SALES MODEL AND LOGISTICS SELECTIONS FOR A MANUFACTURER CONSIDERING CARBON EMISSION AND LOGISTICS LEVELS*

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Abstract. Improving logistics levels requires mobilizing more resources, resulting in more carbon emissions. We consider a manufacturer selling its products through an e-commerce platform that sells them to customers (reselling) or directly accessing customers via the platform by paying a proportional fee (agency selling). Under reselling, the manufacturer first sets a wholesale price and then the platform sets a retail price and a logistics level. Under agency selling, if using third-party logistics, the manufacturer sets the retail price and logistics level. When using the platform logistics, the manufacturer sets the retail price and the platform sets the logistics level; moreover, the manufacturer bears part of the logistics cost. This study examines the manufacturer’s sales model and logistics selection and its impact on the platform’s profit and consumer surplus. We find that if the logistics-improving efficiency is sufficiently high (low), the manufacturer should employ reselling (agency selling and third-party logistics) when the manufacturer bears a sufficient small or large part of the logistics cost. However, regardless of the size of the logistics-improving efficiency, the manufacturer should adopt agency selling and platform logistics when the logistics cost borne by the manufacturer is intermediate. If consumers become more sensitive to carbon emission levels, the manufacturer is less (more) likely to adopt the reselling model and platform logistics (agency selling model and platform logistics). Choosing reselling or agency selling and platform logistics, or agency selling and third-party logistics could benefit both the platform and consumers, generating a “win-win-win” outcome.

Mathematics Subject Classification. 91A10, 91B24, 91B60.

1. Introduction

Nowadays, logistics services are a key link in the competition among various industries and also one of the highest costs incurred, especially in the e-commerce industry [26]. According to a survey conducted by Dotcom Distribution, the delivery time is often more important than price, as 67% of online channel shoppers are willing to pay more for same-day home delivery [31]. It is also evident that consumers are paying more attention to the level of logistics services [7, 20]. In most cases, logistics services are undertaken by e-commerce platforms [56]. Therefore, to meet customers’ expectations of logistics services, e-commerce platforms are actively improving their logistics levels or strengthening the screening and control of third-party logistics [11, 24, 29]. Nevertheless,
such improvements are often accompanied by higher logistics costs and increased carbon emissions. For example, official data from Walmart in 2010 showed that about 90% of carbon emissions were caused by the supply chain (including consumers), and only 10% came from business activities [5]. This implies that the carbon emissions generated from logistics activities should not be underestimated [25]. In addition, with a rise in consumers’ awareness related to environmental protection, more consumers are considering the environmental impact of products when purchasing. According to [42], one-third of more than 20,000 consumers from five countries preferred environment-friendly products and considered the social and environmental impact of products before choosing. Therefore, many logistics companies are determined to reduce carbon emissions in the process of logistics services, such as Sendle\(^1\) (an Australian logistics enterprise) and Cai Niao\(^2\) (a logistics platform under Alibaba), they are keen to publicize their contribution to carbon emission reduction to attract green consumers. To meet the consumers’ needs, logistic bearers have to consider not only the burden of costs but also the close relationship between the level of logistics and the carbon emissions generated when they attempt to improve the quality of their logistics services to meet consumer demand.

According to the 2021 Annual Report of JD.com\(^3\), the number of active buyers of JD reached 417.9 million in the past 12 months as of December 31, 2020. This indicates a net increase of more than 30 million users in a single quarter, and a net increase of nearly 110 million active users in the whole year. This statistical result shows that an increasing number of consumers are buying products through online channels, and online sales are enabling merchants to explore more potential customers [61]. With the vigorous development of e-commerce, a growing number of manufacturers, such as Dell, IBM, Nike, and Apple, have chosen to resort to online sales channels and e-commerce platforms to sell their products [44,64]. In practice, these manufacturers employ two common sales models [34,60]. The first is reselling: manufacturers sell their products through e-commerce platforms that sell them to end customers. For example, Cartier, a famous French watch brand, sells its watches through JD.com, which in turn sells them to customers (i.e., adopts the reselling model). The other is agency selling: manufacturers directly access customers via e-commerce platforms by paying a proportional fee. In this sales model, the manufacturers can choose the platforms’ logistics or third-party logistics to transport products. For instance, two watch brands Ernest Borel and Versus sell through JD by paying a proportional fee (i.e., employ the agency selling model). However, Ernest Borel uses JD’s logistics while Versus uses third-party logistics. This raises an important question of whether the manufacturers should adopt reselling, agency selling and platforms’ logistics, or agency selling and third-party logistics. Furthermore, what is the impact of the manufacturers’ sales model and logistics selection on platforms’ profits and consumer surplus?

In fact, manufacturers find it difficult to choose one of the above three models. The reasons are as follows. First, choosing the reselling model is beneficial for the manufacturers because they could enjoy a higher wholesale price due to the double marginalization effect. However, they lose the power to adjust the logistics levels to appeal to more customers with low carbon or high logistics level preferences. Second, if the manufacturers choose the agency selling model and third-party logistics, although they obtain the right to determine product prices and logistics level, it may be detrimental for them because they should bear all the logistics costs. Finally, if the manufacturer adopts the agency selling model and platform logistics, they still obtain the right to set product prices. However, they transfer the right of determining logistics level to the platform and should bear a part of the logistics cost. Therefore, it is meaningful to help manufacturers to determine the sales model and logistics selections. Moreover, the impacts of the selection on the platforms’ profits and consumer surplus also deserve to be investigated.

To address these issues, we consider an e-commerce supply chain consisting of a manufacturer, an online platform, and environmentally conscious consumers. Considering that improving logistics results in more carbon

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emissions, we study the manufacturer’s sales model and logistics selection and examine the impact of its selection on the platform’s profit and consumer surplus. The manufacturer first chooses between reselling, agency selling and platform logistics, or agency selling and third-party logistics. In the reselling model, the manufacturer sets a wholesale price chargeable to the platform, which then determines a retail price chargeable to customers and the logistics level. In the agency selling and platform logistics, the manufacturer sets the retail price and then the platform sets the logistics level; moreover, the manufacturer bears part of the logistics cost. In the agency selling model and third-party logistics, the manufacturer sets the retail price and logistics level.

Following are the major findings from our analysis. First, if the logistics-improving efficiency is sufficiently high (low), the manufacturer should choose reselling (agency selling and third-party logistics) when the manufacturer bears a sufficiently small or large part of the logistics cost. When consumers are less (more) sensitive to carbon emissions, the manufacturer should choose the reselling model (agent selling and third-party logistics). Second, if the logistics-improving efficiency further increases (decreases), the selection of the reselling (agency selling and platform/third-party logistics) can benefit the platform. Additionally, each sales model and logistics selected by the manufacturer could also benefit consumers, leading to a “win-win” outcome. Nevertheless, the “win-win” outcome depends on not only the logistics-improving efficiency and the fraction of the logistics cost that the manufacturer should bear but also the proportional fee. Finally, we conclude that if the proportional fee is sufficiently small, only the agency selling and platform/third-party logistics could reach a “win-win-win” outcome; if the proportional fee is intermediate, the three sales and logistics model could reach a “win-win-win” outcome; and if the proportional fee is large, only the reselling/agency selling and platform logistics could reach a “win-win-win” outcome.

Our conclusions offer constructive recommendations for the manufacturer of an enrolled e-commerce platform within an environment where consumers have a dual sensitivity to both logistic levels and carbon emissions: as opposed to the general practice, the manufacturer does not necessarily opt to ask the platform to help cover part of the logistics costs when it is less efficient to improve the logistics, but rather to cover them entirely himself. The purpose is to enable the supply chain to maintain a high logistics level to ensure profitability in an environment where logistics spend is higher. However, when the logistics improvement is inefficient, we recommend that the manufacturer move away from the reselling model and opt for the agency selling model. The double marginal effect can be mitigated to the advantage of the overall supply chain in the environment of high logistics costs. In short, the manufacturer being in a market where consumers are sensitive to both carbon emissions and logistics levels, proactively incurring higher logistics costs and striving to relieve the undesirable double marginal effect for the whole supply chain operating effectiveness can yield higher revenues.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 introduces our model settings. Section 4 characterizes and analyzes the optimal solutions under the three sales and logistics models. Section 5 studies the manufacturer’s sales model and logistics selections and their impact on the platform’s profit and consumer surplus. Section 6 concludes the paper. Proofs are presented in Appendix.

2. Literature review

This work relates to three streams of research: logistics services in supply chains, reselling vs. agency selling formats, and operations management in the low-carbon industry.

2.1. Logistics services

The existing research on logistics services in related supply chains can be roughly divided into three types. The first one is research on logistics services in traditional retail channels [12, 22, 48], described by Zhao et al. [63] as the problem of two competing manufacturers and a common retailer jointly providing products and logistics services to final consumers in traditional supply chains. Ali et al. [3] found that price and service level investment decisions are significantly affected by disturbances in retail market demand. The second is research based on the coexistence of self-built online direct sales channels and traditional distribution channels [33, 46, 53], considered by Dan et al. [9] as the situation where a single channel provides services in a dual-channel supply
chain. Wang et al. [45] studied pricing and service decisions for complementary products in a dual-channel supply chain consisting of two manufacturers and a retailer. Qin et al. [34] explored the interaction between various logistics service strategies and their impact on the choice of sales models. The third is research on logistics services provided by supply chains based on the participation of e-commerce platforms [6, 27, 32, 43]. For instance, Zhang and Ma [59] explored the impact of logistics service level on consumer purchasing behavior. Agrawal [2] and Mehrsai et al. [30] studied collaborative logistics services for partner-facing and consumer-facing supply chains, respectively. Shen et al. [37] researched the selection strategies of four different service channels in the online supply chain dominated by e-commerce platforms. He et al. [18] analyzed the strategic impact of various factors on the manufacturer and the retailer for logistics outsourcing.

2.2. Reselling vs. agency selling

The literature on the sales model of agency selling and reselling can be divided into four categories according to the composition of the supply chain: In the first category, research by several scholars focused on one-to-one supply chains [4, 13, 15, 16, 35, 36]. Among them, Yi et al. [55] showed that manufacturers are ditching the resale model and opting for a agency sale model to ease consumer concerns about fairness. Tan and Carrillo [38] analyzed that manufacturers selling vertically differentiated products choose the agency sales model over the wholesale model. Huang et al. [19] found that the platform under the resale mode is more willing to choose a high pricing strategy for national brands, while the manufacturer under the agency sales is the opposite. In the second, Tian et al. [40] found that in a many-to-one supply chain, the competition intensity of upstream suppliers and order fulfillment costs play a decisive role in retailer model selection. In the third category, some scholars have analyzed the one-to-many supply chain with downstream competition. Zhu and Yao [65] analyzed the influence of the wholesale price and substitutability of complementary products on the sales model selection strategy in the duopoly market. Abhishek et al. [1] and Tan et al. [39] confirmed that choosing agency sales can help reduce product prices and increase the profitability of each member of the supply chain. Finally, in the fourth, in research on many-to-many supply chains, Lu [28] explored the benefits of competing manufacturers and retailers under the framework of bilateral duopoly. Johnson [23] mainly analyzed the influence of the agency model on product prices when multiple retail platforms and multiple manufacturers competed.

According to [27, 34, 37], logistics undertakers usually develop different distribution methods based on variations in the quantities or categories of products, or differences in transportation distances, and so on. Thus, most of the related literature considered both third-party and platform logistics levels as decision variables, we include the same setting in our research. Few of them, provided the manufacturer with multiple choices of logistics methods (i.e., third-party or platform logistics) under the reselling and agency selling models. In contrast, we focus more on the marginal utility of consumers with a double marginal effect, as considering the negative impact on the environment that may result from an elevated logistics level, whereby the consumer’s utility is modeled instead, and win-win and win-win-win outcomes among the manufacturer, the platform and consumers are discussed.

2.3. Carbon emission

Studies related to the impact of carbon emission differences between related channels on multi-channel supply chains compare the current differences in carbon emissions from online and traditional channels only [14, 21, 50, 51, 54, 57, 62]. For example, Edwards et al. [10] provided insights into the carbon footprint of online and traditional channels from a “last mile” perspective. Wu et al. [49] considered the low carbon strategy of online stores and their suppliers based on online to offline (O2O) integration when consumers have low carbon awareness. Yuan et al. [58], considering the risk aversion of advertised products in a low carbon supply chain, examined the strategies chosen by retailers. Weideli and Cheikhrouhou [47] compared consumers’ purchasing habits and carbon footprint in the shopping process, and concluded that online shopping is a greener option. Zhang and Hou [60] studied the strategic choices of retailers’ brand sales models from the perspective of environmentally sustainable development. These studies show that existing research focuses on the differences in carbon emission
levels between different sales channels or models. However, they rarely study the impact of different logistics service levels on carbon emissions resulting from e-commerce platforms and their supply chains.

In summary, many scholars considered consumers who are sensitive to the logistics service level or carbon emission respectively. However, logistics activities, as one of the major sources of carbon emissions (from [5, 25], and increasing the logistics level may have an impact on the environment, and little literature has investigated this phenomenon by modeling. We not only consider that the logistics level and carbon emission of products affect the consumer’s demand for purchase concurrently, but also introduce the negative impacts of increasing logistics level for the environment in our model, considering two selling modes, reselling and agency selling, respectively, so as to be able to suggest selling modes and logistic method selection strategies for a new manufacturer who has moved into an e-commerce platform.

Table 1 describes the comparison between the current work and the most relevant studies.

### Table 1. Comparison between the current study and the related literature.

<table>
<thead>
<tr>
<th>Literature</th>
<th>Pricing</th>
<th>Logistics services</th>
<th>Selling Model Selection</th>
<th>Carbon Emission</th>
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<td>Qin et al. [34]</td>
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<td>Li et al. [27]</td>
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<td>Xu et al. [51]</td>
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<td>Yu et al. [57]</td>
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3. Model

To investigate the manufacturer’s strategy for choosing the selling mode and logistics method, we develop the Stankenbery model and consider an online supply chain consisting of a manufacturer (he) and a platform (she), as depicted in Figure 1. Specifically, under reselling model, the manufacturer sells products to the platform at the wholesale price $w$, and is limited to using the platform logistics because of the transfer of ownership of the products. However, under the agency selling model, the manufacturer pays a proportion of the commission fee $\eta$ to the platform and sells the product directly to consumers. Hereafter, we assume that the proportional fee $\eta$ is exogenous and is not relatively large, where, $0 < \eta < \frac{1}{5}$. This assumption has often been adopted by existing literature related to e-commerce or information sharing [40, 52]. In practice, this fee is generally fixed and normally determined by an industry standard at less than 20%, whereas the prices and information-sharing strategies of the intermediary might be updated frequently. For example, most products on JD.com are charged no more than 20% (except for products with special services such as door-to-door services, which are not considered in this study), and this amount has been stable for more than six years [5]. He has the choice of either third-party logistics (which bears the full logistics cost from [27] or platform logistics (which shares the logistics cost with the platform).
cost for the platform in proportion to \( \lambda \). Because of the limited financing capacity of the manufacturer, where \( 0 < \lambda < 1 \). So, the manufacturer chooses one of the following three models: (i) reselling, (ii) agency selling and third-party logistics, and (iii) agency selling and platform logistics. We employ superscripts “rp”, “at”, and “ap” to denote the optimal solutions for the three model configurations (i.e., reselling, agency selling and third-party logistics, and agency selling and platform logistics), respectively.

Following [40] and [52], we assume that the logistics cost at the logistics level \( s \) is given by \( \frac{1}{2} \beta s^2 \), where \( \beta \) is the efficiency coefficient of logistics level improvement and a larger \( \beta \) represents lower efficiency. According to [27], we believe that the improvement efficiencies of platform logistics and third-party logistics are the same. In practice, under the premise of imperfect logistics technology and operation management, improving the logistics level often requires dispatching more resources and expanding the scope of logistics, which will have more negative impacts on the environment [17, 41]. For example, if some express companies want to achieve next day delivery, they often do not use space efficiently when loading trucks, and there will be space left in the carriage of delivery trucks. Here, we describe the negative impact of improving logistics on the environment as an increase in carbon emissions. We assume that the unit carbon emission of products is \( \Delta \), where \( \Delta = \Delta_0 + \alpha s \) indicates that the unit carbon emission consists of the basic unit carbon emission \( \Delta_0 \) from business operation and the additional carbon emission \( \alpha s \) from the improvement of logistics level. The product price and the unit carbon emission negatively affect the consumer utility, while the logistics level is positive. Specifically, the expression of consumer utility function is as follows:

\[
U = v - k\Delta - p + ls.
\]  

(1)

Where, \( v \) is the valuation of the product, which varies across customers. \( k \) is customers’ sensitivity to carbon emissions, and \( l \) is customers’ sensitivity to logistics level. We assume that the service valuation \( v \) is uniformly distributed over range \([0, 1]\), and a rational customer with valuation \( v \) will buy the product if and only if

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6 As the current standard of logistics service charges of major e-commerce platforms are not standardized. For example, Amazon charges the logistics service fees of the stationed manufacturers according to the weight and volume of the products, etc., JD also considers the total turnover of the products on this basis, while Tmall charges the logistics service fees according to the quantity of the products’ orders. Therefore, charging logistics fees is fundamentally a way for manufacturers to share logistics costs for platforms.
Table 2. Notation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meanings</th>
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<tbody>
<tr>
<td>η</td>
<td>Commission rate charged by the platform, $0 &lt; η &lt; \frac{1}{2}$.</td>
</tr>
<tr>
<td>λ</td>
<td>Proportion of logistics costs that the manufacturer shares when using platform logistics, $0 &lt; λ &lt; 1$.</td>
</tr>
<tr>
<td>Δ</td>
<td>Total carbon emission per unit, consisting of the basic unit carbon emission and that resulting from improved logistics, i.e. $Δ = Δ_0 + αs$.</td>
</tr>
<tr>
<td>Δ₀</td>
<td>Basic unit of carbon emission.</td>
</tr>
<tr>
<td>α</td>
<td>Proportion of the increase in carbon emission per unit due to the logistics level.</td>
</tr>
<tr>
<td>k</td>
<td>Consumer valuation of a product obeys a uniform distribution in the range 0 to 1.</td>
</tr>
<tr>
<td>l</td>
<td>Sensitivity coefficient of consumers to logistics level.</td>
</tr>
<tr>
<td>β</td>
<td>Efficiency of improving the logistics level.</td>
</tr>
</tbody>
</table>

Functions

- $U$: Consumers' utility of products.
- $π_m$, $π_r$: Profits of the manufacturer, platform, respectively.
- $CS$: Consumer surplus of the products.

Decision variables

- $s$: Logistics level.
- $w$: Wholesale price.
- $p$: Selling price.

$U(v) ≥ 0$. This implies that the demand is $d = 1 - kΔ - p + ls$ and the corresponding consumer surplus is

$$CS = \int_{kΔ+p-ls}^{1} (v - kΔ - p + ls) dv = \frac{1}{2}d^2.$$ (2)

The notations we are going to adopt throughout the paper are listed in Table 2. Our time sequence is as follows: Under reselling mode, the manufacturer first sets the wholesale price $w$. Thereafter, the platform determines the logistics level $s$ and the retail price $p$, the logistics cost is borne by the platform. Under agency selling and third-party logistics, the manufacturer determines the logistics level $s$ and the retail price $p$, where the logistics cost is borne by the manufacturer. Finally, under the agency selling model and platform logistics, the platform determines the logistics level $s$, and the manufacturer sets the retail price $p$, the logistics cost is shared between the manufacturer and platform.

4. Equilibrium analysis

In this section, we characterize the optimal solutions for the three model configurations. We normalize the manufacturer’s production cost to zero. To ensure that the objective functions for the three models are concave and ensure positive profits, we assume the following:

$$\frac{1 - \sqrt{2β}}{\alpha} ≤ k < \min\left\{\frac{l}{α}, \frac{1}{Δ_0}\right\},$$

$$λ < \frac{1}{4} \left[ 4 + \frac{(l - kα)^2η^2}{β(1 - η)} - \sqrt{\frac{8(l - kα)^2β(1 - η)η^2 + (l - kα)^4η^4}{β^2(1 - η)^2}} \right].$$
4.1. Reselling

In this case, since the platform bears the logistics cost, the profit functions of the manufacturer and e-commerce platform are as follows:

\[ \pi_m = w (1 - k\Delta - p + ls), \]
\[ \pi_p = (p - w) (1 - k\Delta - p + ls) - \frac{1}{2} \beta s^2. \] (3)

We solve this model in a backward induction. Given the wholesale price and logistics level, the platform first determines the product price. Substituting the best response of the price into the manufacturer’s profit function, we find the optimal wholesale price and logistics level. This leads to the following lemma.

**Lemma 4.1.** When the manufacturer adopts the reselling model and platform logistics, the optimal solutions are as follows: \( w^{*p} = \frac{1}{2} (1 - k\Delta_0), s^{*p} = \frac{\beta (1 - k\Delta_0) \beta (1 - k\Delta_0)}{2(2\beta - (l - k\Delta))]}, p^{*p} = \frac{\frac{\beta^2 - (l - k\Delta^2)}{2(2\beta - (l - k\Delta))]}}{2(2\beta - (l - k\Delta))}. \) Moreover, the optimal profits of the manufacturer and the e-commerce platform and the corresponding consumer surplus are, respectively, given by \( \pi_m^{*p} = \frac{\beta (1 - k\Delta_0)^2}{4(2\beta - (l - k\Delta))}, \pi_p^{*p} = \frac{\beta^2 (1 - k\Delta_0)^2}{8(2\beta - (l - k\Delta))^2}, \) and \( CS^{*p} = \frac{\beta^2 (1 - k\Delta_0)^2}{8(2\beta - (l - k\Delta))^2}. \)

From Lemma 4.1, it is easy to understand that when the manufacturer chooses the reselling model and platform logistics, with the increase in the basic emissions from the product (i.e., \( \Delta_0 \) increases) or customers’ sensitivity to carbon emission (i.e., \( k \) increases), the manufacturer should lower the wholesale price. This induces the platform to reduce the logistics level and product price, which ultimately leads to lower profits for both the manufacturer and platform, as well as consumer surplus. Nevertheless, if customers’ sensitivity to logistics level, \( l \), increases, the manufacturer can keep the wholesale price unchanged but the platform can improve the logistics level and the product price such that the three members’ profits increase. Finally, as the logistics-improving efficiency, \( \beta \), increases, the logistics level and product price could be increased, leading to higher profits for the three members.

4.2. Agency selling model

When the manufacturer adopts the agency selling model, he sells through the platform by paying a proportional fee \( \eta \) but sets the retail price. The manufacturer can use either third-party or platform logistics. We consider these two scenarios in the following.

4.2.1. Using third-party logistics

In this case, the logistics cost is borne by the manufacturer. The profit functions of the manufacturer and the e-commerce platform are as follows:

\[ \pi_m = (1 - \eta)p (1 - k\Delta - p + ls) - \frac{1}{2} \beta s^2, \]
\[ \pi_p = \eta p (1 - k\Delta - p + ls). \] (4)

In this model, the manufacturer simultaneously decides the logistics level and the product price \( p \). The optimal logistics level and product selling price and the corresponding optimal profits of both the manufacturer and platform are as follows.

** Lemma 4.2.** When the manufacturer adopts the agency selling model and third-party logistics, the optimal solutions are as follows: \( s^{at} = \frac{\beta (1 - k\Delta_0)(1 - k\Delta_0)}{2\beta - (l - k\Delta_0)(1 - \eta)}, p^{at} = \frac{\beta (1 - k\Delta_0)(1 - \eta)}{2\beta - (l - k\Delta_0)(1 - \eta)}. \) Consequently, the optimal profits of the manufacturer and the e-commerce platform and the corresponding consumer surplus are, respectively, given by \( \pi_m^{at} = \frac{\beta^2 (1 - k\Delta_0)^2 (1 - \eta)}{2(2\beta - (l - k\Delta_0)(1 - \eta))^2}, \pi_p^{at} = \frac{\beta^2 (1 - k\Delta_0)^2 \eta}{2(2\beta - (l - k\Delta_0)(1 - \eta))^2}, \) and \( CS^{at} = \frac{\beta^2 (1 - k\Delta_0)^2 \eta}{2(2\beta - (l - k\Delta_0)(1 - \eta))^2}. \)
Similar to Lemma 4.1, from Lemma 4.2, we can also confirm that when $\Delta_0$ or $k$ increases, the manufacturer should reduce the logistics level and the product price to attract more customers, which results in lower profits of both the manufacturer and platform, as well as consumer surplus. However, if customers’ sensitivity to logistics level $l$ or the logistics-improving efficiency, $\beta$ increases, both the manufacturer and platform can increase the logistics level and product price, respectively, such that the three members’ profits increase. Finally, when the commission rate ($\eta$) increases, the manufacturer has an incentive to lower the logistics level, which induces the platform to reduce the product price. Although the manufacturer earns less profit, the platform can still obtain more profit in this case because the commission rate increases, which is not good for consumers.

4.2.2. Using the platform logistics

In this case, the platform determines the logistics level, and the manufacturer bears the fraction $\lambda$ of the logistics cost. Therefore, the profit functions of the manufacturer and the e-commerce platform can be, respectively, given by

$$\pi_m = (1 - \eta)p (1 - k\Delta - p + ls) - \frac{1}{2} \lambda \beta s^2,$$

$$\pi_p = \eta p (1 - k\Delta - p + ls) - \frac{1}{2} (1 - \lambda) \beta s^2.$$  

We solve this model in a backward induction. Given the logistics level, the manufacturer first determines the product price. Substituting the best response of the price into the platform’s profit function, we find the logistics level set by the platform. This leads to the following lemma.

Lemma 4.3. When the manufacturer adopts the agency selling model and platform logistics, the optimal solutions are as follows: $s^{ap} = \frac{(l - ko)(1 - k\Delta_0)\eta}{2\beta(1 - \lambda) - (l - ko)^2\eta}$, $p^{ap} = \frac{\beta(1 - k\Delta_0)(1 - \lambda)}{2\beta(1 - \lambda) - (l - ko)^2\eta}$. Therefore, the optimal profits of the manufacturer and the e-commerce platform and the corresponding consumer surplus are, respectively, given by $\pi^{ap}_m = \frac{\beta(1 - k\Delta_0)^2[2\beta(1 - \lambda)(1 - \lambda) - (l - ko)^2\eta^2\lambda]}{2[2\beta(1 - \lambda) - (l - ko)^2\eta]^2}$, $\pi^{ap}_p = \frac{\beta(1 - k\Delta_0)^2\eta(1 - \lambda)}{4\beta(1 - \lambda) - 2(l - ko)^2\eta^2}$, and $CS^{ap} = \frac{\beta^2(1 - k\Delta_0)^2(1 - \lambda)^2}{2[2\beta(1 - \lambda) - (l - ko)^2\eta]^2}$.

We are interested in the impact of the fraction $\lambda$ of the logistics cost. In fact, according to Lemma 4.3, one can show that with the increase in $\lambda$, that is, the manufacturer bears a large part of the logistics cost, increases the logistics level to attract more customers, and compensates for the increased logistics cost as much as possible. Nevertheless, this enables the manufacturer to increase the price such that his profit decreases while the platform’s profit increases. However, the price increase will negatively impact consumers. Moreover, the improvement of logistics level positively impacts consumer surplus.

5. Sales model and logistics selections and the impacts

This section studies the manufacturer’s sales model and logistics selections. We also study the impact of his selection on the platform’s profit and consumer surplus.

5.1. Manufacturer’s sales model and logistics selections

The following theorem characterizes the manufacturer’s sales model and logistics selections.

Theorem 5.1. There exist two thresholds $\lambda^l$ and $\lambda^h$ such that the following statements hold:

(i) if $\beta < \frac{(l - ko)^2(1 - \eta)}{2(1 - 2\eta)}$ and $\lambda < \lambda^l$ or $\beta < \frac{(l - ko)^2(1 - \eta)}{2(1 - 2\eta)}$ and $\lambda \geq \lambda^h$, the manufacturer is willing to choose the reselling model and platform logistics (i.e., $\pi^{ap}_m > \max\{\pi^{at}_m, \pi^{ap}_m\}$);

(ii) if $\beta \geq \frac{(l - ko)^2(1 - \eta)}{2(1 - 2\eta)}$ and $\lambda < \lambda^l$ or $\beta \geq \frac{(l - ko)^2(1 - \eta)}{2(1 - 2\eta)}$ and $\lambda \geq \lambda^h$, the manufacturer prefers to adopt the agency selling model and third-party logistics (i.e., $\pi^{at}_m > \max\{\pi^{ap}_m, \pi^{at}_m\}$);

(iii) if $\lambda^l \leq \lambda < \lambda^h$, the manufacturer should employ the agency selling model and platform’s logistics (i.e., $\pi^{ap}_m \geq \max\{\pi^{ap}_m, \pi^{at}_m\}$).
Theorem 5.1 indicates that the manufacturer’s sales model and logistics selection strongly depends on the logistics-improving efficiency and the fraction of the logistics cost that the manufacturer bears. Specifically, when the fraction of logistics cost is sufficiently small or large (i.e., $\lambda < \lambda^l$ or $\lambda \geq \lambda^h$), the manufacturer should choose the reselling model (agency selling model and third-party logistics) if the logistics-improving efficiency is high (low), that is, $\beta < \frac{(l-k\alpha)^2(1-\eta)}{2-4\eta}$ ($\beta \geq \frac{(l-k\alpha)^2(1-\eta)}{2-4\eta}$). See Region I (II) in Figure 2 for a graphical illustration. Nevertheless, when the fraction of the logistics cost is intermediate, the agency selling model and platform logistics is a better choice for the manufacturer (see Region III in Fig. 2). This result stands in sharp contrast to that of [34]. They show that the manufacturer should adopt the reselling model and platform logistics (agency selling model and third-party logistics) if the logistics-improving efficiency is relatively high (low). We explain this result as follows.

First, if logistics is more efficient, the manufacturer is likely to opt for the reselling model. Because higher logistical efficiency tends to imply a higher logistics level (from Lem. 4.3), and more overall market demands,
the manufacturer should choose the reselling model to increase the wholesale price and gain more revenue under the double marginality.

Second, we note that the manufacturer is expected to choose the strategy of agency selling and platform logistics when the proportion of shared logistics costs is in the middle of the range. The reason is that if the manufacturer’s share of logistics costs is low, the platform needs to spend more on logistics costs, but as the decision maker, in which she is prone to reduce the logistics level in order to cut costs. This eventually leads to lower overall demand in the supply chain, which is detrimental to the manufacturer. On the contrary, if the manufacturer shares too much of the logistics costs, it will bring him a serious burden. It is a better choice to choose the reselling model to let the platform bear the logistics costs completely, or to choose the agency selling and third-party logistics to get the decision-making power of the logistics level.

Third, we notice that if consumers are more concerned about carbon emission levels (i.e., $k$ increases), the manufacturer’s profits under the three modes will decrease accordingly, because consumers have a strong awareness of environmental protection, the logistics undertaker will have to reduce the logistics level appropriately, which in turn will have a negative impact on product demand. And the manufacturer is less (more) likely to adopt the reselling model (agency selling model and platform logistics). The intuition is that the wholesale price under the reselling model will decrease if consumers are too sensitive to carbon emission levels. This induces the manufacturer to choose the agency selling model. Besides, since consumers’ high awareness of environmental protection will greatly increase logistics costs, it is better for the manufacturer to share the logistics cost with the platform and thus employ the platform logistics.

5.2. Impact of the selections on the platform’s profit

The following theorem examines the impact of the manufacturer’s selections on the platform’s profit.

Theorem 5.2. There exist two thresholds $\beta^l$ and $\beta^h$ such that the following statements hold:

(i) the selection of the reselling model by the manufacturer increases the platform’s profit if $\beta < \beta^l$ and $\lambda < \lambda^l$ or $\beta < \beta^l$ and $\lambda \geq \lambda^h$ (i.e., $\pi^p_p > \max\{\pi^a_t, \pi^p_p\}$);

(ii) the selection of the agency selling model and third-party logistics by the manufacturer increases the platform’s profit if $\frac{(l-k)a^2(1-n)}{2-4n} < \beta < \beta^h$ and $\lambda < \lambda^l$ or $\beta \geq \frac{(l-k)a^2(1-n)}{2-4n}$ and $\lambda \geq \lambda^h$ (i.e., $\pi^a_t > \max\{\pi^r_p, \pi^p_p\}$);

(iii) the selection of the agency selling model and platform’s logistics by the manufacturer increases the platform’s profit if $\beta \geq \frac{(l-k)a^2(1-n)}{2-4n}$ and $\lambda^l \leq \lambda < \lambda^h$ (i.e., $\pi^r_p \geq \max\{\pi^r_p, \pi^a_t\}$).

Theorem 5.2 reveals that each sales model and logistics selected by the manufacturer could benefit the platform, which leads to a “win-win” outcome. More specifically, when the proportion of logistics cost borne by the manufacturer is small or large enough (i.e., $\lambda < \lambda^l$ or $\lambda \geq \lambda^h$), the selection of the reselling model and platform logistics (the agency selling model and third-party logistics) increases the platform’s profit if the efficiency of raising the logistics level is high (not too low), that is, $\beta < \beta^l$ ($\frac{(l-k)a^2(1-n)}{2-4n} < \beta < \beta^h$ or $\beta \geq \frac{(l-k)a^2(1-n)}{2-4n}$). However, when the proportion of logistics cost borne by the manufacturer is intermediate (i.e., $\lambda^l \leq \lambda < \lambda^h$), only the selection of the agency selling and platform’s logistics can benefit the platform if the efficiency of raising the logistics level is low ($\beta \geq \frac{(l-k)a^2(1-n)}{2-4n}$ and $\lambda^l \leq \lambda < \lambda^h$). See Figure 3 for a graphical illustration.

Our interpretation of the result is as follows. First, recall that in the agency selling and platform logistics model, the manufacturer bearing too much or too little logistics costs is detrimental (see Thm. 5.1). On such a basis, if the logistics is very efficient and the manufacturer chooses the resale model, the platform not only benefits from more market demand due to the high logistics level, but also increases its profit by raising the product price due to the stronger double marginality in the reselling structure. Conversely, if logistics efficiency is low and the manufacturer chooses agency selling and third-party logistics, even if the platform loses pricing
power over the product, it can still cut costs by not bearing logistics costs at all, and ultimately gain revenue under this sales model. Coming to the agency selling and platform logistics model, unlike the manufacturer’s sole profitability, the platform is profitable simultaneously with the manufacturer only if the logistics level is low. Because the price of the product in this model is determined directly by the manufacturer, excessive logistics efficiency prompts him to set a higher price, which negatively affects the market demand and reduces the profitability of the platform.

We find that when consumers pay more attention to the level of carbon emissions (i.e., \( k \) increases), it is easy to see that the profits of the platform will also decline accordingly, because the stronger the consumers’ awareness of environmental protection, the lower the logistics level will be, which is unfavorable to both upstream and downstream of the supply chain. And because of the decline in wholesale price and the increase in logistics cost
under the reselling model, the manufacturer will be prompted (obstructed) to choose agent selling and platform logistics (reselling) model.

5.3. Impact of the selections on consumer surplus

We next analyze the impact of the manufacturer’s selections on consumer surplus, as characterized in the following theorem.

**Theorem 5.3.** There exists a threshold \( \lambda^* \) such that the following statements hold:

(i) the selection of the reselling model by the manufacturer increases consumer surplus if \( \eta \geq \frac{2\beta-(l-\kappa\alpha)^2}{(l-\kappa\alpha)^2} \), \( \beta < \min \left\{ \frac{(l-\kappa\alpha)^2(1-\eta)}{2-4\eta}, \frac{3}{5}(l - \kappa\alpha)^2 \right\} \), and \( \lambda < \lambda^l \) (i.e., \( CS^{sp} > \max\{CS^{at}, CS^{ap}\} \));

(ii) the selection of the agency selling model and third-party logistics by the manufacturer increases consumer surplus if \( \eta < \frac{2\beta-(l-\kappa\alpha)^2}{4\beta-(l-\kappa\alpha)^2} \) and \( \lambda < \lambda^l \) (i.e., \( CS^{at} > \max\{CS^{sp}, CS^{ap}\} \));

(iii) the selection of the agency selling model and platform’s logistics by the manufacturer increases consumer surplus if \( \max \left\{ \frac{1-2n}{1-\eta}, \frac{2\beta-(l-\kappa\alpha)^2}{2\beta-2(l-\kappa\alpha)^2} \right\} \leq \lambda < \lambda^h \) (i.e., \( CS^{ap} > \max\{CS^{sp}, CS^{at}\} \)).

Similar to Theorem 5.2, each sales model and logistics selected by the manufacturer could benefit consumers, which also leads to a “win-win” outcome. Nevertheless, Theorem 5.3 finds that the “win-win” outcome depends on not only the logistics-improving efficiency and the fraction of the logistics cost that the manufacturer bears but also the proportional fee. In fact, choosing the reselling model and platform logistics benefits consumers only if both the proportional fee and the logistics-improving efficiency are large but the proportion of the logistics cost shared by the manufacturer is small (see Thm. 5.3(i)). Because the platform claims a higher commission fee, which divides more of the manufacturer’s revenue, and to avoid the manufacturer raising the price of the product to the detriment of consumers, we recommend that he choose the reselling model. This will transfer the decision-making power to the platform, but the platform also raises the logistics level in this scenario where logistics is more efficient.

According to Theorem 5.3(ii), under agency selling and third-party logistics, if the proportional fee is small, the manufacturer will not set the price of the product very high under the agency selling model. Instead, the decision making will appropriately increase the level of logistics to satisfy the consumers. If the manufacturer in the agency selling and platform logistics model for the platform to share logistics costs in the middle threshold, both to ensure that the manufacturer does not set excessively high product price, but also to ensure that the logistics service is not at a low level (from Thm. 5.3(iii)).

Similarly, when consumers’ awareness of environmental protection is enhanced, the logistics level will decline, which will also have a negative impact on consumer surplus. And in the face of a win-win choice among the three parties, consumers’ higher awareness of environmental protection will also prompt (block) the manufacturer to choose agent selling and platform logistics (reselling) model.

Combining Theorems 5.1–5.3, it is not difficult to see how each sales model and logistics selected by the manufacturer could benefit both the platform and consumers, leading to a “win-win-win” outcome. As depicted in Figure 4, (i) if the proportional fee is sufficiently small, only the agency selling and platform/third-party logistics could reach a “win-win-win” outcome because the reselling model is always unfavorable for the manufacturer (see Fig. 4(a)); (ii) if the proportional fee is intermediate, the three sales and logistics model could reach a “win-win-win” outcome (see Fig. 4(b)); and (iii) if the proportional fee is large, only the reselling/agency selling and platform logistics could reach a “win-win-win” outcome because the logistics level will be reduced under the agency selling and third-party logistics in the presence of the high proportional fee (see Fig. 4(c)).

6. Conclusion and future research

In the market environment of e-commerce, manufacturers increasingly choose to sell products through e-commerce platforms manufacturers settled on the platform is one of the most popular business models, some of which use the resale model while others use the agency sales model. As we all know, logistics service is
Figure 4. The effects of $\beta$ and $\lambda$ on the e-commerce platform's profit and consumer surplus. Notes. “W-W-W” represents that a “win-win-win” outcome can be reached; that is, the manufacturer, the platform, and customers all become better off. The following tested parameter values are used: $l = 0.5$, $\alpha = 0.8$, $k = 0.08$, $\eta = 0.13$. (a) $\eta < \eta^*$. (b) $\eta^* \leq \eta < \frac{2\beta-(l-k\alpha)^2}{4\beta-(l-k\alpha)^2}$. (c) $\eta \geq \frac{2\beta-(l-k\alpha)^2}{4\beta-(l-k\alpha)^2}$. 
among the most expensive businesses in the e-commerce industry. It plays a key role in improving consumer satisfaction, increasing the income of supply chain members, and so on. However, blindly improving the level of logistics will increase the level of carbon emissions in the supply chain and reduce the purchasing desire of environmentally conscious consumers. In practice, the decision-making of sales patterns and logistics methods interact, and carbon emissions have an impact on logistics methods, usually resulting in the three most common sales patterns: reselling, agency selling and third-party logistics, and agency selling and platform logistics. On this basis, we developed a method to explore how manufacturers should decide on logistics and sales models when they are affected by carbon emission levels while on the platform. We found several significant conclusions.

First, when only considering the manufacturer’s profit, if the manufacturer’s share of logistics costs is larger or smaller, and the efficiency of logistics level improvement is high (low), the manufacturer should choose the reselling (agency selling and third-party logistics) model. If the manufacturer shares the logistics cost in the middle threshold, he should choose agency sales and platform logistics. Second, when we further compare the equilibrium solutions of platform profits, we find that if the manufacturer’s share of the logistics cost is very large or small, and the logistics-improving efficiency is very high (slightly lower), the manufacturer should choose the resale (agent sales and third-party logistics) model. If the improvement efficiency is slightly lower, the manufacturer will share logistics costs, no more and no less, he should choose agency sales and platform’s logistics. Third, when we consider the consumer surplus, if the efficiency and commission ratios for improving the logistics level are slightly higher, and the manufacturer shares the logistics cost with a lower cost, he should choose the resale model. If the commission ratio is slightly smaller and the shared cost is also low, he should choose agency sales and third-party logistics. If the improvement efficiency is slightly lower, the manufacturer should choose agency sales and platform logistics. In addition, the platform with social responsibility will not set the manufacturer to share the logistics cost in a higher proportion range. Finally, we combine the above conclusions to explore how manufacturers can make decisions that can benefit all three parties in the supply chain. We find that if the logistics-improving efficiency is high, and the manufacturer needs to pay a higher commission, but the logistics cost is less, he should choose the resale model. If the logistics-improving efficiency and the commission ratio are slightly low, the logistics cost that the manufacturer needs to share is low, and for the manufacturer, agency sales and third-party logistics is the best. If the manufacturer’s cost sharing is intermediate and the level of logistics is slightly lower, he should choose agency sales and platform logistics. It is worth noting that when consumers are more (less) sensitive to carbon emission levels, the manufacturer is more inclined to choose the agency sales and platform logistics (resale) model.

As e-commerce becomes increasingly popular in the commercial market, many manufacturers choose to open online channels to improve the distribution of their products. As a result, manufacturers often need a more rational and scientific approach to the selection of sales models on e-commerce platforms. We believe our model addresses important questions, and our findings provide valuable management implications for manufacturers looking to join e-commerce. The most significant contribution of our study is its contribution to analyzing and studying various sales models in reselling, considering almost all types of stores that manufacturers can choose to enter e-commerce platforms. We put forward suggestions for manufacturers’ specific sales model selection strategies to deal with different market conditions, so that a win-win outcome for all channel members can be achieved.

The research in this study can be expanded from the following aspects. First, this study only considers consumers who are sensitive to the levels of logistics and have environmental protection concepts. It is possible to further divide the types of consumers and consider the heterogeneity in consumers’ sensitivity to logistics levels and environmental protection. Additionally, this study focuses on the sales of a single category of products, and the selection strategies of the manufacturer for only three different sales models. Future research can explore the choice of the unit carbon emission levels and logistics levels of the manufacturer’s sales models strategies, based on the market of two or more complementary and competitive products.
Appendix A.

Proof of Lemmas 4.1, 4.2, and 4.3: The solution is obtained by backward induction. First, find the first-order and second-order derivatives of the product price with respect to the platform profit, respectively:

\[
\frac{\partial \pi_p}{\partial p} = 1 - 2p + ls + w - k(\alpha s + \Delta_0)
\]

\[
\frac{\partial^2 \pi_p}{\partial p^2} = -2 < 0.
\]

When the first-order derivative is 0 and the second-order derivative is less than 0, \(\pi_p(p)\) is a convex function about \(p\). If \(\frac{\partial \pi_p}{\partial p} = 0\), the optimal solution \(p^{rp}\) is

\[
p^{rp} = \frac{1}{2} \left( 1 + ls^{rp} + w - \alpha ks - k\Delta_0 \right) = 0.
\]

Introducing \(p^{rp}\) into the expression of platform profit, and solving the first and second derivatives of \(s\) with respect to the e-commerce platform’s profit:

\[
\frac{\partial \pi_p}{\partial s} = \frac{1}{2} \left[ ts - ko(1 - w) - 2\beta s + k^2 \alpha (\alpha s + \Delta) - l (w + 2k\alpha s + k\Delta_0 - 1) \right]
\]

\[
\frac{\partial^2 \pi_p}{\partial s^2} = \left[ - \beta - \frac{1}{2} (l - ko)^2 \right] .
\]

Here, if the second derivative \(\frac{\partial^2 \pi_p}{\partial s^2}\) is less than 0, and \(\frac{\partial \pi_p}{\partial s} = 0\), then we get the optimal solution for \(s^{rp}\)

\[
s^{rp} = \frac{(l - ko)(1 - w - k\Delta)}{2\beta - (l - ko)^2} .
\]

Finally, introducing \(s^{rp}\) into the expression of the manufacturer’s profit, and using the same method, we find the optimal solution for \(w^{rp}\)

\[
w^{rp} = \frac{1}{2} (1 - k\Delta_0).
\]

Introducing \(w^{rp}\) into \(s^{rp}, p^{rp}, \pi^{rp}\) and \(\pi^{rp}_p\) respectively. Lemma 4.1 requires \(\frac{\partial^2 \pi^{rp}}{\partial s^{rp}^2} < 0\), needs to let \(\beta - \frac{1}{2}(l - ko)^2 > 0\), then it also satisfies \(\frac{\partial^2 \pi^{rp}}{\partial s^{rp}^2} = \frac{2\beta}{2\beta - (l - ko)^2} < 0\).

We adopt the same method of calculating the optimal solution, and require that \(\frac{\partial^2 \pi_m}{\partial s^2}, \frac{\partial^2 \pi_m}{\partial p^2}\) are less than 0, need to let \(\frac{\partial^2 \pi_m}{\partial s^2} = -\beta - [\frac{1}{2}(l - ko)^2 (1 - \eta)] < 0\) and \(\frac{\partial^2 \pi_m}{\partial p^2} = -[\beta(1 - \lambda) - \frac{1}{2}(l - ko)^2 \eta] < 0\) respectively; At this time, the second-order derivatives are respectively less than 0 and each parameter satisfies the range of the model symbol description table. It is also necessary to satisfy the optimal solutions in Lemmas 4.1, 4.2, and 4.3, which are all greater than 0. After simplification, the basic range of each parameter is obtained:

\[
\frac{1 - \sqrt{2\beta}}{\alpha} < k < \min\left\{ \frac{1}{\alpha}, \frac{1}{\Delta_0} \right\},
\]

\[
\lambda < \frac{1}{4} \left[ 4 + \frac{(l - ko)^2 \eta^2}{\beta(1 - \eta)} - \sqrt{\frac{8(l - ko)^2 \beta(1 - \eta)\eta^2 + (l - ko)^4 \eta^4}{\beta^2 (1 - \eta)^2}} \right].
\]

The subsequent calculation and discussion of these parameters are based on the basic value range of each parameter, and will not be repeated when the threshold value of each parameter is involved in the discussion.
Proof of Theorems 5.1, 5.2, 5.3 and Figure 4: We describe $\lambda$ as a piecewise function, when $\beta < \frac{(l-\kappa \alpha)^2(1-\eta)}{2(1-2\eta)}$:

\[
\lambda^l = \frac{1}{4} \left\{ \frac{\beta(2-\eta)\eta(1-2\eta)}{(1-\eta)^2} \left\{ \beta - (l - \kappa \alpha)^2 - \frac{2\beta - (l - \kappa \alpha)^2}{2\beta - (l - \kappa \alpha)^2} \eta \right\} - \frac{\eta^2(2k\alpha + l^2)}{\beta(1-\eta)} \right. - \\
\left. \sqrt{\frac{(l - \kappa \alpha)^2 \left[ \beta - (l - \kappa \alpha)^2 - \frac{2\beta - (l - \kappa \alpha)^2}{2\beta - (l - \kappa \alpha)^2} \eta \right]}{\beta(1-\eta)^2} \right. - \\
\left. \beta \left( \frac{4 + 5(2-\eta)\eta - k^2\alpha^2(1-\eta)\eta^2}{\beta(1-\eta)^2} \right) \right\},
\]

when $\beta \geq \frac{(l-\kappa \alpha)^2(1-\eta)}{2(1-2\eta)}$:

\[
\lambda^h = \frac{1}{4} \left\{ \frac{\beta(2-\eta)\eta(1-2\eta)}{(1-\eta)^2} \left\{ \beta - (l - \kappa \alpha)^2 - \frac{2\beta - (l - \kappa \alpha)^2}{2\beta - (l - \kappa \alpha)^2} \eta \right\} - \frac{\eta^2(2k\alpha + l^2)}{\beta(1-\eta)} \right. + \\
\left. \sqrt{\frac{(l - \kappa \alpha)^2 \left[ \beta - (l - \kappa \alpha)^2 - \frac{2\beta - (l - \kappa \alpha)^2}{2\beta - (l - \kappa \alpha)^2} \eta \right]}{\beta(1-\eta)^2} \right. + \\
\left. \beta \left( \frac{4 + 5(2-\eta)\eta - k^2\alpha^2(1-\eta)\eta^2}{\beta(1-\eta)^2} \right) \right\}.
\]

Among them, $0 < \lambda^l < \lambda^h$ is always established.

\[
\beta^l = \frac{(l - \kappa \alpha)^2(1-3\eta)}{2 - 8\eta} - \sqrt{\frac{(l - \kappa \alpha)^4\eta^2}{(1-4\eta)^2}},
\]

\[
\beta^h = \frac{(l - \kappa \alpha)^2(1-3\eta)}{2 - 8\eta} + \sqrt{\frac{(l - \kappa \alpha)^4\eta^2}{(1-4\eta)^2}}.
\]

Among them, $\beta^l < \beta^h$ is always established.
Among them, \( \eta^r < \frac{2\beta-(l-k\alpha)^2}{4\beta-(l-k\alpha)^2} \) is constantly established.

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References


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