs}$ : Unit retail price of the product under the cost-sharing contract.

$p_c$ : Unit retail price of the product in the fully co-operative case.

$p_{co}$ : Unit retail price of the product under the two-part tariff contract.

$q_i^d$ : i-th green quality criterion under the price-only contract.

$q_i^{cs}$ : i-th green quality criterion under the cost sharing contract

$q_i^c$ : i-th green quality criterion in the fully co-operative case

$q_i^{co}$ : i-th green quality criterion under the two-part tariff contract.

$m_d$ : Marketing effort of the retailer under the price-only contract.

$m_{cs}$ : Marketing effort of the retailer under the cost sharing contract.

$m_c$ : Marketing effort of the retailer in the fully co-operative case.

$m_{co}$ : Marketing effort of the retailer under the two-part tariff contract.

$w_d$ : Unit wholesale price of the product under the price-only contract.

$w_{cs}$ : Unit wholesale price of the product under the cost sharing contract.

$w_{co}$ : Unit wholesale price of the product under the two-part tariff contract.

$c$ : Unit cost of raw materials for manufacturing the product.

$\pi_d^r$ : Profit of the retailer in the price-only contract.

$\pi_{cs}^r$ : Profit of the retailer in the cost sharing contract.

$\pi_{co}^r$ : Profit of the retailer in the two-part tariff contract.

$\pi_d^m$ : Profit of the manufacturer in the price-only contract.
\( \pi_{m}^{c} \) : Profit of the manufacturer in the price-only contract.
\( \pi_{m}^{co} \) : Profit of the manufacturer in the two-part tariff contract.
\( \pi_{c} \) : Profit in the fully co-operative case.
\( \phi_{cs} \) : Fraction of the marketing cost shared by the manufacturer.
\( a \) : Independent part of the functional form of the market demand.
\( b \) : Price sensitivity parameter of the market demand.
\( \alpha_{i} \) : i-th green quality sensitivity parameters of the market demand.
\( \beta_{i} \) : Cost rate for the green quality criterion \( q_{i} \).
\( t \) : Cost rate for marketing.

3.2. Assumptions

The supply chain consists of one manufacturer and one retailer. The manufacturer acts as a leader and first determines the wholesale price, taking into consideration the various costs of manufacturing the product which in turn depends on the level of greenness of the product. The retailer acts as a follower and decides the retail price taking into consideration the cost of green marketing. The manufacturer is considered to be the leader because the manufacturer produces items from the raw materials and can be the best judge to determine the wholesale price. The retailer is considered to be the follower because based on the wholesale price decided by the manufacturer, the retailer can fix the retail price. These assumptions will generate more profit both to the manufacturer and the retailer and also it will ensure social welfare, which is very essential [21]. We analyze the decisions of the manufacturer and the retailer under the contracts namely price-only and green marketing cost sharing in the decentralized scenario and finally the two-part tariff contract in the centralized scenario as these contracts are of increasing level of co-ordination. The cost sharing contract is being practically used by many companies such as Intel, who shared 60% of the retailer’s cost in marketing in 2001 [7]. Walmart received approximately $100 million in advertising funding from its vendors. The two-part tariff contract can perfectly co-ordinate the supply chain whereas revenue sharing contract cannot [29]. Only a single item has been considered in our model.

4. Mathematical formulation of the model

In our model, there is a single manufacturer and a single retailer. The manufacturer produces a green product and sells it to the retailer, who sells it to the environmentally conscious customers. The manufacturer incurs costs such as green technology development, green packaging transport and various other costs to produce and develop the green product. The retailer incurs the cost of green marketing so that it can convert the environmentally aware customers to actual purchasers of the green product. The product greenness is determined by the emission savings as compared to the traditional products.

The demand of the product is given by

\[ D = a - bp + \sum_{i=1}^{n} \alpha_{i}q_{i} + m \]

where, \( p \) is the retail price of the product, \( q_{i} \) are the various quality criteria of the green product like product greenness, packaging etc and \( 0 < q_{i} < 1 \). \( m \) is the marketing effort of the retailer and \( 0 < m < 1 \). \( \alpha_{i} \) are the parameters related to the quality criteria. \( \sum_{i=1}^{n} \beta_{i}q_{i}^{2} \) is the total cost of the manufacturer to produce the green product and \( tm^{2} \) is the cost of marketing of the retailer. \( a \) and \( b \) are as defined earlier with \( a > 0 \) and \( b > 0 \).

The manufacturer and the retailer execute their environmental duties by producing, marketing and selling the green product to the customers. The environmentally aware customers get benefited both from the functional attributes and environmental attributes of the product. The manufacturer invests in green technology to develop the green product. This investment increases with the increase in the greenness of the product. The retailer develops strategies for promoting the green product and thus bears some cost for it, which grows more when the marketing effort increases.

In this section we discuss the cases of the price only and green marketing cost-sharing contract and in the section that follows we discuss the full channel coordination and the two part tariff contract. Finally we analyze the supply chain decisions in the above contracts.
4.1. Price-only contract

Here, the manufacturer and the retailer take their decisions to maximize their profits in a non-cooperative scenario. The manufacturer is the Stackelberg leader, who decides the wholesale price of the product and also the various quality criteria. The retailer acts as a follower and makes decisions regarding the marketing effort and the retail price accordingly. Here the manufacturer and retailer are only connected by the wholesale price.

So the profit of the retailer is given by,

$$\pi_d^m(w_d, q_i^d, q_r^d, \ldots, q_n^d) = (w_d - c) \left( a - bp_d + \sum_{i=1}^{n} \alpha_i q_i^d + m_d \right) - \sum_{i=1}^{n} \beta_i(q_i^d)^2.$$

(3)

By substituting equation (3) on equation (2), we can find the optimal values of \(w_d^*\) and \(q_i^d^*\) which will maximize \(\pi_d^m\). The objective function \(\pi_d^m\) is concave in \(w_d\) and \(q_i^d\). The optimal values of \(w_d^*\) and \(q_i^d^*\) are given by

$$w_d^* = \frac{bct(2bt - 1) \sum_{i=1}^{n} \frac{\alpha_i}{\beta_i} + a(4bt - 1)}{bt(2bt - 1) \sum_{i=1}^{n} \frac{\alpha_i}{\beta_i} - (4bt - 1)(4bt(b - 1) + b)}$$

$$q_i^d^* = \frac{(2bt - 1)[(4bt(b - 1) + b)c + a]}{2\beta_i[(2bt - 1)bt \sum_{i=1}^{n} \frac{\alpha_i}{\beta_i} - (4bt - 1)(4bt(b - 1) + b)]}.$$

(4)

Theorem 4.1. The optimal decisions of the manufacturer and retailer in the Stackelberg equilibrium under the price-only contract are
The profit function of the manufacturer is therefore, 

\[ \pi^m = \left( a - \frac{bt(2bt - 1) \sum_{i=1}^n \frac{\alpha_i^m}{\beta_i^m} - [4bt(b - 1) + b]}{4bt - 1} \right) + \left( b \left[ bct(2bt - 1) \sum_{i=1}^n \frac{\alpha_i^m}{\beta_i^m} + a \right] - \frac{[(4bt(b - 1) + b)c + a](2bt - 1) \sum_{i=1}^n \frac{\alpha_i^m}{\beta_i^m}}{4bt - 1} \right). \]

The following corollaries show the effects of the parameters on the decision variables.

**Corollary 4.2.** The optimal decision variables have the properties

(i) \( \frac{\partial \alpha_i^m}{\partial \alpha_i} > 0 \), (ii) \( \frac{\partial \alpha_i^m}{\partial \alpha_i} > 0 \), (iii) \( \frac{\partial \alpha_i^m}{\partial \alpha_i} > 0 \).

A higher the green quality criterion of the product prompts the manufacturer to produce a greener product and forces the retailer to give more effort in marketing and also to increase the retail price.

**Corollary 4.3.** The optimal decision variables also have the following properties

(i) \( \frac{\partial \alpha_i^m}{\partial \beta_i} < 0 \), (ii) \( \frac{\partial \alpha_i^m}{\partial \beta_i} < 0 \).

A higher green quality development cost or green marketing cost reduces the effort of the manufacturer for producing a green product and the effort of the retailer for green marketing.

Consumers with increasing green consciousness opt for the green items rather than the conventional ones and so want to pay even extra for them. This goes to the favor of the retailer, who can increase the retail price and the green marketing effort. Again, when the cost of green quality improvement grows then the manufacturer can decrease the greenness and the retailer will reduce the green marketing effort if the marketing cost rises.

Putting the optimal values of the decision variables on equations (1) and (3) we get the optimal profits of the retailer and manufacturer \( \pi^r_d \) and \( \pi^m_d \) and thus the overall profit \( \pi^r_d \) is given by \( \pi^r_d = \pi^r_d + \pi^m_d \), can be determined.

**4.2. Green-marketing cost-sharing contract**

As discussed earlier, co-operation in a sustainable supply chain is very beneficial to both the manufacturer and the retailer not only economically but also for environmental improvement. In this section we will consider the contract in which the manufacturer shares a fraction of the green marketing cost with the retailer. We have assumed that the \( \phi_{cs} \) is the fraction of the marketing cost that will be shared by the manufacturer. Thus, the profit function of the retailer will be given by,

\[ \pi^r_{cs} = (p_{cs} - w_{cs}) \left( a - bp_{cs} + \sum_{i=1}^n \alpha_i q_i^{cs} + m_{cs} \right) - (1 - \phi_{cs})t m_{cs}^2. \]  

The profit function of the manufacturer is therefore,

\[ \pi^m_{cs} (w_{cs}, q_1^{cs}, q_2^{cs}, \ldots, q_n^{cs}, m_{cs}, \phi_{cs}) = (w_{cs} - c) \left( a - bp_{cs} + \sum_{i=1}^n \alpha_i q_i^{cs} + m_{cs} \right) - \sum_{i=1}^n \beta_i q_i^{cs^2} - \phi_{cs} t m_{cs}^2. \]  

The optimal values of the decision variables are given in the following Theorem
Theorem 4.4. The optimal values of the decision variables in the green-marketing, cost-sharing contract are

\[
p^*_cs = \frac{2(1 - \phi^*_c) t [a + w^*_c b + \sum_{i=1}^n \alpha_i q^*_i] - w^*_cs}{4bt(1 - \phi^*_c) - 1},
\]

\[
m^*_cs = \frac{2(1 - \phi^*_c) t [a - w^*_c b + \sum_{i=1}^n \alpha_i q^*_i] - w^*_cs}{4bt(1 - \phi^*_c) - 1},
\]

\[
w^*_cs = \left[ \frac{bc2b(1 - \phi^*_c) t - 1 + 2c + 2\phi^*_c t b(1 - \phi^*_c) t}{4bt(1 - \phi^*_c) + b4b(1 - \phi^*_c) t - 2 - 2bt(1 - \phi^*_c) - 1 \sum_{i=1}^n \alpha_i^2}{\beta_i} \right] - \frac{2abt(1 - \phi^*_c) + c2bt(1 - \phi^*_c) - 1 \sum_{i=1}^n \alpha_i^2}{\beta_i}.
\]

We will obtain the optimal values of the profit functions by putting the values of the decision variables in the equations (5) and (6). We thus get the total profit in this case as

\[
\pi^*_cs = \pi^*_c + \pi^*_m
\]

We compare the green qualities and green marketing efforts of the price-only and cost-sharing contracts in the following Proposition

Proposition 4.5. \( q^*_cs > q^*_ls \) and \( m^*_cs > m^*_s \). The green quality of a product under the green-marketing cost-sharing contract is more than that in the price-only contract. The green marketing effort of the retailer is also more in the cost-sharing case than under the price-only contract.

The above Proposition shows that co-ordination between chain members has a positive effect on environmental improvements and allows the members to execute their environmental responsibilities to a large extent. But this sort of co-operation is not at its highest level. In the next section we study the two-part tariff contract which has the highest degree of co-operation for which the economic benefits are similar to that of a centralized supply chain [2].

5. Full Chain Co-ordination

Here, we study the Chain in full co-operative mode and analyze the decisions of the tier members to show how it affects the fulfillment of their environmental responsibilities. In this centralized scenario we also find the optimal values of the decision variables in order to maximize the profit of the chain. This will be a benchmark for comparing all coordination contracts.

The profit function is as follows:

\[
\pi_c(p_c, q^*_1, q^*_2, \ldots, q^*_n, m_c) = (p_c - c) \left( a - bp_c + \sum_{i=1}^n \alpha_i q_i + m_c \right) - \sum_{i=1}^n \beta_i q_i^2 - tm_c^2.
\]

The following Lemma gives the optimal values of the decision variables

Lemma 5.1. The optimal values of the decision variables are

\[
p^*_c = \frac{2a + 2bc - c \sum_{i=1}^n \frac{\alpha_i^2}{\beta_i^2}}{4b - \sum_{i=1}^n \frac{\alpha_i^2}{\beta_i^2} - \frac{1}{t}}, \quad q^*_i = \frac{(a - bc) \alpha_i}{\beta_i \left( 4b - \sum_{i=1}^n \frac{\alpha_i^2}{\beta_i^2} - \frac{1}{t} \right)}, \quad m^*_c = \frac{(a - bc)}{\beta_i \left( 4b - \sum_{i=1}^n \frac{\alpha_i^2}{\beta_i^2} - \frac{1}{t} \right)}.
\]

Proof is given in the Appendix.

Putting the optimal values of the decision variables on the profit functions, we get the maximal overall profit as follows

\[
\pi^*_c = \frac{(a - bc)^2}{4b - \sum_{i=1}^n \frac{\alpha_i^2}{\beta_i^2} - \frac{1}{t}}.
\]
As in this case the wholesale price of the product is actually the cost of manufacturing the product. So, the manufacturer will not be willing to go in this full channel co-ordination case as he is not getting the estimated profit despite of his investments in adopting green technology or other costs for improving the green quality of the product. So, we study the two-part tariff contract so that the manufacturer may get some incentives which will arouse his interest in the contract. This has been utilized and discussed by many researchers in this field [26, 27].

### 5.1. Two-part tariff contract

We assume that the manufacturer offers a two-part tariff contract, in which the manufacturer takes a fixed fee \( T \). So the profit function of the retailer is

\[
\pi_r^{co}(p_{co}, m_{co}) = (p_{co} - w_{co}) \left( a - bp_{co} + \sum_{i=1}^{n} \alpha_i q_i^{co} + m_{co} \right) - tm_{co}^2 - T. 
\]  
(8)

The optimal values of the decision variables of the retailer under this contract are

\[
p_{co}^* = \frac{2[a + w_{co}b + \sum_{i=1}^{n} \alpha_i q_i^{co} - w_{co}]}{4bt - 1}, \quad m_{co}^* = \frac{[a - w_{co}b + \sum_{i=1}^{n} \alpha_i q_i^{co}]}{4bt - 1}.
\]  
(9)

Then the manufacturer optimizes its decision variables by solving the following:

\[
\text{max } \pi_m^{co}(w_{co}, T, q_1^{co}, q_2^{co}, \ldots, q_{m-1}^{co}, q_{m+1}^{co}, \ldots, q_n^{co}) = (w_{co} - c) \left( a - bp_{co}^* + \sum_{i=1}^{n} \alpha_i q_i^{co} + m_{co}^* \right) - \sum_{i=1}^{n} \beta_i q_i^{co2} + T 
\]  
(10)

such that

\[
(p_{co}^* - w_{co}) \left( a - bp_{co}^* + \sum_{i=1}^{n} \alpha_i q_i^{co} + m_{co}^* \right) - T \geq \pi_d^{r*}
\]  
(11)

where \( p_{co}^* \) and \( m_{co}^* \) are given in equation (9). The constraint in equation (11) is a guarantee the acceptance of the contract by the retailer. The optimal values of the decision variables of the manufacturer are depicted in the following Theorem.

**Theorem 5.2.** The manufacturer selects the green quality parameters as \( q_i^{co*} = q_i^*, i = 1, 2, \ldots, n \) and imposes the two-part tariff contract as follows:

\[
\{w_{co}^*, T^*\} = \left\{ c, \frac{(a - bc)^2}{4b - \sum_{i=1}^{n} \frac{\alpha_i^2}{\beta_i} - \frac{1}{\epsilon}} - \pi_d^{r*} \right\}.
\]

**Proof.** The manufacturer can enhance the profit by selecting suitable fixed payment \( T \) which is to be provided by the retailer. Therefore, we let

\[
T = (p_{co}^* - w_{co}) \left( a - bp_{co}^* + \sum_{i=1, i \neq m}^{n} \alpha_i q_i^{co} + m_{co}^* \right) - \pi_d^{r*}.
\]

The manufacturer will have to maximize the profit given by

\[
\pi_m^{co}(w_{co}, T, q_1^{co}, q_2^{co}, \ldots, q_{m-1}^{co}, q_{m+1}^{co}, \ldots, q_n^{co}) = (p_{co}^* - c) \left( a - bp_{co}^* + \sum_{i=1}^{n} \alpha_i q_i^{co} + m_{co}^* \right) - \sum_{i=1, i \neq m}^{n} \beta_i q_i^{co2}.
\]
Putting the values of $p^*_c$ and $m^*_c$ from equation (9) we obtain the optimal values of the decision variables of the manufacturer as follows:

$$w^*_c = c$$

$$q^*_i = \frac{(a - bc)\alpha_i}{\beta_i \left(4b - \sum_{i=1}^{n} \frac{\alpha_i^2}{\beta_i} - \frac{1}{t}\right)}$$

$$m^*_c = \frac{(a - bc)\alpha_i}{t \left(4b - \sum_{i=1}^{n} \frac{\alpha_i^2}{\beta_i} - \frac{1}{t}\right)}$$

and hence, $T^* = \frac{(a - bc)^2}{4b - \sum_{i=1}^{n} \frac{\alpha_i^2}{\beta_i} - \frac{1}{t}} - \pi^*_d$.

This completes the proof. □

Thus under the above contract, the full chain co-ordination is achieved where, $p^*_c = p^*_c$, $q^*_i = q^*_i$. The manufacturer receives all the profits from the increment and the profit of the retailer is the same as in the price only contract.

6. Discussions and analysis

In this section we compare the decision variables in the three contracts discussed earlier and study the effects of co-operation among the tier members in a green production over the decisions under the contracts.

Proposition 6.1. The $n$ green qualities including product greenness, green packaging etc and the green marketing effort and the retail price are related under the three contracts as follows:

(i) $p^*_c < p^*_d < p^*_c$  
(ii) $q^*_i < q^*_i^{cs} < q^*_i^{cos}$  
(iii) $m^*_d < m^*_c < m^*_c$  
(iv) $D^*_d < D^*_c < D^*_c$.

The above Proposition shows that the two-part tariff contract being of the highest degree of co-operation gives the maximum profit with the most environmental friendly product for which the highest marketing effort is required and the product costs the least to the consumers and thus gives rise to the highest demand for the product. Then comes the green marketing cost sharing contract whose degree of co-operation is less than that of the two-part tariff contract also has a positive effect in arousing demand. Here, as discussed earlier, a portion of the marketing cost is shared by the manufacturer, who determines it. Here, both the green quality of the product, the green marketing strategy are greater than that in the price only contract which has the least degree of co-operation. But for producing the greener product, the retail price is greater than that of the price only case.

Proposition 6.2. Let $\Delta q^{csd}_i = q^{cs}_i - q^{ds}_i$ and $\Delta q^{cocs}_i = q^{cos}_i - q^{cs}_i$. Then $\Delta q^{csd}_i$ and $\Delta q^{cocs}_i$ increase in $\alpha_i$ and decrease in $\beta_i$.

In the above Proposition, we note the changes in the green quality of the product with respect to the green quality parameters and the costs incurred for producing and promoting it. It shows that the green quality of the product in the two-part tariff contract is much more than the cost sharing contract which is much more than that of the price only contract as the green quality parameters increase. But this advantage is not so significant whenever the cost of green marketing and green production increases.

We have done the sensitivity analysis upon the quality parameters $\alpha_i$ and $\beta_i$ for providing managerial insights which is portrayed in Figure 1. Here, the basic parameters are $n = 5$, $c = 0.06$, $a = 3$, $b = 2$, $t = 2.75$, $0.25 \leq \alpha_i \leq 1.25$, $1.25 \leq \beta_i \leq 3.25$. A numerical example is given in Table 2 to validate the analysis. It is clearly
noticeable from the figures and the numerical example that the results of the previous two Propositions are true. It is also visible that there are no such differences of product greenness between the cost sharing and price only contracts but there is a significant difference between them and the two-part tariff contract which shows that a high co-operation level in a sustainable chain can produce the most environmental friendly product. In Figure 2, we have portrayed the sensitivity of the green marketing efforts with respect to the green quality parameters and the cost of green technology development. It is noted that the marketing efforts increase with the increase in the quality parameters and decrease with the increase in the cost for the green production. This also justifies the validity of the previous Proposition.

Proposition 6.3. The retailer is motivated by the manufacturer to indulge further in green marketing by the cost sharing contract \((1 - \phi^*_c) t_{m_2}^c > t_{m_2}^d\).

It is evident from the above Proposition that the cost in green marketing of the retailer is more in the cost sharing case than in the price only case. By sharing the green marketing cost, the manufacturer induces the retailer to increase the efforts of green marketing in order to attract more customers. With the increase in the wholesale price also, the retailer’s green marketing effort increases. But the manufacturer minimizes the cost sharing fraction in order to maximize the profit which thus goes in favor of the manufacturer but has adverse effects in the profit of the retailer who gains much more in the price only contract than in this case.

Proposition 6.4. The profit function of the retailer and manufacturer in the three contracts are related as

(i) \(\pi_d^{ms*} < \pi_d^{co*}, \pi_d^{ms*} < \pi_d^{cs*}\)
(ii) \(\pi_c^{cs*} < \pi_c^{co*}\).

It is noted that the manufacturer is a gainer in the co-operative case as it is the decision maker in this case, the retailer however gets the same profit as that of the price only case. In the cost sharing case also the manufacturer is also gains more than that under the price only contract as it is the leader over there, whereas reverse is the case for the retailer as discussed earlier. The sensitivity analysis done for \(\alpha_i\) and \(\beta_i\) is given in Figure 2.

Proposition 6.5. Let \(\Delta \pi_{cad}^m = \pi_{cad}^m - \pi_{cad}^{d*}\) and \(\Delta \pi_{cad}^r = \pi_{cad}^r - \pi_{cad}^{r*}\), then \(\Delta \pi_{cad}^m > 0, \Delta \pi_{cad}^r < 0\) and

(i) \(\Delta \pi_{cad}^m\) increases in \(\alpha_i\) and decreases in \(\beta_i\).
(ii) \(\Delta \pi_{cad}^r\) decreases in \(\alpha_i\) and increases in \(\beta_i\).
As the green awareness of the consumers increases, marketing efforts may be reduced and the cost shared by
the manufacturer reduces so that the manufacturer gains much in the cost sharing case. But as the costs of
producing and promoting the green product increases the profit difference diminishes. But the retailer does not
gain much in the cost sharing case than that of the price only case and the difference decreases as the green
awareness increases and increases as the costs increase which is evident from Figure 3.

7. Managerial insights

1. The two-part tariff contract being of the highest degree of co-operation gives the maximum profit with the
most environmental friendly product for which the highest marketing effort is required and the product costs
the least to the consumers and thus gives rise to the highest demand for the product
2. There are no such differences of product greenness between the cost sharing and price only contracts
Table 2. The optimal solutions of the model under Price Only, Cost Sharing and Two Part-Tariff Contract.

<table>
<thead>
<tr>
<th>Contract</th>
<th>$q_1$</th>
<th>$m$</th>
<th>$p($</th>
<th>$w($</th>
<th>$\pi'($)</th>
<th>$\pi''($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Only (d)</td>
<td>0.018</td>
<td>0.015</td>
<td>0.644</td>
<td>0.142</td>
<td>106.980</td>
<td>173.281</td>
</tr>
<tr>
<td>Cost Sharing (cs)</td>
<td>0.052</td>
<td>0.079</td>
<td>1.548</td>
<td>0.437</td>
<td>8.4</td>
<td>319.159</td>
</tr>
<tr>
<td>Two-Part Tariff (co)</td>
<td>0.389</td>
<td>0.305</td>
<td>0.97</td>
<td>0.06</td>
<td>106.980</td>
<td>249.582</td>
</tr>
</tbody>
</table>

3. Both the green quality of the product, the green marketing strategy in the cost sharing case are greater than that in the price only contract which has the least degree of co-operation. But for producing the greener product, the retail price is greater than that of the price only case.

4. It is noted that the marketing efforts increase with the increase in the quality parameters and decrease with the increase in the cost for the green production.

5. The cost in green marketing of the retailer is more in the cost sharing case than in the price only case. The manufacturer minimizes the cost sharing fraction in order to maximize the profit and thus the retailer gains much less in the cost sharing case than in the price only case.

8. Conclusion

In this work, we studied a supply chain in which a green product is manufactured, promoted and sold to the environmentally aware consumers, where the chain is coordinated in increasing order of the level of cooperation. In this way, the tier members execute their environmental responsibilities. The manufacturer invests in the green quality development of the product and the retailer invests in green marketing strategy. We analyzed the decisions of the chain in three contracts with different levels of cooperation. This comparison among them revealed that for the betterment of environmental performance, supply chain coordination plays a vital role. The environmental consciousness of the consumers is also essential for a green product sustainable supply chain with coordination. We see that the two-part tariff contract which has the highest degree of coordination, generates the most environmental friendly product.

Our work has some limitations. We have taken only a single product in our discussion. This research can be extended by involving multiple products in the chain. It can also be extended by making a coordination where the retailer shares the cost of green technology which is bore only by the manufacturer. There are provisions of extension of this study by varying the attributes of the green qualities of the product.

Appendix A. Proof of the Lemma

The Hessian matrices are given by

$|H_1| = -2b < 0, \quad H_2 = \begin{bmatrix} -2b & \alpha_1 \\ \alpha_1 & -2\beta_1 \end{bmatrix}$ then, $|H_2| = 4b\beta_1 - \alpha_1^2 > 0, \quad i f \ 4b\beta_1 > \alpha_1^2$

$H_3 = \begin{bmatrix} -2b & \alpha_1 & 1 \\ \alpha_1 & -2\beta_1 & 0 \\ 1 & 0 & -2t \end{bmatrix}$ then, $|H_3| = -8bt\beta_1 + 2t\alpha_1^2 + 2\beta_1 < 0, \quad i f \ 4bt\beta_1 > t\alpha_1^2 + \beta_1$

$H_4 = \begin{bmatrix} -2b & \alpha_1 & \alpha_2 & 1 \\ \alpha_1 & -2\beta_1 & 0 & 0 \\ \alpha_2 & 0 & -2\beta_2 & 0 \\ 1 & 0 & 0 & -2t \end{bmatrix}$ then, $|H_4| = -4\alpha_1^2\beta_1\beta_2 + 16bt\beta_1\beta_2 - 4t\alpha_1^2\beta_2 - 4t\alpha_2^2\beta_1.$

Then, $|H_4| > 0, \quad i f \ 4bt\beta_1\beta_2 > t\alpha_1^2\beta_2 + t\alpha_2^2\beta_1 + \alpha_1^2\beta_1\beta_2.$

Therefore, the $n-th$ order Hessian matrix will be negative definite if
4bt_1b_2b_3\ldots b_n > ta_1^2(\beta_2\beta_3\ldots\beta_n) + ta_2^2(\beta_1\beta_3\ldots\beta_n) + \ldots + a_n^2(\beta_1\beta_2\ldots\beta_{n-1}).

Thus \(\pi_c(p_c, q_1^c, q_2^c, \ldots, q_n^c, m_c)\) is jointly concave in \((p_c, q_1^c, q_2^c, \ldots, q_n^c, m_c)\) if the above condition holds.

To obtain the unique optimal solutions \((p_c^*, q_1^c, q_2^c, \ldots, q_n^c, m_c)\), we solve the following:

\[
\begin{align*}
\frac{\partial \pi}{\partial p_c} &= a - 2bp_c + bc + \sum_{i=1}^{n} \alpha_iq_i^c + m_c = 0 \\
\frac{\partial \pi}{\partial q_i^c} &= (p_c - c)\alpha_i - 2\beta_i q_i^c = 0 \\
\frac{\partial \pi}{\partial m_c} &= (p_c - c) - 2tm_c = 0.
\end{align*}
\]

Thus we obtain the following optimal solutions

\[
p_c^* = \frac{2a + 2bc - c}{4b - \sum_{i=1}^{n} a_i^2}, \quad q_i^c = \frac{a_i - bc}{\beta_i \left(4b - \sum_{i=1}^{n} a_i^2 - \frac{1}{4}\right)}, \quad m_c^* = \frac{a - bc}{t \left(4b - \sum_{i=1}^{n} a_i^2 - \frac{1}{4}\right)}
\]

and this completes the proof.

**REFERENCES**


[9] J. Heydari, A.A. Taleizadeh and F. Jolai, Jointly concave in \((p_c, q_1^c, q_2^c, \ldots, q_n^c, m_c)\) if the above condition holds.

To obtain the unique optimal solutions \((p_c^*, q_1^c, q_2^c, \ldots, q_n^c, m_c)\), we solve the following:

\[
\begin{align*}
\frac{\partial \pi}{\partial p_c} &= a - 2bp_c + bc + \sum_{i=1}^{n} \alpha_iq_i^c + m_c = 0 \\
\frac{\partial \pi}{\partial q_i^c} &= (p_c - c)\alpha_i - 2\beta_i q_i^c = 0 \\
\frac{\partial \pi}{\partial m_c} &= (p_c - c) - 2tm_c = 0.
\end{align*}
\]

Thus we obtain the following optimal solutions

\[
p_c^* = \frac{2a + 2bc - c}{4b - \sum_{i=1}^{n} a_i^2}, \quad q_i^c = \frac{a_i - bc}{\beta_i \left(4b - \sum_{i=1}^{n} a_i^2 - \frac{1}{4}\right)}, \quad m_c^* = \frac{a - bc}{t \left(4b - \sum_{i=1}^{n} a_i^2 - \frac{1}{4}\right)}
\]

and this completes the proof.

**REFERENCES**


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