DECISION-MAKING MODEL OF ELDERLY SERVICE COMPANIES UNDER GOVERNMENT SUBSIDIES

JUNFENG DONG\textsuperscript{1,2,*}, MENGYUAN WANG\textsuperscript{1,2}, LI JIANG\textsuperscript{1,3}, WENXING LU\textsuperscript{1,4} and CHANGYONG LIANG\textsuperscript{1,2}

Abstract. This study considers two government subsidy policies, namely, supply-side (subsidising elderly service companies) and demand-side (subsidising elderly customers) subsidies. A Nash game model consisting of elderly customers and two elderly service providers with different levels of infrastructure is developed to present the effects of government subsidies on the price, quality and quantity of demand for elderly services under different subsidy policies. The results show that the economic situation of the region and the strategic objectives of the local government are important factors influencing the effectiveness of the government’s subsidy strategy. For areas with good regional economic conditions, subsidies should be biased in favour of elderly care institutions with poorer infrastructure; for areas with poorer economic conditions, subsidies should be biased in favour of institutions with better infrastructure. Moreover, if the government plans to reduce prices more effectively, then it should adopt a demand-side subsidy policy for areas with a small or large elderly population and a supply-side subsidy policy for areas with a medium elderly population.

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1. Introduction

Today, developing and advanced economies are facing the problems of increasingly ageing populations, including the need to meet diverse elderly healthcare service demands \cite{50}. China is the world’s most populous country. According to figures from the China Economic Net, China’s total population has reached 1.411 billion, with 19.8\% of the total population aged 60 or older and 14.9\% of the total population aged 65 or older. The age structure of China’s population has entered an ageing stage. Furthermore, family size data published in the report show that China’s family structure has changed, with the average household size decreasing from 3.1...
persons a decade ago to 2.62 persons. This case demonstrates a weakening of the family’s elderly care function and that, in the future, there will be many elderly people without children to care for them, creating a great pressure for socialised elderly care. In the current situation, where the problems of elderly services are becoming increasingly serious, elderly services should be significantly developed.

Institutional elderly care services are an important component of the social elderly care service system. This system performs the fundamental elderly care service function of providing centralised living care and specialised aged care services to the elderly. Aggressively encouraging and growing institutional elderly care has become the focus of efforts to address the socialised elderly care problem owing to the changes in China’s population structure and hastened ageing. The recently enacted policies on aged care have proposed some targeted and operational policy measures in six areas: investment and financing, land supply, tax breaks, subsidy support, talent training and employment. Government encourages social forces in organising elderly care institutions through subsidised investment, loan interest reduction, operating subsidies and service purchasing. It also requires localities to accelerate the development of the assessment for elderly services and construct a reliable subsidy system for elderly individuals in financial need. This phenomenon shows how vital the government is in balancing and regulating the market for elderly services. That is, the government not only provides full support to elderly service businesses and ensures the fair development of all types of elderly service institutions but also gives the elderly a sense of security, encourages the growth of the elderly service industry and addresses their social problem.

In the face of the serious trend of insufficient elderly services, the Chinese government has adopted supply-side structural reforms to encourage private investment to actively contribute to the provision of elderly services. At present, government subsidies for elderly care in China mainly focus on subsidising the supply side of elderly care services. They mainly provide financial support for elderly care institutions through construction and operational subsidies [35]. Of course, specific policy reforms vary widely from one region to another, as they are usually enacted by local governments to address the needs of the elderly in their own regions [18, 44]. For example, Harbin City in Heilongjiang Province, China provides a one-time construction subsidy based on the number of beds and an operating subsidy for each elder resident per month [15]. Nanjing City in Jiangsu Province provides a one-time construction subsidy based on the number of nursing care beds and ordinary beds and combines three subsidies (i.e. bed maintenance subsidy, nursing care subsidy and comprehensive insurance subsidy) to form a comprehensive operating subsidy [28]. Furthermore, Hong Kong has introduced a novel means-tested residential care service voucher, which is a demand-side subsidy targeting particular groups, enabling the elderly to purchase services in the private sector and fostering consumer-directed care [42]. Supply-side subsidies can effectively address the pressure on institutions to operate due to high construction investment and low returns. Thus, such subsidies can fully motivate the market and promote elderly service providers to deliver low-cost and high-quality elderly care services.

The existing supply-side subsidy policy lowers the service prices for each elderly customers. However, most elderly care institutions, due to limited subsidies, cannot significantly reduce their prices and effectively increase the purchasing power of the elderly for these care services. Moreover, many low-income elderly people still cannot purchase these services. Hence, the resources of elderly care institutions are not fully utilised, leading to the widespread phenomenon of vacant beds in institutions. The reason is that during policy implementation, the subsidy policy for elderly care services favoured institutions and the supply side more than the actual consumers and the demand side. The insufficient purchasing power of the elderly with a demand for elderly care services results in a lack of internal demand for the development of elderly care services. Therefore, some scholars have suggested that subsidies previously given to facilities and the daily operation of each elderly service provider can be reduced or directly transformed into subsidies for service consumers, achieving a transformation from supply-side to demand-side subsidies.

However, many questions still exist about the change in subsidy approach. Although the disadvantages of supply-side subsidies are undeniable and the advantages of demand-side subsidies are evident, whether demand-side subsidies can fully replace supply-side subsidies is worth discussing [17].
In addition, elderly customers vary considerably in different aspects, such as age, physical conditions, economic conditions and cultural levels. Hence, they will have different demands for products and services. Therefore, a market providing differentiated products + services for elderly customers to choose is critically essential. Recently, to improve the service quality provided by elderly care institutions and facilitate the choice of elderly people and their families who need to stay in elderly care institutions, the Chinese government issued the national standard Classification and Evaluation of Elderly Care Institutions, which provides a new way of thinking about the development of elderly care institutions: adapting to the needs, prioritising quality, reasonable price and multiple supply.

Different from the traditional elderly care system, the product + service model of packaged sales is a system whereby the elderly care service provider purchases elderly care products from its regular upstream suppliers. Then, it packages the products and accompanying services into a product and service system to be sold to elderly customers, better meeting the personalised needs of these customers. For example, at the time of admission to a care-and-attention nursing facility in Yunnan Province, China, two professional assessors in the institution assess the level of care and fees based on the physical condition of the elderly [47]. At the same time, the elderly institutions classify the disabled elderly into three categories, namely, mild, moderate and severe disability, and provide different levels of nursing care accordingly, with additional special nursing care for the severely disabled elderly. Elderly customers can choose products in addition to elderly care services upon institutional admission. For example, when choosing a bed, elderly people who cannot take care of themselves can choose a reclining urinary and defecation care bed with automatic urinary and defecation flushing, automatic drying, turning and other functions to assist in nursing care. Conversely, other elderly people, depending on their conditions, can choose ordinary beds. In addition to nursing care institutions, ordinary institutions are also present in the area. General elderly care institutions only have basic facilities for the elderly and are not yet fully equipped for medical care, so they are more favourably priced and are suitable for the elderly who can take care of themselves physically or whose families are not financially well-off.

Therefore, based on the theoretical basis of the relevant government subsidy policy, this study considers the existence of two elderly service providers in the market, that is, one is poor infrastructure that can only provide low-quality products and the other is better infrastructure that can provide high-quality products, and both elderly service providers providing combination of product + service (abbreviated as services). A game model is constructed for the pricing of elderly service providers in the context of government subsidies on the supply and demand side. The influence of government subsidies on elderly service providers and elderly consumers is examined. Moreover, pertinent suggestions for subsidy policy are made considering relevant study findings.

The remainder of this paper is organised as follows. Section 2 reviews the related literature. Section 3 describes the problem and presents the assumptions. Section 4 formulates the game model in different government subsidy statuses. Section 5 discusses and compares the optimal decisions under different government subsidy statuses. Section 6 gives a numerical example to show the application of the model. Finally, Section 7 concludes the study.

2. Literature review

This study reviews the literature closely related to our research from two aspects, governmental subsidies and elderly services. We briefly review the literature for each.

2.1. Governmental subsidies

Governments frequently implement a few support policies to encourage and promote the quick development of a new industry. Current policies include government investment and finance [9], tax incentives and financial subsidies [2,46,49]. One stream that is closely related to this study is the governments financial subsidy policy. Wang et al. [38] studied that the government provides three different types of environmentally friendly subsidies to encourage the design and promotion of green products, that is, subsidy for the manufacturers design for the environment, the retailers sales effort and the consumers green consumption. Bian et al. [3] studied the effect
of environmental subsidies on the incentives of investing in emission-reducing technologies in manufacturing amidst the environmental concerns of consumers.

Some related studies focused on the issue of optimal government subsidies from different perspectives. From a corporate perspective, government subsidies have a different impact on the existence of companies [26], technological improvement and innovation [20] and social responsibility [24]. Other relevant studies also examined subsidy policies under different distribution channels and market structures [13, 33]. In addition, the impact of government subsidies on corporate performance is two sided [22, 43].

Other research regarded the influence of consumers on government subsidies and companies optimal decisions. For instance, Wang [39] studied optimal subsidy strategies and waste clothing recycling supply chain and found that strong consumer sensitivity to recycling efforts benefits only some supply chain participants. Chen et al. [6] investigated the optimal production and subsidy rate of the supply chain considering that consumer environmental awareness is dynamic. Zaman and Zaccour [48] examined a two-period game between strategic consumers and the government to determine the optimal scrappage subsidy levels. Xu et al. [41] investigated the pricing reduction behaviour of supply chain participants under government subsidy whilst accounting for consumer awareness of carbon emissions.

The government should consider not only consumers and companies but also the budget for financial subsidies when making subsidies [1]. In addition, other studies focused on the character of subsidies to determine what kind of subsidies affect the development of companies. Government subsidies are classified as investment subsidies and usage subsidies [7], linear subsidies and the fixed subsidy model [10], a per-unit production subsidy or an innovation subsidy [5].

In contrast to the literature that demonstrated the vital role of government subsidies in promoting the development of emerging and disadvantaged sectors, the accelerating ageing society in China puts the elderly care industry under tremendous pressure. This situation necessitates the government to encourage private companies to participate in the competition and provide elderly care services. Ren et al. [30] suggested that subsidy policies of governmental departments should focus on improving the professional skills and quality of elderly care domestic workers. Song et al. [35] compared the effects of construction and operating subsidies on achieving policy goals from the perspective of uncertain actual demand. They emphasised the importance of subsidy selection in the context of uncertain actual demand. He et al. [14] analysed the strategic behaviours of two types of participant, the government and private sectors, in providing elderly care services in a long-term and dynamic situation. Song et al. [34] compared the effects of two forms of subsidy in China. The results initially theoretically verified the importance of subsidies in stimulating the private supply of elderly care. Yasuoka [45] examined how a subsidy for elderly care services affects the labour supply in the market of elderly care and other services. Mu et al. [27] analysed the effects of subsidies to private sectors and consumers on the cooperation in the provision of elderly care services from the perspective of public-private partnerships and obtained evolutionary equilibriums of the complex system. In contrast to the literature mentioned above, the present study examines government subsidies to providers and consumers of elderly care services. In addition, concerning the literature on subsidy policies by subsidy recipients [23], this study focuses on comparing the price and quality of elderly services under government supply- and demand- subsidy scenarios.

2.2. Elderly services

The second stream focuses on service, the importance of which has been validated in the relevant literature [16]. Research on services mainly included service quality levels, service improvement and the impact of services. Previous studies mainly focused on the quality and price competition of services. He and Ma [12] focused on exploring the service quality and pricing strategies of duopoly crowdsourced delivery platforms under two different competition scenarios: pure price competition and joint price and service quality competition. Guo et al. [11] examined firms’ post-merger integration strategy under price and service-quality competition when facing either deterministic or stochastic demand. Perdikaki et al. [29] studied how retailers can time their service investments when demand for a product is uncertain and consumers care about price and service when choosing which retailer to buy from.
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Table 1. Related literature.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Pricing</th>
<th>Service quality</th>
<th>Products quality</th>
<th>Individualized demand</th>
<th>Government benefits</th>
<th>Type of policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ren et al. [30]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>Subsidies to clients, subsidies or punishments for enterprises</td>
</tr>
<tr>
<td>Song et al. [35]</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td>Construction subsidy and operating subsidy</td>
</tr>
<tr>
<td>He et al. [14]</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>Implement supervision and implement no supervision</td>
</tr>
<tr>
<td>Song et al. [34]</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td>Construction subsidy and operating subsidy</td>
</tr>
<tr>
<td>Yasuoka [45]</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td>Pay-as-you-go pension</td>
</tr>
<tr>
<td>Mu et al. [27]</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>Subsidies to private sectors and consumers</td>
</tr>
<tr>
<td>Li et al. [21]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>Differentiated subsidies based on the preferences of the customer</td>
</tr>
<tr>
<td>This paper</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>Supply-side and demand-side subsidies</td>
</tr>
</tbody>
</table>

Furthermore, researchers have also studied the impact of services on the profits of participating members. Shan et al. [32] discussed the impact of the cost-sharing ratio, service cost coefficient, government subsidy and service sensitivity coefficient on profits and social net benefits. Ma et al. [25] noted that supply chain members’ reciprocal altruism and consumers’ quality and service reference effects are important behavioural factors that affect the decision-making of supply chain members. Sarkar and Pal [31] examined the impact of retail services on profit and pricing decisions under different return strategies. Li et al. [19] studied retailers’ service capability decisions in a marketplace where they sold products bundled with after-sales services to consumers, and sales were influenced by the retailer’s service level commitment.

In the field of elderly services, Zhao [50, 51] studied the optimal service capacity and service sales efforts of a two-echelon elderly healthcare service supply chain consisting of elderly service integrators and service providers. The research on elderly care services is closely related to this study but differs in that this study examined competition between two different elderly care providers in the elderly care market, and does not consider competition and cooperation between members of the elderly service supply chain.

In Table 1, we compare this paper with the aforementioned literature. As shown in Table 1, numerous scholars have conducted in-depth research on service and price decision making in supply chains, government intervention strategies and other issues. Evidently, within the context of social ageing, numerous studies have been conducted on the subsidisation of elderly services. It is worth noting that, in contrast to supply-side subsidies, which have garnered significant attention in quantitative research among scholars, quantitative research on demand-side subsidies for elderly services and comparative studies of the two types of subsidy are limited. Supply-side subsidies can effectively reduce the overall price of elderly service providers and improve the quality of elderly services. However, they cannot effectively increase the purchasing power of elderly customers, and the lack of internal demand drivers for elderly services hinders the development of the elderly service industry. Therefore, how government departments choose their subsidy policies and how elderly service enterprises make decisions under different subsidy policies are crucial.

The above research has provided a basis for this study to examine government policies on elderly subsidies for different subsidy recipients. Our research contributes to the literature by exploring optimal decision-making for elderly care services under government subsidies. Different from the above studies, this study examines two scenarios of subsidies for elderly services, namely, government subsidies on the supply side and demand side. The mechanism of the government subsidy coefficient on the optimal decision of elderly service providers is
This study examines a duopoly market where two elderly care service providers 1 and 2 sell products + services (hereafter referred to as elderly care services) as a package. Two elderly care providers play a Nash game with concurrent decision making. Provider 1 has limited infrastructure and can only offer elderly customers low-quality products, whereas Provider 2 has a better infrastructure and can offer elderly customers high-quality products. Elderly customers decide to purchase the services of one of the providers based on their sense of the quality of the products, services and prices offered by the providers. As the main body that regulates and controls the market balance, the government will develop financial subsidies to promote the development of the elderly care industry, promote and encourage elderly care institutions to provide higher quality services and ensure the well-being of the elderly in their old age. Moreover, there are two main types of subsidies: supply-side and demand-side subsidies.

The main research considering regional differences in the elderly population is to compare the impact on pricing, elderly service quality and market demand for elderly service providers under two scenarios: government-subsidised supply-side (elderly service providers) and government-subsidised demand-side (elderly customers). Table 2 shows some basic parameters.

In addition, the superscripts N, H and E are used to denote the no-government subsidy scenario, the government-subsidised supply-side scenario and the government-subsidised demand-side scenario, respectively.

3.2. assumptions

Firstly, assume there are two kinds of elderly care providers. Elderly service providers are rational, and all have the goal of maximising their profits. Elderly service provider 1 sells products of quality $v_L$ and services of quality $s_1$ as a package at price $p_1$. Elderly service provider 2 package a product of quality $v_H$ and a service of quality $s_2$ and sells it at price $p_2$. The products sold include rooms, beds, heating and others. The services include care, dining, medical and recreational services of different degree levels. In addition, the cost of the product is assumed to increase linearly with the level of product quality, that is, the cost of the product is $c_v v_i$, $i = \{L, H\}$, where $c_v$ is the cost per unit of product quality and $0 < c_v v_i < p_i$. 

Table 2. Some basic parameters for models.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_i$</td>
<td>Profits for elderly service provider $i$, $i = {1, 2}$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Demand for elderly care services provided by elderly care provider $i$, $i = {1, 2}$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Prices of elderly care services provided by elderly care provider $i$, $i = {1, 2}$</td>
</tr>
<tr>
<td>$s_i$</td>
<td>The level of quality of elderly care services provided by the elderly care provider $i$, $i = {1, 2}$</td>
</tr>
<tr>
<td>$v_i$</td>
<td>The quality of elderly care products offered by elderly care providers, $i = {H, L}$ and $v_H &gt; v_L$</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Cost of quality per unit of product provided by the elderly service provider, $0 &lt; c_v v_i &lt; p_i$</td>
</tr>
<tr>
<td>$D$</td>
<td>Potential size of the demand market for elderly services</td>
</tr>
<tr>
<td>$k$</td>
<td>Coefficient of impact of service quality of elderly service providers on service improvement costs</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The preferences of elderly customers for product quality, obeying a uniform distribution</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The preferences of elderly customers for service quality, obeying a uniform distribution</td>
</tr>
<tr>
<td>$u$</td>
<td>Valuation of products by older customers</td>
</tr>
<tr>
<td>$m$</td>
<td>Government subsidy factor for elderly service providers to improve the quality of services, $m \in (0, 1)$</td>
</tr>
</tbody>
</table>
Secondly, the cost of service improvement is quadratically related to service quality, that is, the cost of service improvement is $\frac{1}{2}k\theta_2^2$, $i = \{1, 2\}$, where $k$ is the coefficient of influence of service quality on the cost of service improvement. The form of quadratic function to indicate the service cost is similar to the existing literature [36, 37].

Thirdly, assume that elderly customers have preferences $\delta$ for products and $\theta$ for services in elderly care services, and $\delta$ and $\theta$ follow a uniform distribution, that is, $\delta \sim [0, 1]; \theta \sim [0, 1]$. In addition, elderly service valuation $u$ is sufficiently large ($u > p_i$, $\{i = 1, 2\}$) that all elderly customers choose to purchase elderly services from elderly service providers 1 and 2. This assumption is consistent with the related literature [4]. The elderly customers efficiency functions obtained after purchasing the elderly care services of elderly service provider 1 and 2 respectively are

$$U_1 = u + \delta v_L + \theta s_1 - p_1$$

$$U_2 = u + \delta v_H + \theta s_2 - p_2.$$  

Fourthly, assume the potential size of the market for elderly services as $D$. If the utility gained by an elderly customer from purchasing elderly services at elderly service provider 1 is higher than the utility gained from purchasing elderly services at elderly service provider 2, then the elderly customer will choose to purchase elderly services at provider 1 when $\delta v_L + \theta s_1 - p_1 > \delta v_H + \theta s_2 - p_2$ (i.e. $U_1 > U_2$). Conversely, when $U_1 < U_2$, elderly customers choose to purchase senior care services from provider 2. From the equation $\delta v_L + \theta s_1 - p_1 = \delta v_H + \theta s_2 - p_2$, we obtain $\delta = \frac{\theta_2 (s_1 - s_2) + p_2 - p_1}{v_L - v_H}$. Therefore, when the elderly customers preference is $\delta = \frac{\theta_2 (s_1 - s_2) + p_2 - p_1}{v_L - v_H}$, the elderly customer chooses to purchase elderly care services from provider 1. Conversely, when $\delta > \frac{\theta_2 (s_1 - s_2) + p_2 - p_1}{v_L - v_H}$, the elderly customer chooses to purchase elderly services from provider 2. These conditions indicate that the elderly customer purchases elderly service $i$ as $U_i(\delta, \theta, p_i) = \max U_j(\delta, \theta, p_j)$. The demand for elderly services 1 is $d_1 = D\delta L$, where $\delta_L = G(\frac{\theta_2 (s_1 - s_2) + p_2 - p_1}{v_L})$; and for elderly services 2 is $d_2 = D\delta H$, where $\delta_H = 1 - G(\frac{\theta_2 (s_1 - s_2) + p_2 - p_1}{v_H})$. This assumption is consistent with the related literature[8]. The demand for elderly service $i$ is expressed as follows,

$$d_i = D\int_{\{\delta, \theta|U_i(\delta, \theta, p_i) = \max U_j(\delta, \theta, p_j)\}} dF(\delta) dG(\theta).$$  

Finally, according to Xiao’s research [40], assume that the government subsidises the service quality of elderly service providers by a factor of $m$ and $m \in (0, 1)$.

4. Model and analysis

For the different government subsidies, the game models to maximise elderly customers and elderly service providers profits are developed under two scenarios: the government-subsidised supply-side model (Scenario H) and the government-subsidised demand-side model (Scenario E). In addition, none of the members is subsidised (N Scenario), is a special case of the above two scenarios (when $m = 0$ is the N Scenario) and is not discussed additionally. The problem can be considered a game problem between the members. The equilibrium solutions are derived, and the effect of the government subsidy coefficient on the solution is analysed.

4.1. Government-subsidised supply-side model

This study first analyses the decision-making process of an elderly service provider in a scenario where the government subsidises the provider. The market demand for each provider is calculated based on the consumer demand function assumed in the previous section. The demand function is then brought into the profit function of each provider to solve for the optimal solution that maximises profit. According to the government-subsidised supply-side model, the demand for elderly services provided by elderly service providers 1 and 2 is as follows.
According to equations (1) and (2), when the elderly customers effectiveness is \( U_1 > U_2 \), the elderly customer purchases the elderly service from the elderly service provider 1 and obtains 0 < \( \delta < \frac{\theta(s_1 - s_2) + p_2 - p_1}{v_H - v_L} \). When the old customers effectiveness is \( U_1 < U_2 \), the elderly customer chooses to purchase the elderly service from the elderly service provider 2 and obtains \( \frac{\theta(s_1 - s_2) + p_2 - p_1}{v_H - v_L} < \delta < 1 \).

Therefore, the demand for each of the two types of elderly care services is

\[
d_1 = DG_1, \quad G_1 = \int_0^1 \int_0^{\frac{\theta(s_1 - s_2) + p_2 - p_1}{v_H - v_L}} d\delta d\theta = \frac{2p_2 - 2p_1 + s_1 - s_2}{2(v_H - v_L)} \tag{4}
\]

\[
d_2 = DG_2, \quad G_2 = \int_0^1 \int_0^{\frac{\theta(s_1 - s_2) + p_2 - p_1}{v_H - v_L}} d\delta d\theta = 1 + \frac{2p_1 - 2p_2 + s_2 - s_1}{2v_H - 2v_L} \tag{5}
\]

Then, we can obtain the profit for each of the elderly care services as follows:

\[
\Pi^H_1 = (p_1 - c_v v_L) d_1 - \frac{1}{2} k (1 - m) s_1^2 \tag{6}
\]

\[
\Pi^H_2 = (p_2 - c_v v_H) d_2 - \frac{1}{2} k (1 - m) s_2^2. \tag{7}
\]

**Proposition 1.** Using the superscript \( H \) to denote the government-subsidised supply-side scenario, the optimal pricing and optimal service quality under scenario \( H \) are obtained

\[
p^H_1 = \frac{(2c_v - 1)(D + 4k(1 - m)v_L) + Av_H - 4B}{2(D - 6k(1 - m)(v_H - v_L))} \tag{8}
\]

\[
p^H_2 = \frac{((1 + 2c_v)D + 4(4 + c_v)k(1 - m)v_L) + Av_H - 8B}{2(D - 6k(1 - m)(v_H - v_L))} \tag{9}
\]

\[
s^H_1 = \frac{D(1 + c_v)k(1 - m)(v_H - v_L)}{4k(1 - m)(D - 6k(1 - m)(v_H - v_L))} \tag{10}
\]

\[
s^H_2 = \frac{D(2 + c_v)k(1 - m)(v_H - v_L)}{4k(1 - m)(D - 6k(1 - m)(v_H - v_L))} \tag{11}
\]

*Note.* \( A = D + 4(2 - c_v)k(1 - m)v_L, B = (1 + c_v)k(1 - m)v_H^2 \).

**Proof.** The proof process of Proposition 1 is shown in the Appendix, and other propositions in the following paragraphs are the same. \( \square \)

According to Proposition 1, we can obtain the demand of elderly customers for the elderly services provided by elderly service provider 1 and elderly service provider 2 is

\[
d^H_1 = \frac{D(1 + c_v)k(1 - m)(v_H - v_L)}{2(D - 6k(1 - m)(v_H - v_L))} \tag{12}
\]

\[
d^H_2 = \frac{D(2 - c_v)k(1 - m)(v_H - v_L)}{2(D - 6k(1 - m)(v_H - v_L))}. \tag{13}
\]

**Lemma 4.1.** In the government-subsidised supply-side scenario, the impact of the government subsidy coefficient on price and demand is as follows:

(i) When \( c_v > \frac{1}{2} \), \( \frac{\partial p^H_1}{\partial m} > 0, \frac{\partial d^H_1}{\partial m} > 0 ; \frac{\partial p^H_2}{\partial m} < 0, \frac{\partial d^H_2}{\partial m} < 0 \); When \( c_v < \frac{1}{2} \), \( \frac{\partial p^H_1}{\partial m} < 0, \frac{\partial d^H_1}{\partial m} < 0 ; \frac{\partial p^H_2}{\partial m} > 0, \frac{\partial d^H_2}{\partial m} > 0 \).

(ii) \( |\frac{\partial p^H_1}{\partial m}| = |\frac{\partial p^H_2}{\partial m}| ; |\frac{\partial d^H_1}{\partial m}| = |\frac{\partial d^H_2}{\partial m}|. \)
Lemma 4.1 suggests that the extent to which the government subsidy coefficient affects the price and demand for elderly services varies, depending on the unit quality cost of the product in elderly services. Specifically, if the cost per unit of quality of the product is above a threshold, the price and demand for elderly care services of elderly care provider 1 will increase with the increase of the government subsidy coefficient. The price and demand for elderly care services of elderly care provider 2 will decrease with the increase of the government subsidy coefficient. Conversely, if the cost per unit of quality of the product is smaller than the threshold, the price and demand for elderly care services of elderly care provider 1 will decrease with the increase of the government subsidy coefficient. The price and demand for elderly care services of elderly care provider 2 will increase with the increase of the government subsidy coefficient. Additionally, the government subsidy component has a similar impact on both types of elderly services prices. The demand has the same effect.

**Lemma 4.2.** In the government-subsidised supply-side scenario, the impact of the government subsidy coefficient on service quality is as follows:

(i) When \( c_v > \frac{1}{2} \), \( \frac{\partial s_1}{\partial m} > 0 \);

   - When \( c_v < \frac{1}{2} \) and \( \frac{D}{k(1 - m)\Delta v} - 6 \) > 12(1 - 2c_v), \( \frac{\partial s_1}{\partial m} > 0 \);
   - When \( c_v < \frac{1}{2} \) and \( \frac{D}{k(1 - m)\Delta v} - 6 \) < 12(1 - 2c_v), \( \frac{\partial s_1}{\partial m} < 0 \).

(ii) When \( c_v < \frac{1}{2} \), \( \frac{\partial s_2}{\partial m} > 0 \);

   - When \( c_v > \frac{1}{2} \) and \( \frac{D}{k(1 - m)\Delta v} - 6 \) > 12(2c_v - 1), \( \frac{\partial s_2}{\partial m} > 0 \);
   - When \( c_v > \frac{1}{2} \) and \( \frac{D}{k(1 - m)\Delta v} - 6 \) < 12(2c_v - 1), \( \frac{\partial s_2}{\partial m} < 0 \).

(iii) \( \left| \frac{\partial s_2}{\partial m} \right| < \left| \frac{\partial s_1}{\partial m} \right| \).

Lemma 4.2 suggests that the extent to which the government subsidy coefficient affects the quality of elderly services varies. If the cost per unit of quality of the product is above a threshold, then the quality of elderly care services of elderly care provider 1 will increase with the increase of the government subsidy coefficient. The quality of elderly care services of elderly care provider 2 will increase, decrease and then increase rapidly as the government subsidy coefficient increases. Conversely, if the cost per unit of quality of the product is smaller than the threshold, the quality of elderly care services of elderly care provider 1 will increase, decrease and then increase rapidly as the government subsidy coefficient increases. The quality of elderly care services of elderly care provider 2 will increase with the increase of the government subsidy coefficient. Moreover, the service quality of elderly care provider 1 is more significantly impacted by the government subsidy coefficient than the case for elderly care provider 2.

Lemmas 4.1 and 4.2 show that when the cost per unit of product quality is above the threshold, that is, when local prices are higher, as the government subsidy coefficient increases, elderly service provider 1 is subsidised to improve its service quality and therefore attracts more quality-conscious elderly customers and setting higher prices to gain more profit. By contrast, elderly service provider 2 bears higher costs for product quality owing to better infrastructure, so government subsidies help to cover some of the cost pressures, and thus, prices fall. However, the overall service quality does not improve significantly, and older customers who focus on quality of service are lost. Conversely, when the unit product quality cost is below the threshold, that is, when local prices are low, and the infrastructure costs of the elderly service providers are low, the pricing, service quality and demand of the two elderly service providers are the exact opposite of the above.

It can be seen that when the overall economic situation of the region is better, government subsidies to improve the quality of services are beneficial to elderly care companies with poorer infrastructure but detrimental to those with better infrastructure. For more economically poor regions, government subsidies to reduce prices to increase the purchasing power of elderly customers are beneficial to elderly care service companies with better infrastructure but detrimental to those with poorer infrastructure. However, companies with poor infrastructure lower their prices. Furthermore, it is more efficient when the government subsidises companies with poor infrastructure.
Therefore, the government should decide according to the local economic situation when making subsidies. The government can increase the subsidies for elderly service provider 1 and strongly support small elderly companies so that they can have more financial support to improve the quality of elderly services. For elderly service provider 2, which represent large and medium-sized companies, the government also needs to subsidise them suitably. However, the purpose of the subsidy is to keep the price of elderly services down and to provide low-cost, high-quality elderly services to elderly customers.

4.2. Government-subsidised demand-side model

In regions with many elderly customers who are price sensitive, the government frequently uses financial subsidies (subsidies to the supply or demand side) to ensure that these elderly customers can access affordable but excellent elderly services. However, supply-side subsidies can result in better-off elderly customers crowding out of the welfare resources for price-sensitive older people. Thus, shifting from supply-side subsidies to demand-side subsidies that directly increase the purchasing power of price-sensitive elderly customers is necessary.

The decision-making process of the elderly service provider in a government-subsidised demand-side scenario is analysed as follows. According to the government-subsidised demand-side model, the demand for elderly services provided by elderly service providers 1 and 2 is as follows,

\[
U_1 = u + \delta v_L + 2 s_1 + m s_1 - p_1 \quad \text{(14)}
\]

\[
U_2 = u + \delta v_H + 2 s_2 + m s_2 - p_2. \quad \text{(15)}
\]

Therefore, the demand for the two types of elderly care services is:

\[
d_1 = DG_1, G_1 = \int_0^1 \int_{\frac{u_1 - u - m(s_1 - s_2) + p_2 - p_1}{v_H - v_L}}^{\frac{u_1 - u - m(s_1 - s_2) + p_2 - p_1}{v_H - v_L}} d\delta d\theta = \frac{2p_2 - 2p_1 + (1 + 2m)(s_1 - s_2)}{2(v_H - v_L)} \quad \text{(16)}
\]

\[
d_2 = DG_2, G_2 = \int_0^1 \int_{\frac{u_2 - u - m(s_2 - s_1) + p_2 - p_1}{v_H - v_L}}^{\frac{u_2 - u - m(s_2 - s_1) + p_2 - p_1}{v_H - v_L}} d\delta d\theta = 1 - \frac{2p_2 - 2p_1 + (1 + 2m)(s_1 - s_2)}{2(v_H - v_L)}. \quad \text{(17)}
\]

Then, we can obtain the profit for each of the elderly care services as follows:

\[
\Pi^E_1 = (p_1 - c_v v_L) d_1 - \frac{1}{2} k s_1^2 \quad \text{(18)}
\]

\[
\Pi^E_2 = (p_2 - c_v v_H) d_2 - \frac{1}{2} k s_2^2. \quad \text{(19)}
\]

Proposition 4.3. Using the superscript E to denote the government-subsidised demand-side scenario, the optimal pricing and optimal service quality under scenario E are obtained as:

\[
p_1^E = \frac{(2c_v - 1) \left( D(1 + 2m)^2 + 4k v_L \right) v_L + C v_H - 4D}{2 \left( D(1 + 2m)^2 - 6k(v_H - v_L) \right)} \quad \text{(20)}
\]

\[
p_2^E = \frac{(1 + 2c_v) D(1 + 2m)^2 + 4(4 + c_v) k v_L v_H - C v_L - 8D}{2 \left( D(1 + 2m)^2 - 6k(v_H - v_L) \right)} \quad \text{(21)}
\]

\[
s_1^E = \frac{D(1 + 2m) \left( D(1 + 2m)^2 - 4(1 + c_v) k(v_H - v_L) \right)}{4k \left( D(1 + 2m)^2 - 6k(v_H - v_L) \right)} \quad \text{(22)}
\]

\[
s_2^E = \frac{D(1 + 2m) \left( D(1 + 2m)^2 - 4(2 - c_v) k(v_H - v_L) \right)}{4k \left( D(1 + 2m)^2 - 6k(v_H - v_L) \right)} \quad \text{(23)}
\]
Note. \( C = D(1 + 2m)^2 + 4(2 - c_v)kv_L; \) \( D = (1 + c_v)kv_H^2. \)

Proof. The proof is similar to the proof of Proposition 1; therefore, it is omitted.

Furthermore, take the result of a Proposition 4.3 into equations (16) and (17), the demand of elderly customers for the elderly services provided by elderly service providers 1 and 2 is obtained as

\[
d^E_1 = \frac{D}{2} \left(\frac{D(1 + 2m)^2 - 4(1 + c_v)k(v_H - v_L)}{2(D(1 + 2m)^2 - 6k(v_H - v_L))}\right)
\]

\[
d^E_2 = \frac{D}{2} \left(\frac{D(1 + 2m)^2 - 4(2 - c_v)k(v_H - v_L)}{2(D(1 + 2m)^2 - 6k(v_H - v_L))}\right).
\]

Lemma 4.4. In the government-subsidised demand-side scenario, the impact of the government subsidy coefficient on price and demand is as follows,

(i) When \( c_v > \frac{1}{2}; \) \( \frac{\partial p_1}{\partial m} > 0, \frac{\partial d_1}{\partial m} > 0; \) \( \frac{\partial p_2}{\partial m} < 0, \frac{\partial d_2}{\partial m} < 0; \)

\( \frac{\partial p_1}{\partial c_v} > 0, \frac{\partial d_1}{\partial c_v} > 0; \) \( \frac{\partial p_2}{\partial c_v} < 0, \frac{\partial d_2}{\partial c_v} < 0; \)

When \( c_v < \frac{1}{2}; \) \( \frac{\partial p_1}{\partial m} < 0, \frac{\partial d_1}{\partial m} < 0; \) \( \frac{\partial p_2}{\partial m} > 0, \frac{\partial d_2}{\partial m} > 0. \)

(ii) \( \frac{\partial p_1}{\partial m} = |\frac{\partial p_1}{\partial m}|; \) \( \frac{\partial d_1}{\partial m} = |\frac{\partial d_1}{\partial m}|. \)

Proof. The proof is similar to the proof of Lemma 4.1, therefore, it is omitted.

Lemma 4.4 suggests that the extent to which the government subsidy coefficient affects the price and demand for elderly services varies, depending on the unit quality cost of the product in elderly services. Specifically, if the cost per unit of quality of the product is above a threshold, then the price and demand for elderly care services of elderly care provider 1 will increase with the increase of the government subsidy coefficient. The price and demand for elderly care services of elderly care provider 2 will decrease with the increases of the government subsidy coefficient. Conversely, if the cost per unit of quality of the product is smaller than the threshold, then the result is the exact opposite of the above.

Comparing Lemmas 4.1 and 4.4 reveals that the trend of the subsidy coefficients on the price and quantity of demand for elderly services is the same under both government subsidy scenarios.

Lemma 4.5. In the government-subsidised demand-side scenario, the impact of the government subsidy coefficient on service quality is as follows,

(i) When \( c_v > \frac{1}{2}; \) \( \frac{\partial s_1}{\partial m} > 0; \) When \( c_v < \frac{1}{2} and \left(\frac{(1+2m)^2D}{k\Delta v} + 2c_v - 7\right)^2 > 25 - 52c_v + 4c_v^2, \) \( \frac{\partial s_1}{\partial m} > 0; \)

When \( c_v < \frac{1}{2} and \left(\frac{(1+2m)^2D}{k\Delta v} + 2c_v - 7\right)^2 < 25 - 52c_v + 4c_v^2, \) \( \frac{\partial s_1}{\partial m} < 0. \)

(ii) When \( c_v < \frac{1}{2}, \) \( \frac{\partial s_2}{\partial m} > 0; \) When \( c_v > \frac{1}{2} and \left(\frac{(1+2m)^2D}{k\Delta v} - 5 - 2c_v\right)^2 > 4c_v^2 + 44c_v - 23, \) \( \frac{\partial s_2}{\partial m} > 0; \)

When \( c_v > \frac{1}{2} and \left(\frac{(1+2m)^2D}{k\Delta v} - 5 - 2c_v\right)^2 < 4c_v^2 + 44c_v - 23, \) \( \frac{\partial s_2}{\partial m} < 0. \)

(iii) When \( c_v > \frac{1}{2}, \) \( \frac{\partial s_1}{\partial m}| > |\frac{\partial s_1}{\partial m}|; \) When \( c_v < \frac{1}{2}, \) \( \frac{\partial s_2}{\partial m}| < |\frac{\partial s_2}{\partial m}|. \)

Proof. It is similar to the proof of Lemma 4.2; therefore, the process of proof is omitted.

Lemma 4.5 suggests that the extent to which the government subsidy coefficient affects the quality of elderly services varies. If the cost per unit of quality of the product is above a threshold, then the quality of elderly care services of elderly care provider 1 will increase with the increase of the government subsidy coefficient. The quality of elderly care services of elderly care provider 2 will increase, decrease and then increase rapidly as the government subsidy coefficient increases. Currently, the service quality of elderly service provider 1 is
more impacted by the government subsidy coefficient than that of elderly service provider 2. Conversely, if the
cost per unit of quality of the product is smaller than the threshold, then the quality of elderly care services of
elderly care provider 1 will increase, decrease and then increase rapidly as the government subsidy coefficient
increases. The quality of elderly care services of elderly care provider 2 will increase with the increase of the
government subsidy coefficient. Furthermore, the government subsidy coefficient has more impact on the service
quality of elderly service provider 2 than elderly service provider 1.

The trend in the subsidy coefficients influence on the quality of elderly care services for the two government
subsidy scenarios is similar, as shown by a comparison of Lemmas 4.2 and 4.5.

Therefore, when the government adopts demand-side subsidies for regions that are poorer, such the subsidy
policy will benefit the elderly service companies with better infrastructure but will be detrimental to those with
poorer infrastructure. In addition, companies with better infrastructure receive subsidised service improvements
more efficiently. In other words, direct government subsidies to the demand side will enable elderly customers to
enjoy better quality elderly care services at the same price. Conversely, for regions with better economic condi-
tions, a government that adopts demand-side subsidies will benefit elderly companies with poorer infrastructure
and will be detrimental to those with better infrastructure. In other words, the development of the lower-based
retirement service providers accelerated at this time.

5. COMPARATIVE ANALYSIS

The elderly customers who purchase elderly services can be broadly divided into two groups: price-insensitive
customers, such as those who are financially well off, and price-sensitive customers, such as those with low
incomes, low pensions or widows and disabled people. Price-sensitive customers tend to focus only on the price
of elderly services and less on other factors such as the quality of elderly services. For regions with a high number
of price-sensitive elderly customers, the local government needs to adopt a subsidy policy that reduces the price
of elderly services. On the contrary, for regions with a high number of price-insensitive elderly customers, the
governments choice of subsidy policy needs to improve the quality of local elderly services.

Therefore, local governments should choose their subsidy policies according to the actual local situation to
promote the stable development of the elderly services market. The following is a comparison of the price and
quality of elderly services under the two government subsidy scenarios discussed above to determine whether
the government should pick supply-side or demand-side subsidies in different conditions.

**Proposition 5.1.** The following is a comparison of the lowest prices for elderly services under the no govern-
ment subsidy and two government subsidy scenarios:

(i) When \( c_v > \frac{1}{2} \) and \( m \in [0, \frac{\sqrt{3}}{2}] \), if \( D < \frac{6k \Delta v}{(1+2m)^2} \) or \( D > 6k \Delta v \), \( p_1^N < \min(p_1^H p_1^E), p_2^E < \min(p_2^H p_2^N) \); if \( \frac{6k \Delta v}{(1+2m)^2} < D < 6k(1-m) \Delta v \), \( p_1^E < \min(p_1^H p_1^N) \), \( p_2^H < \min(p_2^E p_2^N) \); if \( 6k(1-m) \Delta v < D < 6k \Delta v \), \( p_1^H < \min(p_1^N p_1^E), p_2^N < \min(p_2^H p_2^E) \).

(ii) When \( c_v > \frac{1}{2} \) and \( m \in (\frac{\sqrt{3}}{2}, 1] \), if \( D < 6k(1-m) \Delta v \) or \( D > \frac{6k \Delta v}{(1+2m)^2} \), \( p_1^E < p_1^H, p_2^H < p_2^E \); if \( 6k(1-m) \Delta v < D < \frac{6k \Delta v}{(1+2m)^2} \), \( p_1^H < p_1^E, p_2^E < p_2^H \).

(iii) When \( c_v < \frac{1}{2} \) and \( m \in [0, \frac{\sqrt{3}}{2}] \), if \( D < \frac{6k \Delta v}{(1+2m)^2} \) or \( D > 6k \Delta v \), \( p_1^E < \min(p_1^N p_1^H) \), \( p_2^N < \min(p_2^H p_2^E) \); if \( \frac{6k \Delta v}{(1+2m)^2} < D < 6k(1-m) \Delta v \), \( p_1^H < \min(p_1^N p_1^E), p_2^E < \min(p_2^H p_2^N) \); if \( 6k(1-m) \Delta v < D < 6k \Delta v \), \( p_1^N < \min(p_1^H p_1^E), p_2^H < \min(p_2^E p_2^N) \).

(iv) When \( c_v < \frac{1}{2} \) and \( m \in (\frac{\sqrt{3}}{2}, 1] \), if \( D < 6k(1-m) \Delta v \) or \( D > \frac{6k \Delta v}{(1+2m)^2} \), \( p_1^H < p_1^E, p_2^E < p_2^H \); If \( 6k(1-m) \Delta v < D < \frac{6k \Delta v}{(1+2m)^2} \), \( p_1^E < p_1^H, p_2^H < p_2^E \).

For a more visual analysis, the above conclusions are simplified into Figures 1 and 2. N, H and E were used to
denote that none of the members is subsidised, the government-subsidised supply-side and demand-side models,
respectively.
Proposition 5.1 demonstrates that the pricing of two elderly service providers is related to the government subsidy coefficient, product unit quality cost and potential market demand. Firstly, the minimum price of elderly service provider 1 is analysed. When the unit quality cost of the product is greater than the threshold and the government subsidy coefficient is less than the threshold, the price of elderly service is lowest when the potential market demand is high. When the unit quality cost of the product is less than or equal to the threshold, the price of elderly service is lowest when the government subsidy coefficient is high. When the government subsidy coefficient is equal to the threshold, the price of elderly service is lowest when the potential market demand is in the middle.

\[ \frac{6k}{(1 + 2m)^2} \]

**Proof.** The proof process of Proposition 5.1 is in the Appendix. \square
size of the elderly service market is too small or too large without government subsidy. When the potential size of the market is small in the middle, the price of elderly services is lowest when the government subsidises the demand side. When the potential size of the market is large in the middle, the price of elderly services is lowest when the government subsidises the supply side. If the government subsidy is greater than the threshold, then the no government subsidy scenario does not exist. The price of elderly services is lowest when the government subsidises the demand side for too small or too large market size for elderly services; the price is lowest when the market size is between the two thresholds and the government subsidises the supply side.

Conversely, when the cost per unit of product quality is greater than the threshold value, if the government subsidy coefficient is less than the threshold value, then the price of elderly services is lowest when the government subsidises the demand side under the too small or too large potential market size of elderly services. Moreover, when the potential market size is small in the middle, the price of elderly services is lowest when the government subsidises the supply side. Then, when the potential market size is large in the middle, the price of elderly services is lowest without government subsidy. If the government subsidy is above the threshold, then the situation is the opposite of the second situation above.

The minimum price scenario for elderly service provider 2 is completely opposed to that of elderly service provider 1 and is thus omitted.

To sum up, when there are more price-sensitive elderly customers in a certain region, government subsidies are often used to reduce the price of elderly services. In this case, government subsidies are more effective in regions with small or large elderly populations. Supply-side subsidies by the government are more effective in regions with a medium-sized elderly population. On the contrary, if the local government has a very adequate budget for old age subsidies, it is more effective to adopt supply-side subsidies for regions with a small or large elderly population; demand-side subsidies are more effective for regions with a medium elderly population. For economically better-off regions, the minimum price subsidy policy is the complete opposite for economically poorer regions.

For elderly service provider 2, the situation is the complete opposite of that of elderly service provider 1. Moreover, considering its better infrastructure and the relatively higher quality costs of the products it undertakes, the overall price will be higher. Thus, for price-sensitive regions, the demand for elderly care service provider 2 is relatively low.

Therefore, when making decisions on subsidies, the government should consider not only the local price level but also the potential size of the local market for elderly services.

**Proposition 5.2.** The following is a comparison of the quality of elderly care services under the no government subsidy and two government subsidy scenarios:

(i) When $E > F$, $s^H_1 > s^E_1$;

(ii) When $E < F$, $s^H_1 < s^E_1$;

(iii) When $E < -F$, $s^H_2 > s^E_2$;

(iv) When $E > -F$, $s^H_2 < s^E_2$.

**Note.** $E = \frac{4(1-2c)c_kk\Delta v(D(1+2m)+3k(1-2m)\Delta v)}{(D(1+2m)^2-6k\Delta v)(D-6k(1-m)\Delta v)}$; $F = \frac{1-2m}{1-m}$

**Proof.** The proof is similar to the proof of Proposition 5.1; therefore, it is omitted. \qed

According to Proposition 5.2, the service quality of the two elderly care providers is correlated with the government subsidy coefficient, the cost of the product per unit of quality and the potential market demand. As the subsidy coefficient increases, the quality of elderly services, when the government subsidises the elderly service provider, first less than and then greater than the quality of elderly services when the government subsidises the elderly customer. When the government subsidy coefficient is large enough, the quality of elderly services again less than the quality of elderly services when the government subsidises the elderly customer. The change in the quality of elderly care services provided by elderly care provider 2 under the various subsidy scenarios is exactly the same as that of elderly care provider 1.
The findings demonstrate that subsidies can be quite successful in helping the government improve the quality of elderly care. When the government aims to improve the quality of elderly services with financial subsidies, choosing to subsidise elderly customers if the financial subsidy budget is small is the most effective. In addition, when the budget for elderly subsidies is sufficient, subsidising elderly service providers can be more effective in improving the quality of elderly services.

6. Numerical simulation

This section provides numerical analyses to verify the effect of government subsidies on the optimal decision of the elderly service provider under different subsidy scenarios. Considering the parameter meanings and assignment conditions, let the parameters in the model take the values $D = 50; c_v = 1; v_L = 80; v_H = 150; k = 4$. the optimal outcome changes when the government subsidises the supply and demand side, as shown in Figure 3.

Figure 3a shows that the pricing of elderly service provider 1 increases with an increase in the government subsidy coefficient in the government-subsidised supply-side and demand-side scenarios. When the government subsidy coefficient is less than the threshold $\sqrt{3}/2$, the price is lowest when the government subsidies the supply side. Conversely, when the subsidy coefficient is more than the threshold $\sqrt{3}/2$, the price is lowest when the government subsidises the demand side. Figures 3a and 3b show that the price change trend of elderly service provider 2 is exactly opposite that of elderly service provider 1. Furthermore, the price of elderly service provider 2 is always higher than that of elderly service provider 1.

Figures 3c and 3d show that the service quality of both providers improves as the government subsidy coefficient increases, with the supply-side subsidy improving the most. Moreover, the larger the government subsidy coefficient, the greater the difference in service quality between the supply-side and demand-side subsidy scenarios. The results also show that the quality of elderly care services offered by elderly care provider 2 is always worse than that of elderly care provider 1.

Figures 3e and 3f show that as government subsidies increase, the demand for elderly services from elderly service provider 1 increases, whereas that for elderly services from elderly service provider 2 decreases. Moreover, demand-side subsidies stimulate market demand for elderly services significantly better than supply-side subsidies for elderly service provider 1, whereas the opposite is true for elderly service provider 2. Additionally, under the same subsidy scenario, the degree of the positive impact of the subsidy coefficient on the demand of elderly service provider 1 was equal to its degree of negative impact on the demand of elderly service provider 2, with both providers demand showing a reciprocal trend of increasing and decreasing.

As a result, government subsidies are currently favourable for the poorer-infrastructure senior care provider 1 but unfavourable for the better-infrastructure senior care provider 2. We recall all lemmas that the examples of numerical analysis correspond to the economically better-off areas in the four lemmas above. Therefore, the numerical analysis verifies the above conclusions.

7. Management insights and conclusions

7.1. Management insights

Based on the results of the above numerical analysis, the major findings of this study that managers can use in practice are:

(i) The governments decision to focus on subsidising the development of a particular type of elderly care company should be based on the economic development of the local region. In regions with better economic conditions, government subsidies are beneficial to elderly care providers with poorer infrastructure but disadvantageous to those with better infrastructure. Conversely, in regions with poorer economic conditions, government subsidies are beneficial to providers with better infrastructure but disadvantageous to those
with poorer infrastructure. In conclusion, when subsidising different elderly service providers, the government needs to focus on subsidising a certain type of elderly service business based on the local economic situation to ease the financial pressure and fully develop the local elderly companies.

(ii) If the government subsidises to reduce the price of elderly services, then its decision should be based on the potential size of the local elderly market. For regions with backward economic development, if the number of elderly people is low and the supply exceeds the demand, the government does not need to subsidise elderly care businesses. If the number of elderly people is high, there is also no need to subsidise elderly care service providers, with the option of subsidising elderly customers to increase their purchasing power. Then, if the number of elderly people is at a medium level, elderly care companies need to be subsidised to directly reducing prices. Conversely, for areas with better economic conditions, where the number of elderly people is higher or lower, the government does not have to subsidise, and prices are reduced to a minimum. In conclusion, government subsidy policies for different regions need to be based not only on the local economic situation but also on the number of elderly people.

(iii) When the government makes financial subsidies to improve the quality of elderly services, decisions should be based on the financial subsidy budget. If the financial subsidy budget is smaller, then subsidising elderly...
customers is most effective. Then, when the financial subsidy budget is enough, subsidising elderly service providers can be more effective in improving the quality of elderly services.

7.2. Conclusions

As China’s ageing process accelerates, the rapid development of the elderly care industry should be promoted. Therefore, government subsidy policies must be studied and improved to encourage the rapid development of elderly care service enterprises. This study first discusses the optimal decision-making of elderly service providers under two scenarios: the government-subsidised supply side (elderly service providers) and the government-subsidised demand side (elderly customers). Then, the study analyses the impact of government subsidy coefficients on the decision-making outcomes of elderly service providers. Considering the different purposes of government subsidies for elderly care services, the price and quality of elderly care services under different government subsidy scenarios are compared. Moreover, further numerical analysis was combined to verify the impact of the subsidy coefficient on the price, quality and demand for elderly services under two government subsidy scenarios. The results show that the economic situation of the region and the strategic objectives of the local government are important factors influencing the effectiveness of the government’s subsidy strategy. If the areas economic situation is great, then government subsidies will benefit eldercare businesses with poorer infrastructure but be detrimental to those with better infrastructure. Conversely, if the areas economic situation is poor, then the subsidy policy has the opposite effect. Moreover, if the government plans to reduce prices more effectively, then it should adopt a demand-side subsidy policy for areas with a small or large elderly population and a supply-side subsidy policy for areas with a medium elderly population. When the budget for financial subsidies is limited, demand-side subsidies are more effective in enhancing the quality of elderly care services, whereas when the government subsidy budget is sufficient, supply-side subsidies are more effective.

This research can be extended in several directions in the following works. Firstly, there are large differences that exist within the elderly groups in the regions. Therefore, segmenting the elderly groups in a particular region can be considered in future research. That is, decisions on the scope of subsidies, subsidy rates, and others need to be made in the context of the actual situation in developing elderly subsidy policies. In addition, future studies may consider a mix of subsidy policies based on the governments’ financial situation and local elderly care problems to further improve government subsidy policies and promote the development of the elderly care service market.

Appendix A. Proof of Section 4

A.1. Proof of Proposition 1

The decisions between members are game with complete information, and we obtain the solutions of price and quality of elderly care services by backward induction. Firstly, the demand function equations (4) and (5) are taken into the profit function equations (6) and (7) of the elderly service provider. At this time, the Hessian matrix of $\Pi_1^H$ with respect to $p_1, s_1$ is $H(p_1, s_1) = \left( \frac{-v_H - v_L}{D} \right)$. Since $\Delta v = v_H - v_L > \frac{D}{8k(1-m)}$, we can obtain that the Hessian matrix is negative definite, and the function has a maximum value with respect to $p_1, s_1$. Thus, we let $\frac{\partial \Pi_1^H}{\partial p_1} = 0$ and $\frac{\partial \Pi_1^H}{\partial s_1} = 0$, obtaining $p_1 (p_2, s_2)$ and $s_1 (p_2, s_2)$. Similarly, since $\Delta v = v_H - v_L > \frac{D}{8k(1-m)}$, we can obtain that the Hessian matrix $H(p_2, s_2)$ is negative definite. Let $\frac{\partial \Pi_1^H}{\partial p_2} = 0$ and $\frac{\partial \Pi_1^H}{\partial s_2} = 0$, we can obtain $p_2 (p_1, s_1)$ and $s_2 (p_1, s_1)$.

Therefore, the optimal solution for the quality and price of elderly services is obtained by combining equations $p_1 (p_2, s_2)$, $s_1 (p_2, s_2)$, $p_2 (p_1, s_1)$ and $s_2 (p_1, s_1)$. Then, the optimal solution obtained above is then brought into equations (4) and (5) respectively to find the demand under the optimal decision.
A.2. Proof of Lemma 4.1

Taking first-order derivatives on from the result of Proposition 1 as follow:

\[
\frac{\partial p_1}{\partial m} = \frac{\partial d_1}{\partial m} = \frac{(-1 + 2c_v)Dk(v_H - v_L)^2}{(D - 6k(1 - m)(v_H - v_L))^2};
\]

\[
\frac{\partial p_2}{\partial m} = \frac{\partial d_2}{\partial m} = \frac{(1 - 2c_v)Dk(v_H - v_L)^2}{(D - 6k(1 - m)(v_H - v_L))^2}.
\]

As \(\frac{\partial p_1}{\partial m} = \frac{(1 - 2c_v)Dk(v_H - v_L)^2}{(D - 6k(1 - m)(v_H - v_L))^2} > 0\), we can obtain \(2c_v - 1 > 0\), namely \(c_v > \frac{1}{2}\). Similarly, the others can be proven.

When \(c_v > \frac{1}{2}\), we can obtain \(|\frac{\partial p_1}{\partial m} - \frac{\partial p_2}{\partial m}| = 0\), namely \(|\frac{\partial p_1}{\partial m}| = |\frac{\partial p_2}{\partial m}|\). Conversely, when \(c_v < \frac{1}{2}\), \(\frac{\partial p_1}{\partial m} = \frac{\partial p_2}{\partial m}\).

To sum up \(|\frac{\partial p_1}{\partial m}| = |\frac{\partial p_2}{\partial m}|\). Similarly, it can be proven that \(|\frac{\partial p_1}{\partial m}| = |\frac{\partial p_2}{\partial m}|\).

A.3. Proof of Lemma 4.2

Taking first-order derivatives on from the result of Proposition 1 as follow:

\[
\frac{\partial s_1}{\partial m} = \frac{D(D^2 - 12k(1 - m)(v_H - v_L)(D - 2(c_v + 1)k(1 - m)(v_H - v_L)))}{4k(m - 1)^2(D - 6k(1 - m)(v_H - v_L))^2};
\]

\[
\frac{\partial s_2}{\partial m} = \frac{D(D^2 - 12k(1 - m)(v_H - v_L)(D - 2(2 - c_v)k(1 - m)(v_H - v_L)))}{4k(m - 1)^2(D - 6k(1 - m)(v_H - v_L))^2}.
\]

Adjusting the numerator of \(\frac{\partial s_1}{\partial m}\) gives \(D((D - 6k(1 - m)\Delta v)^2 + 12(2c_v - 1)k^2(1 - m)^2\Delta v^2)\). When \(c_v > \frac{1}{2}\), we can obtain \((D - 6k(1 - m)\Delta v)^2 > 12(1 - 2c_v)k^2(1 - m)^2\Delta v^2\), thus \(\frac{\partial s_1}{\partial m} > 0\). When \(c_v < \frac{1}{2}\) and \((D - 6k(1 - m)\Delta v)^2 + 12(2c_v - 1)k^2(1 - m)^2\Delta v^2 > 0\), namely \((\frac{D}{k(1 - m)\Delta v} - 6)^2 > 12(1 - 2c_v)\), \(\frac{\partial s_1}{\partial m} < 0\) can be obtained. Similarly, adjusting the numerator of \(\frac{\partial s_2}{\partial m}\) gives \(D((D - 6k(1 - m)\Delta v)^2 + 12(1 - 2c_v)k^2(1 - m)^2\Delta v^2)\). Other steps are the same as the above proof.

In addition, \(|\frac{\partial s_1}{\partial m}| = \frac{D^2 - 12k(1 - m)(v_H - v_L)(D - 2(c_v + 1)k(1 - m)(v_H - v_L))}{D^2 - 12k(1 - m)(v_H - v_L)(D - 2(2 - c_v)k(1 - m)(v_H - v_L))} > 1\) can be calculated to obtain. Therefore, \(\frac{\partial s_2}{\partial m} < \frac{\partial s_1}{\partial m}\).

APPENDIX B. PROOF OF SECTION 5

B.1. Proof of Proposition 5.1

According to Propositions 1 and 4.3, we can obtain

\[
p_1^H = \frac{(2c_v - 1) Dkm (4m^2 - 3) (v_H - v_L)^2}{(D - 6k(1 - m)(v_H - v_L))(D + 2m)^2 - 6k(v_H - v_L)};
\]

\[
p_1^N = \frac{(1 - 2c_v) Dkm(v_H - v_L)^2}{(D - 6k(1 - m)(v_H - v_L)(D - 6k(v_H - v_L))};
\]

\[
p_1^N = \frac{4(1 - 2c_v) Dkm (1 + m)(v_H - v_L)^2}{(D - 6k(v_H - v_L))(D + 2m)^2 - 6k(v_H - v_L)}.\]
When $2c_v - 1 > 0$ and $4m^2 - 3 < 0$, namely $c_v > \frac{1}{2}$ and $m \in \left[0, \frac{\sqrt{3}}{2}\right]$, since $m$ can be taken to 0, the no government subsidy scenario at $m = 0$ is added together for comparison. At this point, if $D - 6k (1 - m) (v_H - v_L) > 0$ and $D(1 + 2m)^2 - 6k (v_H - v_L) > 0$, we can easily prove $p_H^1 - p_E^1 < 0$, namely $p_H^1 < p_E^1$. Further, compare $p_H^1$ and $p_N^1$, we can obtain when $D - 6k \Delta v > 0$, $p_N^1 = p_H^1 < 0$ and when $D - 6k \Delta v \leq 0$, $p_N^1 = p_H^1 > 0$. When $c_v > \frac{1}{2}$ and $m \in \left[0, \frac{\sqrt{3}}{2}\right]$, if $D - 6k (1 - m) (v_H - v_L) < 0$ and $D(1 + 2m)^2 - 6k (v_H - v_L) > 0$, we can easily prove $p_H^1 - p_E^1 > 0$, namely $p_H^1 > p_E^1$. Then compare $p_H^1$ and $p_N^1$, we can obtain $p_H^1 > p_N^1$, namely $p_H^1$ is the minimum. When $c_v < \frac{1}{2}$ and $m \in \left[0, \frac{\sqrt{3}}{2}\right]$, if $D - 6k (1 - m) (v_H - v_L) < 0$ and $D(1 + 2m)^2 - 6k (v_H - v_L) < 0$, $p_H^1 - p_E^1 < 0$ can be proven, and then compare $p_H^1$ and $p_N^1$. The result is that $p_H^1$ is the minimum. Similarly, three situations can be proved when $c_v > \frac{1}{2}$ and $m \in \left[\frac{\sqrt{3}}{2}, 1\right]$, $c_v < \frac{1}{2}$ and $m \in \left[0, \frac{\sqrt{3}}{2}\right]$, $c_v < \frac{1}{2}$ and $m \in \left[\frac{\sqrt{3}}{2}, 1\right]$. For proof of elderly service provider 2 is similar to the proof of above; therefore, it is omitted.

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