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OPTIMAL STRATEGIES FOR SUPPLY CHAIN WITH CREDIT GUARANTEE USING CVAR

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Abstract. In this paper, we explore optimal strategies of a supply chain consisting of one manufacturer, one capital-constrained retailer and one bank, where the bank provides loans to the retailer due to credit guarantees. However, there are default risks if the retailer can not repay the loans. Using conditional value-at-risk (CVaR) to describe risk aversion of the retailer, optimal order quantities of the retailer and optimal wholesale prices of the manufacturer are obtained by solving a Stackelberg game model, where the manufacturer is a leader and the retailer is a follower, respectively. Our numerical results show that the default probability of the retailer are proportional to optimal wholesale prices of the manufacturer. It implies that when the default probability of the retailer is high, the manufacturer should reduce the default risk by setting higher wholesale prices to avoid a burden of a substantial guarantee. Thus, our results can serve as insights for decision-makers of supply chain management.

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1. Introduction

A credit guarantee (CG) is a financial tool in supply chains (see [34]), where a manufacturer provides a guarantee to a capital-constrained retailer who takes loans from a bank. Moreover, the manufacturer has a responsibility for repaying a portion of the loans if the retailer defaults. The credit guarantee can effectively reduce financial costs of Small and Medium-sized Enterprises (SMEs). For example, Hengfeng Bank has used credit guarantees to provide commercial loans to retailers in machinery and equipment industries located at Ningbo city in China (see [33]).

However, there are default risks from SMEs as special retailers when the credit guarantee is used as a financial contract between manufacturers and retailers in supply chains. Thus, we consider some problems: (1) How do members of supply chains make optimal decisions under a retailer-guaranteed financing model. (2) How do manufacturers make trade-offs between risks and benefits and determine their wholesale prices. (3) How do default risks of retailers affect strategies and profits of supply chains. Therefore, we study an equilibrium

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decision problem of supply chains with default risks of retailers. A capital-constrained retailer chooses a credit guarantee as a financing option for a supply chain consisting of one manufacturer, one retailer and one bank. By applying conditional value-at-risk to represent expected loss of the manufacturer, we examine an effect of credit guarantees on both production and ordering strategies under default risk and operational efficiencies of supply chain.

Thus, this study contributes to the body of research in three different aspects. First, we use credit guarantees as a combination of bank credit financing (BCF) and trade credit financing (TCF) to explore financial strategies of supply chain. Second, we consider default risk when the retailer and the manufacturer take on as part of finance activities of supply chain. Finally, this paper incorporates the default probability into decision-making to explore the impact of default risk on supply chain financing decisions.

The paper is organized as follows. Section 2 reviews related literature, Section 3 introduces assumptions and models, Section 4 shows equilibrium strategies under default risk, and Section 5 examines numerical simulations. Finally, Section 6 summarizes our results.

2. Literature review

In the field of supply chain management, capital constraints have a profound impact on decisions and operations of firms such as production and inventory decisions, debt management and capacity planning. Buzacott and Zhang [1] incorporated capital constraints into operating decisions in a single-period newsvendor model with production and financing decisions. Similarly, Dada and Hu [6] proposed a capital constrained newsboy model where supply chain achieve a partial coordination.

Supply chain finance (SCF) (see [9]) is a solution to capital constraints problems of SMEs. Based on channels of loan acquisitions, SCF has two kinds of financing agreements including both internal financing agreements (*i.e.* TCF) and external financing agreements (*i.e.* BCF) (see [3, 13, 17, 32]). For example, Hua *et al.* [12] considered how capital-constrained retailers deal with uncertain demand through option contracts and obtain loans from banks or trade credit from suppliers. Shen *et al.* [26] explored an impact of BCF and TCF on performance of supply chain when manufacturers are capital constrained. They found that retail competition in supply chain had a significant impact on financing decisions, while different financing contracts affected profit distributions among parties of supply chain. Zheng *et al.* [39] considered capital constraints of retailers in re-manufacture supply chains, and explored an impact of market uncertainties on financing decisions. They found that TCF could derive re-manufacturers and retailers to increase product recalls and ordering with a change of uncertainty.

Furthermore, some papers assumed that some enterprises with enough capital in supply chain provided a credit guarantee to capital-constrained SMEs in order to obtain some loans from banks. For example, Yan *et al.* [33] showed that a partial credit guarantee (PCG) contract in a supply chain finance system maximized benefits of firms, induced channel coordination and achieved hyper-coordination. Li *et al.* [18] extended the study of Yan *et al.* [33] and explored impacts of different risk preferences on optimal decisions using TCF. Lu *et al.* [22] examined two credit guarantee contracts by suppliers as well as other parties. Yan *et al.* [35] investigated optimal strategies and joint contracts in the presence of default costs under credit guarantees. Li *et al.* [19] considered that downstream buyers provided credit guarantees for cash-strapped suppliers.

However, the credit default is an important risk in SCF. Shi and Zhang [27] studied optimal trade credit decisions under an extended structure consisting of constant demand rate with default risk. From the perspective of sellers and taking default risk into consideration, Lou and Wang [21] determined the optimal trade credit and order quantities. Kouvelis and Zhao [15] explored how to design an optimal financial contract for supply chain coordination where manufacturers and retailers are capital-constrained. Wu *et al.* [30] described a manufacturer–retailer noncooperative replenishment system with credit risk. Devalkar and Krishnan [7] focused on the role of TCF in addressing the moral hazard suppliers and coordinating supply chain. Tsao [29] discussed how to implement big data analysis to mitigate default risk in supply chain. Xie *et al.* [31] examined that order

TABLE 1. Literature review.

Authors	SCF contract	Risk measuring	Default risk
Zhang *et al.* [38]	TCF	MV	✓
Chen *et al.* [3]	TCF	CVaR	✓
Zhang *et al.* [37]	BCF	Utility Function	NA
Kouvelis and Zhao [15]	BCF	NA	✓
Peng and Pang [24]	BCF	NA	NA
Shen *et al.* [26]	BCF+TCF	NA	NA
Yan *et al.* [34]	CG	CVaR	NA
Yang *et al.* [36]	BCF+TCF	CVaR	NA
This study	CG	CVaR	✓

Notes. NA means that risk measure is not used.

quantities of dual-channel retailers under the assumption that default risk occurs, and the influence of the default probability on the decision with credit guarantees.

In addition, there exist uncertainties in supply chain management such as uncertainties of market demand, fluctuations of sales price, and so on. Therefore, it is necessary to consider a risk aversion behaviour of members in supply chain. Risk measures for some risk in supply chain include variance [5,38], Value-at-Risk (VaR) [4,8,28] and CVaR [10,11]. Gotoh and Takano [10] used the CVaR to minimize expected costs in the single-cycle kiddie reporting problem. Ma *et al.* [23] used the CVaR to examine profit distribution problems where retailers are risk-averse. Yang *et al.* [36] investigated an equilibrium of two different portfolio financing schemes in the supply chain in which the risk aversion of retailers are represented by CVaR criterion. They showed that both suppliers and retailers obtained Pareto strategies. Liu *et al.* [20] characterized the order quantity decision of retailers under the CVaR criterion. Kang *et al.* [14] found the risk aversion of suppliers was a significant aspect influencing financing decisions using CVaR.

The most relevant studies to our work in this paper are papers of Yan *et al.* [33] and Chen *et al.* [3]. However, Yan *et al.* [33] concentrated on an effect of various guarantee coefficients on optimal decisions of each member in supply chain. Different from them, the objective of this paper is to evaluate features of credit guarantees in addressing credit default risk and associated productions and ordering decisions. Moreover, using the newsvendor model and taking default risk into account, Chen *et al.* [3] explored the system of quantity discount contracts with reference to TCF. Motivated by this study, we consider the credit guarantee financing model that combines BCF and TCF and develop the work of Yan *et al.* [33] by illustrating an impact the default risk of retailers on performances of manufacturers. Table 1 presents some related literature.

3. Model description and assumptions

This paper considers a capital-constrained retailer with risk-neutral preference. Meanwhile, a manufacturer provides the retailer with a credit guarantee and a bank provides commercial loans. Thus, a two-stage Stackelberg game model is constructed with the manufacturer as a leader and the retailer as a follower. The risk-averse manufacturer and the risk-neutral retailer play the game to decide wholesale prices and order quantities of products, respectively. All of supply chain members play the dynamic game with complete information. The sequence of events is presented as Figure 1.

To describe the model more clearly, some notations are defined in Table 2.

Let x be a random market demand with a probability density function $f(x)$ and a distribution function $F(x)$ that satisfies increasing failure rate (IFR). Define $h(x) = f(x)/\overline{F}(x)$ where $\overline{F}(x) = 1 - F(x)$ and denotes $H(x) = xh(x)$ as a generalized lapse rate. These assumptions are usually used to solve optimal solutions for supply chain models (see [2,16]).

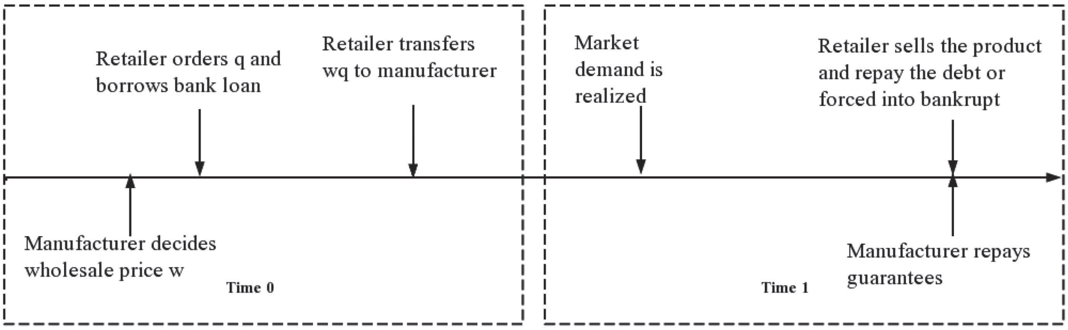

FIGURE 1. The sequence of events.

TABLE 2. Notations.

q	Retailer's order quantity
ω	Manufacturer's wholesale price
c	Unit production cost of manufacturer
p	Unit retail price
r	Bank's interest rate
x	Market demand
k_r	Market demand thresholds in the absence of bankruptcy
λ	Credit guarantee ratio
β	Risk aversion parameter
α	Default probability
π	Expected profit

We do not consider product ending salvage value, inventory costs and out-of-stock losses (see [13,34]). To ensure the economic consistency of the model and the available profit of retailers, the condition $c(1+r) < \omega(1+r) < p$ should be satisfied. This condition is made to ensure the existence and uniqueness of the solution to the Stackelberg game model (see [34]).

4. Equilibrium analysis

4.1. Optimal order quantities of retailers

At the beginning of sale products, the manufacturer firstly gives a wholesale price ω, then the retailer chooses order quantity q and borrows loans ωq from the bank under the manufacturer's a credit guarantee contract in which the bank and the manufacturer agree on a credit guarantee ratio $\lambda(0 \leq \lambda \leq 1)$. At the end of the period, the retailer earns a profit $p\min\{x,q\}$. If the profit is larger than $\omega q(1+r)$, then the retailer can pay the principal and interest of loan to the bank, otherwise the retailer should pay $p\min\{x,q\}$ to the bank and declare her bankruptcy. Moreover, the manufacturer can pay $\lambda[\omega q(1+r) - p\min\{x,q\}]$ to the bank. Therefore, the expected profit of the retailer can be shown as in equation (1).

$$\max_q E(\pi_r) = E(p\min\{x,q\} - \min\{\omega q(1+r), p\min\{x,q\}\})^+ \tag{1}$$

where the symbol $(\,\cdot\,)^+ = \max\{\cdot, 0\}$.

Lemma 4.1. *The threshold of market demand for the capital-constrained retailer without bankruptcy risk is* $k_r = \frac{\omega q(1+r)}{p}$.

The proof of Lemma 4.1 and the other propositions and corollaries are in Appendix A.

From Lemma 4.1, if the market demand is lower than the threshold value k_r, then the retailer will have a large inventory of goods backlog and less revenue, and can not pay back the principal and interest of the loan. Thus, the retailer must declare bankruptcy, and transfers its bankruptcy risk to the manufacturer due to credit guarantees. In addition, the larger the order quantity of the retailer is, the larger the market demand threshold is, which indicates that the size of order quantity of the retailer determines the profitability of the retailer.

Proposition 4.2. *When the market random demand satisfies an IFR condition, the optimal order quantity of the retailer is* $q^* = \overline{F}^{-1}\left(\frac{\omega(1+r)\overline{F}(k_r)}{p}\right)$.

Proposition 4.2 shows that the optimal order quantity of the retailer is correlated with product market prices and risk-free interest and wholesale prices of the manufacturer, respectively.

Corollary 4.3. *The optimal order quantity of the retailer is negative with the wholesale price of the manufacturer, that is,* $\frac{\mathrm{d}q^*}{\mathrm{d}\omega} < 0$.

Corollary 4.3 shows the wholesale price of the manufacturer directly affects the order quantity of the retailer. Therefore, the manufacturer can reduce the bankruptcy risk of the retailer by choosing an appropriate wholesale price by maximizing his profit.

Corollary 4.4. *The threshold of the market demand is negative with the wholesale price of the manufacturer, that is,* $\frac{\mathrm{d}k_r}{\mathrm{d}\omega} < 0$.

Corollary 4.4 shows that if the wholesale price increases, then the threshold of the market demand decreases.

4.2. Optimal wholesale prices of risk aversion manufacturers

4.2.1. Benchmark models without default risk

When the manufacturer is the core firm in the supply chain with sufficient capital to provide a credit guarantee with the bank to the capital-constrained retailer, the manufacturer needs to determine the credit guarantee ratio λ and the wholesale price ω. If the retailer fails to repay its debt to the bank, then the manufacturer must assume a certain percentage of the remaining debt $\lambda[\omega q^*(1+r) - p\min\{x, q^*\}]$. Thus, the profit function of the manufacturer under a credit guarantee can be expressed as follows

$$\pi_m = (\omega - c)q^* - \lambda[\omega q^*(1+r) - p\min\{x, q^*\}]^+. \tag{2}$$

In order to determine the wholesale prices of the manufacturer, the CVaR is used to measure expected losses. Let $g(\omega, \alpha) = P\{L(\omega, \varphi) \leq \alpha\}$, where $L(\omega, \varphi) = -\pi_m$ is a loss function.

Define $\mathrm{VaR}_\beta(\omega) = \min\{\alpha \in R : g(\omega, \alpha) \geq \beta\}$ be a Value-at-Risk, where β is a given confidence level. Thus, the CVaR of the manufacturer is

$$\mathrm{CVaR}_\beta(\omega) = \frac{1}{1-\beta} \int\limits_{L(\omega,\varphi)\geq \mathrm{V}aR_\beta(\omega)} L(\omega, \varphi) f(\varphi)\,\mathrm{d}\varphi.$$

Moreover, the manufacturer can choose an optimal wholesale price using minimizing CVaR, that is,

$$\min_\omega \mathrm{CVaR}_\beta(\omega).$$

To calculate CVaR, an auxiliary function is defined as follows (see [3, 25]):

$$F_\beta(\omega, \varepsilon) = \varepsilon + \frac{1}{1-\beta} E\Big[(L(\omega, \varphi) - \varepsilon)^+\Big]. \tag{3}$$

Thus, the optimization problem of the manufacture is presented as follows

$$\min_{\omega} \mathrm{CVaR}_{\beta}(\omega) = \min_{(\omega,\varepsilon)} F_{\beta}(\omega,\varepsilon) = \min_{(\omega,\varepsilon)} \left\{ \varepsilon + \frac{1}{1-\beta} \mathrm{E}\Big[(-\pi_m - \varepsilon)^+ \Big] \right\} \tag{4}$$

where $F_{\beta}(\omega, \varepsilon)$ is a continuous differentiable convex function with respect to the a real number ε reflecting a specific profit level of the manufacturer, $\beta \in (0, 1]$ represents the confidence level and can be used as a risk aversion parameter for the manufacturer.

Proposition 4.5. *When the market demand satisfies an IFR distribution, the manufacturer's optimal wholesale price is* $\omega_1^* = \frac{c}{1-\eta_1} - \frac{q^*}{\Omega}$, *where* $\eta_1 = \frac{\lambda(1+r)F(k_r)}{p(1-\beta)}$, $\Omega = \frac{\mathrm{d}q^*}{\mathrm{d}\omega} = \frac{p(1+r)\,\bar{F}(k_r)[1-H(k_r)]}{\omega^2(1+r)^2 f(k_r) - p^2 f(q^*)}$.

According to Proposition 4.5, the optimal wholesale prices of the manufacturer is related with some operational factors like order size, product cost, credit guarantee coefficient and interest rate.

4.2.2. Optimal wholesale prices with default risk

When the manufacturer provides a partial credit guarantee for the retailer, if the retailer can repay the principal and interest of loans at the end of the sales period, the profit of the manufacturer is $(\omega - c)q^*$. While the retailer cannot repay the principal and interest of loans, and use all of its sales revenue to repay loans, then the manufacturer will bear a certain percentage of remaining insufficient parts. Suppose that the retailer has the default probability $\alpha (0 \le \alpha \le 1)$. So that the profit of the manufacturer is

$$\pi_m = \begin{cases} (\omega - c)q^* & x > k_r \\ \alpha p x + (1-\alpha)\omega q^* - c q^* - \lambda[\omega q^*(1+r) - p \min\{x, q^*\}] & x < k_r. \end{cases} \tag{5}$$

Therefore, the decision problem of risk-averse manufacturer is given as follows

$$\max E(\pi_m) = \int_0^{k_r} \{\alpha p x + (1-\alpha)\omega q^* - c q^* - \lambda[\omega q^*(1+r) - p x]\} f(x)\,\mathrm{d}x + \int_{k_r}^{+\infty} (\omega - c) q^* f(x)\,\mathrm{d}x.$$

Proposition 4.6. *The optimal wholesale price of the manufacturer is* $\omega_2^* = \frac{c}{1-\eta_2} - \frac{q^*}{\Omega}$, *where* $\eta_2 = \frac{(\lambda+\alpha)(1+r)}{p(1-\beta)} F(k_r)$.

Proposition 4.6 shows that the pricing decision of the manufacturer is very complex and is influenced by several factors such as quantity order, cost of production, ratio of credit guarantee, interest rate and risk aversion of the retailer. Thus, there is an inseparable relationship between operation decisions and financing decisions.

5. Numerical analysis

This section represents some management insights from our model by numerical examples. Suppose that a unit retail price p is 1.5, a unit production cost c is 0.2, and the market demand x satisfies the exponential distribution, that is, $x \sim E(100)$ (see [33]).

According to Corollary 4.3, Figure 2 shows that the optimal order quantity of the retailer is a decrease function with respect to both the wholesale price and the interest rate. This reason is that the order quantity of the rational retailer maybe decrease when the wholesale price of the manufacturer increases. While the higher the interest rate is, such as $r = 7\%$, the lower the order quantity of the retailer is for any wholesale prices. Moreover, from Corollary 4.4, Figure 3 shows that the default threshold of the retailer increases when the wholesale price of the manufacturer goes up. This is because the manufacturer can limit the order quantity of the retailer by adjusting the wholesale price. The larger the wholesale price of the manufacturer is, the larger

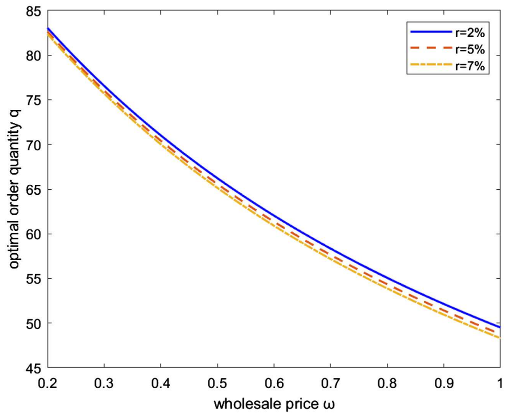

FIGURE 2. Optimal order quantity and wholesale price.

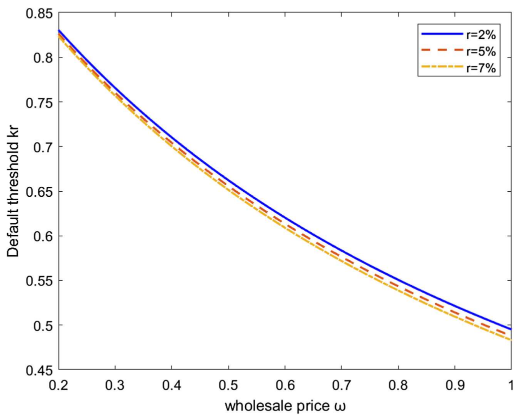

FIGURE 3. Default threshold and wholesale price.

the default threshold of the retailer is. This indicates that the magnitude of the manufacturer's wholesale price determines the ability of the retailer to survive in a market with random demand. Therefore, it is important for the retailer to have a full understanding of the demand pressures and the wholesale price before determining the amount of order quantities.

As illustrated in Figure 4, the default probability of the retailer affects the optimal wholesale prices of the manufacturer with different risk aversion parameters and different credit guarantee rates. Obviously, when the default probability of the retailer goes up, the wholesale price of the manufacturer will raise. As the credit guarantee rate increases ($v.g.$ $\lambda = 0.2$ and $\lambda = 0.5$), the manufacturer chooses to lower wholesale prices in order to reduce the risk of paying off remaining balances, while the retailer will order more goods due to high credit guarantee ratios. A larger credit guarantee means that the manufacturer is taking on more risk for the retailer, so the retailer will increase its revenue by ordering more goods. Moreover, an increase of credit guarantee ratio or a high interest rate will result in a loss of revenues, so the manufacturer will try to minimize the credit guarantee rate or interest rates, otherwise the manufacturer may not provide a credit guarantee to the retailer.

However, for the event of bankruptcy, the retailer with a credit guarantee is freed from her obligations of repaying debts. As a result, the retailer often adopts aggressive ordering strategies. Conversely, the manufacturer with different degrees of risk aversion will increase the wholesale prices. In order to limit the default risk and

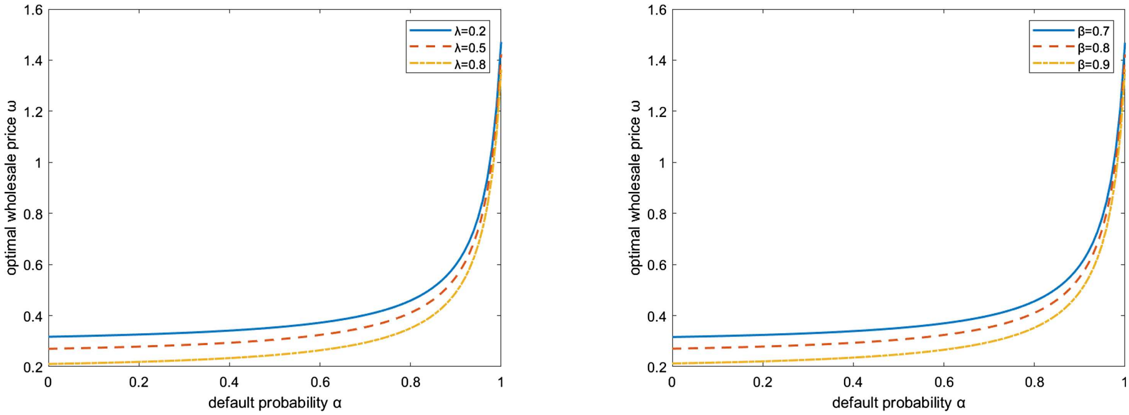

Figure 4. Optimal wholesale prices and default probability.

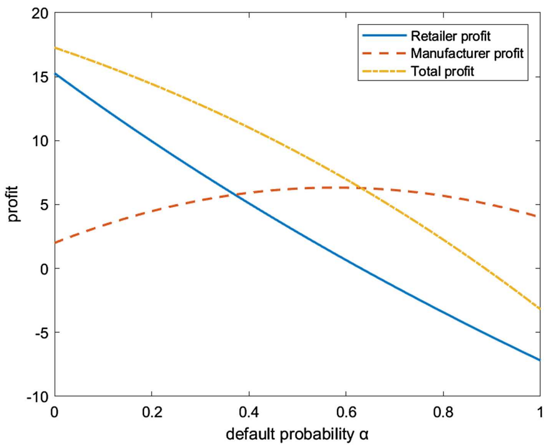

Figure 5. Profits and default probability.

prevent the retailer from overstocking, the manufacturer will raise the wholesale price. As we can see that the three curves in the Figure 4 rise with different rates. When the default probability increases and the risk aversion of the manufacturer is higher (*v.g.* $\beta = 0.7$), he ordering decisions of the retailer are relatively conservative, and it is profitable for the manufacturer to provide a credit guarantee to the retailer. However, when the risk aversion of the manufacturer is lower (*v.g.* $\beta = 0.9$), the ordering decisions of the retailer are relatively aggressive, and the credit guarantee of the manufacturer would bring more risk to himself, and the manufacturer no longer provides the credit guarantee.

Finally, Figure 5 shows the relation between the default probability of the retailer and profit levels of different participants in supply chain. As the default probability of the retailer increases, the manufacturer will accordingly increase wholesale prices such that the profit of the retailer decreases. For the manufacturer, his profit firstly increases and then decreases when the default probability increases. Since the wholesale price is monotonically decreasing with respect to the default probability, the wholesale price decreases gradually. Moreover, there exists a specific default probability that maximizes profits of the manufacturer. In addition, total profits of supply chain decreases with the increase of the default probability of the retailer.

6. Conclusion

This paper investigates the optimal strategies under the credit guarantee financing model in the supply chain where the retailer is capital-constrained. Suppose that the retailer is exposed to the default risk, we construct loss aversion models of the manufacturer using the CVaR criterion. Finally, we explore the impact of the default risk and the credit guarantee ratio on the optimal strategies of the supply chain, and find the following results.

First, when the default probability is set in a credit guarantee, the increase of the credit guarantee ratio will lead to the increase of the optimal wholesale prices and the decrease of the optimal order quantities. The manufacturer can compensate for the loss caused by the bankruptcy of the retailer *via* setting the appropriate wholesale price. As the default probability of the retailer increases, the optimal order quantity is smaller and the optimal wholesale price is larger, respectively. Moreover, the optimal expected profit of both the retailer and the supply chain decreases with the increase of the default probability of the retailer. This indicates that the capital-constrained retailer must consider its default threshold. Similarly, the manufacturer should take the default probability into account in their decision-making with the credit guarantee.

Second, if the risk aversion of the manufacturer is low and the credit guarantee ratio is within a certain range, then both players in supply chain can achieve a win–win situation by choosing the credit guarantee contract. Compared with banks, the core enterprises in the supply chain have a better understanding of the financial situation of SMEs, which will reduce the default risk. When capital-constrained SMEs are more difficult to obtain loans from banks, the credit guarantee contract provided by the core enterprises in supply chain can be applied. Moreover, the decision of the bank often plays an important role and affects the operational decisions of supply chain.

Appendix A.

Proof of Lemma 4.1. From equation (1), if the profits obtained by the retailer at the end of the sale period are more than the sum of the principal and interest from the bank, *i.e.*, $p\min\{x,q\}-\omega q(1+r)\geq 0$, the retailer is not at risk of bankruptcy, so that the threshold of market demands $k_r=\frac{\omega q(1+r)}{p}$. □

Proof of Proposition 4.2. Using the first and second order partial derivative of equation (1) with respect to q, respectively, we obtain

$$\frac{\mathrm{d}E(\pi_r)}{\mathrm{d}q}=p\overline{F}(q)-\omega(1+r)\overline{F}(k_r),\frac{\mathrm{d}^2E(\pi_r)}{\mathrm{d}q^2}=-pf(q)+\frac{\omega^2(1+r)^2}{p}f(k_r). \tag{A.1}$$

Because of $\omega(1+r)<p$, $k_r<q$ and $h(q)>h(k_r)$, we can get

$$\frac{f(q)}{\overline{F}(q)}>\frac{f(k_r)}{\overline{F}(k_r)}=\frac{\omega(1+r)f(k_r)}{\omega(1+r)\overline{F}(k_r)}=\frac{\omega(1+r)f(k_r)}{p\overline{F}(q)}\cdot \tag{A.2}$$

Therefore, the inequality $pf(q)>\omega(1+r)f(k_r)>\omega^2(1+r)^2f(k_r)$ is satisfied, *i.e.*, $\frac{\mathrm{d}^2E(\pi_r)}{\mathrm{d}q^2}<0$. Let $\frac{\mathrm{d}E(\pi_r)}{\mathrm{d}q}=0$. We obtain $q^*=\overline{F}^{-1}\left(\frac{\omega(1+r)\overline{F}(k_r)}{p}\right)$. □

Proof of Corollary 4.3. By the implicit function theorem and $p\overline{F}(q)-\omega(1+r)\overline{F}(k_r)$ in Proposition 4.2, we have

$$\frac{\mathrm{d}q^*}{\mathrm{d}\omega}=\frac{p(1+r)\overline{F}(k_r)[1-H(k_r)]}{\omega^2(1+r)^2f(k_r)-p^2f(q^*)}\cdot \tag{A.3}$$

Since the condition of demand distribution $H(k_r)<H(q^*)\leq 1$, the inequality $p(1+r)\overline{F}(k_r)[1-H(k_r)]<0$ holds and $\omega^2(1+r)^2f(k_r)-p^2f(q^*)<0$. Furthermore, let $\frac{\mathrm{d}q^*}{\mathrm{d}\omega}=\Omega$. Thus, we have $\Omega=\frac{p(1+r)\overline{F}(k_r)[1-H(k_r)]}{\omega^2(1+r)^2f(k_r)-p^2f(q^*)}<0$. □

Proof of Corollary 4.4. Since $\frac{\mathrm{d}k_r}{\mathrm{d}\omega} = \frac{p(1+r)}{p} + \frac{\omega(1+r)}{p}\Omega = \frac{q(1+r)(1-H(q))}{p(H(k_r)-H(q))}$, the inequality $1-H(q)>0$ and $H(k_r)-H(q)<0$, then we have $\frac{\mathrm{d}k_r}{\mathrm{d}\omega}<0$. □

Proof of Proposition 4.5. According to equation (3), we have

$$F_\beta(\omega,\varepsilon) = \varepsilon + \frac{1}{1-\beta}\int_0^{k_r} \{-(\omega-c)q^* + \lambda[\omega q^*(1+r) - px] - \varepsilon\}^+ \,\mathrm{d}F(x) + \frac{1}{1-\beta}\int_{k_r}^{+\infty} [-(\omega-c)q^* - \varepsilon]^+ \,\mathrm{d}F(x). \tag{A.4}$$

Here are three cases that are evaluated as follows.

(1) If $\varepsilon < -(\omega-c)q^*$, then we have

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon} = 1 - \frac{1}{1-\beta} < 0.$$

(2) If $-(\omega-c)q^* \le \varepsilon < \lambda\omega q^*(1+r) - (\omega-c)q^*$,

$$F_\beta(\omega,\varepsilon) = \varepsilon + \frac{1}{1-\beta}\int_0^{\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}} \{-(\omega-q^*) + \lambda[\omega q^*(1+r) - px] - \varepsilon\}\,\mathrm{d}F(x)$$

then we have

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon} = 1 - \frac{1}{1-\beta}F\left(\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}\right).$$

It is known that

$$\left.\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=-(\omega-c)q^*} = 1 - \frac{1}{1-\beta}F\left(\frac{\omega q^*(1+r)}{p}\right)$$

and

$$\left.\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\lambda\omega q^*(1+r)-(\omega-c)q^*} = 1 > 0.$$

(3) When $[\varepsilon \ge \lambda\omega q^*(1+r) - (\omega-c)q^*$, we can obtain

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon} = 1.$$

From the above three cases, derivatives of $F_\beta(\omega,\varepsilon)$ do not exist at $\varepsilon = -(\omega-c)q^*$ because the left derivative of $F_\beta(\omega,\varepsilon)$ is not equivalent to the right one.

Using First-Order-Condition (FOC), that is $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial\varepsilon} = 0$, then we have

$$F\left(\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}\right) = 1-\beta. \tag{A.5}$$

From equation (A.5), we show that the minimum loss value is $\varepsilon^* = \lambda\omega q^*(1+r) - (\omega-c)q^* - \lambda pF^{-1}(1-\beta)$. Furthermore, since $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial q^*} = 0$, it can be seen that

$$-[(\omega-c)q^* - \lambda\omega q^*(1+r)]F\left(\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}\right) = 0. \tag{A.6}$$

From equation (A.5), we get $F(\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}) \ne 0$, so

$$-[(\omega-c)q^* - \lambda\omega q^*(1+r)]F\left(\frac{\varepsilon+(\omega-c)q^*-\lambda\omega q^*(1+r)}{-\lambda p}\right) \ne 0.$$

Which contradicts equation (A.6).

The minimal value can only be attained if there is no derivative because equation (A.5) does not exist while equation (A.6) does, excluding FOC.

Then we bring $\varepsilon^* = -(\omega - c)q^*$ into the following equation

$$F_\beta(\omega, \varepsilon) = -(\omega - c)q^* + \frac{1}{1-\beta}\int_0^{\frac{\omega q^*(1+r)}{p}} \lambda[\omega q^*(1+r) - px]\,\mathrm{d}F(x)$$

and applying the first-order partial derivative of $F_\beta(\omega, \varepsilon)$ with respective to ω, we get

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \omega} = q^*\left(\frac{\lambda(1+r)}{p(1-\beta)}F(k_r) - 1\right) + \left[\omega\left(\frac{\lambda(1+r)}{p(1-\beta)}F(k_r) - 1\right) + c\right]\Omega.$$

Let $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \omega} = 0$. We get $\omega_1^* = \frac{c}{1-\eta_1} - \frac{q^*}{\Omega}$, where $\eta_1 = \frac{\lambda(1+r)F(k_r)}{p(1-\beta)}$. □

Proof of Proposition 4.6. According to the following formula

$$F_\beta(\omega, \varepsilon) = \varepsilon + \frac{1}{1-\beta}\int_0^{k_r} \{-(\alpha+\lambda)px + [\lambda(1+r) - (1-\alpha)]\omega q^* + cq^* - \varepsilon\}^+\,\mathrm{d}F(x)$$
$$+ \frac{1}{1-\beta}\int_{k_r}^{+\infty} [-(\omega - c)q^* - \varepsilon]^+\,\mathrm{d}F(x). \tag{A.7}$$

Three scenarios are evaluated as follows.

(1) If $\varepsilon < -(\omega - c)q^*$ is available, then

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon} = 1 - \frac{1}{1-\beta} < 0.$$

(2) If $-(\omega - c)q^* \le \varepsilon < [\lambda(1+r) - (1-\alpha)]\omega q^* + cq^*$,

$$F_\beta(\omega, \varepsilon) = \varepsilon + \frac{1}{1-\beta}\int_0^{\frac{\varepsilon - [\lambda(1+r)-(1-\alpha)]\omega q^* - cq^*}{-p(\lambda+\alpha)}} \{-p(\lambda+\alpha)x + [\lambda(1+r) - (1-\alpha)]\omega q^* + cq^* - \varepsilon\}\,\mathrm{d}F(x)$$

then we have

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon} = 1 - \frac{1}{1-\beta}F\left(\frac{\varepsilon - [\lambda(1+r) - (1-\alpha)]\omega q^* - cq^*}{-p(\lambda+\alpha)}\right).$$

It is known that

$$\left.\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon = -(\omega-c)q^*} = 1 - \frac{1}{1-\beta}F\left(\frac{[\lambda(1+r)+\alpha]\omega q^*}{p(\lambda+\alpha)}\right)$$

and

$$\left.\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon = [\lambda(1+r)-(1-\alpha)]\omega q^* + cq^*} = 1 > 0.$$

(3) When $\varepsilon \ge [\lambda(1+r) - (1-\alpha)]\omega q^* + cq^*$, we can obtain

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon} = 1.$$

The left derivation does not exist because it is not equivalent to the right derivation at $\varepsilon = -(\omega - c)q^*$, as can be seen from the previous three cases. When FOC holds or there are no variations, the minimal value of $F_\beta(\omega, \varepsilon)$ can be obtained. If the absolute minimum is achieved using FOC, that is $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \varepsilon} = 0$, then we have

$$F\left(\frac{\varepsilon - [\lambda(1+r) - (1-\alpha)]\omega q^* - cq^*}{-p(\lambda+\alpha)}\right) = 1 - \beta. \tag{A.8}$$

Using solving equation (A.8), the minimum loss value is

$$\varepsilon^* = [\lambda(1+r) - (1-\alpha)]\omega q^* + cq^* - p(\alpha+\lambda)F^{-1}(1-\beta).$$

Furthermore, from $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial q^*} = 0$ it can be seen that

$$\{[\lambda(1+r) - (1-\alpha)]\omega + c\}F\left(\frac{\varepsilon - [\lambda(1+r) - (1-\alpha)]\omega q^* - cq^*}{-p(\lambda+\alpha)}\right) = 0. \tag{A.9}$$

From the above equation (A.8), we get $[\lambda(1+r) - (1-\alpha)]\omega + c \neq 0$, so

$$\{[\lambda(1+r) - (1-\alpha)]\omega + c\}F\left(\frac{\varepsilon - [\lambda(1+r) - (1-\alpha)]\omega q^* - cq^*}{-p(\lambda+\alpha)}\right) = 0.$$

Which contradicts equation (A.9).

The minimal can only be attained if there is no derivative because equation (A.8) does not exist while equation (A.9) does, excluding FOC.

Furthermore, we bring $\varepsilon^* = -(\omega - c)q^*$ into the following equation:

$$F_\beta(\omega, \varepsilon) = -(\omega - c)q^* + \frac{1}{1-\beta}\int_0^{\frac{\omega q^*(1+r)}{p}} \{-p(\lambda+\alpha)x + [\alpha + \lambda(1+r)]\omega q^*\}\,\mathrm{d}F(x)$$

and applying the first-order partial derivative of $F_\beta(\omega, \varepsilon)$ with respective to ω, we get

$$\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \omega} = q^*\left(\frac{(\lambda+\alpha)(1+r)}{p(1-\beta)}F(k_r) - 1\right) + \left[\omega\left(\frac{(\lambda+\alpha)(1+r)}{p(1-\beta)}F(k_r) - 1\right) + c\right]\Omega.$$

Let $\frac{\partial F_\beta(\omega,\varepsilon)}{\partial \omega} = 0$. We get $\omega_2^* = \frac{c}{1-\eta_2} - \frac{q^*}{\Omega}$, where $\eta_2 = \frac{(\lambda+\alpha)(1+r)}{p(1-\beta)}F(k_r)$. □

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