

OPTIMAL PRICING AND ENVIRONMENTAL IMPROVEMENT IN A DUAL-CHANNEL HAZARDOUS WASTE DISPOSAL SUPPLY CHAIN UNDER COST SHARING CONTRACT

YAN FENG¹, XIAOSHEN LI^{2,*} AND YOULIN SHANG²

Abstract. A dual-channel hazardous waste supply chain consisting of a disposal facility and a contractor is studied, where the customer demand is sensitive to price and environment impacts. Under the government's emissions penalty, the disposal facility must invest in technology for hazardous waste disposal to reduce waste emissions. Firstly, under the cost-sharing contract, the cooperation model based on a disposal facility and a contractor is constructed, and the influence of cost-sharing ratio on supply chain system is discussed; Secondly, the Stackelberg game models are presented; Finally, the numerical experiments are conducted. The computational results show that the cost sharing contract can improve the total profit of the dual-channel supply chain. When one party is dominant, whether it is the disposal facility-Stackelberg or the contractor-Stackelberg, the profit of that party always increases as the cost sharing proportion increases. Moreover, for the environment and consumers, an increasing cost sharing proportion of the contractor can improve the disposal technology, and the emissions per unit of hazardous waste disposal have been consistently decreasing.

Mathematics Subject Classification. 91A10, 91A40, 91B40.

Received October 12, 2022. Accepted January 22, 2024.

1. INTRODUCTION

With the continuous advancement of China's industrialization and urbanization, hazardous wastes with properties such as flammable, explosive, corrosive are increasing, including solid, semi-solid and liquid wastes. At present, 479 kinds of hazardous wastes in 46 categories are listed in the National Waste List (2021 Edition) [35]. The most abundant hazardous wastes that exist in the environment can be divided into two categories depending on the source: naturally occurring hazardous materials and man-made hazardous materials. Among the man-made hazardous wastes, industrial and manufacturing processes are major sources of waste generation [32]. The rapid increase in the volume and complexity of hazardous waste and the accompanying problems of random discharge, storage and disposal of hazardous waste will not only destroy the ecological environment and affect human health, but will also become an important factor limiting sustainable development. The study of hazardous waste disposal management systems has become critical [31].

Keywords. Dual-channel supply chain, Stackelberg model, hazardous waste, cost sharing contract.

¹ Business School, Henan University of Science and Technology, Luoyang 471000, P.R. China.

² School of Mathematics and Statistics, Henan University of Science and Technology, Luoyang 471000, P.R. China.

*Corresponding author: hnykli@163.com

Hazardous waste management is defined as the collection, transportation, treatment, recycling and disposal of hazardous waste in a safe and cost-effective way. A hazardous waste disposal management system typically consists of a waste contractor, which is responsible for recycling and transporting the waste to a disposal facility, which has the waste treatment technology and permits to dispose of hazardous waste. Here, we use the term “technology” to refer to the infrastructures required for waste sorting, recycling, incineration, composting and landfill [11]. When hazardous waste disposal is carried out by the disposal facility, it is inevitable that pollutants are released into the atmosphere of the environment. Studies show that dioxins and furans, dioxin-like polychlorinated biphenyl and chlorobenzenes have higher emission rates, so it is of great importance to dispose of the hazardous waste in an efficient and economic way [5, 28, 30]. Hazardous waste disposal technology is critical to waste disposal, so it is important to invest in advanced technologies to achieve resourcefulness, reduction and harmlessness in hazardous waste disposal. With the rapid development of science and technology, the treatment of hazardous waste is becoming increasingly advanced, and common disposal methods already available include incineration, pyrolysis, safety landfill, solidification and biological treatment [5, 16]. In recent years, many companies have adopted more environmentally friendly practices in hazardous waste management, participants include waste sources, disposal facilities, contractors who are responsible for collection and transportation of waste. Sometime the disposal facility also serve the waste source by collecting and transporting hazardous waste directly, for which the dual-channel supply chain for hazardous waste disposal is established. Studies have shown that, in some cases, profitability increases when the disposal facility is actively involved in waste collection and transportation under a dual-channel supply chain model [11].

Since both the contractor and the disposal facility are the most important players in the hazardous waste disposal management system, the contractor is prepared to share a percentage of the cost of the technology investment to encourage the disposal facility to dispose of the waste with more advanced technology. This is a real problem that exists. How does the cost sharing proportion of the contractor affect the system? Whether the disposal facility and the contractor would benefit from cost-sharing contract? Will the cost sharing contract be environmentally friendly?

Motivated by above issues, we consider a dual-channel hazardous waste disposal supply chain consisting of a disposal facility and a contractor under a cost sharing contract, develop two Stackelberg game models: disposal facility leadership (CS-D) and contractor leadership (CS-C). Our contributions are as follow.

- (1) We obtain the optimal equilibrium for the disposal facility-Stackelberg and contractor-Stackelberg structures for the dual-channel supply chain under the cost sharing contract, and compare the optimal values under the two structures.
- (2) We explore the impact of the cost sharing proportion on optimal equilibrium decisions and profits, finding that the cost sharing contract can improve environmental impact, increase the service demand and benefit the dominant player.
- (3) We discuss the managerial insights. As the cost-share λ increases, the investment in improving environmental impact increases, which leads to improvements in disposal technology, and furthermore decrease of the emissions per unit of hazardous waste disposal. To reduce emissions in the waste disposal, the manager or government should encourage contractors to take a larger share of investment in disposal technology.

In the rest sections, the paper is organized as follows. In Section 2, we review the relevant literature. In Section 3, the problem description and assumptions are given and the model is obtained. The model formulation is established and solved in Section 4. Section 5 shows the numerical analysis. Section 6 provides the managerial insights. Conclusions and further work are given in Section 7. All the proofs and the abbreviations are listed in “Appendix A”.

2. LITERATURE REVIEW

This work is related to dual-channel supply chain, hazardous waste management system, government intervention and contract design in supply chain.

In contrast to traditional supply chain management, dual-channel supply chain management faces channel competition and various uncertainties, and it is critical to manage participating members and make decisions effectively. In recent years, many scholars have studied the dual-channel structure. The dual-channel structure has been investigated by considering factors such as acceptance of direct channel [9], competing in service of channels [8], and different products of the channels [33]. Based on two suppliers who manufacture alternative commodities in dual-channel supply chains, Cai *et al.* study the changes in suppliers' profits and the consumer utility under different sales structures [4]. Yang *et al.* simulate a dual-channel structural strategy for environmental responsibility behaviors of manufacturers and consumers and further investigates their environmental performance under fuzzy uncertainties [36]. Zhao and Niu consider three types of horizontal mergers in a dual-channel supply chain consisting of three firm types: suppliers, single-channel retailers, and dual-channel retailers [40]. Xie *et al.* investigate a two-stage fresh food supply chain system consisting of producers and retailers through a Stackelberg game theory approach to derive optimal decisions for carbon reduction and pricing, and performs sensitivity analysis and comparison of dual-channel models [34].

Many scholars have studied the models that can achieve the environmental development in the supply chain. Yi *et al.* establish a mixed integer linear model incorporating the reverse logistics network into the forward logistics network for the optimization design of scrap remanufacturing closed-loop supply chain network in retailer-oriented construction machinery, and apply an improved hybrid genetic algorithm [37]. Panja and Mondal study a four-tier green supply chain imperfect production inventory model for green products under fuzzy credit period, and performs sensitivity analysis on the clear and fuzzy scenarios of the model, and finally proposes some management suggestions [25].

Effective treatment of hazardous waste can further achieve sustainable social and economic development. As the generation of hazardous waste continues to grow, the study of hazardous waste-related issues becomes crucial. Hazardous waste management problems can be considered as a supply-chain management problem [24]. In reverse logistics network design, Safdar *et al.* develop a multi-objective reverse logistics network for e-waste management by considering the concept of the triple bottom line approach [29]. Considering carbon cap-and-trade policies, the study suggests trade-offs between conflicting objectives of maximizing profits, minimizing carbon emissions and maximizing employment opportunities. Ma and Li use parallel enumeration method (PEM) and genetic algorithm (GA) to solve the closed-loop supply chain network model of hazardous products with uncertain demands and returns, and evaluate the performance of the model [22]. Importantly, it is the first to consider risk-limiting constraints and reward-penalty mechanisms in one model. Based on green supply chain management approach, Liu *et al.* adopt the GTA hybrid algorithm dominated by genetic algorithm and tabu algorithm to study the double path optimization of transport of industrial hazardous waste in environmental protection enterprises [21]. Their goal is to integrate green purchasing, hazardous waste storage and green disposal under green supply chain management in green purchasing management of environmentally friendly enterprises associated to green production. In a recent study, Ghalekhondabi *et al.* study a hazardous waste management system consisting of a disposal facility and a contractor, in which the government intervenes in the supply chain by imposing emission penalties for reducing disposal process emissions [11].

In addition, not all hazardous waste disposal companies will voluntarily adopt more environmentally friendly disposal processes for waste disposal, so legislation and government intervention are key to mitigating adverse environmental impacts. Some studies have considered the role of government intervention in supply chains [13–15]. Zhang and Yousaf study the coordination problem of a single-channel supply chain consisting of a refinery and retailer considering government intervention [39]. The results show that appropriately planned government intervention can improve supply chain performance and help achieve sustainable development goals. Zand *et al.* consider a dyadic online-to-offline (O2O) closed-loop supply chain (CLSC), which is composed of a manufacturer and a retailer [38]. The authors use specific thresholds to determine the level of product greening to establish direct government restrictions. Chen *et al.* discuss the behavior of supply chain members in green supply chain management under the reward-penalty mechanism of the government [7]. Results show that the reward-penalty mechanism can improve the return rate and the green effort.

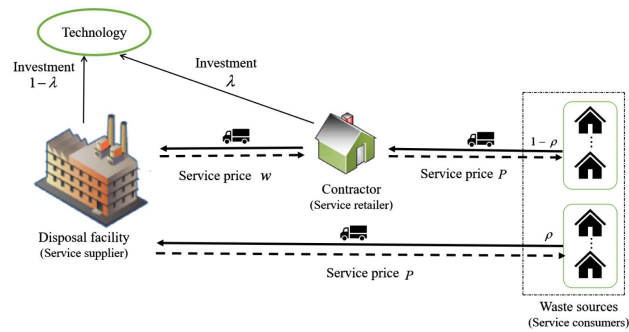


FIGURE 1. Dual-channel supply chain with hazardous waste disposal.

Contracts are a standard tool for improving supply chain efficiency, and many scholars have studied the impact of cooperative relationships between supply chain members on the supply system. It is found that cost-sharing contracts are widely used in the supply chains involving significant investment. For example, in September 2002, Alpha Labs (a small public U.S. biotech company) and Mega Pharmaceuticals (a large Fortune100 pharmaceutical company) agreed to share the cost of investing in drug development equally [2]. Alpha Labs can reduce its costs in drug development, and Mega can profit from the commercialization and distribution of the products. Ma *et al.* propose a cost sharing contract for the green supply chain composed of a manufacturer and a retailer [23]. They study green cost-sharing contracts with the same confidence level and analyze the variation of optimal decisions in this system. In response to supply chain coordination issues arising out of green supply chain initiatives, Ghosh and Shah explore the impact of cost sharing contract on the key decisions of supply chain participants implementing green initiatives [12]. Through a game-theoretic approach, the paper shows how product greening levels, prices and profits are influenced by the cost sharing contract. The results show that cost sharing contracts can improve the level of greening, increase the profit of individual firms, and increase the profit of the supply chain, that is, benefit both firms and the supply chain. All the above studies show that the cooperation among supply chain members in the supply chain system provides the opportunity to integrate internal and external resources for the supply chain, and plays a crucial role in supply chain management [6, 20]. Panja and Mondal investigate a two-layer green supply chain consisting of a manufacturer and a retailer, solving for three decision scenarios: centralized, decentralized, and revenue sharing contract [26].

In summary, this study differs from the above literature. We focus on the dual-channel hazardous waste disposal supply chain under a cost sharing contract, and we mainly investigate the impact of the cost sharing proportion on dual-channel supply chain system. At the same time, we consider the impact of the cost sharing contract on hazardous waste disposal emissions.

3. MODEL

We consider a dual-channel hazardous waste disposal supply chain serving waste sources. The supply chain consists of a contractor and a disposal facility, where the contractor is responsible for waste collection and transportation, and the disposal facility has the technology to dispose of hazardous waste. Based on the performance of the contractor in the market, the disposal facility collects a portion ρ ($0 < \rho < 1$) of hazardous waste directly from waste sources to meet the service demand. The term “dual-channel” refers to two supply channels for the service demand: portion $1 - \rho$ from the contractor and portion ρ from waste sources, which is illustrated in Figure 1.

The disposal of hazardous wastes will produce emissions, and the disposal facility is responsible for paying fines to the government based on the emissions generated. According to the increasing environmental awareness of hazardous waste producers, disposal facilities must invest in technologies to reduce waste disposal emissions, that

TABLE 1. Model notations.

Symbol	Description
d	Potential service demand for hazardous waste disposal (pound)
D	Total service demand for waste disposal (pound)
D_i	Waste sources' demand for service channel i , $i = \text{DF}, \text{CO}$ (pound)
α	Demand price-sensitivity coefficient
β	Demand environmental impact-sensitivity coefficient
a	Emission per unit when environmental improvement is 0, $a > 0$
b	Coefficient of environmental effect on reducing the emission, $b > 0$
I	Environmental impact investment parameter, $I > 0$
c	Unit emission trading price
o	Treatment and disposal cost in the disposal facility (per pound)
λ	The cost sharing proportion
ρ	The portion of service demand which directly deals with the disposal facility
T_{DF}	The disposal facility's profit
T_{CO}	The contractor's profit
T_{SC}	Supply chain's profit
Decision variables	
w	Disposal facility service price asked from the contractor (per pound)
P	Service price asked from waste source (per pound)
m	Marginal profit from collecting waste ($P = w + m$) (per pound)
θ	Level of improving environmental impact (compared to a benchmark)

is, they tend to adopt treatment processes with lower emissions. In order to achieve higher overall performance, an investment cost sharing contract is established. In other words, collectors will share part of the cost of investing ($0 \leq \lambda \leq 1$) in hazardous waste disposal technology. We attempt to understand whether the collector would benefit from a cost sharing contract, and analyze the implications for management.

For convenience, we make the following notation: CS-D and CS-C denote the cost sharing contract under disposal facility leadership and contractor leadership, respectively. D and C denote the model under disposal facility leadership and contractor leadership without cost sharing.

Relevant notations used in this paper are listed in Table 1.

To build the model, some assumptions are provided as follows:

Assumption 3.1. *The total service demand is modeled as a function of the service price and the environmental improvement level as $D = d - \alpha P + \beta \theta$, where $d, \alpha, \beta > 0$ [3, 17–19]. The demand function shows that the potential market demand decreases with increasing price and increases with improving environmental impact. Thus, the service demand functions for the indirect and direct channels in this paper are as follows:*

$$D_{\text{DF}} = \rho(d - \alpha P + \beta \theta),$$

$$D_{\text{CO}} = (1 - \rho)(d - \alpha P + \beta \theta),$$

where D_{DF} and D_{CO} are the waste source demand in the disposal facility channel and the contractor channel, respectively.

Assumption 3.2. *To make financial sense, assume $w \geq o > 0$ and $P \geq w$. Waste sources have a positive demand for disposal service, and both the contractor and the disposal facility must gain non-negative profits.*

Assumption 3.3. *The disposal of unit hazardous waste can cause emissions of amount $(a - b\theta)$ by disposal facility. We assume that there is no technology that can dispose of hazardous waste with zero emissions. Therefore, we have $a - b\theta > 0 \rightarrow 0 \leq \theta < \frac{a}{b}$.*

Assumption 3.4. *Disposal facility should pay the emission penalty of $(c * (a - b\theta))$ for disposing of each pound of waste. As c , a and b are constant parameters of this problem, we can consider ca as a portion of disposal cost o . And just to simplify notation, we assume $\gamma = cb$. Thus, the disposal facility's profit per pound would be $(w - o + \gamma\theta)$.*

Assumption 3.5. *It is assumed that the environmental impact investment cost is taken as an increasing quadratic function as $I\theta^2$ [1, 10, 27].*

According to model formulations and assumptions given above, the contractor's profit function is

$$T_{CO}^{CS} = m(1 - \rho)(d - \alpha(m + w) + \beta\theta) - \lambda I\theta^2. \quad (1)$$

The disposal facility's profit function is

$$T_{DF}^{CS} = [(w - o + \gamma\theta)(1 - \rho) + (w + m - o + \gamma\theta)\rho](d - \alpha(m + w) + \beta\theta) - (1 - \lambda)I\theta^2. \quad (2)$$

4. MODEL ANALYSIS

In this model, hazardous waste from waste sources is collected and transported to the disposal facility for disposal through dual-channel, where the contractor acts as an intermediary for hazardous waste disposal and the disposal facility invests in technology to dispose waste.

In this section, we study the optimal decisions of the contractor and the disposal facility when considering an investment cost sharing contract in the decentralized dual-channel hazardous waste supply chain. In the Stackelberg games, firms interact with each other in a leader-follower relationship. Each firm would make decisions to optimize its own objective function. To obtain some managerial insights, we perform analyses and discussions of optimal decisions.

4.1. Disposal facility-Stackelberg

Due to its long-term presence in the market, the disposal facility is likely to be a leading player in the waste disposal supply chain. It's a Stackelberg game when one firm is in such a strong position that it can make decisions first. In the supply chain system, each participant maximizes its own profit. In this model, the disposal facility determines the disposal price w and the environment impact improvement level θ first. Second, after observing the disposal facility's decision, the contractor decides the marginal profit from collecting waste m .

By using the backward induction, we can obtain the equilibrium decision and the total profit of the supply chain.

Proposition 4.1. *For $(2 - \rho)(1 - \lambda)I\alpha - \frac{1}{4}(\beta + \gamma\alpha)^2 > 0$, the disposal facility's total profit function is concave with respect to w and θ .*

Proposition 4.2. *In disposal facility-Stackelberg model with cost sharing, equilibrium decisions and the profits are given as follows.*

$$\begin{aligned} w^{CS-D*} &= \frac{o(4(1 - \lambda)I\alpha - \beta(\beta + \gamma\alpha)) + d(4(1 - \rho)(1 - \lambda)I - \gamma(\beta + \gamma\alpha))}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ \theta^{CS-D*} &= \frac{(d - o\alpha)(\beta + \gamma\alpha)}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ m^{CS-D*} &= \frac{2(1 - \lambda)I(d - o\alpha)}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ P^{CS-D*} &= \frac{o(2(1 - \lambda)I\alpha - \beta(\beta + \gamma\alpha)) + d((6 - 4\rho)(1 - \lambda)I - \gamma(\beta + \gamma\alpha))}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \end{aligned}$$

$$\begin{aligned}
 T_{DF}^{CS-D^*} &= \frac{(1-\lambda)I(d-\alpha)^2}{4(2-\rho)(1-\lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\
 T_{CO}^{CS-D^*} &= \frac{(4(1-\rho)(1-\lambda)^2I^2\alpha - \lambda I(\beta + \gamma\alpha)^2)(d-\alpha)^2}{(4(2-\rho)(1-\lambda)I\alpha - (\beta + \gamma\alpha)^2)^2}, \\
 T_{SC}^{CS-D^*} &= \frac{(4(3-2\rho)(1-\lambda)^2I^2\alpha - I(\beta + \gamma\alpha)^2)(d-\alpha)^2}{(4(2-\rho)(1-\lambda)I\alpha - (\beta + \gamma\alpha)^2)^2}.
 \end{aligned}$$

Corollary 4.1. *According to Assumption 3.2, we have*

- (1) $d - \alpha \geq 0$;
- (2) $4(1 - \lambda)(1 - \rho)I - \gamma(\beta + \gamma\alpha) \geq 0$;
- (3) $4(1 - \rho)(1 - \lambda)^2I\alpha - \lambda(\beta + \gamma\alpha)^2 \geq 0$.

Corollary 4.2. (1) $\frac{\partial \theta^{CS-D^*}}{\partial \lambda} > 0$; (2) $\frac{\partial m^{CS-D^*}}{\partial \lambda} > 0$.

Corollary 4.2 illustrates that the cost sharing contract will have a positive impact on the level of environmental impact improvement and the marginal profit of collection and transportation. As the cost sharing proportion increases, the continuous improvement in hazardous waste disposal technology leads to an increased level of environmental impact improvement, which is conducive to environmental sustainability.

Corollary 4.3. (1) $\frac{\partial T_{DF}^{CS-D^*}}{\partial \lambda} > 0$; (2) If $\lambda < \frac{(\beta + \gamma\alpha)^2 - 4\rho I\alpha}{4(4 - 3\rho)I\alpha}$, then $\frac{\partial T_{CO}^{CS-D^*}}{\partial \lambda} > 0$; If $\lambda > \frac{(\beta + \gamma\alpha)^2 - 4\rho I\alpha}{4(4 - 3\rho)I\alpha}$, then $\frac{\partial T_{CO}^{CS-D^*}}{\partial \lambda} < 0$; (3) If $\lambda < \frac{1 - \rho}{3 - 2\rho}$, then $\frac{\partial T_{SC}^{CS-D^*}}{\partial \lambda} > 0$; If $\lambda > \frac{1 - \rho}{3 - 2\rho}$, then $\frac{\partial T_{SC}^{CS-D^*}}{\partial \lambda} < 0$.

Corollary 4.4. *According to Propositions 4.1 and 4.2, when $\lambda = 0$, it is a case of this supply chain without cost sharing. For $4I\alpha(2 - \rho) - (\beta + \gamma\alpha)^2 > 0$, disposal facility’s profit function is concave with respect to θ and w , and equilibrium decisions and the profits are given as follows [11].*

$$\begin{aligned}
 w^{D^*} &= \frac{o(4I\alpha - \beta(\beta + \gamma\alpha)) + d(4(1 - \rho)I - \gamma(\beta + \gamma\alpha))}{4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2}, \\
 \theta^{D^*} &= \frac{(d - \alpha)(\beta + \gamma\alpha)}{4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2}, \\
 m^{D^*} &= \frac{2I(d - \alpha)}{4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2}, \\
 P^{D^*} &= \frac{o(2I\alpha - \beta(\beta + \gamma\alpha)) + d((6 - 4\rho)I - \gamma(\beta + \gamma\alpha))}{4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2}, \\
 T_{DF}^{D^*} &= \frac{I(d - \alpha)^2}{4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2}, \\
 T_{CO}^{D^*} &= \frac{4(1 - \rho)I^2\alpha(d - \alpha)^2}{(4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2)^2}, \\
 T_{SC}^{D^*} &= \frac{(4(3 - 2\rho)I^2\alpha - I(\beta + \gamma\alpha)^2)(d - \alpha)^2}{(4(2 - \rho)I\alpha - (\beta + \gamma\alpha)^2)^2}.
 \end{aligned}$$

4.2. Contractor-Stackelberg

If the contractor has greater scale and credibility in the hazardous waste recycling market, then the contractor will be the leader in the Stackelberg game. In this section, we consider a scenario where the contractor has more power in the market. In this scenario, the contractor leads a Stackelberg game, taking into account the optimal value of the disposal facility.

The profits of the disposal facility and the contractor are given by (1) and (2), respectively. By using backward induction, we derive the following proposition.

Proposition 4.3. *For $4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2 > 0$, the disposal facility’s total profit function is concave with respect to w and θ .*

Proposition 4.4. *In contractor-Stackelberg model with cost sharing, equilibrium decisions and the profits are given as follows.*

$$\begin{aligned}
 w^{\text{CS-C}^*} &= \frac{d\left(2(1 - 3\rho)(1 - \lambda)^2I\alpha - (\beta + \gamma\alpha)\left((2\lambda - 1)\rho\beta + (3\lambda\rho - \lambda - 2\rho + 1)\gamma\alpha\right)\right)}{(1 - \rho)\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)\alpha} \\
 &\quad + \frac{o\alpha\left(2(1 - \lambda)^2I\alpha(3 - \rho) - (\beta + \gamma\alpha)(\lambda\rho - 3\lambda - \rho + 2)\beta + (1 - 2\lambda)\gamma\alpha\right)}{(1 - \rho)\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)\alpha}, \\
 \theta^{\text{CS-C}^*} &= \frac{(1 - \lambda)(\beta + \gamma\alpha)(d - o\alpha)}{8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2}, \\
 m^{\text{CS-C}^*} &= \frac{\left(4(1 - \lambda)^2I\alpha - (1 - 2\lambda)(\beta + \gamma\alpha)^2\right)(d - o\alpha)}{(1 - \rho)\alpha\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)}, \\
 P^{\text{CS-C}^*} &= \frac{o\alpha\left(2(1 - \lambda)^2I\alpha - (1 - \lambda)\beta(\beta + \gamma\alpha)\right) + d\left(6(1 - \lambda)^2I\alpha - (\beta + \gamma\alpha)\left((1 - 2\lambda)\beta + (2 - 3\lambda)\gamma\alpha\right)\right)}{(1 - \rho)\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)\alpha}, \\
 T_{\text{DF}}^{\text{CS-C}^*} &= \frac{(1 - \lambda)^3I\left(4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right)(d - o\alpha)^2}{\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)^2}, \\
 T_{\text{CO}}^{\text{CS-C}^*} &= \frac{(1 - \lambda)^2I(d - o\alpha)^2}{8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2}, \\
 T_{\text{SC}}^{\text{CS-C}^*} &= \frac{(1 - \lambda)^2I\left(12(1 - \lambda)^2I\alpha - (3 - 4\lambda)(\beta + \gamma\alpha)^2\right)(d - o\alpha)^2}{\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)^2}.
 \end{aligned}$$

Corollary 4.5. *According to Assumption 3.2, we have*

- (1) $8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2 > 0$;
- (2) $d - o\alpha \geq 0$;
- (3) $4(1 - \lambda)^2I\alpha - (1 - 2\lambda)(\beta + \gamma\alpha)^2 \geq 0$.

Corollary 4.6. (1) *If $\lambda < 1 - \sqrt{\frac{(\beta + \gamma\alpha)^2}{8I\alpha}}$, then $\frac{\partial\theta^{\text{CS-C}^*}}{\partial\lambda} > 0$; If $\lambda > 1 - \sqrt{\frac{(\beta + \gamma\alpha)^2}{8I\alpha}}$, then $\frac{\partial\theta^{\text{CS-C}^*}}{\partial\lambda} < 0$;* (2) *If $\lambda < \sqrt{1 - \frac{(\beta + \gamma\alpha)^2}{4I\alpha}}$, then $\frac{\partial m^{\text{CS-C}^*}}{\partial\lambda} > 0$; If $\lambda > \sqrt{1 - \frac{(\beta + \gamma\alpha)^2}{4I\alpha}}$, then $\frac{\partial m^{\text{CS-C}^*}}{\partial\lambda} < 0$.*

Corollary 4.6 states the impact of the contractor’s cost sharing proportion on the equilibrium decision. As can be seen from the above conclusions that at the beginning, the environmental impact improvement level θ and the contractor’s marginal profit m gradually increase as the cost sharing proportion increases. As cost sharing continues to increase, both the environment impact improvement level and marginal profit of contractor will decrease.

Corollary 4.7. (1) If $\lambda > 1 - \frac{3(\beta+\gamma\alpha)^2}{16I\alpha}$, then $\frac{\partial T_{DF}^{CS-C^*}}{\partial \lambda} > 0$; if $\lambda < 1 - \frac{3(\beta+\gamma\alpha)^2}{16I\alpha}$, then $\frac{\partial T_{DF}^{CS-C^*}}{\partial \lambda} < 0$; (2) If $1 - 3\lambda > 0$, then $\frac{\partial T_{CO}^{CS-C^*}}{\partial \lambda} > 0$; If $1 - 3\lambda < 0$, then $\frac{\partial T_{CO}^{CS-C^*}}{\partial \lambda} < 0$; (3) If $\frac{6\lambda^2 - 6\lambda + 1}{(1-\lambda)^2(1-5\lambda)} < \frac{4I\alpha}{(\beta+\gamma\alpha)^2}$, then $\frac{\partial T_{SC}^{CS-C^*}}{\partial \lambda} > 0$; If $\frac{6\lambda^2 - 6\lambda + 1}{(1-\lambda)^2(1-5\lambda)} > \frac{4I\alpha}{(\beta+\gamma\alpha)^2}$, then $\frac{\partial T_{SC}^{CS-C^*}}{\partial \lambda} < 0$.

Corollary 4.8. In this model, when $\lambda = 0$, it is a case of this supply chain without cost sharing. For $4I\alpha - (\beta + \gamma\alpha)^2 > 0$, disposal facility’s profit function is concave with respect to θ and w , and equilibrium decisions and the profits are given as follows [11].

$$w^{C^*} = \frac{d(2(1 - 3\rho)I\alpha + (\beta + \gamma\alpha)(\rho\beta + (2\rho - 1)\gamma\alpha) + o\alpha(2(3 - \rho)I\alpha + (\beta + \gamma\alpha)((-2 + \rho)\beta - \gamma\alpha))}{(1 - \rho)(8I\alpha - 2(\beta + \gamma\alpha)^2)\alpha},$$

$$\theta^{C^*} = \frac{(\beta + \gamma\alpha)(d - o\alpha)}{8I\alpha - 2(\beta + \gamma\alpha)^2},$$

$$m^{C^*} = \frac{d - o\alpha}{2\alpha(1 - \rho)},$$

$$P^{C^*} = \frac{o\alpha(2I\alpha - \beta(\beta + \gamma\alpha)) + d(6I\alpha - (\beta + \gamma\alpha)(\beta + 2\gamma\alpha))}{(1 - \rho)(8I\alpha - 2(\beta + \gamma\alpha)^2)\alpha},$$

$$T_{DF}^{C^*} = \frac{I(d - o\alpha)^2}{16I\alpha - 4(\beta + \gamma\alpha)^2},$$

$$T_{CO}^{C^*} = \frac{I(d - o\alpha)^2}{8I\alpha - 2(\beta + \gamma\alpha)^2},$$

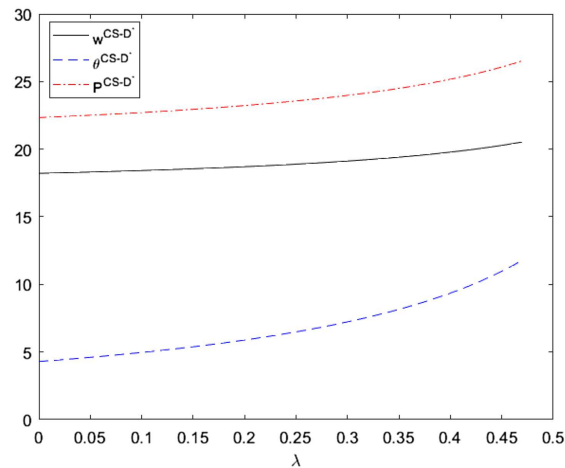
$$T_{SC}^{C^*} = \frac{3I(d - o\alpha)^2}{16I\alpha - 4(\beta + \gamma\alpha)^2}.$$

5. NUMERICAL ANALYSIS

In this section, an extended numerical study of all the proposed optimal strategies under CS-D and CS-C is performed. We present some numerical analyses to explain the performance of the supply chain in different supply chain settings. Based on the study in the literature [11], the following values are assumed: $\lambda = 0.3$, $d = 850$, $I = 16$, $a = 12$, $b = 0.09$, $c = 1$, $\alpha = 37$, $\beta = 30$, $o = ca = 12$, $\gamma = bc = 0.09$, $\rho = 0.2$. The optimal decisions and profits are: $w^{CS-D^*} = 19.12$, $\theta^{CS-D^*} = 7.23$, $m^{CS-D^*} = 4.86$, $P^{CS-D^*} = 23.98$, $T_{DF}^{CS-D^*} = 985.78$, $T_{CO}^{CS-D^*} = 447.41$, $T_{SC}^{CS-D^*} = 1433.19$ in CS-D model, and $w^{CS-C^*} = 15.23$, $\theta^{CS-C^*} = 8.62$, $m^{CS-C^*} = 8.94$, $P^{CS-C^*} = 24.17$, $T_{DF}^{CS-C^*} = 409.73$, $T_{CO}^{CS-C^*} = 1176.26$, $T_{SC}^{CS-C^*} = 1585.99$ in CS-C model. We mainly focus on the impact of the cost sharing proportion λ on optimal decisions and profits. At this point, it is worth noting that, according to Corollary 4.1 and 4.5, in order to ensure the profitability of the members of the supply chain, the ratio λ can be increased to 0.46 and 0.53 in CS-D model and CS-C model, respectively. At the end, we consider the impact of other two important parameters ρ and c on the optimal decisions and profits.

TABLE 2. Optimal results for different λ in CS-D model.

λ	w	θ	m	P	T_{DF}	T_{CO}	T_{SC}
0.05	18.31	4.61	4.20	22.51	852.68	505.28	1357.96
0.15	18.55	5.39	4.40	22.94	892.37	502.35	1394.73
0.25	18.89	6.49	4.67	23.56	948.28	477.57	1425.85
0.35	19.41	8.15	5.09	24.50	1032.91	394.07	1426.98
0.45	20.28	10.97	5.79	26.08	1176.03	126.82	1302.86

FIGURE 2. Impacts of λ on the optimal decisions in CS-D model.

5.1. Impact of λ on the optimal decisions and profits in CS-D model

In this subsection, the impact of the cost sharing proportion λ on the optimal decisions and profits in CS-D model is discussed through extensive numerical experiments. The optimal results of this model are given in Table 2 when λ takes different values, as well as their change curves displayed in Figures 2 and 3.

Figure 2 shows the impact of the cost sharing proportion λ on the optimal decisions of CS-D model. From Figure 2, we can see that when the contractor shares the cost of the technology, a larger λ would directly increase the disposal price w^{CS-D*} and the customer price P^{CS-D*} . So when the disposal facility is in power, it maximizes its profit by asking a higher price from the contractor. In addition, the act of the contractor sharing in the cost of technology investment motivates the disposal facility to invest more in environmental improvements, which raises the level of improving environmental impact and therefore reduces the emissions in the process of hazardous waste disposal. As a result, it is possible to require more service price from customers. It is not difficult to understand that when the contractor shares a portion of the cost of the technology investment, it inevitably loses a portion of its profits. To offset the lost profits, the contractor will take measures to increase his profits and choose to increase the consumer price.

Figure 3 illustrates the impact of the cost sharing proportion λ on the optimal profit per member and the total profit in the dual-channel supply chain system. Intuitively, under the disposal facility-Stackelberg model with cost sharing, the profit of disposal facility increases as the cost sharing proportion increases. These results imply that when the cost sharing proportion is within a certain small range, it can improve the contractor's profit. However, when the cost sharing proportion continues to increase, the contractor's profit is lower than

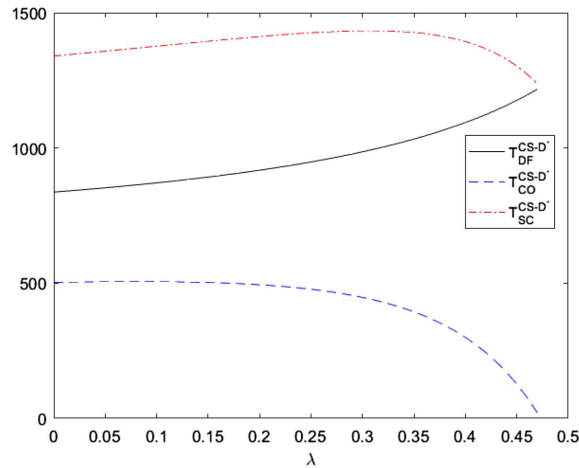


FIGURE 3. Impacts of λ on the optimal profits in CS-D model.

TABLE 3. Optimal results for different λ in CS-C model.

λ	w	θ	m	P	T_{DF}	T_{CO}	T_{SC}
0.05	15.36	5.80	7.03	22.39	522.88	1072.61	1595.50
0.15	15.41	6.77	7.53	22.94	505.54	1120.96	1626.50
0.25	15.34	7.96	8.35	23.70	454.95	1163.19	1618.14
0.35	15.03	9.30	9.68	24.71	346.93	1178.38	1525.31
0.45	14.25	10.47	11.68	25.93	166.44	1122.78	1289.22

that without cost sharing. When the disposal facility is in power, it is easy to understand that a larger λ reduces the contractor’s profit.

As shown in Figure 3, we observe that the total profit of the dual-channel supply chain system increases at the beginning and decreases as the cost sharing proportion continues to increase. Importantly, we can conclude that the contractor sharing a portion of the cost of technology investment is beneficial to the supply chain system. This result provides an answer to the question of why supply chain members sign cost sharing contracts.

5.2. Impact of λ on the optimal decisions and profits in CS-C model

In this subsection, the impact of the cost sharing proportion λ on the optimal decisions and profits in CS-C model is discussed through extensive numerical experiments. The optimal results of this model are given in Table 3 when λ takes different values, as well as their change curves displayed in Figures 4 and 5.

Figure 4 shows the impact of the cost sharing proportion λ on the optimal decisions of CS-C model. As shown in Figure 4, we can observe that the disposal price increases first as the cost sharing proportion increases, but at this time the contractor is in charge, and he will take measures to reduce the disposal price to improve his own profit. From Figure 4, we can observe that the service price asked from waste sources increases under the contractor-Stackelberg model with cost sharing, and the same trend is observed under the disposal facility-Stackelberg model with cost sharing. In fact, it is easy to understand that when the contractor is dominant, the asked disposal price by the disposal facility decreases. At this point, the level of impact of environmental improvements increases, which will result in a more friendly environment, thus increasing the demand for hazardous waste disposal, as shown in Figure 6.

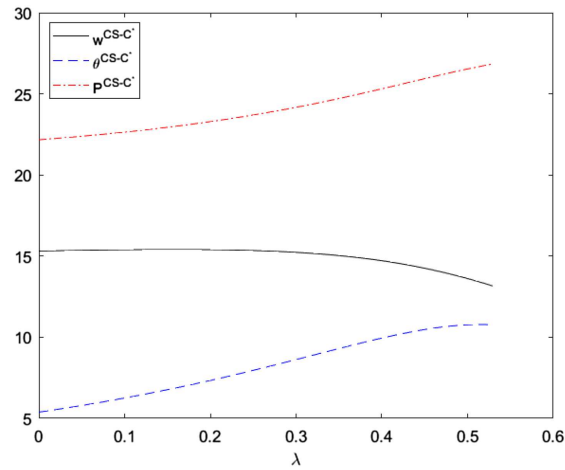


FIGURE 4. Impacts of λ on the optimal decisions in CS-C model.

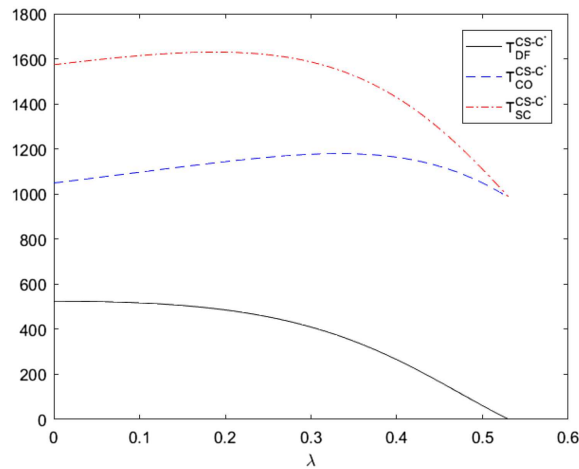


FIGURE 5. Impacts of λ on the optimal profits in CS-C model.

Figure 5 illustrates the effect of the cost sharing proportion λ on the players' profits and the total profit in the dual-channel supply chain system. Obviously, we can find that in CS-C model the profit of the disposal facility decreases in cost sharing proportion. This finding is significantly different from Figure 3, and may be directly related to the fact that the contractor is in a dominant position and the disposal facility is the follower. At this point, we can see that when the contractor shares a portion of the investment cost, both the contractor's profit and the total profit of the supply chain increase, which implies that cost sharing is better than no cost sharing. However, when the share becomes too large, profits start to diminish, suggesting that a higher cost sharing proportion is not always beneficial.

By studying Figures 3 and 5, we can obtain that the cost sharing contract can improve the total profit of the dual-channel supply chain. This result may explain the problem of why supply chain members sign cost sharing contracts. In addition, we find that the profit of the dominant party always increases with the cost sharing portion, whether it is the disposal facility-Stackelberg or the contractor. This finding has important implications for the participants in the supply chain.

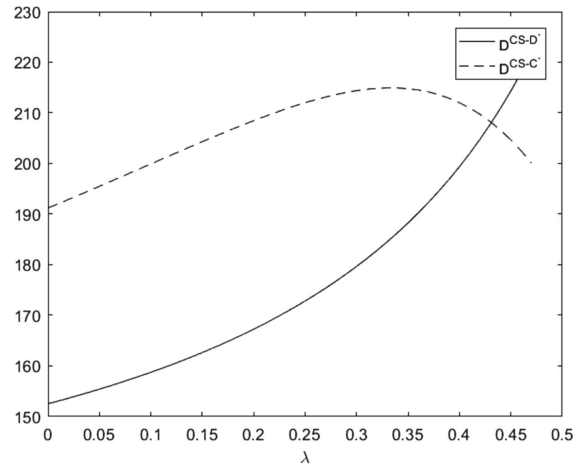


FIGURE 6. Impacts of λ on the total service demand.

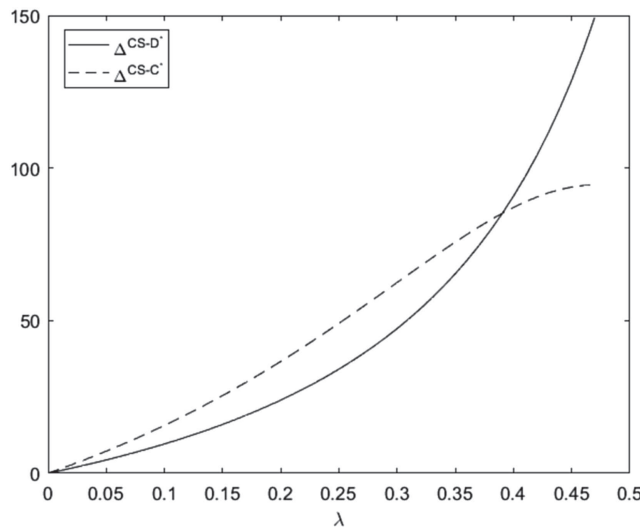


FIGURE 7. Impacts of λ on the emissions reduction.

5.3. Impact of λ on the service demand and emissions reduction

Figure 6 shows when the contractor and the disposal facility share the cost of investment in hazardous waste disposal technology, the total waste disposal demand increases. This suggests that the cost sharing contract raises the service demand for hazardous waste disposal. It is not in line with the intuition that the cost share increases the service prices as in Figures 2 and 4. It is the improvement of environmental impact that leads to the increase of service demand.

As the cost-share λ increases, the investment in improving environmental impact increases, and therefore the emissions reduce. The emission reduction, denoted by Δ , can be calculated by $D^* * b * (\theta^* - \theta_0^*)$ where θ_0^* is the value of θ^* when the cost-share λ is zero. The results for the two models CS-D and CS-C are shown in Figure 7.

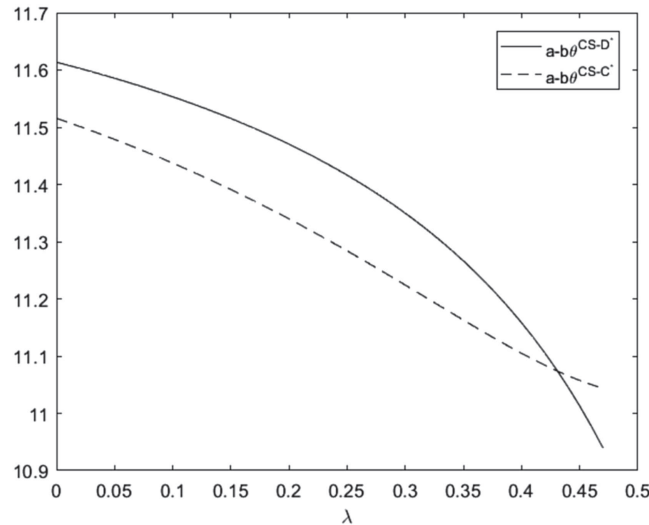


FIGURE 8. Impacts of λ on the emissions (per unit waste disposal).

Figure 8 shows the impact of cost sharing proportion λ on emissions per unit of hazardous waste disposal. Figure 7 shows that as the cost sharing portion increases, the emission reductions keep increasing. As can be seen in Figure 8, when the contractor and the disposal facility share the cost of investment in hazardous waste disposal technology, the emissions per unit of hazardous waste are less than those without the cost-share in the two models.

To sum up, the cost sharing contract can improve environmental impact and increase profits in the supply chain in hazardous waste disposal. Within an appropriate range, as the cost-share λ increases, the level θ of improving environmental impact increases, which leads to decrease of the emissions per unit of hazardous waste disposal and increase of the service demand for waste disposal.

5.4. Comparison of CS-D and CS-C models

Based on the above numerical experimental results, we compare the two models within an appropriate range of the cost share λ .

- (1) From Figures 2 and 4, we observe that the service price w^* asked from the contractor in CS-D model is larger than that in CS-C model, but the service prices P^* in the two models are not obviously different. These imply that the marginal profit m^* of the contractor in CS-D model is less than that in CS-C model. We also find that the level θ^* of improving environmental impact in CS-D is less than that in CS-C model. This implies that the investment in waste disposal technology in CS-D model is less than that in CS-C model.
- (2) From Figures 3 and 5, we observe that the dominant player obtains more profit than the other, a player obtains more profit when he is the leader than when he is not the leader, the dominant player obtains more profit as his cost-share becomes larger, but the total profit T_{SC}^* of the system in CS-C is more than that in CS-D model.
- (3) From Figure 6, we observe that the service demand for waste disposal D^* in CS-C model is more than that in CS-D model. It is the leadership in CS-C model that encourages the contractor to invest more in improving environmental impact. More investment raises the level of improving environmental impact, which increases the service demand. From Figures 7 and 8, the emissions per unit waste disposal in CS-C

TABLE 4. Optimal results for different ρ in CS-D model.

ρ	w	θ	m	P	D	T_{DF}	T_{CO}	T_{SC}
0	19.70	6.14	4.13	23.82	152.65	837.52	448.91	1286.43
0.15	19.28	6.92	4.65	23.93	172.06	944.00	450.30	1394.30
0.30	18.75	7.93	5.33	24.07	197.12	1081.51	433.50	1515.00
0.45	18.02	9.28	6.24	24.26	230.73	1265.89	378.09	1643.98
0.60	17.01	11.19	7.52	24.52	278.15	1526.07	235.83	1761.89

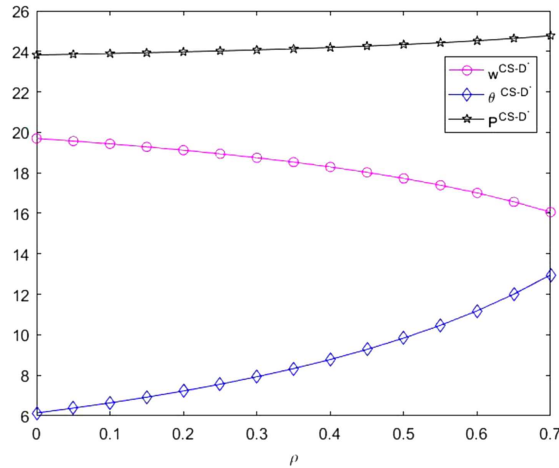


FIGURE 9. Impacts of ρ on the optimal decisions in CS-D model.

model is less than that in CS-D model, the emission reduction in CS-C is more than that in CS-D model. The reason is similar to above with respect to the service demand.

In general, CS-C model is superior to CS-D model.

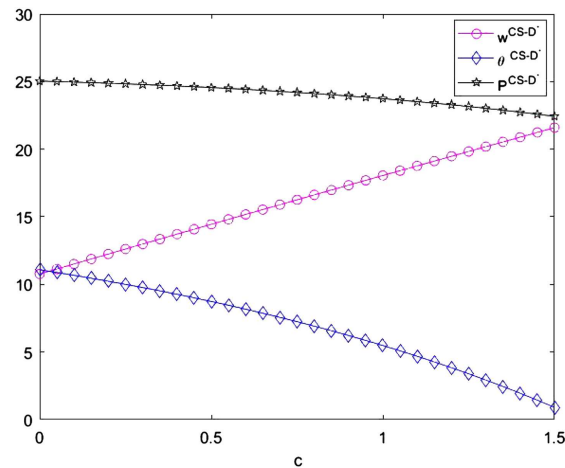
5.5. Impact of ρ , c on the optimal decisions and profits

In this subsection, the impact of the portion ρ of service demand and unit emission trading price c on the optimal results and profits is discussed. We first consider parameter ρ . From Proposition 4.4, we observe that the difference of parameter ρ only affects the prices, not the level of improving environmental impact or the profits in CS-C model. Through experiments we observe that the impact of ρ on the prices in CS-C model is similar to that in CS-D model. So we only investigate the impact of ρ on the optimal decisions and profits in CS-D model. For values of ρ which range from 0 to 0.6, the optimal results are given in Table 4, and the change curves of the optimal decisions are shown in Figure 9.

Table 4 and Figure 9 show that: as ρ increases, the disposal price w decreases; the marginal profit m increases, which leads to increase of the service price P ; the level θ increases, which leads to increase of the total service demand. As ρ increases, the service demand from the contractor decreases, which leads to decrease of the contractor’s profit; the service demand directly from waste sources increases, which leads to increase of the disposal facility’s profit and the total service demand, furthermore the increase of the supply chain’s profit. We conclude that the more a firm’s portion of service demand from waste sources is, the more profitable it is for the firm itself. So its portion is decided by its position in the market.

TABLE 5. Optimal results for different c in CS-D model.

c	w	θ	m	P	D	T_{DF}	T_{CO}	T_{SC}
0	16.07	10.11	7.55	23.62	279.23	2648.87	1195.44	3844.31
0.35	17.67	8.47	6.08	23.75	225.12	1662.82	751.77	2414.59
0.70	19.22	6.52	4.52	23.74	167.27	884.25	400.57	1284.82
1.05	20.74	4.24	2.84	23.58	104.98	334.51	151.87	486.38
1.40	22.22	1.56	1.01	23.23	37.39	40.62	18.49	59.11

FIGURE 10. Impacts of c on the optimal decisions in CS-D model.

Then we consider parameter c . Through experiments we observe that the impact of c on the optimal decisions and profits in CS-C model is similar to that in CS-D model. So we only investigate the impact of ρ on the optimal decisions and profits in CS-D model. For values of ρ which range from 0 to 1.4, the optimal results in CS-D model are given in Table 5, and the change curves of the optimal decisions are shown in Figure 10.

Table 5 and Figure 10 show that: as c increases, the disposal price w increases; the marginal profit m decreases; the level θ decreases, which leads to decrease of the total service demand D , furthermore decrease of the players' profits. We conclude that a higher emission price hinders economic development.

6. MANAGERIAL INSIGHTS

The results of this study can provide a reference for government and business to make decisions. Both total social profits increase and emission reduction are the important tasks of government. First, because CS-C model is superior to CS-D model, the government should take measurements to ensure the contractor's advantageous position in the supply chain, for example, raising the barriers to entry, giving policy and financial support, etc. Second, an appropriately large cost-share, for example, 0.3 in CS-D model and 0.2 in CS-C model, can ensure maximum total social profits while achieving a sufficiently large emission reduction. So the government should make policy or regulation to encourage or require contractors to take a reasonably large share of investment in disposal technology in a range, for example, $[2.5, 3.5]$ in CS-D model and $[1.5, 2.5]$ in CS-C model. Third, the government should establish a reasonable emission price to ensure both emission and economic development meet standards.

To a firm, its position in the market, cost-share of investment in hazardous waste disposal technology, and decision are very important aspects. First, a firm would make its higher profit when it is dominant in the market. This dominance comes from the financial and operational power of a firm. So it should manage to enhance its financial and operational power, for example, reducing operational cost, revising operation management processes, investing in new technologies, maintaining good relations with the government and upstream/downstream firm, etc. Second, a larger cost-share will benefit the dominant player, but not benefit the non-dominant player. So a firm should strive to facilitate a contract with a higher cost-share when it is the dominant, or *vice versa*. Finally, a firm should make the right decision based on its position and the cost-share contract.

7. CONCLUSION

Hazardous waste is a byproduct of the rapid economic development. With the increasing importance of environmental sustainability, research on hazardous waste disposal is of great importance. The disposal of hazardous waste requires technical support. Improving technology requires investment cost. We consider a dual-channel hazardous waste disposal supply chain composed of a disposal facility and a contractor under a cost sharing contract, establish two Stackelberg models CS-D and CS-C, discuss the impact of the cost sharing proportion of the contractor on the optimal decisions and profits in the supply chain.

Analysis and numerical experiments show that:

- (1) Cost sharing contracts can effectively improve the performance of the hazardous waste disposal supply chain. Importantly, the cost sharing contract can improve the level of environmental protection and the total profit of the supply chain. Within an appropriate range, the more the cost sharing proportion of the contractor is, the more the improvements are.
- (2) The dominant player obtains more profit than the other, a player obtains more profit when he is the leader than when he is not the leader, the dominant player obtains more profit as his cost-share becomes larger.
- (3) The level of improving environmental impact and the total profit in CS-C model are more than those in CS-D model. In this sense CS-C model is better than CS-D model.
- (4) A firm’s larger portion of service demand from waste sources will benefit the firm itself. And a higher emission price will hinder economic development.

All of these provide managers and governments references for management.

This study can be extended in several ways. First, it would be interesting to consider the competition between multiple disposal facilities and multiple contractors. Our work could provide a good framework. Second, we consider the penalty as an emission regulation. The consideration of different emission regulations, such as reward-penalty mechanisms, may bring different insights. Third, this study uses a cost sharing contract and the role of other forms of contracts can be considered.

APPENDIX A.

Proof of Proposition 4.1. We should find the principal minors’ sign of the Hessian matrix.

$$H = \begin{bmatrix} \frac{\partial^2 T_{DF}}{\partial w^2} & \frac{\partial^2 T_{DF}}{\partial w \partial \theta} \\ \frac{\partial^2 T_{DF}}{\partial \theta \partial w} & \frac{\partial^2 T_{DF}}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\alpha(-2 + \rho) & \frac{1}{2}(\beta - \rho\beta - \gamma\alpha) \\ \frac{1}{2}(\beta - \rho\beta - \gamma\alpha) & -2(1 - \lambda)I + \gamma\beta + \frac{\rho\beta^2}{2\alpha} \end{bmatrix}.$$

The first principal minor of H is $H^1 = -2\alpha < 0$. The objective function is joint concave in w and θ if $(2 - \rho)(1 - \lambda)I\alpha - \frac{1}{4}(\beta + \gamma\alpha)^2 > 0$. Therefore, Proposition 4.1 is proved. □

Proof of Proposition 4.2. In disposal facility-Stackelberg model, the contractor’s profit function is

$$T_{CO}^{CS-D} = m(1 - \rho)(d - \alpha(m + w) + \beta\theta) - \lambda I\theta^2. \tag{A.1}$$

The disposal facility’s profit function is

$$T_{DF}^{CS-D} = [(w - o + \gamma\theta)(1 - \rho) + (w + m - o + \gamma\theta)\rho](d - \alpha(m + w) + \beta\theta) - (1 - \lambda)I\theta^2. \tag{A.2}$$

First of all, with respect to equation (A.1), the first-order partial derivative of the contractor’s profit function T_{CO}^{CS-D} with respect to m is calculated as $\frac{\partial T_{CO}^{CS-D}}{\partial m} = (1 - \rho)(d - 2\alpha m - \alpha w + \beta\theta)$. Since $0 < \rho < 1$ and assumptions, it can be seen that $\frac{\partial^2 T_{CO}^{CS-D}}{\partial m^2} = -2(1 - \rho)\alpha < 0$. It shows that T_{CO}^{CS-D} is a strictly concave function. Then equating the first order condition to 0, we have

$$m^{CS-D} = \frac{d - \alpha w + \beta\theta}{2\alpha}. \tag{A.3}$$

Substituting equation (A.3) into the disposal facility’s profit function given in equation (A.2), the Hessian matrix of the objective function in equation (A.2) on variables w and θ is given as

$$H = \begin{bmatrix} \frac{\partial^2 T_{DF}}{\partial w^2} & \frac{\partial^2 T_{DF}}{\partial w \partial \theta} \\ \frac{\partial^2 T_{DF}}{\partial \theta \partial w} & \frac{\partial^2 T_{DF}}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\alpha(-2 + \rho) & \frac{1}{2}(\beta - \rho\beta - \gamma\alpha) \\ \frac{1}{2}(\beta - \rho\beta - \gamma\alpha) & -2(1 - \lambda)I + \gamma\beta + \frac{\rho\beta^2}{2\alpha} \end{bmatrix}.$$

Hence, when the condition $(2 - \rho)(1 - \lambda)I\alpha - \frac{1}{4}(\beta + \gamma\alpha)^2 > 0$ is satisfied, T_{DF}^{CS-D} is jointly concave in w and θ .

By setting $\frac{\partial T_{DF}^{CS-D}}{\partial w}$ and $\frac{\partial T_{DF}^{CS-D}}{\partial \theta}$ to zero and solving them with respect to w and θ simultaneously, we can derive the disposal facility’s optimal strategies as the following functions:

$$\begin{aligned} w^{CS-D*} &= \frac{o(4(1 - \lambda)I\alpha - \beta(\beta + \gamma\alpha)) + d(4(1 - \rho)(1 - \lambda)I - \gamma(\beta + \gamma\alpha))}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ \theta^{CS-D*} &= \frac{(d - o\alpha)(\beta + \gamma\alpha)}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ m^{CS-D*} &= \frac{2(1 - \lambda)I(d - o\alpha)}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}. \end{aligned}$$

Accordingly, the optimal values of the profit of the conductor, the disposal facility and the overall supply chain are

$$\begin{aligned} T_{DF}^{CS-D*} &= \frac{(1 - \lambda)I(d - o\alpha)^2}{4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \\ T_{CO}^{CS-D*} &= \frac{(4(1 - \rho)(1 - \lambda)^2 I^2 \alpha - \lambda I(\beta + \gamma\alpha)^2)(d - o\alpha)^2}{(4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2)^2}, \\ T_{SC}^{CS-D*} &= \frac{(4(3 - 2\rho)(1 - \lambda)^2 I^2 \alpha - I(\beta + \gamma\alpha)^2)(d - o\alpha)^2}{(4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2)^2}. \end{aligned}$$

Hence, Proposition 4.2 is proved. □

Proof of Corollary 4.1. According to Assumption 3.2 and Proposition 4.2, we have $m^{CS-D*} \geq 0$, $w^{CS-D*} - o \geq 0$, $T_{CO}^{CS-D*} \geq 0$. Therefore, Corollary 4.1 is proved. □

Proof of Corollary 4.2. The environment impact improvement level θ^{CS-D*} regarding λ :

$$\frac{\partial \theta^{CS-D*}}{\partial \lambda} = \frac{4(2 - \rho)(d - o\alpha)(\beta + \gamma\alpha)I\alpha}{[4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2]^2} > 0.$$

The marginal profit from collecting waste m^{CS-D^*} regarding λ :

$$\frac{\partial m^{CS-D^*}}{\partial \lambda} = \frac{2I(d - o\alpha)(\beta + \gamma\alpha)^2}{\left[4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right]^2} > 0.$$

□

Proof of Corollary 4.3. In the supply chain system under disposal facility-Stackelberg model with cost sharing, we have

$$\begin{aligned} \frac{\partial T_{DF}^{CS-D^*}}{\partial \lambda} &= \frac{I(d - o\alpha)^2(\beta + \gamma\alpha)^2}{\left[4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right]^2} > 0, \\ \frac{\partial T_{CO}^{CS-D^*}}{\partial \lambda} &= -\frac{\left[4I\alpha(\rho(1 - \lambda) + 2\lambda(2 - \rho)) - (\beta + \gamma\alpha)^2\right]I(d - o\alpha)^2(\beta + \gamma\alpha)^2}{\left[4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right]^3}, \\ \frac{\partial T_{SC}^{CS-D^*}}{\partial \lambda} &= \frac{8[(3 - 2\rho)(1 - \lambda) - (2 - \rho)](d - o\alpha)^2(\beta + \gamma\alpha)^2I^2\alpha}{\left[4(2 - \rho)(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right]^3}. \end{aligned}$$

Hence, Corollary 4.3 is proved. □

Proof of Corollary 4.4. See reference [11] for proof. □

Proof of Proposition 4.3. In this model, the Hessian matrix for the objective function of variables w and θ in equation (A.2) is given as follows:

$$H = \begin{bmatrix} \frac{\partial^2 T_{DF}}{\partial w^2} & \frac{\partial^2 T_{DF}}{\partial w \partial \theta} \\ \frac{\partial^2 T_{DF}}{\partial \theta \partial w} & \frac{\partial^2 T_{DF}}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -2\alpha & \beta - \gamma\alpha \\ \beta - \gamma\alpha & -2(1 - \lambda)I + 2\gamma\beta \end{bmatrix}.$$

Hence, when the condition $4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2 > 0$ is satisfied, T_{DF}^{CS-C} is jointly concave in w and θ . Therefore, Proposition 4.3 is proved. □

Proof of Proposition 4.4. In contractor-Stackelberg model, the contractor’s profit function is

$$T_{CO}^{CS-D} = m(1 - \rho)(d - \alpha(m + w) + \beta\theta) - \lambda I\theta^2.$$

The disposal facility’s profit function is

$$T_{DF}^{CS-D} = [(w - o + \gamma\theta)(1 - \rho) + (w + m - o + \gamma\theta)\rho](d - \alpha(m + w) + \beta\theta) - (1 - \lambda)I\theta^2.$$

In this case, given the contractor’s earlier decision m , the first-order partial derivatives of T_{DF}^{CS-C} to w and θ are given as follows:

$$\frac{\partial T_{DF}^{CS-C}}{\partial w} = d - \alpha(m + w) + \beta\theta + (w + \rho m - o + \gamma\theta)(-\alpha), \tag{A.4}$$

$$\frac{\partial T_{DF}^{CS-C}}{\partial \theta} = \gamma(d - \alpha(m + w) + \beta\theta) + (w + \rho m - o + \gamma\theta)\beta - 2(1 - \lambda)I\theta. \tag{A.5}$$

Correspondingly, the Hessian matrix for the objective function of variables w and θ in equation (A.2) is given as follows:

$$H = \begin{bmatrix} \frac{\partial^2 T_{DF}}{\partial w^2} & \frac{\partial^2 T_{DF}}{\partial w \partial \theta} \\ \frac{\partial^2 T_{DF}}{\partial \theta \partial w} & \frac{\partial^2 T_{DF}}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -2\alpha & \beta - \gamma\alpha \\ \beta - \gamma\alpha & -2(1 - \lambda)I + 2\gamma\beta \end{bmatrix}.$$

Hence, when the condition $4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2 > 0$ is satisfied, T_{DF}^{CS-C} is jointly concave in w and θ . Then, let equations (A.4) and (A.5) to 0 simultaneously, disposal facility's optimal strategies are given in equations (A.6) and (A.7):

$$w^{CS-C}(m) = \frac{d(2(1 - \lambda)I - \gamma(\beta + \gamma\alpha)) - 2(1 - \lambda)I\alpha(m - o + m\rho)}{4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2} - \frac{(\beta + \gamma\alpha)(o\beta - m(\gamma\alpha + \rho\beta))}{4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}, \tag{A.6}$$

$$\theta^{CS-C}(m) = \frac{(\beta + \gamma\alpha)(d - (m + o)\alpha + m\alpha\rho)}{4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2}. \tag{A.7}$$

Substituting the best response function of the disposal facility given in equations (A.6) and (A.7) into the contractor's profit function given in equation (A.1), we can get that the objective function $T_{CO}^{CS-C}(m)$ is concave in m . Solving the first order condition, we get

$$m^{CS-C*} = \frac{\left(4(1 - \lambda)^2I\alpha - (1 - 2\lambda)(\beta + \gamma\alpha)^2\right)(d - o\alpha)}{(1 - \rho)\alpha\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)}.$$

Substituting the value of m , we have

$$\begin{aligned} \theta^{CS-C*} &= \frac{(1 - \lambda)(\beta + \gamma\alpha)(d - o\alpha)}{8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2}, \\ w^{CS-C*} &= \frac{d\left(2(1 - 3\rho)(1 - \lambda)^2I\alpha + (\beta + \gamma\alpha)((1 - 2\lambda)\rho\beta + ((1 - \lambda)(3\rho - 1) - \rho)\gamma\alpha)\right)}{(1 - \rho)\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)\alpha} \\ &\quad + \frac{o\alpha\left(2(1 - \lambda)^2I\alpha(3 - \rho) + (\beta + \gamma\alpha)((1 - (3 - \rho)(1 - \lambda))\beta - (1 - 2\lambda)\gamma\alpha)\right)}{(1 - \rho)\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)\alpha}. \end{aligned}$$

Then, the contractor's and the disposal facility's profit function are as follows:

$$\begin{aligned} T_{CO}^{CS-C*} &= \frac{(1 - \lambda)^2I(d - o\alpha)^2}{8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2}, \\ T_{DF}^{CS-C*} &= \frac{(1 - \lambda)^3I\left(4(1 - \lambda)I\alpha - (\beta + \gamma\alpha)^2\right)(d - o\alpha)^2}{\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)^2}, \\ T_{SC}^{CS-C*} &= \frac{(1 - \lambda)^2I\left(12(1 - \lambda)^2I\alpha - (3 - 4\lambda)(\beta + \gamma\alpha)^2\right)(d - o\alpha)^2}{\left(8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right)^2}. \end{aligned}$$

Hence, Proposition 4.4 is proved. □

Proof of Corollary 4.5. According to Assumption 3.2 and Proposition 4.4, we have $m^{CS-C*} > 0$, $\theta^{CS-C*} > 0$, $T_{CO}^{CS-C*} > 0$. Therefore, Corollary 4.5 is proved. □

Proof of Corollary 4.6. The environment impact improvement level θ^{CS-C*} regarding λ :

$$\frac{\partial\theta^{CS-C*}}{\partial\lambda} = \frac{\left[8(1 - \lambda)^2I\alpha - (\beta + \gamma\alpha)^2\right](d - o\alpha)(\beta + \gamma\alpha)}{\left[8(1 - \lambda)^2I\alpha - (2 - 3\lambda)(\beta + \gamma\alpha)^2\right]^2}.$$

The marginal profit from collecting waste m^{CS-C^*} regarding λ :

$$\frac{\partial m^{CS-C^*}}{\partial \lambda} = \frac{\left[4(1-\lambda)(1+\lambda)I\alpha - (\beta + \gamma\alpha)^2\right](d - o\alpha)(\beta + \gamma\alpha)^2}{(1-\rho)\alpha \left[8(1-\lambda)^2I\alpha - (2-3\lambda)(\beta + \gamma\alpha)^2\right]^2}.$$

□

Proof of Corollary 4.7. In the supply chain system dominated by the disposer, when the contractor shares part of the technology cost, we have

$$\begin{aligned} \frac{\partial T_{DF}^{CS-C^*}}{\partial \lambda} &= -\frac{\lambda(1-\lambda)^2I \left[16(1-\lambda)I\alpha - 3(\beta + \gamma\alpha)^2\right](d - o\alpha)^2(\beta + \gamma\alpha)^2}{\left[8(1-\lambda)^2I\alpha - (2-3\lambda)(\beta + \gamma\alpha)^2\right]^3}, \\ \frac{\partial T_{CO}^{CS-C^*}}{\partial \lambda} &= \frac{(1-\lambda)(1-3\lambda)I(d - o\alpha)^2(\beta + \gamma\alpha)^2}{\left[8(1-\lambda)^2I\alpha - (2-3\lambda)(\beta + \gamma\alpha)^2\right]^2}, \\ \frac{\partial T_{SC}^{CS-C^*}}{\partial \lambda} &= \frac{2(1-\lambda)I \left[4(1-\lambda)^2(1-5\lambda)I\alpha - (6\lambda^2 - 6\lambda + 1)(\beta + \gamma\alpha)^2\right](d - o\alpha)^2(\beta + \gamma\alpha)^2}{\left[8(1-\lambda)^2I\alpha - (2-3\lambda)(\beta + \gamma\alpha)^2\right]^3}. \end{aligned}$$

Hence, Corollary 4.7 is proved. □

Proof of Corollary 4.8. See reference [11] for proof. □

Acknowledgements

The work was supported by the National Natural Science Foundation of China (Grant No. 12071112).

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