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PARTNER SELECTION IN BUSINESS MERGERS: A DATA ENVELOPMENT ANALYSIS APPROACH

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Abstract. Business mergers and partnerships could create opportunities for the decision making units (DMUs) involved to collectively enhance their efficiency. Estimating potential merger gains for a set of given merging DMUs using data envelopment analysis (DEA) and inverse DEA have been discussed in the literature. This paper develops new inverse DEA models for partner selection in a merger. The developed models extend the literature by finding optimal sets of partners that would maximize merger gains among a group of potential merging partners. The results of this study are useful to business managers seeking to merge to improve competitiveness. Data from the top US commercial banks is used to show the applicability of the proposed DEA models in this study.

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1. Introduction

The prospect of achieving synergies in partnerships is an important driver of value in the business world. Companies work together to achieve their strategic goals knowing that their combined worth is more valuable than the sum of the separate companies. Strategic alliances may allow companies to achieve optimal decision making when a partnership of two or more companies results in them selling more products and services in total than they would have otherwise achieved separately. Partnerships may also be an opportunity to reduce overall costs of doing business as companies seek to extract cost synergies [12].

In the financial sector, banks may partner with other banks to improve the customer experience, reduce operating expenses, expand their financial products and services, or reach new markets [6]. Banks may also collaborate with financial technology (fintech) companies to find new sources of non-interest income, and help banks offer services in a cost-efficient manner. The fintech companies, in return, benefit from the partnerships by scaling distribution faster and more cheaply than they could on their own.

Many studies in the literature have highlighted the importance of partner selection in a variety of business partnerships. The objectives of partner selection may include to maximize customer satisfaction, maximize revenue, or minimize the operations cost of business partners. Thus, partner selection problems require quantitative models to determine the partner selection options that optimizes partnership objectives. Prior studies have used

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a variety of methods to model partner selection including particle swarm optimization [18], systematic four-stage optimization model for supply chain [17], and a joint distribution optimization model [7].

This study focuses on partner selection in mergers – a special form of partnership that creates a merged entity – using data envelopment analysis (DEA) approach. DEA was successfully applied to estimate the gains from mergers and acquisitions for a given set of decision making units (DMUs) [1,2,4,5,8]. This paper extends the literature by identifying an optimal set of merging partners that would maximize the merger gains.

The literature on DEA and inverse DEA has shown the potential for inverse DEA models in solving difficult decision problems including the intricacies of potential merger gains and partnership gains in general. We introduce new inverse DEA models to identify optimal merging partners among a set of potential DMUs. These inverse DEA models advance the literature of DEA by finding the optimal set of merging partners that would maximize merger gains. We apply the proposed inverse DEA models to the US banking industry. The results show how DMUs could choose an optimal set of merging partners to attain a given efficiency target.

The rest of this paper is organized as follows: Section 2 provides a brief literature review of DEA and inverse DEA on partner selection. Section 3 develops new inverse DEA models to identify optimal merging partners among a set of potential DMUs. Application and data collection are given in Section 4. The section also gives the result and discussion. Section 5 concludes the paper.

2. Literature review

Research on partner selection by DMUs have relied on optimization models to determine the optimal partners. Wang *et al.* [15] studied a partner selection problem where DMUs have private information about their cost to undertake the tasks. For each possible cost combination, they proposed a multi-objective integer nonlinear programming model and an algorithm to obtain the optimal partners. Wu *et al.* [17] developed a systematic four-stage optimization model for constructing partner selection criteria. Their model integrates quantitative and qualitative methodologies to explore potential alternative criteria for partner selection in sustainable supply chains and ensured the best overall benefits to the supply chain, rather than pursuing the interests of only one company.

Zhao *et al.* [18] proposed the use of particle swarm optimization for an enterprise partner selection model with the aims of maximizing average customer satisfaction, maximizing customers' minimum satisfaction, and minimizing project completion cost. By transforming the multi-objective model into a single-objective model they were able to show that there were different influences on the optimal solution. Tavana *et al.* [13] presented a model for an integrated sustainable private partner selection framework in a public-private partnerships.

Cheng *et al.* [7] developed an optimization model for joint distribution partner selection with the objectives of minimizing the total operation cost and maximizing the supply-demand matching degree. Hlioui *et al.* [9] also introduced a joint supplier selection model to minimize total production and inventory costs. They used a combination of optimization, simulation, and mathematical formulation techniques to confirm significant cost savings under their proposed dynamic supplier selection policy model.

There are studies that have modelled partnership selection using DEA. Wu *et al.* [16] presented a cooperative DEA model by which a specific DMU can select its best cooperative partner, another DMU. The selection of the best partner depends on the complementarity between the input resources and the technology level for the two cooperative partners. Lozano [10] introduced a joint venture partner selection using DEA. Each potential partner has a different technology that complements others as the best fit for the strategic goal of a horizontal cooperation. The decision on a joint venture depends on the synergy between the technologies of the potential partners, the input cost structure and target output levels.

Companies often need to select suitable merging partners to work to create a merged entity that would best serve their customers and grow their collective market. Zhu *et al.* [19] proposed a DEA model to support decision making for best partner selection in mergers and acquisitions for DMUs. Their three proposed model were based on different perspectives of efficiency, revenue, and cost to select the best merging partner. Zhu *et al.* [20] combined DEA and big data theories to construct each company's production technology, which

could reflect the new production of the integrated company more precisely, and to select the best partner from different candidates for merger and acquisitions in banking industry.

Amin and Ibn Boamah [3] introduced DEA and inverse DEA models to guide partners in various types of strategic alliance collaborations and partnerships between different DMUs on how redistributing their inputs and outputs could enhance their performance. In this paper, we extend the literature on partner selection by introducing new inverse DEA models to identify optimal merging partners among a set of potential DMUs. The models in this study are the first known inverse DEA approach for optimizing partner selection involving more than two merging DMUs.

3. Partner selection using inverse DEA

Gattoufi *et al.* [8] introduced the following input-oriented inverse DEA model to estimate the minimum level of inputs allowing the merged entity to produce the outputs of the merging DMUs for a given efficiency target, $\bar{\theta}$.

$$
\begin{aligned}
\min\ & \sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il}) \\
\text{s.t.}\ & \\
& \sum_{j\in F}\lambda_j x_{ij}+(\alpha_{ik}+\alpha_{il})\lambda_M-(\alpha_{ik}+\alpha_{il})\bar{\theta}\le 0 && i=1,\cdots,m \\
& \sum_{j\in F}\lambda_j y_{rj}+(y_{rk}+y_{rl})\lambda_M\ge y_{rk}+y_{rl} && r=1,\cdots,s \\
& \sum_{j\in F}\lambda_j+\lambda_M=1 \\
& 0\le\alpha_{ik}\le x_{ik},\quad 0\le\alpha_{il}\le x_{il} && i=1,\cdots,m \\
& \lambda_j\ge 0,\quad j\in F,\quad \lambda_M\ge 0
\end{aligned}
\tag{1}
$$

where F is the set of post-merger DMUs, x_{ij} and y_{rj} are, respectively, the ith input and the rth output of the jth DMU, and α_{ik} and α_{il} represent, respectively, the levels of the ith input from the merging DMU_k and DMU_l that will be used by the merged entity, for each $i=1,\ldots,m$, $r=1,\ldots,s$, and $j=1,\ldots,n$. And λ_j and λ_M are the intensity decision variables for the jth DMU and the merger entity respectively. The quantity of inputs that can be saved from the pre-merger DMU_k and DMU_l are viewed as the potential merger gains. Amin and Ibn Boamah [1] modified the above inverse DEA model as shown below.

$$
\begin{aligned}
\min\ & \sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il}) \\
\text{s.t.}\ & \\
& \sum_{j\in F}\lambda_j x_{ij}+(x_{ik}+x_{il})\lambda_M-(\alpha_{ik}+\alpha_{il})\bar{\theta}\le 0 && i=1,\cdots,m \\
& \sum_{j\in F}\lambda_j y_{rj}+(y_{rk}+y_{rl})\lambda_M\ge y_{rk}+y_{rl} && r=1,\cdots,s \\
& \sum_{j\in F}\lambda_j+\lambda_M=1 \\
& 0\le\alpha_{ik}\le x_{ik},\quad 0\le\alpha_{il}\le x_{il} && i=1,\cdots,m
\end{aligned}
$$

$$\lambda_j \geq 0, \quad j \in F, \quad \lambda_M \geq 0. \tag{2}$$

The modification in model (2) ensures that the merged entity will be presented by itself when it cannot be written as a convex combination of the post-merger DMUs in F.

Following the work of Gattoufi *et al.* [8], the literature of DEA has been enriched by the studies that applied the inverse DEA for estimating merger gains. In all of these studies, the merging DMUs are given and certain. In this paper, we extend the literature to the merging of unknown DMUs when finding partners is part of the optimization.

Estimating potential merger gains using inverse DEA for a given set of merging DMUs has been discussed in the literature. Gattoufi *et al.* [8] was the first study in the literature that developed inverse DEA models for estimating merger gains. Amin *et al.* [4] defined a new concept called generalized restructuring that extends the concept of mergers and acquisitions. A generalized restructuring is defined as a move in a business market when a set of pre-restructuring firms combine their resources to create another set of post-restructuring firms given their efficiency targets.

In what follows, we develop inverse DEA models that can capture the gains of the merger by identification the optimal set of partners. To do this, let us denote P as a set of potential pre-partnership DMUs. For each $p \in P$, we define

$$z_p = \begin{cases} 1 & \text{if DMU } p \text{ is a partner} \\ 0 & \text{otherwise.} \end{cases}$$

To determine the optimal set of partners we extend the input-oriented inverse DEA model (2) as follows:

$$\begin{aligned}
\min \ & \sum_{i=1}^{m} \sum_{p \in P} \alpha_{ip} z_p \\
\text{s.t.} \ & \\
& \sum_{j \in F} \lambda_j x_{ij} + \sum_{p \in P} x_{ip} z_p \lambda_M - \sum_{p \in P} \alpha_{ip} z_p \bar{\theta} \leq 0 && i = 1, \cdots, m \\
& \sum_{j \in F} \lambda_j y_{rj} + \sum_{p \in P} y_{rp} z_p \lambda_M \geq \sum_{p \in P} y_{rp} z_p && r = 1, \cdots, s \\
& \sum_{j \in F} \lambda_j + \lambda_M = 1 \\
& \sum_{p \in P} z_p = t \\
& 0 \leq \alpha_{ip} \leq x_{ip} && i = 1, \cdots, m, \ p \in P \\
& \lambda_j \geq 0, \quad j \in F, \quad \lambda_M \geq 0, \quad z_p \in \{0, 1\} && p \in P
\end{aligned} \tag{3}$$

where t is a parameter satisfying $2 \leq t \leq |P| - 1$. The constraint $\sum_{p \in P} z_p = t$ is added to ensure that the number of merging partners is at least two and at most one unit less than the entire potential DMUs. As defined above, P is the set of all potential DMUs in which a merger is possible. It is worth noting that merging any set of DMUs may not be practically possible due to several limitations including physical distance, cultural difference, different size and scale of operations between DMUs. Model (3) is an extension of the inverse DEA model (2) simply because in model (2) the merging DMUs (k and l) are given and known prior to solving the model. In model (3), however, this is unknown and there is a set of potential DMUs instead and the objective is to find the best partners among them. The defined binary variables $z_p (p \in P)$ identify the optimal set of partners. For a given optimal solution of model (3), they are DMUs with $z_p^* = 1$ for some $p \in P$. It is easy

to verify that constraints in model (3) are an extended version of the constraints in model (2). For example, the term $(x_{ik} + x_{il})\lambda_M - (\alpha_{ik} + \alpha_{il})\bar{\theta}$ in the first set of constraints in model (2) is a special case of the term $\sum_{p\in P} x_{ip}z_p\lambda_M - \sum_{p\in P} \alpha_{ip}z_p\bar{\theta}$ in model (3) simply because in model (2) we know that DMUs k and l are the merging units while this is unknown in model (3) and the objective is to optimize it within a set of potential merging DMUs.

Model (3) is a nonlinear programming problem due to the existence of nonlinear terms $\alpha_{ip}z_p$ and $z_p\lambda_M$, for $i = 1, \ldots, m$, $p \in P$.

To linearize it, we define the following variable changes

$$\alpha_{ip}z_p = \delta_{ip} \;\&\; z_p\lambda_M = \mu_p \qquad \forall i = 1, \ldots, m,\ p \in P.$$

This would result in the following linear input-oriented inverse DEA model (4).

$$\begin{aligned}
\min\ & \sum_{i=1}^{m}\sum_{p\in P} \delta_{ip} \\
\text{s.t.}\ & \\
& \sum_{j\in F} \lambda_j x_{ij} + \sum_{p\in P} x_{ip}\mu_p - \sum_{p\in P} \delta_{ip}\bar{\theta} \le 0 && i = 1, \cdots, m \\
& \sum_{j\in F} \lambda_j y_{rj} + \sum_{p\in P} y_{rp}\mu_p - \sum_{p\in P} y_{rp}z_p \ge 0 && r = 1, \cdots, s \\
& \sum_{j\in F} \lambda_j + \lambda_M = 1 \\
& \sum_{p\in P} z_p = t \\
& 0 \le \delta_{ip} \le Nz_p && i = 1, \cdots, m,\ p \in P \\
& \alpha_{ip} - N(1 - z_p) \le \delta_{ip} \le \alpha_{ip} && i = 1, \cdots, m,\ p \in P \\
& 0 \le \mu_p \le Nz_p && p \in P \\
& \lambda_M - N(1 - z_p) \le \mu_p \le \lambda_M && p \in P \\
& 0 \le \alpha_{ip} \le x_{ip} && i = 1, \cdots, m,\ p \in P \\
& \lambda_j \ge 0, \quad j \in F, \quad \lambda_M \ge 0, \quad z_p \in \{0, 1\} && p \in P
\end{aligned} \tag{4}$$

where N is a large enough number. It is easy to verify that the objective function and the first two sets of constraints in model (4) are derived directly by substituting the above variable changes in model (3). Constraints $0 \le \delta_{ip} \le Nz_p$ and $0 \le \mu_p \le Nz_p$ in model (4) are written to ensure that when $z_p = 0$ then δ_{ip} and μ_p must be zero for each $i = 1, \cdots, m$, $p \in P$. In addition, the constraints $\alpha_{ip} - N(1 - z_p) \le \delta_{ip} \le \alpha_{ip}$ $(\lambda_M - N(1 - z_p) \le \mu_p \le \lambda_M)$ guarantee that when $z_p = 1$ then $\delta_{ip} = \alpha_{ip}$ $(\mu_p = \lambda_M)$.

A special case of the above model is the situation when the objective is to find the optimal partners for a given DMU_k. In this case, $z_k = 1$. Now, we develop inverse DEA models for determining the optimal merger partners under the output-orientation. Akin to the input-orientation, we extend the output-oriented inverse DEA model suggested in Gattoufi *et al.* [8]. Their model is replicated below.

$$\begin{aligned}
\max\ & \sum_{r=1}^{s} \beta_r \\
\text{s.t.}\ &
\end{aligned}$$

$$\sum_{j\in F}\lambda_j x_{ij} + (x_{ik}+x_{il})\lambda_M \le x_{ik}+x_{il} \qquad i=1,\cdots,m$$

$$\sum_{j\in F}\lambda_j y_{rj} + (y_{rk}+y_{rl}+\beta_r)\lambda_M \ge (y_{rk}+y_{rl}+\beta_r)\overline{h} \qquad r=1,\cdots,s$$

$$\sum_{j\in F}\lambda_j + \lambda_M = 1$$

$$\lambda_j \ge 0, \quad j\in F, \quad \lambda_M \ge 0, \quad \beta_r \ge 0, \qquad r=1,\ldots,s \tag{5}$$

where β_r is the rth additional output of the merged entity. What would be the optimal set of merger partners under the output orientation? The following inverse DEA model answers the question.

$$\max \sum_{r=1}^{s}\beta_r$$

s.t.

$$\sum_{j\in F}\lambda_j x_{ij} + \sum_{p\in P} x_{ip}z_p\lambda_M \le \sum_{p\in P} x_{ip}z_p \qquad i=1,\cdots,m$$

$$\sum_{j\in F}\lambda_j y_{rj} + \left(\sum_{p\in P} y_{rp}z_p + \beta_r\right)\lambda_M \ge \left(\sum_{p\in P} y_{rp}z_p + \beta_r\right)\overline{h} \qquad r=1,\cdots,s$$

$$\sum_{j\in F}\lambda_j + \lambda_M = 1$$

$$\sum_{p\in P} z_p = t$$

$$\lambda_j \ge 0, \quad j\in F, \quad \lambda_M \ge 0, \quad \beta_r \ge 0, \qquad r=1,\ldots,s, \quad z_p\in\{0,1\}, \quad p\in P. \tag{6}$$

The above model is certainly nonlinear and can be linearized by substituting the following

$$z_p\lambda_M = \pi_p, \beta_r\lambda_M = \gamma_r \qquad p\in P, \quad r=1,\ldots,s.$$

It should be noted that in determining the optimal set of merger partners under the output orientation, in model (6), we can assume that λ_M is a binary variable simply because the merged entity will be presented based on the efficient DMUs in F or by itself. As with all DEA merger gains' models, the former indicates $\lambda_M = 0$ and that the inefficient merged entity is on the current efficiency frontier. The latter means $\lambda_M = 1$ and that the merged entity is efficient outside the existing frontier, and this would be possible if and only if it is efficient with an efficiency target of one the merged entity could also be fully efficient when it falls on the current efficiency frontier. In this case, the DEA model has alternative optimal solutions consisting of $\lambda_M = 0$ and $\lambda_M = 1$ Hence, the partner selection optimization model under the output-oriented is as follows:

$$\max \sum_{r=1}^{s}\beta_r$$

s.t.

$$\sum_{j\in F}\lambda_j x_{ij} + \sum_{p\in P} x_{ip}\pi_p \le \sum_{p\in P} x_{ip}z_p \qquad i=1,\cdots,m$$

$$\sum_{j\in F}\lambda_j y_{rj} + \sum_{p\in P} y_{rp}\pi_p + \gamma_r \ge \left(\sum_{p\in P} y_{rp}z_p + \beta_r\right)\overline{h} \qquad r=1,\cdots,s$$

$$
\begin{aligned}
&\sum_{j\in F}\lambda_j+\lambda_M=1\\
&\sum_{p\in P}z_p=t\\
&0\le\pi_p\le Kz_p && p\in P\\
&\lambda_M-K(1-z_p)\le\pi_p\le\lambda_M && p\in P\\
&0\le\gamma_r\le K\lambda_M && r=1,\dots,s\\
&\beta_r-K(1-\lambda_M)\le\gamma_r\le\beta_r && r=1,\dots,s\\
&\lambda_j\ge 0,\quad j\in F,\quad \lambda_M\in\{0,1\},\quad \beta_r\ge 0, && r=1,\dots,s,\quad \pi_p\ \&\ z_p\in\{0,1\},\quad p\in P. \qquad (7)
\end{aligned}
$$

Model (7) obtains the optimal set of partners when the merger gains is to maximize the additional production of outputs.

4. Application

The top five commercial banks, including Wells Fargo, JPMorgan Chase, Bank of America, Goldman Sachs, and Citigroup control over 40% of the assets held in the US banking sector. However, the top banks operate in a competitive market with over 4700 Federal Deposit Insurance Corporation (FDIC) insured commercial banks.

Partnerships including mergers among commercial banks, and with investment banks and insurance companies are increasing common as a result banks have grown large and complex in an effort to increase efficiency, and to provide a wide range of financial products. Commercial banks focus on strategic use of their assets in order to expand the products and services they offer, to access markets in new geographic locations, and to be operational efficient.

Many banks have large information technology platforms that need to be updated and maintained to conform to changing regulations, to support growth and cost optimization initiatives. Banks also need to enable integration with emerging technologies at scale. A partnership may be required to shift from the maintenance of legacy platform systems to building the capability to run a scalable agile business model and the ability to integrate new financial technologies. A partnership allows banks to offer their customers useful tools, features, and financial products that they wouldn't be able to provide without a significant investment.

Automation is a big money-saver for businesses in the long term, but overhauling existing infrastructure, developing the technology, training and additional staffing can quickly eat away the profit margins of small and regional banks over the short term. Banks have several options when it comes to updating their technology, but not without friction. Creating products and features from scratch takes time and resources and making massive updates to infrastructure may reduce operating costs, but the big initial investment poses a potential risk. A partnership with other banks could provide the solutions required without the potential risk associated with a massive initial investment upfront.

Banks in the United States can form partnerships that enable lenders to originate loans through their bank partners and avoid having to comply with usury and licensing laws in every state in which they operate. These partnerships enable banks to reach market segments which would otherwise be impractical or uneconomic for them to engage in. This collaboration can bring exponential growth in deposits, loans, interest, and non-interest income through innovative partnerships and mergers while growing the customer base and improving customer engagement.

This paper develops inverse DEA models to determine the optimal set of partners between merging DMUs. As a nonparametric optimization approach that incorporates multiple inputs and multiple outputs at the same time, DEA is an appropriate method for this purpose. An application of the proposed models in the US banking sector highlights optimal merging partners for different efficiency levels.

We apply the models to 2018 consolidated financial statements data of 51 US commercial banks consisting of three inputs and three outputs. The inputs and outputs in the data that was obtained from the FDIC database,

and it is based on the intermediation role that banks play in the economy. Interest expenses, labor cost, and the costs of equipment and premises are inputs necessary to obtain and transform borrowed funds in to earning assets such and securities and loans [2]. Bank loans, investments such as securities, and cash deposits with other depository institutions are considered traditional bank outputs [8,11,14].

This study uses interest expenses (X_1), labor cost (X_2), and costs of equipment and premises (X_3) as inputs in the novel inverse DEA models to determine the optimal set of partners that would maximise merger gains. Cash deposits including balances with other banks (Y_1), securities (Y_2), and loans (Y_3) are outputs. The data is shown in Table 1.

In Table 2, we illustrate the applicability of input-oriented inverse DEA model (4). We assume the potential set of DMUs consists of five inefficient banks listed in $P = \{B_{17}, B_{28}, B_{32}, B_{46}, B_{50}\}$. The following table presents the optimal merging partners for different numbers of DMUs and efficiency targets.

The first row in Table 2 shows how pre-merger inefficient banks B_{46} and B_{50} operating at 59.50% and 75.73% efficiencies respectively in Table 1 could be the merging partners. The two banks are the optimal merging partners at 90% efficiency target based on the input-oriented inverse DEA model (4). Efficiency target could be increased to 95% and 100% as indicated in rows 3 and 4 respectively as the merged entity improves the synergy between the two merging partners by minimizing the use of joint inputs such as interest expenses, labor cost, and costs of equipment and premises. There is no solution to the model for merging partners involving any three banks, out of the set of five inefficient banks, at 90% and 95% efficient targets.

Now, we apply the merging partner selection optimization model (7) with the objective of finding partners to maximize the additional production of outputs. And without loss of generality, we assume the potential set of merging partner DMUs consist of four inefficient banks listed in $P = \{B_{27}, B_{35}, B_{44}, B_{48}\}$. The following table presents the optimal partners for different sizes of partnership as well as various efficiency targets.

Table 3 presents the optimal set of potential merging partners identified under the output-oriented inverse DEA model (7). It is insightful that the only feasible scenario is the case when the efficiency target is 1.00 or the merged entity is fully efficient. The results in Table 3 highlight a scenario when the merged entity is efficient outside the existing frontier, and it would be possible if and only if it is efficient with an efficiency target of 1.00 or 100%. Model (7) is infeasible for all other efficiency targets. At 100% efficiency target, pre-merger inefficient banks B_{27} and B_{48} operating at 75.89% and 80.48% efficiencies respectively in Table 1 are potential optimal merging partners. As shown, the optimal set of three partners are B_{27}, B_{44}, B_{48}.

It is worth to note that the existing partner selection studies in the DEA literature, such as Zhu *et al.* [19,20], are different from the approach in this paper as they are all based on DEA and hence unable to solve the inverse problems that we discussed in this paper.

5. Conclusion

This paper addressed partner selection for a set of potential merging DMUs using inverse DEA. Mergers and acquisitions create opportunities for the DMUs to enhance the efficiency of the merged entity. The novel inverse DEA models introduced in this paper enable DMUs to find the optimal set of partners that would maximize merger gains.

Banks often seek to expand their asset size by merging with other banks to maintain and improve their operational efficiency in the financial landscape that is constantly evolving. The results show the applicability of the proposed novel inverse DEA models to top US commercial banks' dataset. The models highlight how DMUs could determine the optimal merging partners for different efficiency levels in both input and output-oriented models. The paper confirms that pre-merger inefficient banks could be optimal merging partners, and that the merged entity could also improve its efficiency target.

The results from both the input and output-oriented inverse DEA models both have huge managerial implications as banks seek to improve their competitiveness through mergers. Bank managers want to determine the optimal set of partners that would maintain legacy platform systems, build, and integrate new financial features such as online banking to new reach market segments and improve customer engagement. Innovative

TABLE 1. Data for 51 US Banks and their efficiencies (Year 2018).

DMUs	X_1	X_2	X_3	Y_1	Y_2	Y_3	θ_k^*	h_k^*
B_{01}	12 030 000	26 053 000	6 574 000	321 590 000	260 071 000	874 492 000	1.0000	1.0000
B_{02}	6 797 000	17 438 000	4 667 000	159 983 000	418 582 000	930 337 000	1.0000	1.0000
B_{03}	9 433 000	24 857 000	4 847 000	171 910 000	399 795 000	922 214 000	0.9829	1.0041
B_{04}	13 486 000	14 843 000	1 821 000	179 696 000	323 535 000	642 173 000	1.0000	1.0000
B_{05}	2 906 278	7 143 995	1 049 107	21 369 509	111 246 751	284 800 984	1.0000	1.0000
B_{06}	2 694 444	5 251 503	1 299 328	16 477 644	83 180 647	224 541 765	0.6793	1.3912
B_{07}	2 674 738	5 599 066	1 238 290	63 273 007	66 710 323	145 275 378	0.7210	1.3252
B_{08}	2 254 869	2 391 864	724 736	12 493 366	118 413 161	147 169 695	1.0000	1.0000
B_{09}	1 822 000	3 902 000	734 000	84 268 000	116 053 000	26 039 000	1.0000	1.0000
B_{10}	716 282	4 392 756	1 609 084	75 992 027	86 823 543	25 850 933	1.0000	1.0000
B_{11}	1 130 000	3 824 000	732 000	3 611 000	45 833 000	143 259 000	0.8051	1.2053
B_{12}	1 021 835	2 834 754	531 510	5 759 223	31 414 275	151 773 966	1.0000	1.0000
B_{13}	3 065 000	408 000	49 000	30 522 000	2 936 000	75 981 000	1.0000	1.0000
B_{14}	2 134 927	827 474	195 004	16 895 877	46 229 666	66 651 806	0.4188	1.3966
B_{15}	2 355 000	969 000	57 000	4 091 000	22 558 000	120 065 000	1.0000	1.0000
B_{16}	874 000	75 000	17 000	14 999 000	56 740 000	57 972 000	1.0000	1.0000
B_{17}	833 984	2 132 425	415 034	4 510 217	32 304 735	94 709 936	0.7314	1.2801
B_{18}	806 063	2 007 746	396 230	3 295 262	30 936 413	90 940 461	0.7234	1.2915
B_{19}	693 721	2 044 429	304 313	38 913 888	51 197 684	32 377 459	1.0000	1.0000
B_{20}	826 942	2 392 684	342 382	8 213 749	26 942 302	85 234 124	0.6844	1.3387
B_{21}	1 004 168	1 617 353	493 556	4 318 567	12 979 263	98 699 167	0.7051	1.3252
B_{22}	478 939	1 880 615	463 911	3 467 026	24 619 079	82 616 342	0.9437	1.0436
B_{23}	860 888	1 406 981	332 788	13 690 820	17 973 105	80 161 870	0.6679	1.3697
B_{24}	2 767 499	196 097	177 247	2 958 018	15 679 177	95 246 115	1.0000	1.0000
B_{25}	489 153	1 633 131	286 837	9 641 162	12 068 166	87 281 714	1.0000	1.0000
B_{26}	1 948 393	74 089	22 392	20 452 309	1 955 074	91 694 824	1.0000	1.0000
B_{27}	654 536	1 437 922	323 041	2 592 121	22 348 994	74 874 375	0.7207	1.3177
B_{28}	1 962 672	1 058 471	52 803	14 811 896	3 475 449	87 459 200	0.8028	1.1382
B_{29}	530 498	1 122 359	200 962	2 811 159	16 234 808	75 525 219	0.9097	1.0740
B_{30}	648 660	1 040 482	223 393	3 097 780	13 634 835	64 370 078	0.6472	1.4120
B_{31}	688 114	1 088 034	240 598	4 488 941	12 960 589	61 953 747	0.5830	1.5366
B_{32}	455 204	917 125	360 462	4 381 225	13 006 251	51 152 423	0.6501	1.3990
B_{33}	413 000	138 000	27 000	7 223 000	12 503 000	54 765 000	1.0000	1.0000
B_{34}	250 928	967 129	184 088	4 514 874	12 087 410	49 495 150	0.9115	1.0718
B_{35}	250 398	1 070 095	221 349	1 231 922	15 511 046	46 312 248	0.8024	1.1777
B_{36}	43 884	689 880	63 640	3 408 468	23 077 746	28 057 377	1.0000	1.0000
B_{37}	99 028	787 196	157 252	2 658 890	9 934 276	35 463 205	1.0000	1.0000
B_{38}	410 010	307 404	48 832	432 784	9 032 345	36 678 427	0.7109	1.5425
B_{39}	557 418	475 005	47 617	1 412 929	5 967 721	27 828 008	0.5614	2.2447
B_{40}	258 707	375 189	69 824	3 282 377	2 773 056	32 144 142	0.8557	1.3602
B_{41}	294 276	641 672	123 855	2 055 884	4 649 921	28 034 702	0.5919	1.8481
B_{42}	183 000	182 000	28 000	17 588 000	5 000	10 210 000	1.0000	1.0000
B_{43}	31 972	645 894	154 809	1 131 893	6 735 308	25 346 591	1.0000	1.0000
B_{44}	233 451	514 566	91 005	1 256 897	9 114 602	18 637 162	0.6599	2.2894
B_{45}	268 688	482 676	82 363	729 617	6 689 021	22 781 670	0.6642	2.0450
B_{46}	519 103	359 248	44 772	5 514 050	100 813	25 840 618	0.5950	2.0454
B_{47}	219 180	349 895	110 539	493 838	6 595 405	21 974 970	0.8447	1.5830
B_{48}	171 946	418 399	128 227	1 118 529	3 994 800	25 734 653	0.9213	1.2425
B_{49}	85 186	433 798	111 751	3 317 919	12 450 688	13 967 601	1.0000	1.0000
B_{50}	378 901	225 266	70 767	382 072	8 106 359	21 904 069	0.7573	2.1815
B_{51}	291 019	333 816	108 763	428 629	3 817 790	24 918 765	0.7522	1.8539

Table 2. Optimal merging partners (Model 4: Input Orientation).

Efficiency target ($\bar{\theta}$)	Size (t)	Partners
0.90	2	B_{46}, B_{50}
0.90	3	Infeasible
0.95	2	B_{46}, B_{50}
0.95	3	Infeasible
1.00	2	B_{46}, B_{50}

Table 3. Optimal merging partners (Model 7: Output Orientation).

Efficiency target ($\frac{1}{h}$)	Size (t)	Partners
0.85	2	Infeasible
0.85	3	Infeasible
0.95	2	Infeasible
0.95	3	Infeasible
1.00	2	B_{27}, B_{48}
1.00	3	B_{27}, B_{44}, B_{48}

partnerships and mergers bring growth and variety in financial products and services while minimizing the use of joint inputs including labor cost and the costs of equipment and technology.

Further research could extend the proposed models by incorporating the heterogeneity in the DMUs. In the banking sector, heterogeneity is important as differences in business operations, financial products and services, bank exposure in various markets and industries could be sources of inefficiency. The model in this paper proposes identifying optimal merging partners from a set of potential DMUs. The limit on the set of potential DMUs and its impact on the choice of optimal merging partners could be relaxed in future research.

References

[1] G.R. Amin and M. Ibn Boamah, A new inverse DEA cost efficiency model for estimating potential merger gains: a case of Canadian banks. *Ann. Oper. Res.* **295** (2020) 21–36.

[2] G.R. Amin and M. Ibn Boamah, A two-stage inverse data envelopment analysis approach for estimating potential merger gains in the US banking sector. *Manage. Decis. Econ.* **42** (2021) 1454–1465.

[3] G.R. Amin and M. Ibn Boamah, Modeling business partnerships: a data envelopment analysis approach. *Eur. J. Oper. Res.* **305** (2023) 329–337.

[4] G.R. Amin, A. Emrouznejad and S. Gattoufi, Modelling generalized firms' restructuring using inverse DEA. *J. Prod. Anal.* **48** (2017) 51–61.

[5] G.R. Amin, S. Al-Muharrami and M. Toloo, A combined goal programming and inverse DEA method for target setting in mergers. *Expert Syst. App.* **115** (2019) 412–417.

[6] G. Buchak, G. Matvos, T. Piskorski and A. Seru, Fintech, regulatory arbitrage, and the rise of shadow banks. *J. Finan. Econ.* **130** (2018) 453–483.

[7] Y. Cheng, Y. Li and Y. Li, An optimization model of partner selection of joint distribution alliance considering the supply and demand matching degree. *IEEE Access* **7** (2019) 162454–162464.

[8] S. Gattoufi, G.R. Amin and A. Emrouznejad, A new inverse DEA method for merging banks. *IMA J. Manage. Math.* **25** (2014) 73–87.

[9] R. Hlioui, A. Gharbi and A. Hajji, Joint supplier selection, production and replenishment of an unreliable manufacturing-oriented supply chain. *Int. J. Prod. Econ.* **187** (2017) 53–67.

[10] S. Lozano, Using DEA to find the best partner for a horizontal cooperation. *Comput. Ind. Eng.* **66** (2013) 286–292.

[11] F. Niepmann and T. Schmidt-Eisenlohr, International trade, risk and the role of banks. *J. Int. Econ.* **107** (2017) 111–126.

[12] D.T. Robinson, Strategic alliances and the boundaries of the firm. *Rev. Finan. Stud.* **21** (2008) 649–681.

[13] M. Tavana, A.K. Nasr, H. Mina and J. Michnik, A private sustainable partner selection model for green public-private partnerships and regional economic development. *Soc.-Econ. Planning Sci.* **83** (2022) 101189.

[14] K. Wang, W. Huang, J. Wu and Y. Liu, Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. *Omega* **44** (2014) 5–20.

[15] C. Wang, R. Wu and L. Deng, An incentive mechanism for partner selection from a collaborative network with private information. *Comput. Ind. Eng.* **154** (2021) 107053.

[16] J. Wu, Q. Zhu, W. Cook and J. Zhu, Best cooperative partner selection and input resource reallocation using DEA. *J. Oper. Res. Soc.* **67** (2016) 1221–1237.

[17] C. Wu, Y. Zhang, H. Pun and C. Lin, Construction of partner selection criteria in sustainable supply chains: a systematic optimization model. *Expert Syst. App.* **158** (2020) 113643.

[18] J. Zhao, X. Deng, Y. Lin and J. Lu, Multi-objective enterprise partner selection model with different relative superiority parameters based on particle swarm optimization, in Data Processing Techniques and Applications for Cyber-Physical Systems (DPTA 2019). *Advances in Intelligent Systems and Computing*, edited by C. Huang, Y.W. Chan and N. Yen. Springer, Singapore (2020) 1088.

[19] Q. Zhu, J. Wu, J. Chu, A. Amirteimoori and J. Sun, DEA-based models for best partner selection for merger. *RAIRO Oper. Res.* **51** (2017) 1345–1357.

[20] Q. Zhu, X. Li, F. Li and A. Amirteimoori, Data-driven approach to find the best partner for merger and acquisitions in banking industry. *Ind. Manage. Data Syst.* **121** (2021) 879–893.