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THE COST BEARING MECHANISM FOR ADVERTISING IN A CAPITAL-CONSTRAINED SUPPLY CHAIN

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Abstract. With rising market competition, increasing numbers of firms are launching advertising to attract customers and promote product sales. The increase in operating costs caused by advertising places greater pressure on small retail businesses that are prone to capital shortages, leading them to seek financing from upstream firms. However, in the financing process, upstream and downstream firms may not be able to acquire all of one another's real information, which inevitably has a significant impact on their operating strategies. By constructing a Stackelberg game, this paper studies the effects of information asymmetry on the retailer's initial capital and the manufacturer's financing rate on their advertising strategies. We find that in the symmetric information scenario, when the advertising cost coefficient is low, manufacturer advertising is the superior strategy and increases the retailer's and manufacturer's profits and social welfare; when it is moderate, retailer advertising is the superior policy; but when it is high, retailer advertising is more beneficial for the manufacturer's profit and social welfare but is more unfavorable for the retailer's profit. In addition, information asymmetry on the manufacturer's financing rate affects the advertising strategies of the manufacturer and the retailer, but information asymmetry on the retailer's initial capital fails. Additionally, we further extend the model to the Nash game scenario and cooperative advertising scenario and draw some different conclusions. This study contributes to the literature by analyzing the advertising strategies of the retailer and the manufacturer with asymmetric financing information, and guides the design of advertising strategies for companies in practice.

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1. Introduction

With the continuous intensification of market competition, increasing numbers of companies invest heavily in advertising to seize the market [48]. According to the General Administration of Market Supervision in China, in 2020, the advertising industry was valued at 914.39 billion yuan, a 5.4% increase over the previous year. Advertising, as an important marketing approach, conveys product information to consumers to attract their attention and interest and thereby stimulate and induce consumption [8,18]. The clear benefits of advertising in

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promoting product sales incentivize both downstream retailer and upstream manufacturer companies to spare no effort in launching advertising campaigns.

In retailer advertising, also called local advertising, the retailer exploits its understanding of the local market to engage in advertising activity to expand product sales [32, 62]. Examples of this type of advertising include Walmart's advertising activities for Coca-Cola and Best Buy's advertising activities for Samsung UHD TVs [40]. In manufacturer advertising, also known as national advertising, the manufacturer conducts advertising campaigns with a nationwide scope to improve brand awareness, brand image and product sales [20, 36]. From corporate annual reports, in 2020, BYD and By-Health invested 1.49 and 0.96 billion yuan in advertising, increases of 37.75% and 19.21% over the previous year, respectively.

Both retailer advertising and manufacturer advertising expand product sales and improve their sales revenue but also increase operating costs [29,56]. Intuitively, the retailer and the manufacturer usually prefer advertising by the other party, since they can enjoy the increased revenue from advertising without bearing any advertising cost. However, in the process of product sales, the expensive advertising costs of the retailer and the manufacturer are usually transferred to final consumers through increased product prices [24, 65]. Specifically, in retailer advertising, the retailer directly raises the products' retail prices to transfer the advertising cost. In manufacturer advertising, to pass on advertising expenses, the manufacturer increases the wholesale price to the retailer, then the retailer in turn raises the retail price. Considering the impact of advertising cost shifting, is retailer advertising or manufacturer advertising better? This is a real-world problem that urgently needs to be solved.

In actual business activities, downstream retailers, such as Hongfu and other small and medium-sized supermarkets, need to pay substantial money in advance to purchase a variety of products for sales. In such a case, the retailer can easily face a shortage of funds. Faced with this situation, the downstream retailer always finances from the upstream manufacturer and defers payment for its products [54]. Inevitably, the retailer's funding shortfall exerts a significant impact on the operational decisions of the retailer and even the manufacturer [7, 30], which necessarily impose complicated effects on their advertising strategies.

In addition, a variety of business practices and theoretical research indicate that information asymmetry among supply chain participants exerts important effects on firms' operating decisions [5, 37]. When a capital-constrained retailer obtains financing from a manufacturer, the manufacturer may not acquire the capital-constrained retailer's real initial capital information, and the retailer may not obtain the manufacturer's real financing interest rate information. The concealment of such information by the retailer and manufacturer directly affects their operational decisions and profits, which further influence their advertising strategies.

Motivated by the abovementioned considerations, we focus on the following questions:

(1) What are the optimal advertising strategies (*e.g.*, application conditions, appropriate investor and investment level) of the retailer and manufacturer in scenarios featuring symmetric information? (2) Furthermore, how does asymmetric information on the manufacturer's financing interest rate and the retailer's initial capital affect the advertising strategies of the retailer and the manufacturer? (3) How do these two types of asymmetric information affect the performance of the retailer and the manufacturer?

To answer these questions, we consider a supply chain with a capital-constrained retailer and a well-funded manufacturer. The capital-constrained retailer first purchases products from the manufacturer and defers payment for them, then sells them to the consumer, and finally reimburses the manufacturer for the money received from the consumer. To expand product sales, both the retailer and manufacturer consider whether to launch advertising campaigns. In this context, we explore the optimal advertising strategies of the retailer and manufacturer in three scenarios of symmetric and asymmetric information related to financing information (*i.e.*, the manufacturer's financing interest rate and the retailer's initial capital).

Our study shows that in the Stackelberg game, under symmetric information, when the advertising cost coefficient is low, manufacturer advertising is a more advantageous policy than retailer advertising and increases the profits of the retailer and the manufacturer, consumer surplus and social welfare; when it is moderate, retailer advertising is a better strategy; but when it is high, retailer advertising is more advantageous for the manufacturer's profit, consumer surplus and social welfare but is more unfavorable for the retailer's profit. Furthermore, under asymmetric information, the application conditions, appropriate investor and role of advertising for the retailer and the manufacturer are affected by asymmetric information on the manufacturer's financing interest rate but not by asymmetric information on the retailer's initial capital. In addition, in contrast to the symmetric information case, asymmetric information on the manufacturer's financing interest rate may allow both the retailer and manufacturer to obtain higher profits, but asymmetric information on the retailer's initial capital may enable the manufacturer to collect more profit and has no effect on the retailer's profit. However, in the Nash game, whether in the symmetric or asymmetric information scenario, the retailer prefers the manufacturer to advertise and the manufacturer hopes that the retailer will advertise, *i.e.*, they both want the other party to advertise rather than advertising themselves.

The contributions of this work are listed below. Previous studies mainly explore advertising strategies of the retailer and the manufacturer in the context of information symmetry. Although only a few studies focus on advertising strategies of firms under information asymmetry, they basically consider information asymmetry in terms of demand, cost and so on. To the best of our knowledge, this work is the first to analyze the advertising strategies of the retailer and the manufacturer in a capital-constrained supply chain with asymmetric information on the manufacturer's financing interest rate and the retailer's initial capital, revealing the impacts of these two types of information asymmetries on the advertising strategies of the retailer and the manufacturer. In addition, this paper indicates the information disclosure and concealment strategies of the retailer and the manufacturer in the presence of advertising. Our findings not only enrich the literature on information asymmetry and advertising strategies, but also provide guidance for firms in practice to develop appropriate advertising strategies and information disclosure and concealment strategies.

The remainder of this research is organized as follows. Section 2 reviews the literature related to this study. Section 3 elaborates the assumptions, notations and descriptions of the model. In Sections 4 and 5, we analyze the retailer advertising and the manufacturer advertising cases in a capital-constrained supply chain with symmetric information and asymmetric information, respectively. Section 6 extends the scenario where the game between the retailer and the manufacturer is a Stackelberg game to a Nash game and the scenario where the retailer and manufacturer advertise separately to the scenario where they advertise together. Finally, Section 7 concludes the paper.

2. Literature review

We review the literature related to our study in the following four research streams and detail the differences between our study and the previous literature.

The first research stream is on advertising. Zhang *et al.* [62] analyze whether advertising campaigns should be launched by the retailer or the manufacturer in the presence of manufacturer encroachment. When the retailer conducts the advertising campaign, Zhou and Yang [64] investigate the optimal advertising expense and order quantity decisions of the retailer. Khouja and Robbins [26] demonstrate that retailer advertising increases the product sales and profit of the retailer. Different from [64] and [26], who focus on the case of well-funded retailer advertising, Cao *et al.* [3] discuss the advertising situation of a capital-constrained retailer and find that whether the retailer should adopt an "invest-all-or-none" advertising strategy depends on market conditions. Zhou *et al.* [65] further explore the optimal advertising decision and financing mode selection of a capital-constrained retailer. Different from [3] and [65], who concentrate on the scenario of information symmetry, some scholars analyze the retailer advertising problem under various asymmetric information scenarios, such as cost information asymmetry [63] and demand information asymmetry [28]. When the manufacturer launches the advertising campaign, Chan *et al.* [4] examine the complicated interaction between the manufacturer's advertising decision and wholesale price decision and the retailer's retail price decision. Vol [52] conducts an empirical analysis and shows that manufacturer advertising enhances brand awareness and goodwill. In addition, some scholars explore cooperative advertising, such as [17, 20, 31, 34, 56] and so forth. Specifically, Li *et al.* [31] study the effect of different cooperative advertising modes in a dual-channel supply chain composed of an e-platform and a manufacturer and find that the increase in the manufacturer's profit comes at the cost of the decrease in the e-platform's profit under bilateral cooperative advertising. Wu *et al.* [56] investigate the impact

Table 1. Comparison of this study with existing literature.

Existing literature	Advertising (type)	Information symmetry	Information asymmetry (type)
Zhang *et al.* [62]	Retailer advertising manufacturer advertising cooperative advertising	✓	×
Zhou and Yang [64]	Retailer advertising	✓	×
Khouja and Robbins [26]	Retailer advertising	✓	×
Cao *et al.* [3]	Retailer advertising	✓	×
Zhou *et al.* [65]	Retailer advertising	✓	×
Zhao and Zhao [63]	Retailer advertising	×	Cost information
Li *et al.* [28]	Retailer advertising	×	Demand information
Chan *et al.* [4]	Manufacturer advertising	✓	×
Vol [52]	Manufacturer advertising	✓	×
Wu *et al.* [56]	Cooperative advertising	×	Demand information
Li *et al.* [31]	Cooperative advertising	✓	×
Liu and Li [34]	Cooperative advertising	✓	×
Han *et al.* [17]	Cooperative advertising	✓	×
Hong *et al.* [19]	Manufacturer advertising cooperative advertising	×	Product quality information
This paper	Retailer advertising manufacturer advertising	×	Financing information

of information sharing on the optimal advertising expenses of the manufacturer and retailer under cooperative advertising.

Most of the above studies explore retailer advertising and manufacturer advertising in the context of information symmetry. However, in reality, information asymmetry often occurs between the retailer and the manufacturer. Although a few studies such as [63] and [28] have analyzed retailer advertising in the context of information asymmetry, they basically considered information asymmetry on demand and cost. To the best of our knowledge, to far, almost no scholars have considered the retailer and manufacturer advertising problems in the context of information asymmetry on the retailer's initial capital. However, in real commercial trade, some small retail firms can easily face the dilemma of capital shortage and usually finance from the upstream manufacturer. In these circumstances, the manufacturer may not acquire the real initial capital information of the retailer. Hence, it is necessary to investigate the advertising strategies of the retailer and manufacturer in the scenario of information asymmetry on the retailer's initial capital.

The second research stream considers capital-constrained supply chains. Zhan *et al.* [61] investigate the impact of a shortage of funds in a capital-constrained supply chain with one manufacturer and two competitive retailers (one capital constrained and one well funded) and show that as horizontal competitive intensity increases, the capital-constrained retailer can obtain greater benefits than the well-funded retailer. Yuan *et al.* [60] explore the effect of capital constraints on the supplier in a supply chain with a manufacturer, a capital-constrained and unreliable supplier, and a well-funded and reliable supplier. Wang and Chen [53] show that changing the manufacturer's capital situation (i.e, no capital constraints, capital constraints and capital constraints with financing) significantly affects the manufacturer's operational decisions. Xu *et al.* [57] analyze the influence of capital constraints in a dual-channel supply chain and find that they exert important impacts on the channel strategies and pricing decisions of the supplier and retailer. Assuming that a capital-constrained retailer can obtain trade credit financing from a manufacturer, Wu *et al.* [54] examine the role of trade credit financing and show that trade credit financing can encourage the retailer to order more products from the manufacturer. In addition, trade credit can increase the profits of supply chain participants [61] and improve the efficiency of the supply chain [58]. Jena *et al.* [22] extend the single-channel model of [54] to a dual-channel model and a multi-echelon model and reveal that trade credit financing creates more profit for the total supply chain than reverse factoring financing and hybrid reverse factoring financing when the market size is sufficiently large.

Different from the above literature, which mainly analyzes the impact of capital constraints on the operating decisions of capital-constrained firms and their upstream and downstream firms, we explore the impact of capital constraints not only on these firms' operating decisions but also on their advertising strategies (*e.g.*, advertising condition, advertising effort and advertising investor) while considering the complex interactions between operational decisions and advertising strategies. In addition, by comparing the manufacturer's profit when cooperating with the retailer with and without financial constraints, we reveal how the manufacturer should choose the capital-constrained or well-funded retailer to carry out business cooperation.

The third research stream is on asymmetric information. Considering information asymmetry on the production cost using a mathematical model, Meng *et al.* [37] study the impact of cost information sharing in a remanufacturing supply chain and find that sharing information can benefit social welfare and the environment. Different from [33, 37] adopt an empirical model to analyze the effect of the tone of customers' forward-looking disclosures on suppliers' asymmetric cost behaviors. In a scenario with asymmetric market demand information, Chang *et al.* [5] examine the effect of information sharing in a dual-channel supply chain and find that information sharing benefits dual-channel retailing. Different from [5] focusing on a one-period model, Sarkar *et al.* [47] explore the impact of strategic inventory strategy on information sharing strategy in a two-period model and show that holding strategic inventory will not affect the retailer's information sharing strategy. Considering information asymmetry on product quality and consumer preferences, Guan and Chen [15] investigate the interactive relationship between consumer preference information acquisition and quality information disclosure strategies. In addition, some studies explore asymmetric information on collection effort level and collection ability [55], random yield [23] and so forth.

Different from the above studies that focus on information asymmetry on the production cost, product quality, consumer preference, market demand and so forth [5,15,33,37], we focus on information asymmetry on financing, *i.e.*, the lender's (manufacturer) financing interest rate and the debtor's (retailer) initial capital and analyze and compare how these two types of information asymmetry affect the pricing decisions and advertising strategies of the manufacturer and retailer. In addition, by contrasting these two asymmetric information scenarios with the symmetric information scenario, we further indicate whether the manufacturer and retailer should reveal or conceal the financing interest rate and initial capital information.

The fourth literature stream is on operational decision-making. A series of studies on operational decisions can be divided into two aspects: one is based on a deterministic demand scenario, and the other is based on a random demand scenario. For the deterministic demand scenario where market demand depends on price, many scholars discuss the operating decisions of parties of different types of supply chains in this situation [1, 44–46, 49]. Specifically, Sana [46] focuses on a two-level supply chain with one manufacturer and one retailer and explores the retail price and product greenness decisions of the retailer and the wholesale price decision of the manufacturer. Asghari *et al.* [1] examine the pricing and advertising decisions in a closed-loop supply chain. Different from [46] and [1], who focus on a single-channel supply chain, Saha *et al.* [44] assume that firms can sell products to customers through both online and offline channels and discuss the optimal pricing strategies of the manufacturer and retailer in a two-level dual-channel supply chain. In addition, some scholars also conduct in-depth discussions on the operational decisions of supply chain parties under random demand [6, 13, 21, 35, 43]. Specifically, Liu *et al.* [35] analyze the optimal pricing, inventory and sales effort level decisions of one manufacturer and one retailer who face price and effort dependent stochastic demand in the additive demand case. Yang and Liu [59] further compare how the additive and multiplicative formats of price-dependent stochastic demand structures impact supply chain participants' decisions and profits.

Similar to the above literature such as [1, 44] and [46], in this paper, we also explore the decision-making of supply chain participants in the deterministic demand scenario. However, unlike their focus on the symmetric information scenario, we discuss the pricing and advertising effort decisions of the retailer and manufacturer when

TABLE 2. Notation.

p	The unit retail price
w	The unit wholesale price
a	The advertising effort
b	The cost coefficient of advertising effort
u	The market size of the product
k_r	The retailer's initial capital
r_m	The manufacturer's financing interest rate
π_{rj}^t	The retailer's profit
π_{mj}^t	The manufacturer's profit
j	Superscripts $j = n$, $j = r$, $j = m$ and $j = c$ represent no advertising, retailer advertising, manufacturer advertising and cooperative advertising
t	Superscripts $t = s$, $j = ai$ and $j = ac$ represent symmetric information, asymmetric information on the financing interest rate and asymmetric information on the initial capital

market demand is dependent on price in the symmetric and asymmetric information scenarios and investigate the significant impact of information asymmetry on the operational decisions of supply chain participants.

In summary, although a few studies explore the problem of advertising strategies under information asymmetry, they focus on the scenarios of cost information asymmetry, demand information asymmetry, product quality information asymmetry, and so forth. To the best of our knowledge, there is no literature that examines how asymmetric information about financing (*i.e.*, asymmetric information about the debtor's initial capital and the lender's financing interest rate) affect their advertising strategies in a capital-constrained supply chain with trade credit financing. Based on this, we explore the effect of asymmetric information about the retailer's initial capital and the manufacturer's financing interest rate on their advertising strategies and analyze the impact of the advertising strategies on their information sharing strategies, revealing the interaction mechanism between their information sharing and advertising strategies.

3. Model description

This study focuses on a supply chain with one manufacturer ("she") and one retailer ("he"). We model the interaction between the manufacturer and the retailer as a Stackelberg game where the manufacturer acts as the leader and the capital-constrained retailer acts as the follower. In addition, in the extension in Section 6.2, we extend the Stackelberg game between the manufacturer and the retailer to the Nash game between them.

The sequence of events between the retailer and manufacturer is as follows. At the beginning of the period, the capital-constrained retailer purchases products from the manufacturer at unit wholesale price w on a partial deferred payment basis with financing interest rate r_m. Then, the retailer sells products to consumers at unit retail price p. To stimulate market demand, the retailer or manufacturer considers whether to launch an advertising campaign with advertising effort a and bears advertising cost ba^2, where b is the cost coefficient of advertising effort. This quadratic cost form is widely applied by researchers, such as [39,51] and [41]. At the end of the period, the retailer uses the proceeds from product sales to repay the principal and interest owed to the manufacturer. In the process where the capital-constrained retailer finances from the manufacturer, we assume that the retailer will promptly repay the debts owed to the manufacturer without default. This assumption is widely applied in supply chain finance, such as [22,65] and so forth.

In addition, without loss of generality, following the assumption of [36] and [9], we set the production cost and other management costs of the manufacturer to zero. For convenience, we summarize all notations in Table 2.

4. The symmetric information scenario

In this section, to identify the appropriate investor (the retailer or manufacturer) and the role of advertising in a financially constrained supply chain where the capital-constrained retailer finances from the manufacturer under information symmetry, we first analyze the non-advertising case, the retailer advertising case and the manufacturer advertising case and then compare these three cases in pairs.

First, in the capital-constrained supply chain, when neither the retailer nor the manufacturer invests in advertising, the retailer's profit is $\pi^s_{rn} = p(u-p) - [w(u-p) - k_r](1+r_m) - k_r$, and the manufacturer's profit is $\pi^s_{mn} = [w(u-p) - k_r](1+r_m) + k_r$. Obviously, the retailer's optimal retail price and the manufacturer's optimal wholesale price are $p^s_n = \frac{3u}{4}$ and $w^s_n = \frac{u}{2(r_m+1)}$, respectively.

4.1. Retailer advertising

When a capital-constrained retailer advertises, due to the shortage of funds, the retailer may fail to pay off the product payment $w((a+1)u-p)$ and the advertising expense a^2b. In this situation, to extensively promote product sales, the manufacturer allows the retailer to defer the product payment and provides financing to the retailer for advertising expense that may not be paid off but charges a financing interest rate on the retailer's deferred payment and financing. Thus, under this circumstance, the retailer first uses all the initial capital k_r to pay the advertising costs a^2b and the manufacturer's product payment $w((a+1)u-p)$ in turn and then finances the remaining part $w((a+1)u-p) + a^2b - k_r$ from the manufacturer at the expense of the financing interest rate r_m. Hence, the retailer's profit is

$$\pi^s_{rr}(p,a) = p[(a+1)u-p] - k_r - [w((a+1)u-p) + a^2b - k_r](r_m+1). \quad (1)$$

In this scenario, the manufacturer acquires the product payment $k_r - a^2b$ from the retailer at the beginning of the sales period and obtains the delayed payment with interest $[w((a+1)u-p) - (k_r - a^2b)](1+r_m)$ from the retailer at the end of the sales period. Hence, the manufacturer's profit is

$$\pi^s_{mr}(w) = [w((a+1)u-p) - (k_r - a^2b)](r_m+1) + k_r - a^2b. \quad (2)$$

As stated in [27], in the process of purchasing and consuming products, consumers acquire consumer surplus, which is represented by the region under the demand curve above the optimal retail price. That is, consumer surplus can be defined as $\int_{p^s_r}^{(a+1)u}[(a+1)u-t]dt$. In addition, the sum of consumer surplus and firms' profits (*i.e.*, the retailer and manufacturer's profit) constitute social welfare, which is as follows:

$$Social \quad Welfare = Firms' \quad Profits + Consumer \quad Surplus$$

Proposition 4.1. *In the retailer advertising case, when* $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$, *the manufacturer's optimal wholesale price is* $w^s_r = \frac{r_m(8bu-2u^3)+4bur_m^2-(u^2-4b)u}{(r_m+1)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$; *the retailer's optimal retail price and advertising effort are* $p^s_r = \frac{r_m(12bu-2u^3)+6bur_m^2+6bu-u^3}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$ *and* $a^s_r = \frac{u^2(r_m+1)}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$, *respectively.*

Advertising improves product awareness and encourages consumers to purchase, which effectively boosts product demand. The increased demand created by retailer advertising encourages the manufacturer to lower her wholesale price. However, although enjoying the higher product demand and the lower wholesale price from advertising, the retailer raises the retail price of products to transfer the expensive advertising cost. This suggests that the advertising cost of the retailer is partly borne by the manufacturer charging a lower wholesale price and partly borne by consumers bearing a higher retail price for products.

Lemma 4.2. *In the retailer advertising case, compared to the well-funded retailer scenario, in the capital-constrained retailer scenario,*
(i) when $\hat{b}_1 < b$, *the manufacturer's profit, consumer surplus and social welfare are lower, but the retailer's*

profit is higher;
(ii) when $\frac{u^2}{4} < b \leq \hat{b}_1$*, the retailer and manufacturer's profits, consumer surplus and social welfare are lower;*
(iii) when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \leq \frac{u^2}{4}$*, the retailer and manufacturer's profits, consumer surplus and social welfare are higher.*
Due to spatial limitations, we detail the expression for $\hat{b}_1$ *in the Appendix.*

In the retailer advertising case, when b exceeds $\hat{b}_1$, compared to the scenario where the retailer is well-funded, in the scenario where the retailer is capital-constrained, the manufacturer obtains lower profit, but the retailer collects higher profit. Simultaneously, consumer surplus and social welfare exhibit worse performance. This is because although the manufacturer charges the capital-constrained retailer an additional financing interest rate, she sets a lower wholesale price, which allows the capital-constrained retailer to collect higher profit. Consequently, a lower wholesale price reduces the manufacturer's profit. As a result, consumer surplus and social welfare achieve worse performance. That is, the shortage of funds may be beneficial for the retailer when he can obtain trade credit financing from the manufacturer. This result seems contrary to intuition and previous research such as [50] and [2], among others.

Moreover, when b is in the region of $(\frac{u^2}{4}, \hat{b}_1]$, in the capital-constrained retailer scenario, due to the complicated interaction of the financing interest rate and lower wholesale price, both the manufacturer and the retailer acquire less profits, and consumer surplus and social welfare are lower. Furthermore, when b decreases to $(\frac{u^2(3r_m+2)}{8(r_m+1)^2}, \frac{u^2}{4}]$, there is no equilibrium solution in this region in the well-funded retailer scenario; hence, in the capital-constrained retailer scenario, the manufacturer and retailer's profits, consumer surplus and social welfare are higher. The above results imply that in the retailer advertising case, whether the manufacturer should choose the constrained or unconstrained retailer depends on the circumstances at hand.

4.2. Manufacturer advertising

In the manufacturer advertising case, the capital-constrained retailer does not need to pay any advertising expenses, uses all his initial capital k_r to pay the partial product payment at the beginning of the sales period, and pays the remaining product payment $w((a+1)u-p)-k_r$ with the additional financing interest $[w((a+1)u-p)-k_r]r_m$ to the manufacturer at the end of the sales period. In addition, the retailer acquires the sales revenue $p[(a+1)u-p]$. After deducting the cost from sales revenue, the retailer's profit is

$$\pi^s_{rm}(p) = p[(a+1)u-p] - [w((a+1)u-p)-k_r](r_m+1) - k_r. \tag{3}$$

In the scenario where the manufacturer launches the advertising campaign, the manufacturer also spends a^2b on the advertising campaign, and obtains the product sales $(a+1)u-p$. In addition, based on the retailer's payment to the manufacturer as described above in this scenario, the manufacturer obtains the partial product payment k_r at the beginning of the sales period and acquires the remaining product payment and interest $[w((a+1)u-p)-k_r](r_m+1)$ from the retailer at the end of the sales period. Thus, the manufacturer's profit is

$$\pi^s_{mm}(w,a) = [w((a+1)u-p)-k_r](r_m+1) + k_r - a^2b. \tag{4}$$

Proposition 4.3. *In the manufacturer advertising case, when* $\frac{u^2}{8} < b$*, the manufacturer's optimal wholesale price and advertising effort are* $w^s_m = \frac{4bu}{(8b-u^2)(r_m+1)}$ *and* $a^s_m = \frac{u^2}{8b-u^2}$*, respectively; the retailer's optimal retail price is* $p^s_m = \frac{6bu}{8b-u^2}$*.*

In the manufacturer advertising case, since the manufacturer charges the capital-constrained retailer an additional financing interest rate, the manufacturer lowers the wholesale price. In addition, under the combined effect of the financing interest rate and wholesale price, the capital-constrained retailer sets the same retail price as the well-funded retailer. This is different from the retailer advertising case, where the capital-constrained retailer and the well-funded retailer charge different retail prices to consumers.

Corollary 4.4. *In a capital-constrained supply chain,*
(i) when $\frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)} < b$*, consumers prefer the retailer to advertise;*
(ii) when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \leq \frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}$*, consumers prefer the manufacturer to advertise.*

From Corollary 4.4, when the advertising cost coefficient b exceeds $\frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}$, consumers are charged a higher retail price in the manufacturer advertising case. The reason lies in the double-increasing price effect. Specifically, to transfer the expensive advertising cost, the manufacturer raises the wholesale price ($w_m^s - w_n^s > 0$). However, consequently, the retailer not only transfers the increase in the wholesale price to consumers but also further raises the retail price ($p_m^s - p_n^s > w_m^s - w_n^s$).

Furthermore, when b is in the region of $(\frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}, \frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}]$, consumers are exploited to a greater extent in the retailer advertising case. This is because the retailer is more aggressive in advertising investment than the manufacturer ($a_r^s > a_m^s$). Hence, in this region, the heavier advertising expense incentivizes the retailer to charge a more expensive retail price to consumers.

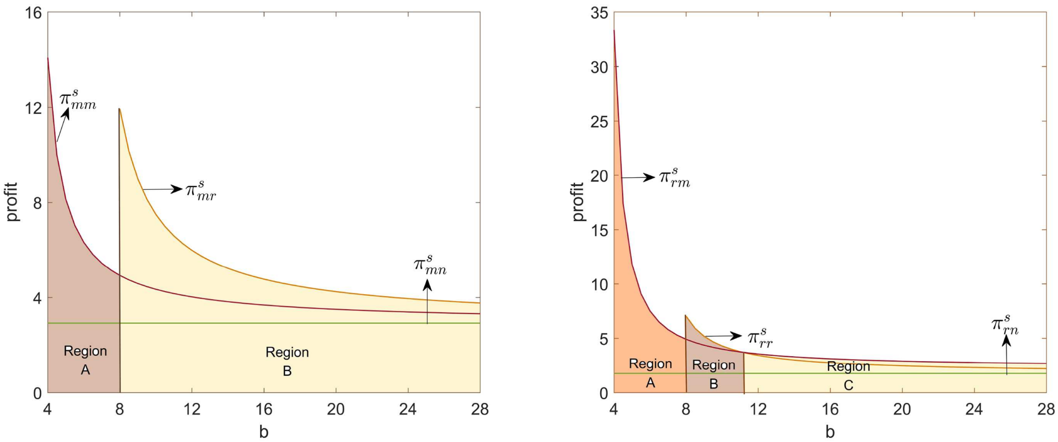
Lemma 4.5. *In the manufacturer advertising case, compared to the well-funded retailer scenario, in the capital-constrained retailer scenario, the retailer obtains a higher profit but the manufacturer obtains a lower profit. However, consumer surplus and social welfare are identical in the well-funded retailer and capital-constrained retailer scenarios.*

In the manufacturer advertising case, compared to the well-funded retailer scenario, in the capital-constrained retailer scenario, the retailer always acquires a higher profit but the manufacturer always obtains a lower profit. However, consumer surplus and social welfare are identical in the two cases. The reason is as follows. In the capital-constrained retailer scenario, although the manufacturer charges the retailer an additional financing interest rate, she sets a lower wholesale price. In this situation, the product of the wholesale price and the financing interest rate equals the wholesale price in the well-funded retailer scenario, *i.e.*, $w_m^s(1 + r_m) = \bar{w}_m^s$. Thus, in the capital-constrained retailer scenario, the manufacturer obtains less profit ($-k_r r_m$), but the retailer collects more profit ($k_r r_m$). Furthermore, in the manufacturer advertising case, the retail price and advertising effort are independent of the retailer's initial capital. Therefore, regardless of whether the retailer is subject to capital constraints, the retail price and advertising effort are identical, as are consumer surplus and social welfare. Lemma 4.5 conveys the insight that in the manufacturer advertising case, the manufacturer should cooperate with the well-funded retailer rather than the capital-constrained retailer.

4.3. The comparison of retailer advertising and manufacturer advertising

Theorem 4.6. *In a capital-constrained supply chain, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2} < b$*, the manufacturer's profit, consumer surplus and social welfare are higher, but the retailer's profit is lower;*
(ii) when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \leq \frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$*, the retailer and manufacturer's profits, consumer surplus and social welfare are higher.*
(iii) when $\frac{u^2}{8} < b \leq \frac{u^2(3r_m+2)}{8(r_m+1)^2}$*, the retailer and manufacturer's profits, consumer surplus and social welfare are lower.*
Due to spatial limitations, we detail the expression for $\bar{r}_m$ *in the Appendix.*

From Theorem 4.6, when the advertising cost coefficient b exceeds $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, compared to the manufacturer advertising case, the retailer launching the advertising campaign reduces his profit but improves the manufacturer's profit, consumer surplus and social welfare. The reason is that the shortage of funds compels the retailer to invest less in advertising, which means that he charges a lower retail price and then acquires a lower profit. Consequently, the lower advertising expenses and retail price improve consumer



(a) (b)

FIGURE 1. Retailer advertising *vs.* manufacturer advertising under symmetric information. (a) The manufacturer's profit. (b) The retailer's profit.

surplus. Furthermore, in the retailer advertising case, the manufacturer does not assume any advertising cost, which allows her to obtain higher profit. In addition, under the combined effect of the profits of the retailer and manufacturer and consumer surplus, social welfare is higher in the retailer advertising case. That is, in this region, the retailer prefers the manufacturer to advertise, but the manufacturer hopes that the retailer will advertise instead.

In addition, when b decreases to $(\frac{u^2(3r_m+2)}{8(r_m+1)^2}, \frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}]$, under the combined effect of the wholesale price, advertising effort and retail price decisions, compared to manufacturer advertising, retailer advertising is a more beneficial policy, which increases the profits of both the retailer and the manufacturer, consumer surplus and social welfare. However, when b is in the region of $(\frac{u^2}{8}, \frac{u^2(3r_m+2)}{8(r_m+1)^2}]$, since there is no solution in this region in the retailer advertising case, manufacturer advertising is a more advantageous strategy. Figure 1 clearly illustrates the results of Theorem 4.6. In Figure 1, we adopt the parameter settings in [3], *i.e.*, the market size of the product $u = 5$, the manufacturer's financing interest rate $r_m = 0.1$, and the retailer's initial capital $k_r = 2$.

5. The asymmetric information scenario

Section 4 above explored the appropriate investor and role of advertising for the capital-constrained retailer and the manufacturer in the symmetric information scenario. Furthermore, considering that the capital-constrained retailer may face asymmetric information in the financing process, in the following Section 5, we examine the impact of asymmetric information (regarding the manufacturer's financing interest rate r_m and the retailer's initial capital k_r) on the appropriate investor and role of advertising.

5.1. The scenario with asymmetric information on the manufacturer's financing interest rate

5.1.1. Retailer advertising

In the asymmetric information scenario regarding the manufacturer's financing interest rate, the manufacturer conceals her real financing interest rate r_m, and the retailer assumes that it follows a uniform distribution on $[r_s, r_t]$. This assumption is widely used in research considering asymmetric information, such as [16]. As stated in the scenario with the retailer advertising under symmetric information, the retailer's profit is $p[(a+1)u-p]-k_r-[w((a+1)u-p)+a^2b-k_r](r_m+1)$. Since the retailer estimates that the financing interest rate r_m follows a uniform distribution on $[r_s, r_t]$, the retailer's profit can be described as

$$\pi_{rr}^{ai}=\int_{r_s}^{r_t}\frac{1}{r_t-r_s}\{p((a+1)u-p)-k_r-[w((a+1)u-p)+a^2b-k_r](r_m+1)\}dr_m. \tag{5}$$

As stated in the scenario with the retailer advertising under symmetric information, the manufacturer's profit is $[w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r$. In the scenario with asymmetric information on the manufacturer's financing interest rate, since the manufacturer clearly knows her own real financing interest rate information, her profit is the same as that in the scenario with symmetric information, which follows

$$\pi_{mr}^{ai}=[w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r. \tag{6}$$

Proposition 5.1. *In the retailer advertising case with asymmetric information on the financing interest rate, when* $\frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}<b$, *the manufacturer's optimal wholesale price is* $w_r^{ai}=\frac{2u(2r_m(br_s+br_t+2b-u^2)+2br_s+2br_t+4b-u^2)}{(r_s+r_t+2)(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)}$; *the retailer's optimal retail price and advertising effort are* $p_r^{ai}=\frac{u(r_m(3br_s+3br_t+6b-2u^2)+3br_s+3br_t+6b-u^2)}{r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2}$ *and* $a_r^{ai}=\frac{u^2(r_m+1)}{r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2}$, *respectively.*

From Proposition 5.1, in the retailer advertising case, under asymmetric information on the financing interest rate, when the retailer underestimates the manufacturer's financing interest rate (*i.e.*, $\frac{1}{2}(r_s+r_t)<r_m$), the retailer invests more in advertising than in the symmetric information scenario. As a consequence, the larger advertising expense encourages the retailer to charge a higher retail price to consumers.

Corollary 5.2. *In the retailer advertising case, compared to the symmetric information scenario, in the scenario with asymmetric information on the financing interest rate,*
(i) when $\frac{1}{2}(r_s+r_t)<r_m$, *the manufacturer's profit and consumer surplus are higher; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *the manufacturer's profit and consumer surplus are lower;*
(ii) when $\frac{1}{2}(r_s+r_t)<r_m$, *if* $k_r<\tilde{k}_r$, *the retailer's profit is higher; otherwise, the retailer's profit is lower; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *if* $k_r<\tilde{k}_r$, *the retailer's profit is lower; otherwise, the retailer's profit is higher;*
(iii) when $\frac{1}{2}(r_s+r_t)<r_m$, *if* $k_r<\bar{k}_r$, *social welfare is higher; otherwise, social welfare is lower; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *if* $k_r<\bar{k}_r$, *social welfare is lower; otherwise, social welfare is higher.*
Given spatial limitations, we detail the expression for $\tilde{k}_r, \bar{k}_r$ *in the Appendix.*

As analyzed in Proposition 5.1, compared to the symmetric information scenario, in the asymmetric information scenario, when $\frac{1}{2}(r_s+r_t)<r_m$, the retailer invests more in advertising to encourage the manufacturer to achieve higher product sales, thereby allowing the manufacturer to obtain a higher profit. Moreover, in this situation, the larger advertising effort also raises consumer surplus. Furthermore, under this circumstance, in the asymmetric information scenario, under the combined effect of the advertising effort, retail price and wholesale price, the retailer's profit and social welfare are higher when the retailer's initial capital is low. On the contrary, when $0<r_m\le\frac{1}{2}(r_s+r_t)$, the result is exactly the opposite of the above result. This finding indicates that information confidentiality regarding the manufacturer's financing interest rate may be advantageous for the retailer, manufacturer and society.

5.1.2. Manufacturer advertising

As stated in the scenario with manufacturer advertising under information symmetry, the retailer's profit is $p[(a+1)u-p]-k_r-[w((a+1)u-p)-k_r](r_m+1)$. In the scenario with asymmetric information on the manufacturer's financing interest rate, the retailer estimates that the financing interest rate r_m follows a uniform distribution on $[r_s, r_t]$, and the retailer's profit can be written as

$$\pi_{\mathrm{rm}}^{ai}=\int_{r_s}^{r_t}\frac{1}{r_t-r_s}\{p((a+1)u-p)-(r_m+1)[w((a+1)u-p)-k_r]-k_r\}dr_m. \tag{7}$$

As indicated in the scenario where the manufacturer advertises under symmetric information, the manufacturer's profit is $[w((a+1)u-p)-k_r](r_m+1)+k_r-a^2b$. In the scenario with asymmetric information on the manufacturer's financing interest rate, since the manufacturer clearly knows her own real financing interest rate information, her profit is the same as that in the scenario with symmetric information, which follows

$$\pi_{\mathrm{mm}}^{ai}=(r_m+1)[w((a+1)u-p)-k_r]+k_r-a^2b. \tag{8}$$

Proposition 5.3. *In the manufacturer advertising case with asymmetric information on the financing interest rate, when* $\frac{u^2(r_m+1)}{4(2+r_s+r_t)}<b$, *the manufacturer's optimal wholesale price and advertising effort are* $w_m^{ai}=\frac{4bu}{4br_s+4br_t+8b-u^2r_m-u^2}$ *and* $a_m^{ai}=\frac{u^2(r_m+1)}{4br_s+4br_t+8b-u^2r_m-u^2}$, *respectively; the retailer's optimal retail price is* $p_m^{ai}=\frac{3bu(r_s+r_t+2)}{4br_s+4br_t+8b-u^2r_m-u^2}$.

Similar to the retailer advertising case, in the manufacturer advertising case, compared to the scenario where the retailer can obtain all of the manufacturer's real information, when the retailer fails to acquire the manufacturer's real financing interest rate information and underestimates the financing interest rate (*i.e.*, $\frac{1}{2}(r_s+r_t)<r_m$), the manufacturer selects a larger advertising effort and charges a higher wholesale price to the retailer, and the retailer sets a higher retail price.

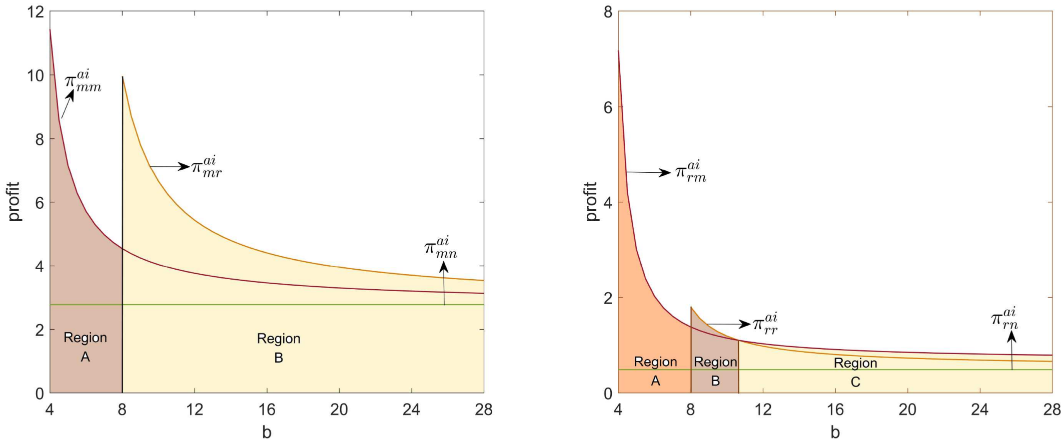
Corollary 5.4. *In the manufacturer advertising case, compared to the symmetric information scenario, in the scenario with asymmetric information on the financing interest rate,*
(i) when $\frac{1}{2}(r_s+r_t)<r_m$, *the manufacturer's profit and consumer surplus are higher; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *the manufacturer's profit and consumer surplus are lower;*
(ii) when $\frac{1}{2}(r_s+r_t)<r_m$, *if* $k_r<\hat{k}_r$, *the retailer's profit is higher; otherwise, the retailer's profit is lower; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *if* $k_r<\hat{k}_r$, *the retailer's profit is lower; otherwise, the retailer's profit is higher;*
(iii) when $\frac{1}{2}(r_s+r_t)<r_m$, *if* $k_r<\boldsymbol{k}_r$, *social welfare is higher; otherwise, social welfare is lower; when* $0<r_m\le\frac{1}{2}(r_s+r_t)$, *if* $k_r<\boldsymbol{k}_r$, *social welfare is lower; otherwise, social welfare is higher.*
Given spatial limitations, we detail the expression for $\hat{k}_r,\boldsymbol{k}_r$ *in the Appendix.*

Analogous to the retailer advertising case, in the manufacturer advertising case, when $\frac{1}{2}(r_s+r_t)<r_m$, that is, when the retailer underestimates the manufacturer's financing rate, the manufacturer can acquire higher profit by concealing her real financing rate information, which may motivate her to deliberately conceal the real information. Furthermore, in this situation, in the manufacturer advertising case, asymmetric information on the financing rate also allows for higher consumer surplus. Moreover, if $\frac{1}{2}(r_s+r_t)<r_m$, when the retailer's initial capital is at a different low level, information asymmetry increases the retailer's profit and social welfare. This outcome shows that compared with a policy of concealing information, an information disclosure policy may harm the retailer, which differs from the result of [12] that an information disclosure strategy always benefits the retailer in the manufacturer advertising case.

5.1.3. *The comparison of retailer advertising and manufacturer advertising*

Theorem 5.5. *In the scenario with asymmetric information on the financing interest rate, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{16r_m^2(r_m+1)(r_s+r_t+2)}<b$, *the retailer's profit is lower but the manufacturer's profit is higher;*
(ii) when $\frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}<b\le\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{16r_m^2(r_m+1)(r_s+r_t+2)}$, *the retailer and manufacturer's profits are higher;*
(iii) when $\frac{u^2(r_m+1)}{4(r_s+r_t+2)}<b\le\frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}$, *the retailer and manufacturer's profits are lower.*

In contrast to the scenario with symmetric information in Theorem 4.6, as stated in Theorem 5.5, in the scenario with asymmetric information on the manufacturer's financing interest rate, when $\frac{1}{2}(r_s+r_t)<r_m$, compared to manufacturer advertising, the probability that the retailer advertises voluntarily increases, that is, the probability of retailer advertising creating a win-win situation that improves the profits of both the



(a) (b)

FIGURE 2. Retailer advertising *vs.* manufacturer advertising under asymmetric information on the financing interest rate. (a) The manufacturer's profit. (b) The retailer's profit.

retailer and the manufacturer increases. In addition, when $\frac{1}{2}(r_s + r_t) < r_m$, since $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{16r_m^2(r_m+1)(r_s+r_t+2)}$ exceeds $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, the probability that the retailer wants the manufacturer to advertise and the manufacturer wants the retailer to advertise decreases. Furthermore, under this circumstance, due to the length of interval $(\frac{u^2(r_m+1)}{4(r_s+r_t+2)}, \frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}]$ being higher than that of interval $(\frac{u^2}{8}, \frac{u^2(3r_m+2)}{8(r_m+1)^2}]$, the probability that the manufacturer voluntarily advertises also increases. On the contrary, when $\frac{1}{2}(r_s + r_t) > r_m$, the result is just the opposite of the above. That is, when the retailer underestimates the manufacturer's financing rate, the probability that the retailer and manufacturer voluntarily advertise increases, but the probability that the retailer prefers the manufacturer to advertise and the manufacturer prefers the retailer to advertise decreases. Conversely, when the retailer overestimates the manufacturer's financing rate, the result is reversed. In addition, by comparing the scenarios with symmetric and asymmetric information, it can be observed that whether the manufacturer discloses or conceals the financing rate information affects the advertising strategies of the retailer and the manufacturer. Figure 2 graphically illustrates the results of Theorem 5.5 with the following parameterization: $u = 5$, $r_m = 0.1$, $k_r = 2$, $r_s = 0.01$, and $r_t = 0.3$.

5.2. The scenario with asymmetric information on the retailer's initial capital

5.2.1. Retailer advertising

In the scenario with asymmetric information regarding the retailer's initial capital, the retailer keeps his initial capital information confidential. In this situation, we assume that the manufacturer estimates that the retailer's initial capital k_r follows a uniform distribution on $[k_s, k_t]$. As stated in the scenario where the retailer advertises under symmetric information, the retailer's profit is $p[(a+1)u - p] - k_r - [w((a+1)u - p) + a^2b - k_r](r_m + 1)$. In the scenario with asymmetric information on the retailer's initial capital, since the retailer clearly knows his own real initial capital information, his profit is the same as that in the scenario with symmetric information, which follows

$$\pi_{\rm rr}^{ac} = p((a+1)u - p) - k_r - [w((a+1)u - p) + a^2b - k_r](r_m + 1). \tag{9}$$

As stated in the scenario where the retailer advertises under symmetric information, the manufacturer's profit is $[w((a+1)u - p) - (k_r - a^2b)](r_m + 1) - a^2b + k_r$. As the manufacturer estimates that the retailer's initial capital information k_r follows a uniform distribution on $[k_s, k_t]$, the manufacturer's profit can be described as

$$\pi_{\text{mr}}^{ac} = \int_{k_s}^{k_t} \frac{1}{k_t - k_s}\{[w((a+1)u-p) - (k_r - a^2 b)](r_m+1) - a^2 b + k_r\} dk_r. \tag{10}$$

Proposition 5.6. *In the retailer advertising case with asymmetric information on the retailer's initial capital, when* $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$*, the optimal wholesale price is* $w_r^{ac} = \frac{u(8b-2u^2)r_m+4bur_m^2-u(u^2-4b)}{(r_m+1)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$*; the retailer's optimal retail price and advertising effort are* $p_r^{ac} = \frac{u(12b-2u^2)r_m+6bur_m^2+6bu-u^3}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$ *and* $a_r^{ac} = \frac{u^2(r_m+1)}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$*, respectively.*

From Proposition 5.6, in the retailer advertising case, regardless of whether the retailer keeps the initial capital information secret, the retailer's retail price and advertising effort decisions and the manufacturer's wholesale price decision remain the same. This is because the retail price, advertising effort and wholesale price are independent of the initial capital and thus will not be affected by the concealment of the retailer's initial capital information.

Corollary 5.7. *In the retailer advertising case,*
(i) the retailer's profit and consumer surplus are identical in the scenarios of symmetric and asymmetric information on the retailer's initial capital;
(ii) compared to the symmetric information scenario, in the scenario with asymmetric information on the retailer's initial capital, when $\frac{k_s+k_t}{2} < k_r$*, the manufacturer's profit and social welfare are higher; otherwise, the manufacturer's profit and social welfare are lower.*

As demonstrated in Proposition 5.6, regardless of whether the retailer discloses or conceals his initial capital information, the retailer's retail price and advertising effort remain the same, and thus consumer surplus also remains unchanged. Moreover, in the asymmetric information scenario, since the retailer is familiar with his real initial capital information, he can obtain the same profit as in the symmetric information scenario. However, under the effect of information asymmetry, when $\frac{k_s+k_t}{2} < k_r$, the manufacturer acquires more profit than under information symmetry, which improves social welfare.

5.2.2. Manufacturer advertising

As stated in the scenario with manufacturer advertising under information symmetry, the retailer's profit is $p((a+1)u-p) - k_r - (r_m+1)[w((a+1)u-p) - k_r]$. In the scenario with information asymmetry on the retailer's initial capital, since the retailer clearly knows his own real initial capital information, his profit is the same as that in the scenario with symmetric information, which follows

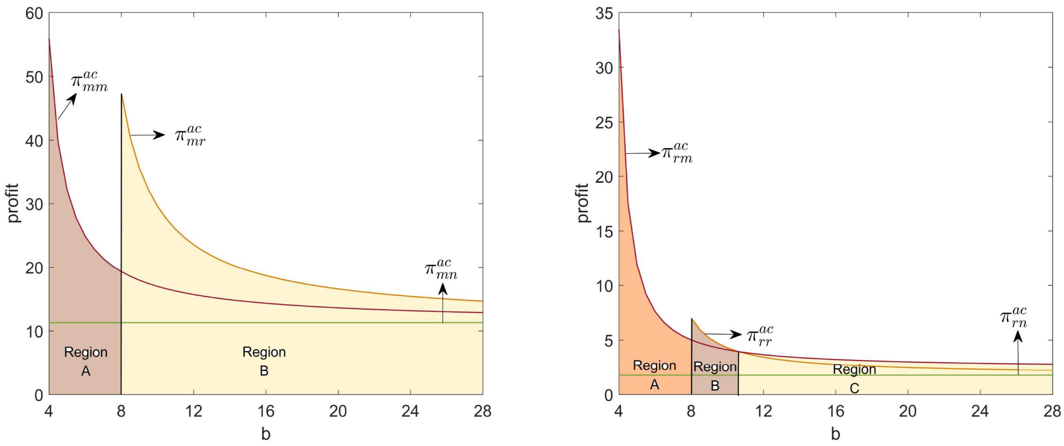
$$\pi_{\text{rm}}^{ac} = p((a+1)u-p) - k_r - (r_m+1)[w((a+1)u-p) - k_r]. \tag{11}$$

As indicated in the scenario of manufacturer advertising under symmetric information, the manufacturer's profit is $(r_m+1)[w((a+1)u-p) - k_r] + k_r - a^2 b$. As the manufacturer estimates that the retailer's initial capital information k_r follows a uniform distribution on $[k_s, k_t]$, the manufacturer's profit can be described as

$$\pi_{\text{mm}}^{ac} = \int_{k_s}^{k_t} \frac{1}{k_t - k_s}\{(r_m+1)[w((a+1)u-p) - k_r] + k_r - a^2 b\} dk_r. \tag{12}$$

Proposition 5.8. *In the manufacturer advertising case with asymmetric information on the retailers initial capital, when* $\frac{u^2}{8} < b$*, the manufacturer's optimal wholesale price and advertising effort are* $w_m^{ac} = \frac{4bu}{(8b-u^2)(r_m+1)}$ *and* $a_m^{ac} = \frac{u^2}{8b-u^2}$*, respectively; the retailer's optimal retail price is* $p_m^{ac} = \frac{6bu}{8b-u^2}$*.*

From Proposition 5.8, similar to the retailer advertising case, in the manufacturer advertising case, since the retail price, advertising effort and wholesale price decisions are not related to the retailer's initial capital, the retailer and the manufacturer will not make any changes to their operational decisions regardless of whether the retailer's initial capital information is disclosed or concealed.



(a) (b)

FIGURE 3. Retailer advertising *vs.* manufacturer advertising under asymmetric information on the retailer's initial capital. (a) The manufacturer's profit. (b) The retailer's profit.

Corollary 5.9. *In the manufacturer advertising case,*
(i) the retailer's profit and consumer surplus are the same in the scenarios of symmetric and asymmetric information on the retailer's initial capital;
(ii) compared to the symmetric information scenario, in the scenario with asymmetric information on the retailer's initial capital, when $\frac{k_s+k_t}{2} < k_r$, *the manufacturer's profit and social welfare are higher; otherwise, the manufacturer's profit and social welfare are lower.*

Similar to the retailer advertising case, in the manufacturer advertising case, since whether the retailer discloses or conceals his real initial capital information, the retail price, advertising effort, and wholesale price decisions of the retailer and the manufacturer remain the same, then the retailer's profit and consumer surplus remain unchanged. In addition, in the manufacturer advertising case, when $\frac{k_s+k_t}{2} < k_r$, the retailer concealing his initial capital information improves the manufacturer's profit and social welfare. That is, in the manufacturer advertising case, the probability of the manufacturer obtaining a higher profit without seeking the retailer's real initial capital information is the same as in the retailer advertising case. Corollaries 5.2, 5.4, 5.7 and 5.9 show that in both the retailer advertising and manufacturer advertising cases, whether the supply chain participants choose to conceal or disclose private information depends on which type of information is private.

5.2.3. The comparison of retailer advertising and manufacturer advertising

Theorem 5.10. *In the scenario with asymmetric information on the retailer's initial capital, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2} < b$, *the manufacturer's profit, consumer surplus and social welfare are higher, but the retailer's profit is lower;*
(ii) when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \leq \frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, *the manufacturer and retailer's profits, consumer surplus and social welfare are higher;*
(iii) when $\frac{u^2}{8} < b \leq \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, *the manufacturer and retailer's profits, consumer surplus and social welfare are lower.*

Interestingly, when combining Theorems 4.6 and 5.10, regardless of whether the retailer discloses or conceals his initial capital information, the retailer and manufacturer do not change their advertising strategies. The reason for this result is as follows. As stated in Propositions 5.6 and 5.8, the retail price, advertising effort and wholesale price decisions are not affected by the retailer's initial capital information in either the retailer advertising or the manufacturer advertising case; thus, the application conditions and appropriate investor in advertising for the retailer and manufacturer in the scenario with asymmetric information on the retailer's initial capital are same as those in the scenario with symmetric information. However, in these two scenarios, although the advantages and disadvantages of retailer advertising compared to manufacturer advertising are roughly the same (*i.e.*, increasing or decreasing the manufacturer's profit and social welfare), the specific extent of the advantages and disadvantages are different (*i.e.*, the extent to which the manufacturer's profit and social welfare increase or decrease differs). Figure 3 shows the results of Theorem 5.10 with the following parameter settings: $u = 5$, $r_m = 0.1$, $k_r = 2$, $k_s = 1$, $k_t = 5$.

6. Extension

6.1. Nash game

In the previous section, we discussed the scenario where there is a Stackelberg game between the retailer and manufacturer. In this section, we focus on the scenario where there is a Nash game between the retailer and manufacturer and compare the changes in their advertising strategies under these two game scenarios.

6.1.1. The symmetric information scenario

Theorem 6.1. *In the symmetric information scenario, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(12-r_m^4-4r_m^3+3r_m^2+22r_m)} \leq b$*, the manufacturer's profit is higher, but the retailer's profit, consumer surplus and social welfare are lower;*
(ii) when $\frac{u^2(r_m+1)}{2r_m+6} < b < \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(12-r_m^4-4r_m^3+3r_m^2+22r_m)}$*, the manufacturer's profit and social welfare are higher, but the retailer's profit and consumer surplus are lower.*

In the symmetric information scenario, different from the Stackelberg game, under the Nash game, the retailer prefers the manufacturer to advertise, while the manufacturer prefers the retailer to advertise. Both of them desire the other party to advertise, rather than advertising themselves. In addition, from the perspective of social welfare, when the cost coefficient of advertising effort b is high, social welfare brought by retailer advertising is lower, and manufacturer advertising should be advocated at this case. On the contrary, when b is low, social welfare produced by manufacturer advertising is lower, and retailer advertising should be promoted in this situation. Combining Theorems 4.6 and 6.1, it is clear that the difference in the types of games between the retailer and manufacturer impose a significant impact on their advertising strategies.

6.1.2. The scenario with asymmetric information on the manufacturer's financing interest rate

Theorem 6.2. *In the scenario with asymmetric information on the manufacturer's financing interest rate, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\bar{b}_1 \leq b$*, the manufacturer's profit is higher, but the retailer's profit, consumer surplus and social welfare are lower;*
(ii) when $\frac{u^2(r_m+1)}{2r_m+6} < b < \bar{b}_1$*, the manufacturer's profit and social welfare are higher, but the retailer's profit and consumer surplus are lower.*
Due to spatial limitations, we detail the expression for $\bar{b}_1$ *in the Appendix.*

In the Nash game, in the scenario with asymmetric information on the manufacturer's financing interest rate, the retailer and manufacturer's advertising strategies are the same as in the symmetric information scenario, *i.e.*, the retailer prefers the manufacturer to advertise, but the manufacturer wants the retailer to advertise instead. However, in the Stackelberg game, as stated in Theorem 5.5, the retailer and manufacturer's advertising strategies differ in the scenario of asymmetric information on the manufacturer's financing interest rate from the symmetric information scenario. This result suggests that the effect of the manufacturer disclosing or concealing financing interest rate information on the retailer and manufacturer's advertising strategies depends on the type of game between them.

6.1.3. The scenario with asymmetric information on the retailer's initial capital

Theorem 6.3. *In the scenario with asymmetric information on the retailer's initial capital, compared to the manufacturer advertising case, in the retailer advertising case,*
(i) when $\frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)} \le b$*, the manufacturer's profit is higher, but the retailer's profit, consumer surplus and social welfare are lower;*
(ii) when $\frac{u^2(r_m+1)}{2r_m+6} < b < \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)}$*, the manufacturer's profit and social welfare are higher, but the retailer's profit and consumer surplus are lower.*

Taking Theorems 6.1–6.3 together, we find that in the Nash game, information symmetry, asymmetric information on the manufacturer's financing rate and asymmetric information on the retailer's initial capital do not exert any effect on the retailer and manufacturer's advertising strategies, *i.e.*, both the retailer and the manufacturer prefer the other party to advertise rather than doing so themselves. This finding differs from the Stackelberg game case in which asymmetric information on the manufacturer's financing rate leads to different advertising strategies for the manufacturer and the retailer from the scenarios with symmetric and asymmetric information on the retailer's initial capital. This result indicates that the different types of games between the retailer and the manufacturer affect their advertising strategies, whether under symmetric or asymmetric information.

6.2. Cooperative advertising

In the previous section, we discussed the scenarios where the retailer or the manufacturer advertises. In the following section, we focus on the situation where the retailer and the manufacturer advertise jointly, *i.e.*, the retailer and the manufacturer share the advertising cost.

6.2.1. The symmetric information scenario

Different from the scenario where the manufacturer and the retailer advertise separately, in the scenario where the manufacturer and the retailer advertise together, the retailer only pays $a^2b(1-\delta)$ for advertising, and the manufacturer bears the advertising cost $a^2b\delta$. Hence, in the cooperative advertising scenario, the retailer's profit is $\pi_{rc}^s = p((a+1)u-p) - [w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) - (k_r - a^2b(1-\delta)) - a^2b(1-\delta)$, and the manufacturer's profit is $\pi_{mc}^s = [w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) + k_r - a^2b(1-\delta) - a^2b\delta$. Obviously, in this scenario, when $\frac{9u^2}{32} < b$, the manufacturer's optimal wholesale price is $w_c^s = \frac{16bu-3u^3}{(32b-9u^2)(r_m+1)}$, the optimal proportion of the advertising cost shared by the manufacturer is $\delta_c^s = \frac{3r_m+1}{3(r_m+1)}$, the retailer's optimal retail price is $p_c^s = \frac{24bu-3u^3}{32b-9u^2}$, and the retailer's optimal advertising effort is $a_c^s = \frac{6u^2}{32b-9u^2}$.

Theorem 6.4. *In the symmetric information scenario, compared to the non-advertising case, in the cooperative advertising case,*
(i) when $\frac{27u^2}{64} \le b$*, the manufacturer and retailer's profits, consumer surplus and social welfare are higher;*
(ii) when $\frac{9u^2}{32} < b < \frac{27u^2}{64}$*, the manufacturer's profit, consumer surplus and social welfare are higher, but the retailer's profit is lower.*

Theorem 6.4 shows that in the symmetric information scenario, when $\frac{9u^2}{32} < b < \frac{27u^2}{64}$, the retailer refuses to launch advertising because it reduces his benefit. However, when $\frac{27u^2}{64} \le b$, the retailer is willing to advertise, which improves the profits of the retailer and manufacturer, consumer surplus and social welfare, achieving a win-win-win-win situation. In addition, in this scenario, the retailer has to bear $\frac{2}{3(r_m+1)}$ of all advertising expenses, but the manufacturer only pays $\frac{3r_m+1}{3(r_m+1)}$. The proportion of the advertising cost borne by the retailer and the manufacturer depends only on the financing interest rate set by the manufacturer. That is, the higher the financing interest rate is, the smaller the proportion of the advertising cost borne by the retailer and the larger the proportion of the advertising cost borne by the manufacturer. Furthermore, the advertising effort is higher in the scenario with cooperative advertising than in that with separate advertising.

6.2.2. The scenario with asymmetric information on the manufacturer's financing interest rate

In the scenario with asymmetric information on the financing interest rate, similar to the scenario where the manufacturer and the retailer advertise separately, in the scenario where the manufacturer and the retailer advertise together, we still assume that the retailer does not know the manufacturer's real financing interest rate information, and the retailer estimates that the manufacturer's financing interest rate follows a uniform distribution on $[r_s, r_t]$. According to the profit functions of the retailer and manufacturer in the symmetric information scenario, under asymmetric information on the manufacturer's financing interest rate, we can obtain that the retailer's profit is $\pi_{rc}^{ai} = \int_{r_s}^{r_t} \frac{1}{r_t - r_s}\{p((a+1)u-p) - [w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) - (k_r - a^2b(1-\delta)) - a^2b(1-\delta)\}dr_m$, and the manufacturer's profit is $\pi_{mc}^{ai} = [w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) + k_r - a^2b(1-\delta) - a^2b\delta$. Obviously, in this scenario, when $\frac{9u^2(r_m+1)}{16(r_s+r_t+2)} < b$, the manufacturer's optimal wholesale price is $w_c^{ai} = \frac{2u(8br_s+8br_t+16b-3u^2r_m-3u^2)}{(r_s+r_t+2)(16br_s+16br_t+32b-9u^2r_m-9u^2)}$, the optimal proportion of advertising cost shared by the manufacturer is $\delta_c^{ai} = \frac{3r_m+1}{3(r_m+1)}$, the retailer's optimal retail price is $p_c^{ai} = \frac{3u(-4br_s-4br_t-8b+u^2r_m+u^2)}{-16br_s-16br_t-32b+9u^2r_m+9u^2}$, and the retailer's optimal advertising effort is $a_c^{ai} = \frac{6u^2(r_m+1)}{16br_s+16br_t+32b-9u^2r_m-9u^2}$.

Theorem 6.5. *In the scenario with asymmetric information on the financing interest rate, compared to the non-advertising case, in the cooperative advertising case,*
(i) when $\frac{27u^2(r_m+1)}{32(r_s+r_t+2)} \leq b$, *the manufacturer and retailer's profits, consumer surplus and social welfare are higher;*
(ii) when $\frac{9u^2(r_m+1)}{16(r_s+r_t+2)} < b < \frac{27u^2(r_m+1)}{32(r_s+r_t+2)}$, *the manufacturer's profit, consumer surplus and social welfare are higher, but the retailer's profit is lower.*

In the case with cooperative advertising between the retailer and manufacturer, compared with the symmetric information scenario, asymmetric information on the manufacturer's financing rate affects the retailer's advertising condition and advertising effort but does not influence the sharing proportion of advertising costs between the retailer and the manufacturer. Specifically, when $\frac{1}{2}(r_s+r_t) > r_m$, in the scenario with asymmetric information on the manufacturer's financing rate, the probability that the retailer advertises aggressively rises, increasing from when $\frac{27u^2}{64} \leq b$ to when $\frac{27u^2(r_m+1)}{32(r_s+r_t+2)} \leq b$, but the retailer's advertising effort declines, decreasing from $\frac{6u^2}{32b-9u^2}$ to $\frac{6u^2(r_m+1)}{16br_s+16br_t+32b-9u^2r_m-9u^2}$. On the contrary, when $\frac{1}{2}(r_s+r_t) < r_m$, the retailer is less likely to advertise, but his advertising effort is stronger. This outcome suggests that when the retailer overestimates the manufacturer's financing rate, the retailer is more eager to advertise, but his advertising effort decreases; otherwise, when the retailer underestimates the manufacturer's financing rate, the retailer is less eager to advertise, but his advertising effort increases.

6.2.3. The scenario with asymmetric information on the retailer's initial capital

In the scenario with asymmetric information on the initial capital, we still assume that the manufacturer does not know the retailer's real initial capital information k_r, and the manufacturer estimates that the retailer's initial capital follows a uniform distribution on $[k_s, k_t]$. According to the profit functions of the retailer and the manufacturer in the scenario with symmetric information, in the scenario with asymmetric information on the retailer's initial capital, we obtain that the retailer's profit is $\pi_{rc}^{ac} = p((a+1)u-p) - [w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) - (k_r - a^2b(1-\delta)) - a^2b(1-\delta)$, and the manufacturer's profit is $\pi_{mc}^{ac} = \int_{k_s}^{k_t} \frac{1}{k_t-k_s}\{[w((a+1)u-p) - (k_r - a^2b(1-\delta))](r_m+1) + k_r - a^2b(1-\delta) - a^2b\delta\}dk_r$. Obviously, in this scenario, when $\frac{9u^2}{32} < b$, the manufacturer's optimal wholesale price is $w_c^{ac} = \frac{16bu-3u^3}{(32b-9u^2)(r_m+1)}$, the optimal proportion of the advertising cost shared by the manufacturer is $\delta_c^{ac} = \frac{3r_m+1}{3(r_m+1)}$, the retailer's optimal retail price is $p_c^{ac} = \frac{24bu-3u^3}{32b-9u^2}$, and the retailer's optimal advertising effort is $a_c^{ac} = \frac{6u^2}{32b-9u^2}$.

Theorem 6.6. *In the scenario with asymmetric information on the initial capital, compared to the non-advertising case, in the cooperative advertising case,*
(i) when $\frac{27u^2}{64} \leq b$*, the manufacturer and retailer's profits, consumer surplus and social welfare are higher;*
(ii) when $\frac{9u^2}{32} < b < \frac{27u^2}{64}$*, the manufacturer's profit, consumer surplus and social welfare are higher, but the retailer's profit is lower.*

Similar to the scenario where the retailer and the manufacturer advertise separately, in the scenario where the manufacturer and the retailer advertise together, compared to the symmetric information scenario, asymmetric information on the retailer's initial capital does not affect the retailer's advertising condition or advertising effort or even the sharing proportion of advertising costs between the retailer and the manufacturer. In addition, combining the scenarios with asymmetric information on the manufacturer's financing interest rate and the retailer's initial capital in the case of cooperative advertising between the manufacturer and the retailer in Theorems 6.5 and 6.6 reveals that the different types of information asymmetry between the manufacturer and the retailer exert different effects on advertising condition, advertising effort and advertising cost-sharing. This result indicates that the manufacturer and the retailer should appropriately adjust their advertising strategies based on the types of information that they cannot acquire.

7. Concluding remarks

7.1. Conclusions

In this paper, we analyze the equilibrium pricing and advertising decisions of a capital-constrained retailer and a manufacturer in scenarios with symmetric and asymmetric information with respect to financing information (*i.e.*, the manufacturer's financing interest rate and the retailer's initial capital). By exploring these three scenarios, we answer the following two questions and arrive at some important results.

First, we examine the application conditions, appropriate investor and role of advertising in a capital-constrained supply chain with symmetric and asymmetric information. Specifically, in the symmetric information scenario, when the advertising cost coefficient is low, manufacturer advertising is a more favorable strategy that improves the profits of both the retailer and the manufacturer, consumer surplus and social welfare. When the advertising cost coefficient is moderate, retailer advertising is a more advantageous scheme. However, when it is high, retailer advertising improves the manufacturer's profit, consumer surplus and social welfare but worsens the retailer's profit relative to manufacturer advertising. In addition, in the scenario with asymmetric information on the retailer's initial capital, the results for the application condition, appropriate investor and role of advertising of the retailer and the manufacturer are the same as those in the symmetric information scenario. However, in the scenario with asymmetric information on the manufacturer's financing rate, when the retailer underestimates the financing rate, the probability that the retailer and the manufacturer voluntarily advertise increases, but the probability that the retailer prefers the manufacturer to advertise and the manufacturer prefers the retailer to advertise decreases. Conversely, when the retailer overestimates the financing rate, this result is be reversed.

Second, we analyze the multiple effects of asymmetric information. Specifically, in the scenario with asymmetric information on the financing interest rate, the manufacturer keeping the financing interest rate confidential has complex effects on the retailer's profit and enables the manufacturer to collect a higher profit when the retailer underestimates the financing interest rate. However, in the scenario with asymmetric information on the retailer's initial capital, the retailer concealing the initial capital information does not exert any influence on the retailer's profit and consumer surplus but improves the manufacturer's profit and social welfare when the manufacturer underestimates the retailer's initial capital.

In the extension section, we extend the Stackelberg game case between the retailer and the manufacturer to the Nash game case. We find that in the Nash game, the retailer hopes that the manufacturer will advertise and the manufacturer wants the retailer to advertise, *i.e.*, they both prefer the other party to advertise rather than advertising themselves, regardless of whether the information about the manufacturer's financing interest rate and the retailer's initial capital is symmetric or asymmetric.

Furthermore, we extend the scenario where the retailer and the manufacturer advertise separately to the scenario where the retailer and the manufacturer advertise together. In both the symmetric and asymmetric information scenarios, when the advertising cost coefficient is high, cooperative advertising increases the retailer and manufacturer's profits, consumer surplus and social welfare, but when it is low, cooperative advertising increases the manufacturer's profit, consumer surplus and social welfare but decreases the retailer's profit. In addition, in the cooperative advertising case, compared with the symmetric information scenario, information asymmetry on the manufacturer's financing rate affects the retailer's advertising condition and advertising effort and does not influence the sharing proportion of advertising costs between the retailer and the manufacturer, but information asymmetry on the retailer's initial capital does not affect these three aspects.

7.2. Managerial insights

In view of the conclusions drawn from the above study, we provide managerial insights from these conclusions from the following two aspects.

First, we address the formulation of the advertising strategies of the retailer and the manufacturer. When the retailer and manufacturer formulate their advertising strategies, they should not only pay attention to the market size, advertising cost coefficient, retailer's initial capital and manufacturer's financing interest rate but also on whether the retailer and the manufacturer have adequate access to all the information. In addition, for the retailer and manufacturer, the pursuit of maximizing their own interests may lead to them preferring different advertising strategies in some situations, even if they benefit more or less from all these advertising strategies. Under this circumstance, rational decision-makers should conduct active negotiation and may consider that the party with more profit should compensate the party with less profit to achieve a win-win situation for both parties and thus avoid being unable to benefit from advertising due to inconsistencies in negotiation.

Second, we address the information disclosure and concealment strategies of the retailer and manufacturer. Since concealing the real financing interest rate information may allow the manufacturer to obtain higher profits, this will induce the manufacturer to deliberately conceal its financing interest rate information in cooperating with the retailer. In addition, the concealment of the real financing rate information may benefit both the retailer and the manufacturer, which may create a win-win situation. Moreover, since the concealment of the real initial capital information does not affect the retailer's profit but improves the manufacturer's profit and social welfare when the manufacturer underestimates the retailer's initial capital, the retailer does not need to intentionally conceal its initial capital information.

7.3. Limitations

This research is subject to certain limitations that recommend further study. First, this study analyzes the problem of advertising strategies of the retailer and manufacturer in a scenario with determined demand dependent on price. However, in reality, the retailer generally faces the random market demand. In future research, we can assume that the market demand faced by the retailer is random and adopt the newsvendor model to explore the advertising strategies of the retailer and manufacturer in the random demand scenario. Second, this study primarily adopts game theory to analyze the pricing and advertising decisions. However, in reality, there are many different domains where advanced optimization algorithms have been applied as solution approaches, such as online learning, scheduling, multiobjective optimization, transportation, medicine, data classification, and so forth. In future research, we can adopt advanced optimization algorithms for this decision problem, such as adaptive heuristics and metaheuristics, self-adaptive algorithms, diffused algorithms, and island algorithms [10, 11, 14, 25, 38, 42].

Appendix A.

Proof of Proposition 4.1.
From equation (1), taking the first derivative of π_{rr}^s with respect to p and a, we have $\frac{\partial \pi_{rr}^s}{\partial p} = (a+1)u + w(r_m + 1) - 2p$ and $\frac{\partial \pi_{rr}^s}{\partial a} = -2ab(r_m+1) - uw(r_m+1) + pu$. Since $\frac{\partial^2 \pi_{rr}^s}{\partial p^2} = -2$, $\frac{\partial^2 \pi_{rr}^s}{\partial a^2} = -2b(r_m+1)$, $\frac{\partial^2 \pi_{rr}^s}{\partial p \partial a} = \frac{\partial^2 \pi_{rr}^s}{\partial a \partial p} = u$, Hesse matrix is $\frac{\partial^2 \pi_{rr}^s}{\partial p^2}\frac{\partial^2 \pi_{rr}^s}{\partial a^2} - \frac{\partial^2 \pi_{rr}^s}{\partial p \partial a}\frac{\partial^2 \pi_{rr}^s}{\partial a \partial p} = 4b(r_m+1) - u^2$. When $b > \frac{u^2}{4(r_m+1)}$, given the wholesale price w, the optimal retail price and advertising effort are $p_r^s = -\frac{2b(r_m+1)(w(r_m+1)+u) - u^2 w(r_m+1)}{u^2 - 4b(r_m+1)}$, $a_r^s = -\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2}$, respectively.

According to equation (2), the manufacturer's profit is $\pi_{mr}^s(w) = [w((a+1)u-p)-(k_r-a^2b)](r_m+1)+k_r-a^2b$. When the manufacturer (the leader) decides the optimal wholesale price, she considers the decisions of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi_{mr}^s(w) = [w((a+1)u-p)-(k_r-a^2b)](r_m+1)+k_r-a^2b = [w((-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2}+1)u + \frac{2b(r_m+1)(w(r_m+1)+u)-u^2w(r_m+1)}{u^2-4b(r_m+1)}) - (k_r - (-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2})^2 b)](r_m+1) + k_r - (-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2})^2 b$. Taking the first derivative of $\pi_{mr}^s(w)$ with respect to w, we can obtain that $\frac{\partial \pi_{mr}^s}{\partial w} = \frac{2b(r_m+1)[-r_m^2(4b(u-6w)+3u^2w)+r_m(u^2(2u-5w)-8b(u-3w))-(4b-u^2)(u-2w)+8bwr_m^3]}{-(-4br_m-4b+u^2)^2}$, $\frac{\partial^2 \pi_{mr}^s}{\partial w^2} = -\frac{2b(r_m+1)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)}{(-4br_m-4b+u^2)^2} < 0$ when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$. For $\frac{3r_m+2}{8(r_m+1)^2} > \frac{1}{4(r_m+1)}$, when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, the manufacturer's optimal wholesale price is $w_r^s = \frac{r_m(8bu-2u^3)+4bur_m^2-(u^2-4b)u}{(r_m+1)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$. Thus, by inverting the manufacturer's wholesale price decision into the retailer's decisions p_r^s and a_r^s, we can get that the optimal advertising effort and retail price are $a_r^s = \frac{-u^2(r_m+1)}{(3u^2-16b)r_m-8br_m^2-8b+2u^2}$, $p_r^s = \frac{r_m(12bu-2u^3)+6bur_m^2+6bu-u^3}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$, respectively.

Proof of Lemma 4.2.
From Proposition 4.1, we have $\frac{\partial \pi_{mr}^s}{\partial r_m} = -\frac{k_r((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2+bu^4(3r_m^2+4r_m+1)}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2} < 0$, $\frac{\partial CS_r^s}{\partial r_m} = -\frac{4b^2u^4(r_m+1)^3(3r_m+1)}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^3} < 0$, $\frac{\partial SW_r^s}{\partial r_m} = -\frac{2bu^4(r_m+1)((58b-6u^2)r_m^2+(38b-6u^2)r_m+26br_m^3+6b-u^2)}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^3} < 0$. In addition, as $\frac{\partial \pi_{rr}^s}{\partial r_m} = \frac{k_r((16b-3u^2)r_m+8br_m^2+8b-2u^2)^3-bu^4r_m(r_m+1)^2(16br_m+16b-3u^2)}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^3}$, we can get that when $b \leq \hat{b}_1 = \frac{\frac{u^6r_m(r_m+1)^6(6k_r(9r_m^2+14r_m+5)+u^2r_m)}{\sqrt[3]{H}} + \sqrt[3]{H} + u^2(r_m+1)^3(12k_r(3r_m^2+5r_m+2)+u^2r_m)}{96k_r(r_m+1)^6}$, $\frac{\partial \pi_{rr}^s}{\partial r_m} \leq 0$; when $b > \hat{b}_1$, $\frac{\partial \pi_{rr}^s}{\partial r_m} > 0$; where $H = u^8r_m(r_m+1)^9(108k_r^2(r_m+1)^2(9r_m^2+9r_m+2) + u^4r_m^2 + 3\sqrt{3}k_r(r_m+1)\sqrt{(-r_m-1)(8u^2k_rr_m(81r_m^2+108r_m+35) - 432k_r^2(r_m+1)(9r_m^2+9r_m+2)^2 + 9u^4r_m^2(r_m+1))} + 9u^2k_rr_m(9r_m^2+14r_m+5))$. Note that the case of non-financing with sufficient funds is a special case when $r_m = 0$ in the case of financing with capital shortage. In addition, in the capital-constrained retailer scenario, the condition for the existence of the optimal solution is $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$. In the well-funded retailer scenario, the condition for the existence of the optimal solution is $\frac{u^2}{4} < b$. Thus, compared to the well-funded retailer scenario, in the capital-constrained retailer scenario, (1) when $\hat{b}_1 < b$, the manufacturer's profit, consumer surplus and social welfare are lower, but the retailer's profit is higher; (2) when $\frac{u^2}{4} < b \leq \hat{b}_1$, the retailer's and the manufacturer's profits, consumer surplus and social welfare are lower; (3) when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \leq \frac{u^2}{4}$, the retailer's and the manufacturer's profits, consumer surplus and social welfare are higher.

Proof of Proposition 4.3.
According to equation (3), taking the first derivative of π_{rm}^s with respect to p, we have $\frac{\partial \pi_{rm}^s}{\partial p} = (a+1)u + w(r_m+1) - 2p$ and $\frac{\partial^2 \pi_{rm}^s}{\partial p^2} = -2 < 0$. Given the wholesale price w and the advertising effort a, the optimal retail price is $p_m^s = \frac{1}{2}(au + wr_m + u + w)$.

From equation (4), the manufacturer's profit is $\pi^s_{mm}(w,a) = [w((a+1)u-p)-k_r](r_m+1)+k_r-a^2b$. When the manufacturer (the leader) decides the optimal wholesale price and advertising effort, she considers the decision of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi^s_{mm}(w,a) = [w((a+1)u-p)-k_r](r_m+1)+k_r-a^2b = [w((a+1)u-\frac{1}{2}(au+wr_m+u+w))-k_r](r_m+1)+k_r-a^2b$. Taking the first derivative of π^s_{mm} with respect to w and a, we get $\frac{\partial \pi^s_{mm}}{\partial w} = (r_m+1)[\frac{1}{2}(-au-wr_m-u-w)+(a+1)u+\frac{1}{2}w(-r_m-1)]$, $\frac{\partial \pi^s_{mm}}{\partial a} = -2ab-\frac{1}{2}cu+\frac{1}{2}uw(r_m+1)$. Since $\frac{\partial^2 \pi^s_{mm}}{\partial w^2} = -(1+r_m)^2$, $\frac{\partial^2 \pi^s_{mm}}{\partial a^2} = -2b$, $\frac{\partial^2 \pi^s_{mm}}{\partial w \partial a} = \frac{\partial^2 \pi^s_{mm}}{\partial a \partial w} = \frac{1}{2}u(1+r_m)$, Hesse matrix is $\frac{\partial^2 \pi^s_{mm}}{\partial w^2}\frac{\partial^2 \pi^s_{mm}}{\partial a^2} - \frac{\partial^2 \pi^s_{mm}}{\partial w \partial a}\frac{\partial^2 \pi^s_{mm}}{\partial a \partial w} = 2b(1+r_m)^2 - \frac{1}{4}u^2(1+r_m)^2$. Thus, when $b > \frac{u^2}{8}$, the optimal wholesale price and advertising effort are $w^s_m = \frac{4bu}{(8b-u^2)(r_m+1)}$, $a^s_m = \frac{u^2}{8b-u^2}$, respectively. Thus, by inverting the manufacturer's wholesale price and advertising effort decisions into the retailer's decision p^s_m, the optimal retail price is $p^s_m = \frac{-6bu}{u^2-8b}$.

Proof of Corollary 4.4.
In the capital-constrained supply chain, the optimal solutions of retailer advertising exist when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$. Thus, we have $p^s_r - p^s_n = \frac{u^3(r_m+2)}{4((16b-3u^2)r_m+8br_m^2+8b-2u^2)} > 0$. The optimal solutions of manufacturer advertising exist when $b > \frac{u^2}{8}$, then we obtain $p^s_m - p^s_n = \frac{3u^3}{32b-4u^2} > 0$. Note that $p^s_m - p^s_r = \frac{u^3(2(5b-u^2)r_m+6br_m^2+2b-u^2)}{(8b-u^2)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$. We can obtain that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b \le \frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}$, $p^s_m - p^s_r \le 0$; when $b > \frac{u^2(2r_m+1)}{2(3r_m^2+5r_m+1)}$, $p^s_m - p^s_r > 0$.

Proof of Lemma 4.5.
From Proposition 4.3, we can easily get $\frac{\partial \pi^s_{rm}}{\partial r_m} = k_r > 0$, $\frac{\partial \pi^s_{mm}}{\partial r_m} = \frac{(u^2-8b)k_r}{8b-u^2} < 0$, $\frac{\partial CS^s_m}{\partial r_m} = 0$, $\frac{\partial SW^s_m}{\partial r_m} = 0$. Note that the case of non-financing with sufficient funds is a special case when $r_m = 0$ in the case of financing with capital shortage. Then, we can obtain that $\pi^s_{rm} - \bar{\pi}^s_{rm} > 0$, $\pi^s_{mm} - \bar{\pi}^s_{mm} < 0$, $CS^s_m = \bar{CS}^s_m$, $SW^s_m = \bar{SW}^s_m$.

Proof of Theorem 4.6.
(1) Note that $\pi^s_{rm} - \pi^s_{rr} = \frac{1}{(u^2-8b)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}[bu^4((128b^2-32bu^2+u^4)r_m^3+(64b^2-36bu^2+3u^4)r_m^2+4b(16b-u^2)r_m^4+(3u^4-16bu^2)r_m-4bu^2+u^4)]$. For the convenience of calculation, let $\bar{r}_m = \sqrt{(-r_m^2+r_m+1)^2(r_m^4+2r_m^3+7r_m^2+6r_m+1)}$. Obviously, we can obtain that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b < \frac{u^2\bar{r}_m+u^2(r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, $\pi^s_{rm} - \pi^s_{rr} < 0$; when $\frac{u^2}{8} < b < \frac{u^2(3r_m+2)}{8(r_m+1)^2}$ or $b > \frac{u^2\bar{r}_m+u^2(r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, $\pi^s_{rm} - \pi^s_{rr} > 0$.
(2) Note that $\pi^s_{mm} - \pi^s_{mr} = \frac{bu^4(r_m^2-r_m-1)}{(8b-u^2)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$ and $q^s_m - q^s_r = \frac{2bu^3(r_m^2-r_m-1)}{(8b-u^2)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$. We can get that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$, $\pi^s_{mm} - \pi^s_{mr} < 0$, $q^s_m - q^s_r < 0$; when $\frac{u^2}{8} < b \le \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $\pi^s_{mm} - \pi^s_{mr} > 0$, $q^s_m - q^s_r > 0$.
(3) Note that $CS^s_m - CS^s_r = \frac{2b^2u^4(r_m^2-r_m-1)((16b-u^2)r_m^2+(32b-5u^2)r_m+16b-3u^2)}{(u^2-8b)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}$. Obviously, we can get that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$, $CS^s_m - CS^s_r < 0$; when $\frac{u^2}{8} < b \le \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $CS^s_m - CS^s_r > 0$.
(4) Note that $SW^s_m - SW^s_r = \frac{bu^4}{(u^2-8b)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}[4(56b^2-18bu^2+u^4)r_m^3-2(64b^2+3bu^2-u^4)r_m^2-2(144b^2-32bu^2+u^4)r_m-96b^2+2b(80b-7u^2)r_m^4+26bu^2-u^4]$. Obviously, we can get that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$, $SW^s_m - SW^s_r < 0$; when $\frac{u^2}{8} < b \le \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $SW^s_m - SW^s_r > 0$.

Proof of Proposition 5.1.
From equation (5), taking the first derivative of π^{ai}_{rr} with respect to p and a, we have $\frac{\partial \pi^{ai}_{rr}}{\partial p} = ((a+1)u-2p+w(\frac{1}{2}(r_s+r_t)+1))$ and $\frac{\partial \pi^{ai}_{rr}}{\partial a} = (pu-(\frac{1}{2}(r_s+r_t)+1)(2ab+uw))$. Since $\frac{\partial^2 \pi^{ai}_{rr}}{\partial p^2} = -2$, $\frac{\partial^2 \pi^{ai}_{rr}}{\partial a^2} = -2b(\frac{1}{2}(r_s+r_t)+1)$, $\frac{\partial^2 \pi^{ai}_{rr}}{\partial p \partial a} = \frac{\partial^2 \pi^{ai}_{rr}}{\partial a \partial p} = u$, Hesse matrix is $\frac{\partial^2 \pi^{ai}_{rr}}{\partial p^2}\frac{\partial^2 \pi^{ai}_{rr}}{\partial a^2} - \frac{\partial^2 \pi^{ai}_{rr}}{\partial p \partial a}\frac{\partial^2 \pi^{ai}_{rr}}{\partial a \partial p} = 2br_s+2br_t+4b-u^2$. When $b > \frac{u^2}{2(r_s+r_t+2)}$, given

the wholesale price w, the optimal retail price and advertising effort are $p_r^{ai} = \frac{(r_s+r_t+2)(bwr_s+bwr_t+2bu+2bw-u^2w)}{4br_s+4br_t+8b-2u^2}$, $a_r^{ai} = \frac{u(-wr_s-wr_t+2u-2w)}{4br_s+4br_t+8b-2u^2}$, respectively.

From equation (6), the manufacturer's profit is $\pi_{mr}^{ai} = [w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r$. When the manufacturer (the leader) decides the optimal wholesale price, she considers the decisions of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi_{mr}^{ai} = [w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r = [w((\frac{u(-wr_s-wr_t+2u-2w)}{4br_s+4br_t+8b-2u^2}+1)u - \frac{(r_s+r_t+2)(bwr_s+bwr_t+2bu+2bw-u^2w)}{4br_s+4br_t+8b-2u^2})-(k_r-(\frac{u(-wr_s-wr_t+2u-2w)}{4br_s+4br_t+8b-2u^2})^2b)](r_m+1)-(\frac{u(-wr_s-wr_t+2u-2w)}{4br_s+4br_t+8b-2u^2})^2b+k_r$. Taking the first and second derivatives of $\pi_{mr}^{ai}(w)$ with respect to w, we can get that $\frac{\partial \pi_{mr}^{ai}}{\partial w} = -\frac{1}{2(2br_s+2br_t+4b-u^2)^2}[b(r_s+r_t+2)(r_m(r_s(8bwr_t-4bu+16bw-3u^2w)+4bwr_s^2+r_t(-4bu+16bw-3u^2w)+4bwr_t^2-8bu+16bw+4u^3-6u^2w)-2(2br_s+2br_t+4b-u^2)(-wr_s-wr_t+u-2w))]$, $\frac{\partial^2 \pi_{mr}^{ai}}{\partial w^2} = -\frac{b(r_s+r_t+2)^2(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)}{2(2br_s+2br_t+4b-u^2)^2} < 0$ when $b > \frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}$. For $\frac{3r_m+2}{4(r_m+1)} > \frac{1}{2}$, when $b > \frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}$, the manufacturer's optimal wholesale price is $w_r^{ai} = \frac{2u(2r_m(br_s+br_t+2b-u^2)+2br_s+2br_t+4b-u^2)}{(r_s+r_t+2)(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)}$. Thus, by inverting the manufacturer's wholesale price decision into the retailer's decisions p_r^{ai} and a_r^{ai}, we can get that the optimal advertising effort and retail price are $a_r^{ai} = \frac{u^2(r_m+1)}{r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2}$, $p_r^{ai} = \frac{u(r_m(3br_s+3br_t+6b-2u^2)+3br_s+3br_t+6b-u^2)}{r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2}$, respectively.

Proof of Corollary 5.2.

Note that $\pi_{\text{mr}}^{ai} - \pi_{\text{mr}}^{s} = \frac{4b^2u^2(r_m+1)^3(2r_m-r_s-r_t)}{AB}$, where $A = (16b-3u^2)r_m+8br_m^2+8b-2u^2$, $B = r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2$. We can get that when $2r_m > r_s+r_t$, $\pi_{\text{mr}}^{ai} - \pi_{\text{mr}}^{s} > 0$; when $2r_m \le r_s+r_t$, $\pi_{\text{mr}}^{ai} - \pi_{\text{mr}}^{s} \le 0$.

In addition, when $2r_m > r_s+r_t$, if $k_r < \tilde{k}_r$, $\pi_{\text{rr}}^{ai} - \pi_{\text{rr}}^{s} > 0$; if $k_r \ge \tilde{k}_r$, $\pi_{\text{rr}}^{ai} - \pi_{\text{rr}}^{s} \le 0$; when $2r_m \le r_s+r_t$, if $k_r < \tilde{k}_r$, $\pi_{\text{rr}}^{ai} - \pi_{\text{rr}}^{s} < 0$; if $k_r \ge \tilde{k}_r$, $\pi_{\text{rr}}^{ai} - \pi_{\text{rr}}^{s} \ge 0$, where $\tilde{k}_r = \frac{1}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)^2}[bu^4(r_m+1)^2(r_m^2(320b^2+2b(80b-9u^2)r_s+2b(80b-9u^2)r_t-120bu^2+9u^4)+4br_m^3(16br_s+16br_t+32b-9u^2)+4(16b-3u^2)r_m(2br_s+2br_t+4b-u^2)+4(4b-u^2)(2br_s+2br_t+4b-u^2))]$.

According to Propositions 4.1 and 5.1, we have $CS_r^{ai} - CS_r^{s} = \frac{1}{2A^2B^2}[b^2u^4(r_m+1)^2(3r_m+2)(2r_m-r_s-r_t)(2r_m^2(8br_s+8br_t+16b-3u^2)+r_m((32b-3u^2)r_s+(32b-3u^2)r_t+64b-16u^2)+2((8b-u^2)r_s+(8b-u^2)r_t+16b-4u^2))]$. We can get that when $\frac{1}{2}(r_s+r_t) < r_m$, $CS_r^{ai} - CS_r^{s} > 0$; when $r_m \le \frac{1}{2}(r_s+r_t)$, $CS_r^{ai} - CS_r^{s} \le 0$.

Let $\bar{k}_r = \frac{1}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)^2}[bu^2(r_m+1)^2(64b^2r_m^4(4br_s+4br_t+8b-3u^2)+2br_m^3(1024b^2+8b(64b+u^2)r_s+8b(64b+u^2)r_t-336bu^2+9u^4)+r_m^2(3072b^3+b(1536b^2+32bu^2-27u^4)r_s+b(1536b^2+32bu^2-27u^4)r_t-896b^2u^2-12bu^4+9u^6)+4r_m(512b^3+b(256b^2+4bu^2-9u^4)r_s+b(256b^2+4bu^2-9u^4)r_t-136b^2u^2-10bu^4+3u^6)+4((64b^3-3bu^4)r_s+(64b^3-3bu^4)r_t+128b^3-32b^2u^2-4bu^4+u^6))]$. We can get that when $2r_m > r_s+r_t$, if $k_r < \bar{k}_r$, $SW_r^{ai} - SW_r^{s} > 0$; if $k_r \ge \bar{k}_r$, $SW_r^{ai} - SW_r^{s} \le 0$; when $2r_m \le r_s+r_t$, if $k_r < \bar{k}_r$, $SW_r^{ai} - SW_r^{s} < 0$; if $k_r \ge \bar{k}_r$, $SW_r^{ai} - SW_r^{s} \ge 0$.

Proof of Proposition 5.3.

According to equation (7), taking the first derivative of π_{rm}^{ai} with respect to p, we have $\frac{\partial \pi_{rm}^{ai}}{\partial p} = \frac{1}{2}(2(au-2p+u+w)+wr_s+wr_t)$ and $\frac{\partial^2 \pi_{rm}^{ai}}{\partial p^2} = -2 < 0$. Given the wholesale price w and advertising effort a, the optimal retail price is $p_m^{ai} = \frac{1}{4}(2(au+u+w)+wr_s+wr_t)$.

From equation (8), the manufacturer's profit is $\pi_{mm}^{ai} = (r_m+1)[w((a+1)u-p)-k_r]+k_r-a^2b$. When the manufacturer (the leader) decides the optimal wholesale price and advertising effort, she considers the decision of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi_{mm}^{ai} = (r_m+1)[w((a+1)u-p)-k_r]+k_r-a^2b = -a^2b+(r_m+1)(w((a+1)u-\frac{1}{2}(au+\frac{1}{2}w(r_s+r_t)+u+w))-k_r)+k_r$. Taking the first derivative of π_{mm}^{ai} with respect to w and a, we get $\frac{\partial \pi_{mm}^{ai}}{\partial w} = \frac{1}{2}(r_m+1)(au-wr_s-wr_t+u-2w)$,

$\frac{\partial \pi_{mm}^{ai}}{\partial a} = \frac{1}{2}uw(r_m+1)-2ab$. Since $\frac{\partial^2 \pi_{mm}^{ai}}{\partial w^2} = \frac{1}{2}(r_m+1)(-r_s-r_t-2)$, $\frac{\partial^2 \pi_{mm}^{ai}}{\partial a^2} = -2b$, $\frac{\partial^2 \pi_{mm}^{ai}}{\partial w \partial a} = \frac{\partial^2 \pi_{mm}^{ai}}{\partial a \partial w} = \frac{1}{2}u(r_m+1)$, Hesse matrix is $\frac{\partial^2 \pi_{mm}^{ai}}{\partial w^2}\frac{\partial^2 \pi_{mm}^{ai}}{\partial a^2} - \frac{\partial^2 \pi_{mm}^{ai}}{\partial w \partial a}\frac{\partial^2 \pi_{mm}^{ai}}{\partial a \partial w} = -\frac{1}{4}(r_m+1)(-4br_s-4br_t-8b+u^2r_m+u^2)$. Thus, when $b > \frac{u^2(r_m+1)}{4(r_s+r_t+2)}$, the wholesale price and advertising effort are $w_m^{ai} = \frac{4bu}{4br_s+4br_t+8b-u^2r_m-u^2}$, $a_m^{ai} = -\frac{u^2(r_m+1)}{-4br_s-4br_t-8b+u^2r_m+u^2}$, respectively. Thus, by inverting the manufacturer's wholesale price and advertising investment decisions into the retailer's decision p_m^{ai}, the optimal retail price is $p_m^{ai} = \frac{3bu(r_s+r_t+2)}{4br_s+4br_t+8b-u^2r_m-u^2}$.

Proof of Corollary 5.4.

Note that $\pi_{\text{mm}}^{ai} - \pi_{\text{mm}}^{s} = \frac{4b^2u^2(2r_m-r_s-r_t)}{(8b-u^2)(4br_s+4br_t+8b-u^2r_m-u^2)}$. We can get that when $2r_m > r_s+r_t$, $\pi_{\text{mm}}^{ai} - \pi_{\text{mm}}^{s} > 0$; when $2r_m \le r_s+r_t$, $\pi_{\text{mm}}^{ai} - \pi_{\text{mm}}^{s} \le 0$.

In addition, obviously, we can get that when $2r_m > r_s+r_t$, if $k_r < \hat{k}_r$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rm}}^{s} > 0$; if $k_r \ge \hat{k}_r$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rm}}^{s} \le 0$; when $2r_m \le r_s+r_t$, if $k_r < \hat{k}_r$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rm}}^{s} < 0$; if $k_r \ge \hat{k}_r$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rm}}^{s} \ge 0$, where $\hat{k}_r = \frac{2b^2u^4((16b-u^2)r_s+16br_t+32b-2u^2r_m-u^2r_t-4u^2)}{(u^2-8b)^2(-4br_s-4br_t-8b+u^2r_m+u^2)^2}$.

Note that $CS_m^{ai} - CS_m^{s} = -\frac{b^2u^4(2r_m-r_s-r_t)(-16br_s-16br_t-32b+2u^2r_m+u^2r_s+u^2r_t+4u^2)}{2(u^2-8b)^2(-4br_s-4br_t-8b+u^2r_m+u^2)^2}$. We can get that when $r_m \le \frac{1}{2}(r_s+r_t)$, $CS_m^{ai} - CS_m^{s} \le 0$; when $\frac{1}{2}(r_s+r_t) < r_m$, $CS_m^{ai} - CS_m^{s} > 0$.

Moreover, let $\boldsymbol{k}_r = \frac{b^2u^2((256b^2+16bu^2-3u^4)r_s+256b^2r_t+512b^2+(2u^4-64bu^2)r_m+16bu^2r_t-32bu^2-3u^4r_t-4u^4)}{(u^2-8b)^2(-4br_s-4br_t-8b+u^2r_m+u^2)^2}$. We can get that when $2r_m > r_s+r_t$, if $k_r < \boldsymbol{k}_r$, $SW_m^{ai} - SW_m^{s} > 0$; if $k_r \ge \boldsymbol{k}_r$, $SW_m^{ai} - SW_m^{s} \le 0$; when $2r_m \le r_s+r_t$, if $k_r < \boldsymbol{k}_r$, $SW_m^{ai} - SW_m^{s} < 0$; if $k_r \ge \boldsymbol{k}_r$, $SW_m^{ai} - SW_m^{s} \ge 0$.

Proof of Theorem 5.5.

(1) Note that $\pi_{\text{mm}}^{ai} - \pi_{\text{mr}}^{ai} = \frac{bu^4(r_m^3-2r_m-1)}{(4br_s+4br_t+8b-u^2r_m-u^2)B} < 0$. We can get when $\frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)} < b$, $\pi_{\text{mm}}^{ai} - \pi_{\text{mr}}^{ai} < 0$; when $\frac{u^2(r_m+1)}{4(r_s+r_t+2)} < b \le \frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}$, $\pi_{\text{mr}}^{ai} = 0$, $\pi_{\text{mm}}^{ai} - \pi_{\text{mr}}^{ai} > 0$.

(2) Note that $\pi_{\text{rm}}^{ai} - \pi_{\text{rr}}^{ai} = \frac{1}{2(-4br_s-4br_t-8b+u^2r_m+u^2)^2(r_m(4br_s+4br_t+8b-3u^2)+4br_s+4br_t+8b-2u^2)^2}[bu^4(r_s+r_t+2)(2r_m^2(8b^2r_s^2+8b^2r_t^2+32b^2+br_s(16br_t+32b-9u^2)+b(32b-9u^2)r_t-18bu^2+3u^4)+u^2r_m^4(-2br_s-2br_t-4b+u^2)+4r_m^3(2br_s+2br_t+4b-u^2)^2+4u^2r_m(-2br_s-2br_t-4b+u^2)+u^2(-2br_s-2br_t-4b+u^2))]$. We can get that when $b \ge \frac{u^2(r_m^4+8r_m^3+9r_m^2+4r_m+1-(r_m^2-r_m-1)\sqrt{r_m^4+2r_m^3+7r_m^2+6r_m+1})}{16r_m^2(r_m+1)(r_s+r_t+2)}$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rr}}^{ai} \ge 0$; when $\frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)} < b < \frac{u^2(r_m^4+8r_m^3+9r_m^2+4r_m+1-(r_m^2-r_m-1)\sqrt{r_m^4+2r_m^3+7r_m^2+6r_m+1})}{16r_m^2(r_m+1)(r_s+r_t+2)}$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rr}}^{ai} < 0$; when $\frac{u^2(r_m+1)}{4(r_s+r_t+2)} < b \le \frac{u^2(3r_m+2)}{4(r_m+1)(r_s+r_t+2)}$, $\pi_{\text{rm}}^{ai} - \pi_{\text{rr}}^{ai} > 0$.

Proof of Proposition 5.6.

From equation (9), taking the first derivative of π_{rr}^{ac} with respect to p and a, we have $\frac{\partial \pi_{rr}^{ac}}{\partial p} = (a+1)u+w(r_m+1)-2p$ and $\frac{\partial \pi_{rr}^{ac}}{\partial a} = -2ab(r_m+1)-uw(r_m+1)+pu$. Since $\frac{\partial^2 \pi_{rr}^{ac}}{\partial p^2} = -2$, $\frac{\partial^2 \pi_{rr}^{ac}}{\partial a^2} = -2b(r_m+1)$, $\frac{\partial^2 \pi_{rr}^{ac}}{\partial p \partial a} = \frac{\partial^2 \pi_{rr}^{ac}}{\partial a \partial p} = u$, Hesse matrix is $\frac{\partial^2 \pi_{rr}^{ac}}{\partial p^2}\frac{\partial^2 \pi_{rr}^{ac}}{\partial a^2} - \frac{\partial^2 \pi_{rr}^{ac}}{\partial p \partial a}\frac{\partial^2 \pi_{rr}^{ac}}{\partial a \partial p} = 4b(r_m+1)-u^2$. When $b > \frac{u^2}{4(r_m+1)}$, given the wholesale price w, the optimal retail price and advertising effort are $p_r^{ac} = -\frac{2b(r_m+1)(w(r_m+1)+u)-u^2w(r_m+1)}{u^2-4b(r_m+1)}$, $a_r^{ac} = -\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2}$, respectively.

From equation (10), the manufacturer's profit is $\pi_{mr}^{ac} = \int_{k_s}^{k_t}\frac{1}{k_t-k_s}\{[w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r\}dk_r$. When the manufacturer (the leader) decides the optimal wholesale price, she considers the decisions of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi_{mr}^{ac} = \int_{k_s}^{k_t}\frac{1}{k_t-k_s}\{[w((a+1)u-p)-(k_r-a^2b)](r_m+1)-a^2b+k_r\}dk_r = \int_{k_s}^{k_t}\frac{1}{k_t-k_s}\{[w((-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2}+1)u+\frac{2b(r_m+1)(w(r_m+1)+u)-u^2w(r_m+1)}{u^2-4b(r_m+1)})-(k_r-(-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2})^2b)](r_m+1)-(-\frac{u(-wr_m+u-w)}{-4br_m-4b+u^2})^2b+k_r\}dk_r$. Taking the first derivative of $\pi_{mr}^{ac}(w)$ with respect to w, we have $\frac{\partial \pi_{mr}^{ac}}{\partial w} = \frac{1}{k_t-k_s}\frac{2b(r_m+1)[-r_m^2(4b(u-6w)+3u^2w)+r_m(u^2(2u-5w)-8b(u-3w))-(4b-u^2)(u-2w)+8bwr_m^3]}{-(-4br_m-4b+u^2)^2}$, $\frac{\partial^2 \pi_{mr}^{ac}}{\partial w^2} =$

$-\frac{1}{k_t-k_s}\frac{2b(r_m+1)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)}{(-4br_m-4b+u^2)^2} < 0$ when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$. For $\frac{3r_m+2}{8(r_m+1)^2} > \frac{1}{4(r_m+1)}$, when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, the manufacturer's optimal wholesale price is $w_r^{ac} = \frac{r_m(8bu-2u^3)+4bur_m^2-(u^2-4b)u}{(r_m+1)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$. Thus, by inverting the manufacturer's wholesale price decision into the retailer's decisions p_r^{ac} and a_r^{ac}, we can get that the optimal advertising effort and retail price are $a_r^{ac} = \frac{-u^2(r_m+1)}{(3u^2-16b)r_m-8br_m^2-8b+2u^2}$, $p_r^{ac} = \frac{r_m(12bu-2u^3)+6bur_m^2+6bu-u^3}{(16b-3u^2)r_m+8br_m^2+8b-2u^2}$, respectively.

Proof of Corollary 5.7.
From Propositions 4.1 and 5.6, we can get $\pi_{rr}^{ac} = \pi_{rr}^{s} = \frac{1}{A^2}[r_m^3((384b^2-128bu^2+9u^4)k_r + bu^2(16b-u^2)) + r_m^2(4(64b^2-28bu^2+3u^4)k_r+3bu^2(8b-u^2))+64b^2k_rr_m^5+4br_m^4(4(16b-3u^2)k_r+bu^2)+r_m(4(u^2-4b)^2k_r+bu^2(16b-3u^2))+bu^2(4b-u^2)]$, $CS_r^{ac} = CS_r^{s} = \frac{2b^2u^2(r_m+1)^4}{((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}$. In addition, $\pi_{mr}^{ac} - \pi_{mr}^{s} = \frac{1}{2}r_m(2k_r-k_s-k_t)$, $SW_r^{ac} - SW_r^{s} = \frac{1}{2}r_m(2k_r-k_s-k_t)$. Thus, when $k_r > \frac{k_s+k_t}{2}$, $\pi_{mr}^{ac} - \pi_{mr}^{s} > 0$, $SW_r^{ac} - SW_r^{s} > 0$; when $k_r \le \frac{k_s+k_t}{2}$, $\pi_{mr}^{ac} - \pi_{mr}^{s} \le 0$, $SW_r^{ac} - SW_r^{s} \le 0$.

Proof of Proposition 5.8.
According to equation (11), taking the first derivative of π_{rm}^{ac} with respect to p, we have $\frac{\partial \pi_{rm}^{ac}}{\partial p} = (a+1)u + w(r_m+1) - 2p$ and $\frac{\partial^2 \pi_{rm}^{ac}}{\partial p^2} = -2 < 0$. Given the wholesale price w and the advertising effort a, the optimal retail price is $p_m^{ac} = \frac{1}{2}(au + wr_m + u + w)$.

From equation (12), the manufacturer's profit is $\pi_{mm}^{ac} = \int_{k_s}^{k_t} \frac{1}{k_t-k_s}\{(r_m+1)[w((a+1)u-p)-k_r]+k_r-a^2b\}dk_r$. When the manufacturer (the leader) decides the optimal wholesale price and the advertising effort, she considers the decision of the retailer (the follower), so the manufacturer's objective function is transformed into $\pi_{mm}^{ac} = \int_{k_s}^{k_t} \frac{1}{k_t-k_s}\{(r_m+1)[w((a+1)u-p)-k_r]+k_r-a^2b\}dk_r = \int_{k_s}^{k_t} \frac{1}{k_t-k_s}\{(r_m+1)[w((a+1)u-\frac{1}{2}(au+wr_m+u+w))-k_r]+k_r-a^2b\}dk_r$. Taking the first derivative of π_{mm}^{ac} with respect to w and a, we get $\frac{\partial \pi_{mm}^{ac}}{\partial w} = \frac{1}{k_t-k_s}(r_m+1)[\frac{1}{2}(-au-wr_m-u-w)+(a+1)u+\frac{1}{2}w(-r_m-1)]$, $\frac{\partial \pi_{mm}^{ac}}{\partial a} = \frac{1}{k_t-k_s}(-2ab-\frac{1}{2}cu+\frac{1}{2}uw(r_m+1))$. Since $\frac{\partial^2 \pi_{mm}^{ac}}{\partial w^2} = -\frac{1}{k_t-k_s}(1+r_m)^2$, $\frac{\partial^2 \pi_{mm}^{ac}}{\partial a^2} = \frac{-2b}{k_t-k_s}$, $\frac{\partial^2 \pi_{mm}^{ac}}{\partial w \partial a} = \frac{\partial^2 \pi_{mm}^{ac}}{\partial a \partial w} = \frac{1}{2(k_t-k_s)}u(1+r_m)$, Hesse matrix is $\frac{\partial^2 \pi_{mm}^{ac}}{\partial w^2}\frac{\partial^2 \pi_{mm}^{ac}}{\partial a^2} - \frac{\partial^2 \pi_{mm}^{ac}}{\partial w \partial a}\frac{\partial^2 \pi_{mm}^{ac}}{\partial a \partial w} = \frac{1}{(k_t-k_s)^2}(2b(1+r_m)^2 - \frac{1}{4}u^2(1+r_m)^2)$. Thus, when $b > \frac{u^2}{8}$, the wholesale price and advertising effort are $w_m^{ac} = \frac{4bu}{(8b-u^2)(r_m+1)}$, $a_m^{ac} = \frac{u^2}{8b-u^2}$, respectively. Thus, by inverting the manufacturer's wholesale price and advertising investment decisions into the retailer's decision p_m^{ac}, the optimal retail price is $p_m^{ac} = \frac{-6bu}{u^2-8b}$.

Proof of Corollary 5.9.
According to Propositions 4.3 and 5.8, we can obtain $\pi_{rm}^{ac} = \pi_{rm}^{s} = \frac{4b^2u^2+(u^2-8b)^2k_rr_m}{(u^2-8b)^2}$, $CS_m^{ac} = CS_m^{s} = \frac{2(3bu-4bu)^2}{(u^2-8b)^2}$. Note that $\pi_{mm}^{ac} - \pi_{mm}^{s} = \frac{1}{2}r_m(2k_r-k_s-k_t)$ and $SW_m^{ac} - SW_m^{s} = \frac{1}{2}r_m(2k_r-k_s-k_t)$. Hence, we can get that when $k_r > \frac{k_s+k_t}{2}$, $\pi_{mm}^{ac} - \pi_{mm}^{s} > 0$ and $SW_m^{ac} - SW_m^{s} > 0$; when $k_r \le \frac{k_s+k_t}{2}$, $\pi_{mm}^{ac} - \pi_{mm}^{s} \le 0$ and $SW_m^{ac} - SW_m^{s} \le 0$.

Proof of Theorem 5.10.
(1) Note that $\pi_{mm}^{ac} - \pi_{mr}^{ac} = -\frac{bu^4(-r_m^2+r_m+1)}{(8b-u^2)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$ and $q_m^{ac} - q_r^{ac} = \frac{2bu^3(r_m^2-r_m-1)}{(8b-u^2)((16b-3u^2)r_m+8br_m^2+8b-2u^2)}$. We can get that when $\frac{u^2(3r_m+2)}{8(r_m+1)^2} < b$, $\pi_{mm}^{ac} - \pi_{mr}^{ac} < 0$, $q_m^{ac} - q_r^{ac} < 0$; when $\frac{u^2}{8} < b \le \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $\pi_{mm}^{ac} - \pi_{mr}^{ac} > 0$, $q_m^{ac} - q_r^{ac} > 0$.
(2) Note that $\pi_{rm}^{ac} - \pi_{rr}^{ac} = \frac{1}{(u^2-8b)^2A^2}[bu^4((128b^2-32bu^2+u^4)r_m^3 + (64b^2-36bu^2+3u^4)r_m^2 + 4b(16b-u^2)r_m^4 + (3u^4-16bu^2)r_m - 4bu^2+u^4)]$. We can get that when $b \ge \frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, $\pi_{rm}^{ac} - \pi_{rr}^{ac} \ge 0$; when $\frac{(3r_m+2)u^2}{8(r_m+1)^2} < b < \frac{u^2(\bar{r}_m+r_m^4+8r_m^3+9r_m^2+4r_m+1)}{32r_m^2(r_m+1)^2}$, $\pi_{rm}^{ac} - \pi_{rr}^{ac} < 0$; when $\frac{u^2}{8} < b \le \frac{(3r_m+2)u^2}{8(r_m+1)^2}$, $\pi_{rm}^{ac} - \pi_{rr}^{ac} > 0$.

(3) Note that $CS_r^{ac} - CS_m^{ac} = -\frac{2b^2u^4(r_m^2-r_m-1)((16b-u^2)r_m^2+(32b-5u^2)r_m+16b-3u^2)}{(u^2-8b)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}$. We can get that when $\frac{u^2}{8} < b \leq \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $CS_r^{ac} - CS_m^{ac} < 0$; when $b > \frac{u^2(3r_m+2)}{8(r_m+1)^2}$, $CS_r^{ac} - CS_m^{ac} > 0$.
(4) As $SW_r^{ac} - SW_m^{ac} = \frac{bu^4(-4(56b^2-18bu^2+u^4)r_m^3+2(64b^2+3bu^2-u^4)r_m^2+2(144b^2-32bu^2+u^4)r_m+96b^2-2b(80b-7u^2)r_m^4-26bu^2+u^4)}{(u^2-8b)^2((16b-3u^2)r_m+8br_m^2+8b-2u^2)^2}$, we can get that when $\frac{(3r_m+2)u^2}{8(r_m+1)^2} < b$, $SW_r^{ac} - SW_m^{ac} > 0$; when $\frac{u^2}{8} < b \leq \frac{(3r_m+2)u^2}{8(r_m+1)^2}$, $SW_r^{ac} - SW_m^{ac} < 0$.

Proof of Theorem 6.1.
Obviously, $\hat{\pi}_{\mathrm{rr}}^s - \hat{\pi}_{\mathrm{rm}}^s = -\frac{bu^4((204b^2-44bu^2+3u^4)r_m^2+(156b^2-44bu^2+3u^4)r_m+4b(4b-u^2)r_m^4+(u^2-10b)^2r_m^3+(u^2-6b)^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} < 0$, $\hat{\pi}_{\mathrm{mr}}^s - \hat{\pi}_{\mathrm{mm}}^s = \frac{bu^4((68b^2-24bu^2+3u^4)r_m^2+4b^2r_m^6+4b(6b+u^2)r_m^5+4b(11b+4u^2)r_m^4+(6b+u^2)^2r_m^3+3(u^2-6b)^2r_m+(u^2-6b)^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} > 0$, $\hat{CS}_r^s - \hat{CS}_m^s = -\frac{2b^2u^4r_m(r_m+2)((4b-u^2)r_m^2+2(8b-u^2)r_m+12b-2u^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} < 0$.

As $S\hat{W}_r^s - S\hat{W}_m^s = \frac{2b^2u^4r_m(r_m+2)(2(4b+u^2)r_m^3+(7u^2-6b)r_m^2+(6u^2-44b)r_m+2br_m^4+4(u^2-6b))}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2}$, we can get that when $\frac{u^2(r_m+1)}{2r_m+6} < b < \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)}$, $S\hat{W}_r^s - S\hat{W}_m^s > 0$; when $b \geq \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)}$, $S\hat{W}_r^s - S\hat{W}_m^s \leq 0$.

Proof of Theorem 6.2.
Obviuosly, $\hat{\pi}_{\mathrm{mr}}^{ai} - \hat{\pi}_{\mathrm{mm}}^{ai} = \frac{1}{(2br_s(r_t+4)+br_s^2+br_t^2+8br_t+12b-2u^2)^2(br_s+br_t+6b+u^2(-r_m)-u^2)^2}[bu^4(r_m^2(4b^2r_s^3(r_t+2) + b^2r_s^4 + b^2r_t^4 + 8b^2r_t^3 + 12(-4b^2 - 4bu^2 + u^4) + 2br_s^2(3br_t^2 + 12br_t + 4(b+u^2)) + 4br_s(4(b+u^2)r_t + br_t^3 + 6br_t^2 - 8b + 2u^2) + 8b(b+u^2)r_t^2 + 8b(u^2-4b)r_t) + 2r_m(4b^2r_s^3(r_t+2) + b^2r_s^4 + b^2r_t^4 + 8b^2r_t^3 + 120b^2 + 2br_s^2(3br_t^2 + 12br_t + 9b + u^2) + 4br_s((9b+u^2)r_t + br_t^3 + 6br_t^2 + 14b - 3u^2) + 2b(9b+u^2)r_t^2 + 4b(14b - 3u^2)r_t - 56bu^2 + 6u^4) + 6b^2r_s^2(r_t+2)^2 + 4b^2r_s^3(r_t+2) + b^2r_s^4 + b^2r_t^4 + 8b^2r_t^3 + 24b^2r_t^2 + 96b^2r_t + 144b^2 + 4u^2r_m^3(2br_s(r_t+2) + br_s^2 + br_t^2 + 4br_t + 4b + u^2) + 4br_s(br_t^3 + 6br_t^2 + 12br_t + 24b - 4u^2) - 16bu^2r_t - 48bu^2 + 4u^4)] > 0$, $\hat{\pi}_{\mathrm{rr}}^{ai} - \hat{\pi}_{\mathrm{rm}}^{ai} = -\frac{1}{(2br_s(r_t+4)+br_s^2+br_t^2+8br_t+12b-2u^2)^2(br_s+br_t+6b+u^2(-r_m)-u^2)^2}[2bu^4(2r_m(r_s + r_t + 2)(2b^2r_s^2 + 2b^2r_t^2 + 24b^2 + br_s(4br_t + 16b - 3u^2) + b(16b - 3u^2)r_t - 10bu^2 + u^4) + r_s(15b^2r_t^2 + 108b^2 + 4b(23b - 2u^2)r_t - 24bu^2 + u^4) + 5b^2r_s^3 + 5b^2r_t^3 + 46b^2r_t^2 + 108b^2r_t + 72b^2 + u^2r_m^2(r_s + r_t + 2)(-2br_s - 2br_t - 4b + u^2) + br_s^2(15br_t + 46b - 4u^2) - 4bu^2r_t^2 - 24bu^2r_t - 24bu^2 + u^4r_t + 2u^4)] < 0$, $\hat{CS}_r^{ai} - \hat{CS}_m^{ai} = \frac{2b^2u^4(r_m(r_s+r_t+2)+r_s+r_t)(r_s(-4br_t-16b+u^2)-2br_s^2-2br_t^2-16br_t-24b+u^2r_m(r_s+r_t+2)+u^2r_t+4u^2)}{(2br_s(r_t+4)+br_s^2+br_t^2+8br_t+12b-2u^2)^2(br_s+br_t+6b+u^2(-r_m)-u^2)^2} < 0$.

As $S\hat{W}_r^{ai} - S\hat{W}_m^{ai} = \frac{1}{(2br_s(r_t+4)+br_s^2+br_t^2+8br_t+12b-2u^2)^2(br_s+br_t+6b+u^2(-r_m)-u^2)^2}[bu^4(r_m^2(2r_s(2b^2r_t^3 + 12b^2r_t^2 - 16b^2 + 2b(4b+7u^2)r_t + 16bu^2 - u^4) + 4b^2r_s^3(r_t+2) + b^2r_s^4 - 2(16b^2 - 16bu^2 + u^4)r_t + b^2r_t^4 + 8b^2r_t^3 + 8(-6b^2 - 3bu^2 + u^4) + 2br_s^2(3br_t^2 + 12br_t + 4b + 7u^2) + 2b(4b+7u^2)r_t^2) + 2r_m(2r_s(2b^2r_t^3 + 3b^2r_t^2 - 56b^2 + 2b(5u^2 - 21b)r_t + 14bu^2 - u^4) + 2b^2r_s^3(2r_t+1) + b^2r_s^4 - 2(56b^2 - 14bu^2 + u^4)r_t + b^2r_t^4 + 2b^2r_t^3 - 24b^2 + 2br_s^2(3br_t^2 + 3br_t - 21b + 5u^2) + 2b(5u^2 - 21b)r_t^2 - 8bu^2 + 2u^4) + (r_s + r_t)(3b^2r_s^2(r_t - 2) + b^2r_s^3 + b^2r_t^3 - 6b^2r_t^2 - 2(84b^2 - 20bu^2 + u^4) + br_s(3br_t^2 - 12br_t + 10(u^2 - 10b)) + 10b(u^2 - 10b)r_t) + 4u^2r_m^3(2br_s(r_t+2) + br_s^2 + br_t^2 + 4br_t + 4b + u^2))]$, we can get that when $b \geq \bar{b}_1$, $S\hat{W}_r^{ai} - S\hat{W}_m^{ai} \leq 0$; when $b < \bar{b}_1$, $S\hat{W}_r^{ai} - S\hat{W}_m^{ai} > 0$, where $\bar{b}_1 = \sqrt{\Phi} - \frac{1}{(r_s+r_t+6)\Gamma}[u^2(2r_m^3(r_s+r_t+2)^2 + r_m^2(2r_s(7r_t+8) + 7r_s^2 + 7r_t^2 + 16r_t - 12) + 2r_m(2r_s(5r_t+7) + 5r_s^2 + 5r_t^2 + 14r_t - 4) + 5(2r_s(r_t+2) + r_s^2 + r_t(r_t+4))))]$, $\Phi = \frac{1}{(r_s+r_t+6)^2\Gamma^2}[u^4(r_m(r_s+r_t+2) + r_s + r_t)^2(4r_m^4(r_s+r_t+2)^2 + 16r_m^3(2r_s(r_t+1) + r_s^2 + r_t(r_t+2)) + r_m^2(r_s^2(6r_t + 45) + 2r_s(3r_t^2 + 45r_t + 50) + 2r_s^3 + 2r_t^3 + 45r_t^2 + 100r_t + 148) + 2r_m(r_s^2(6r_t+23) + r_s(6r_t^2 + 46r_t + 42) + 2r_s^3 + 2r_t^3 + 23r_t^2 + 42r_t + 96) + 2r_sr_t(3r_t + 13) + r_s^2(6r_t + 13) + 2r_s^3 + 2r_t^3 + 13r_t^2 + 64)]$, $\Gamma = (r_m^2(r_s + r_t - 2)(r_s + r_t + 2)^2 + 2r_m(r_s^2(3r_t-4) + r_s(3r_t^2 - 8r_t - 18) + r_s^3 + r_t^3 - 4r_t^2 - 18r_t - 4) + 3r_s^2(r_t-4) + r_s(3r_t^2 - 24r_t - 28) + r_s^3 + r_t(r_t^2 - 12r_t - 28))$.

Proof of Theorem 6.3.
Obviously, $\hat{\pi}_{\mathrm{rr}}^{ac} - \hat{\pi}_{\mathrm{rm}}^{ac} = -\frac{bu^4((204b^2-44bu^2+3u^4)r_m^2+(156b^2-44bu^2+3u^4)r_m+4b(4b-u^2)r_m^4+(u^2-10b)^2r_m^3+(u^2-6b)^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} < 0$, $\hat{\pi}_{\mathrm{mr}}^{ac} - \hat{\pi}_{\mathrm{mm}}^{ac} = \frac{bu^4((68b^2-24bu^2+3u^4)r_m^2+4b^2r_m^6+4b(6b+u^2)r_m^5+4b(11b+4u^2)r_m^4+(6b+u^2)^2r_m^3+3(u^2-6b)^2r_m+(u^2-6b)^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} > 0$, $\hat{CS}_r^{ac} - \hat{CS}_m^{ac} = -\frac{2b^2u^4r_m(r_m+2)((4b-u^2)r_m^2+2(8b-u^2)r_m+12b-2u^2)}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2} < 0$.

As $S\hat{W}_r^{ac} - S\hat{W}_m^{ac} = \frac{2b^2u^4r_m(r_m+2)(2(4b+u^2)r_m^3+(7u^2-6b)r_m^2+(6u^2-44b)r_m+2br_m^4+4(u^2-6b))}{((2b-u^2)r_m+6b-u^2)^2(2br_m^2+8br_m+6b-u^2)^2}$, we can get that when $b \geq \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)}$, $S\hat{W}_r^{ac} - S\hat{W}_m^{ac} \leq 0$; when $\frac{u^2(r_m+1)}{2r_m+6} < b < \frac{u^2(2r_m^3+7r_m^2+6r_m+4)}{2(-r_m^4-4r_m^3+3r_m^2+22r_m+12)}$, $S\hat{W}_r^{ac} - S\hat{W}_m^{ac} > 0$.

Proof of Theorem 6.4.
(1) Note that $\pi_{rc}^s - \pi_{rn}^s = \frac{3(64bu^4-27u^6)}{16(32b-9u^2)^2}$. Considering the region of the optimal solutions of cooperative advertising with $b > \frac{9u^2}{32}$, we can get that when $b \geq \frac{27u^2}{64}$, $\pi_{rc}^s - \pi_{rn}^s \geq 0$; when $\frac{9u^2}{32} < b < \frac{27u^2}{64}$, $\pi_{rc}^s - \pi_{rn}^s < 0$.
(2) Note that $\pi_{mc}^s - \pi_{mn}^s = \frac{9u^4}{256b-72u^2}$. We can get that when $b > \frac{9u^2}{32}$, $\pi_{mc}^s - \pi_{mn}^s > 0$.
(3) Note that $CS_c^s - CS_n^s = \frac{32b^2u^2}{(32b-9u^2)^2} - \frac{u^2}{32}$. We can obtain that when $b > \frac{9u^2}{64}$, $CS_c^s - CS_n^s > 0$. Hence, when $b > \frac{9u^2}{32}$, $CS_c^s - CS_n^s > 0$.
(4) Note that $SW_c^s - SW_n^s = -\frac{3u^4(189u^2-704b)}{32(32b-9u^2)^2}$. We can obtain that when $b > \frac{189u^2}{704}$, $SW_c^s - SW_n^s > 0$. Thus, considering the region of the optimal solutions of cooperative advertising and $\frac{9u^2}{32} > \frac{189u^2}{704}$, we can get that when $b > \frac{9u^2}{32}$, $SW_c^s - SW_n^s > 0$.

Proof of Theorem 6.5.
(1) Note that $\pi_{rc}^{ai} - \pi_{rn}^{ai} = \frac{3u^4(r_m+1)(32br_s+32br_t+64b-27u^2r_m-27u^2)}{16(16br_s+16br_t+32b-9u^2r_m-9u^2)^2}$. Considering the region of the optimal solutions of cooperative advertising with $b > \frac{9u^2(r_m+1)}{16(r_s+r_t+2)}$, we can get that when $b \geq \frac{27u^2(r_m+1)}{32(r_s+r_t+2)}$, $\pi_{rc}^{ai} - \pi_{rn}^{ai} \geq 0$; when $\frac{9u^2(r_m+1)}{16(r_s+r_t+2)} < b < \frac{27u^2(r_m+1)}{32(r_s+r_t+2)}$, $\pi_{rc}^{ai} - \pi_{rn}^{ai} < 0$.
(2) Note that $\pi_{mc}^{ai} - \pi_{mn}^{ai} = \frac{9u^4(r_m+1)^2}{4(r_s+r_t+2)(16br_s+16br_t+32b-9u^2r_m-9u^2)}$. We can get that when $b > \frac{9u^2(r_m+1)}{16(r_s+r_t+2)}$, $\pi_{mc}^{ai} - \pi_{mn}^{ai} > 0$.
(3) Note that $CS_c^{ai} - CS_n^{ai} = \frac{9u^4(r_m+1)(32br_s+32br_t+64b-9u^2r_m-9u^2)}{32(16br_s+16br_t+32b-9u^2r_m-9u^2)^2}$. We can get that when $b > \frac{9u^2(r_m+1)}{32(r_s+r_t+2)}$, $CS_c^{ai} - CS_n^{ai} > 0$. Hence, when $b > \frac{9u^2(r_m+1)}{16(r_s+r_t+2)}$, $CS_c^{ai} - CS_n^{ai} > 0$.
(4) Note that $SW_c^{ai} - SW_n^{ai} = \frac{1}{32(r_s+r_t+2)(16br_s+16br_t+32b-9u^2r_m-9u^2)^2}[3u^4(r_m+1)(r_m((384b-81u^2)r_s+(384b-81u^2)r_t+768b-594u^2)+r_s(320br_t+1024b-81u^2)+160br_s^2+160br_t^2+1024br_t+1408b-216u^2r_m^2-81u^2r_t-378u^2)]$. We can get that when $b > \frac{9u^2(r_m+1)}{16(r_s+r_t+2)}$, $SW_c^{ai} - SW_n^{ai} > 0$.

Proof of Theorem 6.6.
(1) Note that $\pi_{rc}^{ac} - \pi_{rn}^{ac} = \frac{3(64bu^4-27u^6)}{16(32b-9u^2)^2}$. Considering the region of the optimal solutions of cooperative advertising with $b > \frac{9u^2}{32}$, we can get that when $b \geq \frac{27u^2}{64}$, $\pi_{rc}^{ac} - \pi_{rn}^{ac} \geq 0$; when $\frac{9u^2}{32} < b < \frac{27u^2}{64}$, $\pi_{rc}^{ac} - \pi_{rn}^{ac} < 0$.
(2) Note that $\pi_{mc}^{ac} - \pi_{mn}^{ac} = \frac{9u^4}{256b-72u^2}$. We can get that when $b > \frac{9u^2}{32}$, $\pi_{mc}^{ac} - \pi_{mn}^{ac} > 0$.
(3) Note that $CS_c^{ac} - CS_n^{ac} = \frac{32b^2u^2}{(32b-9u^2)^2} - \frac{u^2}{32}$ .We can get that when $b > \frac{9u^2}{64}$, $CS_c^{ac} - CS_n^{ac} > 0$. Hence, when $b > \frac{9u^2}{32}$, $CS_c^{ac} - CS_n^{ac} > 0$.
(4) Note that $SW_c^{ac} - SW_n^{ac} = \frac{3(704bu^4-189u^6)}{32(32b-9u^2)^2}$. We can get that when $b > \frac{9u^2}{32}$, $SW_c^{ac} - SW_n^{ac} > 0$.

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