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A LOCATION-INVENTORY-PRICING MODEL FOR A THREE-LEVEL SUPPLY CHAIN DISTRIBUTION NETWORK

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Abstract. This article applies competition-based pricing method to a location-inventory-pricing problem, something which had not yet been investigated as of this study. It reveals that examining the three problems of location, inventory and pricing in an integrated and simultaneous manner can be the best approach to optimize the supply chain (SC) and increase the profitability of its companies. The investigated three-level SC distribution network model includes one supplier, several distribution centers (DCs), and several customers. The model is structured in form of a mixed integer nonlinear programming to maximize the long-term average profit of the distribution network. It considers constraints of inventory capacity along with continuous review inventory policy in each open DC. For its DCs' retail price determination, two competing pricing approaches of higher than and lower than the average price of competitors have been used. The approach has been investigated on several potential DCs and customers and its numerical results show that retail price from the DCs to the customers is dependent on the average price of the market and its fluctuations. The major finding of this research is the fact that the type of demand function can have a substantial effect on the obtained results in the location-inventory-pricing model and greatly influence the amount of the profit of each company. Its applied strategy along with innovative competition-based pricing approach and projection of customers' demand *via* linear and exponential functions show how companies can increase their profit through price adjustment and demand projection.

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1. Introduction

In our competitive environment, design of a suitable SC network is one of the important strategies for increasing the profits of companies. This is mainly because product and service pricing play an essential role in business competitiveness in the market, and strategic pricing decisions can have a substantial effect on business profitability since pricing affects customers' demand and their purchasing decisions in various locations. Decisions related to the location of facilities are in the category of strategic decisions and decisions related to inventory control are included in the category of operational and tactical decisions [17]. In general, location-inventory problems determine location, allocation, and inventory decisions simultaneously. The idea is to integrate SC's

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strategic decisions (*e.g.*, determining the location and capacity of facilities), along with tactical decisions (*e.g.*, production and distribution planning, demand planning), and operational decisions (*e.g.*, inventory planning, transportation planning) [14]. Through location-inventory optimization, companies can reduce costs and increase their profitability.

This article investigates the location-inventory-pricing problem in a three-level SC distribution network. Its model considers a three-level SC which can be used to design distribution networks of similar industries. In this approach, the problem of location-inventory becomes a problem of location-inventory-pricing by adding pricing methods. It uses competition-based pricing method and adopts two methods of pricing above the average price of competitors and pricing below the average price of competitors to maintain profitability under different scenarios, details of which is explained in Section 3. The idea is that if a company incurs higher costs than competitors to provide a product or service, by using a pricing method higher than the average price of competitors, it can maintain its profitability. On the other hand, when a company incurs lower costs than competitors, it can attract price-sensitive customers, gain market share, and maintain its profitability. This pricing method can also be used to quickly sell surplus products in the warehouse. In general, the use of these two pricing methods depends on various factors, including the goals of the company which depend on the competitive environment of the market, and the perceived value of the products or services.

The three-level SC of this study includes one supplier, several DCs and several customers. Its case study is a sugar product which was implemented for a sample company in Iran based on the data of the year 2022. It assumes that each DC has a limited storage capacity, and such limits affect the allocation of customers to open DCs. In location-inventory-pricing issues, after reviewing decisions related to location and inventory, the product pricing process is done. After determining the retail price of the product by the DCs for customers, the amount of customer demand is determined. In the investigated problem, each DC can offer different retail prices to customers. Furthermore, the demand of customers is sensitive to the price, and investigated based on two patterns of linear and exponential functions.

2. Literature review

Since suitable SC networks are quite important in our business environment, numerous studies have investigated simultaneous consideration of location, allocation, and inventory decisions in their design. This section reviews some previous studies that have investigated these issues. It initially provides a general overview and then examines the details of its two forms of location-inventory problem and location-inventory-pricing problem, the second category of which considers price sensitive customers and is more directly relevant to this study.

In a SC, when an inventory is not suitably located, it can lead to loss and additional costs. Furthermore, if the pricing of products and services are not done appropriately, it may reduce profitability. Hence, examining the three problems of location, inventory and pricing in an integrated and simultaneous manner can be the best approach to optimize the SC and increase the profitability of companies. Several researchers have investigated the location-inventory issues in this structure [15, 24, 26].

It should be noted that the type of demand function has a substantial effect on the results obtained in inventory-pricing models. Previous studies show that a very small change in the appearance of the demand curve can result in a very large change in the optimal solution of the model [22]. Pricing methods are considered as explicit steps or procedures by which companies arrive at pricing decisions. They are divided into three general categories, based on cost, competition, and demand. There are five cost-based, four competition-based, and three demand-based pricing methods, which sums up to twelve pricing methods [6]. The cost-based pricing strategy has a sense of financial prudence, in which the profit margin is generally added to the costs. Productions that use cost-based pricing are always looking for measures to reduce costs. In general, the import of raw materials and needed supplies can improve the profit margin [10]. In the demand-based pricing strategy, the seller adjusts product's price according to the customer's demand and the perceived value of the product. This strategy is used in the airline industry to price tickets in different seasons of the year. Competitive pricing strategy uses competitors' price levels to determine retail prices. Its main advantage is consideration of the real pricing state of competitors. It should be noted that strong competition can create a price war among competitors in the market [19].

The four pricing methods of competition-based are, pricing similar to competitors or according to the average market price, pricing higher than the average price of competitors, pricing lower than the average price of competitors and pricing based on the prevailing price in the market [6]. Pricing based on competition is mostly used in markets that offer similar products or services. Its main criterion is the average market price, and the product may be sold at a higher or lower price than this criterion. In this method, the retail prices are determined according to the competitors' prices. In this pricing method, companies with an average share compete with competitors that have a higher share. As an example, we can cite the price competition between local hotels and international chain hotels. This method is also used for products with the least differences, such as types of gasoline [28].

In pricing models based on competition, the products are either the same or distinguishable in terms of quality by at least one parameter. For similar products, their retail price serves as the only variable in purchase decision. However, many companies try to differentiate their products, and this changes the focus from price as a competitive lever to other product-related features. This way, superior product design creates differentiation [16]. Vidrova *et al.* investigated numerous advantages and limits of competitive pricing [32]. The study considers adoption of competitive pricing easy and safe. It is mainly because in this approach competitors have already determined the optimal price and considered that around 60% of customers consider pricing first in their purchase decisions. Hence, companies can easily have access to prices of competitors, such pricing can keep customers satisfied and help business to grow. When most retailers use this pricing method, the market can also reach to an equilibrium pricing level. As such, competitive pricing is considered a very important part of revenue management. It can serve as a management tool that provides the possibility of selling suitable products and services to customers in such a way that the highest income is obtained with the best retail price. It should be noted that pricing higher than the average price of competitors leads to higher income. For example, when marine tourism ports (marinas) charge higher prices than competitors for renting docks, the number of applicants for renting them decreases. Of course, due to the high price compared to competitors, the revenue per docks increases [33]. The fact that some customers perceive higher prices as an indicator for superior products or services, by using pricing higher than the average price of competitors, companies can attract such customers. In general, the strategy works when companies can differentiate themselves from others by providing superior products or services.

On the other hand, pricing lower than the average price of competitors can lead to commercial success. For example, as a retailer, Amazon has gained a significant share of the global book market by reducing retail prices by 40–50%. The strategy of this online store is to use online sales for direct distribution of product, which leads to a sharp reduction in costs and eventually a price reduction. When this pricing method is used, profit of a company would depend on the number of customers attracted, since low profit should be compensated by high sales figures [11]. This pricing method works when customers can trust a company and obtain similar quality products or services at a lower price.

Some case studies have also investigated competitive pricing in many sectors such as the automobile market in the United States [30], using social networks to sell products [8], hotel of Europe [12], and the golf industry, which includes all businesses, products and services related to the sport of golf [13].

2.1. Location-inventory problem

Tancrez *et al.* presented a three-level SC which included factories, DCs and customers [31]. In this problem, they integrated the decisions related to the location of DCs, flow allocation and shipment size. They considered fixed demand rate and applied the economic order inventory policy (EOQ) to the problem. They investigated a nonlinear continuous mathematical model and their goal of modeling the problem was to minimize costs including periodic transportation, inventory maintenance, fixed distribution, and handling center.

Ross *et al.* presented a three-level SC that included main warehouse, DCs, retail stores, and exclusive supply centers [27]. In this model, they simultaneously determined the number of open sales centers, the locations of

DCs, and the orders of retailers from several open DCs. They considered random demand and supply, and their goal of modeling this integer nonlinear programming problem was to minimize the total annual cost. Dai *et al.* presented a three-level SC that included factories, warehouses, and retailers [9]. They considered an optimization model for perishable products with phase capacity constraints and carbon emissions. They used mixed integer nonlinear programming model, and their purpose was to minimize the total cost of the SC network.

Raoofpanah & Ghezavati [26] presented an integrated stochastic SC model. In this integer nonlinear programming problem, simultaneous optimization of location and inventory decisions were done. In this model, the number of customers assigned to each DC and the number of open DCs were limited. Their goal of modeling this problem was to select a set of DCs and allocate customers to open DCs to minimize costs, such as DC location, transportation, and inventory costs. Fathi *et al.* [15] presented a three-level SC including a supplier, several DCs, and several retailers. They presented an integer nonlinear programming model and considered customer demand and re-delivery time randomly. In this system, each DC has a limited storage capacity and follows the continuous review inventory policy (s, Q). Their goal of modeling this problem was to minimize the cost of the entire system.

Bassey & Zelibe [7] presented a two-level inventory-location system. The system consists of a factory at a high level and a set of service centers and customers at a low level. In this mixed integer nonlinear programming model, there are limitations of the response time. They combined this two-level system with lateral transport. In this system, factories and service centers have limited storage spaces for keeping inventory, the inventory system of which is under the control of continuous review policy (s-1, s). The purpose of presenting this model was to minimize the cost of the entire system. Wang *et al.* [35] investigated a supply network design problem for cross-border e-commerce. They presented a three-level SC including suppliers, regional DCs and DCs. It should be noted that the capacity of regional DCs has an important effect. They formulated a mixed integer nonlinear programming model and the goal of modeling this problem was to minimize the total cost. Some researchers have also combined such problems with other decisions. For instance, Aghili *et al.* [1] considered the addressed problem for distribution of perishable items with stochastic demand and travelling times while Song and Wu [29] examined it for perishable items with the possibility of direct shipment from suppliers to retailers.

2.2. Location-inventory-pricing problem

Ahmadi-Javid & Hoseinpour [3] studied a three-level SC problem including one supplier, several DCs, and several customers. They formulated a mixed integer nonlinear programming problem and investigated it with the capacity constraints of DCs. They used the continuous review inventory policy (Q, R) and markup pricing, and investigated profit maximization in this single-product SC with price-sensitive customer demands. In another study, they investigated DCs in two cases of with and without capacity. They also used the continuous review inventory policy (Q, R), mark-up pricing and investigated profit maximization in this multi-product SC with price-sensitive customer demands.

Kaya & Urek [20] presented a three-level closed-loop SC including customers, facilities, and re-manufacturing center. In this model, used products are collected by opened facilities and transported to the re-manufacturing center for re-manufacturing/recycling. They combined the collection of used products with the distribution of new products. The demand for new products was price sensitive and they used the economic order model to determine the optimal values of the inventory cycle time. They presented a mixed integer nonlinear model to decide on the optimal location of facilities, inventory quantities, new product prices, and incentive value to collect the appropriate amount of used products to maximize the profit of the entire SC.

Ahmadzadeh & Vahdani [4] presented a three-level closed loop SC including factories, collection and DCs, and customer areas. They formulated a mixed integer nonlinear programming model and investigated it in a single product mode. In this problem, they used the periodic review inventory policy (R, T). The demand of customers depended on the population of the regions, the price of new products and the distance between customers and distribution and collection centers were uncertain. The objective of their model was to maximize the profit of the SC. Wang *et al.* [34] considered a green inventory-location problem for designing a sustainable low-carbon SC network. They presented a three-level SC including suppliers, DCs and demand areas. They considered the periodic review inventory policy (t, s, S) for the inventory system in each open DC. They formulated a mixed integer linear programming model and the goal of modeling this problem was to maximize profit based on carbon trading income.

Asghari *et al.* [5] investigated a location-inventory-pricing model considering the limitations of warehouse capacity, disruption, and perishable products. They presented a three-level SC including a factory, several warehouses, and several retailers. In this problem, the demand was uncertain and mark-up pricing and EOQ strategy have been used. They formulated a mixed integer nonlinear programming model and the goal of modeling this problem was to maximize the total profit of warehouses. Rabbani *et al.* [25] incorporated pricing decisions into the traditional location-inventory problem in municipal solid waste management. Moghadam *et al.* [23] studied location-inventory-pricing problem along with routing for a four-level closed-loop supply chain in the e-commerce context. They focused on hybrid heuristic approach for solving this problem.

Haghshenas *et al.* [18] presented a three-level closed loop SC including a factory, retailer, and collection centers. They formulated a mixed integer nonlinear programming model and the goal of modeling this multi-product SC was to maximize the profit of the SC. In this case, the demand was certain and the EOQ strategy has been used for the costs of transportation and inventory in sales and collection centers.

In summary, the literature review clearly shows that numerous studies have investigated the design of suitable SC network for cost reduction and profit maximization. Studies under location-inventory problem, mostly focused on cost minimization strategies on different locations for various products. Studies under location-inventory-pricing problem, focused on profit maximization and simultaneously considered pricing. Both approaches utilized commonly used modelling approach of mixed integer nonlinear programming under different configurations, specific assumptions, and policies. However, it also clearly shows that as of this study, competition-based pricing method had not been investigated in location-inventory-pricing problems. Furthermore, this study also forecasts customers' demand *via* linear and exponential functions and shows that examining the three problems of location, inventory and pricing in an integrated and simultaneous manner can be the best approach to optimize the SC and increase the profitability of companies. In fact, previous research in the literature review only considered price as the decision variable in the model except in Asghari *et al.* [5] investigation, in which a pricing policy (*i.e.*, mark-up policy), was also assumed and behavior of the model investigated. Hence, distinguishing feature of this research compared to previous studies is adoption of two pricing policies in the SC as well as two forms of demand function. For each of their associated combination, it has investigated the behavior of the profit maximization model. The remaining sections of the paper describe the problem, its formulation and modeling, a numerical example, generated results of the model, its associated sensitivity analysis and finally the conclusion.

3. Problem description and formulation

In this research, a three-level SC which includes one supplier, several DCs and several customers has been investigated. The product is sent from the supplier to the open DCs and from the open DCs to the customers. In this problem, the product distributed in all DCs is the same and the delivery times for all DCs are considered fixed. Furthermore, each DC has a limited storage capacity and can offer different retail prices to its customers. DCs use competitive pricing to determine retail prices. Two methods of pricing above the average price of competitors and pricing below the average price of competitors have been used to determine the retail price separately. Customer demand depends on the retail price offered by DCs. Furthermore, the demand rate of each customer from DCs is estimated based on two linear and exponential functions. The input variable of these functions is the retail price offered to the customer by the DCs. In this problem, it is not necessary to satisfy the demand of all customers and to provide services. Thus, a desired group of customers can be selected. This model simultaneously calculates the location of DCs, the allocation of customers to open DCs, the order quantities in DCs from suppliers, the percentage change in the retail price compared to the average price of competitors, and the retail price offered by each center. It determines the distribution to customers and the amount of customer demand from the DCs. The SC network in this model for six potential DCs and five customers is shown in

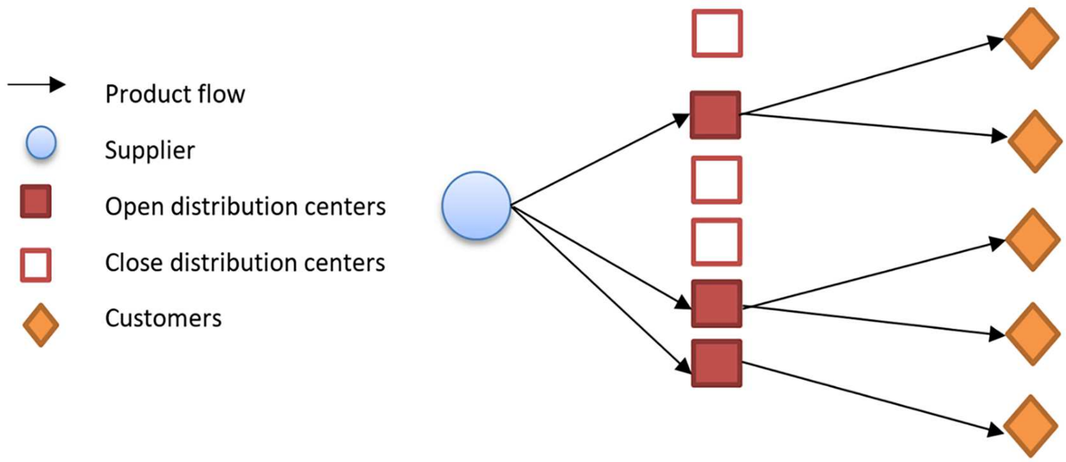

FIGURE 1. SC Network.

Figure 1. The purpose of this modeling is maximization of the long-term average profit of the SC distribution network.

3.1. Assumptions

The following assumptions are made for the location-inventory-pricing problem:

1) Demand of each customer should be supplied from only one open DC.
2) Each DC follows the continuous review inventory policy.
3) A fixed fee for order registration and an inventory holding fee are paid in each DC.
4) Transportation costs from the supplier to each open DC and from that DC to the assigned customers are paid by the DC.

It should be mentioned that all the indicated fees, costs, and prices are in Persian currency Toman, the commonly used black market exchange rate of which was 28,000 Tomans per US Dollar at the beginning of this study in 2022.

3.2. Notations

The indices are as follows:

Indexes
i : Index of potential DCs
j : Index related to customers

Parameters
$\overline{rp}$: Average retail price in the market
f_i : Fixed cost for establishing the$i$-th DC per time unit
t_{ij} : Transportation cost per product unit between the i-th DC and the j-th customer
c_i : Inventory holding capacity in the$i$-th DC
o_i : Fixed cost of ordering in the i-th DC
e_i : Fixed shipping cost for shipping from supplier to i-th DC
a_i : Transportation cost per product unit from the supplier to i-th DC
h_i : Holding cost per product unit in the i-th DC per time unit
wp_i : Wholesale price of each product unit in the i-th DC

Decision variables

X_i : A binary variable that is equal to 1 if the i-th DC is established and is equal to 0 otherwise.

Y_{ij} : A binary variable that is equal to 1 if the j-th customer is assigned to the i-th DC to satisfy the associated demand.

Q_i : A non-negative continuous variable that represents the order size of the i-th DC.

rp_{ij} : The retail price of each product at the i-th DC for the j-th customer.

d_{ij} : Demand rate (per time unit) of the j-th customer from the i-th DC.

In the case of pricing higher than the average price of competitors, the decision variable γ_{ij} is defined as follows:

γ_{ij} : The variable increasing the average price considered for the j-th customer by the i-th DC.
In the case of pricing lower than the average price of competitors, the decision variable γ_{ij} is defined as follows:

γ_{ij} : The variable decreasing the average price considered for the j-th customer by the i-th DC.

4. SC modeling

This section shows the relationships between model symbols and formulation of the objective function.

4.1. Relationships between model symbols

This subsection describes the mathematical representation of our two pricing schemes based on the retail price and types of demand function. Although we may think that a higher price will decrease the number of units sold and vice versa, such impacts depend on the price elasticity of demand, which measures the responsiveness of demand to price changes. When demand is relatively elastic, a small increase in price can lead to a significant decrease in demand. However, when demand is relatively inelastic (*i.e.*, less responsive to price changes) demand would decline less and it can lead to higher income and profit.

4.1.1. Pricing higher than the average price of competitors-Retail price

The retail price for each product unit in the i-th DC for the j-th customer is calculated through equation (1).

$$rp_{ij} = \overline{rp}\left(1+\gamma_{ij}\right). \tag{1}$$

Equation (2) shows the relationship between the average retail price in the market, the increasing variable of the considered average price (γ_{ij}) and the demand rate.

$$d_{ij} = \pi_{ij}\left(\overline{rp}\left(1+\gamma_{ij}\right)\right) \quad i \in I, j \in J. \tag{2}$$

Equation (3) shows the final retail price which is determined based on a value higher than the average market price and must always be higher than the wholesale price.

$$\overline{rp}\left(1+\gamma_{ij}\right) > wp_i \quad i \in Ij \in J. \tag{3}$$

Equation (4) indicates the limit for γ_{ij}.

$$1+\gamma_{ij} > \frac{wp_i}{\overline{rp}} \rightarrow \gamma_{ij} > \frac{wp_i}{\overline{rp}} - 1 \quad i \in I, j \in J. \tag{4}$$

$\pi_{ij}(\cdot)$ is a non-increasing function that may change with the i-th DC and the j-th customer. Its two possible forms, which are linear and exponential, are shown in Equations (5) and (6), respectively.

$$\pi_{ij}(x) = \alpha_{ij} - \beta_{ij}x \qquad 0 \le x \le \frac{\alpha_{ij}}{\beta_{ij}} \tag{5}$$

$$\pi_{ij}(x) = \alpha_{ij}e^{-\beta_{ij}x} \qquad x \ge 0. \tag{6}$$

The variable$x$ represents the retail price specified by the i-th DC to the j-th customer andα_{ij}, β_{ij}>0 are given constants. In the case that the retail price is equal to zero, the parameterα_{ij} indicates the point of intersection of the demand curve with the vertical axis for the i-th DC and the j-th customer. This parameter also indicates influencing factors other than price that cause changes in product demand.

The parameter β_{ij} is the elasticity coefficient of the price. In the linear demand function, it indicates the slope of the demand curve for the i-th DC and the j-th customer and shows the amount of demand change due to the price variation. In the exponential demand function, the higher the parameter β_{ij}, the more sensitive consumers become to price change. Of course, its lower value indicates less customers' sensitivity to price change.

Linear demand function is used when the relationship between price and quantity of demand is relatively simple, and the price elasticity of demand is constant in a certain range of prices. When the relationship between price and demand quantity is more complex, exponential demand function is used. The main difference between these functions is in how demand responds to price changes.

4.1.2. Pricing lower than the average price of competitors-Retail Price

The retail price for each product unit in the i-th DC for the j-th customer is calculated using the following equation (7).

$$rp_{ij} = \overline{rp}\,(1-\gamma_{ij})\,. \tag{7}$$

From an analytical point of view, there is a relationship between the average retail price in the market, the reducing variable of the considered average price (γ_{ij}), and the demand rate. The following equation (8) shows such relationship:

$$d_{ij} = \pi_{ij}\left(\overline{rp}\,(1-\gamma_{ij})\right) \quad i \in I, j \in J. \tag{8}$$

Hence, the final retail price is determined based on a value lower than the average market prices and must always be higher than the wholesale price in the i-th DC. Equation (9) shows this constraint:

$$\overline{rp}\,(1-\gamma_{ij}) > wp_i \quad i \in I, j \in J. \tag{9}$$

The constraint on γ_{ij} is shown in equation (10):

$$1-\gamma_{ij} > \frac{wp_i}{\overline{rp}} \rightarrow \gamma_{ij} < 1-\frac{wp_i}{\overline{rp}} \quad i \in I, j \in J. \tag{10}$$

The following two subsections examine the constants α_{ij} and $\beta_{ij}$for the linear demand function in two cases of pricing higher than the average price of competitors and pricing lower than the average price of competitors, using the approach of Ahmadi-Javid & Hoseinpour [2].

4.1.3. Pricing higher than the average price of competitors-Linear Demand

Based on the above-mentioned equations (2) and (5), the linear demand function is $d_{ij} = \alpha_{ij} - \beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})$. Since the variable d_{ij} is the demand value of the j-th customer from the i-th DC in units of weight (tons, kilograms, etc.), then α_{ij} and $\beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})$ should also be in terms of weight units. As mentioned earlier, γ_{ij} is the increasing variable of the average price considered for the j-th customer by the i-th DC and its value is between zero and one. The parameter ($\overline{rp}$) is in terms of unit price (Tomans) per unit of product, so for the unit $\beta_{ij} \times \overline{rp}$ to be the same as the unit of demand, it is necessary that β_{ij} be defined as a fraction and the denominator of this fraction is the value $\overline{rp}$. This way, the price unit gets removed. We take the form of the fraction related to β_{ij} in terms of weight unit equal to the value of θ_{ij} and based on equation (11), the minimum and maximum value that can be specified for θ_{ij} can be determined. According to the constraint $0 \le x \le \frac{\alpha_{ij}}{\beta_{ij}}$ in equation (5) and by replacing x and β_{ij}, the constraint $\overline{rp} \times (1+\gamma_{ij}) \le \frac{\alpha_{ij}}{\frac{\theta_{ij}}{\overline{rp}}}$ is established. The expression becomes $(1+\gamma_{ij}) \times \theta_{ij} \le \alpha_{ij}$, and since γ_{ij} is at most equal to one, then α_{ij} must be at least equal to $2\theta_{ij}$. Thus, we consider the value of α_{ij} to be equal to $2\theta_{ij}$. As such, the linear demand function in the case of pricing higher than the average price of competitors can be represented by equation (11).

$$\begin{aligned} & d_{ij} = \alpha_{ij} - \beta_{ij} \times \overline{rp} \times (1 + \gamma_{ij}) \xrightarrow{\beta_{ij} = \frac{\theta_{ij}}{\overline{rp}}} d_{ij} = \alpha_{ij} - \theta_{ij}\,(1 + \gamma_{ij}) \\ & \xrightarrow{\alpha_{ij} = 2\theta_{ij}} d_{ij} = 2\theta_{ij} - \theta_{ij}\,(1 + \gamma_{ij}) \rightarrow \\ & d_{ij} = \theta_{ij} - \theta_{ij} \times \gamma_{ij} \rightarrow d_{ij} = \theta_{ij}\,(1 - \gamma_{ij})\,. \end{aligned} \tag{11}$$

In the above equation, γ_{ij} is a variable between zero and one. Therefore, d_{ij} can be between zero and θ_{ij}. We also set θ_{ij} in such a way that the demand of the j-th customer from the i-th DC is less than the storage capacity of the i-th DC ($d_{ij} \leq c_i$). This way, the DC can supply the demand, and the maximum value of θ_{ij} can be equal to the storage capacity of the i-th DC ($\theta_{ij} \leq c_i$).

In the linear demand function, elasticity is constant in the demand curve. When the demand is relatively inelastic, it declines less, and this can lead to a higher income and profit even at a higher price compared to competitors.

4.1.4. Pricing lower than the average price of competitors-Linear Demand

Using equations (5) and (7) and based on the arguments made in the case of pricing lower than the average price of competitors, we consider β_{ij} equal to $\frac{\theta_{ij}}{\overline{rp}}$. Considering the constraint $0 \leq x \leq \frac{\alpha_{ij}}{\beta_{ij}}$ in equation (5) and by replacing x and β_{ij}, the constraint $\overline{rp} \times (1 - \gamma_{ij}) \leq \frac{\alpha_{ij}}{\frac{\theta_{ij}}{\overline{rp}}}$ can be established. This simplifies to $(1 - \gamma_{ij}) \times \theta_{ij} \leq \alpha_{ij}$, and since maximum value of γ_{ij} is one, then α_{ij} must be at least equal to θ_{ij}. For this reason, we consider the value of α_{ij} to be equal to $2\theta_{ij}$, higher than the average price of competitors. Therefore, the linear demand function in the case of pricing lower than the average price of competitors can be represented by equation (12).

$$\begin{aligned} d_{ij} = \alpha_{ij} - \beta_{ij} \times \overline{rp} \times (1 - \gamma_{ij}) & \xrightarrow{\beta_{ij} = \frac{\theta_{ij}}{\overline{rp}}} d_{ij} = \alpha_{ij} - \theta_{ij}\,(1 - \gamma_{ij}) \\ & \xrightarrow{\alpha_{ij} = 2\theta_{ij}} d_{ij} = 2\theta_{ij} - \theta_{ij}\,(1 - \gamma_{ij}) \rightarrow \\ & \qquad d_{ij} = \theta_{ij} \times (1 + \gamma_{ij})\,. \end{aligned} \tag{12}$$

In the above equation, γ_{ij} is a variable between zero and one, hence, d_{ij} is between θ_{ij} and $2\theta_{ij}$. We also set θ_{ij} in such a way that the demand of the j-th customer from the i-th DC is less than the storage capacity of the i-th DC ($d_{ij} \leq c_i$) so that the DC can meet the demand. As such, the maximum value that θ_{ij} can have, is equal to half of the storage capacity of the i-th DC $\left(\theta_{ij} \leq \frac{c_i}{2}\right)$.

The following two subsections examine the constants α_{ij} and $\beta_{ij}$for the exponential demand function in two cases of pricing higher than the average price of competitors and pricing lower than the average price of competitors, using the approach of Ahmadi-Javid & Hoseinpour [2].

If the price decreases, demand, and income will increase and if the increase in income is higher than the increase in costs, the profit will increase. By pricing lower than the average price of competitors, the SC may attract more customers and increase its market share.

4.1.5. Pricing higher than the average price of competitors-Exponential Demand

Based on equations (6) and (8), the exponential demand function is $d_{ij} = \alpha_{ij} e^{-\beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})}$. Since the variable d_{ij} is the demand value of the j-th customer from the i-th DC in terms of weight units (tons, kilograms, etc.), then $\alpha_{ij}$and $e^{-\beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})}$ are also per weight unit. The parameter ($\overline{rp}$) is in terms of unit price (Toman) per unit of product, so for the unit to be the same as the unit $e^{-\beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})}$with the demand unit, it is necessary that β_{ij} is defined as a fraction, and the denominator of this fraction is the value $\overline{rp}$ and with this action the price unit gets removed. We take the form of the fraction related to β_{ij} in terms of weight unit equal to the value of θ_{ij} and according to the equation (13), we determine the minimum and maximum value that θ_{ij}

can have. Since in the expression $d_{ij} = \alpha_{ij} e^{-\theta_{ij}(1+\gamma_{ij})}$, the power of e is negative according to Ahmadi-Javid & Hoseinpour [2], in order for e to have a positive power, we consider α_{ij} exponential and raise to the power of $2\theta_{ij}$. Also, for the demand function to have a faster growth rate, we multiply by 2. Therefore, we set α_{ij} equal to $2\mathrm{e}^{2\theta_{ij}}$. This way, the exponential demand function in the case of pricing higher than the average price of competitors can be expressed as equation (13).

$$\begin{aligned} d_{ij} = \alpha_{ij} e^{-\beta_{ij} \times \overline{rp} \times (1+\gamma_{ij})} &\xrightarrow{\beta_{ij} = \frac{\theta_{ij}}{\overline{rp}}} d_{ij} = \alpha_{ij} e^{-\theta_{ij}(1+\gamma_{ij})} \\ &\xrightarrow{\alpha_{ij} = 2\mathrm{e}^{2\theta_{ij}}} d_{ij} = 2\mathrm{e}^{2\theta_{ij}} e^{-\theta_{ij}(1+\gamma_{ij})} \to \\ d_{ij} = 2\mathrm{e}^{2\theta_{ij}} e^{-\theta_{ij}} e^{-\theta_{ij}\gamma_{ij}} &\to d_{ij} = 2\mathrm{e}^{\theta_{ij}} e^{-\theta_{ij}\gamma_{ij}} \\ &\to d_{ij} = 2\mathrm{e}^{\theta_{ij}(1-\gamma_{ij})}. \end{aligned} \tag{13}$$

In the above Equation, γ_{ij} is a variable between zero and one. Therefore, d_{ij} can be between 2 and $2e^{\theta_{ij}}$, and we also set θ_{ij} in such a way that the demand of the j-th customer from the i-th DC is less than the storage capacity of the i-th DC ($d_{ij} \le c_i$) so that the DC can meet the demand. Therefore, $2e^{\theta_{ij}}$ is at most equal to the storage capacity of the i-th DC $\left(2e^{\theta_{ij}} \le c_i\right)$, thus θ_{ij} can at most be equal to $\ln \frac{c_i}{2}$.

The exponential demand function shows that demand is more sensitive to price changes. Its demand elasticity is high, and customers are more responsive to price changes. A small increase in price can lead to a significant decrease in demand. Therefore, pricing higher than the average price of competitors may lead to a significant reduction in the number of units sold. As such, a careful analysis of demand elasticity and cost structure is critical in determining whether a higher price can generate enough revenue to offset any potential decline in sales.

4.1.6. Pricing lower than the average price of competitors-Exponential Demand

Based on equations (6) and (8) and according to the arguments made in the case of pricing lower than the average price of competitors, we consider β_{ij} equal to $\frac{\theta_{ij}}{\overline{rp}}$ and α_{ij} equal to $2\mathrm{e}^{2\theta_{ij}}$. Therefore, the exponential demand function in the case of pricing lower than the average price of competitors can be represented by equation (14).

$$\begin{aligned} & d_{ij} = \alpha_{ij} e^{-\beta_{ij} \times \overline{rp} \times (1-\gamma_{ij})} \xrightarrow{\beta_{ij} = \frac{\theta_{ij}}{\overline{rp}}} d_{ij} = \alpha_{ij} e^{-\theta_{ij}(1-\gamma_{ij})} \\ & \xrightarrow{\alpha_{ij} = 2e^{2\theta_{ij}}} d_{ij} = 2e^{2\theta_{ij}} e^{-\theta_{ij}(1-\gamma_{ij})} \\ & \to d_{ij} = 2e^{2\theta_{ij}} e^{-\theta_{ij}} e^{\theta_{ij}\gamma_{ij}} \to d_{ij} = 2e^{\theta_{ij}} e^{\theta_{ij}\gamma_{ij}} \\ & \to d_{ij} = 2e^{\theta_{ij}(1+\gamma_{ij})}. \end{aligned} \tag{14}$$

In the above Equation, considering that γ_{ij} is a variable between zero and one, d_{ij} can be between $2e^{\theta_{ij}}$ and $2e^{2\theta_{ij}}$. Also, we set θ_{ij} in such a way that the demand of the j-th customer from the i-th DC is less than the storage capacity of the i-th DC ($d_{ij} \le c_i$) so that the DC can meet the demand. As such, the maximum value of $2e^{2\theta_{ij}}$ is equal to the storage capacity of the i-th DC $\left(2e^{2\theta_{ij}} \le c_i\right)$, so the maximum θ_{ij} is equal to $\frac{1}{2} \ln \frac{c_i}{2}$.

As mentioned earlier, the demand elasticity is high in this case and customers react strongly to price changes. As such, when the price decreases, the demand increases at a high rate and results in higher income since the lower prices are compensated by the increase in sales. It should also be noted that with the increase in sales, the cost of production also increases. If the cost per unit decreases with the increase in production, the SC can benefit from lower costs and higher profit margin.

It is important to note that the impact of pricing decisions on the market depends on market conditions and customer preferences. Analyzing these factors along with consideration of the company's objectives and cost structure is very important when determining the optimal pricing strategy.

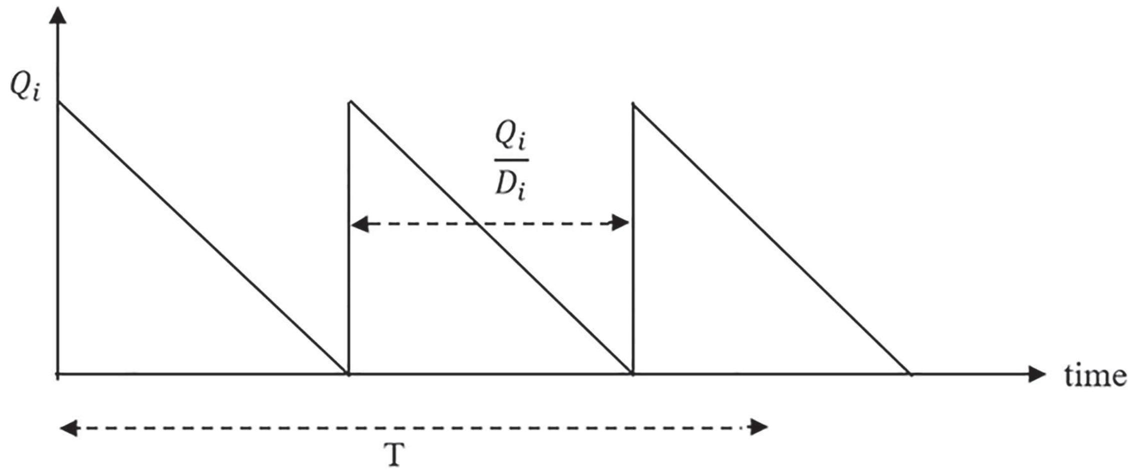

FIGURE 2. Variations of inventory with respect to time in an inventory system.

4.2. Formulation of the objective function

As mentioned earlier, the objective function in this problem represents the long-term average profit of the SC, in which the long-term average cost is deducted from the long-term average revenue. As it was stated in the previous sections, the inventory system at each i-th DC follows a continuous review inventory policy in which a fixed order quantity at the i-th DC is ordered from the supplier when the inventory position at the DC reaches the reorder point R_i or below. According to Ahmadi-Javid & Hoseinpour [2], Figure 2 shows inventory changes for a continuous review system $(Q_i.R_i)$ in which duration of two complete cycles and one incomplete cycle is considered T.

According to Ahmadi-Javid & Hoseinpour [2], the long-term average cost of inventory in the i-th DC is obtained through equations (15) to (19). When the demand rate in the i-th DC (D_i) is constant, equation (15) is used to calculate the average cost of long-term inventory in the i-th DC $(\Psi_i(Q_i))$.

$$\Psi_i(Q_i) = \lim_{T\to+\infty} \frac{N_i(Q_i.T) \times \emptyset_i(Q_i.T) + \delta_i(Q_i.T)}{T}. \tag{15}$$

In the above equation (15), $N_i(Q_i.T)$is the number of complete cycles in the period$[0.T]$, $\emptyset_i(Q_i.T)$is the probable cost in one cycle and $\delta_i(Q_i.T)$ is the cost of an incomplete cycle, which does not end with time T respectively. Therefore, the number of complete cycles, the probable cost in one cycle, and the cost of an incomplete cycle are calculated through the equations (16), (17) and (18), respectively.

$$N_i(Q_i.T) = \left\lfloor \frac{D_i T}{Q_i} \right\rfloor \tag{16}$$

$$\emptyset_i(Q_i.T) = o_i + (e_i + a_i Q_i) + h_i \frac{1}{2} \times \frac{Q_i}{D_i} \times Q_i \tag{17}$$

$$\begin{aligned}\delta_i(Q_i.T) = {} & o_i + (e_i + a_i Q_i) \\ & + h_i\left(\frac{1}{2} \times \frac{Q_i}{D_i} \times Q_i - \frac{1}{2} \times \left(\frac{Q_i}{D_i} - \left(T - \left\lfloor \frac{D_i T}{Q_i} \right\rfloor \times \frac{Q_i}{D_i}\right)\right)^2 \times D_i\right).\end{aligned} \tag{18}$$

Using $x-1 \leq \lfloor x \rfloor \leq x$ and the sandwich rule of Kuhn & MacKinnon, the average cost of long-term inventory in the i-th DC is obtained through equation (19) [21].

$$\Psi_i\left(Q_i\right) = o_i \frac{D_i}{Q_i} + h_i \frac{Q_i}{2} + \left(e_i + a_i Q_i\right) \frac{D_i}{Q_i} \cdot \tag{19}$$

It should be noted that the demand rate in the i-th DC is equal to the total demand rate for customers assigned to it and is calculated as $D_i = \sum_{j \in J} d_{ij} Y_{ij}$. Finally, we formulate the problem as a mixed integer nonlinear programming in the following manner:

$$\begin{aligned} \max \sum_{i \in I} \sum_{j \in J} rp_{ij} d_{ij} Y_{ij} - \sum_{i \in I} f_i X_i - \sum_{i \in I} \sum_{j \in J} wp_i d_{ij} Y_{ij} - \sum_{i \in I} \sum_{j \in J} t_{ij} d_{ij} Y_{ij} \\ - \sum_{i \in I} \left(o_i \frac{\sum_{j \in J} d_{ij} Y_{ij}}{Q_i} + \frac{h_i Q_i}{2} + \left(e_i + a_i Q_i\right) \frac{\sum_{j \in J} d_{ij} Y_{ij}}{Q_i} \right) \end{aligned} \tag{20}$$

s.t.

$$\sum_{i \in I} Y_{ij} \leq 1 \qquad j \in J \tag{21}$$

$$Y_{ij} \leq X_i \qquad i \in I, j \in J \tag{22}$$

$$Q_i \leq c_i \qquad i \in I \tag{23}$$

$$0 \leq \gamma_{ij} \leq 1 \qquad i \in I, j \in J \tag{24}$$

$$X_i \in \{0.1\} \text{ and } Y_{ij} \in \{0.1\} \qquad i \in I, j \in J \tag{25}$$

$$Q_i \geq 0 \text{ and } d_{ij} \geq 0 \text{ and } rp_{ij} \geq 0 \qquad i \in I, j \in J. \tag{26}$$

In the above model, equation (20) represents the objective function. The objective function is long-term average revenue minus long-term average cost. The long-term average cost includes the fixed costs of establishing DCs, procurement costs, transportation costs from open DCs to customers, and inventory costs in open DCs. Inventory cost in each DC includes fixed cost of ordering, inventory holding cost and shipping cost. For better feasibility of the objective function, the convention $0/0 = 0$ is utilized.

Equation (21) states that each customer can be assigned to a maximum of one DC, which implies that some customers may not be assigned. Equation (22) guarantees that a customer is not assigned to a DC unless the DC is opened. Equation (23) shows the limitation of the capacity level for the order amount. Equation (24) states that the variable increasing or decreasing the price is a value between zero and one. Equations (25) and (26) define the status of the variables used.

5. Numerical examples

To show the applied and practical nature of our approach, numerical values for two different configurations of potential DCs and customers in specific regions are presented here.

5.1. Three potential DCs and five customers

The investigated SC is related to a sample company with sugar product, which includes one supplier, three potential DCs and five customers. The supplier is located in the center of Tehran province and the potential DCs are located in the centers of Golestan, Isfahan and Bushehr provinces. Customers are located in the centers of Khorasan-Razavi, South Khorasan, Hormozgan, Hamadan and Khuzestan provinces. Figure 3 shows the flow of the product from the supplier to the potential DCs. According to the notification of the Road Traffic and Road Transport Organization, the price of moving goods is 423.3 Tomans per ton-kilometer. Since the

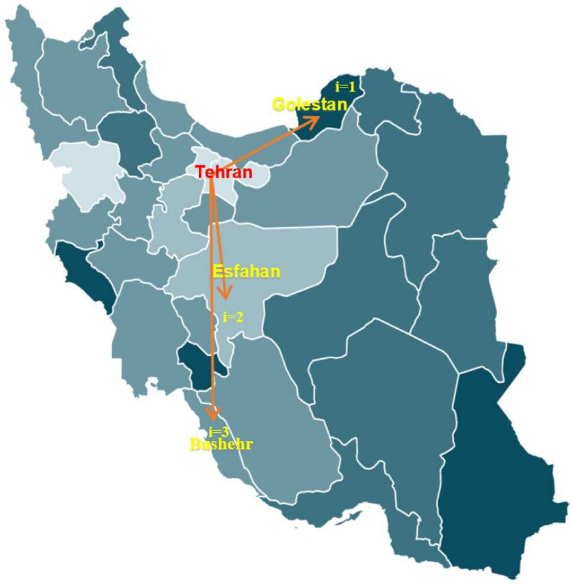

FIGURE 3. Product flow from supplier to potential DCs.

distance from Tehran to Golestan is 415 km, Tehran to Isfahan is 448 km, and Tehran to Bushehr is 1031 km, per ton transportation cost of sugar between the supplier and these DCs can be obtained by multiplying the supplier index rate and these distances. The associated transportation costs are: $a_1 = 175669.5$, $a_2 = 189638.4$, $a_3 = 436422.3$. The product flow from potential DCs to customers is shown in Figure 4. The value of t_{ij} can be obtained by multiplying the distance between the i-th DC and the j-th customer with 423.3 (the price of moving goods). The generated results are shown in Table 1.

We consider DCs as containers with specific dimensions. We assume that the first DC, the second DC, and the third DC are in the size of a standard 10-foot, 20-foot, and 20-foot container, respectively. It should be noted that the 10-foot container has an internal length of 2.79 meters, an internal width of 2.35 meters, and an internal height of 2.38 meters. This results in a total volume of 15.60447 cubic meters and since the density of sugar is 1.59 grams per cubic centimeter, it is equivalent to 1.59 tons per cubic meter. Therefore, 24.8111073 tons of sugar can be stored in 15.60447 cubic meters. In a similar manner, the 20-foot container has an internal length of 5.9 meters, an internal width of 2.35 meters, and an internal height of 2.38 meters. This results in a total volume of 32.9987 cubic meters; thus, it can store 52.467933 tons of sugar. Therefore, the inventory holding capacity is considered equal to $c_1 = 24.8111073$, $c_2 = 52.467933$, $c_3 = 52.467933$.

If we assume that the useful life of each container is 25 years, the purchase price of a 10-foot and 20-foot container for one year will be equal to 1,500,000 Tomans and 3,000,000 Tomans, respectively. For each of the DC, it is necessary to prepare land which is compatible with size of the containers. The price of renting 6.5565 square meters of land in Gorgan for one year is equal to 1,311,300 Tomans. Also, the rental price of 13.865 square meters of land for one year is 3,327,600 Tomans and 2,495,700 Tomans respectively for Isfahan and Bushehr. Based on these prices, we consider 30% of the land rental price as the cost of preparation and purchase of equipment. As such, the fixed cost in one year for the establishment of DCs is considered equal to $f_1 = 3204690$, $f_2 = 7325880$, $f_3 = 6244410$. In this problem, we consider the cost of third-party insurance in road transportation as a fixed cost of transportation. Considering that the first DC is a 10-foot container and can store a maximum of 24.8111073 tons of sugar, for carrying this amount of cargo (a maximum criterion), we must use a ten-wheeled truck with a loading capacity of 28 tons. Since the insurance cost of a truck with a capacity of more than 20 tons for one year is equal to 6,916,000 Tomans, the fixed transportation cost of the

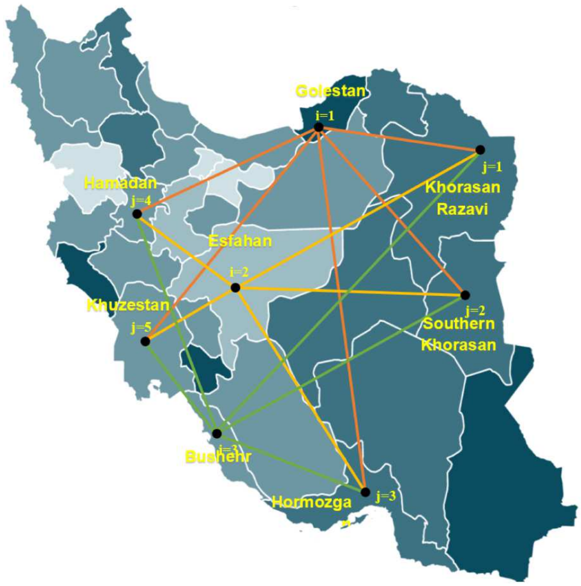

FIGURE 4. Product flow from potential DCs to customers.

first DC is equal to 6,916,000. Since the second DC and the third DC are 20-foot containers and a maximum of 52.467933 tons of sugar can be stored in them, we must use two ten-wheeled trucks for carrying this amount of cargo (in the maximum mode). Considering that the insurance cost of two trucks for one year is equal to 13,832,000 Tomans, the fixed transportation cost of the second and third DC is equal to 13,832,000 Tomans. The wholesale price of sugar produced within the factory (supplier) for the year 2022 is 18,000 Tomans per kilogram and has been approved. We assume that the total stock value of one ton of sugar P_i in each DC is equal to $P_1 = 17900000$, $P_2 = 18000000$, $P_3 = 18100000$. Holding costs typically comprise about 20 to 30 percent of total inventory value (P_i), although it varies by industry and business size. We assume that the interest rate is equal to 20% (a common rate in Iran), so the holding cost is equal to $h_1 = 3580000$, $h_2 = 3600000$, $h_3 = 3620000$. The fixed ordering cost is the cost that the i-th DC incurs every time an order is placed, regardless of the size of the order. This cost is usually related to administrative and processing costs for order registration and execution. Things like labor costs for order processing, order processing software, and overhead costs for each order registration are included in the category of fixed ordering costs. Based on the findings of experts in this industry, we consider the fixed cost of ordering to be between 500,000 and 800,000 Tomans. Therefore, we consider the fixed cost of ordering as $o_1 = 500000$, $o_2 = 700000$, $o_3 = 600000$. We consider the wholesale price of each ton of sugar in DCs to be 300,000 to 400,000 Tomans higher than the value of one ton of sugar when it is kept at the supplier. Therefore, the wholesale price of each product unit in DCs as $wp_1 = 18200000$, $wp_2 = 18300000$, $wp_3 = 18500000$. It should be noted that the price of one kilo of white sugar in chain stores is from 26,500 to 30,000 Tomans and for one ton of sugar it is 2,650,000 Toman to 3,000,000 Toman. Hence, we consider the average retail price in the market to be around 28,000,000. In the case of linear demand, for the pricing method above the average price of the competitors and the pricing method below the average price of the competitors, the value of θ_{ij} should be less than c_i and $\frac{c_i}{2}$, respectively. Therefore, in Tables 2 and 3, values for θ_{ij} are considered randomly in these cases.

In the case of exponential demand for the method of pricing above the average price of competitors and pricing below the average price of competitors, the value of θ_{ij} should be less than $\ln \frac{c_i}{2}$ and $\frac{1}{2} \ln \frac{c_i}{2}$, respectively. Therefore, in Tables 4 and 5, values for θ_{ij} are considered randomly in these cases.

TABLE 1. Values of t_{ij}.

i	j = 1	2	3	4	5
1	267102.3	360651.6	592196.7	308585.7	523198.8
2	496954.2	368271.	398325.3	198104.4	218422.8
3	695481.9	576534.6	306045.9	430919.4	183712.2

TABLE 2. θ_{ij} values (higher than competitors-linear demand).

i	j = 1	2	3	4	5
1	4	7	2	10	3
2	13	21	9	13	22
3	31	11	20	17	8

TABLE 3. θ_{ij} values (lower than competitors-linear demand).

i	j = 1	2	3	4	5
1	2	8	3	5	9
2	22	5	9	14	14
3	9	11	21	19	6

TABLE 4. θ_{ij} values (higher than competitors-exponential demand).

i	j = 1	2	3	4	5
1	1	0.5	0.7	0.2	1.2
2	1.6	0.9	0.5	1.8	2
3	0.8	0.4	2.2	1.2	0.4

5.2. Eight potential DCs and fifteen customers

The investigated SC is related to a sample company with sugar product, which includes one supplier, eight potential DCs and fifteen customers. The supplier is located in the center of Tehran province, and the potential DCs are located in the centers of Golestan, Isfahan, Bushehr, Yazd, Lorestan, Sistan and Baluchistan, East Azerbaijan and Zanjan respectively. Customers are located in the centers of Khorasan-Razavi, South Khorasan, Hormozgan, Hamadan, Khuzestan, West Azerbaijan, Kurdistan, Ilam, Kermanshah, Fars, Kerman, Semnan, Qom, Gilan, North Khorasan provinces. We assume that the first DC, the fourth DC, the fifth DC and the eighth DC are the size of a standard 10-foot container. On the other hand, the second, third, sixth, and seventh DCs are assumed to be the size of a standard 20-foot container. The parameters of this problem are calculated according to the procedure explained in Sections 5–1.

TABLE 5. θ_{ij} values (lower than competitors-exponential demand).

i	j				
	1	2	3	4	5
1	0.1	0.3	0.1	0.2	0.2
2	0.7	0.4	0.3	0.4	0.5
3	0.9	1.1	1.3	0.1	0.1

TABLE 6. Values of γ_{ij} (higher than competitors-linear demand).

i	j				
	1	2	3	4	5
1	0.968	0.773	0.0002893	0.737	0.452
2	0.356	0.354	0.484	0.351	0.351
3	0.364	0.356	0.357	0.360	0.392

6. RESULTS OF MODEL EXECUTION AND PROBLEM SOLVING

This section shows generated results of our model for the patterns of pricing and demand functions for the two scenarios of: "three potential DCs and five customers", and "eight potential DCs and fifteen customers", the numerical values of which were presented in the previous section.

6.1. Solving the problem of three potential DCs and five customers

It should be noted that Baron's solver was used in solving this problem.

6.1.1. Pricing higher than the average price of competitors

A) Linear demand

The duration of solving this simulated model was 17 minutes and 47 seconds and the results are as follows:

$$z = 1207248000 \qquad X_2 = 1 \qquad X_3 = 1$$
$$Y_{22} = 1 \qquad Y_{25} = 1 \qquad Y_{31} = 1 \qquad Y_{33} = 1 \qquad Y_{34} = 1$$
$$Q_1 = 0 \qquad Q_2 = 14.991 \qquad Q_3 = 18.611$$

The above results indicate that the established DC in the center of Isfahan province meets the demand of customers in the centers of South Khorasan and Khuzestan provinces. Also, a DC has been established in the center of Bushehr province and it provides the demand of customers in the centers of Khorasan-Razavi, Hormozgan and Hamadan provinces. The variable value of the average price increase considered for the customers by the DCs is specified in Table 6 and based on these values, the obtained value of the retail price for the customers by the DCs is shown in Table 7. Furthermore, the demand values for customers from DCs are shown in Table 8 based on equation (11).

B) Exponential demand

The duration of solving this simulated model was 16 minutes and 49 seconds and the results are as follows:

$$z = 522057300 \qquad X_2 = 1 \qquad X_3 = 1$$
$$Y_{21} = 1 \qquad Y_{22} = 1 \qquad Y_{24} = 1 \qquad Y_{25} = 1 \qquad Y_{33} = 1$$
$$Q_1 = 0 \qquad Q_2 = 14.353 \qquad Q_3 = 9.68$$

Table 7. Values of rp_{ij} (higher than competitors-linear demand).

i	j				
	1	2	3	4	5
1	55112910	49642860	28008100	48631240	40647070
2	37978050	37913660	41557920	37837700	37838740
3	38203680	37962630	38008970	38071400	38980650

Table 8. Values of d_{ij} (higher than competitors-linear demand).

i	j				
	1	2	3	4	5
1	0.127	1.589	1.999	2.632	1.645
2	8.367	13.565	4.642	8.432	14.270
3	19.703	7.086	12.851	10.885	4.255

Table 9. Values of γ_{ij} (higher than competitors-exponential demand).

i	j				
	1	2	3	4	5
1	0.369	0.480	1	0.799	0.996
2	0.339	0.821	1	0.259	0.204
3	0.755	1	0.195	0.484	1

The above results show that in the center of Isfahan province, a DC has been established and it meets the demand of customers in the centers of Khorasan- Razavi, South Khorasan, Hamadan, and Khuzestan provinces. Also, a DC has been established in the center of Bushehr province and it provides the customer demand in the center of Hormozgan province. The variable value of the average price increase considered for customers by DCs is specified in Table 9 and according to these values, the obtained value of retail price for customers by DCs is shown in Table 10. Also, the demand values for customers from DCs are specified in Table 11 based on equation (13). According to the results of solving the problem, the profit of the SC in the case of the exponential demand function is more than the case of the linear demand function.

6.1.2. Pricing lower than the average price of competitors

A) Linear demand

The duration of solving this simulated model was 17 minutes and 15 seconds and the results are as follows:

$$z = 621725400 \qquad X_2 = 1 \qquad X_3 = 1$$
$$Y_{21} = 1 \qquad Y_{25} = 1 \qquad Y_{32} = 1 \qquad Y_{33} = 1 \qquad Y_{34} = 1$$
$$Q_1 = 0 \qquad Q_2 = 17.048 \qquad Q_3 = 20.166$$

The above results indicate that the established DC in the center of Isfahan province meets the demand of customers in the centers of Khorasan-Razavi, South Khorasan, Hormozgan, Hamedan and Khuzestan provinces. The variable value of the average price reducer considered for the customers by the DCs is specified in Table 12 and based on these values, the obtained value of the retail price for the customers by the DCs is shown in Table 13. Furthermore, the demand values for customers from DCs are specified in Table 14 based on equation (12).

Table 10. Values of rp_{ij} (higher than competitors-exponential demand).

i	j				
	1	2	3	4	5
1	38345000	41447830	56000000	50382410	55898620
2	37499070	50981500	56000000	35255780	33720540
3	49128680	56000000	33460580	41555440	56000000

Table 11. Values of d_{ij} (higher than competitors-exponential demand).

i	j				
	1	2	3	4	5
1	3.757	2.593	2	2.082	2.009
2	5.757	2.350	2	7.589	9.821
3	2.434	2	11.753	3.714	2

Table 12. Values of γ_{ij} (lower than competitors-linear demand).

i	j				
	1	2	3	4	5
1	0.201	0.149	0.289	0.201	0
2	0	0	0	0.213	0
3	0.223	0	0	0	0

Table 13. Values of rp_{ij} (lower than competitors-linear demand).

i	j				
	1	2	3	4	5
1	22365000	23835000	19906580	22365000	28000000
2	28000000	28000000	28000000	22032040	28000000
3	21767600	28000000	28000000	28000000	28000000

B) Exponential demand

The duration of solving this simulated model was 16 minutes and 56 seconds and the results are as follows:

$z = 140201900 \quad X_3 = 1$

$Y_{31} = 1 \quad Y_{32} = 1 \quad Y_{33} = 1 \quad Y_{34} = 1 \quad Y_{35} = 1$

$Q_1 = 0 \quad Q_2 = 0 \quad Q_3 = 13.450$

The above results show that in the center of Bushehr province, a DC has been established and it meets the demand of customers in the centers of Khorasan- Razavi, South Khorasan, Hormozgan, Hamedan and Khuzestan provinces. The variable value of the average price reducer considered for the customers by the DCs is specified in Table 15 and according to these values, the obtained value of retail price for customers by DCs is shown in Table 16. Table 16 shows that the highest retail price set for customers by DCs is 28,000,000 Tomans. Based on the parameter values and demand values in Table 17, this company determines the best allocation of customers

TABLE 14. Values of d_{ij} (lower than competitors-linear demand).

i	j 1	2	3	4	5
1	2.402	9.190	3.867	6.006	9
2	22	5	9	16.984	14
3	11.003	11	21	19	6

TABLE 15. Values of γ_{ij} (lower than competitors-exponential demand).

i	j 1	2	3	4	5
1	0.083	0	0.092	0	0
2	0.133	0.159	0.162	0.144	0.141
3	0	0	0	0	0

TABLE 16. Values of rp_{ij} (lower than competitors-exponential demand).

i	j 1	2	3	4	5
1	25668980	28000000	25436630	28000000	28000000
2	24264480	23549350	23475320	23966800	24054280
3	28000000	28000000	28000000	28000000	28000000

to DCs and maximizes the profit of the SC. The obtained results show that the profit of the SC in the case of the exponential demand function is lower than the case of the linear function.

6.2. Solving the problem of eight potential DCs and fifteen customers

Baron's solver is used in solving this problem.

6.2.1. Pricing higher than the average price of competitors

A) Linear demand

The duration of solving this simulated model was 16 minutes and 43 seconds and the results are as follows:

$$z = 3835851000$$

$$Q_1 = 0 \qquad Q_2 = 23.314 \qquad Q_3 = 25.313 \qquad Q_4 = 9.593$$

$$Q_5 = 0 \qquad Q_6 = 15.384 \qquad Q_7 = 13.586 \qquad Q_8 = 0$$

Indicating that open DCs are 2, 3, 4, 6, and 7; together with customers that receive services from these DCs which are (2-5-6-9-13), (1-3-8-12-14), (4-10), (11-15), and (7), respectively.

TABLE 17. Values of d_{ij} (lower than competitors-exponential demand).

i	j = 1	2	3	4	5
1	2.229	2.700	2.231	2.443	2.443
2	4.422	3.180	2.834	3.161	3.538
3	4.919	6.008	7.339	2.210	2.210

B) Exponential demand
The duration of solving this simulated model was 16 minutes and 41 seconds and the results are as follows:

$z = 1959656000$

$Q_1 = 5.520$	$Q_2 = 22.739$	$Q_3 = 9.680$	$Q_4 = 0$
$Q_5 = 8.006$	$Q_6 = 8.733$	$Q_7 = 20.075$	$Q_8 = 0$

Indicating that open DCs are 1, 2, 3, 5, 6, and 7; together with customers that receive services from these DCs which are (11), (4-5-7-12-14-15), (3-6), (8-9), (6), and (1-2-10-13), respectively.

Based on the obtained results, profit of the SC in the case of exponential demand function, is lower than that of the linear function.

6.2.2. Pricing lower than the average price of competitors

A) Linear demand
The duration of solving this simulated model was 16 minutes and 50 seconds and the results are as follows:

$z = 1989222000$

$Q_1 = 0$	$Q_2 = 29.115$	$Q_3 = 20.750$	$Q_4 = 0$
$Q_5 = 0$	$Q_6 = 28.658$	$Q_7 = 0$	$Q_8 = 0$

Indicating that open DCs are 2, 3, and 6; together with customers that receive services from these DCs which are (1-6-7-10-13-15), (3-4-9), and (2-5-8-11-12-14), respectively.

B) Exponential demand
The duration of solving this simulated model was 16 minutes and 41 seconds and the results are as follows:

$z = 475596300$

$Q_1 = 0$	$Q_2 = 0$	$Q_3 = 0$	$Q_4 = 0$
$Q_5 = 0$	$Q_6 = 11.027$	$Q_7 = 13.804$	$Q_8 = 11.440$

Indicating that open DCs are 6, 7, and 8; together with customers that receive services from these DCs which are (10-13-15), (3-5-7-8-9), and (1-2-4-6-11-12-14), respectively.

Based on the obtained results, the profit of the SC in the case of the exponential demand function is lower than that of the linear function.

The obtained results of the model for the two scenarios in this problem can be summarized in the following manner:

– In each pricing method, the type of demand function has a substantial effect on the obtained results in the location-inventory-pricing model and can greatly change the amount of profit of each company.

TABLE 18. SC profit in relation to the interest rate parameter.

i	z				
	0.1	0.15	0.2	0.25	0.3
mode (a)	1242850000	1223517000	1207248000	1192936000	1180015000
mode (b)	547620200	533709400	522057300	511849100	502666800
mode (c)	661082200	639727900	621725400	605864900	591525900
mode (d)	154462200	146724800	140201900	134455100	129259600

- When a significant number of customers in the market are sensitive to the offered prices, higher prices may reduce the number of customers and sales. Consequently, the loyalty of customers will be lost, and it will become a challenge for companies to retain customers in the long run. Thus, it is critical for companies to carefully consider the potential impacts of such decisions and carefully analyze them before their implementation.
- The main advantage of pricing above the average price of competitors is to earn more profit. By using this pricing method, the sales revenue of the products increases and if the production and operating costs are controlled, the company can earn more profit.
- The main advantage of pricing lower than the average price of competitors, is competition in the market and attracting more customers. However, it may also adversely affect the value or quality of products or services in the minds of customers wherein some customers associate lower prices with lower quality products or services and cause sales reduction. Nonetheless, effective marketing can help reduce such risks and increase sales and profit margins.
- When choosing any of the competitive pricing methods, several factors need to be considered, the competitors in the market should be analyzed and the market response ought to be identified. The cost structure must also be evaluated to assure the SC about the financial viability of the competitive pricing methods. Marketing research should also be conducted to understand customer preferences, their willingness to pay, and price sensitivity. Finally, profitability should also be evaluated in each of the methods.

7. Sensitivity analysis

This section evaluates the sensitivity of SC profits, to the parameters of interest rate, average price, wholesale price, fixed cost of DC establishment, and distance between supplier and DC, for the abovementioned two scenarios of potential DCs and customers.

7.1. Sensitivity of SC profits to the interest rate parameter

The following two subsections evaluate such sensitivity in the two configurations of our DCs and customers.

7.1.1. Three potential DCs and five customers

The profit values of the SC in relation to the interest rate parameter are specified in Table 18. Modes (a) and (b) correspond to pricing mode higher than the average price of competitors, and modes (c) and (d) correspond to pricing mode lower than the average price of competitors, in the linear and exponential demand respectively.

Their associated graphical data are shown in the following four elements of Figure 5 (a, b, c, d).

As can be seen from the above Figure 5, an increase in the interest rate from 0.1 to 0.3, results in the profit decline. Therefore, in this SC, in both pricing methods, the maximum amount of profit can be achieved with the lowest interest rate of 0.1.

Eight potential DCs and fifteen customers

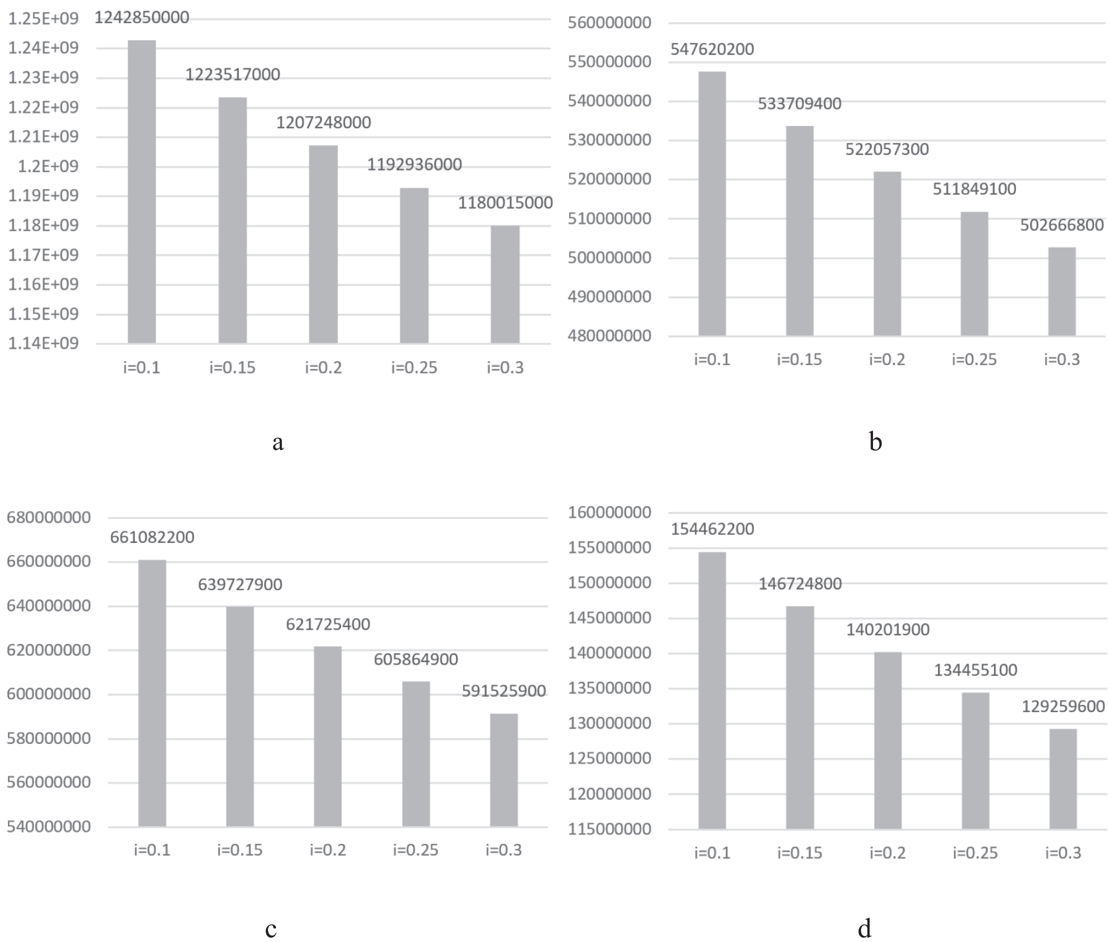

FIGURE 5. SC's profit variation with respect to change in interest rate.

TABLE 19. SC's profit in relation to the interest rate parameter.

i	z				
	0.1	0.15	0.2	0.25	0.3
mode (a)	3846640000	3877998000	3835851000	3723065000	3755014000
mode (b)	2020869000	1981021000	1959656000	1927932000	1872639000
mode (c)	2075171000	2027304000	1989222000	1955672000	1925339000
mode (d)	522391400	485227300	475596300	466880700	447587400

The profit values of the SC in relation to the interest rate parameter are specified in Table 19. In this table, the definition of mode (a) to mode (d) is the same as that of Table 18.

According to Table 19 in modes (b), (c) and (d), by increasing the interest rate parameter, the maintenance cost increases, and the profit of the SC decreases. Furthermore, in these modes, by decreasing the interest rate parameter, the maintenance cost decreases and the profit of the SC increases. In mode (a), with the increase

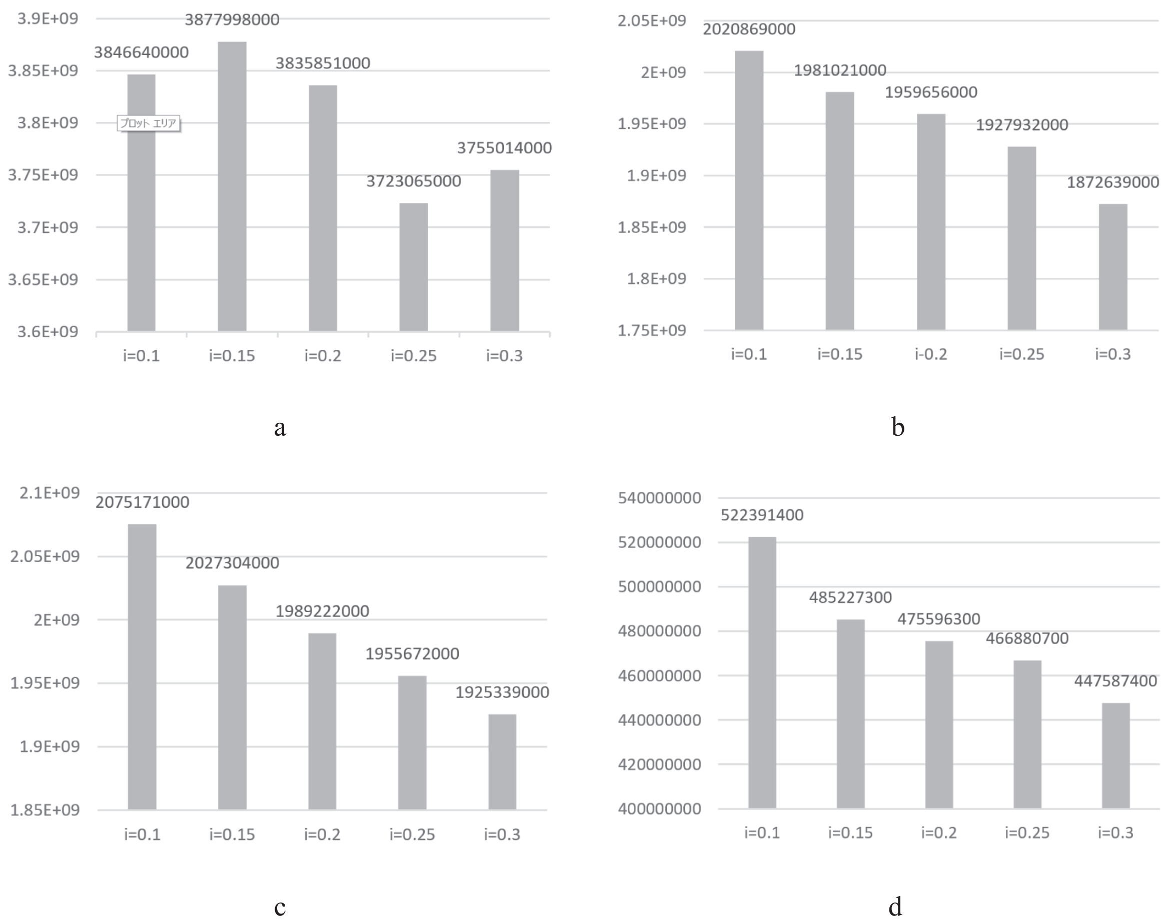

FIGURE 6. SC's profit variation with respect to change in interest rate.

of the interest rate parameter from 0.25 to 0.3, the profit of the SC has increased by about 31 million Tomans. On the other hand, with the reduction of the interest rate parameter from 0.15 to 0.1, the profit of the SC has decreased by about 31 million Tomans. The changes in SC profit in these cases are due to the change of actual DCs and assigned customers and the amount of variables in the optimal solution of the problem. Therefore, in mode (a) with an interest rate of 0.15, the SC earns the most amount of profit.

The obtained SC's profit variations are graphically shown in the following four elements of Figure 6 (a, b, c, d).

As can be noticed, in the above Figures 6b, 6c and 6d, with the increase of interest rate from 0.1 to 0.3, the profit changes are in downward patterns. In Figure 6a, with an increase in the interest rate from 0.15 to 0.25, the profit changes are also downward, but with an increase in the interest rate from 0.1 to 0.15 and from 0.25 to 0.3, the profit changes are in upward patterns, which are due to the change in DCs and assigned customers.

7.2. Sensitivity of SC profits to the average price parameter in the market

In the following two subsections, we increase and decrease the average price of the market by 5% and 10% and compute the associated profit of the SC in the two configurations of our DCs and customers.

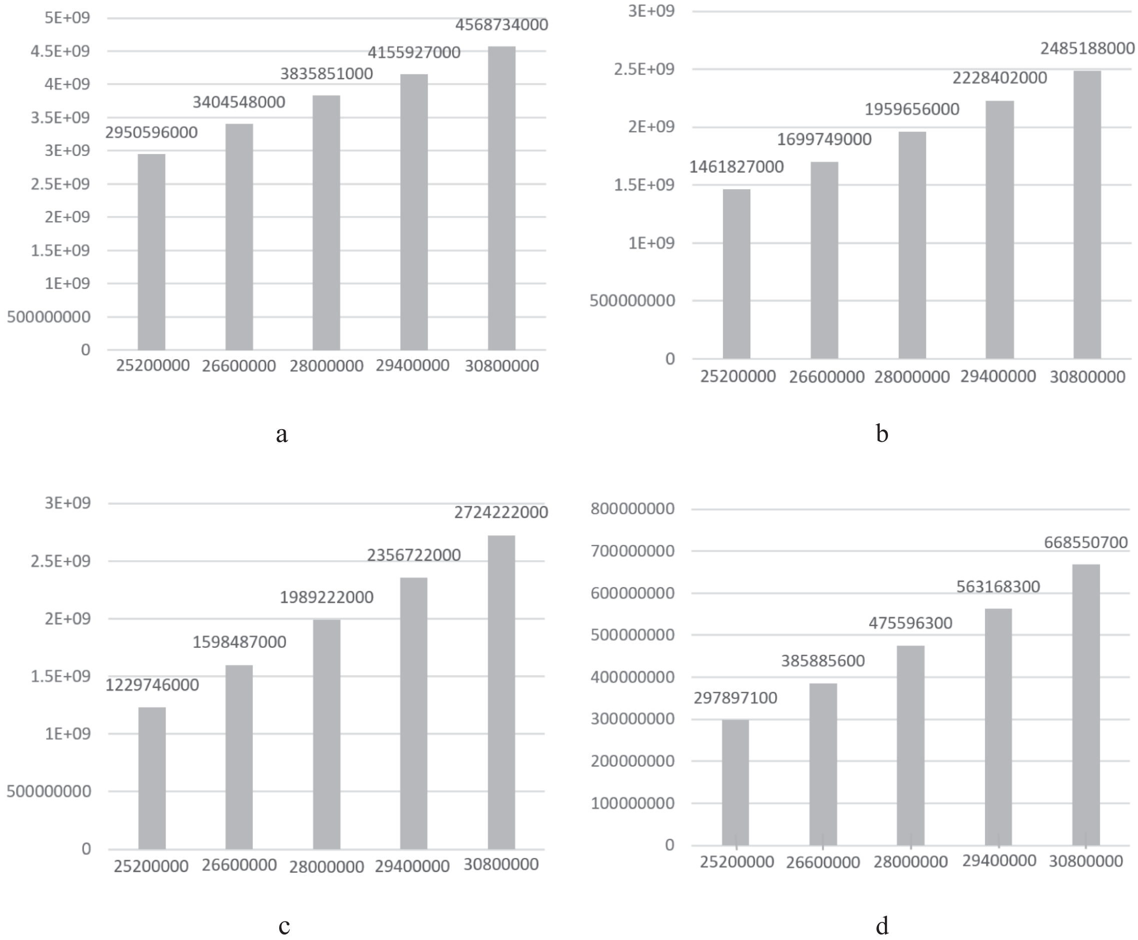

FIGURE 7. SC's profit variation with respect to the average price in the market.

7.2.1. Three potential DCs and five customers

In the case of SC with three potential DCs and five customers, an increase or decrease in the average market price, affects the profit of SC accordingly.

7.2.2. Eight potential DCs and fifteen customers

Details of the obtained profit of the SC against the average price variation in the market with eight potential DCs and fifteen customers are graphically shown in the following four elements of Figure 7 (a, b, c, d).

In the method of pricing above the average price of competitors in the market, the lowest retail price is equal to the average price in the market, and with an increase in the average price in the market, the retail price, and the profit of the SC increase. In the method of pricing lower than the average price of competitors in the market, the maximum retail price is equal to the average price in the market, and with an increase in the average price in the market, the retail price, and the profit of the SC increase. As can be seen from the above Figure 7, the changes in the amount of profit with the increase of the average price in the market from 25,200,000 to 30,800,000 are upward, and with the range of 5% changes in the average price in the market, the profit of the SC has changed by several hundred million.

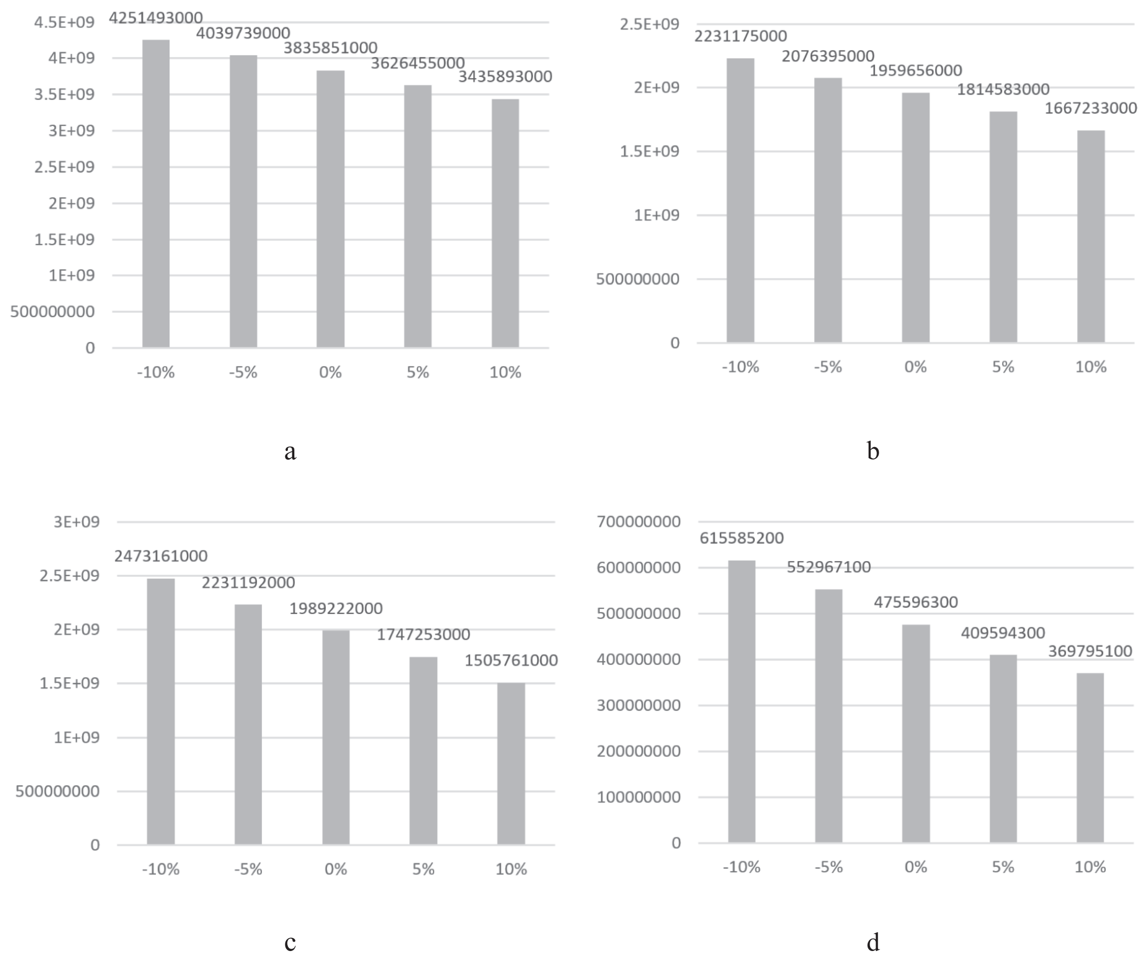

FIGURE 8. SC's profit variation with respect to wholesale price.

7.3. Sensitivity of SC profits to the wholesale price parameter

The following two subsections evaluate such sensitivity in the two configurations of our DCs and customers.

7.3.1. Three potential DCs and five customers

In this scenario, an increase or decrease in the wholesale price parameter, inversely affects the profit of SC.

7.3.2. Eight potential DCs and fifteen customers

Details of the profit variation of SC with respect to the average price in the market along with an increase and decrease of the wholesale price by 5% and 10% are graphically shown in four elements of Figure 8 (a, b, c, d). The horizontal axis of these figures indicates the wholesale price variations, in which 0% shows its baseline, 5% and −5% along with 10% and −10%, indicate a 5% and 10% increase and decrease respectively.

As can be seen in the above Figure 8, changes in the amount of profit are downward with the increase in the wholesale price and upward with the decrease in the wholesale price. The range of 5% change in the wholesale price in Figures 8a, 8b and 8c has caused a change in the profit of the SC up to several hundred million, and in

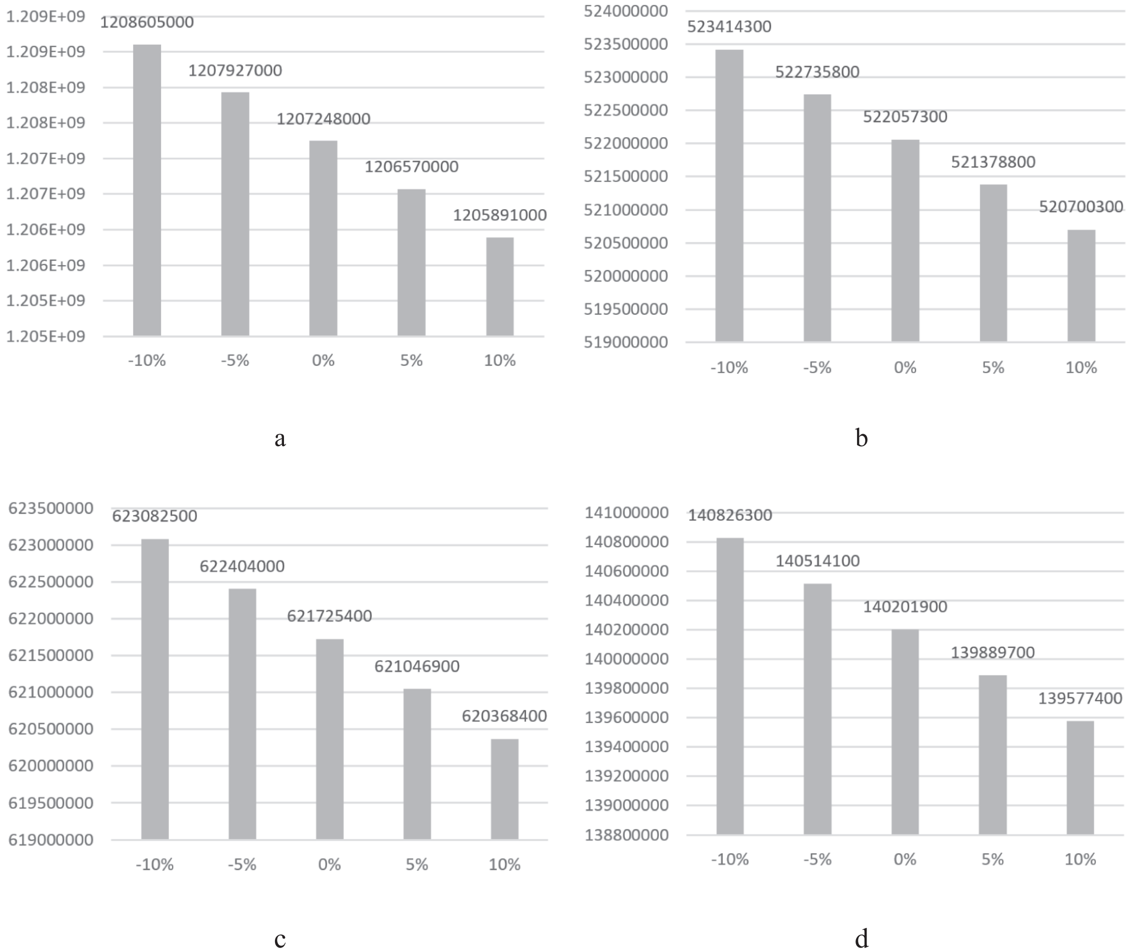

FIGURE 9. SC's profit variation with respect to the fixed cost parameter of establishing DCs.

Figure 8d it has caused a change of less than one hundred million in the profit of the SC. Hence, changing this parameter causes a significant change in the profit of the SC.

7.4. Sensitivity of SC profits to the fixed cost parameter of establishing DCs

The following two subsections evaluate such sensitivity in the two configurations of our DCs and customers.

7.4.1. Three potential DCs and five customers

In this section, we increase and decrease the fixed cost of establishing DCs by 5% and 10% and the associated SC's profit variation are graphically shown in the four elements of the following Figure 9 (a, b, c, d).

As can be seen from the above Figure 9, increase and decrease of the fixed cost of establishing DCs, inversely affects the profit.

7.4.2. Eight potential DCs and fifteen customers

The results of the SC's profit variation against the average price in the market are graphically shown in the following four elements of Figure 10 (a, b, c, d).

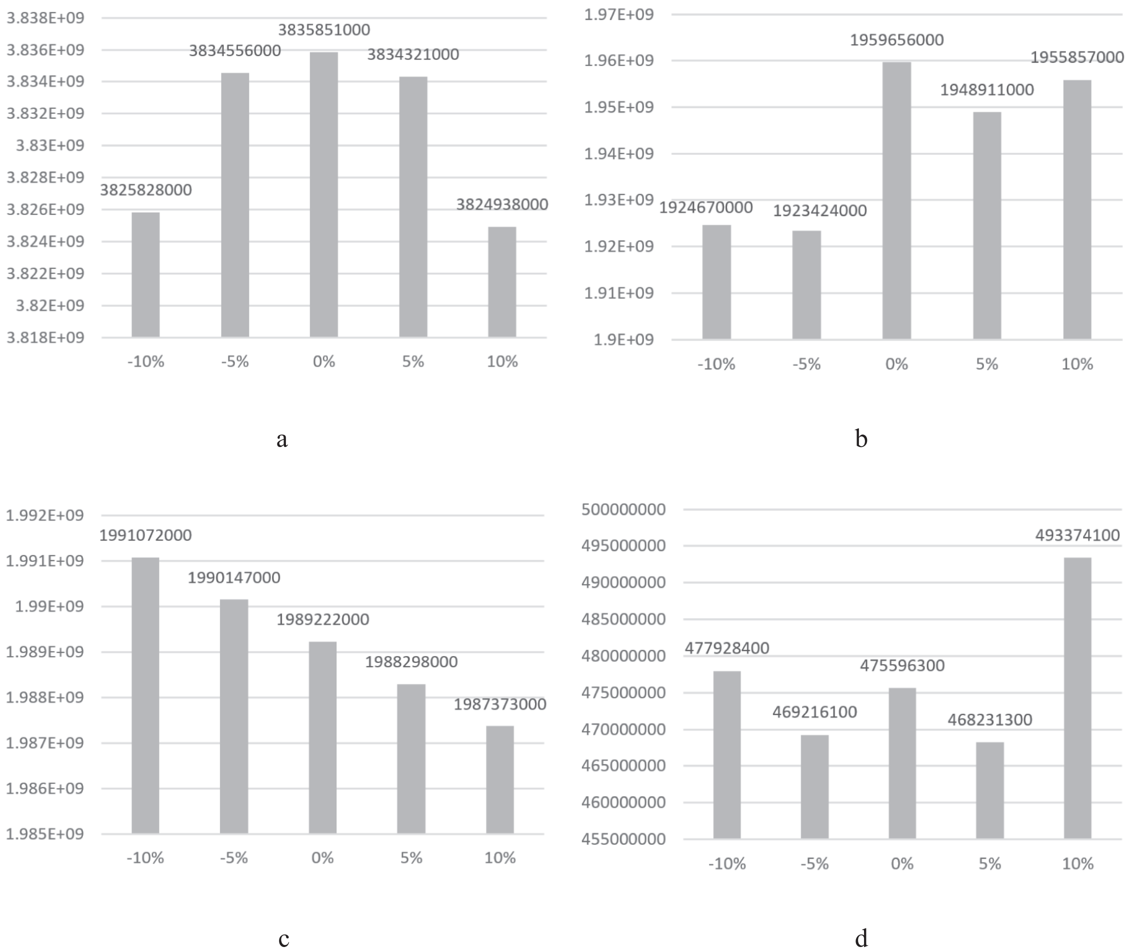

FIGURE 10. SC's profit variation with respect to the fixed cost parameter of establishing DCs.

As can be seen in the above Figure 10a, with the increase in the fixed cost of establishing DCs, the profit of the SC decreases. With a 5% decrease in this parameter, the profit of the SC has decreased. The decrease in profit is due to the decrease in the number of one of the DCs with respect to a base level, as well as to the values of the variables of the problem. A 10% decrease in this parameter, based on the values of parameters and variables, has reduced the profit of the SC by about 10 million. In Figure 10b, with the increase and decrease of the fixed cost parameter of the establishment of DCs, the profit of the SC has decreased by considering the amount of parameters, variables and established DCs. Therefore, in the method of pricing higher than the average price of competitors in the market in exponential demand mode and by not changing this parameter, the maximum profit is obtained. In Figure 10c, with the increase and decrease of this parameter, the profit of the SC has decreased and increased, respectively. Figure 10d shows that in the pricing method lower than the average price of competitors in the market in exponential demand mode, a 10% increase in this parameter results in the highest profit for the SC.

TABLE 20. SC profit in relation to the distance parameter.

z	Distance		
	Main route	The second route: 10 km more than the main path	The third route: 80 km more than the main path
mode (a)	1207248000	1206946000	1204836000
mode (b)	522057300	521899500	520796700
mode (c)	621725400	621357200	618779300
mode (d)	140201900	140105800	139433600

7.5. Sensitivity of SC profits to the parameter of distance between supplier and DC

If the vehicles use alternative routes with longer distances to send the product from the supplier to the DCs, transportation cost between the supplier and the DCs increases. The following two subsections examine such impacts thru increase of delivery routes by 10 km and 80 km in the two configurations of our DCs and customers.

7.5.1. Three potential DCs and five customers

The profit values of the SC in relation to the parameter of the distance between the supplier and the DC are specified in Table 20. It is to be noted that the definition of mode (a) to mode (d) is the same as that of Table 18.

The graphical results of SC's profit variation with respect to the distance between suppliers and DCs are shown in the four elements of Figure 11 (a, b, c, d).

As can be seen from the above Figure 11, the profit of the SC decreases by increasing the distance between the supplier and the DCs by 10 km and 80 km. It shows that sending the product from the supplier to the DCs through the main route has the maximum profit.

7.5.2. Eight potential DCs and fifteen customers

The profit values of the SC in relation to the parameter of the distance between the supplier and the DC are specified in Table 21. It is to be noted that the definition of mode (a) to mode (d) is the same as that of Table 18.

The graphical results of SC's profit variation with respect to the distance between suppliers and DCs are shown in the four elements of Figure 12 (a, b, c, d).

As can be seen from Figures 12a, 12c and 12d, if the distance between the supplier and the DCs is increased by 10 km and 80 km, the cost of transportation gets increased, and the profit of the SC gets decreased compared to the main path. However, in the second path of Figure 12b, the profit of the SC is even less than that of the third path, which is due to the opening of the DC in Golestan and the associated change in the assigned costumers to Kerman and Semnan in the third path.

The sensitivity analysis clearly shows that an increase in average market price parameter, or a decrease in SCs' cost related parameters (*e.g.*, interest rate, wholesale price, fixed cost of establishing DCs, distance between supplier and DC) positively influences the profit of SCs. Nonetheless, such may not be the case in exponential demand or occurrence of structural changes in the SC network. Hence, accurate forecasting of demand and timely optimization can play an essential role in the profit maximization of the SC network. The results obtained from this study rely on the distinguishing feature of its method which is adoption of two pricing policies in the SC as well as two types of demand functions. For each of their associated combination, it has investigated the behavior of the profit maximization model.

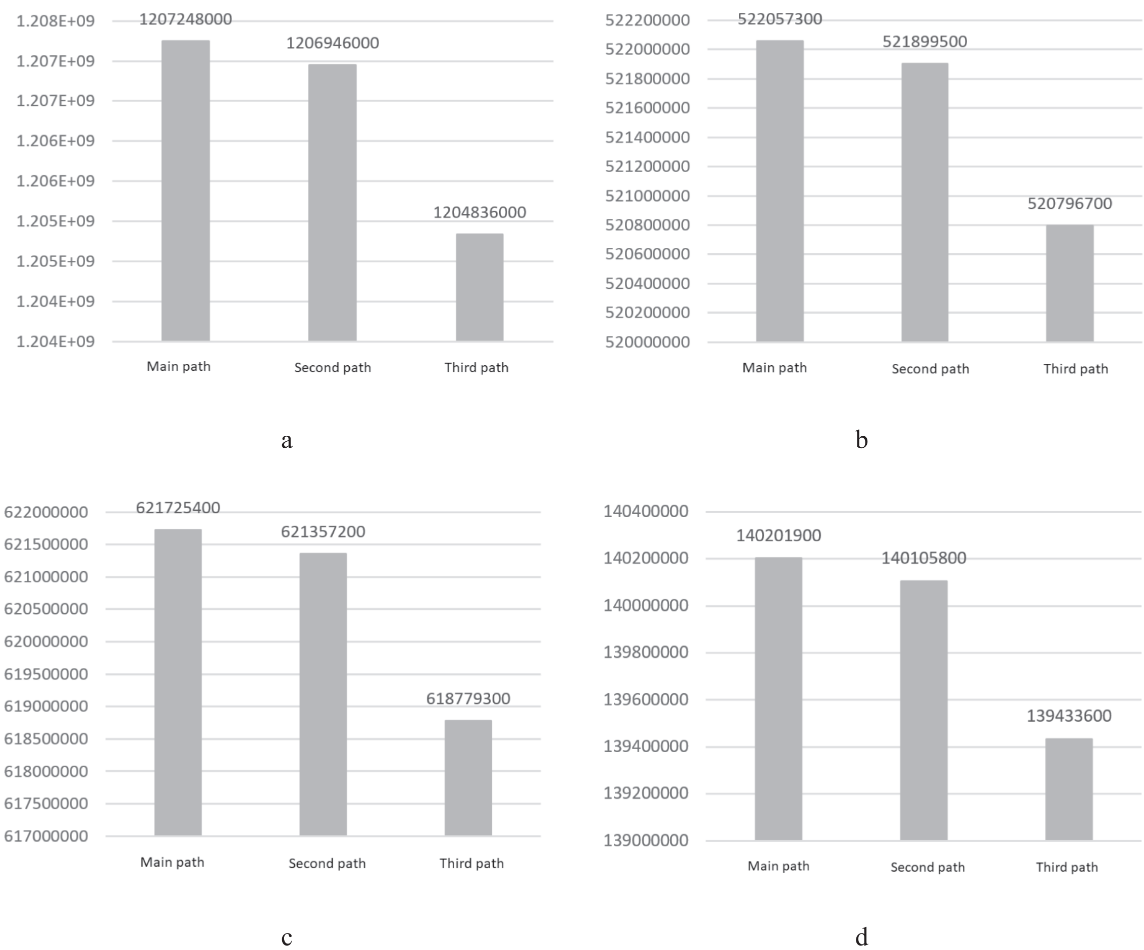

FIGURE 11. SC's profit variation with respect to the distance parameter of supplier and DCs.

TABLE 21. SC profit in relation to the distance parameter.

z	Distance		
	Main route	The second route: 10 km more than the main path	The third route: 80 km more than the main path
mode (a)	3835851000	3834910000	3828326000
mode (b)	1959656000	1930873000	1955438000
mode (c)	1989222000	1988111000	1980333000
mode (d)	475596300	468519000	466606000

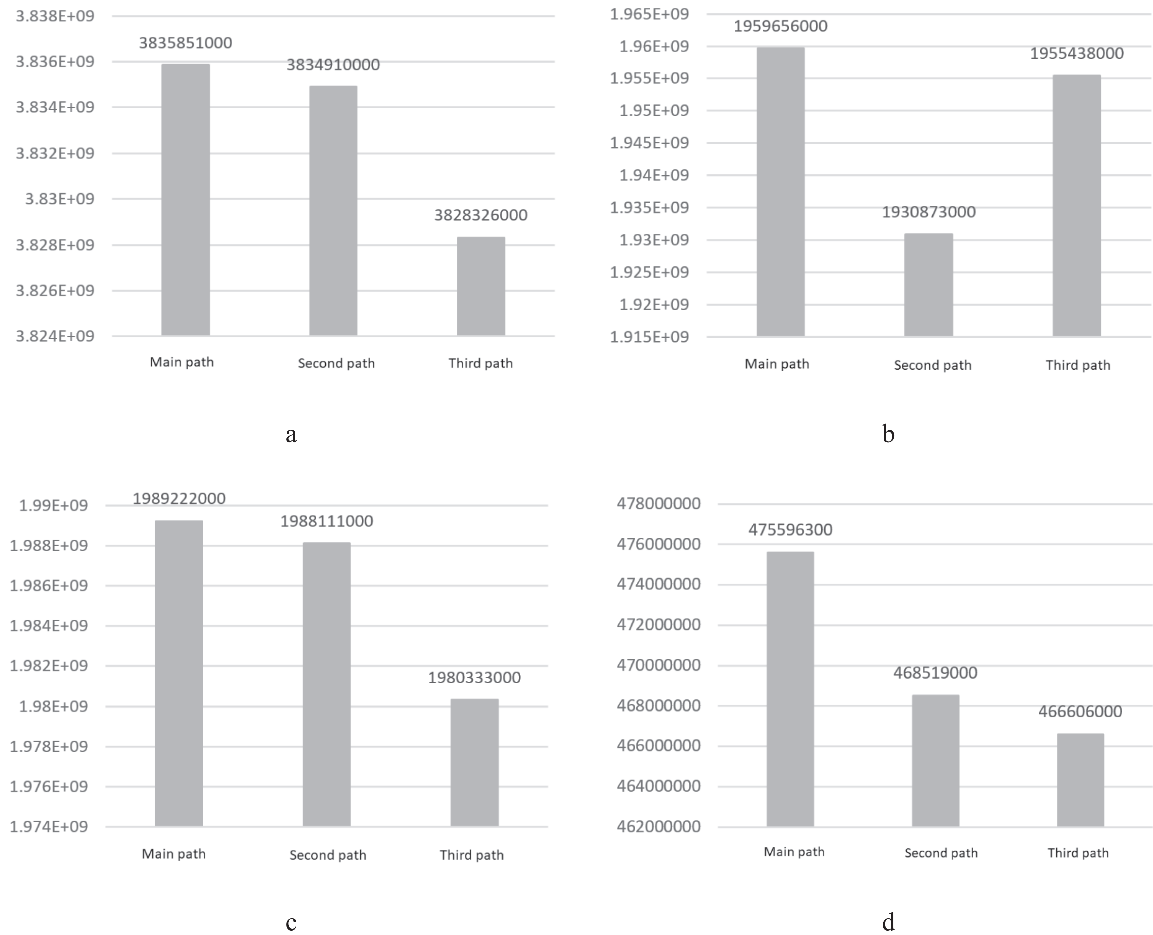

Figure 12. SC's profit variation with respect to the distance parameter of supplier and DCs.

8. Conclusions and future research

The presented location-inventory-pricing model of a three-level SC distribution network, which includes one supplier, several DCs, and several customers, utilized a mixed integer nonlinear programming approach in the process. Its overall goal, which is to maximize the long-term average profit of the distribution network, was achieved through an objective function that includes numerous types of costs. The model simultaneously included the location of DCs, the allocation of customers to open DCs, the amount of orders in DCs from suppliers, and the percentage change in the retail price compared to the average price of competitors in the market. The retail price of each DC determined the customers and the amount of customer demand from the DCs. The problem was presented considering the constraint of the inventory capacity, and the inventory system in each open DC followed the continuous review inventory policy. Two pricing methods based on competition, including pricing higher than the average price of competitors and pricing lower than the average price of competitors, were used by DCs in determining the retail price. In this SC, customer demand from DCs was sensitive to price. The numerical results showed that the retail price determined by the DCs to the customers was dependent on the average price in the market and the amount of increasing and decreasing variables. A higher competitors' price in the market resulted in a higher average market price and led to a significant increase in the income and profit of the SC. The study also revealed that patterns of customer demand can significantly influence profit of a company. When exponential function was used for customer demand, changes in price or other influencing factors led to a significant change in demand and its associated profit, compared to utilization of the linear demand function. In summary, like previous studies, this research also focused on profit maximization of the SC network but applied competition-based pricing method to location-inventory-pricing problems, something that had not been explored as of this study. It showed the advantage of this unique approach, which is to enable companies to maintain profitability under different scenarios, details of which were explored through sensitivity analysis of its numerous parameters. Nonetheless, this study focused on selecting a desired group of customers rather than fulfilling all customers' demands. Hence, its approach may not be optimal when fulfilling of all customers' demands becomes a necessary criterion. Furthermore, its delivery time were based on distance rather than realtime trafficspecific delivery time. A few recommendations for future research on this topic are as follows: Formulating the problem as a multi-product system is one approach. Considering the effect of time and random demand on the problem is another perspective. Adding an optimized routing with respect to real time traffic and travel time to the model can also further improve its results.

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