RAIRO-Oper. Res. 58 (2024) 1899–1925
https://doi.org/10.1051/ro/2024062

RAIRO Operations Research
www.rairo-ro.org

EXAMINING SUPPLY CHAIN FINANCING: A COMPARATIVE AND COORDINATION ANALYSIS UNDER A BUYBACK CONTRACT

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Abstract. This paper focuses on financing schemes for a supply chain with high salvage values of unsold products. Combining the buyback contract with a partial credit guarantee (PCG) contract and trade credit (TC) contract, we propose two financing schemes, PCG-Buyback and TC-Buyback, to provide flexible financing services for a capital-constrained retailer and obtain equilibrium strategies of each supply chain member. Furthermore, for PCG-Buyback, this paper obtains a Pareto coordination frontier consisting of the credit guarantee coefficient and the buyback price at different initial capital levels, which is more flexible to achieve supply chain coordination. Finally, we analyze the manufacturer's strategic choices of PCG-Buyback and TC-Buyback in terms of credit guarantee coefficient, buyback price, and financing rate. Our results show that PCG-Buyback is the optimal financing scheme for the manufacturer when both the credit guarantee coefficient and buyback price (or financing rate) are low; otherwise, TC-Buyback is the dominant strategy. This study explores how the risk-sharing mechanism combination of PCG and buyback contracts can provide a more flexible approach to supply chain coordination. Additionally, it highlights the significance of the manufacturer selecting a better financing scheme based on its individual characteristics.

Mathematics Subject Classification. 90B06, 91A65.

Received June 6, 2023. Accepted March 6, 2024.

1. Introduction

1.1. Financing schemes

Small and medium-sized enterprises (SMEs) often face difficulty in securing loans from financial institutions due to their lack of credit records and asset mortgages. As reported by the SME Finance Forum in 2020, out of the 131 million SMEs in developing countries, 41% have not been able to meet their financing needs, with the unmet credit demand estimated to be worth US$4.5 trillion[1]. In order to address this issue, a new financing scheme, supply chain finance (SCF), has been introduced to help SMEs solve their financing problems during supply chain operations. SCF can be divided into two categories: Bank Credit Finance (BCF) and Trade Credit (TC) [40]. With BCF, a bank utilizes the credit of a core enterprise to provide loans to its upstream and downstream SMEs. Nevertheless, Basel III has introduced a stricter definition of capital, as well as more

Keywords. Supply chain management, supply chain coordination, partial credit guarantee, trade credit, buyback contract.

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[1] https://www.smefinanceforum.org/about/annual-report.

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risk-sensitive banking regulations, for the purpose of increasing bank transparency and disclosure, as well as enhancing risk management and governance [33]. Unfortunately, this may lead to an increase in the interest rates on bank loans to SMEs, which may in turn exacerbate the financial difficulties of SMEs. To avoid this situation, under the Basel Accord, banks are allowed to use a variety of collateral and guarantees to protect themselves from the risks associated with lending to small businesses [11]. This means that the partial credit guarantee (PCG) contract has become the tool of choice for policymakers to improve SMEs' access to financing. For example, Inner Mongolia Yili Industrial Group Company Limited (Yili Group) provides joint and several liabilities guarantee for the bank loans incurred by its wholly-owned subsidiary, Westland Dairy Company Limited, with HSBC, ASB and CCB. In the event of default by Westland Dairy Company Limited, Yili Group is committed to indemnifying the lending banks for the principal amount of the loan, interest, default interest, damages and any other expenses and costs[2].

For SMEs, the use of Partial Credit Guarantees (PCG) offers an effective way to improve the relationship between borrowers and lenders. This facilitates the development of long-term trust between the two parties by transferring the details of the SMEs to the banks. Furthermore, PCG helps alleviate the problem of credit market information asymmetry [21, 22], as well as reduce the bank's funding costs [15] and default risk [1]. In the event of loan defaults, the bank is compensated from the guarantor. Lower funding costs allow the bank to increase its lending capacity, which is based on market imperfections or market forces [15]. Additionally, banks can also increase their earnings by selling the risk-free portion of guaranteed loans in the secondary market [3]. Therefore, PCG encourages banks to capitalize on the capital effect by maximizing their own interests.

TC is another financing service whereby a core enterprise provides credit to downstream SMEs partners through short-term loans, enabling the latter to purchase goods or services from the former without needing to make immediate payments from financial institutions [40, 41]. It can provide SMEs with flexible financing services and is the widely used financing scheme. For instance, General Motors, Procter & Gamble, and Unilever offer trade credit to their downstream cash-strapped retailers who cannot immediately pay for their goods, which effectively alleviates their capital shortages and reduces transaction costs and inventory risk [38].

1.2. Motivations

SMEs may obtain loans through PCG or TC for production or ordering, yet the uncertainty of market demand and customer preferences can lead to an overstock of inventory, making repayment of loans difficult and potentially resulting in bankruptcy. Liquidation of the borrower is then carried out by the lender. Although existing literature does not address the value of unsold products when studying TC or PCG financing bankruptcy liquidation, this does not apply to bulk commodities or luxury goods, as their unsold value remains considerable.

The motivation for this research stems from the observed role of buyback contracts in addressing these financing and inventory challenges. Luxury companies, for example, may enter into a buyback contract with downstream distributors, enabling them to provide unsold products to luxury rental platforms for rental services [9]. If the commodity is of high value and the borrower's loan amount is large, the lender will usually require the upstream company to sign a buyback contract. This ensures that the downstream company will have the necessary resources to repay the loan at the end of the sales season. For example, a secondary downstream distributor of Hengda steel had an initial capital of only 3 million yuan, but needed 10 million yuan to purchase steel. The distributor, Hengda steel, and the bank then entered into a tripartite agreement, which stipulated that the distributor had to deposit at least 30% of the purchase funds in the bank as a security deposit, and Hengda steel had to promise to buy back any remaining steel at the end of the sales season, thus guaranteeing the distributor had enough money to repay the loan [14].

Furthermore, the motivation for this research is rooted in the recognition that both PCG contracts [40] and buyback contracts [35, 39] serve as coordination mechanisms in supply chains. The former considers the risk-sharing between the bank and core enterprise, while the latter is a profit transfer between the core enterprise and downstream retailer, which means that both of them only consider the relationship between two supply chain

[2]https://pdf.dfcfw.com/pdf/H2_AN202108161510483714_1.pdf?1629138010000.pdf.

members. An intuitive question then arises: Is a single contract flexible enough to coordinate the supply chain when considering the bank, the core enterprise, and the downstream retailer simultaneously? For instance, when Basel III requires banks to increase their Tier 1 capital adequacy ratio to 7% [4], they must set an adequate guarantee coefficient to meet the risk requirement. However, this may not be enough to coordinate the supply chain. Therefore, it may be beneficial to explore combining two or more contracts, in order to be more flexible in coordinating the supply chain where a single contract cannot.

Another key aspect of the research motivation pertains to scenarios where PCG is not available, potentially due to bank requirements a guarantee coefficient that exceeds the core firm's affordability. In such case, whether TC can be a tool for SMEs to obtain financing, and how the buyback contract affects supply chain members' decision making and the choice of financing schemes at this moment.

1.3. Topic issues

The research problem addressed in this study revolves around the financing challenges faced by SMEs in the context of market demand uncertainty. Specifically, the problem centers on SMEs' ability to obtain loans through PCG or TC financing for ordering, and the associated risks that may lead to overstocked inventory, loan repayment difficulties, and the potential for bankruptcy. Notably, existing literature has not explored the valuation of unsold products within the framework of TC or PCG financing bankruptcy liquidation (see, [25–27], etc.), while the unsold product value remains significant.

Driven by the observations made above, this paper's primary focus is on exploring supply chain strategies and coordination within the context of different financing schemes, namely PCG and TC, under the influence of the buyback contract. We delve into three key aspects:

(1) How does the buyback contract affect the risk allocation and the optimal decisions among supply chain members under different financing schemes?
(2) Under PCG financing scheme, can the buyback contract and PCG contract jointly determine a Pareto region to realize supply chain coordination?
(3) Under the buyback contract, how does the manufacturer choose the optimal financing scheme to maximize profits?

To answer these questions, we design two financing schemes, PCG-Buyback and TC-Buyback, for a capital-constrained retailer, and use the Stackelberg game to identify each participant's optimal decision. Our main findings and contributions are as follows:

Research findings

(1) In our study, we observe that the buyback contract has a distinct effect on the two financing schemes. In PCG-Buyback, PCG and buyback contracts work together to evenly distribute the retailer's bankruptcy risk; however, this comes at the expense of the manufacturer's profits. Conversely, in TC-Buyback, the manufacturer is both the seller and the lender, allowing it to gain both sales revenue and financing income. Leveraging the buyback contract in this situation helps to minimize the risk of the retailer's insolvency and maximize the manufacturer's profits.
(2) Our research reveals that the PCG-Buyback enables the coordination of the supply chain through a Pareto region composed of the credit guarantee coefficient and the buyback price. This approach offers more flexibility in supply chain coordination, compared to traditional methods relying on a single PCG contract [40] or single buyback contract [35, 39].

Research contributions

Our study contributes to the field by offering valuable insights into the selection of financing schemes for manufacturers, primarily focusing on profit maximization. We define selection regions that encompass the credit guarantee coefficient, buyback price, and financing interest rate. Moreover, we investigate the influence of the initial capital level on these financing selection regions. The results suggest that when the guarantee risk is low, PCG-Buyback is more beneficial to manufacturers; however, when the commodity buyback value is high, TC-Buyback is the better option.

Our research, focusing on the area of supply chain decision making, takes a financing perspective, with a special focus on the role of buyback contracts under various financing scenarios and how to achieve supply chain coordination and reach optimal decision making. Our research extends and enriches the traditional research on pricing and inventory management based on EOQ modeling, such as Rahaman *et al.* [32] and Momena *et al.* [29], which each explored the impact of pricing, display inventory, shortages, and memory on retailers' decision making, as well as the impact of customer demand uncertainty, pricing strategies, and empirical learning on EOQ models. We look at the challenges of finance in supply chains, while also focusing on potential bankruptcy risks. This research extends our understanding of supply chain management and decision making and aims to provide additional insights and best practices for real business operations.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 introduces two financing schemes under a buyback contract, and establishes the corresponding parameters. In Section 4, we analyze the optimal strategy and supply chain coordination under PCG-Buyback. Section 5 focuses on the optimal strategy for the supply chain under TC-Buyback. We conduct numerical experiments to evaluate supply chain coordination and financing selection in Section 6, followed by the main results and managerial insights. Finally, the paper is concluded in Section 7, with the proofs included in the appendix.

2. Literature review

Our paper is related to three research streams: PCG financing and TC financing, buyback contract, and the coordination of a capital-constrained supply chain.

2.1. PCG and TC

PCG provided by large corporations can help high-risk and collateral-starved SMEs obtain loans from the bank, which can lower the financing costs of their smaller partners [16,21]. Yan *et al.* [40] analyze the equilibrium strategy of PCG in capital-constrained supply chains and the influence of the credit guarantee coefficient on financing decisions and supply chain coordination. Our paper complements theirs by considering the residual value and discussing the joint impact of the buyback contract and PCG contract on supply chain financing decisions and coordination and provides a Pareto region composed of the credit guarantee coefficient and the buyback price for more flexible supply chain coordination. Lu *et al.* [27] compare PCG contracts offered by suppliers or third-party institutions and discuss the effect of initial capital level and market fluctuation on their choice. Our paper focuses on the supplier-provided PCG contract and compares it with TC, providing the dominant area of the two financing methods based on the guarantee coefficient, buyback price and financing interest rate. Li and Jiang [25] research the influence of the wholesale price on the supplier's credit guarantee coefficient, finding that when the wholesale price is a decision variable, the best strategy for the supplier is a full guarantee, while a limited guarantee may be more suitable when the wholesale price is exogenous. In contrast to their perspective, our study of the guarantee coefficient focuses on supply chain coordination.

TC provides an attractive alternative financing scheme for SMEs who are unable to obtain credit guarantees or bank loans. Chen and Wang [13] and Cao and Yu [10] find that retailers with lower initial capital can obtain more orders through TC financing, which means that TC will create additional value for the supply chain. Burkart and Ellingsen [6] and Cai *et al.* [8] point out that TC is essentially product lending, which reduces the opportunistic behavior of borrowers by linking with product transactions. Babich and Tang [2] and Rui and Lai [34] demonstrate that the delayed payment of TC can mitigate the problem of supplier product quality adulteration. Wu *et al.* [37] offer two buyback financing mechanisms for a capital constrained supplier based on buyback contracts: BSBPOF (viewed as BCF) and BSAPD (viewed as TC). Their results indicate that the financing equilibrium is BSBPOF if the buyback price is above a certain level and the difference between the buyback price and the residual value is less than a certain range, otherwise, BSAPD is preferred. Our own financing model for retailers has opposite results: TC-Buyback is selected when the buyback price is above a certain level, and PCG-Buyback is favored otherwise. This is because our financing target is the downstream retailer, a higher buyback price reduces the retailer's losses and risk of bankruptcy, and the manufacturer is more likely to grant TC when the retailer is financially constrained.

2.2. Buyback contract

The second stream of literature is about the buyback contract, which mainly focuses on supply chain operation and financing management. Padmanabhan and Png [30] think that manufacturers may want to offer buyback contracts to prevent retailers from discounting remaining items, which damages the manufacturer's brand image. For example, if a fashion clothing brand has a large number of clothes on discounting sales, it is difficult to convince consumers that your clothes are popular. More importantly, they argue that manufacturers can use the buyback contract to increase their profit margins by influencing price competition among retailers [31]. As a complement to them, in our paper, the buyback contract can also be employed to recover merchandise with a high salvage value and alleviate retailer inventory pressure, thus lowering the risk of bankruptcy. Recent literature even explores how buyback contracts can be used to address capital constraints in supply chain finance and its effects on financing decisions and supply chain coordination. For example, when the retailer is dealing with inventory-level-dependent demand, Devangan *et al.* [18] design an individual rational buyback contract to coordinate the supply chain, providing management insights into the design of the contract and the influence of shelf space inventory on contract parameters. Kouvelis and Zhao [23] shows that, taking bankruptcy costs into account, although the buyback contract can coordinate a capital constrained supply chain, revenue sharing contract is Pareto dominated over buyback contract, as the buyback amount must be used to cover fixed default costs in the event of low demand. Chen *et al.* [14] study a retailer obtain loans from a bank through the manufacturer's buyback guarantee and give optimal pricing or ordering decisions. Shi *et al.* [35], which offers a buyback contract to compensate the bank when the retailer goes bankrupt. They set the buyback price as a ratio of the wholesale price, while our buyback price is independent of the wholesale price, enabling us to set a flexible sufficient buyback price to coordinate the supply chain together with the guarantee coefficient. This setting is also common in various buyback literature such as Wu [36], Chen *et al.* [14] and Wu *et al.* [37].

2.3. Supply chain coordination with buyback and PCG contracts

The third stream of literature focuses on the coordination of capital-constrained supply chains, with the goal of avoiding the double marginalization effect and maximizing the utility of the entire supply chain through contract design [7]. For example, Dada and Hu [17] proposed a nonlinear bank loan plan to coordinate the newsvendor model under capital constraints. Moreover, the double or even triple marginalization of supply chain members has caused them to seek out contract design solutions to better coordinate the supply chain, especially under random demand. Related to our study, such as Chen and Bell [12], Wu [36] and Shi *et al.* [35] study the coordination of buyback contract, and Yan *et al.* [40] study the coordination of PCG contract. Other articles design a set of coordination contracts for the supply chain, such as Kouvelis and Zhao [23] and Xiao *et al.* [38]. They develop a general contract menu for capital-constrained retailers, which includes revenue sharing, buyback, and quantity discount. In addition, these studies examine how different factors, such as market demand, buyback amounts, default cost, and total supply chain working capital, can affect the design of contracts and, ultimately, the coordination of the supply chain. Most existing literature on the coordination of PCG and buyback contracts focus on adjusting a single parameter. For instance, Yan *et al.* [40] adjust the credit guarantee coefficient, Bi *et al.* [5] update the buyback rate to coordinate the supply chain. However, this approach has the disadvantage of being unable to account for the different bargaining powers of the contract signatories. To fill this gap, this paper considers two contracts, PCG and buyback, to coordinate the supply chain jointly and gives a Pareto coordination region composed of the credit guarantee coefficient and the buyback price. In this regard, the work of Shi *et al.* [35] is close to ours. They use a combination of a buyback contract and a wholesale price contract to coordinate the supply chain, which mainly considers profit transfer and risk

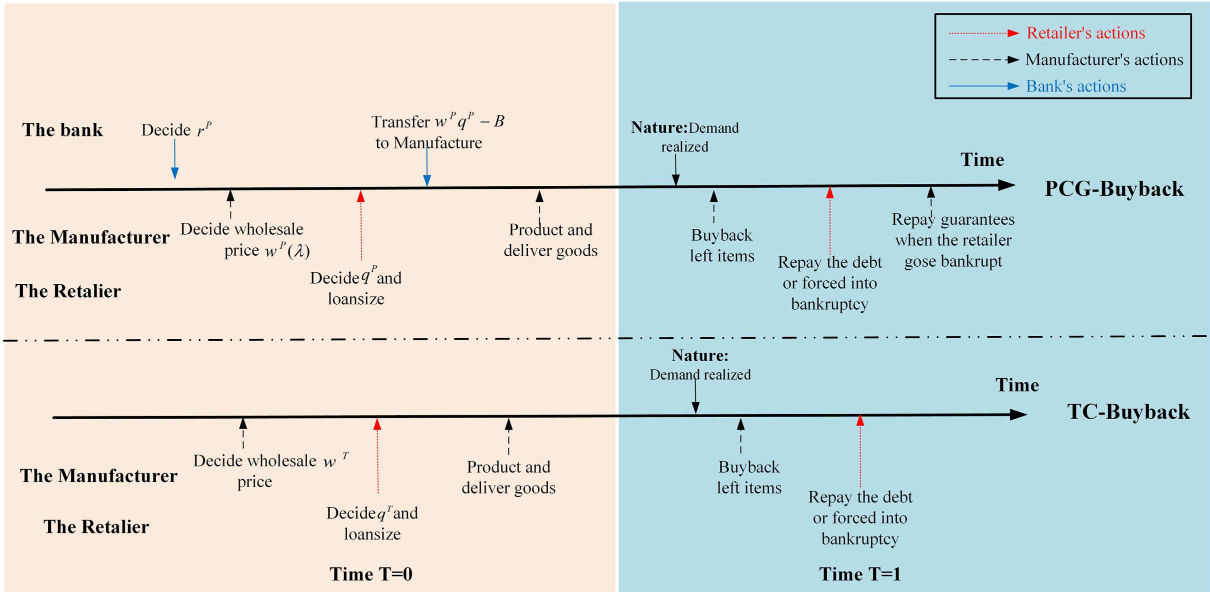

Figure 1. The time sequence of PCG-Buyback and TC-Buyback.

sharing between the supplier and retailer. Our SCF system, on the other hand, employs a PCG contract to share the bank's risk with the supplier and a buyback contract to transfer the supplier's profit with the retailer, thus coordinating the supply jointly. In addition, they set the buyback price as a ratio of the wholesale price, where we differ by setting the exogenous buyback to ensure sufficient flexibility to coordinate the supply chain with the guarantee coefficient.

3. Model description and assumptions

This section introduces the design of the financing scheme, the definition of variables and parameters, and related assumptions. The supply chain studied in this paper consists of a manufacturer (M) with sufficient funds and a retailer (R) with financial constraints. The manufacturer produces a single product with a unit cost c and sells it to the retailer with a wholesale price w^P. The retailer sells it to consumers at a unit sales price p. The retailer's capital is not enough to complete the order and needs financing. We offer two financing schemes. One is that the manufacturer provides PCG for the retailer, and the retailer borrows loans from the bank. The other is that the manufacturer provides TC for the retailer. At the end of the selling season, the manufacturer should buy back the unsold products at a unit price of $e(e < c < w^P)$. We name these two schemes as PCG-Buyback and TC-Buyback, respectively, and the time sequences of these two schemes are shown in Figure 1. In addition, the supply chain structure under the two financing schemes are diagrammed in Figure 2, which consists of three main supply chain agents, cash flow and product flow.

Under PCG-Buyback, the manufacturer acts as the seller and the bank serves as the lender. As shown in Figure 1, at Time $T = 0$, the bank first signs PCG contract with the manufacturer and announces the financing interest rate r^P, the manufacturer signs buyback contracts with the retailer. Afterward, the manufacturer decides a wholesale price $w^P(w^P(1 + r^P) < p)$, and the retailer decides an order quantity q^P. Then the bank transfers the loan to the manufacturer. After the demand is realized at Time $T = 1$, the manufacturer buys back unsold products and the retailer repays loans to the bank. If the retailer goes bankrupt, the manufacturer needs to pay the proportion of λ to the bank for the remaining loans.

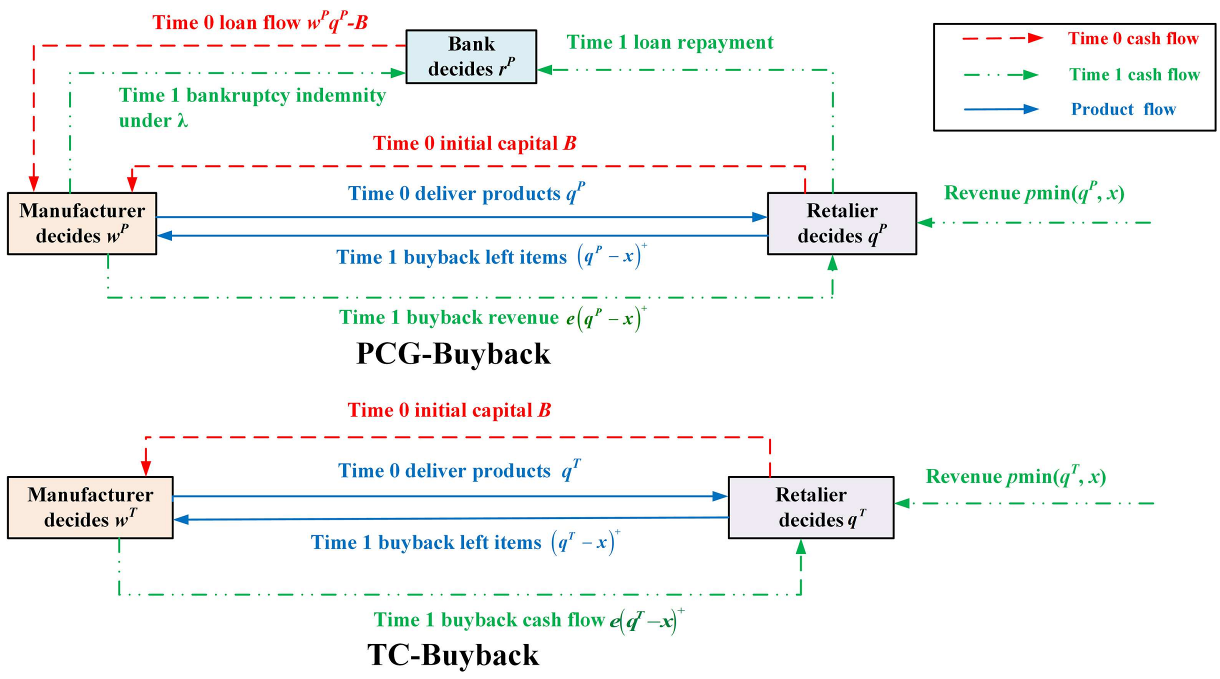

FIGURE 2. The supply chain structure of PCG-Buyback and TC-Buyback.

It should be noted that we do not use the classic Modigliani–Miller (MM) theorem (1958) to view the relationship between bank capital and lending, which assumes that in a frictionless world with perfect information and perfect markets, a firm's capital structure does not affect its investment policy [28]. The assumptions underlying the MM theorem are flawed due to the cost associated with financial distress [15, 19] and debt risk [20]. PCG can indirectly reduce the bank's funding costs. If a loan defaults, the bank can receive compensation from the guarantor and thus reduce the costs associated with financial distress, which allows the bank to provide funds for more loans. In other words, under PCG, the bank has an incentive to capitalize based on maximizing its own interest. Therefore, in this paper, we develop a framework for endogenous financing decisions in which the bank wishes to maximize capital effects and price loans in a monopolistic rather than a perfectly competitive market and in which bank financing influences firms decisions.

Under TC-Buyback, the manufacturer acts as both the seller and the lender. As shown in Figure 1, at Time $T = 0$, the manufacturer announces a financing interest rate r^T and decides a wholesale price w^T. The retailer decides an order quantity q^T. At Time $T = 1$, the manufacturer buys back unsold products and the retailer repays loans to the manufacturer. For the convenience of expression, we summarize the related variables and parameters in Table 1.

Note that λ is agreed in advance by the bank and manufacturer. Market demand x is a nonnegative random variable, and its probability density function (PDF) is $f(x)$ and $f(x) > 0$. The cumulative distribution function (CDF) is $F(x)$, and complementary CDF is $\bar{F}(x)$. $F(x)$ is differentiable and monotonically increasing.

Without losing generality, we list some assumptions for A_1 to A_4.

A_1 All participants in the supply chain are risk-neutral and aim to maximize their expected profits, and the risk-free interest rate $r_f = 0$.

A_2 There is no information asymmetry, *i.e.*, production cost, initial capital, demand distribution, etc., are common knowledge.

TABLE 1. List of notations.

Decision variables	Definitions
q^P	Retailer's order quantity under PCG-Buyback
w^P	Manufacturer's unit wholesale price under PCG-Buyback
r^P	Bank's interest rate under PCG-Buyback
q^T	Retailer's order quantity under TC-Buyback
w^T	Manufacturer's unit wholesale price under TC-Buyback
Parameters	Definitions
p	Retailer's unit retail price
c	Manufacturer's unit production cost
e	Manufacturer's unit buyback price
r^T	Manufacturer's trade credit rate under TC-Buyback
λ	The credit guarantee coefficient under PCG-Buyback
k	Demand threshold of bankruptcy
π_i^P	The expected profit of supply chain members under PCG-Buyback, $i = R, M$ or B
π_i^T	The expected profit of supply chain members under TC-Buyback, $i = R, M$
B	Retailer's initial budget
x	Random market demand

A_3 There is no moral hazard problem, that is, the retailer has no tendency to default in advance and will repay the loan as much as possible.

A_4 Let $h(x) = \frac{f(x)}{\bar{F}(x)}$ represent the failure rate and $H(x) = xh(x)$ denote the generalized failure rate. Similar to Kouvelis and Zhao [24], we also assume that the demand distribution includes a strictly increasing and convex failure rate, *i.e.*, $\frac{\mathrm{d}h(x)}{\mathrm{d}x} > 0$ and $\frac{\mathrm{d}x^2 h(x)}{\mathrm{d}xx^2} > 0$, and then $\frac{\mathrm{d}xH(x)}{\mathrm{d}xx} > 0$.

4. Equilibrium strategy of PCG-Buyback

In this section, we construct a three-level Stackelberg game model, in which the bank is the leader (L), then the manufacturer is the sub-leader (SL), and the retailer is the follower (F). First, the leader announces an interest rate of r^P under the guarantee from the manufacturer. Then the sub-leader decides the wholesale price w^P. Finally, as the follower, the retailer decides the order quantity q^P in response to the manufacturer's wholesale price and bank's loan rate. The Stackelberg game is expressed as follows:

$$\begin{cases} (L)\max_{r^P} \pi_B\left(r^P; \lambda, e, q^P, w^P\right) \\ (\mathrm{SL})\max_{w^P} \pi_M^P\left(w^P; \lambda, e, q^P, r^P\right) \\ (F)\max_{q^P} \pi_R^P\left(q^P; w^P, r^P, \lambda, e\right). \end{cases} \tag{1}$$

Then, we solve it *via* backward induction to determine the equilibrium of the above Stackelberg game model.

4.1. Retailer's optimal strategy under PCG-Buyback

At Time $T = 0$, the retailer decides the order quantity q^P according to the wholesale price w^P and borrows a loan $(w^P q^P - B)$ from the bank at the interest rate r. At Time $T = 1$, the retailer yields a revenue $p\mathbf{E}\min(x, q^P)$ from the random demand market and gets buyback funds $e\mathbf{E}(q^P - x)^+$ from the manufacturer if there are unsold products. The retailer's decision problem can be expressed as follows:

$$\operatorname*{Max}_{q^P} \pi_R^P\big(q^P; w^P, r, e\big) = \mathbf{E}\left[\underbrace{p\min\big(x, q^P\big)}_{\text{Sales revenue}} + \underbrace{e\big(q^P - x\big)^+}_{\text{Buyback revenue}} - \underbrace{\big(w^P q^P - B\big)\big(1 + r^P\big)}_{\text{Loan principal and interest}}\right]^+ - B, \tag{2}$$

where $y^+ = \max(y, 0)$. It means that if the retailer's sales revenue and buyback income are insufficient to repay the loan and interest, bankruptcy will happen, and the expected return is zero.

For the retailer, the higher the bank's financing interest rate, the higher the loan cost. When the interest rate exceeds an upper threshold, the retailer will use the own capital to avoid too high financing costs. We give the following lemma.

Lemma 1. *The feasible range of the interest rate for retailers to accept bank financing is* $[0, \hat{r}]$*, where* $\hat{r} = \frac{(p-e)\bar{F}(\frac{B}{w^P})+e-w^P}{w^P}$.

The proof of Lemma 1 is in Appendix A.

The manufacturer has an upper threshold for the credit guarantee coefficient agreed with the bank. If the credit guarantee coefficient surpasses this threshold, the manufacturer's profits will be affected and they will not provide a guarantee for the retailer. Therefore, we present the following lemma.

Lemma 2. *The credit guarantee coefficient agreed by the manufacturer and the bank should satisfy* $\lambda(1+r^P) < 1$.

Lemma 2 demonstrates that if a retailer goes bankrupt, the manufacturer cannot provide the bank with more compensation than the uncollected loan, meaning the bank should not gain any extra advantage from the manufacturer's excessive allocation of bank losses. Given the unpredictability of market demand, the retailer may not be able to pay back the loans and become insolvent if the actual retail sales are low. Let k denote the minimum market demand threshold, which is specified in Lemma 3.

Lemma 3. *When the market demand is not lower than the demand threshold* k*, the retailer will not go bankrupt,* $k \le q^P$ *and* $k = \frac{(w^P q^P - B)\left(1+r^P\right)-eq^P}{p-e}$.

The proof of Lemma 3 is in Appendix B.

Lemma 3 indicates that if the market demand is too low, the retailer's sales revenue will be insufficient to cover the loan and interest, and then lead to bankruptcy. This bankruptcy risk will transfer to the manufacturer through PCG contract.

The retailer's optimal order quantity under PCG-buyback is given by Proposition 1.

Proposition 1. *Given the wholesale price* w^P *and the interest rate* r*, the retailer's optimal order quantity* q^{P^*} *satisfies* $\bar{F}(q^{P^*}) = \Omega\bar{F}(k(q^{P^*}))$*, where* $\Omega = \frac{w^P(1+r^P)-e}{p-e}$.

The proof of Proposition 1 is in Appendix C.

Proposition 1 indicates that the guarantee coefficient does not directly affect the retailer's order quantity. In fact, the guarantee does not directly change the behavior of the borrower because it is a contract between the lender and the guarantor and therefore should not affect the borrower directly but through the behavior of the lender or the guarantor [3].

The retailer's optimal order quantity q^{P^*} is directly affected by wholesale price w^P, interest rate r, initial capital B and buyback price e. We deduce the influence of these factors on the optimal order quantity and give the following corollary.

Corollary 1. *Based on Proposition 1, we have*

(i) *given the interest rate* r*, the optimal order quantity* q^{P^*} *decreases in the wholesale price* w^P*, i.e.,* $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} < 0$*;*

(ii) *given the wholesale price* w^P, *the optimal order quantity* q^{P^*} *decreases in the bank interest rate* r, *i.e.,* $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r^P} < 0$;

(iii) *given the interest rate* r *and the wholesale price* w^P, *the optimal order quantity* q^{P^*} *decreases in the initial capital* B, *i.e.,* $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}B} < 0$.

The proof of Corollary 1 is in Appendix D.

Corollary 1 suggests that when the manufacturer's wholesale price rises, the retailer's purchase cost per unit product will also increase, leading to a reduction in the order quantity to mitigate the risk of demand uncertainty. Additionally, when the bank raises the interest rate, the retailer's loan cost will increase, prompting a reduction in the order quantity and loan size to avoid being unable to repay the loan. Furthermore, if the retailer's initial capital is relatively high, the retailer will adopt a more conservative ordering strategy to protect the initial capital. Conversely, if the retailer's initial capital is relatively low, the retailer will take a more aggressive ordering strategy.

4.2. Manufacturer's optimal strategy under PCG-Buyback

At Time $T = 0$, the manufacturer sells the goods at the wholesale price w^P and expends cq^{P^*} on production. At Time $T = 1$, the manufacturer will buy back all the remaining products at the price e. If the retailer goes bankrupt, the manufacturer must undertake some proportion of the remaining debt as $\lambda\mathbf{E}[(w^P q^{P^*} - B)(1 + r^P) - p\min(q^{P^*}, x) - e(q^{P^*} - x)^+]^+$. The manufacturer's decision problem can be formulated as:

$$\underset{w^P}{\mathrm{Max}}\, \pi_M^P(w^P; \lambda, q^{P^*}, r, e) = \underbrace{(w^P - c)q^{P^*}}_{\text{Operating profit}} - \underbrace{e\mathbf{E}\Big[q^{P^*} - x\Big]^+}_{\text{Buyback expenses}} \\ - \underbrace{\lambda\mathbf{E}\Big[\Big(w^P q^{P^*} - B\Big)\big(1 + r^P\big) - p\min\Big(q^{P^*}, x\Big) - e\Big(q^{P^*} - x\Big)\Big]^+}_{\text{Bankruptcy indemnity}}. \tag{3}$$

Proposition 2. *Given the optimal order quantity* q^{P^*} *and the interest rate* r, *then the optimal wholesale price under the credit guarantee coefficient* λ *is* $w^{P^*} = \frac{\xi(w^{P^*})}{\mu(w^{P^*})\delta(w^{P^*})} + \frac{e}{1+r^P}$, *where* $\delta(w^{P^*}) = \frac{1-H(q^{P^*})}{1-\Omega q^{P^*}h(k(q^{P^*}))}$, $\mu(w^{P^*}) = 1 - \lambda F(k(q^{P^*}))(1 + r^P)$ *and* $\xi(w^{P^*}) = c + eF(q^{P^*}) - \frac{e}{1+r^P}$.

The proof of Proposition 2 is in Appendix E.

Proposition 2 illustrates that the manufacturer's pricing decision in PCG-buyback is more complex than that of the traditional supply chain. The optimal wholesale price w^P is related not only to order quantity, production cost, and buyback price but also to the bank interest rate and the credit guarantee coefficient.

Corollary 2. *Given the optimal order quantity* q^{P^*} *and the interest rate* r, *the manufacturer's optimal wholesale price increases in the credit guarantee coefficient, i.e.,* $\frac{\mathrm{d}w^{P^*}}{\mathrm{d}\lambda} > 0$.

The proof of Corollary 2 is in Appendix F.

Corollary 2 indicates that the larger the λ is, the higher bankruptcy risk will be shared by the manufacturer for the bank. To make up for the loss caused by the risk-sharing, the manufacturer will set a higher wholesale price. According to Honohan [21], a credit guarantee program can be likened to selling a put option on the financed item. The fair price of this option rises in tandem with the riskiness and maturity of the loan. Consequently, when the bank stipulates a higher guarantee coefficient, it provides the supplier with an incentive to elevate the wholesale price, thereby boosting their profits [26].

Corollary 3. *Given the order quantity* q^{P^*} *and interest rate* r, *the manufacturer's optimal expect profit decreases in the credit guarantee coefficient and the buyback price, i.e.,* $\frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}\lambda} < 0$ *and* $\frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}e} < 0$.

The proof of Corollary 3 is in Appendix G.

Corollary 3 indicates that although PCG contract and buyback contract can share bankruptcy risk of the retailer for the bank, they are all at the cost of the manufacturer's profit. The higher the credit guarantee coefficient and the buyback price, the smaller the manufacturer's optimal profit.

4.3. Bank's optimal strategy under PCG-Buyback

In this paper, similar to Chu *et al.* [15] and Bachas *et al.* [3], we develop a framework for endogenous financing decisions in which bank wishes to maximize capital effects based on their own utilities and price loans in a monopolistic rather than a perfectly competitive market under credit guarantee.

At Time $T=0$, the bank lends a loan $(w^{P^*}q^{P^*}-B)$ to the retailer. At Time $T=1$, if the retailer does not go bankrupt, the bank takes back the principal and interest $(w^{P^*}q^{P^*}-B)(1+r^P)$. If the retailer goes bankrupt, the bank will get some proportion of the remaining debt as $\lambda\mathbf{E}[(w^Pq^{P^*}-B)(1+r^P)-p\min(q^{P^*},x)-e(q^{P^*}-x)^+]^+$ from the manufacturer. The bank's decision problem can be formulated as:

$$\underset{r}{\operatorname{Max}}\ \pi_B^P\Big(r;\lambda,q^{P^*},w^{P^*},e\Big)=\mathbf{E}\min\Bigg[\underbrace{(1-\lambda)\Big[p\min\Big(q^{P^*},x\Big)+e\Big(q^{P^*}-x\Big)^+\Big]+\lambda\Big(w^{P^*}q^{P^*}-B\Big)\big(1+r^P\big)}_{\text{Retailer goes bankrupt}},$$

$$\underbrace{\Big(w^{P^*}q^{P^*}-B\Big)\big(1+r^P\big)}_{\text{Retailer dosen't go bankrupt}}\Bigg]-\underbrace{\Big(w^{P^*}q^{P^*}-B\Big)}_{\text{Loan principal}}. \tag{4}$$

Proposition 3. *Given the credit guarantee coefficient λ, the optimal quantity q^{P^*} and the optimal wholesale price w^*, the bank's optimal interest rate is* $r^{P^*}=\frac{c+eF(q^{P^*})-eF(k)}{[\lambda F(k)+\bar{F}(k)][c+eF(q^{P^*})]-\lambda eF(k)}-1.$

The proof of Proposition 3 is in Appendix H.

Note that when $\lambda=1$, $r^{P^*}=0$. In other words, when the manufacturer fully guarantees the retailer, the bank will finance at the risk-free interest rate (since we assume that the risk-free interest rate $r_f=0$).

4.4. Coordination analysis of PCG-Buyback

In this subsection, we explore the use of both PCG and buyback contracts to facilitate more flexible coordination of the supply chain. The supply chain literature is particularly concerned about "coordinating" contracts that achieve "first best" for the decentralized system [23]. To this end, we consider the supply chain as a centralized system with no capital constraints or buybacks. Let q_C denote the centralized order quantity and π_C the centralized total profit, then the profit function of the centralized system can be expressed as follows:

$$\pi_C=p\mathbf{E}\min(x,q_C)-cq_C. \tag{5}$$

Equation (5) can be written as $\pi_C=p\int_0^{q_C}\bar{F}(x)\,\mathrm{d}x-cq_C$. Taking the first derivative of π_C with respect to q_C yields $\frac{\mathrm{d}\pi_C}{\mathrm{d}q_C}=p\bar{F}(q_C)-c$. Moreover, $\frac{\mathrm{d}^2\pi_C}{\mathrm{d}q_C^2}=-pf(q_C)<0$, so the optimal order quantity q_C^* satisfies $\bar{F}(q_C^*)=\frac{c}{p}$, which is only related to production cost and market price.

If a contract can realize the coordination of a supply chain, the mechanism must ensure that the profit of a member is not lower than the profit level in the centralized system [7,23].

Definition 1. Let $\psi=\{(\lambda,e)|\pi_{\mathrm{SC}}(\theta^*(\lambda,e))=\pi_c(q_C^*)\}$, where

$$\theta^*(\lambda,e)=\Big(q^{P^*}(\lambda,e),w^{P^*}(\lambda,e),r^{P^*}(\lambda,e)\Big)^\top,$$

$$\pi_{\mathrm{SC}}\left(\theta^*(\lambda, e)\right)=\pi_R^P\left(\theta^*(\lambda, e)\right)+\pi_M^P\left(\theta^*(\lambda, e)\right)+\pi_B^P\left(\theta^*(\lambda, e)\right).$$

In our paper, the contract agreed by supply chain members is said to coordinate the supply chain if the parameters pair $(\lambda, e) \in \psi$. We refer to ψ as a Pareto-optimal frontier.

When the optimal decision of an individual in a decentralized system is the same as that in a centralized system, *i.e.*, $q^{P^*}=q_C^*$, the contract can coordinate the supply chain [7]. Propositions 1 and 2 demonstrate that q^{P^*} is related to the contract parameters λ and e. By comparing q^{P^*} with q_C^*, we can determine the coordination condition, *i.e.*, the Pareto coordination region formed by λ and e as shown in Proposition 4.

Proposition 4. *There is a Pareto region composed of* $(\hat{\lambda}, \hat{e})$ *to coordinate SCF system, and (a) when the credit guarantee coefficient* $\lambda=\widehat{\lambda}$, $q^{P^*}=q_C^*$; *(b) when* $\lambda>\widehat{\lambda}$, $q^{P^*}<q_C^*$; *(c) when* $\lambda<\widehat{\lambda}$, $q^{P^*}>q_C^*$, *where* $\hat{\lambda}=\frac{1-(1+r^{P^*})\bar{F}(k)}{\delta(w^{P^*})(1+r^{P^*})F(k)}-\frac{p\hat{e}r^{P^*}\bar{F}(k)}{c(p-\hat{e})\delta(w^{P^*})(1+r^{P^*})F(k)}$.

The proof of Proposition 4 is in Appendix I.

From Proposition 4, we can find that the manufacturer can jointly coordinate the supply chain by adjusting the contract parameters λ and e. Compared with the case where only the PCG contract is used (*i.e.*, $e=0$), supply chain coordination can be achieved with the deployment of the buyback contract by setting only a smaller λ, which, in turn, suggests that the buyback contract co-bears the risk that the PCG contract needs to bear.

Our coordination mechanism implies that the manufacturer can adjust its risk allocation to the retailer through the buyback contract and to the bank through PCG contract to help the supply chain better enhance its utility, which is more flexible than traditional single contracts such as Yan *et al.* [40] and Bi *et al.* [5].

5. Equilibrium strategy of TC-Buyback

The retailer may not be able to obtain financing from the bank through the manufacturer providing PCG. On the one hand, from Lemma 1, if the bank's financing interest rate is too high (financing is expensive), the retailer may reduce the order and use the initial funds. On the other hand, from Corollary 3, if the credit guarantee coefficient set by the bank is too high, it will damage the interests of the manufacturer, and it will not provide PCG for the retailer. In such cases, some manufacturers with financial strength prefer to finance for retailers themselves, *i.e.*, providing TC financing.

In this section, we analyze the best decisions for each participant in the supply chain under TC-Buyback by constructing a two-level Stackelberg game model, with the manufacturer as the leader (L) and the retailer as the follower (F).

First, the manufacturer announces the financing interest rate r^T and decides the wholesale price w^T, and then the retailer decides the order quantity q^T in response to the manufacturer's wholesale price. The Stackelberg game is expressed as follows:

$$\left\{\begin{array}{l}(L) \max_{w^T} \pi_M^T\left(w^T ; e, q^T, r^T\right) \\ (F) \max_{q^T} \pi_R^T\left(q^T ; w^T, r^T, e\right).\end{array}\right. \tag{6}$$

5.1. Retailer's optimal strategy under TC-Buyback

As a follower, the retailer's decision problem under TC-Buyback is similar to PCG-Buyback, *i.e.*,

$$\underset{q^T}{\operatorname{Max}}\, \pi_R^T\left(q^T ; w^T, e\right)=\mathbf{E}\left[\underbrace{p \min\left(x, q^T\right)}_{\text{Sales revenue}}+\underbrace{e\left(q^T-x\right)^{+}}_{\text{Buyback revenue}}-\underbrace{\left(w^T q^T-B\right)\left(1+r^T\right)}_{\text{Loan principal and interest}}\right]^{+}-B. \tag{7}$$

Similarly, let k^T denote the minimum market demand threshold under TC-Buyback when the retailer does not experience bankruptcy, which is specified in Lemma 4.

Lemma 4. *Under TC-Buyback, when the market demand is not lower than the demand threshold k^T, the retailer will not go bankrupt, $k^T \leq q^T$ and $k^T = \frac{(w^T q^T - B)(1+r^T) - eq^T}{p-e}$.*

We can get the optimal order quantity from the following proposition.

Proposition 5. *Given the wholesale price w^T and the financing interest rate r^T, the retailer's optimal order quantity q^{T^*} under TC-Buyback satisfies $\bar{F}(q^{T^*}) = \Omega \bar{F}(k(q^{T^*}))$, where $\Omega = \frac{w^T(1+r^T)-e}{p-e}$.*

The proof of Proposition 5 is the same as Proposition 1.

In TC-Buyback, the retailer's optimal order quantity is affected by the wholesale price, financing interest rate, buyback price, and the retailer's initial capital level as well, and the effect is similar to that under PCG-Buyback.

Corollary 4. *Based on Proposition 5, we have*

(i) *given the interest rate r^T, the optimal order quantity decreases in the wholesale price w^T, i.e., $\frac{\mathrm{d}q^{T*}}{\mathrm{d}w^T} < 0$;*
(ii) *given the wholesale price w^T, the optimal order quantity q^{T^*} decreases in the interest rate r^T, i.e., $\frac{\mathrm{d}q^{T*}}{\mathrm{d}r^T} < 0$;*
(iii) *given the interest rate r^T and the wholesale price w^T, the optimal order quantity q^{T^*} decreases in the initial capital B, i.e., $\frac{\mathrm{d}q^{T*}}{\mathrm{d}B} < 0$.*

The proof of Corollary 4 is the same as Corollary 1.

5.2. Manufacturer's optimal strategy under TC-Buyback

At Time $T = 0$, the manufacturer receives the retailer's initial capital B and expends cq^{T^*} on production. At Time $T = 1$, the manufacturer buys back unsold products at $e\mathbf{E}(q^{T^*} - x)^+$. The manufacturer will receive $(w^T q^{T^*} - B)(1 + r^T)$ if the retailer does not go bankrupt. Otherwise, he can only receive $\mathbf{E}[p\min(q^{T^*}, x) + e(q^{T^*} - x)^+]$. The manufacturer's decision problem under TC-Buyback can be expressed as follows:

$$\begin{aligned} \operatorname*{Max}_{w^T} \pi_M^T\left(w^T; q^{T^*}, e\right) = B + \mathbf{E}\min&\left[\underbrace{p\min\left(x, q^{T^*}\right) + e\left(q^{T^*} - x\right)^+}_{\text{Retailer gose bankrupt}}, \underbrace{\left(w^T q^{T^*} - B\right)\left(1 + r^T\right)}_{\text{Retailer doesn't go bankrupt}}\right] \\ &- \underbrace{e\mathbf{E}\left(q^{T^*} - x\right)^+}_{\text{Buybackexpenses}} - \underbrace{cq^{T^*}}_{\text{Productioncost}}. \end{aligned} \tag{8}$$

Proposition 6. *Given the order quantity q^{T^*} and financing interest rate r^T, the optimal wholesale price of the manufacturer under TC-Buyback satisfies $w^{T^*} = \frac{1}{1+r^T} \frac{c + eF(q^{T^*}) - e}{\delta(w^{T^*})\bar{F}(k(q^{T^*}))} + \frac{e}{1+r^T}$, where $\delta(w^{T^*}) = \frac{1 - H(q^{T^*})}{1 - \Omega q^{T^*} h(k(q^{T^*}))}$.*

Proof of Proposition 6 is in Appendix J.

Under TC-Buyback, the retailer's bankruptcy risk can also be reduced by the buyback contract. However, unlike PCG-Buyback, the manufacturer acts as a lender in TC-Buyback, reducing the retailer's bankruptcy risk would advantage the manufacturer even if he needs to pay the buyback cost. We give Corollary 5 as follows.

Corollary 5. *Under TC-Buyback, given the retailer's order quantity q^{T^*} and the manufacturer's interest rate r^T, the optimal expected profit of manufacturer increases in buyback price, i.e., $\frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}e} > 0$.*

Proof of Corollary 5 is in Appendix K.

Corollary 5 illustrates that under TC-Buyback, the manufacturer acts as both seller and lender, and the increased sales and reduced loan risk from the higher buyback price can cover its buyback cost, thus increasing the manufacturer's revenue.

5.3. Comparison between PCG-Buyback and TC-Buyback

Next, we focus on the circumstances under which the manufacturer is willing to provide PCG-Buyback or TC-Buyback to the retailer. Comparing $\pi_M^{P^*}$ and $\pi_M^{T^*}$, we have the following proposition.

Proposition 7. *For the manufacturer, there exists a guarantee coefficient threshold $\bar{\lambda}_M$ and a buyback price threshold $\bar{e}_M$ so that*

(i) *when $\lambda > \bar{\lambda}_M$, $\pi_M^{P^*} < \pi_M^{T^*}$, the manufacturer chooses TC-buyback, otherwise chooses PCG-Buyback;*
(ii) *when $e > \bar{e}_M$, $\pi_M^{P^*} < \pi_M^{T^*}$, the manufacturer chooses TC-buyback, otherwise chooses PCG-Buyback,*

where $\bar{\lambda}_M = \frac{(w^{P^*}-c)q^{P^*}-(w^{T^*}(1+r^T)-c)q^{T^*}+e\int_0^{q^{T^*}-q^{P^*}} F(x)\,\mathrm{d}x+(p-e)\int_0^{k(q^{T^*})} F(x)\,\mathrm{d}x+Br^T}{(p-e)\int_0^{k(q^{P^*})} F(x)\,\mathrm{d}x}$ *and* $\bar{e}_M = \frac{(w^{P^*}-c)q^{P^*}-(w^{T^*}(1+r^T)-c)q^{T^*}-\lambda p\int_0^{k(q^{P^*})} F(x)\,\mathrm{d}x+p\int_0^{k(q^{T^*})} F(x)\,\mathrm{d}x+Br^T}{\int_0^{q^{P^*}} F(x)\,\mathrm{d}x-\lambda\int_0^{k(q^{P^*})} F(x)\,\mathrm{d}x-\int_{k(q^{T^*})}^{q^{T^*}} F(x)\,\mathrm{d}x}$.

The Proof of Proposition 7 is in Appendix L.

Proposition 7 indicates that the manufacturer prefers to finance the retailer itself when the bank requires a higher guarantee factor or when the buyback price is higher. This is because when the guarantee coefficient is larger, the more the manufacturer has to pay out to the bank in the event of the retailer's bankruptcy, and when the guarantee coefficient exceeds a certain threshold, the manufacturer prefers to finance the retailer itself. Additionally, when the buyback price is higher, the manufacturer, as the lender and seller, gains more by reducing the probability of retailer bankruptcy under TC-Buyback than under PCG-Buyback.

The findings provide important suggestions for manufacturers when choosing which type of financing to provide to a retailer. Relatedly, Wu *et al.* [37] also provides assistance on the choice of financing method under a buyback contract, although the subject of their financing is the upstream supplier, and thus draws some different conclusions from ours. Nonetheless, our study still has practical applications and helps to make more informed decisions in the supply chain field.

6. Numerical experiments

In this section, we conduct some numerical experiments to demonstrate the results derived in the previous sections, which are divided into two main parts. Firstly, we analyze the coordination under PCG-Buyback and give the coordination region consisting of contract parameters λ and e. Secondly, we compare PCG-Buyback and TC-Buyback from the perspective of the manufacturer's profit maximization and identify the selection regions for the two financing schemes. For initialization, we let $p = 1$, $c = 0.65$ and x be uniformly distributed over an interval $[0, 100]$.

6.1. Numerical performance of PCG-Buyback

This subsection describes the influence of credit guarantee coefficient λ, buyback price e and initial budget B on supply chain coordination. Given the bank interest rate $r^P = 10\%$, credit guarantee coefficient $\lambda \in [0, 0.8]$, buyback price $e = 0$, 0.2, 0.3 and initial budget $B = 1$, Figure 3 depicts the trend of optimal order quantity (Fig. 3a) and total profits (Fig. 3b) based on different λ.

From Figure 3a, when $\lambda > \hat{\lambda}$, the optimal order quantity of a decentralized SCF system is lower than that of a centralized system, and when $\lambda < \hat{\lambda}$, the opposite is true, when $\lambda = \hat{\lambda}$, they keep the same value. From Figure 3b, when $\lambda = \hat{\lambda}$, the total profit of supply chain of decentralized SCF system is equal to that of centralized system,which is consistent with our conclusion in Proposition 4. In other words, when $\lambda = \hat{\lambda}$, $q^{P^*} = q_C^*$ and $\pi_{\mathrm{SC}}(q^{P^*}) = \pi_C(q_C^*)$, the supply chain achieves coordination. In addition, after the buyback contract is deployed ($e = 0.2$, 0.3), $\hat{\lambda}$ moves to the left compared without buyback contract ($e = 0$), which means the manufacturer only needs to share less risk (a smaller λ) for the bank to achieve supply chain coordination.

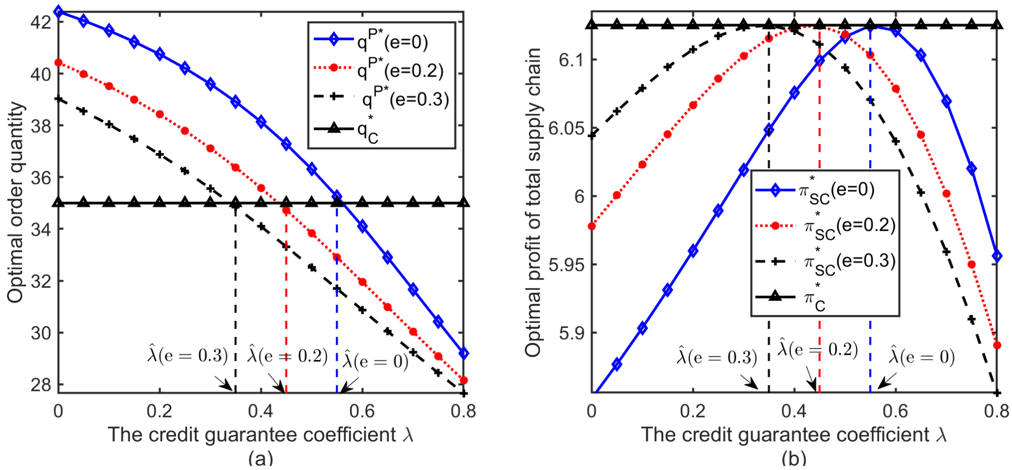

FIGURE 3. Optimal order quantity and profits under different values of λ.

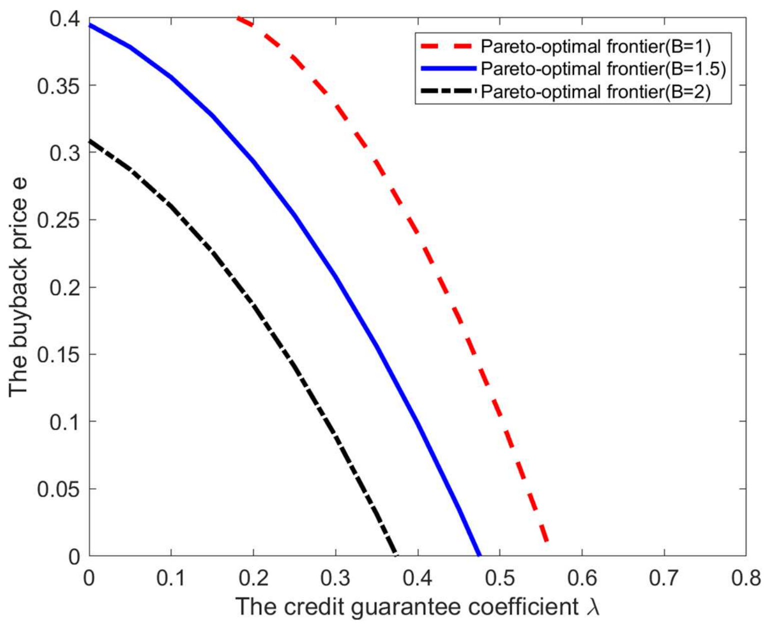

FIGURE 4. Pareto-optimal frontiers with different values of λ and e.

Figure 4 describes the changing trend of (λ, e) under the coordination more intuitively, in which we capture the Pareto-optimal frontiers with λ on the vertical axis and e on the horizontal axis under different B. Each point on the Pareto-optimal frontiers satisfies $\pi^*_{\mathrm{SC}} = \pi^*_C$.

As illustrated in Figure 4, an increase in λ allows for coordination of the supply chain with a smaller e, while a decrease in λ requires a larger e to achieve coordination. Additionally, the Pareto-optimal frontier is seen to move in response to changes in initial capital B. A rise in B causes the frontier to move downward and to the left, while a decrease in B causes the frontier to move upwards and to the right. The above results can be explained in three ways. Firstly, a larger λ implies a higher bankruptcy risk for the retailer, which is shared by the

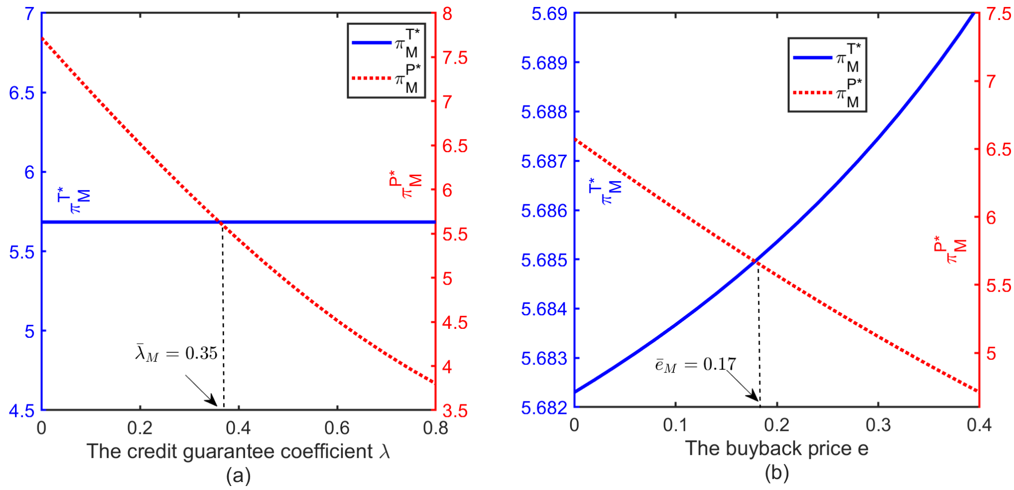

FIGURE 5. Manufacturer's optimal profit under PCG-Buyback and TC-Buyback with different values of λ and e.

manufacturer through the PCG contract. To reduce the risk shared through the buyback contract and achieve supply chain coordination, the manufacturer can lower the buyback price e. Secondly, when the manufacturer increases the buyback price e, the retailer's end-of-period payback improves and the bankruptcy risk decreases, as well as the risk transferred to the bank. Thus, a smaller λ can enable supply chain coordination. Finally, if the retailer is well-capitalized and does not require high financing needs, the bankruptcy risk is reduced, and the risk shared between the bank and the manufacturer is also reduced. Consequently, the bank does not need to share a greater risk with the manufacturer, and the manufacturer does not need to increase the buyback price to guarantee the retailer's end-of-period payback. Therefore, smaller λ and e can enable supply chain coordination. Otherwise, we need bigger λ and e to achieve supply chain coordination.

6.2. Performance profiles between PCG-Buyback and TC-Buyback

In this subsection, we compare PCG-Buyback and TC-Buyback in terms of profit maximization for the manufacturer. We highlight the impact of the contract parameters λ and e on selecting the financing schemes and therefore assume that the bank rate r in PCG-Buyback and the manufacturer rate r^T in TC-Buyback are the same value.

Given $r^P = r^T = 10\%$, $\lambda \in [0, 0.8]$, $e \in [0, 0.4]$, $B = 1$, Figure 5 depicts the variation of the manufacturer's optimal profit with different values of λ (Fig. 5a) and e (Fig. 5b) for the two financing schemes.

In Figure 5, the red dashed line represents $\pi_M^{P^*}$ under PCG-Buyback and the blue line represents the $\pi_M^{T^*}$ under TC-Buyback. Figure 5 shows that the manufacturer's profit under TC-Buyback is greater than that under PCG-Buyback when the credit guarantee coefficient λ and buyback price e exceed the thresholds $\bar{\lambda}_M$ and $\bar{e}_M$, respectively. Explanations are as follows.

Firstly, Corollary 3 reveals that the manufacturer's optimal profit is inversely proportional to the credit guarantee coefficient λ. Thus, when λ increases, the manufacturer will have to pay out more money if the retailer goes bankrupt, resulting in a decrease in their optimal profit. If λ is greater than the threshold $\bar{\lambda}_M$, the manufacturer's profit in PCG-Buyback will be lower than in TC-Buyback, as illustrated in Figure 5a. Therefore, a manufacturer with sufficient financial resources may choose to forgo providing a credit guarantee for the retailer and opt for TC-Buyback instead.

Secondly, under PCG-Buyback, although the manufacturer's increase in the buyback price reduces the retailer's bankruptcy risk, its optimal profit is reduced by the higher buyback price (Cor. 3). However, under

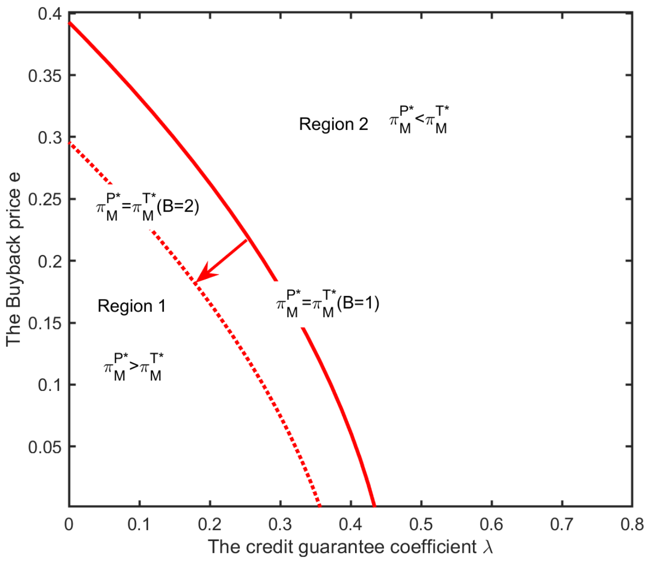

FIGURE 6. Selection regions of the financing schemes with different values of λ and e.

TC-Buyback, since the manufacturer acts as a lender and a seller, thus receiving both the sales revenue and the financing proceeds. This makes it more advantageous for the manufacturer to reduce the retailer's bankruptcy risk by increasing the buyback price. According to Corollary 5, the higher the buyback price, the higher the manufacturer's optimal profit, and therefore the more willing the manufacturer is to choose TC-Buyback.

6.3. Financing selection

Subsequent numerical experiments give the financing selection regions consisting of the credit guarantee coefficient and the buyback price as well as the credit guarantee coefficient and the financing rate.

By substituting different values of λ and e, we calculate the optimal profits of the manufacturer under two financing schemes with $r^P = r^T = 10\%$ and $B = 1$, 2. The comparison of the profits is illustrated in Figure 6, which shows two distinct regions.

When (λ, e) lies within Region 1 of Figure 6, $\pi_M^{P^*} > \pi_M^{T^*}$, the manufacturer will opt for PCG-Buyback over TC-Buyback. This is because both the credit guarantee coefficient λ and the buyback price e are lower in Region 1, so the manufacturer does not need to compensate the bank much for the remaining loan and the retailer does not need to bear much of the order cost transferred from the manufacturer.

On the other hand, when (λ, e) lies within Region 2 of Figure 6, $\pi_M^{P^*} < \pi_M^{T^*}$, the manufacturer will choose TC-Buyback instead of PCG-Buyback. This is because λ and e are higher in Region 2, resulting in an increased guaranteed expense for the manufacturer when selecting PCG-Buyback. Therefore, it is relatively better for the manufacturer to choose TC-Buyback in Region 2.

Finally, as the retailer's initial capital level increases from $B = 1$ to $B = 2$, Figure 6 illustrates how the manufacturer's profit equivalence curves shift. This shift to the lower left indicates that Region 1 decreases while Region 2 expands, making TC-Buyback the more likely choice for manufacturers. This is because the higher initial capital of the retailer reduces the risk of bankruptcy, allowing manufacturers with the financial capacity to provide TC-Buyback to take less risk and receive higher returns than those who offer PCG-Buyback, thus resulting in a decrease in Region 1.

Similarly, given $r^P = r^T \in [0, 0.1]$, $\lambda \in [0, 0.8]$, $B = 3$ and $e = 0.2$, 0.3, Figure 7 gives the manufacturer financing selection regions consisting of λ and r (or r^T).

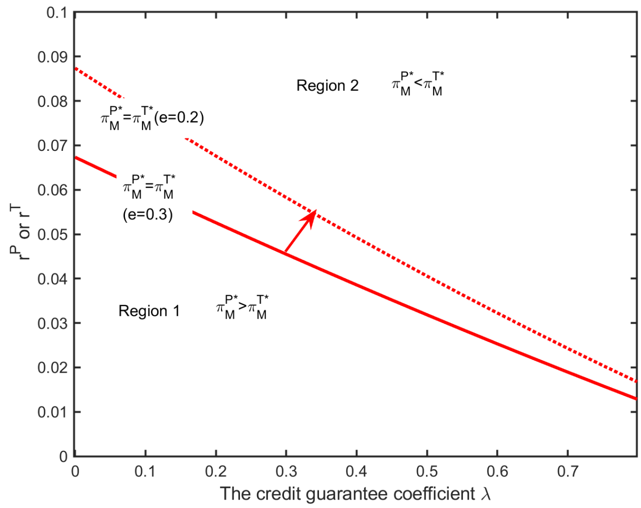

FIGURE 7. Selection regions of the financing schemes with different values of λ and r^P or r^T.

When (λ, r) lies within Region 1 of Figure 7, the manufacturer benefits from this choice as the lower bank rate reduces the risk transferred from the retailer through the PCG contract, thus increasing their profit. However, if the manufacturer sets a financing rate as low as the bank rate under TC-Buyback, the additional financing benefit obtained is not particularly high. When (λ, r) lies within Region 2 of Figure 7, the manufacturer can gain a higher financing return if they set a relatively high financing rate.

Figure 7 also demonstrates that as the manufacturer's buyback price decreases from $e = 0.3$ to $e = 0.2$, the manufacturer's profit equivalence curves move to the upper right, indicating that the manufacturer is more likely to select PCG-Buyback. This is because the lower the buyback price, the higher the profit for the manufacturer under PCG-Buyback, even surpassing that of TC-Buyback. These results confirm our observations in Figure 5a.

7. Conclusion

In this paper, two financing models, PCG-Buyback and TC-Buyback, are designed for supply chains with high residual value of unsold products and financial constraints, and the main findings are as follows.

First, our study introduces a dual contract featuring both PCG and buyback provisions to effectively coordinate the capital-constrained supply chain. Within this framework, we identify a Pareto coordination frontier defined by the credit guarantee coefficient and the buyback price. This frontier highlights a key finding: supply chain managers can streamline coordination by modifying only one contract parameter, causing it to move in the opposite direction from the other, thus simplifying the coordination process.

Second, the buyback contract serves as a risk mitigation tool for the financially constrained retailer, effectively reducing the likelihood of bankruptcy. However, our research highlights a critical distinction for the manufacturer, hinging on its specific roles within the supply chain. Unlike its position as a seller in the context of PCG-Buyback, under TC-Buyback, the manufacturer takes on a dual role as both the seller and financier. This dynamic results in manufacturer benefiting from both sales and financing revenues, and the inclusion of the buyback contract enhances optimal profit.

Finally, we conduct a comparative analysis and delineate the preferred regions for the two financing schemes. Our findings lead to a practical recommendation: when the bank stipulates a lower credit guarantee coefficient or offers a reduced financing rate, manufacturers have the flexibility to either consider providing a guarantee for the retailer or offer self-financing to the retailer. This strategic flexibility enables the manufacturer to make informed decisions.

This paper has several managerial implications. First, this paper highlights the role of PCG and buyback contract in the supply chain. We argue that the potential of both contracts goes beyond what has been discussed in supply chain operations. For example, in this paper, we effectively combine PCG and buyback together to help SMEs finance and reduce lending risk. Second, the manager can use different risk-sharing mechanisms of PCG and buyback for more flexible supply chain coordination. This takeaway is an important one for policymakers facing funding constraints, providing a compensation strategy when a single coordination mechanism cannot coordinate the supply chain. Finally, we emphasize to manufacturers the need for a comprehensive perspective. They should consider not only how different contract parameters affect their own profitability but also the pivotal role they play when offering various forms of financial support to SMEs. This broader awareness empowers manufacturers to make more informed decisions in selecting the most fitting financing scheme for the given circumstances.

There are several potential directions for further research. First, this paper sets exogenous buyback price and guarantee coefficients, whose impact on the decision would be very interesting if we were their endogenous. Secondly, supply chain research under risk aversion has received a lot of attention in recent years, thus it is another research priority to study the decision and coordination of supply chain financing under risk aversion. Finally, while our study focuses on the role of PCG in mitigating downstream market risks as well as financing and coordination, we cannot ignore the important challenges that upstream yield risks pose to the production decisions of SMEs. Therefore, it is of great practical importance to extend the PCG program to upstream SMEs and to study in depth its impact on supply chain decision-making.

Appendix A. Proof of Lemma 1

Proof. Assuming that the retailer has unlimited liability to repay after bankruptcy, he will take a conservative order decision. The profit function of the retailer is

$$\begin{aligned}\pi_R^P &= \mathbf{E}\Big[p\min\big(x,q^P\big)+e\big(q^P-x\big)^+ - \big(w^Pq^P-B\big)\big(1+r^P\big)\Big]-B\\ &= p\int_0^{q^P}\bar F(x)\,\mathrm{d}x + e\int_0^{q^P}F(x)\,\mathrm{d}x - \big(w^Pq^P-B\big)\big(1+r^P\big)-B.\end{aligned}$$

It is easy to get that the optimal order quantity of retailer $q^{P^*}=\bar F^{-1}(\frac{w^P(1+r^P)-e}{p-e})$, if $\frac{B}{w}>q^{P^*}$, the retailer will choose to order with its own funds. In this case, $r>\frac{(p-e)\bar F(\frac{B}{w})+e-w}{w}=\hat r$. □

Appendix B. Proof of Lemma 3

Proof. If $x>q^P$, then $p\min\{x,q^P\}-(w^Pq^P-B)(1+r^P)\geq 0$, there is no bankruptcy occurring to the retailer. If $x<q^P$, when $px+e(q^P-x)-(w^Pq^P-B)(1+r^P)\geq 0$, *i.e.*, $x\geq\frac{(w^Pq^P-B)(1+r^P)-eq^P}{p-e}$, the retailer will not go bankrupt. Therefore, the demand threshold $k=\frac{(w^Pq^P-B)(1+r^P)-eq^P}{p-e}$ and $k<q^P$ since $w^P(1+r^P)<p$. □

Appendix C. Proof of Proposition 1

Proof. According to Lemma 3, the retailer's expected profit can be transformed as

$$\pi_R^P=(p-e)\int_k^{q^P}\bar F(x)\,\mathrm{d}x-B. \tag{C.1}$$

Taking the first-order and second-order derivative of π_R^P with respect to q^P, we can obtain that

$$\frac{\mathrm{d}\pi_R^P}{\mathrm{d}q^P}=\frac{\partial\pi_R^P}{\partial q^P}+\frac{\partial\pi_R^P}{\partial k(q^P)}\frac{\mathrm{d}k\big(q^P\big)}{\mathrm{d}q^P}=(p-e)\bar F\big(q^P\big)-\big(w\big(1+r^P\big)-e\big)\bar F\big(k\big(q^P\big)\big), \tag{C.2}$$

$$\begin{aligned}\frac{\mathrm{d}^2\pi_R^P}{\mathrm{d}q^{P^2}} &= -(p-e)f\left(q^P\right) + \left(w\left(1+r^P\right)-e\right)f\left(k\left(q^P\right)\right)\frac{w\left(1+r^P\right)-e}{p-e} \\ &= -(p-e)\bar{F}\left(q^P\right)h\left(q^P\right) + \left(w^P\left(1+r^P\right)-e\right)\Omega\bar{F}\left(k\left(q^P\right)\right)h\left(k\left(q^P\right)\right).\end{aligned} \tag{C.3}$$

Let $\frac{\mathrm{d}\pi_R^P}{\mathrm{d}q^P}=0$, then $(p-e)\bar{F}(q^P)=(w(1+r^P)-e)\bar{F}(k(q^P))$ and $\frac{\mathrm{d}^2\pi_R^P}{\mathrm{d}q^{P^2}}=-(p-e)\bar{F}(q^P)[h(q^P)-\Omega h(k(q^P))]$, where $\Omega=\frac{w(1+r^P)-e}{p-e}$. It is easy to get $k(q^P)\le q^P$, $h(q^P)\ge h(k(q^P))$ and $\Omega\le 1$, so $h(q^P)-\Omega h(k(q^P))\ge 0$, $\frac{\mathrm{d}^2\pi_b}{\mathrm{d}q^{P^2}}\le 0$. Therefore, the retailer's profit function is unimodal with respect to the order quantity q^P. Then, the first-order condition is $\frac{\mathrm{d}\pi_b}{\mathrm{d}q^P}=0$, the retailer's optimal order q^{P^*} satisfies that $\bar{F}(q^{P^*})=\Omega\bar{F}(k(q^{P^*}))$. Proposition 1 is proved. □

Appendix D. Proof of Corollary 1

Proof. **Case (i).** Proof of $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w}<0$.

Taking the derivative with respect to w^P on both sides of $\bar{F}\left(q^{P^*}\right)=\Omega\bar{F}\left(k\left(q^{P^*}\right)\right)$ in Proposition 1, we have

$$-f\left(q^{P^*}\right)\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}=\frac{\mathrm{d}\Omega}{\mathrm{d}w^P}\bar{F}\left(k\left(q^{P^*}\right)\right)-\Omega f\left(k\left(q^{P^*}\right)\right)\left(q^{P^*}\frac{\mathrm{d}\Omega}{\mathrm{d}w^P}+\Omega\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}\right), \tag{D.1}$$

which can be transformed into

$$\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}=\frac{\mathrm{d}\Omega}{\mathrm{d}w^P}\frac{\bar{F}\left(k\left(q^{P^*}\right)\right)\left(1-\Omega q^{P^*}h\left(k\left(q^{P^*}\right)\right)\right)}{\Omega^2 f(k(q^{P^*}))-f(q^{P^*})}=\frac{1+r^P}{w^P(1+r^P)-e}\frac{1-\Omega q^{P^*}h\left(k\left(q^{P^*}\right)\right)}{\Omega h(k(q^{P^*}))-h(q^{P^*})}. \tag{D.2}$$

Similar to the proof of Chen and Wang [13], we use the proof by contradiction to prove $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}<0$. Assume that $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}\ge 0$ for all $w^P\in[c,\frac{p}{1+r^P}]$ holds. By the Total Derivative Theorem, we obtain that

$$\frac{\mathrm{d}k\left(q^{P^*}\right)}{\mathrm{d}w^P}=\frac{q^{P^*}\left(1+r^P\right)}{p-e}+\frac{w^P\left(1+r^P\right)-e}{p-e}\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}=\frac{1+r^P}{p-e}\frac{\left(1-H\left(q^{P^*}\right)\right)}{\Omega h(k(q^{P^*}))-h(q^{P^*})}. \tag{D.3}$$

Therefore, if $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}\ge 0$, then $\frac{\mathrm{d}k(q^{P^*}(w^P))}{\mathrm{d}w^P}>0$ holds as well. Let $\frac{\mathrm{d}k(q^{P^*}(w^P))}{\mathrm{d}w^P}|_{w^P=w_0}=0$, then $1-H(q^{P^*}(w_0))=0$. For $c\le w^P\le\frac{p}{(1+r^P)}$, we classify w^P into three cases to discuss that $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}\ge 0$ is contradictory.

(1) If $w_0\ge\frac{p}{(1+r^P)}$, which means $w^P<w_0$, since $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}>0$ and the market demand satisfies IFR distribution, it can be seen that

$$H\left(q^{P^*}\left(w^P\right)\right)\le H\left(q^{P^*}\left(\frac{p}{(1+r^P)}\right)\right)\le H\left(q^{P^*}(w_0)\right)=1,$$

which means $1-\Omega q^{P^*}h(k(q^{P^*}))>1-H(q^{P^*})\ge 0$ holds for any w^P in this case. Then, due to $\Omega h(k(q^{P^*}))-h(q^{P^*})<0$, we have $\frac{\mathrm{d}k(q^{P^*}(w^P))}{\mathrm{d}w^P}\le 0$ and $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}<0$, which contradicts our assumption.

(2) If $w_0=c$, *i.e.*, $w^P\ge c$, because $k(q^{P^*})\le q^{P^*}$ and $\Omega\le 1$, we have

$$\Omega q^{P^*}(c)h\left(k\left(q^{P^*}(c)\right)\right)<q^{P^*}(c)h\left(q^{P^*}(c)\right)=H\left(q^{P^*}(w_0)\right)=1.$$

Thus, $\frac{\mathrm{d}q^{P^*}(w^P)}{\mathrm{d}w^P}|_{w^P=c}<0$, which is also contracted to the assumption.

(3) Otherwise, in case of $c<w_0\le\frac{p}{(1+r^P)}$, similarly we know that when $c\le w^P\le w_0$, $\frac{\mathrm{d}k(q^{P^*}(w^P))}{\mathrm{d}w^P}\le 0$ and $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}<0$ holds. Again, it is a contradiction.

In summary, it can be proved that in the three cases studied above, where $c \leq w^P \leq \frac{p}{(1+r^P)}$, the assumption $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} \geq 0$ is violated, *i.e.*, $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} < 0$ holds. This illustrates that q^{P^*} decreases in w^P.

Case (ii). Proof of $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r} < 0$.

Applying the Implicit Function Theorem and taking the derivative with respect to r on both sides of $\bar{F}(q^{P^*}) = \Omega\bar{F}(k(q^{P^*}))$ yields

$$-f\Big(q^{P^*}\Big)\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r} = \frac{\mathrm{d}\Omega}{\mathrm{d}r}\bar{F}\Big(k\Big(q^{P^*}\Big)\Big) - \Omega f\Big(k\Big(q^{P^*}\Big)\Big)\bigg(\frac{\partial k(q^{P^*})}{\partial r} + \Omega\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r}\bigg), \tag{D.4}$$

which can be transformed into

$$\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r} = \frac{w^P}{w^P(1+r^P)-e}\frac{1-\big(q^{P^*}-\frac{B}{w^P}\big)\Omega h\big(k\big(q^{P^*}\big)\big)}{\Omega h(k(q^{P^*}))-h(q^{P^*})}. \tag{D.5}$$

We can get $1-(q^{P^*}-\frac{B}{w^P})\Omega h(k(q^{P^*})) > 1-q^{P^*}\Omega h(k(q^{P^*})) > 0$ and $\Omega h(k(q^{P^*})) - h(q^{P^*}) < 0$ from the proof of case (i), so $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}r} < 0$. Furthermore, we have $\frac{\mathrm{d}k(q^{P^*})}{\mathrm{d}r} = \frac{w^P}{p-e}\frac{1-(q^{P^*}-\frac{B}{w^P})h(q^{P^*})}{\Omega h(k(q^{P^*}))-h(q^{P^*})} < 0$.

Case (iii). Proof of $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}B} < 0$.

Similar to the proof of Case (ii), taking the derivative with respect to B on both sides of $\bar{F}(q^{P^*}) = \Omega\bar{F}(k(q^{P^*}))$, respectively, we can obtain that

$$\frac{\mathrm{d}q^{P^*}}{\mathrm{d}B} = \frac{1+r^P}{p-e}\frac{h\big(k\big(q^{P^*}\big)\big)}{\Omega h(k(q^{P^*}))-h(q^{P^*})}. \tag{D.6}$$

Since $\Omega h(k(q^{P^*})) < h(q^{P^*})$, we can prove that $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}B} < 0$.

□

Appendix E. Proof of Proposition 2

Proof. The manufacturer's expected profit can be transformed as

$$\pi_M^P\big(w^P\big) = \big(w^P-c\big)q^{P^*} - \lambda(p-e)\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x - e\int_0^{q^{P^*}} F(x)\,\mathrm{d}x. \tag{E.1}$$

Taking the first derivative of π_M^P with respect to w^P yields

$$\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} = \left[\frac{w^P(1+r^P)-e}{1+r^P}\frac{1-H\big(q^{P^*}\big)}{1-\Omega q^{P^*}h(k(q^{P^*}))}\Big(1-\lambda F\Big(k\Big(q^{P^*}\Big)\Big)\big(1+r^P\big)\Big) - \bigg(c+eF\Big(q^{P^*}\Big)-\frac{e}{1+r^P}\bigg)\right]\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}.$$

Let $\delta(w^P) = \frac{1-H(q^{P^*})}{1-\Omega q^{P^*}h(k(q^{P^*}))}$, $\mu(w^P) = 1-\lambda F(k(q^{P^*}))(1+r^P)$, $\xi(w^P) = c+eF(q^{P^*})-\frac{e}{1+r^P}$, then

$$\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} = \left[\frac{w^P\big(1+r^P\big)-e}{1+r^P}\delta\big(w^P\big)\mu\big(w^P\big)-\xi\big(w^P\big)\right]\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P}. \tag{E.2}$$

When $\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} = 0$, there is a unique solution, *i.e.*, $w^{P^*} = \frac{\xi(w^{P^*})}{\mu(w^{P^*})\delta(w^{P^*})} + \frac{e}{1+r^P}$.

Next, we prove that w^{P^*} is optimal. It is straightforward that $\mu(w^P) > 0$ and $\xi(w^P) > 0$. From the proof of Corollary 1, we have $\frac{\mathrm{d}(k(q^{P^*}))}{\mathrm{d}w^P} < 0$ and $\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} < 0$. Thus, we can obtain $\frac{\mathrm{d}\mu(w^P)}{\mathrm{d}w^P} = -f(k(q^{P^*}))\frac{\mathrm{d}(k(q^{P^*}))}{\mathrm{d}w^P} > 0$ and

$-\frac{\mathrm{d}\xi(w^P)}{\mathrm{d}w^P} = -f(q^{P^*})\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} > 0$. For $\delta(w^P)$, we take the first derivative with respect to w^P yields

$$\frac{\mathrm{d}\delta(w^P)}{\mathrm{d}w^P} = \frac{-H'\left(q^{P^*}\right)\left(1-\Omega q^{P^*}h\left(k\left(q^{P^*}\right)\right)\right)\frac{\mathrm{d}q^{P^*}}{\mathrm{d}w^P} + \left(1-H\left(q^{P^*}\right)\right)\left(h\left(k\left(q^{P^*}\right)\right)+\Omega q^{P^*}h'\left(k\left(q^{P^*}\right)\right)\right)\frac{\mathrm{d}\left(k\left(q^{P^*}\right)\right)}{\mathrm{d}w^P}}{\left(1-\Omega q^{P^*}h(k(q^{P^*}))\right)^2},$$

where the molecular part can be converted into

$$\begin{aligned}\frac{1+r^P}{p-e}\frac{\left(1-H\left(q^{P^*}\right)\right)^2}{\Omega h(k(q^{P^*}))-h(q^{P^*})}\left(h\left(k\left(q^{P^*}\right)\right)+\Omega q^{P^*}h'\left(k\left(q^{P^*}\right)\right)\right)\\ -\frac{1+r^P}{w^P(1+r^P)-e}\frac{\left(1-\Omega qh\left(q^{P^*}\right)\right)^2}{\Omega h(k(q^{P^*}))-h(q^{P^*})}H'\left(q^{P^*}\right).\end{aligned} \tag{E.3}$$

Note that

(1) $\frac{1+r^P}{p-e} < \frac{1+r^P}{w^P(1+r^P)-e}$ due to $p > w^P(1+r^P)$;
(2) $(1-H(q^{P^*}))^2 < (1-\Omega q^{P^*}h(q^{P^*}))^2$ due to $\Omega q^{P^*}h(q^{P^*}) < q^{P^*}h(q^{P^*}) = H(q^{P^*})$;
(3) $\Omega h(k(q^{P^*})) - h(q^{P^*}) < 0$ since $h(x)$ is increasing;
(4) $H'(q^{P^*}) = h(q^{P^*}) + q^{P^*}h'(q^{P^*}) > h(k(q^{P^*})) + \Omega q^{P^*}h'k((q^{P^*}))$ since $\Omega < 1$ and $h(x)$ is increasing and convex.

From (1) to (4), we can prove that equation (E.3) >0. Therefore $\frac{\mathrm{d}\delta(w^P)}{\mathrm{d}w^P} > 0$. In addition, $\frac{\mathrm{d}\mu(w^P)}{\mathrm{d}w^P} > 0$ and $-\frac{\mathrm{d}\xi(w^P)}{\mathrm{d}w^P} > 0$, so the term $[\frac{w^P(1+r^P)-e}{1+r^P}\delta(w^P)\mu(w^P) - \xi(w^P)]$ increases in w^P.

When $w^P \to 0$, we have $[\frac{w^P(1+r^P)-e}{1+r^P}\delta(w^P)\mu(w^P) - \xi(w^P)] < 0$ and $\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} > 0$; when $w^P \to \frac{p}{1+r^P}$, we have $\Omega \to 1$, $q \to k \to 0$, then $[\frac{w^P(1+r^P)-e}{1+r^P}\delta(w^P)\mu(w^P) - \xi(w^P)] \to \frac{p}{1+r^P} - c > 0$ and $\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} < 0$. Therefore there is a unique solution for $\frac{\mathrm{d}\pi_M^P}{\mathrm{d}w^P} = 0$, π_M^P is a unimodal function with respect to w^P and $w^{P^*} = \frac{\xi(w^P)}{\mu(w^P)\delta(w^P)} + \frac{e}{1+r^P}$ is manufacturer's optimal solution. □

Appendix F. Proof of Corollary 2

Proof. Taking the first derivative with respect to λ on both sides of the equation $w^{P^*} = \frac{\xi(w^{P^*})}{\mu(w^{P^*})\delta(w^{P^*})} + \frac{e}{1+r^P}$, we can obtain that

$$\frac{\mathrm{d}w^{P^*}}{\mathrm{d}\lambda} = \frac{F\left(k\left(q^{P^*}\right)\right)\left(w^{P^*}\left(1+r^P\right)-e\right)}{1-\lambda F(k(q^{P^*}))(1+r^P)} > 0. \tag{F.1}$$

□

Appendix G. Proof of Corollary 3

Proof. According to the envelope theorem, we have

$$\begin{aligned}\frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}\lambda} &= \left.\frac{\partial\pi_M^P}{\partial\lambda}\right|_{w^P=w^{P^*}} = -(p-e)\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x < 0, \\ \frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}e} &= \left.\frac{\partial\pi_M^P}{\partial e}\right|_{w^P=w^{P^*}} = \lambda\left(q^{P^*} - k\left(q^{P^*}\right)\right)F\left(k\left(q^{P^*}\right)\right) + \lambda\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x - \int_0^{q^{P^*}} F(x)\,\mathrm{d}x \\ &< \left(q^{P^*} - k\left(q^{P^*}\right)\right)F\left(k\left(q^{P^*}\right)\right) - \int_{k\left(q^{P^*}\right)}^{q^{P^*}} F(x)\,\mathrm{d}x\end{aligned} \tag{G.1}$$

$$= \left(q^{P^*} - k\left(q^{P^*}\right)\right)\left(F\left(k\left(q^{P^*}\right)\right) - F(\varepsilon)\right) < 0, \tag{G.2}$$

where $\varepsilon \in (k(q^{P^*}), q^{P^*})$. □

Appendix H. Proof of Proposition 3

Proof. The bank's expected profit can be transformed as

$$\pi_B^P = (p-e)k - (1-\lambda)(p-e)\int_0^k F(x)\,\mathrm{d}x + eq^{P^*} - \left(w^{P^*}q^{P^*} - B\right)\cdot \tag{H.1}$$

Taking the first derivative of π_B^P with respect to r, we have $\frac{\mathrm{d}\pi_B^P}{\mathrm{d}r} = \frac{\mathrm{d}\pi_B^P}{\mathrm{d}k}\frac{\mathrm{d}k}{\mathrm{d}r}$. According to Corollary 1, one has $\frac{\mathrm{d}k}{\mathrm{d}r} \neq 0$. Hence the first order condition of $\frac{\mathrm{d}\pi_B^P}{\mathrm{d}r} = 0$ can be transformed to $\frac{\mathrm{d}\pi_B^P}{\mathrm{d}k} = 0$.

$$\frac{\mathrm{d}\pi_B^P}{\mathrm{d}k} = (p-e)\left[\left(\lambda F(k) + \bar{F}(k)\right) - \frac{1}{1+r^P}\left(1 - \frac{re}{\delta(w^{P^*})(w^{P^*}(1+r^P)-e)}\right)\right]\cdot \tag{H.2}$$

From Proposition 2, we have $\delta(w^{P^*})(w^{P^*}(1+r^P)-e) = \frac{(1+r^P)(c+eF(q^{P^*}))-e}{1-\lambda F(k)(1+r^P)}$. Substituting it into equation (H.2) yields

$$\frac{\mathrm{d}\pi_B^P}{\mathrm{d}k} = (p-e)\left[\lambda F(k) + \bar{F}(k) - \frac{1}{1+r^P}\left(1 - re\frac{\left(1-\lambda F(k)\left(1+r^P\right)\right)}{(1+r^P)(c+eF(q^{P^*}))-e}\right)\right]\cdot \tag{H.3}$$

Let $\frac{\mathrm{d}\pi_B^P}{\mathrm{d}k} = 0$, we have

$$\left(1 + r^{P^*}\right) = \frac{1 - \frac{r^{P^*}e\left(1-\lambda F(k)\left(1+r^{P^*}\right)\right)}{\left(1+r^{P^*}\right)\left(c+eF\left(q^{P^*}\right)\right)-e}}{\lambda F(k) + \bar{F}(k)}\cdot$$

Next, we prove the existence and optimality of the first derivative condition.

The first derivative condition exists if and only if $1 - re\frac{(1-\lambda F(k)(1+r^P))}{(1+r^P)(c+eF(q^{P^*}))-e} > 0$, *i.e.*, $(1+r^P)[c-(1-F(q^{P^*})-\lambda rF(k))e] > 0$, which is obvious since $e < c$. What's more,

$$\begin{aligned}\frac{\mathrm{d}^2\pi_B}{\mathrm{d}k^2} =& -(p-e)\bar{\lambda}F(k)\\ &-re(p-e)\left\{\frac{\lambda f(k)}{(1+r^P)(c+eF(q^{P^*}))-e} + ef\left(q^{P^*}\right)\frac{1-\lambda F(k)\left(1+r^P\right)}{[(1+r^P)(c+eF(q^{P^*}))-e]^2\frac{\mathrm{d}q^{P^*}}{\mathrm{d}k}}\right\} < 0.\end{aligned} \tag{H.4}$$

Hence, for every k, the bank's best response r^{P^*} is unique and r^{P^*} can be solved as $(1+r^{P^*}) = \frac{1-\frac{r^{P^*}e(1-\lambda F(k)(1+r^{P^*}))}{(1+r^{P^*})(c+eF(q^{P^*}))-e}}{\lambda F(k)+\bar{F}(k)}$, which can be rearranged to

$$r^{P^*} = \frac{c + eF\left(q^{P^*}\right) - eF(k)}{\left[\lambda F(k) + \bar{F}(k)\right][c + eF(q^{P^*})] - \lambda eF(k)} - 1. \tag{H.5}$$

□

Appendix I. Proof of Proposition 4

Proof. Comparing q^{P^*} and q_C^*, we have

$$\bar{F}\left(q^{P^*}\right) - \bar{F}(q_C^*) = \frac{w^{P^*}(1+r)-e}{p-e}\bar{F}(k) - \frac{c}{p}\cdot \tag{I.1}$$

From the proof of Proposition 3, we have

$$\left(w^{P^*}\left(1+r^P\right)-e\right) = \frac{1}{\delta(w^{P^*})}\frac{er^{P^*}}{1-(1+r^P)\left[\lambda F(k)+\bar{F}(k)\right]}\cdot \tag{I.2}$$

Substituting equations (I.2) into (I.1) yields

$$\bar{F}\left(q^{P^*}\right) - \bar{F}(q_C^*) = \frac{1}{(p-e)\delta(w^{P^*})}\frac{er^{P^*}\bar{F}(k)}{1-(1+r^P)\left[\lambda F(k)+\bar{F}(k)\right]} - \frac{c}{p}\cdot \tag{I.3}$$

Therefore $\bar{F}(q^{P^*}) - \bar{F}(q_C^*)\begin{cases} <0, \lambda<\widehat{\lambda} \\ =0, \lambda=\widehat{\lambda} \\ >0, \lambda>\widehat{\lambda}\end{cases}$ and $q^{P^*} - q_C^*\begin{cases} >0, \lambda<\widehat{\lambda} \\ =0, \lambda=\widehat{\lambda} \\ <0, \lambda>\widehat{\lambda}\end{cases}$, where $\hat{\lambda} = \frac{1-(1+r^{P^*})\bar{F}(k)}{\delta(w^{P^*})(1+r^{P^*})F(k)} - \frac{p\hat{e}r^{P^*}\bar{F}(k)}{c(p-\hat{e})\delta(w^{P^*})(1+r^{P^*})F(k)}$. □

Appendix J. Proof of Proposition 6

Proof. The manufacturer's expected profit can be rewritten as

$$\pi_M^T(w^T) = \left(w^T(1+r^T)-c\right)q^{T^*} - Br^T - p\int_0^{k\left(q^{T^*}\right)} F(x)\,\mathrm{d}x - e\int_{k\left(q^{T^*}\right)}^{q^{T^*}} F(x)\,\mathrm{d}x. \tag{J.1}$$

Taking the first derivative with respect to w^T, we have

$$\frac{\mathrm{d}\pi_M^T(w^T)}{\mathrm{d}w^T} = (1+r^T)q^{T^*} + \left(w^T(1+r^T)-c-eF\left(q^{T^*}\right)\right)\frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T} - (p-e)F\left(k\left(q^{T^*}\right)\right)\frac{\mathrm{d}k\left(q^{T^*}\right)}{\mathrm{d}w^T}\cdot \tag{J.2}$$

Similar to the proof of Corollary 1, we have

$$\frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T} = \frac{1+r^T}{w^T(1+r^T)-e}\frac{1-\Omega q^{T^*}h\left(k\left(q^{T^*}\right)\right)}{\Omega h(k(q^{T^*}))-h(q^{T^*})} \tag{J.3}$$

$$\frac{\mathrm{d}k\left(q^{T^*}\right)}{\mathrm{d}w^T} = \frac{\left(1+r^T\right)}{p-e}\frac{1-H\left(q^{T^*}\right)}{\Omega h(k(q^{T^*}))-h(q^{T^*})}\cdot \tag{J.4}$$

Substituting equations (J.3) and (J.4) into equations (J.2) yields

$$\frac{\mathrm{d}\pi_M^T(w^T)}{\mathrm{d}w^T} = \frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T}\left\{\left(w^T\left(1+r^T\right)-e\right)\delta(w^T)\bar{F}\left(k\left(q^{T^*}\right)\right) - \left(c-e\bar{F}\left(q^{T^*}\right)\right)\right\}, \tag{J.5}$$

where $\delta(w^{T^*}) = \frac{1-H(q^{T^*})}{1-\Omega q^{T^*}h(k(q^{T^*}))}$. Then the first-order condition is $w^{T^*} = \frac{1}{1+r^T}\frac{c+eF(q^{T^*})-e}{\delta(w^{T^*})\bar{F}(k(q^{T^*}))} + \frac{e}{1+r^T}$.

Next, we prove the optimality of the first-order condition. It is straightforward that

(1) $w^T(1+r^T)-e>0$ and increases with r^T;
(2) $\bar{F}(k(q^{T^*}))>0$ and increases with w^T;

(3) $-(c - e\bar{F}(q^{T^*}))$ increases with w^T;
(4) Similar to the proof of Corollary 2, we have $\frac{\mathrm{d}\delta(w^T)}{\mathrm{d}w^T} > 0$.

Let $l(w^T) = \{(w^T(1+r^T) - e)\delta(w^T)\bar{F}(k(q^{T^*})) - (c - e\bar{F}(q^{T^*}))\}$, then $\frac{\mathrm{d}\pi_M^T(w^T)}{\mathrm{d}w^T} = l(w^T)\frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T}$. From (1) to (4), we have $l(w^T)$ increases in w^T, *i.e.*, $\frac{dl(w^T)}{\mathrm{d}w^T} < 0$. What's more $\frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T} < 0$. Let $l(w^{T^*}) = 0$, then

$$\left.\frac{\mathrm{d}^2\pi_M^T(w^T)}{\mathrm{d}w^{T^2}}\right|_{w^T=w^{T^*}} = \left.\frac{\mathrm{d}q^{T^*}}{\mathrm{d}w^T}\frac{\mathrm{d}l(w^T)}{\mathrm{d}w^T}\right|_{w^T=w^{T^*}} + l(w^T)\left.\frac{\mathrm{d}^2q^{T^*}}{\mathrm{d}w^{T2}}\right|_{w^T=w^{T^*}} < 0. \tag{J.6}$$

Therefore, $w^{T^*} = \frac{1}{1+r^T}\frac{c+eF(q^{T^*})-e}{\delta(w^{T^*})\bar{F}\left(k(q^{T^*})\right)} + \frac{e}{1+r^T}$ is the manufacturer's optimal solution. □

Appendix K. Proof of Corollary 5

Proof. According to the envelope theorem, we have

$$\frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}e} = \left.\frac{\partial\pi_M^T}{\partial e}\right|_{w^T=w^{T^*}} = \left(q^{T^*} - k\left(q^{T^*}\right)\right)F\left(k\left(q^{T^*}\right)\right) > 0. \tag{K.1}$$

□

Appendix L. Proof of Proposition 7

Proof. Let $\Delta\pi_M^P = \pi_M^{P^*} - \pi_M^{T^*}$, then $\frac{\mathrm{d}\Delta\pi_M^P}{\mathrm{d}\lambda} = \frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}\lambda} - \frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}\lambda}$, $\frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}\lambda} = 0$ and from Corollary 3, $\frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}\lambda} < 0$, thus$\frac{\mathrm{d}\Delta\pi_M^P}{\mathrm{d}\lambda} < 0$. Let $\pi_M^{P^*} = \pi_M^{T^*}$, we have

$$\bar{\lambda}_M = \frac{(w^{P^*} - c)q^{P^*} - \left(w^{T^*}\left(1+r^T\right) - c\right)q^{T^*} + e\int_0^{q^{T^*}-q^{P^*}} F(x)\,\mathrm{d}x + (p-e)\int_0^{k(q^{T^*})} F(x)\,\mathrm{d}x + Br^T}{(p-e)\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x},$$

when $\lambda > \bar{\lambda}_M$, we have $\pi_M^{P^*} < \pi_M^{T^*}$; when $\lambda < \bar{\lambda}_M$, we have $\pi_M^{P^*} > \pi_M^{T^*}$.

Similarly, $\frac{\mathrm{d}\Delta\pi_M^P}{\mathrm{d}e} = \frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}e} - \frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}e}$. From Corollaries 3 and 5, we have $\frac{\mathrm{d}\pi_M^{P^*}}{\mathrm{d}e} < 0$ $\frac{\mathrm{d}\pi_M^{T^*}}{\mathrm{d}e} > 0$. Therefore $\frac{\mathrm{d}\Delta\pi_M^P}{\mathrm{d}\lambda} < 0$. Let $\pi_M^{P^*} = \pi_M^{T^*}$, we have

$$\bar{e}_M = \frac{(w^{P^*} - c)q^{P^*} - \left(w^{T^*}\left(1+r^T\right) - c\right)q^{T^*} - \lambda p\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x + p\int_0^{k\left(q^{T^*}\right)} F(x)\,\mathrm{d}x + Br^T}{\int_0^{q^{P^*}} F(x)\,\mathrm{d}x - \lambda\int_0^{k\left(q^{P^*}\right)} F(x)\,\mathrm{d}x - \int_{k\left(q^{T^*}\right)}^{q^{T^*}} F(x)\,\mathrm{d}x},$$

when $e > \bar{e}_M$, we have $\pi_M^{P^*} < \pi_M^{T^*}$; when $\lambda < \bar{e}_M$, we have $\pi_M^{P^*} > \pi_M^{T^*}$. □

Acknowledgements

This work was supported by the National Social Science Foundation of China (Grant no. 21BGL125), the National Natural Science Foundation of China (Grant no. 11601142).

References

[1] C. Altavilla, A. Ellul, M. Pagano, A. Polo and T. Vlassopoulos, Loan guarantees, bank lending and credit risk reallocation (November 14, 2021). Available at SSRN: https://ssrn.com/abstract=3963246.

[2] V. Babich and C.S. Tang, Managing opportunistic supplier product adulteration: deferred payments, inspection, and combined mechanisms. *Manuf. Serv. Oper. Manage.* **14** (2012) 301–314.

[3] N. Bachas, O.S. Kim and C. Yannelis, Loan guarantees and credit supply. *J. Finan. Econ.* **139** (2021) 872–894.

[4] B. Basel III, A Global Regulatory Framework for More Resilient Banks and Banking Systems. Bank for International Settlements, Basel, Switzerland (2011).

[5] G. Bi, P. Chen and Y. Fei, Optimal decisions and coordination strategy of a capital-constrained supply chain under customer return and supplier subsidy. *J. Model. Manage.* **13** (2018) 278–301.

[6] M. Burkart and T. Ellingsen, In-kind finance: a theory of trade credit. *Am. Econ. Rev.* **94** (2004) 569–590.

[7] G.P. Cachon, Supply chain coordination with contracts, in Supply Chain Management: Design, Coordination and Operation. Vol. 11 of *Handbooks in Operations Research and Management Science*. Elsevier (2003) 227–339.

[8] G. Cai, X. Chen and Z. Xiao, The roles of bank and trade credits: theoretical analysis and empirical evidence. *Prod. Oper. Manage.* **23** (2014) 583–598.

[9] Y.-J. Cai, T.-M. Choi and J. Zhang, Platform supported supply chain operations in the blockchain era: supply contracting and moral hazards. *Decis. Sci.* **52** (2021) 866–892.

[10] E. Cao and M. Yu, Trade credit financing and coordination for an emission-dependent supply chain. *Comput. Ind. Eng.* **119** (2018) 50–62.

[11] C. Cardone-Riportella, A. Trujillo-Ponce and A. Briozzo, Analyzing the role of mutual guarantee societies on bank capital requirements for small and medium-sized enterprises. *J. Econ. Policy Reform* **16** (2013) 142–159.

[12] J. Chen and P.C. Bell, Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy. *Eur. J. Oper. Res.* **212** (2011) 293–300.

[13] X. Chen and A. Wang, Trade credit contract with limited liability in the supply chain with budget constraints. *Ann. Oper. Res.* **196** (2012) 153–165.

[14] J. Chen, Y.W. Zhou and Y. Zhong, A pricing/ordering model for a dyadic supply chain with buyback guarantee financing and fairness concerns. *Int. J. Prod. Res.* **55** (2017) 5287–5304.

[15] Y. Chu, D. Zhang and Y.E. Zhao, Bank capital and lending: Evidence from syndicated loans. *J. Finan. Quant. Anal.* **54** (2019) 667–694.

[16] F. Corredera-Catalán, F. di Pietro and A. Trujillo-Ponce, Post-COVID-19 SME financing constraints and the credit guarantee scheme solution in Spain. *J. Banking Regulation* **22** (2021) 250–260.

[17] M. Dada and Q. Hu, Financing newsvendor inventory. *Oper. Res. Lett.* **36** (2008) 569–573.

[18] L. Devangan, R. Amit, P. Mehta, S. Swami and K. Shanker, Individually rational buyback contracts with inventory level dependent demand. *Int. J. Prod. Econ.* **142** (2013) 381–387.

[19] M.J. Flannery and K.P. Rangan, What caused the bank capital build-up of the 1990s? *Rev. Finan.* **12** (2008) 391–429.

[20] N. Halov and F. Heider, Capital structure, risk and asymmetric information. *Q. J. Finan.* **1** (2011) 767–809.

[21] P. Honohan, Partial credit guarantees: principles and practice. *J. Finan. Stab.* **6** (2010) 1–9.

[22] V. Ioannidou, N. Pavanini and Y. Peng, Collateral and asymmetric information in lending markets. *J. Finan. Econ.* **144** (2022) 93–121.

[23] P. Kouvelis and W. Zhao, Supply chain contract design under financial constraints and bankruptcy costs. *Manage. Sci.* **62** (2016) 2341–2357.

[24] P. Kouvelis and W. Zhao, Who should finance the supply chain? Impact of credit ratings on supply chain decisions. *Manuf. Serv. Oper. Manage.* **20** (2018) 19–35.

[25] Y. Li and X. Jiang, The supplier's optimal guarantee policy in newsvendor finance. *Int. Trans. Oper. Res.* **27** (2020) 2370–2395.

[26] B. Li, S.M. An and D.P. Song, Selection of financing strategies with a risk-averse supplier in a capital-constrained supply chain. *Transp. Res. Part E: Logistics Transp. Rev.* **118** (2018) 163–183.

[27] Q. Lu, J. Gu and J. Huang, Supply chain finance with partial credit guarantee provided by a third-party or a supplier. *Comput. Ind. Eng.* **135** (2019) 440–455.

[28] F. Modigliani and M.H. Miller, The cost of capital, corporation finance and the theory of investment. *Am. Econ. Rev.* **48** (1958) 261–297.

[29] A.F. Momena, M. Rahaman, R. Haque, S. Alam and S.P. Mondal, A learning-based optimal decision scenario for an inventory problem under a price discount policy. *Systems* **11** (2023) 235.

[30] V. Padmanabhan and I.P. Png, Returns policies: make money by making good. *Sloan Manage. Rev.* **37** (1995) 65–65.

[31] V. Padmanabhan and I.P. Png, Reply to "do returns policies intensify retail competition?". *Marketing Sci.* **23** (2004) 614–618.

[32] M. Rahaman, R.M.S. Abdulaal, O.A. Bafail, M. Das, S. Alam and S.P. Mondal, An insight into the impacts of memory, selling price and displayed stock on a retailer's decision in an inventory management problem. *Fractal Fractional* **6** (2022) 531.

[33] C.C. Riportella and A.T. Ponce, LGAs collateral' effects on the smes financing and capital requirements of basel II. *Spanish J. Finan. Acc. / Revista Española de Financiación y Contabilidad* **36** (2007) 757–789.

[34] H. Rui and G. Lai, Sourcing with deferred payment and inspection under supplier product adulteration risk. *Prod. Oper. Manage.* **24** (2015) 934–946.

[35] J. Shi, Q. Du, F. Lin, Y. Li, L. Bai, R.Y. Fung and K.K. Lai, Coordinating the supply chain finance system with buyback contract: a capital-constrained newsvendor problem. *Comput. Ind. Eng.* **146** (2020) 106587.

[36] D. Wu, Coordination of competing supply chains with newsvendor and buyback contract. *Int. J. Prod. Econ.* **144** (2013) 1–13.

[37] S.M. Wu, F.T. Chan and S. Chung, The impact of buyback support on financing strategies for a capital-constrained supplier. *Int. J. Prod. Econ.* **248** (2022) 108457.

[38] S. Xiao, S.P. Sethi, M. Liu and S. Ma, Coordinating contracts for a financially constrained supply chain. *Omega* **72** (2017) 71–86.

[39] L. Xie, J. Ma and M. Goh, Supply chain coordination in the presence of uncertain yield and demand. *Int. J. Prod. Res.* **59** (2021) 4342–4358.

[40] N. Yan, B. Sun, H. Zhang and C. Liu, A partial credit guarantee contract in a capital-constrained supply chain: financing equilibrium and coordinating strategy. *Int. J. Prod. Econ.* **173** (2016) 122–133.

[41] S.A. Yang and J.R. Birge, Trade credit, risk sharing, and inventory financing portfolios. *Manage. Sci.* **64** (2018) 3667–3689.

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