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SOLVING A REAL WORLD NON-CONVEX QUADRATIC ASSIGNMENT PROBLEM

Badri Toppur

Abstract. The India-Japan Lighting company operates three plants in India. These three plants manufacture headlamps and taillights for the automotive industry. This study examines the facility location problem in one of these plants where 12 facilities must be placed in a two-column multi-row cellular layout. The machining sequences for the 20 parts conveyed among the 12 facilities were specified. The Quadratic Assignment Problem (QAP) is classified as an NP-hard problem in large instances. We modelled the specific instance as a QAP and reported the solution obtained by an easily available generalised reduced gradient (GRG) nonlinear solver and the solution obtained from the *Gurobi* optimiser. The *Gurobi* optimiser provides an excellent incumbent solution in quick time, but takes exponential time to reduce the duality gap.

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1. Introduction

Three plants belonging to the India-Japan Lighting company manufacture headlamps, rear combination lamps (RCL), and other small lamps. They produce approximately 2.8 million headlamps and 2.7 million RCL annually, and therefore optimized throughput is an important part of their mission. The factory layout at Thirumzhisai, near Chennai in India is undergoing redesign for this purpose. Designing or redesigning a facility requires considerable resources, and a good design entails sufficient analysis to maintain low material handling costs and prevent future bottlenecks. By facility location one usually means the geographical location of the factory considered as one unit. The plural form of "facility" refers to entities that facilitate the performance of a job. A facility layout is concerned with the location and arrangement of the these facilities within the cells of the factory.

The factory space in question is rectangular and cellular, with an aisle running between two columns of cells. Several facilities where parts are machined must be placed in the best cell location so that the products do not require frequent long-distance conveyance. One to three cells may be grouped together to form a work centre. In effect one is assigning the facilities to the work centres. Some of the products may have a higher priority or may be hazardous and must be treated differently. This is known to be a very difficult industrial problem to solve in terms of optimising material flow, and many methods and algorithms have been developed to obtain good solutions. Benchmark problems are used to test the efficiency of a new algorithm against the current best

Keywords. Facility Location, Quadratic Assignment Problem, non-convex objective, non-linear solvers, incumbent solution.

Rajalakshmi School of Business, Chennai, India.

algorithms. A company team solved this problem using evolutionary and metaheuristic algorithms. Our goal was to obtain a solution using traditional nonlinear programming techniques, which had not been attempted at this company using modern-day hardware and algorithms. Observing and conveying the benefits of the optimal assignment to the company was one of the goals of this study. This empirical study also contributes to theory building by solving instances of a class of Quadratic Assignment Problems (QAPs) that arise in modelling facility location problems in the industry.

Section 2 presents a literature review relevant to this study. Section 3 describes the industrial problems automotive manufacturing companies face. Section 4 explains the general formulation of the QAP and the concrete instances for which data are available. The last two sections are Section 5 which contains the computational results from the solver executions and Section 6 which contains general conclusions.

2. Literature review

Rockafeller's treatise on Convex Analysis was a milestone in mathematical programming [19]. His classification of the objective function and constraints as convex, pseudoconvex, quasi-convex, strictly convex, strictly pseudoconvex, or strictly quasiconvex helped in structuring the space of nonlinear programs, with the goal of developing a methodology for solving them. This codification implies that one can determine the suitable algorithm for a certain class of problems. According to Bazaraa *et al.* [5], "the reduced gradient method was developed by Wolfe to solve nonlinear programming problems with linear constraints. This method was later generalised by Abadie and Carpentier [1] to handle nonlinear constraints".

Quadratic assignment problems are evident in several industrial settings. The Lawler form of the QAP, which is a generalisation of the Koopmans–Beckman form, is the traditional way to solve this problem [17]. Pioneering works were reported by Rosenblatt [20] and Gerd *et al.* [11]. Rosenblatt examined the assignment for five different flow matrices as a simulation of a dynamic layout. This is a good approach for sensitivity analysis of the QAP solution and is followed in this study. A survey of dynamic layout problems was done in 1998 by Balakrishnan and Cheng [3]. Drira *et al.* conducted a recent literature survey in 2006 [10]. Solutions that approach the simplest instances of the QAP were explained in a textbook by Burkhard *et al.* [8]. Burkhard *et al.* maintained QAPLib, which is a library of benchmark quadratic assignment problems [7]. An example of the formation of a partitioned matrix Φ in the quadratic objective function $x^t\Phi x$ is provided on page 7 of the NSF report by Bazaraa [4]. This methodology is used to formulate the quadratic objective function for our problem. Advanced algorithms for QAP were reported by Adams and Waddell [2] who further developed the reformulation–linearisation technique (RLT), which is suitable for the convexification of a non-convex objective function. Hahn *et al.* designed an algorithm for general QAPs [14]. The QAP is NP-Hard in terms of computational complexity, and only small-sized problems can be solved optimally in a reasonable amount of time [13]. Kothari and Ghosh studied a single-row facility layout problem, and this problem was itself very complex [15]. This intractability has led many researchers to design heuristic and meta-heuristic approaches, such as genetic algorithms, simulated annealing, ant colony optimisation, and particle swarm optimisation, to obtain near-optimal solutions quickly. Tari *et al.* demonstrated a tabu search approach for cellular layout design [21]. In addition, Hariprasad *et al.* presented their CRAFT software package written in Java programming language to solve facility layout problems [18]. Recent studies which document instances of facility layout problems formulated as QAPs include those by Brosch and de Klerk [6], Cubukcuoglu *et al.* [9], Fu *et al.* [12] and Wu *et al.* [22].

This study focuses on using current commercial solvers for a specific real-world instance of the problem. The first algorithm used was the generalised reduced gradient, which was designed by Lasdon *et al.* and implemented in Frontline Systems [16]. The GRG solver is available as an add-in to the Office tools of the Windows operating system. It has been an algorithm trusted for nonlinear optimisation for many decades. Other non-linear solvers available from the COIN-OR project are the BonMin solver and the Couenne solver which are available for use by researchers through Open Solver, and also remotely through the NEOS server in Wisconsin. The commercial *Gurobi* optimiser incorporates several optimisation methods. The academic version of the *Gurobi* optimiser is available for comparison and other research purposes. It uses the multiple cores of the processor for parallel

TABLE 1. Work Centre area.

Number	Work Centre	Area (Sq.ft)
1	A	200
2	B	300
3	C	100
4	D	200
5	E	100
6	F	200
7	G	200
8	H	100
9	I	200
10	J	200
11	K	100
12	L	300

processing of the subproblems of the branch and bound algorithm[1]. This implies that except at possible the root node, the branch-and-bound can solve various subproblems in parallel. The contribution of this study is a comparison of the performances of the GRG solver with the *Gurobi* optimizer in solving the given industrial problem.

3. Problem definition

The 12 work centres occupy area, that are given in Table 1. The area of these work centres, are either 100 ft^2, 200 ft^2, or 300 ft^2. The overall structure of the shop floor, is two-column, and multi-row, with each column having a width of 20 ft as shown in Figure 1. There is an aisle between the two columns that is 10 ft wide. Thus the workspace in the two columns can be seen as a grid of 10 ft $\times$ 10 ft cells. One rule, that we have enforced to model the system, is that the 100 ft^2 work centres occupy one of these 10 ft $\times$ 10 ft cells, the 200 ft^2 work centres occupy two of the 10 ft $\times$ 10 ft cells, and the 300 ft^2 work centres occupy three 10 ft $\times$ 10 ft cells. This rules out the placement of a 200 ft^2 work centre in an area that has dimension 40 ft $\times$ 5 ft or 25 ft $\times$ 8 ft by straddling two or more cells partially. One can easily obtain a compact layout that accommodates these 12 work centres within 2200 ft^2. The minimum vertical length required for the two columns is 60 ft.

Figures 1 and 2 show two feasible layouts for the 12 work centres. The work centres are labelled according to the specifications in Table 1. For example, the two work centres that are 300 ft^2 labelled "B" and "L" on the diagram are placed vertically since, if placed horizontally, they would obstruct the movement in the aisle. Given their long vertical orientation and large area, one would place them first and then place the smaller ones in the gaps above or below them. Such a combinatorial argument considerably simplifies the number of layouts; however, thousands remain. Computer programs can exhaustively list all feasible layouts for an existing plot. In Layout A, six work centres that are 200 ft^2 in area (A, D, F, G, I, J), were stacked vertically in the second column. The four work centres that are 100 ft^2 in area (C, E, H, K) were placed below the two work centres B and L in the first column. Subsequent to the enumeration of the layouts, rectilinear distances between the work centres were calculated from the centroids. The facilities are considered to be located at these centroids. They are small enough to fit any of the work centres, and their size and shape is not a constraint for any of the work centres. Rectilinear distance is the more appropriate metric than the Euclidean norm because there may be automatic vehicles and robots that move along straight lines, such as along the aisle, and turn into work centres at right angles to the aisle. In the distance matrix, the distance to the centre of the aisle, from

[1]The preliminary results obtained with the GRG solver were presented at the virtual conference in 2020 of the Serbian OR Society, wherein a participant suggested the use of the *Gurobi* optimiser.

Table 2. Production sequence and load.

Product	Sequence	Load	Product	Sequence	Load
α	1–2–6–7–8–10–12	40	λ	1–6–8–5–4–7–12	55
β	1–2–6–5–4–11–12	100	μ	1–4–5–10–11–12	35
γ	1–3–4–5–6–12	25	ν	1–2–5–6–4–7–12	65
δ	1–2–3–6–7–9–12	50	ξ	1–9–10–7–8–12	95
ϵ	1–3–5–4–8–11–12	90	o	1–8–6–7–11–5–2	150
ζ	1–3–5–4–7–10–12	30	π	1–4–6–10–8–12	20
η	1–5–3–6–7–8–12	45	ρ	1–8–11–10–7–9–12	35
θ	1–8–6–5–4–10–12	60	σ	1–9–6–5–4–10–12	45
ι	1–4–5–9–11–12	75	τ	1–2–3–4–5–6–11–12	50
κ	1–8–3–6–7–9–12	40	υ	1–2–6–8–10–11–12	40

the centroid of the cell, and the vertical distance along the aisle were added. The obtained rectilinear distance matrix varied depending on the layout. Because our focus was more on facility location than on facility layout, we only selected two layouts that were not equivalent by any symmetry. Given these layouts the task is to assign the 12 facilities to the work centres, so that the movement of parts between the facilities is as smooth as possible.

We have also assumed that all material handling and movement are within the confines of the factory perimeter and do not allow parts to be conveyed outside the factory perimeter from a work centre and brought in again from the outside into another work centre. Twenty parts must be machined at one or more of the 12 facilities. The job sequences in the 12 facilities are listed in Table 2. The loads that must be moved are listed in this table. These numbers can be consolidated to obtain the overall number of parts that need to be moved between facilities without differentiating between the parts. These data are displayed in Table 3, where the values in the first row indicate the number of parts moved from facility 1 to the other 11 facilities. The other rows correspond to flows from other facilities. Note that, sometimes, the flow of parts occurs from a higher-numbered facility to a lower-numbered facility. This is because the parts are different and require different processes. This means that the flow matrix is not strictly upper triangular but is sparse in the lower triangle.

4. General quadratic assignment model and concrete instance

The optimization goal for a problem involving n facilities is to minimise the sum of the fixed cost of assigning facility i to cell r, and the total material handling cost (TMHC) incurred between pairs of assigned facilities.

$$\sum_{i=1}^{n}\sum_{r=1}^{n} f_{ir}x_{ir} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{r=1}^{n}\sum_{s=1}^{n} u_{ij}d_{rs}x_{ij}x_{rs}$$

where f_{ir} is the fixed cost of locating facility i in cell r. Because these fixed costs were not specified for the problem instance, they were assumed constant for each i and r. The matrix element u_{ij} is the workflow from facility i to facility j. Matrix element d_{rs} is the rectilinear distance in feet between facilities r and s. A binary variable with double subscripts such as x_{ij} is given the value 1 if facility i is assigned to work centre j and is given value 0 otherwise. This summation is better viewed as a matrix Ψ of dimension $n^2 \times n^2$. The elements of the matrix are obtained using the following formula:

$$\Psi_{ij} = \left(\Psi_{rs}^{ij}\right) \quad r = 1, 2, \ldots, n; s = 1, 2, \ldots, n \tag{1}$$

where if $i = j$, then

$$\Psi_{rs}^{ij} = \begin{cases} f_{ir} & \text{if } r = s \\ 0 & \text{if } r \neq s \end{cases} \tag{2}$$

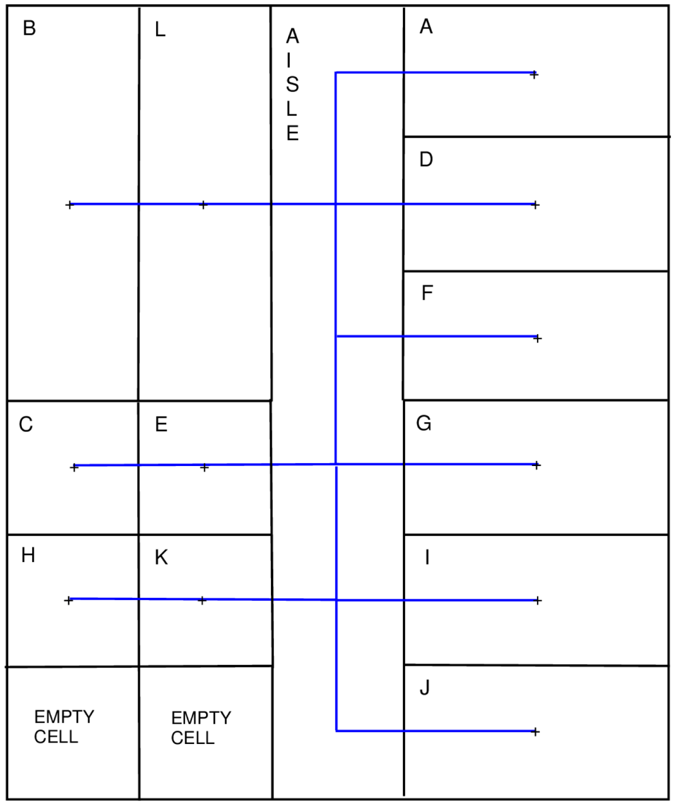

FIGURE 1. Layout A.

and if $i \neq j$, then

$$\Psi^{ij}_{rs} = \begin{cases} 0 & \text{if } r = s \\ u_{ij} d_{rs} & \text{if } r \neq s. \end{cases} \tag{3}$$

The partitioned matrix Ψ has the following structure.

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \dots & \Psi_{1n} \\ \Psi_{21} & \Psi_{22} & \dots & \Psi_{2n} \\ \dots & \dots & \dots & \dots \\ \Psi_{n1} & \Psi_{n2} & \dots & \Psi_{nn} \end{bmatrix}.$$

Let e_n be an $n \times 1$ vector of ones, and let I_n be an $n \times n$ identity matrix. Furthermore, let Φ be a $2n \times n^2$ assignment matrix.

$$\begin{bmatrix} e_n^t & & & & \\ & e_n^t & & & \\ & & e_n^t & & \\ & & & \ddots & e_n^t \\ I_n & I_n & I_n & \dots & I_n \end{bmatrix}.$$

The location problem, or QAP, that remains to be solved is:

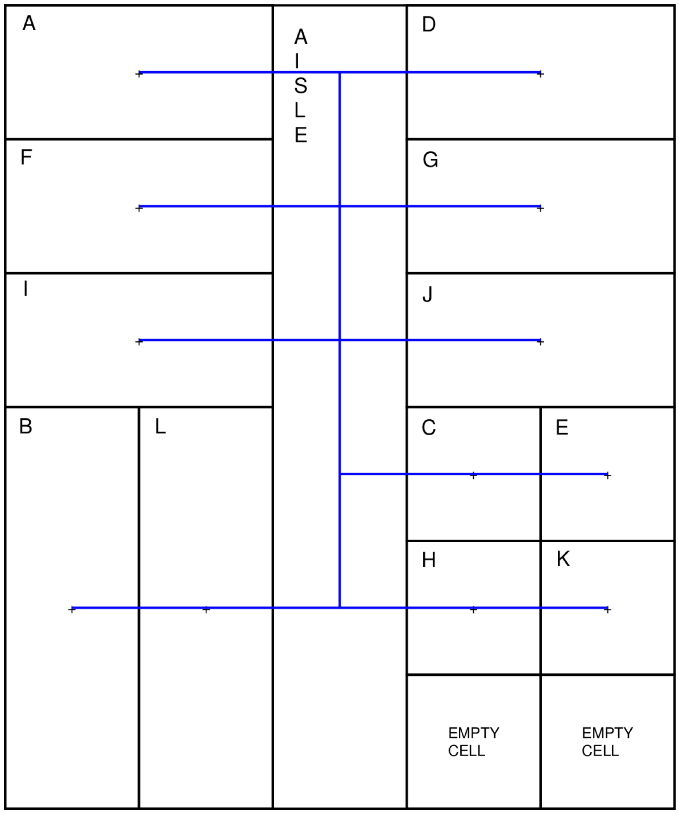

FIGURE 2. Layout B.

$$\begin{aligned} &\text{Minimize } x^t \Psi x \\ &\text{subject to } \Phi x = e_{2n} \\ &\qquad\qquad x \in \{0,1\}^{n^2}. \end{aligned}$$

This succinct formulation belies the computational intractability of the problem, and in the remaining part of the study, we look at the various solutions attempted to solve it.

4.1. Concrete instance

The flow matrix in Table 3 was obtained by summing all the flows between facilities across all 12 parts. For example, between facilities 1 and 2, note the flow of 40 units of part α, 100 units of part β, 50 units of part δ, 65 units of part ν, 50 units of part τ, and 40 units of part υ, giving a total of 345 units. Thus, the aggregated flow between the two facilities could be calculated. For a few facility pairs, the index of the receiving facility is lower than that of the sending facility, and these instances must also be initialised. Tables 4 and 5 list the rectilinear distances between the facilities based on the layout diagrams in Figures 1 and 2.

5. Computational results

We first ran Frontline Systems' GRG Solver on a Lenovo desktop computer equipped with an Intel core i5, 1.0 GHz, four-core processor and 8 GB RAM to solve the mixed binary problem for the first layout. The

TABLE 3. Flow matrix for parts.

Fac.	1	2	3	4	5	6	7	8	9	10	11	12
1	0	345	145	130	45	55	0	285	140	0	0	0
2	0	0	100	0	65	180	0	0	0	0	0	0
3	0	0	0	75	120	135	0	0	0	0	0	0
4	0	0	0	0	185	20	150	90	0	105	100	0
5	0	0	0	380	0	140	0	0	75	35	0	0
6	0	0	0	65	205	0	325	95	0	20	50	25
7	0	0	0	0	0	0	0	180	125	30	150	120
8	0	0	40	0	55	210	0	0	0	80	90	160
9	0	0	0	0	0	45	0	0	0	95	75	125
10	0	0	0	0	0	0	0	0	0	0	40	175
11	0	0	0	0	150	0	0	0	0	35	0	390
12	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 4. Rectilinear distance between facilities for first layout.

D_{ij}	1	2	3	4	5	6	7	8	9	10	11	12
1	–	45	65	40	55	50	60	75	70	80	65	35
2		–	60	35	50	45	55	70	65	70	60	10
3			–	55	10	45	35	50	45	55	40	50
4				–	45	40	50	65	60	70	55	25
5					–	35	25	40	35	45	30	40
6						–	40	55	50	60	45	35
7							–	45	40	50	35	45
8								–	35	45	10	60
9									–	40	25	55
10										–	35	75
11											–	50
12												–

minimum material-handling cost was 248 703. The highest material handling cost obtained by maximising the objective function was 293 528. The difference between the best and worst solutions was 44 875, with a percentage error of 18.04%. This implies a significant gain from optimising the location of the facility, and not just an improvement of a few percentage points. Even if one would not be starting from a worst case placement scenario, the improvement would be about 9% over the average case scenario which is significant. For the second layout, the best solution obtained was 248 553, which is 150 less than that of the first layout. The difference between the best and worst solutions was 62 850, which is a percentage error of 25.29%. The gain over the average case scenario would be about 12%. Thus, it appears that whichever of the two layouts one chooses at the start, optimising the assignment of facilities saves over 9%. Of course, thousands of layouts have not been explored, and the percentage improvement will vary across them.

One limitation of the non-linear solvers are that there are no post-optimality sensitivity analysis reports. The volume of demanded parts may change dynamically with the season and business cycles, and the stability of the solution in response to such changes can be examined by simulating a dynamic layout. Five or more different flow matrices are used to represent the demand across a planning horizon. The changes in the optimal solution and corresponding assignment is observed. The randomised flow values were created by the Excel formula $= \text{INT}(\text{B6} + \mathit{Rand1} * 0.25 * \text{MAX}(0, \text{RANDBETWEEN}(0, \text{B6})))$ where B6 is a typical flow value and *Rand1* is a

TABLE 5. Rectilinear distance between facilities for second layout.

D_{ij}	1	2	3	4	5	6	7	8	9	10	11	12
1	–	75	55	30	65	40	40	65	50	50	75	65
2		–	40	75	50	65	65	30	55	55	40	10
3			–	55	10	45	45	30	35	35	40	30
4				–	65	40	40	65	50	50	75	65
5					–	55	55	40	45	45	50	40
6						–	30	55	40	40	65	55
7							–	55	40	40	65	55
8								–	45	45	10	20
9									–	30	55	45
10										–	55	45
11											–	30
12												–

TABLE 6. Assignment of facilities for simulated flows in layout A.

Simulated flow matrix	Solution	Facility permutation
Original	264 053	(10, 9, 8, 12, 7, 11, 6, 5, 3, 2, 1, 4)
1	252 018	(10, 9, 8, 3, 7, 11, 6, 5, 4, 2, 1, 12)
2	259 038	(10, 9, 8, 3, 7, 11, 6, 5, 4, 2, 1, 12)
3	255 638	(10, 9, 8, 3, 7, 11, 6, 5, 4, 2, 1, 12)
4	261 003	(10, 9, 8, 3, 7, 11, 6, 5, 4, 2, 1, 12)
5	253 253	(10, 9, 8, 3, 7, 11, 6, 5, 4, 2, 1, 12)

uniform random variate that decides the sign of the perturbation, and the second random number decides the quantum of the perturbation which has been set with an upper bound of 25% of the original value. The results are displayed in Table 6 for the first layout. One can see that the assignment permutations are extremely stable for flows that are within ±25% of the original values; nine out of the twelve assignments are invariant.

5.1. Results from the Gurobi optimizer

Non-convexity of the objective function in the quadratic assignment problem is an issue that should not be overlooked. This issue is important because different local minima are obtained when solving multiple starting solutions for the algorithm. The eigenvalues of matrix Ψ in the quadratic term $x^t\Psi x$ indicate whether the matrix is positive definite (PD) or positive semi-definite (PSD). In addition, the constraint set may contain only equality constraints or may have nonlinear constraints. These factors suggest that a nonlinear solver is best suited for obtaining a solution. If some of the eigenvalues for the Q matrix are complex numbers, as discovered in this problem instance, this implies that the matrix is neither PD nor PSD. This means that the objective function is not strictly convex and that the solution found by an algorithm may not be the global minimum. One method to determine whether a solution to a nonlinear problem exists is to solve the Lagrangian dual problem and check for a duality gap. Because matrix Ψ is not PSD, one cannot form Dorn's dual quadratic program or a similar Lagrangian dual and solve it. Moreover, because the variables are binary valued for this QAP, a dual-integer program is difficult to interpret. The reformulation-linearization/convexification technique (RLT) is a cutting plane method that has been used successfully for QAPs with non-convex objective functions. The *Gurobi* solver uses RLT along with various other cutting plane techniques to solve the primal QAP and a dual formulation.

The computer used for these experiments was the same as the one on which the Frontline solver was used. The *Gurobi* optimiser used was version 10.0.1. After obtaining the first few incumbent solutions quickly, the remaining duality gap reduces very slowly over many hours of execution time. For the first layout, the incumbent solution or best integer feasible solution of 208 468 is obtained within a few iterations and this is much better than the optimal solution obtained from the GRG solver with a percentage improvement of 19%. For the second layout the incumbent solution of 208 585 is obtained almost immediately, which is also a percentage improvement of about 19% over the corresponding GRG solver solution. If the global optimal solution with zero duality gap is required the *Gurobi* optimizer will have to be used on a powerful multi-threaded workstation computer.

6. Conclusions

Solving industrial problems using system modelling and quantitative techniques is very complex. Mathematical modelling of a system is a difficult task and requires ingenuity. The next step for solving this problem is equally difficult, which is of picking an appropriate algorithm. An insight from all the numerical computations is that a good solution can be obtained with a generic solver; however, if an appropriate nonlinear solver is used for the mathematical model, the savings can be substantially improved. Optimisation with a generic solver reduced the material handling cost by over 9% compared to an average solution. The non-convex branch and bound algorithm showed an improvement of about 19% over the local optimal solution, although it only provides an incumbent solution in the first few minutes.

A limitation of the study is the issue of part priority, hazardous materials, or the clockwise or counterclockwise flow of parts in the facility. Customarily, a clockwise flow is preferred over a counterclockwise flow. These qualitative policy issues can be incorporated into a more elaborate quantitative model, once the most suitable algorithm has been identified for solving this class of problems. The flow matrix can be easily randomised to perform a simulation. The solutions obtained from this method may be compared with nature-inspired metaheuristic methods such as artificial neural networks (ANNs), genetic algorithms (GAs), particle swarm optimisation, and the bee colony algorithm, which are recommended for large problem instances owing to the exponential time complexity of the problem.

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