Abstract. In this paper, the differential game model is constructed to study the coordination problem of platform supply chain by introducing the impacts of Big Data marketing and reference promotion effect on consumer conversion rate. Firstly, the optimal strategies and the profits under centralized and decentralized modes are given by applying the optimal control theory, and the comparative analyses are carried out. Subsequently, in order to coordinate the platform supply chain, a combined contract including a two-part tariff scheme and a promotion cost sharing scheme is designed. Finally, the effects of system parameters on equilibrium strategies and coordination contract are analyzed. The results show that the respective decisions of the manufacturer and the e-commerce platform as well as the total profit of the supply chain are higher under centralized mode. Moreover, within the feasible region, the combined contract not only achieves channel coordination but also improves the economic situations of channel members. It can be also observed that the coordination capacity of the proposed contract reduces with an increase in the memory parameter and improves with an increase in parameters such as the effectiveness of the Big Data marketing and the effectiveness of the reference promotion effect.

Mathematics Subject Classification. 91A40, 90B60.

Received November 18, 2023. Accepted February 3, 2024.

1. Introduction

Over the past two decades, the online shopping has become increasingly popular with the development of e-commerce. The scale of online shopping users in China has reached 845 million by December 2022. Accordingly, the e-commerce platforms such as Tmall, JD.com, Suning.com, have changed their business modes, i.e., from a traditional retail to platform business one. Under this mode, the manufacturers are allowed to sell their products to the users on platform through a revenue-sharing scheme [1]. So far, this platform mode has been widely adopted because it not only meets the consumers’ needs of fast and convenient shopping, but also provides more opportunities for manufacturers to explore the market. According to statistics in 2022, the number of new merchants on JD.com has revealed 34% year-on-year growth and the revenue brought by the platform mode have accounted for 51.5% of the total platform revenue, which reflect that the platform mode has become
an important business mode for JD.com. In addition, with the introduction of a large number of sellers and buyers on the platform, a huge online marketplace has gradually formed, and the data on various transactions and user behavior have been recorded. In order to help manufacturers improve their transaction rates and optimize the purchase experience of consumers, the e-commerce platform adopts its own Big Data technology to provide the marketing services [2]. Through its own accumulation of massive user sales information, the e-commerce platform can use Big Data technology to identify potential consumer groups, provide accurate personalized recommendation service and enhance the purchase intentions for them [3]. For instance, JD.com use Big Data technology to implement meticulous analysis and in-depth evaluation of the data on consumers’ online browsing records, transaction records and others. On this basis, a personalized recommendation system has been established to effectively assist the consumers in discovering their preferred products quickly. The fact shows that the purchasing efficiency of consumers and product sales of merchants can be improved, and the degree of customer satisfaction can increase.

In practice, in order to achieve win–win cooperation with e-commerce platforms, the manufacturers also carry out price discounts, coupon discounts, buy-gift activities, member rewards and other promotional activities to attract potential consumer in the holidays such as 6.18, Double 11, and Double 12 [4]. According to the statistics, the transaction value of the whole Internet on Double 11 in 2022 amounted to 557.1 billion RMB, which set a new record. That is to say, this campaign significantly boosted the product sales and proved the effectiveness of online promotional strategies. However, frequent promotional activities will produce a reference promotion effect in the minds of consumers and affect the effectiveness of future promotional activities [5]. When making a purchase decision, the consumers will compare the current promotion level with the reference one. If the current promotion level is lower than the reference one, the consumers will think that it is a small promotion and postpone or abandon the purchase. Conversely, the consumers will regard it as an attractive promotion activity and choose to buy immediately. For instance, the daily discount of Lancome beauty brand in Chinese counters is 10% off, which is always regarded as a reference promotion point. However, during the Double 11 holiday, the brand offers 20% off all its products, and even most of the products carry out a buy-one-get-one free campaign. At this time, the promotional activities successfully satisfy the consumers' inner desires and drive them to make a purchase decision quickly. Therefore, it is necessary for managers to consider the consumers’ reference promotion effect when conducting the promotional activities.

Although the abundant efforts have been devoted concerning on promotion problem in marketing channels, the promotion reference effect has been ignored. In fact, the reference promotion effect not only affects on current sales, but also has a lasting impact on future profits. Moreover, in the past decade, the combination of the Big Data marketing and promotional campaigns is used to influence consumer purchasing behavior. At this point, the channel members are likely to adjust their promotion and Big Data marketing strategies to eliminate the adverse impact of promotion reference effect. In this study, we develop a differential game in a platform supply chain with reference promotion effect and Big Data marketing. Our objective is to provide a better understanding of reference promotion and Big Data marketing. In particular, we address the following questions:

(1) In the presence of reference promotion effect, what are the equilibrium strategies of the manufacturer and the e-commerce platform? What is the influence of the reference promotion effect on the channel profit?
(2) How to design an effective coordination contract? Under what conditions does the proposed contract make both parties better off?
(3) How do the key parameters such as reference promotion factor and memory parameter affect equilibrium strategies and channel profits as well as coordination contract?

Based on the above issues, the main contributions of the research in this paper are summarized as follows:
- A differential game model is constructed with the consumers’ reference promotion point and consumer conversion rate as state variables;
- By applying the optimal control theory, the Big Data marketing strategies, promotion strategies, time trajectories of reference promotion point, time trajectories of consumer conversion rate and profits of supply
chain under centralized and decentralized decision modes are given respectively, and comparative analyses are carried out:
– A combined contract including a two-part tariff scheme and a promotion cost sharing scheme is designed to improve the performance of the decentralized platform supply chain and achieve the channel coordination.

The rest of the paper is organized as follows. Section 2 provides the literature review and research gaps. Section 3 provides the problem statement and model establishment. In Section 4, the equilibrium solutions in the centralized and decentralized modes are stated, and their relationship is revealed. In Section 5, the combined contract is designed. Section 6 presents the numerical results. Section 7 describes the managerial insights and practical implications. Finally, the conclusions are drawn in Section 8.

2. Literature review

The literature involved in this paper is mainly reflected in three aspects: platform supply chain, Big Data marketing, and reference effects. After analyzing the literature covered, we try to determine research gaps and propose the issue of this study.

2.1. Platform supply chain

In recent years, the platform supply chain management mode has attracted an increasing research attention due to its wide applications in the e-commerce [6–8]. For example, the optimal promotional strategies under three scenarios are given in [9], and it has revealed that launching promotion activities proactively is not always advantageous for the platform or seller. In [10], the optimal pricing problem has been discussed by considering the effects of online reviews and competition between manufacturers. Moreover, the optimal logistics service strategy has been provided in [11], and it has found that the logistics service cost plays an essential role in formulating the optimal logistics strategy. In addition, the issues of design and coordination for supply chain network have attracted considerable attention and some effective results have been given in [12–17]. For instance, the supply chain network design problem has been studied in [16] by constructing an interesting multi-objective mathematical programming model, where the scenario-based robust optimization method has been used to deal with uncertainty. In [17], the influence of manufacturers’ fairness concern behavior on optimal decisions and coordination in the platform supply chain has been discussed by using the game theory, where the results have revealed that the manufacturers’ fairness concern behavior has negative impact on the incomes of supply chain members. Further, the problem of demand information sharing in the platform supply chain has been discussed in [18], where it has found that whether an e-tailer chooses to share demand information is closely related to the supplier’s offline entry cost, channel substitution rate, and information uncertainty. Subsequently, by introducing the preference behavior of consumer, the optimal service quality strategy has been given in [19]. The results indicated that the service quality level of the e-commerce platform under decentralized decision mode is higher than the one in the centralized decision mode, which is different from previous conclusions. By reviewing the relevant literature, we have found that there is a lack of attention to Big Data marketing strategies and consumers’ reference behavior in platform supply chain.

2.2. Big Data marketing

With the advent of the Internet era, the Big Data technology has been widely adopted in many areas such as market, business, and so on [20, 21]. The application of Big Data technology in intelligent transportation system has been analyzed in [22], and it has revealed that the efficiency and safety of intelligent transportation system can be improved by using Big Data technology. In [23], an application framework of Big Data technology has been proposed to build a robust data-driven agri-food supply chain. Also, the early warning effect of Big Data technology on disease prevention of users has been proved in [24]. In recent years, many studies have addressed the issue related to application of Big Data in the marketing field [25]. Specifically, in [26], how to use the Big Data technology to gain insights with respect to behaviors and preferences of consumers and then
help the enterprises carrying out marketing activities more accurately has been discussed. The results in [27] demonstrated that the use of Big Data not only helped companies in marketing analysis but also improved their operational capabilities. In brief, the above studies have focused primarily on the Big Data from the standpoint of empirical analysis. In [28], the pricing method for the green supply chain with Big Data marketing has been given, and the corresponding change trend of prices has been provided from a theoretical perspective. Furthermore, in [29], the dynamic cooperation problem in a closed-loop supply chain with the involvement of an internet service platform has been further investigated. The results show that the Big Data marketing from Internet service platform can increase product sales and facilitate the recall of used products, which is conducive to the environmental sustainability as well as business operations. However, although the Big Data services of third-party platforms have been analyzed in the above-mentioned references, the fact that the e-commerce platforms in platform supply chain also are providers of Big Data marketing services has been ignored.

2.3. Reference effects

Currently, the examinations of reference effect mainly focuses on reference price, reference quality, and reference low-carbon [30–32]. The reference price refers to a comprehensive product evaluation in consumer’s mind and is affected by a large number of variables including the historical price, advertising campaign, product quantity, etc [33]. Therefore, the reference price has received considerable research attention due to its important influence on consumers’ purchase behavior. In [34], the price promotion and advertising investment strategies have been discussed for the supply chain with the reference price effect, where it has revealed that the impact from memory factor onto the equilibrium strategy and the total channel profits has been revealed. In [35], the reference price effect has been introduced into the green supply chain and the optimal results under different contracts have been compared and analyzed. The results showed that the cost-sharing contract is superior to the wholesale price contract in improving product greenness and demand when considering the reference price effect of consumers. Subsequently, the idea of the reference effect has been extended to product quality and low-carbon. In [36], the dynamic quality investment program has been given for a distributed channel by taking the reference quality into account. In addition, the coordination problem of dual-channel supply chain considering the reference quality effect has been further discussed in [37]. On the other hand, under the background of a low-carbon economy, the reference low-carbon effect of consumers has become a factor that cannot be ignored. In [38], the impact of reference low-carbon effect on the joint carbon reduction strategies has been examined, the results showed that the reference low-carbon effect can help companies achieve their low-carbon management goals better. In addition, the joint influences of reference low-carbon effect and altruistic behavior on carbon emission reduction decisions of the supply chain have been considered in [39]. In the previous studies, it has been observed that the reference effect in promotion is ignored. In [40], the dynamic evolution process of reference promotion point has been described, and the equilibrium strategies have been given by introducing impact of reference promotion effect on the demand. So far, the studies on the coordination problem of the supply chain with reference promotion effect are still relatively few.

2.4. Research gaps

The related studies and the systematic comparison between this paper and related studies are tabulated in Table 1. As shown in Table 1, it should be noted that it is very important to discuss the promotion and the Big Data marketing campaigns due to the growing transparency of information in the Internet era. Unfortunately, there is insufficient attention to platform supply chain with reference promotion effect and Big Data marketing. Moreover, the complexity of reference effect and marketing process often leads to the situation that the traditional contracts lose their coordination ability. Therefore, in order to shorten this research gap, the coordination contract design issue of a platform supply chain subject to reference promotion effect and Big Data marketing will be discussed by using differential game theory in this paper.
Table 1. A comparison of the present work with related previous works.

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3. Problem statement

3.1. Problem statement

We consider a platform supply chain consisting of a manufacturer \((M)\) and an e-commerce platform \((R)\). Under the platform selling mode, the manufacturer directly distributes its products to consumers through the e-commerce platform and compensates the platform with a fixed commission for each unit sold. In order to gain more benefits, the manufacturer and the e-commerce platform invest in promotion and Big Data marketing respectively to increase the conversion rate of potential consumers. Let the manufacturer’s promotion effort is \(P(t)\) and the e-commerce platform’s Big Data marketing effort is \(B(t)\) at time \(t\). In addition, the e-commerce platform charges the manufacturer for the Big Data marketing services, which is denoted as \(F\). The model structure is shown in Figure 1.
3.2. Assumptions

Before establishing modes, we first present the following assumptions:

**Assumption 3.1.** As in [41], the costs of both promotion effort and Big Data marketing effort can be described by quadratic functions, i.e.,

\[
C(P(t)) = \frac{1}{2}c_P P^2(t), \quad C(B(t)) = \frac{1}{2}c_B B^2(t),
\]

where \(c_P > 0\) and \(c_B > 0\) represent the cost coefficients.

**Assumption 3.2.** As described in [40], when consumers decide whether to purchase a product, they will compare the current level of promotion effort with the expected level. The consumers’ expectation of the promotion effort level is called the reference promotion point, which is dynamically adjusted with the difference between promotion effort and reference point. Thus, the evolution equation of reference promotion point \(r(t)\) can be described as

\[
\dot{r}(t) = \eta(P(t) - r(t)),
\]

where \(\eta > 0\) is the “memory parameter”. A larger \(\eta\) indicates that the consumer has a shorter memory of past promotional effort and purchase experiences. Besides, \(r(0) = r_0\) depicts the initial reference promotion point.

**Assumption 3.3.** The platform’s Big Data marketing and the manufacturer’s promotion play an important role in increasing consumer conversion rate. On this basis, we further introduce the influence of reference promotion effect on the consumer conversion rate. Thus, the consumer conversion rate \(S(t)\) is given by:

\[
\dot{S}(t) = \alpha B(t) + \beta P(t) + \gamma (P(t) - r(t)) - \delta S(t),
\]

where the consumer conversion rate satisfies \(0 \leq S(t) \leq 1\), \(S(0) = S_0\) is the initial conversion rate, \(\alpha > 0\) denotes the effectiveness of the Big Data marketing effort, \(\beta > 0\) is the effectiveness of the promotion effort, and \(\gamma > 0\) stands for the effectiveness of the reference promotion effect. If \(P(t) - r(t) > 0\), it indicates that the reference promotion effect positively influences the consumer conversion rate. \(\delta > 0\) represents the decay rate of the consumer conversion rate.

3.3. Model establishment

The product demand \(D(t)\) is assumed to linearly depend on the consumer conversion rate \(S(t)\), i.e.,

\[
D(t) = D_0 + NS(t),
\]

where \(D_0 > 0\) represents base demand, \(N > 0\) is the number of potential consumers who may be induced to consume through Big Data marketing and promotional activities.

Let \(\pi\) be the manufacturer’s marginal revenue for the unit product sold and \(\varphi\) be the commission paid by the manufacturer to the e-commerce platform with the commission rate \(\varphi\). Therefore, for an infinite horizon with discount rate \(\rho > 0\), the profit functions of the whole supply chain can be formulated as follows: The profit functions of the manufacturer and the e-commerce platform can be respectively formulated as follows:

\[
J_M = \int_{0}^{+\infty} e^{-\rho t} [ (1 - \varphi) \pi D(t) - C(P(t)) - F ] dt,
\]

\[
J_R = \int_{0}^{+\infty} e^{-\rho t} [ \varphi \pi D(t) - C(B(t)) + F ] dt,
\]

\[
J_C = \int_{0}^{+\infty} e^{-\rho t} [ \pi D(t) - C(P(t)) - C(B(t))] dt.
\]

Moreover, in order to enhance the readability, the notations used in this research are summarized in Tables 2 and 3.
Table 2. Indices and variables.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
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<tbody>
<tr>
<td>Superscript</td>
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<tr>
<td>C</td>
<td>Centralized decision mode</td>
</tr>
<tr>
<td>D</td>
<td>Decentralized decision mode</td>
</tr>
<tr>
<td>E</td>
<td>Combined contract mode</td>
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<tr>
<td>*</td>
<td>Optimal</td>
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<td>M</td>
<td>Manufacturer</td>
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<td>R</td>
<td>E-commerce platform</td>
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<td>Decision variables</td>
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<td>Promotion effort</td>
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<td>B(t)</td>
<td>Big Data marketing effort</td>
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<td>Dependent variables</td>
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<td>r(t)</td>
<td>Reference promotion point</td>
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<td>S(t)</td>
<td>Consumer conversion rate</td>
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<td>J_M</td>
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<td>J_R</td>
<td>E-commerce’s profit</td>
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<tr>
<td>J_C</td>
<td>Whole supply chain’s profit</td>
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Table 3. Parameters.

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<th>Notations</th>
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<tr>
<td>η</td>
<td>Memory parameter</td>
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<tr>
<td>α</td>
<td>Effectiveness of the Big Data marketing effort</td>
</tr>
<tr>
<td>β</td>
<td>Effectiveness of the promotion effort</td>
</tr>
<tr>
<td>γ</td>
<td>Effectiveness of the reference promotion effect</td>
</tr>
<tr>
<td>δ</td>
<td>Decay rate of consumer conversion rate</td>
</tr>
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<td>D_0</td>
<td>Market base demand</td>
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<td>N</td>
<td>Number of potential consumers</td>
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<td>c_P</td>
<td>Cost coefficient of promotion effort</td>
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<td>c_B</td>
<td>Cost coefficient of Big Data marketing effort</td>
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<td>Discount rate</td>
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<td>F</td>
<td>Big Data marketing service fee</td>
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4. Modelling and solutions

4.1. Centralized decision mode (C mode)

In the centralized decision mode, the manufacturer and the e-commerce platform decide promotional strategy \( P(t) \) and Big Data marketing strategy \( B(t) \) by maximizing the profit of the whole supply chain. In order to obtain the optimal strategies under centralized decision mode, we need to solve the following optimal control problem:

\[
\max_{P(t)>0, B(t)>0} J_C = \int_0^{+\infty} e^{-\rho t} [\pi D(t) - C(P(t)) - C(B(t))] \, dt \\
\text{s.t. } \dot{r}(t) = \eta (P(t) - r(t)), r(0) = r_0
\]
\( \dot{S}(t) = \alpha B(t) + \beta P(t) + \gamma (P(t) - r(t)) - \delta S(t), S(0) = S_0. \) \hspace{1cm} (8)

**Theorem 4.1.** In the centralized decision mode, the optimal strategies of the supply chain are

\[
P_M^* (t) = \frac{\pi N \beta (\rho + \eta + \rho \gamma)}{c_p (\rho + \delta)(\rho + \eta)}, \hspace{1cm} (9)
\]

\[
B_R^* (t) = \frac{\pi N \alpha}{c_B (\rho + \delta)}, \hspace{1cm} (10)
\]

the optimal trajectories of the reference promotion point and the consumer conversion rate are

\[
r^{CS^*} (t) = r^{CS} + (r_0 - r^{CS}) e^{-\eta t}, \hspace{1cm} (11)
\]

\[
S^{CS^*} (t) = S^{CS} + \frac{\gamma}{\eta - \delta} (r_0 - r^{CS}) e^{-\eta t} + \left[ S_0 - S^{CS} - \frac{\gamma}{\eta - \delta} (r_0 - r^{CS}) \right] \times e^{-\delta t}, \hspace{1cm} (12)
\]

where \( r^{CS} = P_M^* (t) \) and \( S^{CS} = \frac{\alpha B_R^* + \beta P^*}{\alpha} \) represent the steady states of reference promotion point and consumer conversion rate, respectively.

Furthermore, the optimal profit of the whole supply chain is

\[
J_C^* = \frac{\pi D_0}{\rho} + \frac{\pi N}{\rho} S^{CS} + \frac{\pi N (S_0 - S^{CS})}{\rho + \delta} - \frac{\pi N \gamma (r_0 - r^{CS})}{\rho + \eta + \rho \delta} - \frac{c_p}{2 \rho} \left( P_M^* \right)^2 - \frac{c_B}{2 \rho} \left( B_R^* \right)^2. \hspace{1cm} (13)
\]

**Proof.** Introducing the costate variables \( \mu_C, \) and \( \mu_{CS} \) associated with \( r(t) \) and \( S(t), \) the corresponding current-value Hamiltonian is given by:

\[
H_C = \pi (D_0 + N S(t)) - \frac{c_p P^2(t)}{2} - \frac{c_B}{2} B^2(t) + \mu_C \alpha (\beta P(t) + \gamma (P(t) - r(t))) + \mu_{CS} [\alpha B(t) + \beta P(t) + \gamma (P(t) - r(t)) - \delta S(t)]. \hspace{1cm} (14)
\]

According to literature [18], the necessary conditions for promotion and Big Data marketing efforts are as follows:

\[
\frac{\partial H_C}{\partial P} = -c_p P(t) + \mu_C \eta + \mu_{CS} (\beta + \gamma) = 0, \hspace{1cm} (15)
\]

\[
\frac{\partial H_C}{\partial B} = -c_B B(t) + \mu_{CS} \alpha = 0, \hspace{1cm} (16)
\]

\[
\dot{\mu}_C = \rho \mu_C - \frac{\partial H_C}{\partial r}, \hspace{1cm} (17)
\]

\[
\dot{\mu}_{CS} = \rho \mu_{CS} - \frac{\partial H_C}{\partial S}. \hspace{1cm} (18)
\]

The equations (15) and (16) imply

\[
P_M(t) = \frac{\mu_C \eta + \mu_{CS} (\beta + \gamma)}{c_p}, \hspace{1cm} (19)
\]

\[
B_R(t) = \frac{\mu_{CS} \alpha}{c_B}. \hspace{1cm} (20)
\]

In order to solve \( \mu_C \) and \( \mu_{CS}, \) substituting (19) and (20) into (17) and (18), we have

\[
\begin{bmatrix}
\dot{\mu}_C \\
\dot{\mu}_{CS}
\end{bmatrix} = B_1 \begin{bmatrix}
\mu_C \\
\mu_{CS}
\end{bmatrix} + b_1, \hspace{1cm} (21)
\]
where \( B_1 = \begin{bmatrix} \rho + \eta & \gamma \\ 0 & \rho + \delta \end{bmatrix} \), \( b_1 = \begin{bmatrix} 0 \\ -\pi N \end{bmatrix} \).

The two characteristic roots of the matrix \( B_1 \) can be obtained by calculation as: \( \lambda_1 = \rho + \eta \) and \( \lambda_2 = \rho + \delta \). The eigenvector matrix corresponding to the two characteristic roots is

\[
H_1 = \begin{bmatrix} 1 & \frac{\gamma}{-\pi \eta} \\ 0 & 1 \end{bmatrix}.
\] (22)

Then, the solution of differential equation (21) can be expressed as

\[
\begin{bmatrix} \mu_{Cr} \\ \mu_{CS} \end{bmatrix} = H_1 \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \end{bmatrix} c_1 - B_1^{-1} b_1 = \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \begin{bmatrix} \frac{\gamma}{\delta - \eta} e^{\lambda_1 t} \\ -\frac{\pi N \gamma}{(\rho + \delta)} \end{bmatrix}.
\]

(23)

According to conditions \( \lim_{t \to \infty} e^{-\rho t} \mu_{Cr}(t) = 0 \) and \( \lim_{t \to \infty} e^{-\rho t} \mu_{CS}(t) = 0 \), we arrive at that \( c_1 = c_2 = 0 \), i.e., \( \mu_{Cr} = -\frac{\pi N \gamma}{(\rho + \delta)} \) and \( \mu_{CS} = \frac{\pi N \gamma}{(\rho + \delta)} \). Substituting \( \mu_{Cr} \) and \( \mu_{CS} \) into (19) and (20), the optimal decisions can be obtained.

Furthermore, substituting the obtained optimal decisions into equations (2) and (3), one has

\[
\begin{bmatrix} \dot{r}(t) \\ \dot{S}(t) \end{bmatrix} = B_2 \begin{bmatrix} r(t) \\ S(t) \end{bmatrix} + b_2,
\]

(24)

where \( B_2 = \begin{bmatrix} -\eta & 0 \\ -\gamma - \delta \end{bmatrix} \), \( b_2 = \begin{bmatrix} \alpha B_{kr}^{CS} + (\beta + \gamma) P_{M}^{CS} \end{bmatrix} \).

Similarly, the solution of differential equation (24) can be expressed as

\[
\begin{bmatrix} r(t) \\ S(t) \end{bmatrix} = H_2 \begin{bmatrix} e^{m_1 t} \\ e^{m_2 t} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} - B_2^{-1} b_2 = \begin{bmatrix} 0 \\ e^{m_1 t} \end{bmatrix} \begin{bmatrix} 0 \\ e^{m_2 t} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} - \begin{bmatrix} -P_{M}^{CS} \\ -\frac{\alpha B_{kr}^{CS} + \beta P_{M}^{CS}}{\delta} \end{bmatrix},
\]

(25)

where \( m_1 = -\delta \), \( m_2 = -\eta \), \( H_2 = \begin{bmatrix} 0 & \frac{\delta - \eta}{\gamma} \\ 1 & 1 \end{bmatrix} \).

According to the initial conditions \( r(0) = r_0 \) and \( S(0) = S_0 \), we can derive that \( d_1 = S_0 - \frac{\alpha B_{kr}^{CS} + \beta P_{M}^{CS}}{\delta} - \frac{\gamma (r_0 - P_{M}^{CS})}{\delta - \eta} \) and \( d_2 = \frac{\gamma (r_0 - P_{M}^{CS})}{\eta - \delta} \). Substituting (9)–(12) into (7), we get the optimal profit of the whole supply chain as (13).

From Theorem 4.1, the optimal strategies of the manufacturer and the e-commerce platform are time-independent, which make it easier to realize the above strategies in practice. Through further analysis of the optimal strategies, the following proposition can be obtained.

Proposition 4.2. Under centralized decision mode, the impacts of system parameters on the optimal strategies can be obtained for the both channel members:

1. \( \frac{\partial P_{r}^{CS}}{\partial \beta} > 0 \), \( \frac{\partial P_{r}^{CS}}{\partial \eta} > 0 \), \( \frac{\partial P_{r}^{CS}}{\partial N} > 0 \), \( \frac{\partial P_{r}^{CS}}{\partial \eta} < 0 \);
2. \( \frac{\partial B_{kr}^{CS}}{\partial \alpha} > 0 \), \( \frac{\partial B_{kr}^{CS}}{\partial \eta} > 0 \);
3. \( \frac{\partial S_{CS}^{CS}}{\partial \alpha} > 0 \), \( \frac{\partial S_{CS}^{CS}}{\partial \eta} > 0 \), \( \frac{\partial S_{CS}^{CS}}{\partial N} > 0 \), \( \frac{\partial S_{CS}^{CS}}{\partial \eta} < 0 \).

From Proposition 4.2, we obtain the following managerial implications. Firstly, the manufacturer should invest more in the promotion effort owing to the existence of promotion effect (i.e., \( \gamma \)) in the promotion action, which further induces the higher reference promotion point and consumer conversion rate. Furthermore, the higher promotion cost and consumer conversion rate brought by higher promotion effort jointly affect the supply
chain profits. Secondly, when the effectiveness of the promotion effort onto the consumer conversion rate (i.e., $\beta$) becomes higher, the manufacturer will invest more in promotion effort. For the memory parameter $\eta$, a larger $\eta$ means that the consumers have shorter memory of past promotion experience, which leads to lower promotion effort. Therefore, it is beneficial for the whole channel to invest less in the promotion effort when the consumers have a larger $\eta$. In addition, the parameter $\alpha$ reflects the positive effect from the Big Data marketing effort on consumer conversion rate, which means that the expansionary effect of Big Data marketing effort on consumer conversion rate will become larger accompanying with the increasing of $\alpha$. Accordingly, it is better to invest more in Big Data marketing in order to improve the consumer conversion rate and then increase the demand. Finally, as the number of potential consumers (i.e., $N$) increases, both the Big Data marketing effort by the e-commerce platform and the promotion effort by the manufacturer will increase. Moreover, the higher number of potential consumers will result in the case that the manufacturer and the e-commerce platform should invest more promotion effort and Big Data marketing effort, respectively.

In practice, the decision makers might not take promotional reference effects into account when making long-term promotional strategies, which leads to reduced profit for the whole channel. In order to explore more managerial insights, we give the optimal strategies and profit of the supply chain without the promotion reference effect by setting $\gamma = 0$ in the Theorem 4.1.

**Theorem 4.3.** When the promotion reference effect is not considered, the optimal strategies of the supply chain in the centralized decision mode are

$$P^*_M(t) = \frac{\pi N \beta}{c_P(\rho + \delta)},$$

$$B^*_R(t) = \frac{\pi N \alpha}{c_B(\rho + \delta)},$$

the optimal profit of the whole supply chain is

$$J^*_C = \frac{\pi D_0}{\rho} + \frac{\pi N}{\rho} S^{CNS} + \frac{\pi N}{\rho} (S_0 - S^{CNS}) - \frac{c_P}{2\rho} \left( p^*_M \right)^2 - \frac{c_B}{2\rho} \left( B^*_R \right)^2,$$

where $S^{CNS} = \frac{\alpha B^*_R}{\delta} + \frac{\beta P^*_M}{\delta}$ is the steady state of consumer conversion rate.

Subsequently, the following propositions recapitulate some relative findings.

**Proposition 4.4.** In centralized supply chain without and with the reference effect, the optimal strategies and profits are related as follows:

1. $P^*_M(t) > P^*_M(t)$, $B^*_R(t) = B^*_R(t)$;
2. $J^*_C > J^*_C$, if $\gamma > \frac{2(\rho + \rho)(\rho + \delta) - \pi N \beta}{\rho \pi N}$.

Proposition 4.4 indicates that the entire supply chain will augment the promotion effort in the presence of the promotion reference effect. Regarding the channel profits, there exists a threshold value for the effectiveness of the reference promotion effect $\gamma$. Specifically, the entire supply chain benefits from the presence of the promotion reference effect when the effectiveness of the reference promotion effect exceeds the threshold. The reason is that the higher effectiveness of the reference promotion effect induces the entire supply chain to augment their promotion effort to attract additional consumers. Otherwise, the entire supply chain should ignore the presence of the promotion reference effect in order to obtain higher profit when the effectiveness of the reference promotion effect $\gamma$ is smaller than the threshold. The reason is that although the existence of reference effect can promote the whole supply chain to increase promotion efforts to attract more consumers, it will also lead to an increase in the promotion cost of the entire supply chain. Ultimately, the presence of the promotion reference effect causes decreased profit for the entire supply chain. The above findings can be seen as a generalization of the existing conclusions regarding the promotion.
### 4.2. Decentralized decision mode (D mode)

In the decentralized decision mode, the manufacturer and the e-commerce platform conduct the Nash differential game. Under this game, both manufacturer and e-commerce platform make their decisions simultaneously. In particular, the manufacturer determines the promotion effort \( P(t) \) and the e-commerce platform determines the Big Data marketing effort \( B(t) \). Accordingly, the optimization problems of the manufacturer and the e-commerce platform can be respectively formulated as follows:

\[
\begin{align*}
\max_{P(t) > 0} J_M &= \int_0^{+\infty} e^{-rt}[(1 - \varphi)\pi D(t) - C(P(t)) - F(t)] dt \\
\text{s.t.} \quad \dot{r}(t) &= \eta(P(t) - r(t)), \quad r(0) = r_0 \\
\dot{S}(t) &= \alpha B(t) + \beta P(t) + \gamma(P(t) - r(t)) - \delta S(t), \quad S(0) = S_0, \quad (29)
\end{align*}
\]

and

\[
\begin{align*}
\max_{B(t) > 0} J_R &= \int_0^{+\infty} e^{-rt}[(\varphi\pi D(t) - C(B(t)) + F(t)] dt \\
\text{s.t.} \quad \dot{r}(t) &= \eta(P(t) - r(t)), \quad r(0) = r_0 \\
\dot{S}(t) &= \alpha B(t) + \beta P(t) + \gamma(P(t) - r(t)) - \delta S(t), \quad S(0) = S_0. \quad (30)
\end{align*}
\]

**Theorem 4.5.** In the decentralized decision mode, the optimal equilibrium strategies are given by

\[
\begin{align*}
P_M^* &= \frac{(1 - \varphi)\pi N[\beta(\rho + \eta) + \rho\gamma]}{c_P(\rho + \delta)(\rho + \eta)}, \quad (31) \\
B_R^* &= \frac{\varphi\pi N\alpha}{c_B(\rho + \delta)}, \quad (32)
\end{align*}
\]

the optimal trajectories of the reference promotion point and the consumer conversion rate are

\[
\begin{align*}
r^D(t) &= r^{DS} + (r_0 - r^{DS})e^{-\eta t}, \quad (33) \\
S^D(t) &= S^{DS} + \frac{\gamma}{\eta - \delta}(r_0 - r^{DS})e^{-\eta t} + \left[ S^{DS} - S^{DS} - \frac{\gamma}{\eta - \delta}(r_0 - r^{DS}) \right] \times e^{-\delta t}, \quad (34)
\end{align*}
\]

where \( r^{DS} = P_M^* \) and \( S^{DS} = \frac{\alpha B_R^* + \beta P_M^*}{\delta} \) represent the steady states of reference promotion point and consumer conversion rate, respectively.

Furthermore, the optimal profits of both players and the whole supply chain are given by

\[
\begin{align*}
J_M^* &= \frac{(1 - \varphi)\pi D_0}{\rho} + \frac{(1 - \varphi)\pi N S^{DS}}{\rho} + \frac{(1 - \varphi)\pi N(S_0 - S^{DS})}{\rho + \delta} - \frac{(1 - \varphi)\pi N\gamma(r_0 - r^{DS})}{(\rho + \eta)(\rho + \delta)} - \frac{c_P}{2\rho}\left( P_M^* \right)^2 \\
&\quad - \frac{F}{\rho}, \quad (35) \\
J_R^* &= \frac{\varphi\pi D_0}{\rho} + \frac{\varphi\pi N S^{DS}}{\rho} + \frac{\varphi\pi N(S_0 - S^{DS})}{\rho + \delta} - \frac{\varphi\pi N\gamma(r_0 - r^{DS})}{(\rho + \eta)(\rho + \delta)} - \frac{c_B}{2\rho}\left( B_R^* \right)^2 + \frac{F}{\rho}, \quad (36) \\
J_D^* &= J_M^* + J_R^* = \frac{\pi D_0}{\rho} + \frac{\pi N S^{DS}}{\rho} + \frac{\pi N(S_0 - S^{DS})}{\rho + \delta} - \frac{\pi N\gamma(r_0 - r^{DS})}{(\rho + \eta)(\rho + \delta)} - \frac{c_P}{2\rho}\left( P_M^* \right)^2 - \frac{c_B}{2\rho}\left( B_R^* \right)^2. \quad (37)
\end{align*}
\]

**Proof.** In order to solve problem (29), the Hamiltonian for the manufacturer is presented as follows:

\[
H_M = (1 - \varphi)\pi(D_0 + NS(t)) - \frac{c_P}{2}P^2(t) - F + \mu_{M^*}(t)[\eta(P(t) - r(t))]
\]
+ \mu_{MS}(t)[\alpha B(t) + \beta P(t) + \gamma(P(t) - r(t)) - \delta S(t)], \quad (38)

where \mu_{Mr}(t) and \mu_{MS}(t) are the costate variables associated with r(t) and S(t).

Similar to proof of Theorem 4.1, the manufacturer’s optimal decision satisfies the following conditions:

\[ \frac{\partial H_M}{\partial P} = -c_P P(t) + \mu_{Mr} \eta + \mu_{MS}(\beta + \gamma) = 0, \]
\[ \hat{\mu}_{Mr} = \rho \mu_{Mr} - \frac{\partial H_M}{\partial r}, \quad (40) \]
\[ \hat{\mu}_{MS} = \rho \mu_{MS} - \frac{\partial H_M}{\partial S}. \quad (41) \]

From equation (39), we get

\[ P_M(t) = \frac{\mu_{Mr} \eta + \mu_{MS}(\beta + \gamma)}{c_P}, \quad (42) \]

In addition, equations (40) and (41) are combined as follows:

\[ \begin{bmatrix} \hat{\mu}_{Mr} \\ \hat{\mu}_{MS} \end{bmatrix} = B_3 \begin{bmatrix} \mu_{Mr} \\ \mu_{MS} \end{bmatrix} + b_3, \quad (43) \]

where \( B_3 = \begin{bmatrix} \rho + \eta & \gamma \\ 0 & \rho + \delta \end{bmatrix}, \)
\[ b_3 = \begin{bmatrix} 0 \\ -(1 - \varphi)\pi N \end{bmatrix}. \]

The two characteristic roots of the matrix \( B_3 \) are \( \lambda_3 = \rho + \eta \) and \( \lambda_4 = \rho + \delta \). The eigenvector matrix corresponding to the two characteristic roots is

\[ H_3 = \begin{bmatrix} 1 & \frac{\gamma}{\delta - \eta} \\ 0 & 1 \end{bmatrix}. \quad (44) \]

Then, the solution of differential equation (43) can be expressed as:

\[ \begin{bmatrix} \mu_{Mr} \\ \mu_{MS} \end{bmatrix} = H_3 \begin{bmatrix} e^{\lambda_3 t} \\ 0 \\ e^{\lambda_4 t} \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} - B_3^{-1} b_3 = \begin{bmatrix} e^{\lambda_3 t} \\ 0 \\ e^{\lambda_4 t} \end{bmatrix} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} - \left( \frac{(1 - \varphi)\pi N \gamma}{(\rho + \delta)(\rho + \eta)} \right). \quad (45) \]

According to conditions \( \lim_{t \to \infty} e^{-\rho t} \mu_{Mr}(t) = 0 \) and \( \lim_{t \to \infty} e^{-\rho t} \mu_{MS}(t) = 0 \), we can determine \( c_3 = c_4 = 0 \), i.e.,
\[ \mu_{Mr} = -\frac{(1 - \varphi)\pi N \gamma}{(\rho + \delta)(\rho + \eta)} \quad \text{and} \quad \mu_{MS} = \frac{(1 - \varphi)\pi N}{(\rho + \delta)}. \]
Substituting \( \mu_{Mr} \) and \( \mu_{MS} \) into equation (42), the optimal promotion decision can be obtained.

On the other hand, the current-value Hamiltonian for the platform is given by

\[ H_R = \varphi \pi(D_0 + NS(t)) - \frac{c_B}{2} B(t)^2 + F + \mu_{Rr}(t)[\eta(P(t) - r(t))] + \mu_{RS}(t)[\alpha B(t) + \beta P(t) + \gamma(P(t) - r(t)) - \delta S(t)], \quad (46) \]

where \( \mu_{Rr}(t) \) and \( \mu_{RS}(t) \) are costate variables.

According to the same solving process, we have \( \mu_{Rr} = -\frac{\varphi \pi N \gamma}{(\rho + \delta)(\rho + \eta)} \) and \( \mu_{RS} = \frac{\varphi \pi N}{(\rho + \delta)} \). Further, the optimal Big Data marketing effort of the e-commerce platform can be obtained. Substituting (31)–(34) into (5) and (6), the optimal profits of the manufacturer, the e-commerce platform and the whole supply chain can be obtained. \( \square \)
Theorem 4.5 shows that the optimal strategies of the manufacturer and the e-commerce platform under decentralization have similar properties to those under centralization. However, the optimal strategies derived from the decentralized mode are also associated with the commission rate $\varphi$. The e-commerce platform should increase the Big Data marketing effort while the manufacturer should decrease the promotional effort when the percentage $\varphi$ increases. To further investigate the impact of different decision modes on the marketing strategies and profits of supply chain members, the comparative analysis is carried out and the following proposition is obtained.

**Proposition 4.6.** Comparing the optimal results of centralized and decentralized modes, we have

1. $P_M^C(t) > P_M^D(t)$, $B_R^C(t) > B_R^D(t)$;
2. $r^{CS} > r^{DS}$, $r^C(t) > r^D(t)$, $S^{CS} > S^{DS}$, $J^C(t) > J^D(t)$;

**Proof.** According to the results of Theorems 4.1 and 4.5, it follows that

$$P_M^C(t) - P_M^D(t) = \frac{\varphi \pi N[\beta(\rho + \eta + \rho \gamma)]}{cp(\rho + \delta)(\rho + \eta)} > 0,$$

$$B_R^C(t) - B_R^D(t) = \frac{(1 - \varphi)N\alpha}{cB(\rho + \delta)} > 0,$$

$$r^{CS} - r^{DS} = P_M^C(t) - P_M^D(t) > 0,$$

$$r^C(t) - r^D(t) = (1 - e^{-\eta t})(r^{CS} - r^{DS}) > 0,$$

$$S^{CS} - S^{DS} = \frac{A}{\delta} \left( B_R^C(t) - B_R^D(t) \right) + \frac{\beta}{\delta} \left( P_M^C(t) - P_M^D(t) \right) > 0,$$

$$S^C(t) - S^D(t) = (1 - e^{-\delta t})(S^{CS} - S^{DS}) + \frac{\gamma}{\delta - \eta}(r^{CS} - r^{DS}) \times (e^{-\eta t} - e^{-\delta t}) > 0,$$

$$J_C^* - J_D^* = \frac{(1 - \varphi)^2cB}{2\rho} \left( B_R^C(t) \right)^2 + \frac{\varphi^2cP}{2\rho} \left( P_M^C(t) \right)^2 > 0,$$

which end the proof. □

Proposition 4.6 shows that the promotion effort of the manufacturer, the Big Data marketing effort of the e-commerce platform, the reference promotional point, the consumer conversion rate and the total profit of the supply chain are relatively higher in the centralization compared to the decentralization. This finding suggests that the centralized structure not only allows the consumers to enjoy higher promotional benefits but also improves the consumer conversion rate and the profit of the whole supply chain. Therefore, it is necessary to design an efficient contract to restrain the behavior of two parties under the decentralized mode.

### 5. Design of the Combined Contract (E Mode)

From the preceding analysis, it is clear that the marketing efforts of the two parties and the total profits of the supply chain in the decentralized structure are relatively lower. In order to get more profits, a combined contract including a two-part tariff scheme and a promotion cost sharing scheme is developed in section to encourage both channel parties to increase their marketing investments. The provisions of this combined contract are designed as follows: the e-commerce platform charges the manufacturer a fixed fee and a variable fee based on Big Data marketing spend. At the same time, the e-commerce platform shares a portion of the manufacturer’s promotional cost. Specifically, we assume that the e-commerce platform charges the manufacturer for Big Data marketing services in the form of $F^E = nB(t) + L$, where $L$ is the fixed fee and $n$ is the coefficient of variable fee. The promotional cost sharing ratio of the e-commerce platform is recorded as $\phi$ ($0 < \phi < 1$). Therefore,
the profit functions of the manufacturer and the e-commerce platform under the combined contract can be expressed as follows:

\[
\begin{align*}
J_{M}^{E} &= \int_{0}^{+\infty} e^{-\rho t} \left[ (1 - \varphi) \pi D(t) - \frac{1}{2} (1 - \phi) c_P P^2(t) - nB(t) - L \right] dt, \\
J_{R}^{E} &= \int_{0}^{+\infty} e^{-\rho t} \left[ \varphi \pi D(t) - \frac{1}{2} \phi c_P P^2(t) - \frac{1}{2} c_B B^2(t) + nB(t) + L \right] dt.
\end{align*}
\]

(47) \hspace{2cm} (48)

**Theorem 5.1.** If the parameters of combined contract satisfy \( \phi = \varphi \) and \( n = \frac{(1 - \varphi) \pi N \alpha}{\rho + \delta} \), the combined contract can realize the coordination of the supply chain. Further, when \( L_1 < L < L_2 \), then the proposed contract is acceptable for both players and the interval \( (L_1, L_2) \) is called the feasible region of the contract, where

\[
L_1 = \rho \left[ J_{R}^{E^*} - \varphi J_{C}^{*} + \frac{c_B}{2 \rho} (1 - \varphi) \left( B_{R}^{C^*} \right)^2 - \frac{(1 - \varphi) \pi N \alpha}{\rho (\rho + \delta)} B_{R}^{C^*} \right] \quad \text{and} \quad L_2 = \rho \left[ (1 - \varphi) J_{C}^{*} - J_{M}^{E^*} + \frac{c_B}{2 \rho} (1 - \varphi) \left( B_{R}^{C^*} \right)^2 - \frac{(1 - \varphi) \pi N \alpha}{\rho (\rho + \delta)} B_{R}^{C^*} \right].
\]

**Proof.** The optimal control problems of manufacturer and e-commerce platform under the combined contract are expressed as follows:

\[
\begin{align*}
\max_{P(t) > 0} J_{M}^{E} &= \int_{0}^{+\infty} e^{-\rho t} \left[ (1 - \varphi) \pi D(t) - \frac{1}{2} (1 - \phi) c_P P^2(t) - nB(t) - L \right] dt \\
\text{s.t.} \quad \dot{r}(t) &= \eta(P(t) - r(t)), r(0) = r_0 > 0 \\
\dot{S}(t) &= \alpha B(t) + \beta P(t) + \gamma (P(t) - r(t)) - \delta S(t), S(0) = S_0 > 0,
\end{align*}
\]

(49)

and

\[
\begin{align*}
\max_{B(t) > 0} J_{R}^{E} &= \int_{0}^{+\infty} e^{-\rho t} \left[ \varphi \pi D(t) - \frac{1}{2} \phi c_P P^2(t) - \frac{1}{2} c_B B^2(t) + nB(t) + L \right] dt \\
\text{s.t.} \quad \dot{r}(t) &= \eta(P(t) - r(t)), r(0) = r_0 > 0 \\
\dot{S}(t) &= \alpha B(t) + \beta P(t) + \gamma (P(t) - r(t)) - \delta S(t), S(0) = S_0 > 0.
\end{align*}
\]

(50)

Similar to the solution process of Theorem 4.5, the optimal promotion effort of the manufacturer and the Big Data marketing effort of the e-commerce platform can be obtained, i.e., \( P_{M}^{E^*} (t) = \frac{(1 - \varphi) \pi N \alpha + n (\rho + \delta)}{\rho (\rho + \delta)} \) and \( B_{R}^{E^*} (t) = \frac{\varphi \pi N \alpha + n (\rho + \delta)}{\rho (\rho + \delta)} \). Let \( P_{M}^{E^*} (t) = P_{M}^{E^*} (t), B_{R}^{E^*} (t) = B_{R}^{E^*} (t) \), we can get \( \phi = \varphi \) and \( n = \frac{(1 - \varphi) \pi N \alpha}{\rho + \delta} \).

Substituting the optimal decisions with \( \phi = \varphi \) and \( n = \frac{(1 - \varphi) \pi N \alpha}{\rho + \delta} \) into (47) and (48), the optimal profits of the manufacturer and the e-commerce platform under the proposed contract can be obtained as follows:

\[
\begin{align*}
J_{M}^{E^*} &= (1 - \varphi) J_{C}^{*} + \frac{c_B}{2 \rho} (1 - \varphi) \left( B_{R}^{C^*} \right)^2 - \frac{(1 - \varphi) \pi N \alpha}{\rho (\rho + \delta)} B_{R}^{C^*} - \frac{L}{\rho}, \\
J_{R}^{E^*} &= \varphi J_{C}^{*} - \frac{c_B}{2 \rho} (1 - \varphi) \left( B_{R}^{C^*} \right)^2 + \frac{(1 - \varphi) \pi N \alpha}{\rho (\rho + \delta)} B_{R}^{C^*} + \frac{L}{\rho}.
\end{align*}
\]

(51) \hspace{2cm} (52)

Besides, it should be pointed out that the designed contract should satisfy \( J_{M}^{E^*} > J_{M}^{D^*} \) and \( J_{R}^{E^*} > J_{R}^{D^*} \). Then, the manufacturer and the e-commerce platform can adopt this combined contract. By solving the above inequalities, we can achieve the conclusions in Theorem 5.1. \( \square \)

As mentioned in [42], the control-dependent incentive is an inefficient mechanism to improve the profits of all players under the decentralized decision mode. In this research, a combined contract based on the control-dependent incentive and cost-sharing contract is proposed to coordinate a platform supply chain, where the
cost of Big Data marketing service is composed of the fixed fee $L$ and variable fee (control-dependent incentive) $nB(t)$. In addition, it follows from Theorem 5.1 that the optimal strategies under the combined contract are independent of fixed fee $L$ and the profits of the e-commerce platform and the manufacturer depend on the fixed fee $L$. Further, when the value of the fixed fee $L$ locates in the interval $(L_1, L_2)$, the combined contract can not only achieve perfect coordination of the supply chain, but also improve the profits of both parties to ensure that they have the willingness to agree on this contract. In other words, the e-commerce platform and the manufacturer may allocate the profits by adjusting the fixed fee $L$ within the feasible region. Subsequently, the proposed combined contract will be validated by comparing its efficiency with the available result in subsequent numerical analysis.

6. Numerical results

On the basis of the obtained theoretical results, we explore the effects of several system parameters on the Big Data marketing effort, the promotion effort, the profits and the feasible region through specific numerical examples. In response to this analysis process, we can further derive valuable managerial insights. Referring to [19, 40], the values of parameters are set as shown in Table 4. Correspondingly, all computation results are outlined in Figures 2–18 and Table 5.
6.1. Optimal trajectories of state variables

According to the results of Theorems 4.1 and 4.3, we can obtain Figures 2 and 3. From those figures, we can see that the optimal trajectories of the reference promotion point and the consumer conversion rate under the centralized and decentralized modes tend to be stable when time $t \to \infty$. It can be seen from Figure 2 that, regardless of the value of the initial reference promotion point $r_0$, the stable value of the reference promotion point in the same decision mode is identical and the trajectories of the reference promotion point are monotonic. In addition, we can also observe that the reference promotion point under centralized mode is consistently higher than the one under decentralized mode. Figure 3 shows that the initial values have no effect on the stable value of the consumer conversion rate and the consumer conversion rate under centralized mode is higher than one under decentralized mode. However, the trajectory of the consumer conversion rate is not always monotonic, as it is affected by the initial reference promotion point $r_0$ and the initial consumer conversion rate $S_0$. For example, when $r_0 = 0$ and $S_0 = 3$, $S^C(t)$ and $S^D(t)$ decrease monotonically and then tend to be stable. However, when $r_0 = 6$ and $S_0 = 3$, $S^C(t)$ and $S^D(t)$ firstly decrease and then increase, and eventually achieve the stability.

6.2. The effect of system parameters on optimal decisions and profits

The effects of parameters $\alpha$, $\beta$, $\eta$, $\gamma$, and $N$ on the optimal Big Data marketing decisions, the optimal promotion decisions, the stable values of the reference promotion point and the consumer conversion rate as well as the profits for the centralized and decentralized modes are given in Figures 4–13, respectively. Because of $r^{CS} = P^C(t)$ and $r^{DS} = P^D(t)$, the variation curves for the stabilized values of the reference promotion point will not repeat in the subsequent simulations.
As shown in Figures 4 and 5, the higher effectiveness of the Big Data marketing effort $\alpha$ will lead to more Big Data marketing efforts and channel profits under centralized and decentralized modes. This result indicates that, regardless of the centralization and decentralization, the e-commerce platform is motivated by high effectiveness of the Big Data marketing effort to invest more in the Big Data marketing service, which results in a higher consumer conversion rate. Furthermore, the higher consumer conversion rate by higher Big Data marketing effort can contribute to higher supply chain profits. However, the profit of the e-commerce platform decreases accompanying with the increasing effectiveness of the Big Data marketing effort. The main reason is that the increase of Big Data marketing effort under decentralization leads to a higher cost of the Big Data marketing. On the other hand, when the parameter $\alpha$ increases, the gaps of the Big Data marketing efforts/profits among...
the two decision modes will become large. Therefore, in order to better achieve win–win cooperation among supply chain members, it is necessary to implement the coordination contract.

From Figures 6 and 7, regardless of the decision modes, the effectiveness of the promotion effort $\beta$ has positive impacts onto the promotion efforts, reference promotion points, consumer conversion rates and profits. That is, a higher $\beta$ induces the higher consumer conversion rates and the higher market demands, which leads to higher profits for all members and whole supply chain. The above results show that even face the adverse impact of a higher reference promotion point on consumer conversion rate, a higher promotion effort can still effectively improve the consumer conversion rate, thereby expanding the market demand and profits. Moreover, the gaps of the promotion efforts/profits among the two decision modes enlarge with increasing of effectiveness of the

\begin{figure}
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\caption{Profits of supply chain members with different $\eta$.}
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\caption{Promotion decision and consumer conversion rate with different $\gamma$.}
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\includegraphics[width=0.5\linewidth]{fig11}
\caption{Profits of supply chain members with different $\gamma$.}
\end{figure}
promotion effort. Therefore, the coordination contract should be implemented by the manufacturer and the e-commerce platform to improve the profits of both members under the decentralized mode.

Figures 8 and 9 show that the influences of the memory parameter \( \eta \) on the optimal results are opposite to the effectiveness of the promotion effort \( \beta \). In addition, Figures 10 and 11 show that the influences from the effectiveness of the reference promotion effect \( \gamma \) onto the optimal strategies, the consumer conversion rates and supply chain profits. The specific observations are as follows: accompanying with the increasing the effectiveness of the reference promotion on the consumer conversion rate (i.e., a higher \( \gamma \)), the manufacturer’s promotion effort increases. A higher manufacturer’s promotion effort results in higher consumer conversion rate and market demand, which in turn leads to higher profits of the supply chain. The above results show that the high promotion effort can still offset the promotion cost by effectively improving the consumer conversion rate when the impact of reference promotion effect on the consumer conversion rate is significant, thereby expanding market demand and increasing profits. Furthermore, the gaps of the promotion efforts/profits between two decision modes enlarge with increasing of \( \gamma \). Figures 12 and 13 illustrate that an increase in the number of potential consumers \( N \) results in higher promotion and Big Data marketing efforts by both parties. The higher marketing efforts lead to a superior consumer conversion rate and greater profits. A higher \( N \) will also lead to a larger gap of the promotion investments/Big Data marketing efforts/profits of the two decision modes. Therefore, the coordination contract in this situation is more attractive to the manufacturer and the e-commerce platform.

6.3. Validation of the proposed combined contract

In order to verify the advantages of the proposed contract, the combination of cost-sharing contract and control-dependent incentive in [42] (\( S \) mode) is used for comparison in this Section. Correspondingly, \( J^*_M \) and \( J^*_R \) represent the profits of the manufacturer and the e-commerce platform under \( S \) mode, respectively.
Table 5. The profit comparisons of two members under different contracts.

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Figure 14. The influence of α on the feasible region.

In addition, the fixed fee L is selected based on the feasible region of combined contract in different F and is provided in Table 5. Subsequently, according to the related results in the above sections, the profits and profit gaps of channel members under different modes also are presented in Table 5. It can be seen from Table 5 that, regardless of the value of the fixed fee L, the two contracts can effectively improve the system performance under the decentralized mode and realize the channel coordination when F = 30 or 40. Therefore, both contracts are likely to be accepted by two parties. However, when F = 50 or 60, we find that compared with the decentralized mode, the contract corresponding to E mode still improves the profits of both parties, but the contract corresponding to S mode is ineffective. More specifically, although the S mode increases the profit of the manufacturer, but it damages the profit of the e-commerce platform. It means that the S mode cannot be accepted by e-commerce platform. To sum up, the obtained results demonstrate that the contract proposed in this paper not only effectively coordinates the supply chain, but also shows strong flexibility.
6.4. Impact of system parameters on feasible regions

Figures 14–18 illustrate the effects of parameters $\alpha$, $\beta$, $\gamma$, $N$ and $\eta$ onto the feasible region of combined contract. In particular, we observe from those figures that when $L$ locates between $L_1$ and $L_2$, the combined contract including two-part tariff contract and promotion cost sharing contract can coordinate the supply chain well and improve the economic situations of both parties. Moreover, when the effectiveness of the Big Data marketing effort $\alpha$ (or the effectiveness of the promotion effort $\beta$, or the effectiveness of the reference promotion effect $\gamma$, or the number of potential consumers $N$) increases, the feasible region of combined contract becomes larger. Those implies that, for larger $\alpha$, $\beta$, $\gamma$ and $N$, the combined contract provides more space for cooperation between two parties. Therefore, the manufacturer and the e-commerce platform should strengthen
their partnership by choosing more effective ways to increase these system parameters. However, the feasible region will gradually shrink with the increasing of the memory parameter $\eta$, which leads to larger constraints on the cooperative relationship between the two members. In this case, the willingness of the manufacturer and the e-commerce platform accepting the combined contract can be reduced.

7. Managerial insights and practical implications

Throughout the operation of the supply chain, the e-commerce platform has realized that the advanced Big Data technology can help manufacturers improve the efficiency and precision of marketing. This is especially true because manufacturers can better promote product information to target customer groups by using the Big Data marketing services provided by the platform. At the same time, in order to improve the sales volume and profits of the platform supply chain, the manufacturers and e-commerce platforms have to invest in promotion and Big Data marketing efforts respectively. Notably, supply chain managers should pay attention to the influencing degree of reference promotion onto the demand to determine whether to consider the reference promotion effect when making marketing strategies. Furthermore, the combined contract can be implemented in order to maintain sustained and stable cooperative relationship between channel members.

This study has a high scientific value. Academically, the dynamic decision problem of the platform supply chain has been discussed from perspective of reference effect. Firstly, this paper not only considered the dynamic evolution of reference promotion point but also the effects of Big Data marketing effort, promotion effort and reference promotion effect on consumer conversion rate. Secondly, the impacts from reference promotion effect onto the channel profit and the equilibrium solutions are revealed. Finally, this paper has designed a new combined contract, and the results confirm that this contract can realize the perfect coordination of the supply chain.

In practical applications, the conclusions derived from this research can be used to guide implementation of Big Data marketing and promotional strategies. Furthermore, the designed combined contract provides a new framework for optimization of mode like the Platform Open Plan (POP) for JD.com in China, because it can make a coordinated supply chain having more resilience.

8. Conclusions and outlook

8.1. Conclusions

In this paper, we have investigated the problems of optimal decisions and coordination in the presence of the reference promotion effect for a platform supply chain consisting of a manufacturer and an e-commerce platform. By applying the optimal control theory, the optimal equilibrium under centralized and decentralized
decision modes have been given. In addition, a new combined contract has been proposed to coordination this platform supply chain. We have obtained the following conclusions:

1. The centralized decision mode can improve the manufacturer’s promotion effort, the e-commerce platform’s Big Data marketing effort, and total profits of the whole supply chain;
2. The comparison between the two situations (without and with the reference effect) implied that it is advantageous to consider reference effect when the effects from the reference promotion effect onto the consumer conversion rate exceeds the critical threshold;
3. The combined contract including a two-part tariff scheme as well as a promotion cost sharing scheme can coordinate the platform supply chain, and there exists a suitable feasible region where all channel members are involved in the execution of the contract;
4. The coordination ability of the proposed combined contract will become weaker with an increase in the memory parameter. On the contrary, it will become stronger with the increasing of the effectiveness of the Big Data marketing effort, the reference promotion effect and other parameters.

In summary, it can be seen that the obtained conclusions can assist the managers reducing their decision deviations in both the promotion and Big Data marketing. In addition, the novel coordination mechanism proposed in this study can better solve the profit distribution problem for coordinated supply chain.

8.2. Limitations

There are also some limitations in this study. Firstly, the e-commerce platform is considered only as a medium for manufacturers to sell their products. It is worth noting that the e-commerce platforms like JD.com often have their own self-operated channel, and the products they sell may be substitutes for products sold by manufacturers. Therefore, the competition between the manufacturers and the e-commerce platforms cannot be ignored. Secondly, the immediate effects of the Big Data marketing and the reference promotion on the consumer conversion rate are considered simultaneously in this paper. However, the delayed effects are widely observed in the customers’ purchasing behaviors. Additionally, it is more meaningful explore the coordination problem for the platform supply chain with input delays in future.

8.3. Future research

To better address the limitations of this study, we can extend in many directions as follows: (i) The single-channel sales system consisting of a manufacturer and an e-commerce platform can be expanded to a dual-channel system considering price competition factors. (ii) The delayed effects can be incorporated into the decision process and appropriate decision models can be constructed to capture the impact of both immediate and delayed effects accordingly, then more effective marketing strategies can be developed to better cope with the delayed effects.

Acknowledgements

The authors thank the editor-in-chief, associate editor and the anonymous reviewers for their insightful comments on the earlier version of this manuscript. They also acknowledge the financial support provided by Natural Science Foundation of Heilongjiang Province of China (No. YQ2020A004) and the National Natural Science Foundation of China (No. 72071065).

References


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