DEMAND FORECAST INFORMATION SHARING WITH MANUFACTURER ENCROACHMENT

YAN YAN, FUJUN HOU* AND HUIMIN ZHANG

Abstract. This study explores the interplay between the manufacturer’s encroachment strategy and the retailer’s information sharing strategy in a supply chain, wherein both the upstream manufacturer and downstream retailer possess private demand forecast information. The manufacturer has the option to establish a direct selling channel to encroach on the end market, and the retailer can decide whether to share private information with the manufacturer. We consider four scenarios and derive the corresponding equilibrium outcomes of firms. Theoretical research results show that when the manufacturer opts not to encroach, neither the manufacturer nor the retailer will voluntarily share their demand information. In contrast, if the manufacturer encroaches, they will reach an information sharing agreement under certain conditions. Once such an agreement is reached, the manufacturer can benefit more from encroachment. If information sharing is not achieved, the manufacturer encroaches only if his unit direct selling cost is lower than a certain threshold. In addition, fierce competition among channels encourages the manufacturer to encroach. Based on the abovementioned works, we conduct numerical studies to analyze the impact of forecast accuracy on the profits and information sharing value of the manufacturer, the retailer and the whole supply chain. These results offer valuable management insights for firms. For example, the improved forecast accuracy is beneficial to both firms. Moreover, as the channel substitution rate increases, not only the possibility of manufacturer encroachment increases, but both the manufacturer and the whole supply chain also get more profits from it.

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1. Introduction

The demand information possessed by members of the supply chain is very important for them to make effective operational management decisions (e.g., [19, 28]). With advances in technology, it is easier for companies to obtain market demand information. For example, Walmart and Procter & Gamble have invested a lot in information systems and technologies to obtain and analyze product sales information [33]. According to a Chinese bank survey, 63% of the 821 firms think that SC firms should share forecast information and production plans [3]. Li [18] gives an example of vertical information sharing, i.e., the transmission of point-of-sales data from the retailer to the manufacturer. In reality, the upstream manufacturer (or supplier) in the supply chain

Keywords. Information sharing, manufacturer encroachment, supply chain, demand forecast.
is far from the end-consumer market, so the downstream retailer has better market demand information than the upstream company. However, the retailer is usually unwilling to proactively share demand information with the upstream firm to maintain its own information advantage. Under such circumstances, some studies [35] show that under certain conditions, firms can achieve a win-win outcome by sharing demand information. For example, upstream firms can use the forecasts of third-party companies to access market demand information or may induce retailers to share private information by offering a side-payment contract [32], a bargaining mechanism [6] or subsidies [9]. In addition, Ha and Tong [9] shows that an upstream manufacturer can offer subsidies to retailers to compensate for the losses caused by information sharing. Therefore, information sharing can help supply chain members better grasp the market demand information, thereby helping managers make better pricing and management decisions.

In addition to selling products to the end consumer market via retailers, upstream companies also consider establishing their own direct selling channel to encroach on the retail channel [18]. Arya et al. [2] defined this activity as supplier encroachment, which has occurred in all walks of life. For example, in the cosmetics industry, some high-end cosmetics companies, such as Estee Lauder, Chanel and L’Oreal, not only sell their products through intermediate retail stores (e.g., Sephora and Watsons) and cosmetics counters but also consider selling directly to consumers through their official websites. Under such circumstances, upstream firms need to bear a certain cost for establishing a direct selling channel, including building online websites and providing services, warehousing, staffing and funds. The existed literature on channel encroachment characterizes the encroachment cost through a fixed entry cost [15–17] or a per-unit selling cost [13]. It would be helpful if the upstream firm knows more market demand information before deciding whether to encroach. In this case, we are encouraged to study the influence of information sharing on the upstream firm encroachment decisions.

Existing literature has examined the interaction between information sharing and channel encroachment. However, most of the literature considers unilateral information sharing, that is, only the retailer has private information and considers whether to share it with the upstream firm. For example, Huang et al. [15] discuss the motivation of the retailer to share private demand information with the supplier, and find that the retailers information sharing can be the strategic effect to deal with upstream enterprise encroachment. However, in reality, we find that upstream manufacturers may also collect information about demand through various channels. For example, JD.com, an e-commerce platform, has been selling the products of P&G since 2013. In addition to offline physical stores, the former also chooses to collect product demand information through online channels. Motivated by these examples, we examine the impact of the manufacturer’s private demand information on the whole supply chain and each player. Compared with the existing studies on information sharing and channel encroachment, the difference in our study is that we consider double-sided demand information asymmetry rather than unilateral information asymmetry. This provides us with some new insights into the issue of bilateral information asymmetry and channel encroachment.

Motivated by the above discussion, we investigate the upstream manufacturer’s optimal encroachment strategy and the downstream retailer’s information sharing decision based on existing research. In addition, we examine the value of information sharing (VIS) by calculating the profit difference between firms with and without information sharing. Both players have private forecast information about primary demand. We aim to address the following questions:

1. If the manufacturer and retailer share (or do not share) their forecasts of demand, should the manufacturer decide to encroach? If the manufacturer encroaches, how does the competition between a direct selling channel and a retail channel affect the profits of the firms?

2. If the manufacturer encroaches (or does not encroach), is it an optimal decision for the retailer to share personal demand information with the manufacturer? Can the manufacturer, the retailer, and the whole supply chain all benefit from information sharing?

3. How do the parameters involved in the model affect the value from information sharing for the manufacturer, the retailer, and the whole supply chain?
To this end, we investigate the interaction between manufacturer encroachment and retailer information sharing. We find that if the manufacturer encroaches, under certain conditions, information sharing makes a win-win for both firms. Now, the manufacturer is willing to take measures to encourage the retailer to share private forecasts under some conditions. Moreover, as the competition between these two channels intensifies, the manufacturer’s possibility of encroaching will be greater. Specifically, when the retailer’s forecast is sufficiently low, the conditions for both firms to achieve information sharing are stricter than when the retailer’s forecast is sufficiently high. Since the manufacturer gains more value from information sharing than the retailer, he is willing to encourage information sharing to improve forecast accuracy. In summary, manufacturer encroachment and the retailer’s information sharing not only help the manufacturer access demand information but also improve the demand forecast accuracy of both firms. Therefore, our study contributes to the literature on channel encroachment and information sharing in supply chain management, and provides some management implications for firms.

This article makes several contributions to extend and complement the literature on supply chain information sharing and channel encroachment. (1). Most of the existing studies on vertical demand information sharing focus on unilateral demand information sharing, that is, the downstream retailer has demand information and consider whether to share it with the upstream firm. However, few studies pay attention to bilateral information sharing, i.e., the supply chain members both have demand information and decide whether to share it with each other. We focus on bilateral information sharing in a vertical supply chain, in which both the retailer and the manufacturer forecast primary demand; (2). Our study enriches the literature on information sharing and channel encroachment in supply chain management. In addition to examining vertical demand information sharing, we also analyze the impact of information sharing on the manufacturer’s encroachment decision. We mainly explore the interplay between the retailer’s information sharing and channel encroachment; (3). We find that it is difficult for the retailer to reach a forecast information sharing agreement under the premise that the manufacturer does not encroach. On the contrary, if the manufacturer encroaches, they can achieve a win-win situation and reach an information sharing agreement under certain conditions; (4). We derive new managerial insights that can provide theoretical guidance for both firms by analyzing the manufacturer’s best encroachment decisions and the retailer’s information sharing decisions.

The remainder of this article is organized as follows. Section 2 provides a brief review of related studies, and Section 3 describes our basic model. Section 4 presents the equilibrium results of firms. In Section 5, we analyze the retailer’s information sharing decisions and the manufacturer’s encroachment decisions. Section 6 performs the sensitivity analysis of key parameters by numerical study. Finally, we conclude this article in Section 7. Proofs of the results are in the Appendix.

2. Literature review

Our study is related to three streams of literatures: information sharing, channel encroachment, and the interplay between information sharing and channel encroachment.

2.1. Information sharing

This study belongs to the literature on vertical information sharing (see, e.g., [4,9,10,18,25]). Most existing literature on information sharing is one-sided demand information asymmetry, that is, downstream firms process private demand information [18]. For example, Zhang [32] examines the impact of competition types of two downstream retailers on their private demand information sharing strategies. Li [18] and Li and Zhang [19] extend from two retailers to multiple retailers. In addition to asymmetric information in a single supply chain, Ha and Tong [9] and Ha et al. [10] extend vertical information sharing between supply chains and study the competitive reaction of information sharing from one supply chain to another supply chain.

Although much research has been devoted to unilateral information asymmetry, less attention has been paid to bilateral information asymmetry. Specifically, Mishra et al. [25] is the first to investigate the incentives of the manufacturer and retailer to share their partial demand forecasts. Bian et al. [3] and Wei et al. [29] further
extend the models to a channel-to-channel environment. They consider bilateral information sharing in two competing supply chains, in which the manufacturer in each supply chain sells substitutable or complementary products to the corresponding retailer. In the study of bilateral information asymmetry, in addition to the asymmetric demand information, Wang et al. [28] and Zissis et al. [35] consider other types of asymmetric information. For example, Wang et al. [28] investigate that in a single supply chain, the manufacturer and the retailer have private information about manufacturing costs and the degree of risk aversion, respectively. Vosooghidizaji et al. [26] study the coordination mechanism in a two-stage supply chain where two firms both possess own private corporate social responsibility (CSR) cost information. In this paper, we consider that two players in the supply chain have private asymmetrical demand information, and examines the influence of upstream firm encroachment behavior on firm information sharing strategy.

2.2. Channel encroachment

Our study is also related to the literature on the encroachment of the upstream supplier (or the manufacturer). Initial studies show that the upstream firm’s direct invasion of the end consumer market might sacrifice the retailer’s profit and maximize its own profit (see, e.g., [22,23]). The downstream retailer can adopt strategies to deter upstream firm encroachment, such as haggling price policy [6], information sharing [12], quality differentiation [11], and the dual-purpose structure [27]. However, follow-up studies also show that the encroachment of an upstream company can benefit both itself and the downstream retailer under certain conditions (see, e.g., [2, 9, 15, 21]). Specifically, Arya et al. [2] demonstrate that encroachment does not promote product differentiation or price discrimination, and the retailer can also benefit from encroachment. The reason is that both the decrease in the whole price, coupled with intense competition with the downstream retailer, alleviate the problem of double marginalization to achieve Pareto improvement. Several studies have investigated the factors that affect the encroachment strategy of upstream firms, such as investment spillover [16], the number of downstream retailers [23], information sharing [16], pursuit of consumer surplus [22], product quality ([11,17,18,31]) and advertising investment [24,34].

2.3. The interplay between information sharing and channel encroachment

Both information sharing and channel encroachment are key factors in supply chain management, so it is valuable to analyze the interaction between them. Although it is a traditional view that upstream firm encroachment and downstream retailer’s information sharing may both harm the retailer’s payoff, there are studies showing that the retailer may voluntarily share its information with the upstream firm or welcome him to encroach. Li et al. [20,21] first study the interplay between supplier encroachment and retailer information sharing and show that encroachment and information sharing can be complementary. Ha et al. [12] also verified this conclusion by investigating the encroachment of the manufacturer into the online retail platform. Huang et al. [15] show that the retailer can prevent manufacturer encroachment by sharing low demand information. Unlike the information symmetry between the players, when the information is asymmetric, the encroachment equilibrium strategies of the upstream firm and other decisions will be affected. Huang et al. [17] study how the interaction between product quality information disclosure and encroachment affects the equilibrium strategy of the upstream firm. For example, the upstream manufacturer proceeds with demand-enhancing investment, which will spill over to the retailer. Huang et al. [16] find that if the investment is sufficiently effective, the retailer is more willing to incentivize the manufacturer to encroach on the endogenous channel structure through information sharing. Although information sharing is not the motivation of manufacturer encroachment, the encroachment equilibrium strategies are affected by information sharing.

The two main differences between our study and the existing related literature are as follows. First, different from the traditional unilateral information asymmetry, we analyze the bilateral information asymmetry in a single supply chain. We assume that the upstream and downstream firms holding own private demand forecast. Second, we aim to analyze the impact of information sharing on encroachment decisions under bilateral demand information asymmetry. Simultaneously, we examine the impact of the interplay between information sharing
and upstream manufacturer encroachment on the equilibrium strategies of supply chain members and the impact of the manufacturer’s per-unit direct selling cost on the firm’s expected revenue.

3. Model

We investigate a traditional two-level supply chain in which the upstream manufacturer (he) sells a single product to the downstream retailer (she). The manufacturer sets the wholesale price \( w \), and then the retailer decides the retail price \( p \) and sells the product to end market consumers. In addition to the traditional retail channel, the manufacturer can also choose to build a direct selling channel to directly sell products to the end-consumer market at retail price \( p_m \). Now, the retailer engages in retail price competition by setting the retail price \( p_r \) with the manufacturer. For example, in addition to selling products through e-commerce platforms (e.g., Tmall and JD.com), P&G has also established its own official website to sell products directly to consumers. It inevitably leads to price competition with downstream retailers.

If the manufacturer encroaches, we assume that he needs to bear a variable per-unit selling cost \( x \). This cost is thought to include all the manpower, material resources, and capital costs he needs when establishing the channel. Meanwhile, the production cost of the manufacturer and the selling cost of the retailer are standardized to zero. Such an assumption of nonzero direct-selling costs has been widely used in the literature on encroachment (e.g., [2,11,20,21,23,27]). The intuition behind this assumption is that the manufacturer is generally considered to be inferior to the retailer in terms of sales and operating ability.

When the manufacturer does not encroach on the retail market, the retailer’s market demand function is \( q_r(p_r) = a - p_r \), where \( a \) is the primary demand. Considering the uncertainty of demand, we assume that is a random variable. As such, we define \( a = \pi + \theta \), where \( \pi \) is the base demand that is known to both members in the supply chain. The uncertainty level for the market \( \theta \) is normally distributed with a mean \( E[\theta] = 0 \) and a variance \( Var[\theta] = \sigma^2_\theta \). This linear demand function and parameter settings are generally used in the literature (see, e.g., [3,15,18,25,29]). If the manufacturer encroaches, the responding demand functions of the manufacturer and retailer are represented by \( q_m(p_m, p_r) = a - p_m + \beta p_r \) and \( q_r(p_m, p_r) = a - p_r + \beta p_m \), respectively. We define \( \beta \in (0,1) \) is the channel substitution rate, and a higher \( \beta \) represents greater intense competition. Our channel demand functions are widely used in the previous literature on channel encroachment; such as Huang et al. [15], Huang et al. [17] and Guan et al. [8].

For the primary demand, \( a \), we assume that both the manufacturer and the retailer can observe a forecast for it and define as \( f_m \) and \( f_r \), respectively. Before observing accurate market demand, firms can decide whether to share with each other based on their forecasts. As such, our forecast information sharing model belongs to the bilateral information sharing structure. The forecast errors of demand are defined as \((f_m - a)\) and \((f_r - a)\) and are normally distributed with a mean 0 and variances \( \sigma_m^2 \) and \( \sigma_r^2 \).

In addition, we use parameters \( \sigma_m \) and \( \sigma_r \) to describe the accuracy of the firms’ market demand forecast, and \( \rho \) to measure the correlation between the two forecasts. Specifically, a higher variance for the forecast error means that the forecast is less accurate; in contrast, a smaller variance for the forecast error indicates that the forecast is more accurate. As such, if \( \sigma_m > \sigma_r \), it indicates that the manufacturer’s demand forecast is more accurate than that of the retailer; if \( \sigma_m < \sigma_r \), it indicates that the retailer’s demand forecast is more accurate than that of the manufacturer. The correlation \( \rho \) is related to the data and methods adopted by both firms in their process of forecasting market demand. A higher \( \rho \) (closer to 1) indicates that the data and methods used by the manufacturer and the retailer are more similar and relevant when forecasting. A smaller \( \rho \) implies that the data sources and methodology are more different. Consistent with Mishra et al. [25], we first assume that the forecast errors of the manufacturer and the retailer on demand can be correlated but must be independent of \( a \). Second, the covariance should not be greater than the individual variances, i.e., \( \rho \sigma_m \sigma_r \leq \sigma_m^2 \) and \( \rho \sigma_r \sigma_m \leq \sigma_r^2 \). Finally, the probability distribution parameters in the supply chain are assumed to be common knowledge. Similar assumptions are adopted by Mishra et al. [25], Bian et al. [3] and Wei et al. [29].

We assume that both game players are risk neutral and make the effort to maximize their expected profits. It is not enough to only know the prior probability distribution of \( a \) for the manufacturer and the retailer, they
Table 1. Model notations.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
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<tbody>
<tr>
<td>(p(p_r))</td>
<td>Retail price charged by the retailer (selling price in the retail channel)</td>
</tr>
<tr>
<td>(p_m)</td>
<td>Retail price in the direct selling channel</td>
</tr>
<tr>
<td>(w)</td>
<td>Unit wholesale price of the manufacture</td>
</tr>
<tr>
<td>(f_m, f_r)</td>
<td>Forecasts of the manufacturer and the retailer about primary demand</td>
</tr>
<tr>
<td>(E[\pi_M</td>
<td>f_m], E[\pi_R</td>
</tr>
<tr>
<td>(v_R, v_M)</td>
<td>Values of information sharing of the retailer and the manufacturer</td>
</tr>
<tr>
<td>(V, V_R, V_M)</td>
<td>Expected values of information sharing of the whole supply chain, the retailer, and the manufacturer</td>
</tr>
</tbody>
</table>

Parameters

| \(a\) | The primary demand |
| \(\pi\) | The base demand that is known to both players |
| \(x\) | A per-unit selling cost when the manufacturer decides to encroach |
| \(\beta\) | The channel substitution rate, \(\beta \in (0, 1)\) |
| \(\theta\) | Random market demand normally distributed with a mean of zero and a variance of \(\sigma_\theta^2\) |
| \(\sigma_m^2, \sigma_r^2\) | Variance of the manufacturer’s and the retailer’s forecast errors |
| \(\rho\) | Correlation between the manufacturer and the retailer’s forecast errors |

Indices

| Subscript | \(r, R(m, M)\) | The retailer (the manufacturer) |
| Superscript | \(SN(\pi N)\) | The case that the manufacturer and the retailer (do not) share their information under non-encroachment |
| Superscript | \(SE(\pi E)\) | The case that the manufacturer and the retailer (do not) share their information under encroachment |

The sequence of events is depicted in Figure 1. As shown in Figure 1, the manufacturer and the retailer first determine whether to share their forecasts, and then the manufacturer decides whether to establish a direct selling channel. If the manufacturer does not encroach, he first determines the wholesale price \(w\), and then the retailer determines the retail price \(p\). If the manufacturer encroaches, he first sets \(w\) and the direct selling price \(p_m\), and then the retailer sequentially determines the retail price \(p_r\). From the above assumptions, we consider four scenarios, namely, \(NN, SN, NE\) and \(SE\) (see Fig. 2 for more illustration). We define \(NE\) and \(NN\) to represent scenarios in which the manufacturer decides to encroach and not encroach under no information.
sharing, respectively. In addition, we use the superscripts \( SE \) and \( SN \) to indicate scenarios that the manufacturer and the retailer share their forecasts under encroachment and non-encroachment conditions, respectively.

4. Equilibrium Analysis

In this section, in order to ensure that the subgame is perfect, we adopt backward induction to solve the equilibrium solution of the firm in four different scenarios. We require conditions \( E[a|f_m] > 0 \) and \( \frac{1}{2}E[a|f_m] < E[a|f_m, f_r] < \frac{3}{2}E[a|f_m] \) to be satisfied in scenario NN to guarantee that the firm gets a nonnegative ex ante expected payoff of firms.

4.1. Scenario NN

In this case, we first examine the benchmark model (i.e., scenario NN). When the manufacturer and the retailer do not share their demand forecasts information, the retailer’s expected payoff function is given by \( \text{Max}_w E[\pi^\text{NN}_R | f_r] = (p^\text{NN} - w^\text{NN})(E[a|f_r, w^\text{NN}] - p^\text{NN}) \). Because both firms do not share their forecasts. Given \( w^\text{NN} \) and \( f_r \), the retailer’s optimal price is given by \( p(f_r, w^\text{NN}) = (w + E[a|f_r, w^\text{NN}])/2 \). The manufacturer’s payoff function is given by \( \text{Max}_w E[\pi^\text{NN}_M | f_m] = w^\text{NN}(E[a|f_m] - E[p(f_r, w^\text{NN})|f_m]) \). Using the concept of Bayesian Nash equilibrium and backward induction, we obtain the unique Bayesian Nash equilibria for the manufacturer and the retailer as \( w^\text{NN} = \frac{1}{2}E[a|f_m] \) and \( p^\text{NN} = \frac{1}{4}(2E[a|f_m, f_r] + E[a|f_m]) \), respectively. The manufacturer’s and the retailer’s payoffs are \( E[\pi^\text{NN}_M | f_m] = \frac{1}{8}E[a|f_m](3E[a|f_m] - 2E[a|f_m, f_r]) \) and \( E[\pi^\text{NN}_R | f_r] = \frac{1}{16}(2E[a|f_m, f_r] - E[a|f_m])^2 \), respectively.

4.2. Scenario SN

We now examine the case of information sharing under non-encroachment (i.e., scenario SN), in which both players share their demand forecast information before the manufacturer sets \( w \). Because both players possess symmetrical demand forecast information, the expected payoff of the retailer is \( \text{Max}_w E[\pi^\text{SN}_R | f_m, f_r] = (p^\text{SN} - w^\text{SN})(E[a|f_m, f_r, w^\text{SN}] - p^\text{SN}) \), and the expected payoff function of the manufacturer is \( \text{Max}_w E[\pi^\text{SN}_M | f_m, f_r] = w^\text{SN}(E[a|f_m, f_r] - E[p|f_m, f_r, w^\text{SN}]) \). We still adopt backward induction to solve the above questions and obtain the equilibrium price and expected payoffs. The optimal strategy of the manufacturer is \( w^\text{SN} = E[a|f_m, f_r]/2 \), and the optimal strategy of the retailer is \( p^\text{SN} = \frac{3E[a|f_m, f_r]}{4} \). The equilibrium profit of the manufacturer and the retailer are \( E[\pi^\text{SN}_M | f_m, f_r] = \frac{1}{8}(E[a|f_m, f_r])^2 \) and \( E[\pi^\text{SN}_R | f_m, f_r] = \frac{1}{16}(E[a|f_m, f_r])^2 \), respectively.

4.3. Scenario NE

Next, we investigate scenario NE, where the retailer and the manufacturer do not share their forecasts under manufacturer encroachment. This scenario becomes a similar traditional dual-channel supply chain with asymmetric demand information. In this case, the manufacturer has only \( f_m \) in his information set to determine
The optimal prices for the manufacturer and the retailer in scenario SE are as follows:

\[ p_{SE} = \frac{E[a|f_m, f_t]}{2(1-\beta)} \]

Proposition 4.1. Under scenario NE, the unique Bayesian Nash equilibrium \((p^NE_m, p^NE_r, w^NE)\) is represented as follows:

\[
\begin{align*}
E[\pi^NE_m | f_m] &= w^NE - \frac{E[a|f_m]}{2(1-\beta)} - \frac{1}{2}(\alpha + 2E[a|f_m])^2 + \frac{1}{2}x^2 - \frac{5(1-\beta)(E[a|f_m])^2}{8(1-\beta)} \\
E[\pi^NE_r | f_m, f_t] &= \frac{1}{2}(\alpha + 2E[a|f_m, f_t] - E[a|f_m])^2 + \frac{1}{4}x^2 - \frac{5(1-\beta)(E[a|f_m])^2}{8(1-\beta)} \\
E[\pi^NE_r | f_m, f_t] &= \frac{1}{2}(\alpha + 2E[a|f_m, f_t] - E[a|f_m])^2 + \frac{1}{4}x^2 - \frac{5(1-\beta)(E[a|f_m])^2}{8(1-\beta)} \\
\end{align*}
\]

Table 2. The profits of the manufacturer and the retailer under scenarios NE and SE.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value in scenarios NE and SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E[\pi^NE_m</td>
<td>f_m])</td>
</tr>
<tr>
<td>(E[\pi^NE_r</td>
<td>f_m, f_t])</td>
</tr>
</tbody>
</table>

the expected value of demand \(a\). And then, he sets the wholesale price \(w\) and the direct selling price \(p^NE_m\). The retailer determines her retail price \(p^NE_r\) based on \(f_m\) and the wholesale price \(w\). Therefore, the profits of the retailer and the manufacturer are given as follows:

\[
\begin{align*}
\max_{p^NE_m} E[\pi^NE_r | f_m] &= (p^NE_r - w^NE)(E[a|f_r, w^NE] - p^NE_r + \beta p^NE_m), \\
\max_{(w^NE, p^NE_m)} E[\pi^NE_m | f_m] &= w^NE - E[p^NE_r | f_m] + \beta p^NE_m + (p^NE_m - x)(E[a|f_m] - p^NE_m + \beta E[p^NE_r | f_m]).
\end{align*}
\]

Proposition 4.1. Under scenario NE, the unique Bayesian Nash equilibrium \((p^NE_m, p^NE_r, w^NE)\) is represented as follows:

\[
\begin{align*}
w^NE &= \frac{E[a|f_m]}{2(1-\beta)}, \\
p^NE_r &= \frac{1}{2}(\alpha + 2E[a|f_m, f_t]) + \frac{1+\beta}{1-\beta}E[a|f_m], \quad \text{and} \quad p^NE_m &= \frac{(1-\beta)x + E[a|f_m]}{2(1-\beta)}. \\
\end{align*}
\]

From Proposition 4.1, we find that the channel substitution rate \((\beta)\) and the manufacturer’s per-unit selling cost \(x\) affect the decisions of the manufacturer and the retailer. The equilibrium profits of the manufacturer and retailer under scenarios NE and SE are presented in Table 2.

4.4. Scenario SE

In this subsection, we analyze the equilibrium results of the firms under scenario SE, that is, the manufacturer encroaches on the retailer after the manufacturer and the retailer share their demand forecast information. Unlike scenario NE, in scenario SE both players have perfectly symmetrical demand forecasts. Therefore, both the manufacturer and the retailer make decisions based on the same information set \(f_m\) and \(f_t\). The payoff functions of the manufacturer and the retailer are shown as follows:

\[
\begin{align*}
\max_{p^SE_m} E[\pi^SE_m | f_m] &= (p^SE_m - w^SE)(E[a|f_r, f_m, w^SE] - p^SE_m + \beta p^SE_m), \\
\max_{(w^SE, p^SE_m)} E[\pi^SE_r | f_m, f_t] &= w^SE - E[p^SE_r | f_m] + \beta p^SE_m + (p^SE_m - x)(E[a|f_m, f_t] - p^SE_m + \beta E[p^SE_r | f_m]).
\end{align*}
\]

We summarize the equilibrium prices of firms in Proposition 4.2, and the optimal profits of the retailer and the manufacturer are shown in Table 2 above.

Proposition 4.2. The optimal prices for the manufacturer and the retailer in scenario SE are as follows:

\[
\begin{align*}
w^SE &= \frac{E[a|f_m, f_t]}{2(1-\beta)}, \\
p^SE_r &= \frac{1}{2}(\alpha + 2E[a|f_r, f_m]), \\
p^SE_m &= \frac{(1-\beta)x + E[a|f_m, f_t]}{2(1-\beta)}. \\
\end{align*}
\]

Next, from the above discussion, we can derive the value of information for the manufacturer, the retailer and the whole supply chain.

5. Decision analysis

In this section, we examine the value of information sharing to firms and the whole supply chain, and then analyze firms’ information sharing and the manufacturer’s encroachment decisions. We begin with the retailer’s information-sharing decisions. First, we compare the retailer’s optimal expected profit under encroachment and non-encroachment to derive her information sharing decision. Then, we point out the value of information sharing to firms and the whole supply chain.
5.1. The value of information sharing under non-encroachment

Following Mishra et al. [25] and Bian et al. [3], we denote the value of information sharing (VIS) for the manufacturer (the retailer) as $v^N_{M}(f_m, f_r) = E[\pi^N_M | f_m, f_r] - E[\pi^N_W | f_m] = E[\pi^N_W | f_m, f_r] - E[\pi^N_R | f_r]$. The VISs of the manufacturer and retailer are given below:

$$v^N_{M}(f_m, f_r) = E[\pi^N_M | f_m, f_r] - E[\pi^N_W | f_m] = \left(\frac{E[a|f_m, f_r]^2 - E[a|f_m]3E[a|f_m, f_r] - 2E[a|f_m]^2}{16}\right),$$

$$v^N_{R}(f_m, f_r) = E[\pi^N_W | f_m, f_r] - E[\pi^N_R | f_r] = \left(\frac{E[a|f_m, f_r]^2 - 2E[a|f_m, f_r] - E[a|f_m]^2}{16}\right).$$

In addition, $V^N_{M}(f_m, f_r) (V^N_{R}(f_m, f_r))$ is the expected value of information sharing (EVIS) of the manufacturer and retailer are expressed as follows:

$$V^N_{M}(f_m, f_r) = \int_{0}^{\infty} \int_{0}^{\infty} v^N_{M}(f_m, f_r)g(f_m, f_r)df_m df_r,$$

$$V^N_{R}(f_m, f_r) = \int_{0}^{\infty} \int_{0}^{\infty} v^N_{R}(f_m, f_r)g(f_m, f_r)df_m df_r.$$

where $g(f_m, f_r)$ is the joint probability density of $f_m$ and $f_r$. 

Proposition 5.1. We obtain the following results about the values of information sharing (EVIS) under non-encroachment:

(i). $v^N_{M}(f_m, f_r) \geq 0$ if and only if $E[a|f_m, f_r] \leq E[a|f_m] \leq 3E[a|f_m, f_r]$, that is, $f_r \leq E[f_r|f_m]$; $v^N_{M}(f_m, f_r) \geq 0$ if $f_r \geq E[f_r|f_m]$.

(ii). $V^N_{M}(f_m, f_r) \geq 0, V^N_{R}(f_m, f_r) \leq 0, V^N \leq 0$.

(iii). If $f_r < E[f_r|f_m]$, then $w^N > w^SN, p^NN > p^SN$, and $E[\pi^N_W | f_r] < E[\pi^N_R | f_m, f_r]$; otherwise, $f_r > E[f_r|f_m]$, then $w^N < w^SN, p^NN < p^SN$, and $E[\pi^N_W | f_m] < E[\pi^N_R | f_m, f_r]$.

Proposition 5.1(2) shows that under the premise that the manufacturer does not encroach, information sharing benefits the manufacturer (because $V^N_{M}(f_m, f_r) \geq 0$) but hurts the retailer and the supply chain (because $V^N_{R}(f_m, f_r) \leq 0$ and $V^N \leq 0$). Therefore, the retailer does not voluntarily share her forecast with the manufacturer, and the manufacturer also has no incentive to actively induce the retailer to share forecast. This result is similar to that of Mishra et al. [25], who assumed that the manufacturer and the retailer engage in the “make-to-order” case.

It is concluded in Proposition 5.1(1) that the manufacturer obtains a nonnegative value from information sharing if and only if the retailer’s demand forecast is high enough (i.e., $f_r \geq E[f_r|f_m]$). It indicates that the retailer’s actual forecast is higher than the manufacturer’s expectation of the retailer’s forecast. Compared with the conclusion in the “make-to-order” scenario in Mishra et al. [25], the conditions under which the manufacturer can benefit from information sharing are different. They found that the manufacturer always benefits from forecast information sharing, regardless of their forecasts. Consistently, the retailer’s expected profit under information sharing is uncertain compared to that under no information sharing. It depends on the retailer’s forecast, which may be higher or lower.

Two cases are investigated under the condition that the manufacturer does not encroach and no forecasts information sharing. On the one hand, when the retailer’s forecast of demand is low, i.e., $f_r < E[f_r|f_m]$, forecast information sharing benefits the retailer but hurts the manufacturer. Moreover, the manufacturer and retailer cannot benefit simultaneously from information sharing, regardless of the value of the forecasts of the manufacturer and retailer. The reason may be that when $f_r < E[f_r|f_m]$, the manufacturer’s expectation of the retailer’s forecast is greater than the retailer’s actual forecast, the optimal wholesale price $w$ under no information sharing is higher than the optimal price under no information sharing (since $w^N > w^SN$). The retailer will raise her retail prices accordingly (since $p^NN > p^SN$). It reduces the market demand and decreases the profits of the manufacturer and retailer. Subsequently, if they share their forecast information, the manufacturer can
lower his wholesale price to close the optimal level. The manufacturer’s payoff has not increased compared to that under no information sharing, and the retailer can benefit from the reduced wholesale price $w$. Moreover, the increase in the retailer’s profit is less than the decrease in the manufacturer’s profit under information sharing, which leads to a smaller profit of the entire supply chain.

On the other hand, when the retailer’s forecast is high, for example, $f_r > E[f_r | f_m]$, forecast information sharing benefits the manufacturer but hurts the retailer. Since the retailer’s profit will be reduced after sharing information, it is difficult for both parties to reach an information sharing agreement if the manufacturer does not compensate for the loss of the retailer from sharing her demand forecast information. Unless the profit of the whole supply chain increases, and the manufacturer has enough to pay the retailer for the loss of information sharing. The reason may be that when the manufacturer’s expectation of the retailer’s forecast is sufficiently high, the manufacturer is willing to share information, but the retailer is unwilling. When the retailer’s prediction is sufficiently low, the retailer is also unwilling to cooperate with the manufacturer and hopes to maintain the information advantage over the latter and benefit from it. Therefore, under the condition that the manufacturer has not established a direct selling channel, it is difficult for both parties to reach an information sharing agreement.

### 5.2. The value of information sharing under encroachment

Here, if the manufacturer encroaches, we denote $v^E_M(f_m, f_r) = E[π^E_M | f_m, f_r] − E[π^N_M | f_m]$ and $v^E_R(f_m, f_r) = E[π^E_R | f_m, f_r] − E[π^N_R | f_r]$ as the values of information sharing (VISs) for the manufacturer and retailer, respectively. We obtain the following results:

$$v^E_M(f_m, f_r) = \frac{E[a | f_m, f_r] - E[a | f_m]}{\beta(1-\beta)},$$

$$v^E_R(f_m, f_r) = E[\pi^E_R | f_m, f_r] - E[\pi^N_R | f_r] = \frac{E[a | f_m, f_r] - E[a | f_m]}{16} + 4(\beta - 1)\beta.$$

Specifically, $V^E_M(f_m, f_r)$ and $V^E_R(f_m, f_r)$ are the expected values of information sharing (EVISs) for the manufacturer and the retailer, respectively:

$$V^E_M(f_m, f_r) = \int_0^\infty \int_0^\infty v^E_M(f_m, f_r)g(f_m, f_r)df_mdf_r.$$

$$V^E_R(f_m, f_r) = \int_0^\infty \int_0^\infty v^E_R(f_m, f_r)g(f_m, f_r)df_mdf_r.$$

Similarly, $V^E = V^E_M(f_m, f_r) + V^E_E(f_m, f_r)$ is the expected value of information sharing for the whole supply chain under manufacturer encroachment. Here, if the manufacturer encroaches, the following results about the values of information sharing hold.

**Proposition 5.2.** We obtain the following results about the values of information sharing (EVIS) under encroachment:

(i). If $f_r > E[f_r | f_m]$, $v^E_M(f_m, f_r) > 0$ if and only if $x < \frac{(5-\beta)E[a | f_m] + (\beta + 3)E[a | f_m, f_r]}{4(1-\beta)}$, $v^E_R(f_m, f_r) > 0$ if and only if $x > \frac{(5-\beta)E[a | f_m] + (\beta + 3)E[a | f_m, f_r]}{4(1-\beta)}$.

(ii). When $f_r < E[f_r | f_m]$, $v^E_R(f_m, f_r) > 0$ if and only if $x < \frac{E[a | f_m] - 3E[a | f_m, f_r]}{2\beta}$, $v^E_M(f_m, f_r) > 0$ if and only if $x > \frac{E[a | f_m] - 3E[a | f_m, f_r]}{2\beta}$, $\beta \in (0, \frac{7-\sqrt{41}}{2})$.

(iii). $V^E_M(f_m, f_r) \geq 0$, $V^E_R(f_m, f_r) \geq 0$, $V^E \geq 0$.

If the manufacturer encroaches, the firms’ forecasts information sharing decisions are influenced by the channel substitution rate $\beta$, the per-unit selling cost $x$, and the manufacturer’s forecast. Proposition 5.2(3) shows that from the supply chain’s perspective, if the manufacturer encroaches, information sharing can benefit the whole
supply chain, including the manufacturer and the retailer. Therefore, the forecast information sharing agreement between the retailer and the manufacturer can be realized through a simple side payment contract or a subsidy.

When the retailer’s forecast is sufficiently high (i.e., \( f_r > E[f_r | f_m] \)), the retailer can always obtain a non-negative value from information sharing regardless of the value of the retailer and the manufacturer’s forecasts. However, the manufacturer can benefit from information sharing if and only if his per-unit selling cost is less than the threshold \( x < \frac{(5-\beta)E[a|f_m] + (\beta + 3)E[a|f_m, f_r]}{4(1-\beta)} \).

Next, we consider the case that the retailer’s forecast is sufficiently low (i.e., \( f_r < E[f_r | f_m] \)). First, if the channel substitution rate \( \beta \) is sufficiently high (i.e., \( \beta < 1 \)), there is no unit direct selling cost intersection that enables both the manufacturer and the retailer to benefit from information sharing. This shows that the manufacturer cannot benefit from information sharing with the retailer in this scenario. Specifically, when the per-unit direct selling cost is less than the threshold \( x < \frac{E[a|f_m] - 2\beta E[a|f_m, f_r]}{2\beta} \), the retailer can benefit from the sharing of forecast information; in contrast, when the per-unit selling cost is larger than the threshold (i.e., \( x > \frac{(5-\beta)E[a|f_m] + (\beta + 3)E[a|f_m, f_r]}{4(1-\beta)} \)), the manufacturer can benefit from forecast information sharing. This conclusion is counterintuitive.

Then, if the channel substitution rate is sufficiently low (i.e., \( \beta < \frac{7-\sqrt{41}}{2} \)), the manufacturer and the retailer can both benefit from their forecast information sharing if and only if the per-unit selling cost falls into the range of \( (\frac{(5-\beta)E[a|f_m] + (\beta + 3)E[a|f_m, f_r]}{4(1-\beta)}, \frac{E[a|f_m] - 3E[a|f_m, f_r]}{2\beta}) \). Moreover, because information sharing can also increase the retailer’s profit, she has great motivation to actively share it with the manufacturer. A possible explanation for this is that if the retailer’s forecast is sufficiently low, and channel competition is not intense. The retailer prefers share her forecast with the manufacturer to increase the forecast accuracy.

In summary, when the retailer’s actual forecast is larger than the manufacturer’s expectation of the retailer’s forecast, the manufacturer can only benefit from information sharing when his per-unit selling cost is sufficiently low. Therefore, both the retailer and the manufacturer have great motivation to reach a forecast information sharing agreement; however, once the manufacturer’s per-unit selling cost exceeds a certain threshold, information sharing will not increase the profit for the manufacturer. So, the manufacturer is unwilling to actively share forecast information with the retailer. When the actual forecast of the retailer is lower than the manufacturer’s expectation of the retailer’s forecast, the retail channel and a direct channel are very competitive. It is almost impossible for the retailer and the manufacturer to achieve information sharing because they cannot benefit from it. When channel competition is not very fierce, as long as the manufacturer’s per-unit direct selling cost falls into a certain range, the retailer and the manufacturer can reach an information sharing agreement and benefit from information sharing at the same time.

5.3. Encroachment Decision

In this subsection, we investigate the manufacturer’s encroachment decision under information sharing and no information sharing. We begin with the manufacturer’s encroachment decisions under no information sharing. In this case, we obtain the following lemma by comparing the manufacturer’s expected payoffs under encroachment and no-encroachment.

**Lemma 5.3.** Given that the retailer and the manufacturer do not share their forecast information, the manufacturer encroaches if and only if his per-unit selling cost falls into the range of \( (0, x_1) \), and the manufacturer cannot benefit from encroachment if \( x \) falls into the range of \( (x_1, \hat{x}) \).

If firms do not share their forecasts, the inequality \( \frac{1}{2} E[a|f_m, f_r] < E[a|f_m] < 2E[a|f_m, f_r] \) needs to be satisfied in order to ensure that the \( ex \ ante \) expected profits of the retailer and the manufacturer are positive. Because the representation of the thresholds in Lemma 5.3 is complicated, we use \( x_1 \) and \( \hat{x} \) as substitutes, and the specific representation is as follows:

\[
x_1 = \frac{(1-\beta)(2A_1 + \beta A_2) - \sqrt{\Delta}}{(\beta^2 - 4)(\beta - 1)}, \quad \hat{x} = \frac{2A_1 + (2-\beta)A_2}{2\beta^2},
\]
where \( A_1 = E[a|f_m] \), \( A = E[a|f_m, f_r] \), and \( \Delta = \beta(\beta - 1)[-2(\beta^2 + \beta - 4)A_1^2 + 4(\beta - 1)\beta A_1 + \beta(\beta - 1)A^2] \).
Lemma 5.3 shows that the manufacturer’s encroachment decision will be affected by his per-unit direct selling cost under no information sharing. This conclusion is consistent with the cutoff policy shown in the supplier’s encroachment strategy obtained by Huang et al. [15]. The difference is that Huang et al. [15] consider that the supplier needs to bear a fixed entry cost $I$ when establishing a direct selling channel. We study the effect of the manufacturer’s per-unit selling cost $x$ on his equilibrium strategy. Specifically, when the per-unit selling cost is sufficiently low (e.g., $x < x_1$), he can benefit from his encroachment decision. If $x$ is sufficiently high (e.g., $x_1 < x < \hat{x}$), encroachment is not profitable for him. This conclusion is consistent with our intuition. To ensure that the market demand is positive when the manufacturer encroaches, the per-unit direct selling cost cannot exceed the threshold $\hat{x}$. In addition, the greater the channel substitution rate $\beta$, the more intense the channel competition, and the greater the threshold $\hat{x}$, indicating a greater possibility of encroachment.

Next, we compare the manufacturer’s expected payoffs under encroachment and no-encroachment conditions when firms share their forecasts information. Since the inequality $E[\pi_{SE}^M | f_m, f_r] - E[\pi_{SN}^M | f_m, f_r] > 0$ holds, it indicates that the manufacturer always obtains a nonnegative value from forecast information sharing. The manufacturer and the retailer share their forecasts before make their pricing decisions. The manufacturer not only relies on the per-unit selling cost, but also on the retailer’s forecast $f_r$ to makes pricing and encroachment decisions. He has both forecasts information $f_m$ and $f_r$ in his information set to determine the expected value of $a$, which increases the forecasting accuracy and can more efficiently evaluate the benefit of encroachment.

6. Sensitivity analysis

In this section, we not only analyze the impact of forecast accuracy on the expected profit of the firms, but also discuss the impact of relevant parameters on the manufacturer and the retailer by conducting numerical study. The parameters include the channel substitution rate $\beta$, demand variance $\sigma_0$ and the correlation between firms’ forecasts $\rho$. Through numerical analysis, it helps us to obtain useful management implications for firms.

6.1. Impact of forecast accuracy

In this subsection, we analyze the impact of forecast accuracy, the correlation and the demand variance on the expected profits of the manufacturer and retailer under different scenarios. The analytical results are summarized in Table 3. To facilitate analysis and comparison, the first part of the table summarizes the analysis results of no information sharing and information sharing in the “make-to-order” scenario of Mishra et al. [25].

For example, the results show that increasing forecasts accuracy has a positive impact on their own expected payoffs. In three scenarios, the improvement of the retailer’s forecast accuracy and the correlation will not affect the manufacturer’s expected profit. The three scenarios are as follows: no information sharing in the “make-to-order” scenario of Mishra et al. [25], scenario NN and scenario NE. The above results are all intuitive and are consistent with Mishra et al. [25]. Because in scenarios NN and NE, the manufacturer and the retailer belong to bilateral forecast information asymmetry, and they do not achieve demand sharing. Therefore, the improvement of the retailer’s forecast accuracy and the correlation between two forecasts have no influence on the manufacturer.

As mentioned before, it is an intuitive result that the correlation does not affect the manufacturer’s expected payoff under no forecast sharing. An increase in correlation will adversely affect both the retailer and the manufacturer, regardless of whether the retailer shares her forecast information. When the retailer does not share information, an increase in correlation has a negative impact on the manufacturer’s expected profit. The reason is that the greater the correlation between the retailer’s and the manufacturers forecasts, the more similar data or methods used by the manufacturer and the retailer in forecasting. It will increase the similarity and substitutability of forecasts, thereby reducing the incremental value of each forecast.

An increase in the manufacturer’s forecast accuracy is different for the retailer. If players do not share their forecasts with each other, an increase in the manufacturer’s forecast accuracy is beneficial or unfavorable to
Table 3. Impact of forecast accuracy under different scenarios.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\frac{\partial E[\pi^*]}{\partial \sigma_0}$</th>
<th>$\frac{\partial E[\pi^*]}{\partial \rho}$</th>
<th>$\frac{\partial E[\pi^*]}{\partial \sigma_0}$</th>
<th>$\frac{\partial E[\pi^*]}{\partial \rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>$ \leq 0$</td>
<td>$\leq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>$R$</td>
<td>$&lt; 0$ or $&gt; 0$</td>
<td>$\leq 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
</tr>
</tbody>
</table>

Notes. The values of the table represent the first-order derivatives of the expected profits of the manufacturer and retailer with respect to different forecast accuracies and correlations in different scenarios. "$=0$" indicates that the value is zero, "$\leq 0$" indicates that the value is not positive, "$\geq 0$" indicates that the value is not negative, and "$< 0$ or $> 0$" indicates that the value can be positive or negative.

The retailer. If the retailer and the manufacturer share their forecast with each other, an increase in the manufacturer’s forecast accuracy is beneficial to both the manufacturer and the retailer. This is because the retailer can infer the manufacturer’s forecast from the wholesale price set by the manufacturer, and then determine the retail price based on her forecast and the manufacturer’s forecast. In addition, when the retailer does not share her forecast with the manufacturer, an increase in the manufacturer’s forecast accuracy can enable the manufacturer to set a more reasonable wholesale price. Thus, more supply chain profits can be obtained, although it may hurt the retailer. The improvement in the accuracy of the manufacturer’s forecast can also improve the accuracy of the comprehensive forecast, which in turn will have a positive impact on the retailer in determining the retail price. In Table 3, there are three scenarios where the retailer and the manufacturer share their forecast information. The reason is that due to an increase in the manufacturer’s forecast accuracy resulting in the improvement of comprehensive forecast accuracy, which in turn has a positive impact on the pricing decisions of the retailer and the manufacturer. It is well illustrated by the results shown in the four different scenarios and “make-to-order” scenario [25] shown in Table 3.

In Table 3, the effect of demand variance $\sigma_0$ on the manufacturer’s and retailer’s expected profits is positive for all the scenarios. This is because under the condition of the same mean, the larger the variance of $a$, the higher the probability of high and low demand will increase. The expected profit functions of the manufacturer and the retailer are increasing quadratic functions of $a$, and the larger profit corresponding to the larger $a$ is superior to the smaller profit corresponding to the smaller $a$. Therefore, the expected benefits of the manufacturer and the retailer will also increase as the variance of demand increases.
6.2. Impact of \( \beta \)

In order to better illustrate the impact of the channel substitution rate \( \beta \) on the firms and the whole supply chain, we use numerical experiments to compare with existing literatures. Similar to Mishra et al. [25] and Bian et al. [3], we adopt the following parameter settings: \( \bar{a} = 250, \sigma_0 = 60, \sigma_m = 20, \sigma_r = 20 \) and \( \rho = 0.1 \).

The impact of the channel substitution rate \( \beta \) on EVISs for the manufacturer, the retailer and the whole supply chain is shown in Figure 3. It shows that the effect of \( \beta \) on the EVIS of the manufacturer and the whole supply chain are positive, while the effect on the retailer is positive and remain unchanged. This is because \( \frac{\partial V^E_M}{\partial \beta} > 0, \frac{\partial V^E_R}{\partial \beta} = 0 \) and \( \frac{\partial V^E}{\partial \beta} > 0 \). First, if the manufacturer encroaches, the retailer’s EVIS is independent of \( \beta \), which shows that the channel substitution rate will not affect the retailer’s expected values of information sharing (EVIS); then, as the channel substitution rate \( \beta \) increases, the expected values of information sharing for the manufacturer and the whole supply chain increase, and the impact of \( \beta \) on the manufacturer is greater than that on the whole supply chain because \( \frac{\partial V^E_M}{\partial \beta} < \frac{\partial V^E}{\partial \beta} \). This shows that when the manufacturer encroaches, information sharing is not necessarily harmful to the manufacturer and the retailer. As the channel substitution rate increases, it becomes more important for the retailer to share her forecast. Since the expected information sharing value of the manufacturer and the entire supply chain are both positive, it indicates that the supply chain members have sufficient motivation to coordinate. When the manufacturer encroaches, the impact of the manufacturer and the retailer on the expected information sharing value is mainly retail price competition. Therefore, both firms’ preassessment of the channel substitution rate is important because it will affect their information sharing and encroachment strategies.

6.3. Impact of \( \sigma_m \)

Next, we analyze the impacts of both firms’ forecasting accuracy (\( \sigma_m \) and \( \sigma_r \)) on information sharing, which are shown in Figures 4–6. We set \( \beta = 0.9 \), and consider the values of \( \sigma_m \) and \( \sigma_r \) from 0 to 50. Specifically, we consider how the EVISs of the manufacturer, retailer and whole supply chain change.

(1). Figure 6 shows that \( \sigma_m \) and \( \sigma_r \) have opposite effects on the EVIS of the supply chain. From the perspective of the whole supply chain, if the manufacturer encroaches, as \( \sigma_m \) increases, the EVIS of the whole supply chain increases and is always positive. Because both the manufacturer and the retailer can obtain a positive
EVIS from information sharing. The manufacturer’s forecast variance $\sigma_m$ has a greater impact on the whole supply chain’s expected profit than the retailer’s forecast variance $\sigma_r$. When the manufacturer does not encroach, a higher $\sigma_m$ decreases the EVIS of the whole supply chain, and the EVIS is negative. Because the manufacturer’s positive EVIS is lower than the retailer’s negative EVIS. The effect of $\sigma_m$ on EVIS is also greater than that of $\sigma_r$ similarly.

(2) Now, we analyze the impact of $\sigma_m$ on the game members under non-encroachment. First, the impact of $\sigma_m$ on the manufacturer and the retailer is opposite. As $\sigma_m$ increases, the manufacturer’s EVIS increases, and the growing tendency is similar to the trend of the whole supply chain’s EVIS when the manufacturer encroaches. However, a higher $\sigma_m$ decreases the retailer’s EVIS, and the downtrend is similar to the trend of the whole supply chain’s EVIS under non-encroachment. In this scenario, the manufacturer holds a positive EVIS, and the retailer’s EVIS is negative.

(3) If the manufacturer encroaches, the impact of $\sigma_m$ on the EVISs for the manufacturer and the retailer is identical, i.e., as $\sigma_m$ increases or $\sigma_r$ decreases, both the manufacturer’s and the retailer’s EVISs increase. This conclusion is consistent with Mishra et al. [25]. Moreover, the growth trend of EVISs for both the
manufacturers and the retailer is similar to the trend of EVIS for the whole supply chain when the manufacturer encroaches.

We now analyze the impact of $\sigma_m$ on information sharing. First, as the manufacturer is the Stackelberg leader in the supply chain, the retailer can infer the manufacturer’s forecast $f_m$ from the wholesale price $w$. This is because of the inference effect (see, e.g., [3, 25, 29]). When aggregating firms’ demand forecasts, the relative weights of forecasts $f_m$, $f_r$ and $\pi$ are mainly affected by their relative variances $\sigma_m$ and $\sigma_r$. According to Winkler [30], we have $\rho(\sigma_m^2 + \sigma_r^2) < 2\sigma_m\sigma_r$, and then $\frac{\partial L}{\partial \sigma_m} > 0$ and $\frac{\partial K}{\partial \sigma_m} < 0$ holds. Thus, when aggregating forecasts, the weight of $f_m$ decreases, and the forecast will be affected more by $f_r$. Finally, the retailer cannot obtain more new useful knowledge from their forecast information sharing, while the manufacturer can obtain more accurate demand forecasts from their information sharing and increase his expected profit. This conclusion holds both under manufacturer encroachment and non-encroachment scenarios.

6.4. Impact of $\sigma_r$

Figures 4–6 show that the impacts of $\sigma_m$ and $\sigma_r$ on the EVISs of the manufacturer, the retailer and the whole supply chain are opposite, as follows:

(I). As shown in Figure 6, $\sigma_r$ has the opposite impact on the EVIS for the whole supply chain under both encroachment and non-encroachment scenarios. If the manufacturer encroaches, the EVIS of the whole supply chain decreases as $\sigma_r$ increases; if the manufacturer does not encroach, a higher $\sigma_r$ increases the whole supply chain’s EVIS.

(II). The impact of $\sigma_r$ on the manufacturer and the retailer is also opposite. That is, a higher $\sigma_r$ decreases the manufacturer’s EVIS and increases the retailer’s EVIS under non-encroachment scenario.

(III). Under the condition that the manufacturer encroaches, the impact of $\sigma_r$ on the manufacturer and the retailer is identical. As shown in Figure 5, as $\sigma_r$ increases, both the EVISs for the manufacturer and the retailer decrease, and the effect of $\sigma_r$ on the manufacturer is larger than that on the retailer.

From the above assumptions, a higher retailer’s variance $\sigma_r$ implies less useful information. If the manufacturer encroaches, the EVISs of the manufacturer, the retailer and the whole supply chain decrease as $\sigma_r$ increases. This is because $\frac{\partial L}{\partial \sigma_r} > 0$ and $\frac{\partial K}{\partial \sigma_r} < 0$. Therefore, as $\sigma_r$ increases, the manufacturer and the aggregate forecast rely more on the forecast information $f_m$. As a result, the manufacturer and the whole supply chain obtain fewer EVISs from the retailer’s information sharing. When the manufacturer does not encroach, the impact of information sharing on the manufacturer and the retailer is different. That is, as $\sigma_r$ increases, the EVISs for the retailer and the whole supply chain increase, but the manufacturer’s EVIS decreases. The reason for this result is as follows: the EVIS of the whole supply chain is lower than 0 under the condition that the manufacturer does not encroach, which means that it is difficult to achieve the balance of the supply chain by relying on the manufacturer’s sharing information.

6.5. Impact of $\sigma_0$

The effect of demand variation $\sigma_0$ on the manufacturer’s and the retailer’s EVISs is shown in Figure 7. When the manufacturer encroaches, a higher $\sigma_0$ increases the expected values of information sharing (EVISs) for the manufacturer, the retailer, and the whole supply chain. However, the increase in the retailer’s EVIS is not significant. Under the condition that the manufacturer does not encroach, as $\sigma_0$ increases, the EVISs for both the retailer and the whole supply chain decreases, while the manufacturer’s EVIS increases. This is because $\frac{\partial V_N^N}{\partial \sigma_0} < 0$, $\frac{\partial V_N^R}{\partial \sigma_0} < 0$ and $\frac{\partial V_M^N}{\partial \sigma_0} > 0$. In comparison, whether the manufacturer encroaches or does not encroach, the effect of $\sigma_0$ on the manufacturer and the entire supply chain is obviously different, but the degree of impact on the retailer is the same.
6.6. Impact of $\rho$

The impact of the correlation between the manufacturer’s and the retailer’s forecast errors on the EVISs is shown in Figure 8. As shown in Figure 8, under non-encroachment scenario, as $\rho$ increases, the EVISs for the manufacturer and the whole supply chain increase, while the retailer’s EVIS decreases. If the manufacturer encroaches, the EVISs for the manufacturer, the retailer and the whole supply chain decrease as $\rho$ increases.

When $\rho$ increases, it indicates that the relevance and substitutability between the manufacturer’s and the retailer’s demand forecasts increase, and the value of information sharing weakens. If the manufacturer does not encroach, the EVISs for the retailer and the whole supply chain increase as $\rho$ increases because $\frac{\partial V^N}{\partial \rho} > 0$ and $\frac{\partial V^E}{\partial \rho} > 0$ while enjoying a negative EVIS. This indicates that information sharing is unvaluable for the retailer and the whole supply chain but is valuable for the manufacturer because $\frac{\partial V^M}{\partial \rho} < 0$. The results show that the retailer’s relative information superiority can create more value for her.

When the manufacturer encroaches, the EVISs for the manufacturer, the retailer and the whole supply chain all increase as $\rho$ decreases and always enjoy a positive EVIS because $\frac{\partial V^E}{\partial \rho} < 0$, $\frac{\partial V^R}{\partial \rho} < 0$ and $\frac{\partial V^N}{\partial \rho} < 0$. Therefore, the increase in information substitutability makes the EVISs for the manufacturer, the retailer, and the whole supply chain lower as $\rho$ increases. However, the difference in the retailer’s profit between information sharing and no information sharing cases is not large. This implies that the retailer can benefit from information sharing with the manufacturer.

Regardless of whether the manufacturer encroaches or not, the manufacturer’s EVISs decrease as $\rho$ increases. The decrease in the manufacturer’s EVIS is greater when the manufacturer encroaches, which shows that it is more beneficial for the manufacturer to encroach than not to encroach when the retailer shares information. If the manufacturer encroaches, the retailer’s EVIS decreases as $\rho$ increases but can enjoy a positive EVIS; when the manufacturer does not encroach, the retailer’s EVIS decreases as $\rho$ increases but enjoys a negative EVIS. Therefore, information sharing in the no-encroachment scenario is not valuable to the retailer.
7. Conclusion

In this study, we investigate the manufacturer’s encroachment decisions and the retailer’s information sharing decisions in a supply chain with bilateral information asymmetry, in which both firms forecast the primary demand. We find that neither the manufacturer nor the retailer can reach an information sharing agreement under non-encroachment, and the manufacturer has no incentive to induce the retailer to share her personal information. If the manufacturer chooses to encroach, he has a great incentive to induce the retailer (e.g., contract or subsidy) to achieve bilateral forecast information sharing. In this sense, our research provides guidance for both the manufacturer and the retailer.

Through the study, we derive some managerial implications for both firms. First, from the retailer’s perspective, the retailer will not necessarily volunteer to share her forecast, and she may welcome the manufacturer’s encroachment. Specifically, if the manufacturer does not encroach, both firms cannot voluntarily reach a forecast information sharing agreement; if the manufacturer encroaches, their forecast sharing is profitable to both firms under certain conditions. Additionally, from the manufacturer’s perspective, if they reach an information sharing agreement, the manufacturer will benefit more from information sharing because of his first-mover advantage. Here, once they share demand forecasts, encroachment is always profitable to the manufacturer. However, if they do not share forecast information, the manufacturer’s encroachment is profitable only when the direct selling cost is lower than a certain threshold; The higher the channel substitution rate between the manufacturer and the retailer, the manufacturer is more likely prefer to encroach.

Our computational results first shows that an increase in firms’ forecast accuracy is beneficial to both sides. Second, the increase in the manufacturer’s forecast accuracy has an uncertain impact on the retailer, while the increase in the variance of the demand forecast is beneficial to both firms. Third, regardless of whether the manufacturer and retailer share their forecast information, a higher correlation is unfavorable. Finally, we find that the competition between the two channels will not affect the retailer’s expected value from information sharing under manufacturer encroachment, but as competition intensifies, the manufacturer and the whole supply chain obtain higher profits.

Given that our study has certain limitations, several research directions can be extended.

First, we only consider the price competition between the manufacturer and the retailer, in which the former is the Stackelberg leader and the latter is the follower. In reality, it is difficult for the manufacturer to lead the retailer when the retailer’s market power and bargaining power are large enough, and future research may verify whether our derived results still hold in other competitive structure (e.g., Cournot quantity competition, Bertrand competition and the retailer-leader Stackelberg game).

Second, we examine the bilateral forecast information sharing decisions, where a single manufacturer and a single retailer possessing asymmetric demand information. In the future, further investigation can consider how different risk attitudes of firms or the situation that the manufacturer is constrained by production cost [14] will affect their information sharing decisions.

Third, in the two-stage game process, whether the firms provide service to consumers during the selling period [8], as well as the number of downstream retailers may also be worth studying.

Finally, we find that it is difficult for firms to reach a forecast information sharing agreement under non-encroachment. On the contrary, if the manufacturer encroaches, it is easier for firms to realize information sharing under certain conditions. Since the entire supply chain can benefit from information sharing, how firms guide information sharing and design information sharing revenue contracts may be worth studying as well.

Appendix A.

Winkler’s consensus model. To aggregate forecast information from different sources, the following quantities are obtained according to Winkler’s consensus model.

\[
E[a|f_m] = (1 - t_m)\pi + t_m f_m,
\]

(A.1)
\[ E[a|f_r] = (1 - t_r)\pi + t_r f_r, \]  
\[ E[a|f_m, f_r] = I\pi + K f_m + J f_r, \]

\[ \text{Var}[a|f_m, f_r] = \frac{(1 - \rho^2)\sigma_m^2 \sigma_r^2}{(1 - \rho^2)\sigma_m^2 + \sigma_r^2 (\sigma_m^2 - 2\rho\sigma_m\sigma_r)}, \]

\[ E[f_m|f_r] = (1 - d_r)\pi + d_r f_r, \]

\[ E[f_r|f_m] = (1 - d_m)\pi + d_m f_m, \]

where \( t_m = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_r^2}, \) \( t_r = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_m^2}, \) \( d_m = \frac{\sigma_a^2 + \rho \sigma_m \sigma_r}{\sigma_a^2 + \sigma_m^2}, \) \( d_r = \frac{\sigma_a^2 + \rho \sigma_m \sigma_r}{\sigma_a^2 + \sigma_r^2}, \) \( I = \frac{1 - \rho^2}{(1 - \rho^2)\sigma_m^2 + \sigma_r^2 (\sigma_m^2 - 2\rho\sigma_m\sigma_r)}, \) \( J = \frac{1 - \rho^2}{(1 - \rho^2)\sigma_r^2 + \sigma_m^2 (\sigma_r^2 - 2\rho\sigma_m\sigma_r)}. \)

Proof of Proposition 4.1. In scenario NE, given the wholesale price and the direct selling price, the retailer solves her payoff function \( E[\pi^N_E|f_r]. \) We obtain her optimal price \( p_r(f_r, w^{NE}, p_m) = E[a|f_r, w^{NE}] + \beta p_m^{NE} + w^{NE})/2. \) Similarly, based on the Bayesian Nash equilibrium concept and backward induction, we subsequently substitute the manufacturer’s expectation of the retailer’s retail price \( E[p_r|f_m] = (w + p_m + E[a|f_m])/2 \) into the manufacturer’s payoff function \( E[\pi^N_M|f_m]. \) The manufacturer maximizes his payoff function by determining the
optimal wholesale price and the direct selling price as \( w^NE = \frac{E[a|f_m]}{2(1-\beta)} \) and \( p^NE = \frac{(1-\beta)x+E[a|f_m]}{2(1-\beta)} \). Then, substituting \( w^NE \) and \( p^NE \) into \( p_r(f_r, w^NE, p^NE) \), we obtain \( p^*_r = \frac{1}{4}(x\beta + \frac{1+\beta}{1-\beta}E[a|f_m] + 2E[a|f_m, f_r]) \). As a result, we obtain their expected payoffs are \( E[\pi^NE|f_m] = \left(\frac{(2-\delta^2)x^2}{8} - \frac{\delta xE[a|f_m]}{2} - \frac{(x\beta+E[a|f_m])E[a|f_m, f_r]}{4} + \frac{(5-\delta)(E[a|f_m])^2}{8(1-\beta)}\right) \) and \( E[\pi^NE|f_r] = \frac{1}{16}(x\beta + E[a|f_r, f_m])^2 \).

**Proof of Proposition 4.2.** In scenario SE, the manufacturer and the retailer have symmetry information. Given the wholesale price and the direct selling price, the retailer solves her payoff function \( E[\pi^SE|f_m, f_r] \). We obtain her optimal retail price \( p_r(f_m, f_r, w^SE, p^SE_m) = (E[a|f_r, w^SE] + \beta p^SE_m + w^SE)/2 \). We substitute the manufacturer’s expectation of the retailer’s response function \( E[p_r|f_m] = (E[a|f_r, f_m] + \beta p^SE_m + w^SE)/2 \) into the manufacturer’s expected profit function \( \text{Max}_{w^SE, p^SE_m} E[\pi^SE|f_m, f_r] \), leading to his optimal wholesale price and direct selling price as \( w^SE = \frac{E[a|f_m, f_r]}{2(1-\beta)} \) and \( p^SE_m = \frac{(1-\beta)x+E[a|f_m, f_r]}{2(1-\beta)} \). Then, substituting \( w^SE \) and \( p^SE_m \) into \( p_r(f_m, f_r, w^SE, p^SE_m) \), we obtain \( p^*_r = \frac{1}{4}(x\beta + \frac{3+\beta}{1-\beta}E[a|f_m, f_r]) \). We can obtain both firms’ optimal profits by taking \( w^SE, p^SE_m \) and \( p^*_r \) into \( E[\pi^SE|f_m, f_r] \) and \( E[\pi^SE|f_m, f_r] \).

**Proof of Proposition 5.1.** (1) After calculation, the manufacturer’s and the retailer’s values from information sharing are given as:

\[
v^N_M(f_m, f_r) = E[\pi^SN_M|f_m, f_r] - E[\pi^SN_N|f_r] = \frac{1}{4}(E[a|f_m, f_r] + 3E[a|f_m])(E[a|f_m, f_r] - E[a|f_m]), \quad v^N_R(f_m, f_r) = E[\pi^SN_B|f_m, f_r] - E[\pi^SN_N|f_r] = \frac{1}{16}(3(E[a|f_m, f_r] - E[a|f_m])(E[a|f_m, f_r] - E[a|f_m, f_r])).
\]

\( v^N_M(f_m, f_r) \geq 0 \) only if \( E[a|f_m, f_r] \geq E[a|f_m] \), from the linear-expectation information structure assumption and the consensus model of Winkler (1981) [30] and Clemen and Winkler (1985) [5], we have \( E[a|f_m, f_r] \geq E[a|f_m] \leftrightarrow f_r \geq E[f_r|f_m] \).

Therefore, \( v^N_R(f_m, f_r) \geq 0 \) \( \iff \) \( E[a|f_m, f_r] \leq E[a|f_m] \) and \( E[a|f_m] \leq 3E[a|f_m, f_r] \leftrightarrow f_r \leq E[f_r|f_m] \).

(2) From our assumptions and the results of Mishra et al. [25], we have \( a = \theta + \beta \), \( E[f_r] = E[f_r] = \pi \) and \( f_m = a + \varepsilon, f^2_m = a^2 + 2a\varepsilon + \varepsilon^2 \). Thus,

\[
E[f^2_m] = E[a^2] + E[\varepsilon^2] = (\theta^2) + \sigma_0^2 + \sigma^2_m,
\]

(\text{A.6})

\[
E[f^2_r] = E[a^2] + E[\varepsilon^2] = (\theta^2) + \sigma_0^2 + \sigma^2_r,
\]

(A.7)

\[
E[f_m f_r] = (\theta^2) + \sigma^2_0 + \rho \sigma_m \sigma_r.
\]

(A.8)

In (A.6), \( E[a^2] = (E[a])^2 + \text{Var}(a) = (\theta^2) + \sigma_0^2 \), \( E[\varepsilon^2] = E^2(\varepsilon) + \text{Var}(\varepsilon) = \sigma^2_m \), and \( E[a \varepsilon_m] = E[a \varepsilon_r] = 0 \). This is the same for \( f_r \). In (A.8), \( E[\varepsilon_m \varepsilon_r] = \rho \sigma_m \sigma_r \) holds. We substitute (A.6)–(A.8), \( t_m, I, J \) and \( K \) into \( V^N_M(f_m, f_r) \) and \( V^N_R(f_m, f_r) \), and we have:

\[
V^N_M(f_m, f_r) = \frac{\sigma^4_0 \sigma^2_m (\sigma_m - \rho \sigma_r)^2}{8(\sigma_0^2 + \sigma^2_m)((1 - \rho^2)\sigma^2_m \sigma^2_r + \sigma^2_0 (\sigma^2_m - 2\rho \sigma_m \sigma_r + \sigma^2_r))} \geq 0,
\]

\[
V^N_R(f_m, f_r) = -\frac{3\sigma^4_0 \sigma^2_m (\sigma_m - \rho \sigma_r)^2}{16(\sigma_0^2 + \sigma^2_m)((1 - \rho^2)\sigma^2_m \sigma^2_r + \sigma^2_0 (\sigma^2_m - 2\rho \sigma_m \sigma_r + \sigma^2_r))} \leq 0,
\]

\[
V^N = V^N_M(f_m, f_r) + V^N_R(f_m, f_r) = -\frac{\sigma^4_0 \sigma^2_m (\sigma_m - \rho \sigma_r)^2}{16(\sigma_0^2 + \sigma^2_m)((1 - \rho^2)\sigma^2_m \sigma^2_r + \sigma^2_0 (\sigma^2_m - 2\rho \sigma_m \sigma_r + \sigma^2_r))} \leq 0.
\]

(3) The relationships between \( w^{NN} \) and \( w^{SN} \), \( p^{NN} \) and \( p^{SN} \), \( E[\pi^N_M|f_r] \) and \( E[\pi^N_R|f_m, f_r] \), \( E[\pi^N_M|f_m] \) and \( E[\pi^N_M|f_m, f_r] \) are easy to prove.

\( \square \)
The calculation process is similar to the case in which the manufacturer does not encroach. In addition, this can be proven by simple algebraic calculation. The calculation process is similar to the case in which the manufacturer does not encroach. In addition, the expected profit in scenario NN as an example to illustrate the derivation process.

between encroachment and no encroachment cases is given by $\Delta \pi_M = E[\pi^N_M|f_m] - E[\pi^E_M|f_m]$.

Proof of Lemma 5.3. If the retailer and the manufacturer do not share their forecasts information with each other, the manufacturer’s encroachment takes place if and only if the manufacturer’s expected payoff under encroachment is larger than that under non-encroachment. The difference of the manufacturer’s expected payoff under encroachment and non-encroachment is

$$\Delta \pi_M^N = E[\pi^N_M|f_m] - E[\pi^E_M|f_m] = \frac{(\beta^2 - 2)(\beta - 1)x^2 + 2(\beta - 1)(2E[a|f_m] + \beta E[a|f_m, f_r])x + 2(\beta + 1)(E[a|f_m]^2)}{8(1 - \beta)}.$$  

We mainly examine the numerator of $\Delta \pi_M^N$ because the denominator is always greater than 0. Note that the numerator is a quadratic function with respect to the per-unit selling cost $x$, the discriminant is $\Delta = 4\beta(\beta - 1)(-2(\beta^2 + \beta - 4)E(a|f_m)^2 + 4(\beta - 1)E(a|f_m)E(a|f_m, f_r) + \beta(\beta - 1)E(a|f_m, f_r)^2)$, and $\Delta > 0$ holds. Hence, $\Delta \pi_M^N \geq 0$ if and only if the per-unit direct selling cost $x$ falls into a range of $(0, x_1)$, where $x_1$ is the smaller root of the numerator. The manufacturer’s profit decreases as $x$ increases. Once the per-unit cost exceeds a threshold $x_1$, the manufacturer cannot benefit from his encroachment. According to the retailer’s optimal quantity is strictly positive, we have $x < \frac{E[\pi^N_M|f_m]}{\beta}$.

If the retailer and the manufacturer share their forecasts, the manufacturer’s expected payoff difference between encroachment and no encroachment cases is given by

$$\Delta \pi_M^N = E[\pi^N_M|f_m] - E[\pi^E_M|f_m] = \frac{(\beta^2 - 2)(\beta - 1)x^2 + 2(\beta - 1)(2E[a|f_m] + \beta E[a|f_m, f_r])x + 2(\beta + 1)(E[a|f_m]^2)}{8(1 - \beta)}.$$  

Because the numerator of $\Delta \pi_M^N$ is always greater than 0, this means that encroachment is always profitable for the manufacturer if they reach a forecast information sharing arrangement.

Proof of the results in Table 3. We substitute (A.6)–(A.8) and some related results into the expected payoffs of the retailer and the manufacturer in scenarios NN, NE, SN, and SE and take first-order partial derivatives of the expressions with respect to the parameters $\sigma_m$, $\sigma_r$, $\sigma_0$, and $\rho$. We take the manufacturer’s expected profit in scenario NN as an example to illustrate the derivation process.

The manufacturer’s expected profit is $E[\pi^N_M|f_m] = \frac{1}{8}(a^2 + \frac{\sigma_0^4}{\sigma_m^4 + \sigma_0^4})$, it is easy to show that $\frac{\partial E[\pi^N_M|f_m]}{\partial \sigma_m} = \frac{-2\sigma_m\sigma_0^4}{(\sigma_0^4 + \sigma_m^4)} \leq 0$, and $\frac{\partial E[\pi^N_M|f_m]}{\partial \sigma_0} = \frac{2(\sigma_0^6 + 2\sigma_0^2\sigma_m^2)}{(\sigma_0^4 + \sigma_m^4)} \geq 0$ hold. Obviously, $\frac{\partial E[\pi^N_M|f_m]}{\partial \beta} = \frac{\partial E[\pi^N_M|f_m]}{\partial \rho} = 0$ holds.
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