RAIRO-Oper. Res. 58 (2024) 2927–2949
https://doi.org/10.1051/ro/2024092

RAIRO Operations Research
www.rairo-ro.org

THE INTERACTION BETWEEN MANUFACTURER ENCROACHMENT AND GRAY MARKET

Fengmei Xu[1], Feng Yang[1], Feifei Shan[1,*] and Ting Chen[2]

Abstract. With the rapid development of retail platforms, many manufacturers use retail platforms to encroach into the retail market (*i.e.*, a type manufacturer encroachment) and gray market is increasingly prevalent in various industries. This paper considers a manufacturer who directly sells product 1 in the domestic market and sells product 2 through an incumbent retailer in the overseas market and a gray marketer can divert product 1 across markets without authorized (we call it gray market). In this paper, we examine the interaction between manufacturer encroachment and gray market. We find that when the domestic price of product 1 is sufficiently low, the gray marketer can successfully enter the overseas market without and with encroachment. Second, regardless of whether there is a gray market threat, the manufacturer has an incentive to encroach through retail platform when the commission rate is low. Moreover, the manufacturer's incentive to encroach varies with the domestic price of product 1 and gray market threat. Finally, manufacturer encroachment can reduce the scale of the gray market and even eliminate the gray market under certain conditions. The gray marketer always suffers from manufacturer encroachment while the retailer can benefit from manufacturer encroachment.

Mathematics Subject Classification. 90B06.

Received April 14, 2023. Accepted April 25, 2024.

1. Introduction

In recent years, manufacturers are increasingly establishing direct-selling channel to sell their products to consumers in addition to the incumbent retail channel. This practice of manufacturers establishing direct-selling channels to sell products is called "manufacturer encroachment" in previous literature [29, 36]. With rapid development of information technology and e-commerce, it has become easier for manufacturers to establish direct sales channels (especially *via* retail platforms). For example, many well-known manufacturers have been encroaching into the retail market through retail platforms to boost their profitability, such as Dell, Apple, Adidas and so on [35, 39].

However, the rapid development of global logistics service and information technology has also facilitated the formation of gray markets [14]. Gray market refers to selling genuine branded products outside authorized distribution channels without the consent of the brand owner [2, 10]. This phenomenon is prevalent and growing rapidly in various industries, such as electronics, cosmetics, pharmaceuticals, infant formula and so on [6]. For

Keywords. Supply chain, manufacturer encroachment, gray market, retail platform, game theory.

[1] School of Management, University of Science and Technology of China, 96 Jinzhai Road, Hefei 230026, P.R. China.
[2] School of Foreign Language, Hefei Normal University, 1688 Lianhua Road, Hefei 230601, P.R. China.
*Corresponding author: sasff@ustc.edu.cn

example, KPMG [17] reported that in global IT industry, 5–30% of the total IT sales are gray-market products, which are valued at approximately $58 billion in 2008. It is estimated that milk powder sourced from gray market accounts for 6% to 10% of Chinese market, with annual sales of about 7 billion yuan [23].

In order to better meet consumer demand, international brand manufacturers usually sell differentiated substitutes in different markets, particularly in many industries such as IT, infant milk powder and cosmetics [8, 21]. Each product is designed according to specific consumer segment. For example, Nutricia, a subsidiary of Danone Group, sells the Aptamil infant formula powder in different markets with different formulations[1]. However, due to price discrimination in different markets, gray marketers still may have an incentive to resell products from low-priced markets to high-priced markets for arbitrage. In particular, the development of social media and e-commerce has enabled private individuals and retailers to engage in gray market speculation, which further exacerbates the problem of gray markets.

To mitigate such the gray market threat, in addition to the local version sold by retailers, manufacturers have the option to directly sell another version through retail platforms in high-priced markets. For instance, Nutricia not only sells the Chinese version of milk powder through traditional retailers in Chinese Mainland, but also introduces some European versions of milk powder through retail platforms such as Alibaba's Tmall.hk, involving various brands such as "Aptamil", "Aptamil Profutura" and "Nutrilon"[2]. Despite using retail platform to encroach as a solution to the gray market problem in practice, it is still unclear how manufacturers should use encroachment to deter gray market in the theoretical research.

Previous literature on encroachment typically assumes that manufacturers sell products directly through their own online channels rather than retail platforms [12,22,31]. However, with the rapid development of e-commerce, manufacturers are increasingly selling their products directly through retail platforms in practice. In addition, most of the studies focus on the interaction between manufacturer encroachment and downstream retailers and do not incorporate the gray market into manufacturer's encroachment decision. Therefore, to fill these research gaps in the literature, we are motivated to investigate the interplay between manufacture encroachment and gray market in global supply chain. In this paper, we develop a model in which a manufacturer directly sells product 1 in domestic market and sells product 2 through an incumbent retailer in overseas market, and a gray marketer may divert product 1 from domestic market to overseas market if it is profitable (we call it gray market). Facing such potential threat, the manufacturer decides whether to encroach into the overseas market by selling product 1 through the retail platform. Specifically, we attempt to address the following questions:

(1) Under what market conditions can the gray marketer enter the market? And how does manufacturer encroachment affect the gray marketer's entry decision?
(2) Under what market conditions should the manufacturer encroach into the retail market? And how does the gray market affect the manufacturer's incentive to encroach?
(3) How does manufacturer encroachment affect the gray market as well as the profitability of both the gray marketer and the retailer?

Our analysis gives several managerial insights. First, regardless of the manufacturer's encroachment choice, the gray marketer can successfully enter the overseas market if the domestic price of product 1 is sufficiently low. However, the gray marketer is less likely to enter the market under manufacturer encroachment. This is because when the manufacturer encroaches into the overseas market, it sells product 1 through retail platform in addition to selling product 2. This will further intensify retail competition and then result in the gray marketer being unable to make a positive margin from reselling product 1 unless the domestic price of product 1 becomes lower.

Second, regardless of whether there is a gray market threat, the manufacturer has an incentive to encroach through retail platform when the commission rate is low. This is because when the commission rate is low, due to the high efficiency of the encroachment channel, the manufacturer can obtain more additional retail profit

[1] https://sg.theasianparent.com/baby-milk-complaints.

[2] https://www.dairyreporter.com/Article/2016/11/18/Danone-seeks-to-thwart-China-infant-formula-grey-market-with-online-retailer-link-up.

from its encroachment channel, which outweighs the loss in wholesale profit, and thereby the manufacturer can benefit from encroachment. Furthermore, the manufacturer's incentive to encroach varies with the domestic price of product 1 and gray market threat. In particular, the existence of the gray market threat reduces the incentive for the manufacturer to encroach when the domestic price of product 1 is relatively moderate.

Third, we reveal that manufacturer encroachment can reduce the scale of the gray market or even eliminate the gray market when the domestic price of product 1 is relatively low and when the commission rate is low. This is because when the purchase cost of the gray marketer is not too low, due to manufacturer encroachment, the increased retail competition leads to the gray marketer cannot earn a positive margin from selling unauthorized product 1. Thus, the gray marketer has no incentive to enter market with manufacturer encroachment. Besides, the gray marketer always suffers from manufacturer encroachment due to the increased retail competition. Nevertheless, the retailer can benefit from manufacturer encroachment due to the wholesale price reduction when the domestic price of product 1 is relatively high and the commission rate is moderate.

The remainder of the paper is as follows. Section 2 reviews the related literature. Section 3 gives model description. Next, we present our analytical results in Sections 4 and 5. Finally, Section 6 concludes the paper and suggests future research directions.

2. Literature review

Our paper is related to the literature on manufacturer encroachment and gray market.

2.1. Manufacturer encroachment

Manufacturer encroachment has been extensively studied from various perspectives, such as retail cost [5], product quality [34, 36], asymmetric information [11, 32], channel power [40], and store brand competition [19, 24]. Arya *et al.* [5] show that the manufacturer has an incentive to encroach into retail market when its direct selling cost disadvantage is not too large and the retailer can benefit from manufacturer encroachment due to a reduction in the wholesale price when the manufacturer's selling cost is intermediate. Huang *et al.* [12] investigate retailer's information sharing decision with supplier encroachment and show that the retailer may be willing to share low demand information with supplier to discourage supplier from introducing direct channel. Li *et al.* [19] investigates the strategic interplay between manufacturer encroachment and retailer' store brand introduction, and find that the retailer's store brand introduction has a significant influence on the manufacturer's encroachment decision. Wan *et al.* [31] examine the interaction between retailer's in-store service and manufacturer encroachment and find that the manufacturer is less likely to encroach when the retailer provides the in-store service and the consumer sensitivity to in-store service is sufficiently high. Yang *et al.* [35] focus on whether manufacturers should implement encroachment with new product or same product in the presence of network externality and shows that under certain conditions, new-product encroachment can mitigate channel conflict relative to same-product encroachment. Liu *et al.* [22] extend the work of Arya *et al.* [5] by incorporating multi-retailer competition into manufacturer encroachment and find that the retailers can never benefit from manufacturer encroachment when the number of downstream retailers exceeds four. Ponnachiyur Maruthasalam and Balasubramanian [25] consider a setting where two rival retailers have different selling costs and study the impact of such asymmetric retail competition on the manufacturer's encroachment decision. They find that the manufacturer is more likely to encroach in the presence of asymmetric retail competition.

Our paper differs from the aforementioned studies in two respects. First, these papers assume that the manufacturer encroaches into the retail market through manufacturer-owned online channel. However, with the rapid development of e-commerce, manufacturers are increasingly selling their products directly through retail platforms rather than self-built online channels in practice. Second, unlike these papers, we focus on the interaction between manufacturer encroachment and gray market, and highlight the role of manufacturer encroachment in resisting the gray market. In this paper, we find that regardless of whether there is a gray market threat, the manufacturer has an incentive to encroach through retail platform when the commission rate is low and manufacturer encroachment can be used as an effective tool to combat the gray market.

2.2. Gray market

Most of literature on gray market focuses on the countermeasures of the gray market, such as pricing [2, 20], coordination contract [3, 28], retail services [14], organizational structure [16], information technology [9, 26] and distribution channel [27]. Ahmadi and Yang [2] examine price competition between one manufacturer and one gray marketer in global supply chain and find that the manufacturer can adjust prices to allow or prevent gray market. Ahmadi *et al.* [3] demonstrate that wholesale price contract may outperform a quantity discount contract on supply chain coordination in the presence of gray market. Kim and Park [16] study a manufacturer's organization choice between centralization and decentralization structures and show that decentralization is the optimal structure under certain conditions. Zhang *et al.* [37] examine the impact of money-back guarantees (MBGs) on gray market and find that offering an MBG in the high-price market can help the manufacturer to deter gray market. Chen *et al.* [7] study whether manufacturers adopt price strategy or quantity strategy in the presence of gray products and find that the quantity strategy is more profitable. Recently, Ding *et al.* [9] and Zhang *et al.* [38] reveal that the manufacturer's RFID and blockchain adoption can deter the entry of the gray marketer under certain conditions.

The aforementioned studies fail to examine manufacturers' distribution channel decisions with gray market. Our work is mostly related to the work of Shi *et al.* [27]. They consider a manufacturer who already sell its product in the domestic market and decides whether to enter the overseas market through self-built store or cross-border e-commerce in the presence of gray market. Our research differs from Shi *et al.* [27] in two aspects. First, they consider that the gray marketer had already entered the overseas market before manufacturer. Differently, we consider the manufacturer had already sold two differentiated products in two markets before the gray marketer entered the overseas market. Second, they focus on whether the manufacturer enter the overseas market through self-built store or cross-border e-commerce platform. However, we examine whether the manufacturer should encroach through retail platform in addition to retail channel in the overseas market. Our paper enriches this stream literature by investigating the impact of manufacturer encroachment on gray market.

2.3. Contributions

Our paper makes the following contributions. First, to our knowledge, this paper is the first to examine the interaction between manufacturer encroachment and gray market, which enriches the previous literature. Second, we find that regardless of whether there is a gray market threat, the manufacturer has an incentive to encroach through retail platform when the commission rate is low. However, the existence of the gray market threat may reduce the incentive for the manufacturer to encroach. More importantly, manufacturer encroachment can be used as an effective tool to reduce the scale of the gray market and even eliminate the gray market under certain conditions, which is absent in the current literature. Our results provide practical guidance for manufacturers to manage gray markets and introduce online direct-selling channels.

3. Model description

3.1. Firms

We consider a manufacturer who directly sells product 1 at price p in the domestic market and sells product 2 to an incumbent retailer at wholesale price w in the overseas market. Similar to Li *et al.* [18], we define domestic market with a low consumer valuation as "low-priced market" and overseas market with a high consumer valuation as "high-priced market". This assumption is reasonable in practice. For example, Friesland Campina sells its "Friso" milk powder at a lower price in the Netherlands (domestic market) than it does in China (overseas market). Each product is designed according to a specific consumer segment [21]. The consumer valuation difference across markets usually leads to a price difference between two markets. Hence, in order to arbitrage, a gray marketer may purchase product 1 from domestic market and resell them in overseas market. Such speculation of the gray marketer will generate a gray market.

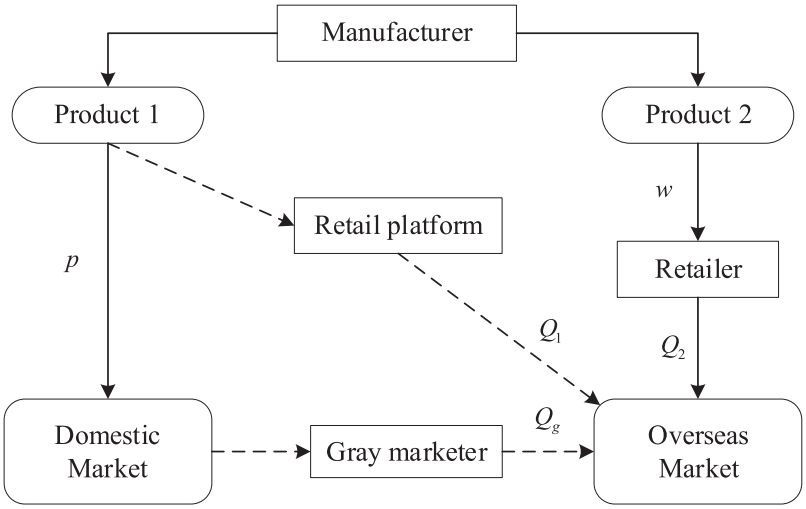

FIGURE 1. The supply chain structure.

Faced with the gray market threat, the manufacturer has the option to encroach into the overseas market by selling product 1 through the retail platform (referred to as "*manufacturer encroachment*" strategy). In this paper, we examine the interplay of manufacturer encroachment and gray market, which is not discussed in previous literature. When the manufacturer decides to encroach into the retail market, the manufacturer need pay a commission rate γ (a certain percentage of revenue) to the retail platform for each product sold on platform. Here, we suppose that the commission rate is exogenous, which is commonly used in retailing platform [1]. In practice, retail platforms such as Amazon and JD, charge the same commission for all items within a certain category for each product [30]. To simplify exposition, we standardize all production costs to zero. Figure 1 illustrates the supply chain structure.

3.2. Consumer utility and demand

We assume that the market size of each market is normalized to 1. This assumption is commonly adopted in gray market literature [13, 33]. In the overseas market, consumers are heterogeneous in product valuation. The overseas consumer's valuation for product 2, denoted by v, is uniformly distributed over $[0, 1]$. The overseas consumer has a discounted value (θv) for product 1 because product 2 is a better match to their needs than product 1, where $0 < \theta < 1$. Here, we use θ to represent the overseas consumers' acceptance level of product 1. Following gray market literature [13], due to lack of after-sales service or warranty, we assume the overseas consumer's valuation on unauthorized product with a discounted factor $\delta \in (0, 1)$ relative to authorized product. Similarly, δ reflects the overseas consumers' acceptance level of gray channel (or gray market). In overseas market, a consumer who (a) purchases product 2 at price p_2 has the utility: $U_2 = v - p_2$; (b) purchases authorized product 1 at price p_1 has the utility $U_1 = \theta v - p_1$; (c) purchases unauthorized product 1 at price p_g has the utility $U_g = \delta\theta v - p_g$; or (d) purchases nothing has zero utility. Following the literature [15, 24], we derive the inverse demand functions based on consumer's utility functions.

When the manufacturer does not encroach and the gray marketer has no option to enter in the overseas market (*i.e.*, no encroachment and no gray market threat), the overseas consumer can purchase product 2 from retailer or nothing. Based on consumer utility functions, the demands for product 2 is $Q_2 = 1 - p_2$. Thus, we derive the inverse demand function for product 2: $p_2 = 1 - Q_2$. When the manufacturer encroaches the retail market and the gray marketer has no option to enter in the overseas market (*i.e.*, encroachment and no gray market threat), the overseas consumer can purchase product 2 from retailer, authorized product 1 from

TABLE 1. Notations.

Notations	Description
Parameters	
v	The overseas consumer's valuation for product 2
θ	The overseas consumers' acceptance level of product 1
δ	The overseas consumers' acceptance level of gray channel (*i.e.*, gray market)
γ	The commission rate charged by retail platform
p	The price of product 1 in domestic market
Variables	
w	The wholesale price of product 2 in overseas market
Q_2	The order quantity of product 2 in overseas market
Q_1	The order quantity of authorized product 1 in overseas market
Q_g	The order quantity of unauthorized product 1 in overseas market
Prices	
p_2	The retail price of product 2 in overseas market
p_1	The retail price of authorized product 1 in overseas market
p_g	The retail price of unauthorized product 1 in overseas market
Profits	
π_m	Profit of manufacturer
π_r	Profit of retailer
π_g	Profit of gray marketer
π_p	Profit of retail platform

retail platform or nothing. Using the same method, we derive the inverse demand functions for product 2 and authorized product 1: $p_2 = 1 - Q_2 - \theta Q_1$ and $p_1 = \theta(1 - Q_1 - Q_2)$.

When the manufacturer does not encroach the retail market and the gray marketer has an option to enter in the overseas market (*i.e.*, no encroachment and gray market threat), the overseas consumer can purchase product 2 from retailer, unauthorized product 1 from gray market or nothing. Based on consumer utility functions, we also derive the inverse demand functions for product 2 and unauthorized product 1: $p_2 = 1 - Q_2 - \delta\theta Q_g$ and $p_g = \delta\theta(1 - Q_g - Q_2)$. When the manufacturer encroaches the retail market and the gray marketer has an option to enter in the overseas market (*i.e.*, encroachment and gray market threat), the overseas consumer can purchase product 2 from retailer, authorized product 1 from retail platform, unauthorized product 1 from gray market or nothing. The inverse demand functions for product 2, authorized product 1 and unauthorized product 1 are as follows: $p_2 = 1 - Q_2 - \theta(Q_1 + \delta Q_g)$, $p_1 = \theta(1 - Q_1 - Q_2 - \delta Q_g)$ and $p_g = \delta\theta(1 - Q_g - Q_1 - Q_2)$.

In practice, the price of a product in a market is determined by a combination of various factors and does not change frequently. To maintain the stability of the domestic market, the manufacturer rarely changes its domestic product price after entering the overseas market, even if the gray market exists. For simplicity and tractability, following Altug and Sahin [4] and Shi *et al.* [26], we suppose the retail price of product 1 in the domestic market (p) is exogenous, where $0 < p < 1$. Hence, the manufacturer obtains a constant profit from the domestic market, and thereby we only need to consider the manufacturer's profit from the overseas market in each case. Table 1 summarizes the notations.

3.3. Sequence of events

The sequence of events is illustrated in Figure 1. In stage 1, the manufacturer decides its channel strategy, encroaching or not. In stage 2, the manufacturer decides a wholesale price (w) to the retailer. In stage 3, the retailer decides its order quantity of product 2 (Q_2). In stage 4, the manufacturer decides order quantity of authorized product 1 (Q_1) if encroaches and the gray marketer decides order quantity of unauthorized product 1 (Q_g) if it has the option to enter.

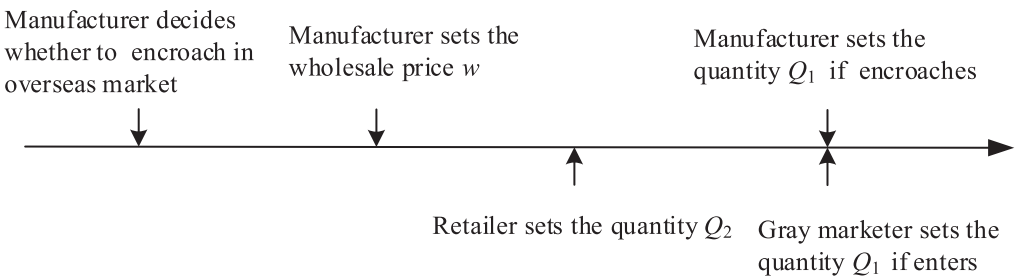

FIGURE 2. The timeline of the game.

4. Models and equilibrium solutions

In this section, we study the four subgames separately and give the corresponding equilibrium solutions. Specifically, these four subgames are denoted as *NN* (no encroachment and no gray market threat), *EN* (encroachment and no gray market threat), *NG* (no encroachment and gray market threat), and *EG* (encroachment and gray market threat).

4.1. No encroachment and no gray market threat (*NN*)

When the gray marketer has no option to enter the overseas market (*i.e.*, without gray market threat), the manufacturer does not encroach the retail market and only sells product 2 through the retailer in overseas market. The profits of the retailer and the manufacturer are as follows: $\pi_r^{NN} = (1 - Q_2 - w)Q_2$ and $\pi_m^{NN} = wQ_2$. Given the wholesale price w, the retailer orders the quantity $Q_2 = \frac{1-w}{2}$ to maximize its profit π_r^{NN}. Anticipating this, the manufacturer sets the wholesale price $w^{NN} = \frac{1}{2}$ to maximize its profit π_m^{NN}. By substitution, we have $Q_2^{NN} = \frac{1}{4}$, $p_2^{NN} = \frac{3}{4}$, $\pi_r^{NN} = \frac{1}{16}$ and $\pi_m^{NN} = \frac{1}{8}$.

4.2. Encroachment and no gray market threat (*EN*)

When the gray marketer has no option to enter the overseas market, the manufacturer encroaches into the retail market. In this case, the manufacturer sells product 1 through retail platform in addition to selling product 2 through the retailer in overseas market. The profits of the retailer, the manufacturer, and the retail platform are as follows:

$$\pi_r^{EN} = (1 - Q_2 - \theta Q_1 - w)Q_2, \tag{1}$$
$$\pi_m^{EN} = wQ_2 + (1 - \gamma)\theta(1 - Q_1 - Q_2)Q_1, \tag{2}$$
$$\pi_p^{EN} = \gamma\theta(1 - Q_1 - Q_2)Q_1. \tag{3}$$

We derive the equilibrium solutions by backward induction, which are given in Proposition 4.1. All proofs are given in the Appendix A.

Proposition 4.1. *Under scenario EN, equilibrium outcomes are summarized in Table 2.*

4.3. No encroachment and gray market threat (*NG*)

When the gray marketer has the option to enter the overseas market (*i.e.*, with gray market threat), and the manufacturer does not encroach into the retail market. In this case, the manufacturer only sells product 2 through the retailer in the overseas market, and the gray marketer may divert product 1 from the domestic

TABLE 2. Equilibrium outcomes under scenario *EN*.

Scenario *EN*
$w^{EN} = \frac{(-2+\theta)(-4+\theta+\gamma\theta)}{16+2(-5+\gamma)\theta}$, $Q_2^{EN} = \frac{2+(-2+\gamma)\theta}{8+(-5+\gamma)\theta}$, $Q_1^{EN} = -\frac{3(-2+\theta)}{2(8+(-5+\gamma)\theta)}$;
$p_2^{EN} = \frac{3(-2+\theta)^2}{2(8+(-5+\gamma)\theta)}$, $p_1^{EN} = -\frac{3(-2+\theta)\theta}{2(8+(-5+\gamma)\theta)}$; $\pi_r^{EN} = -\frac{(-2+\theta)(2+(-2+\gamma)\theta)^2}{2(8+(-5+\gamma)\theta)^2}$,
$\pi_p^{EN} = \frac{9\gamma(-2+\theta)^2\theta}{4(8+(-5+\gamma)\theta)^2}$, $\pi_m^{EN} = \frac{(-2+\theta)(-2+(-1+2\gamma)\theta)}{32+4(-5+\gamma)\theta}$

TABLE 3. Equilibrium outcomes under scenario *NG*.

Scenario *NG*	*NG-E*	*NG-D*
	$p < \frac{3\delta\theta}{4}$	$\frac{3\delta\theta}{4} < p < \delta\theta$
w^{NG}	$\frac{1}{4}(2+2p-\delta\theta)$	$\frac{1}{2}\left(-2+p\left(-1+\frac{4}{\delta\theta}\right)+\delta\theta\right)$
Q_2^{NG}	$\frac{1}{4}$	$1-\frac{p}{\delta\theta}$
Q_g^{NG}	$\frac{3}{8}-\frac{p}{2\delta\theta}$	0
p_2^{NG}	$\frac{1}{8}(6+4p-3\delta\theta)$	$\frac{p}{\delta\theta}$
p_g^{NG}	$\frac{1}{8}(4p+3\delta\theta)$	p

market to the overseas market if it is profitable. The profits of gray marketer, the retailer and the manufacturer are as follows:

$$\pi_g^{NG} = (\delta\theta(1-Q_g-Q_2)-p)Q_g, \tag{4}$$

$$\pi_r^{NG} = (1-Q_2-\delta\theta Q_g-w)Q_2, \tag{5}$$

$$\pi_m^{NG} = wQ_2+pQ_g. \tag{6}$$

We derive the equilibrium solutions by backward induction, which are given in Proposition 4.2.

Proposition 4.2. *Under scenario NG, equilibrium outcomes are summarized in Table 3.*

As shown in Table 3, when the manufacturer does not encroach the retail market and the gray marketer has the option to enter the overseas market, we find that supply chain players' optimal decisions vary depending on the domestic price of product 1 (p). This is because the domestic price of product 1 is the purchase cost of the gray marketer, thus affecting the gray marketer's entry decision. When the domestic price of product 1 is low (*i.e.*, $p < \frac{3\delta\theta}{4}$), the gray marketer can successfully enter the overseas market and orders a positive quantity Q_g^{NG}. In this case, gray marketer enters the market and competes with the manufacturer, we refer to this region as "no encroachment and entry" region denoted by *NG-E*. However, when the domestic price of product 1 is moderate (*i.e.*, $\frac{3\delta\theta}{4} < p < \delta\theta$), the manufacture sets its wholesale price in such way that the gray marketer cannot earn a positive margin from selling unauthorized product 1 and thus the gray marketer's order quantity reduces to zero. In this case, the manufacturer can deter the entry of gray marketer, and hence, we refer to this region as "no encroachment and deterrence" region denoted by *NG-D*.

TABLE 4. Equilibrium outcomes under scenario *EG*.

Scenario *EG*	*EG-E*	*EG-D*
	$p < \bar{p} = \dfrac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)}$	$\bar{p} < p < \dfrac{\delta\theta}{2}$
w^{EG}	$w^{EG\text{-}E}$	$w^{EG\text{-}D}$
Q_2^{EG}	$\dfrac{(-4+\delta)^2-2p(-1+\gamma)(-2+\delta)+2\big(-8+\gamma(-2+\delta)^2-(-5+\delta)\delta\big)\theta}{4(-4+\delta)^2+2\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta}$	$1-\dfrac{2p}{\delta\theta}$
Q_1^{EG}	$\dfrac{(-4+\delta+2\theta)(-4p+3(-2+\delta)\theta)}{2\theta\big(2(-4+\delta)^2+\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta\big)}$	$\dfrac{p}{\delta\theta}$
Q_g^{EG}	$\dfrac{8p(-4+\delta)+2p(10+\gamma(-2+\delta)-\delta)\theta-3\delta\theta(-4+\delta+2\theta)}{2\delta\theta\big(2(-4+\delta)^2+\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta\big)}$	0
p_2^{EG}	$\dfrac{3(-4+\delta+2\theta)^2+2p(10-3\delta-\gamma(-2+\delta)(-1+\theta)+(-6+\delta)\theta)}{4(-4+\delta)^2+2\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta}$	$-\dfrac{p(-2+\theta)}{\delta\theta}$
p_1^{EG}	$\dfrac{(-4+\delta+2\theta)(-4p+3(-2+\delta)\theta)}{4(-4+\delta)^2+2\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta}$	$\dfrac{p}{\delta}$
p_g^{EG}	$\dfrac{-3\delta\theta(-4+\delta+2\theta)+2p(-2+\delta)(-8+5\theta-\gamma\theta+\delta(2+(-1+\gamma)\theta))}{4(-4+\delta)^2+2\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta}$	p

Notes. $w^{EG\text{-}E}$ and $w^{EG\text{-}D}$ are given in the Appendix A.

4.4. Encroachment and gray market threat (*EG*)

When the gray marketer has the option to enter the overseas market, the manufacturer encroaches into the retail market. In this case, the manufacturer sells product 1 through retail platform in addition to selling product 2 through the retailer in overseas market, and the gray marketer may divert product 1 from the domestic market to the overseas market if it is profitable. The profits of the gray marketer, the retailer, the manufacturer and the retail platform are as follows:

$$\pi_g^{EG} = (\delta\theta(1-Q_g-Q_1-Q_2)-p)Q_g, \tag{7}$$
$$\pi_r^{EG} = (1-Q_2-\theta(Q_1+\delta Q_g)-w)Q_2, \tag{8}$$
$$\pi_m^{EG} = wQ_2+(1-\gamma)\theta(1-Q_1-Q_2-\delta Q_g)Q_1+pQ_g, \tag{9}$$
$$\pi_p^{EG} = \gamma\theta(1-Q_1-Q_2-\delta Q_g)Q_1. \tag{10}$$

We derive the equilibrium solutions by backward induction, which are given in Proposition 4.3.

Proposition 4.3. *Under scenario EG, equilibrium outcomes are summarized in Table 4.*

Table 4 shows that the equilibrium outcomes in scenario *EG* are similar to those in scenario *NG*. Specifically, when the manufacturer encroaches the retail market, the gray marketer can successfully enter the overseas market when the domestic price of product 1 is low (*i.e.*, $p < \bar{p}$) and can be deterred from entering the overseas market when the domestic price of product 1 is moderate (*i.e.*, $\bar{p} < p < \frac{\delta\theta}{2}$). The economic intuition behind this result is consistent with that of scenario *NG* and we omit it here. For simplicity, we refer to these two regions as "encroachment and entry" region and "encroachment and deterrence" region, denoted by *EG-E* and *EG-D*, respectively.

5. Comparative analysis

5.1. Gray marketer's entry decision

Given the manufacturer's encroachment strategy, we analyse the condition under which the gray marketer can enter the overseas market, shown in Proposition 5.1.

Proposition 5.1. (i) *When the manufacturer never encroaches in the overseas market, the gray marketer can successfully enter the market only if* $p < \frac{3\delta\theta}{4}$.
(ii) *When the manufacturer encroaches in the overseas market, the gray marketer can successfully enter the market only if* $p < \bar{p}$.
(iii) *The gray marketer is less likely to enter the market under manufacturer encroachment, i.e.,* $\bar{p} < \frac{3\delta\theta}{4}$.

Proposition 5.1 shows that regardless of whether the manufacturer has encroached on the overseas market or not, the gray marketer can successfully enter the overseas market if the domestic price of product 1 is sufficiently low. This is intuitive. Because the domestic price of product 1 (p) is the purchase cost of the gray marketer. when the domestic price of product 1 is sufficiently low, the gray marketer has a stronger incentive to resell product 1 in the overseas market due to a low purchase cost. In this case, if the manufacturer intends to drive the gray marketer out of the market, it will have to charge a lower wholesale price, which in turn will significantly reduce its profit. In order to avoid losing more profit, the manufacturer is forced to accommodate the gray market under such circumstance.

Interestingly, Proposition 5.1(iii) suggests that the gray marketer is less likely to enter the market under manufacturer encroachment. This is because when the manufacturer encroaches into the overseas market, it sells product 1 through retail platform in addition to product 2. This will further intensify retail competition and results in the gray marketer being unable to make a positive margin from reselling product 1 unless the domestic price of product 1 becomes lower. As a result, manufacturer encroachment can narrow the range in which the gray marketer can enter the overseas market.

5.2. Manufacturer's encroachment decision

In this subsection, comparing the manufacturer's profits under different scenarios, we give the conditions under which the manufacturer chooses to encroach without and with gray market threat, shown in Propositions 5.2 and 5.3. To provide a more intuitive presentation, we depict Figure 3 to illustrate the thresholds below which the manufacturer chooses to encroach and how these thresholds vary with the parameter p.

Proposition 5.2. *Without gray market threat (i.e.,* $p > \delta\theta$*), the manufacturer chooses to encroach if* $\gamma < \gamma_1 = \frac{5-2\theta}{9-4\theta}$. *Otherwise, the manufacturer chooses not to encroach.*

As shown in Figure 3, when the domestic price of product 1 is high (*i.e.*, $p > \delta\theta$), the gray marketer always has no incentive to enter market due to high purchase cost (*i.e.*, without gray market threat). In this case, the manufacturer has an incentive to encroach on the retail market when the commission rate γ is low. The reason is as follows. To uphold the retailer's order incentive, the encroaching manufacturer encroaches will charge a lower wholesale price, which reduces its wholesale profit. On the other hand, the manufacturer earns the added retail profit from selling authorized product 1. Moreover, as the commission rate decreases, the efficiency of encroachment channel becomes high, and thereby the manufacturer can get more profit from encroachment channel. Thus, the manufacturer benefits from encroachment when the commission rate is low because the gain in retail profit can offset the loss in wholesale profit.

Proposition 5.3. *With gray market threat (i.e.,* $p < \delta\theta$*),*

(i) *The manufacturer chooses to encroach when any of the following conditions are satisfied:*
  (a) $p < \tilde{p}$ *and* $\gamma < \gamma_2$*;*
  (b) $\tilde{p} < p < \frac{\delta\theta}{2}$ *and* $\gamma < \gamma_3$*;*

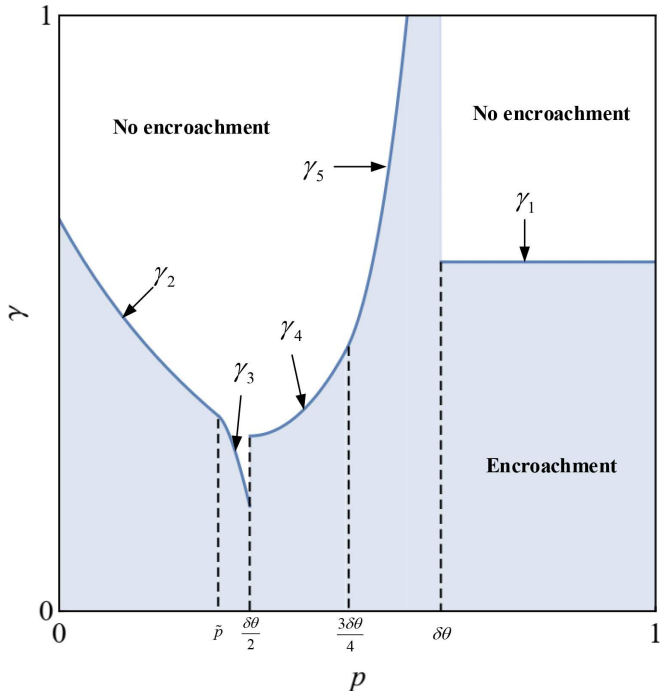

FIGURE 3. Manufacturer's encroachment decision ($\theta = 0.8$ and $\delta = 0.8$).

(c) $\frac{\delta\theta}{2} < p < \frac{3\delta\theta}{4}$ *and* $\gamma < \gamma_4$;
(d) $\frac{3\delta\theta}{4} < p < \delta\theta$ *and* $\gamma < \gamma_5$.

(ii) *Otherwise, the manufacturer chooses not to encroach.*

This result indicates that in the presence of gray market threat, manufacturers still have an incentive to encroach when the commission rate is low (as shown in Fig. 3). The economic intuition behind this is as follows. When the domestic price of product 1 is low (*i.e.*, $p < \frac{\delta\theta}{2}$), the gray marketer has an incentive to enter market regardless of whether the manufacturer chooses to encroach or not. In this case, when the manufacturer chooses to encroach, it can directly sell product 1 through retail platform in addition to gray market. On the one hand, to ensure that his encroachment does not excessively reduce the retailer's order incentive, the encroaching manufacturer must reduce the wholesale price. Moreover, due to manufacturer encroachment, the increased retail competition leads the gray marketer to order less unauthorized product 1. Combining these two effects, the manufacturer gets a lower total wholesale profit from product 2 and unauthorized product 1. On the other hand, the manufacturer earns the added retail profit from its encroachment channel. Moreover, as the commission rate decreases, the manufacturer can get more profit from the encroachment channel due to the increased efficiency of this channel. Consequently, when the commission rate is low, the gain in retail profit (from encroachment channel) can offset the loss in wholesale profit (from product 2 and unauthorized product 1), and thus the manufacturer benefits from encroachment. Similarly, when the domestic price of product 1 is moderate (*i.e.*, $\frac{\delta\theta}{2} < p \leq \delta\theta$), the manufacturer still can benefit from encroachment when the commission rate is low. The economic intuition behind this is similar to that when the domestic price of product 1 is low and the commission rate is low, we omit it here.

Combining Propositions 5.2 and 5.3, regardless of whether there is a gray market threat, the manufacturer should choose to encroach through retail platform when the commission rate is low. This finding provides a

plausible explanation for why brand manufacturers are increasingly encroaching on the retail market through retail platforms in practice.

Next, we analyse the impact of the parameter p on the manufacturer's incentive to encroach (*i.e.*, the thresholds below which the manufacturer chooses to encroach).

Corollary 5.1. *The impact of the parameter p on the manufacturer's incentive to encroach,*

(i) *Without gray market threat (i.e., $p > \delta\theta$), the manufacturer's incentive to encroach remains constant with p (i.e., $\frac{\partial \gamma_1}{\partial p} = 0$).*
(ii) *With gray market threat (i.e., $p < \delta\theta$), the manufacturer's incentive to encroach first decreases and then increases with p (i.e., $\frac{\partial \gamma_2}{\partial p} < 0$, $\frac{\partial \gamma_3}{\partial p} < 0$, $\frac{\partial \gamma_4}{\partial p} > 0$, and $\frac{\partial \gamma_5}{\partial p} > 0$).*

When the domestic price of product 1 is high (*i.e.*, $p > \delta\theta$), the gray marketer has no incentive to enter market due to high purchase cost (*i.e.*, without gray market threat). In such case, the domestic price of product 1 does not affect the manufacturer's decision and, therefore, the manufacturer's incentive to encroach remains constant with p.

However, when the domestic price of product 1 is not high (*i.e.*, $p < \delta\theta$), the manufacturer's incentive to encroach decreases first and then increases with the domestic price of product 1 (as shown in Fig. 3). Specifically, when the domestic price of product 1 is low (*i.e.*, $p < \frac{\delta\theta}{2}$), the gray marketer has a strong incentive to enter the overseas market due to the low cost. In this case, the manufacturer only gets a low margin from unauthorized product 1 (*i.e.*, p at a low level). In order to get a higher margin from product 1, the manufacturer is willing to encroach through retail platform when the commission rate is below a certain threshold. However, as the domestic price of product 1 increases, the manufacturer is less likely to benefit from encroachment and thus manufacturer's incentive to encroach decreases with p. When the domestic price of product 1 is moderate (*i.e.*, $\frac{\delta\theta}{2} < p < \delta\theta$), the manufacturer can drive the gray marketer out of the overseas market through encroachment and thus the manufacturer's total profit remains constant with p if encroaching. In contrast, if not encroaching, as the domestic price of product 1 increases, the reduction in order quantity of unauthorized product 1 makes the manufacturer get less profit from gray market. Thus, the manufacturer is more likely to benefit from encroachment as the domestic price of product 1 increases and accordingly this increases manufacturer's incentive to encroach.

The following Corollary 5.2 study how the gray market threat affects the manufacturer's incentive to encroach.

Corollary 5.2. *When $\underline{p} < p < \bar{\bar{p}}$, the manufacturer is less likely to encroach with gray market threat.*

As shown in Corollary 5.2, the existence of the gray market threat reduces the incentive for the manufacturer to encroach when the domestic price of product 1 is relatively moderate (*i.e.*, $\underline{p} < p < \bar{\bar{p}}$). Under such situation, if not encroaching, the manufacturer can obtain a relatively high wholesale from both retail channel and gray market and the competition between product 2 and unauthorized product 1 is not intense. In contrast, manufacturer encroachment will substantially increase retail competition, and thereby significantly reduce the manufacturer's total wholesale profit. Therefore, when the encroaching channel is not sufficiently efficient (*i.e.*, the commission rate is not sufficiently low), the additional retail profit from the encroachment channel cannot offset the loss in wholesale profit. As a result, the manufacturer is less likely to encroach with gray market threat when the domestic price of product 1 is relatively moderate.

Combining Corollaries 5.2 and 5.1, we find that the manufacturer's incentive to encroach varies with the domestic price of product 1 and gray market threat.

5.3. Equilibrium strategy

Combining the analysis of the gray marketer's entry decision and the manufacturer's encroachment decision, we give the equilibrium strategies as shown below. Figure 4 provides a graphical illustration regarding the equilibrium strategies.

Proposition 5.4. *In equilibrium,*

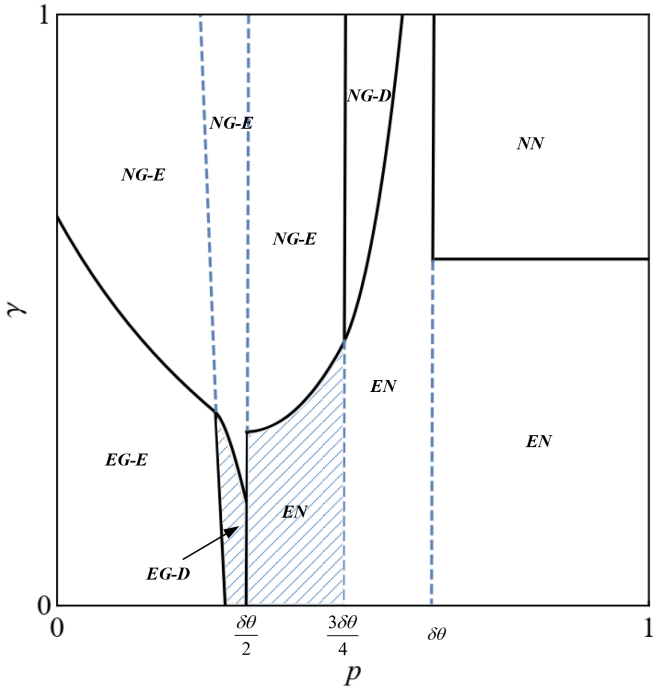

FIGURE 4. Equilibrium strategy ($\theta = 0.8$ and $\delta = 0.8$).

(i) *EG-E is the equilibrium strategy when* $p < \frac{\delta\theta}{2}$ *and* $\gamma < \min\{\bar{\gamma}, \gamma_2\}$.

(ii) *EG-D is the equilibrium strategy when* $p < \frac{\delta\theta}{2}$ *and* $\bar{\gamma} < \gamma < \gamma_3$.

(iii) *EN is the equilibrium strategy when* $\frac{\delta\theta}{2} < p < \frac{3\delta\theta}{4}$ *and* $\gamma < \gamma_4$, $\frac{3\delta\theta}{4} < p < \delta\theta$ *and* $\gamma < \gamma_5$, *or* $p > \delta\theta$ *and* $\gamma < \gamma_1$.

(iv) *NN is the equilibrium strategy when* $p > \delta\theta$ *and* $\gamma > \gamma_1$.

(v) *NG-D is the equilibrium strategy when* $\frac{3\delta\theta}{4} < p < \delta\theta$ *and* $\gamma > \gamma_5$.

(vi) *Otherwise, NG-E is the equilibrium strategy.*

As shown in Figure 4, when the domestic price of product 1 is low (*i.e.*, $p < \frac{\delta\theta}{2}$), the manufacturer chooses to encroach into the retail market when the commission rate is low (*i.e.*, $\gamma < \gamma_2$ or $\gamma < \gamma_3$). In such situation, gray marketer still can successfully enter the market when the commission rate is below the threshold low $\bar{\gamma}$; otherwise, gray marketer is being driven out of the market because of manufacturer encroachment. Therefore, *EG-E* and *EG-D* occur in equilibrium. On the contrary, when the commission rate is high, the manufacturer does not encroach and then gray marketer always can successfully enter the market due to low purchase cost, thus *NG-E* occurs in equilibrium.

When the domestic price of product 1 is moderate (*i.e.*, $\frac{\delta\theta}{2} < p < \delta\theta$), the manufacturer chooses to encroach when the commission rate is low (*i.e.*, $\gamma < \gamma_4$ or $\gamma < \gamma_5$). In this situation, manufacturer encroachment can completely remove the incentive for the gray marketer to enter market, thus *EN* occurs in equilibrium. On the contrary, the manufacturer does not encroach when the commission rate is high, and the gray marketer only can enter the market when the domestic price of product 1 is below a threshold (*i.e.*, $p < \frac{3\delta\theta}{4}$). Otherwise, the manufacturer still can deter the gray marketer to enter through wholesale price. As a result, *NG-E* and *NG-D*

occur in equilibrium. When the domestic price of product 1 is high (*i.e.*, $p > \delta\theta$), the gray marketer always has no incentive to enter market due to high purchase cost, and the manufacturer chooses to encroach when the commission rate is low and chooses not encroach otherwise. Therefore, *EN* and *NN* occur in equilibrium.

5.4. The impacts of manufacturer encroachment

In this subsection, we study the role of manufacturer encroachment in resisting the gray market and the impacts of manufacturer encroachment on the gray marketer and the retailer.

We investigate the effectiveness of manufacturer encroachment in resisting the gray market. We examine the above effectiveness from two perspectives: (1) the effectiveness on eliminating the gray market, and (2) the effectiveness on reducing the scale of the gray market.

Proposition 5.5. *With respect to the gray market,*

(i) *Manufacturer encroachment can eliminate the gray market when* $\frac{\delta\theta}{2} < p < \frac{3\delta\theta}{4}$ *and* $\gamma < \gamma_4$ *or* $\tilde{p} < p < \frac{\delta\theta}{2}$ *and* $\bar{\gamma} < \gamma < \gamma_3$.

(ii) *Manufacturer encroachment can reduce the scale of the gray market when* $\tilde{p} < p < \frac{\delta\theta}{2}$ *and* $\gamma < \bar{\gamma}$ *or* $p < \tilde{p}$ *and* $\gamma < \gamma_2$.

Proposition 5.5(i) shows that manufacturer encroachment can eliminate the gray market effectively when the domestic price of product 1 is relatively low and when the commission rate is low (*i.e.*, the filled areas *EG-D* and *EN* shown in Fig. 4). Within the filled areas *EG-D* and *EN*, the gray marketer can successfully enter the overseas market if the manufacturer does not encroach. This is because the manufacturer would need to charge a lower wholesale price to drive the gray marketer out of the market, which in turn would significantly reduce its wholesale profit from retail channel. Thus, it is more profitable for the manufacturer to accommodate the gray market in this case. However, if the manufacturer chooses to encroach, the gray marketer would face more intense competition as it shifts from competing directly with product 2 to competing with authorized product 1. Moreover, the manufacturer can coordinate both the wholesale price and the selling quantity of authorized product 1. Combining these two effects, encroachment can drive the gray marketer out of the market. Proposition 5.5(ii) shows that manufacturer encroachment can reduce the scale of the gray market when the domestic price of product 1 is low and when the commission rate is low (*i.e.*, the area *EG-E* shown in Fig. 4). In this area, due to a low purchase cost, the gray marketer can successfully enter the market regardless of whether the manufacturer chooses to encroach or not. However, if the manufacturer chooses to encroach, the gray marketer still would face more intense competition and this would lead the gray marketer to order less product 1.

This finding suggests that manufacturer encroachment can be used as an effective tool to reduce the scale of the gray market and even eliminate the gray market under certain conditions, which is absent in the current literature [5, 22]. This provides a feasible solution for manufacturers to deter the gray market.

Proposition 5.6. *When the manufacturer encroaches,*

(i) *Manufacturer encroachment always hurts the gray marketer.*

(ii) *Manufacturer encroachment benefits the retailer when* $\frac{3\delta\theta}{4} < p < \delta\theta$ *and* $\gamma_R < \gamma < \gamma_5$ *and hurts the retailer otherwise.*

Proposition 5.6 shows that manufacturer encroachment always hurts the gray marketer while manufacturer encroachment can benefit the retailer in certain conditions. As shown in Proposition 5.5, due to increased retail competition, manufacturer encroachment can reduce the scale of the gray market and even eliminate the gray market. Therefore, manufacturer encroachment hurts the gray marketer.

Interestingly, the retailer can benefit from manufacturer encroachment when the domestic price of product 1 is relatively high and the commission rate is moderate (see Fig. 5). When the domestic price of product 1 is relatively high, manufacturer encroachment can completely remove the incentive for the gray marketer to enter

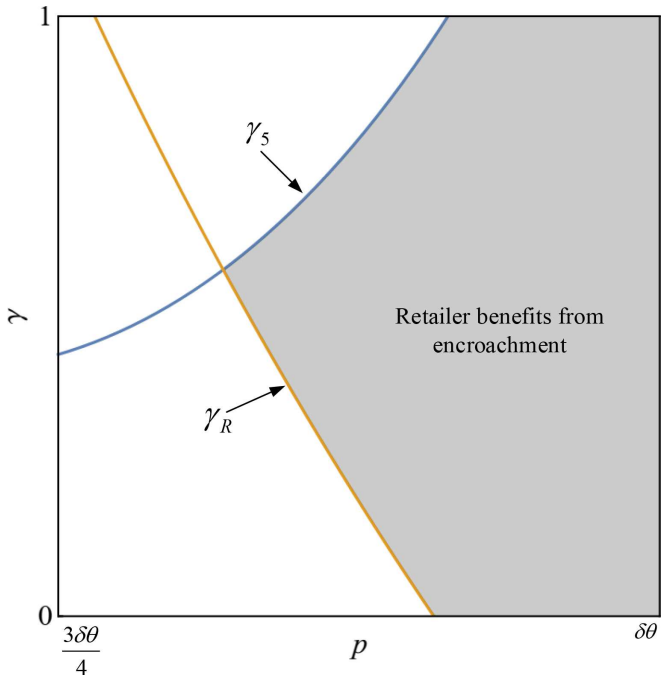

FIGURE 5. The impact of manufacturer encroachment on the retailer ($\theta = 0.8$ and $\delta = 0.8$).

market due to increased retail competition. However, to ensure that encroachment does not excessively reduce the retailer's order incentive, the manufacturer must reduce the wholesale price. Moreover, as the commission rate increases, the manufacturer makes less profit from encroachment channel because the efficiency of encroachment channel decreases. Accordingly, the manufacturer will sell less product 1 at its encroachment channel and charge a lower wholesale price. As a result, when the commission rate is moderate (*i.e.*, $\gamma_R < \gamma < \gamma_5$), the wholesale price reduction dominates the reduction in the demand for product 2 as the competition between product 1 and product 2 is not very intense and the retailer benefits from manufacturer encroachment. In other cases, the increased competition reduces the retailer's demand for product 2 substantially, which hurts the retailer. This result indicates that in presence of gray market threat, the retailer still can benefit from manufacturer encroachment when the domestic price of product 1 is relatively high and the commission rate is moderate, which complements the result of Arya *et al.* [5].

6. Conclusion

We develop a model in which a manufacturer directly sells product 1 product in domestic market and sells product 2 product through a retailer in overseas market, and a gray marketer may divert product 1 from domestic market to overseas market (we call it gray market). With the rapid development of e-commerce, the manufacturer has an option to encroach into the overseas market by selling product 1 through retail platform (a type of manufacturer encroachment). This paper aims to examine the interplay of manufacturer encroachment on gray market. We first identify the condition under which the gray market can enter the market and how does manufacturer encroachment affect the gray marketer's entry decision. Then, we examine whether manufacturer has an incentive to encroach through retail platform and how does the gray market affect the manufacturer's incentive to encroach. Lastly, we analyse the impacts of manufacturer encroachment on the gray market as well as the profitability of both the gray marketer and the retailer.

First, regardless of manufacturer encroachment or not, the gray marketer can successfully enter the overseas market if the domestic price of product 1 is sufficiently low. When the domestic price of product 1 is sufficiently low, the gray marketer has a stronger incentive to resell product 1 due to a low purchase cost. To drive the gray marketer out of the market, the manufacturer must reduce the wholesale price, which in turn will significantly reduce its profit. Therefore, it is more profitable for the manufacturer to accommodate the gray market under such situation.

Second, regardless of whether there is a gray market threat, the manufacturer has an incentive to encroach through retail platform when the commission rate is low. On the one hand, to ensure that his encroachment does not excessively reduce the retailer's order incentive, the encroaching manufacturer must reduce the wholesale price. Moreover, due to the increased retail competition, the gray marketer also orders fewer product 1 from the domestic market. Combining these two effects, the manufacturer's total wholesale profit from retail channel and gray market decreases. On the other hand, the manufacturer can earn the additional retail profit from encroachment channel, and this retail profit increases as the commission rate decreases due to the increased efficiency of encroachment channel. Consequently, when the commission rate is low, the manufacturer benefits from encroachment because the gain in retail profit can offset the loss in wholesale profit.

Third, we find that both the domestic price of product 1 and gray market threat have an impact on the manufacturer's incentive to encroach. Specifically, the manufacturer's incentive to encroach first decreases, then increases and finally remains constant with the domestic price of product 1. In addition, the existence of the gray market threat reduces the incentive for the manufacturer to encroach when the domestic price of product 1 is relatively moderate. Under such situation, in the absence of encroachment, the manufacturer can obtain a relatively high wholesale profit from both retail channel and gray market because the competition between product 2 and unauthorized product 1 is not intense. However, manufacturer encroachment will substantially increase retail competition, and thereby significantly reduce the manufacturer's total wholesale profit. Therefore, the additional retail profit from the encroachment channel can offset the loss in wholesale profit only when the commission rate becomes lower and thus the manufacturer is less likely to encroach under such situation.

Finally, manufacturer encroachment can reduce the scale of the gray market or even eliminate the gray market. Especially when the domestic price of product 1 is relatively low and when the commission rate is low, manufacturer encroachment can effectively eliminate the gray market. This is because when the purchase cost of the gray marketer is not too low, due to manufacturer encroachment, the increased retail competition leads to the gray marketer cannot earn a positive margin from selling unauthorized product 1. Thus, the gray marketer has no incentive to enter market with manufacturer encroachment. Besides, the gray marketer always suffers from manufacturer encroachment due to the increased retail competition. Nevertheless, the retailer can benefit from manufacturer encroachment due to the wholesale price reduction when the domestic price of product 1 is relatively high and the commission rate is moderate.

Our findings provide several new managerial insights for manufacturers to manage gray markets and introduce online channels. First, regardless of manufacturer encroachment or not, the gray marketer can successfully enter the overseas market if the domestic price of product 1 is sufficiently low. This suggests that manufacturers should try to sell their products in similar markets to prevent the prevalence of gray markets. Second, regardless of whether there is a gray market threat, the manufacturer should encroach through retail platform when the commission rate is low. This is because when the efficiency of encroachment channel is high (*i.e.*, a low commission rate), the additional retail profit from the encroachment channel is overwhelming and the manufacturer can obtain higher profit with encroachment. Furthermore, the manufacturer's incentive to encroach varies with the domestic price of product 1 and gray market threat. In particular, the existence of the gray market threat reduces the incentive for the manufacturer to encroach when the domestic price of product 1 is relatively moderate. We suggest that manufacturers should identify the market conditions under which it is advantageous to implement encroachment through retail platforms, such as data analysis, industry surveys, and so on. Finally, our findings show that manufacturer encroachment can be used as an effective tool to combat the gray market. Because manufacturer encroachment can reduce the scale of the gray market and even eliminate the gray market under certain conditions.

Our research leaves several directions for future research. First, we assume that a manufacturer operates as a monopoly in both markets. It would be worthwhile to incorporate the multi-manufacturer competition into encroachment decision in the presence of gray market. Second, we assume that information is complete between upstream and downstream members of the supply chain. It would be interesting to investigate the effect of information asymmetry on gray market as well as manufacturer encroachment.

Appendix A.

Proof of Proposition 4.1. We solve the equilibrium solutions for subgame EN by backward induction. In the last stage, the manufacturer sets the quantity of product 1 sold through retail platform to maximize its total profit from both channels. It can be shown that π_m^{EN} is concave in Q_1. Solving the first-order condition of $\frac{\partial \pi_m^{EN}}{\partial Q_1} = 0$ yields $Q_1(Q_2) = \frac{1-Q_2}{2}$. Then, substituting $Q_1(Q_2)$ into π_r^{EN}, we have that π_r^{EN} is concave in Q_2. Solving the first-order condition of $\frac{\partial \pi_r^{EN}}{\partial Q_2} = 0$ yields $Q_2(w) = \frac{1}{2} + \frac{w}{-2+\theta}$. Finally, we solve the optimization problem for the wholesale price w by substituting $Q_1(Q_2)$ and $Q_2(w)$ into π_m^{EN}. Besides, to ensure $Q_1(Q_2) \geq 0$ and $Q_2(w) \geq 0$, we derive the condition of $2 - 2w - \theta \geq 0$. Then, we introduce the corresponding Lagrangian function is as follows:

$$L_m = \pi_m^{EN} + \lambda(2 - 2w - \theta).$$

We need to discuss the following two cases.

Case 1. When $\lambda = 0$ and $2 - 2w - \theta > 0$, we have $w^{EN} = \frac{(-2+\theta)(-4+\theta+\gamma\theta)}{16+2(-5+\gamma)\theta}$. Since $2 - 2w^{EN} - \theta > 0$, the solution is feasible.

Case 2. When $\lambda > 0$ and $2 - 2w - \theta = 0$, we have $w^{EN} = 1 - \frac{\theta}{2}$ and $\lambda = \frac{2+(-2+\gamma)\theta}{4(-2+\theta)}$. Since $\lambda = \frac{2+(-2+\gamma)\theta}{4(-2+\theta)} < 0$, the solution is infeasible.

Therefore, we can obtain the equilibrium outcomes by substitution, which are summarized in Table 2. □

Proof of Proposition 4.2. we solve the equilibrium solutions for subgame NG by backward induction. In the last stage, the gray marketer sets the quantity of unauthorized product 1. It can be shown that π_g^{NG} is concave in Q_g. Solving the first-order conditions of $\frac{\partial \pi_g^{NG}}{\partial Q_g} = 0$ yields $Q_g(Q_2) = \frac{1}{2}\left(1 - Q_2 - \frac{p}{\delta\theta}\right)$. To ensure $Q_g(Q_2) \geq 0$, we derive the condition of $1 - Q_2 - \frac{p}{\delta\theta} \geq 0$. Then, we introduce the corresponding Lagrangian function is as follows:

$$L_r = \pi_r^{NG} + \alpha\left(1 - Q_2 - \frac{p}{\delta\theta}\right).$$

When $\alpha = 0$, then $Q_2(w) = \frac{2+p-2w-\delta\theta}{4-2\delta\theta}$. $1 - Q_2 - \frac{p}{\delta\theta} > 0$ requires $1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta} > 0$. When $\alpha > 0$, then $Q_2(w) = 1 - \frac{p}{\delta\theta}$ and $\alpha = \frac{1}{2}\left(-2 - 2w + p\left(-1 + \frac{4}{\delta\theta}\right) + \delta\theta\right)$. $\alpha = \frac{1}{2}\left(-2 - 2w + p\left(-1 + \frac{4}{\delta\theta}\right) + \delta\theta\right) > 0$ requires $0 < w < \frac{1}{2}\left(-2 + p\left(-1 + \frac{4}{\delta\theta}\right) + \delta\theta\right)$ if $2 + \delta\theta + \frac{8}{-4+\delta\theta} < p \leq \delta\theta$ or $0 < w < 1$ if $\delta\theta < p < 1$ or. Therefore,

$$Q_2(w) = \begin{cases} \frac{2+p-2w-\delta\theta}{4-2\delta\theta} & \text{if } 1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta} \geq 0 \\ 1 - \frac{p}{\delta\theta} & \text{if } 1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta} < 0. \end{cases}$$

Case 1. For $Q_2(w) = \frac{2+p-2w-\delta\theta}{4-2\delta\theta}$, we introduce the corresponding Lagrangian function is as follows:

$$L_m = \pi_m^{NG} + \alpha_1\left(1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta}\right).$$

We need to discuss the following two cases.

a. When $\alpha_1 = 0$ and $1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta} > 0$, we have $w^{NG} = \frac{1}{4}(2 + 2p - \delta\theta)$. $1 - \frac{2p}{\delta\theta} + \frac{p-2w}{-2+\delta\theta} > 0$ requires $p < \frac{3\delta\theta}{4}$. Thus, the solution is feasible when $p < \frac{3\delta\theta}{4}$.

b. When $\alpha_1 > 0$ and $1-\frac{2p}{\delta\theta}+\frac{p-2w}{-2+\delta\theta}=0$, we have $w^{NG}=\frac{1}{2}\left(-2+p\left(-1+\frac{4}{\delta\theta}\right)+\delta\theta\right)$ and $\alpha_1=p\left(-1+\frac{2}{\delta\theta}\right)+\frac{3}{4}(-2+\delta\theta)$. $\alpha_1=p\left(-1+\frac{2}{\delta\theta}\right)+\frac{3}{4}(-2+\delta\theta)>0$ requires $p>\frac{3\delta\theta}{4}$. Besides, $Q_2^{NG}=1-\frac{p}{\delta\theta}>0$ requires $p<\delta\theta$. Thus, the solution is feasible when $\frac{3\delta\theta}{4}<p<\delta\theta$.

Case 2. For $Q_2(w)=1-\frac{p}{\delta\theta}$, by substitution, we have $\pi_m^{NG}=w-\frac{pw}{\delta\theta}$. It can be shown that π_m^{NG} is monotonically increasing in w.

a. When $2+\delta\theta+\frac{8}{-4+\delta\theta}<p\le\delta\theta$, then $0<w<\frac{1}{2}\left(-2+p\left(-1+\frac{4}{\delta\theta}\right)+\delta\theta\right)$. We derive $w^{NG}=\frac{1}{2}\left(-2+p\left(-1+\frac{4}{\delta\theta}\right)+\delta\theta\right)$. Thus, the solution is feasible when $2+\delta\theta+\frac{8}{-4+\delta\theta}<p\le\delta\theta$.

b. When $\delta\theta<p<1$, then $0<w<1$. We derive $w^{NG}=1$. Since $Q_2^{NG}=1-\frac{p}{\delta\theta}<0$, the solution is infeasible.

Comparing the manufacturer's under these three cases, we derive the equilibrium outcomes for subgame NG, which are summarized in Table 3.

□

Proof of Proposition 4.3. We solve the equilibrium solutions for subgame EG by backward induction. In the last stage, the manufacturer sets the quantity of product 1 sold through retail platform and the gray marketer sets the quantity of unauthorized product 1. It can be shown that π_m^{EG} is concave in Q_1 and π_g^{EG} is concave in Q_g. Solving the first-order conditions of $\frac{\partial\pi_m^{EG}}{\partial Q_1}=0$ and $\frac{\partial\pi_g^{EG}}{\partial Q_g}=0$ yields $Q_1(Q_2)=-\frac{p+(-1+Q_2)(-2+\delta)\theta}{(-4+\delta)\theta}$ and $Q_g(Q_2)=\frac{2p+(-1+Q_2)\delta\theta}{(-4+\delta)\delta\theta}$. To ensure $Q_1(Q_2)\ge 0$ and $Q_g(Q_2)\ge 0$, we derive the condition of $1-\frac{2p}{\delta\theta}-Q_2\ge 0$. Then, we introduce the corresponding Lagrangian function is as follows:

$$L_r=\pi_r^{EG}+\beta\left(1-\frac{2p}{\delta\theta}-Q_2\right).$$

When $\beta=0$, then $Q_2(w)=\frac{1}{2}\left(1-\frac{p+w(-4+\delta)}{-4+\delta+2\theta}\right)$. $1-\frac{2p}{\delta\theta}-Q_2>0$ requires $1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}>0$. When $\beta>0$, then $Q_2(w)=1-\frac{2p}{\delta\theta}$ and $\beta=\frac{p(-\delta(-4+\theta)+8(-2+\theta))-\delta\theta((1+w)(-4+\delta)+2\theta)}{(-4+\delta)\delta\theta}$. $\beta=\frac{p(-\delta(-4+\theta)+8(-2+\theta))-\delta\theta((1+w)(-4+\delta)+2\theta)}{(-4+\delta)\delta\theta}>0$ requires $0<w<-\frac{p\delta(-4+\theta)-8p(-2+\theta)+\delta\theta(-4+\delta+2\theta)}{(-4+\delta)\delta\theta}$ if $-\frac{\delta\theta(-4+\delta+2\theta)}{\delta(-4+\theta)-8(-2+\theta)}<p<-\frac{2\delta\theta(-4+\delta+\theta)}{\delta(-4+\theta)-8(-2+\theta)}$ or $0<w<1$ if $-\frac{2\delta\theta(-4+\delta+\theta)}{\delta(-4+\theta)-8(-2+\theta)}<p<1$. Therefore,

$$Q_2(w)=\begin{cases}\frac{1}{2}\left(1-\frac{p+w(-4+\delta)}{-4+\delta+2\theta}\right) & \text{if } 1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}\ge 0\\ 1-\frac{2p}{\delta\theta} & \text{if } 1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}<0.\end{cases}$$

Case 1. For $Q_2(w)=\frac{1}{2}(1-\frac{p+w(-4+\delta)}{-4+\delta+2\theta})$, we introduce the corresponding Lagrangian function is as follows:

$$L_m=\pi_m^{EG}+\beta_1\left(1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}\right).$$

We need to discuss the following two cases.

a. When $\beta_1=0$ and $1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}>0$, we have

$$w^{EG}=\frac{\begin{array}{l}2p(6+\gamma(-2+\delta)-2\delta)(-4+\delta)+p(28-\gamma(-6+\delta)(-2+\delta)+(-12+\delta)\delta)\theta\\ -(-4+\delta+2\theta)\left(-(-4+\delta)^2+\left(4+\gamma(-2+\delta)^2-(-2+\delta)\delta\right)\theta\right)\end{array}}{2(-4+\delta)^3+(-4+\delta)\left(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\right)\theta}.$$

$1-\frac{4p}{\delta\theta}+\frac{p+w(-4+\delta)}{-4+\delta+2\theta}>0$ requires $p<\frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)}$. Thus, the solution is feasible when $p<\frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)}$. Note that $p<\frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)}$ is equivalent to $\gamma<\bar{\gamma}=\frac{2p(16+\delta(-4+\theta)-10\theta)+3\delta\theta(-4+\delta+2\theta)}{2p(-2+\delta)\theta}$.

b. When $\beta_1 > 0$ and $11 - \frac{4p}{\delta\theta} + \frac{p+w(-4+\delta)}{-4+\delta+2\theta} = 0$, we have $w^{EG} = -\frac{p\delta(-4+\theta)-8p(-2+\theta)+\delta\theta(-4+\delta+2\theta)}{(-4+\delta)\delta\theta}$ and $\beta_1 = \frac{8p(-4+\delta)+2p(10+\gamma(-2+\delta)-\delta)\theta-3\delta\theta(-4+\delta+2\theta)}{2(-4+\delta)\delta\theta}$. $\beta_1 = \frac{8p(-4+\delta)+2p(10+\gamma(-2+\delta)-\delta)\theta-3\delta\theta(-4+\delta+2\theta)}{2(-4+\delta)\delta\theta} > 0$ requires $p > \frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)}$. Besides, $Q_2^{EG} = 1 - \frac{2p}{\delta\theta} > 0$ requires $p < \frac{\delta\theta}{2}$. Thus, the solution is feasible when $\frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)} < p < \frac{\delta\theta}{2}$. Note that $\frac{3\delta\theta(-4+\delta+2\theta)}{-4(8+(-5+\gamma)\theta)+2\delta(4+(-1+\gamma)\theta)} < p < \frac{\delta\theta}{2}$ is equivalent to $p < \frac{\delta\theta}{2}$ and $\gamma > \bar{\gamma} = \frac{2p(16+\delta(-4+\theta)-10\theta)+3\delta\theta(-4+\delta+2\theta)}{2p(-2+\delta)\theta}$.

Case 2. For $Q_2(w) = 1 - \frac{2p}{\delta\theta}$, by substitution, we have $\pi_m^{EG} = w + \frac{p(p-p\gamma-2w\delta)}{\delta^2\theta}$. It can be shown that π_m^{EG} is monotonically increasing in w.

a. When $-\frac{\delta\theta(-4+\delta+2\theta)}{\delta(-4+\theta)-8(-2+\theta)} < p < -\frac{2\delta\theta(-4+\delta+\theta)}{\delta(-4+\theta)-8(-2+\theta)}$, then $0 < w < -\frac{p\delta(-4+\theta)-8p(-2+\theta)+\delta\theta(-4+\delta+2\theta)}{(-4+\delta)\delta\theta}$. We derive $w^{EG} = -\frac{p\delta(-4+\theta)-8p(-2+\theta)+\delta\theta(-4+\delta+2\theta)}{(-4+\delta)\delta\theta}$. Besides, $Q_2^{EG} = 1 - \frac{2p}{\delta\theta} > 0$ requires $p < \frac{\delta\theta}{2}$. Thus, the solution is feasible when $-\frac{\delta\theta(-4+\delta+2\theta)}{\delta(-4+\theta)-8(-2+\theta)} < p < \frac{\delta\theta}{2}$.

b. When $-\frac{2\delta\theta(-4+\delta+\theta)}{\delta(-4+\theta)-8(-2+\theta)} < p < 1$, then $0 < w < 1$. We derive $w^{EG} = 1$. Since $Q_2^{EG} = 1 - \frac{2p}{\delta\theta} < 0$, the solution is infeasible.

Comparing the manufacturer's under these three cases, we derive the equilibrium outcomes for subgame EG, which are summarized in Table 4. Note that within Table 4, we have

$$w^{EG} = w^{EG\text{-}E} = \frac{\begin{array}{l}2p(6+\gamma(-2+\delta)-2\delta)(-4+\delta)+p(28-\gamma(-6+\delta)(-2+\delta)+(-12+\delta)\delta)\theta \\ -(-4+\delta+2\theta)\Big(-(-4+\delta)^2+\Big(4+\gamma(-2+\delta)^2-(-2+\delta)\delta\Big)\theta\Big)\end{array}}{2(-4+\delta)^3+(-4+\delta)\Big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\Big)\theta}$$

if $\gamma < \bar{\gamma}$ and $w^{EG} = w^{EG\text{-}D} = -\frac{p\delta(-4+\theta)-8p(-2+\theta)+\delta\theta(-4+\delta+2\theta)}{(-4+\delta)\delta\theta}$ if $p < \frac{\delta\theta}{2}$ and $\gamma > \bar{\gamma} = \frac{2p(16+\delta(-4+\theta)-10\theta)+3\delta\theta(-4+\delta+2\theta)}{2p(-2+\delta)\theta}$.

□

Proof of Proposition 5.1. Based on the equilibrium outcomes under scenarios NG and EG, we can directly derive the results in Proposition 5.1(i) and (ii). In addition, we have $\bar{p} < \frac{3\delta\theta}{4}$. □

Proof of Proposition 5.2. We have $\pi_m^{EN} - \pi_m^{NN} = \frac{\theta(5-2\theta+\gamma(-9+4\theta))}{64+8(-5+\gamma)\theta} > 0$ and $\frac{\partial(\pi_m^{EN}-\pi_m^{NN})}{\partial\gamma} < 0$. Hence, $\pi_m^{EN} - \pi_m^{NN}$ is monotonically decreasing in γ. Therefore, we have $\pi_m^{EN} - \pi_m^{NN} > 0$ if $\gamma < \gamma_1 = \frac{5-2\theta}{9-4\theta}$ and $\pi_m^{EN} - \pi_m^{NN} < 0$ otherwise. □

Proof of Proposition 5.3. When $p < \frac{\delta\theta}{2}$, we compare π_m^{EG} and π_m^{NG} within the comparable range (*i.e.*, feasible range). This comparable range can be divided into two regions: (a) $\gamma < \bar{\gamma}$ and (b) $p < \frac{\delta\theta}{2}$ and $\gamma > \bar{\gamma}$.

(a) We have $\pi_m^{EG\text{-}E} - \pi_m^{NG\text{-}E} = \frac{\begin{array}{l}\delta^2\theta^2(-18(-4+\delta)+(-8+\delta)(4+\delta)\theta)+8p\delta\theta\big(24(-2+\theta)-\delta^2\theta+2\delta(6+\theta)\big) \\ +8p^2(64+8(-5+\gamma)\theta+\delta(-16+(2-2\gamma+\delta)\theta))\end{array}}{16(-4+\delta)\delta^2\theta^2}$ and $\frac{\partial(\pi_m^{EG\text{-}E}-\pi_m^{NG\text{-}E})}{\partial\gamma} = -\frac{p^2}{\delta^2\theta} < 0$. Hence, $\pi_m^{EG\text{-}E} - \pi_m^{NG\text{-}E}$ is monotonically decreasing in γ. Therefore, we have $\pi_m^{EG\text{-}E} - \pi_m^{NG\text{-}E} > 0$ if $\gamma < \gamma_2$ and $\pi_m^{EG\text{-}E} - \pi_m^{NG\text{-}E} < 0$ otherwise.

(b) We have $\pi_m^{EG\text{-}D} - \pi_m^{NG\text{-}E} = \frac{f(\gamma)}{-16(-4+\delta)\theta\big(2(-4+\delta)^2+\big(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\big)\theta\big)}$ and $\frac{\partial f(\gamma)}{\partial\gamma} < 0$. It can be shown that $-16(-4+\delta)\theta(2(-4+\delta)^2+(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta)\theta) > 0$. Hence, $f(\gamma)$ is monotonically decreasing in γ. Therefore, we have $f(\gamma) > 0$ if $\gamma < \gamma_3$ and $f(\gamma) < 0$ otherwise. Hence, we derive that $\pi_m^{EG\text{-}D} - \pi_m^{NG\text{-}E} > 0$ if $\gamma < \gamma_3$ and $\pi_m^{EG\text{-}D} - \pi_m^{NG\text{-}E} < 0$ otherwise.

When $\frac{\delta\theta}{2} < p < \delta\theta$, we compare π_m^{EN} and π_m^{NG} within the comparable range (*i.e.*, feasible range). This comparable range can be divided into two regions: (c) $\frac{\delta\theta}{2} < p \le \frac{3\delta\theta}{4}$ and (d) $\frac{3\delta\theta}{4} < p < \delta\theta$.

(c) We have $\pi_m^{EN}-\pi_m^{NG\text{-}E}=\frac{(-2+\theta)(-2+(-1+2\gamma)\theta)}{32+4(-5+\gamma)\theta}-\frac{1}{16}(2+8p-\frac{8p^2}{\delta\theta}-\delta\theta)$ and $\frac{\partial(\pi_m^{EN}-\pi_m^{NG})}{\partial\gamma}=-\frac{9(-2+\theta)^2\theta}{4(8+(-5+\gamma)\theta)^2}<0$. Hence, $\pi_m^{EN}-\pi_m^{NG\text{-}E}$ is monotonically decreasing in γ. Solving $\pi_m^{EN}-\pi_m^{NG\text{-}E}=0$, we get $\gamma=\gamma_4$. Therefore, we have $\pi_m^{EN}-\pi_m^{NG\text{-}E}>0$ if $\gamma<\gamma_4$ and $\pi_m^{EN}-\pi_m^{NG\text{-}E}<0$ otherwise.

(d) We have $\pi_m^{EN}-\pi_m^{NG\text{-}D}=\frac{(-2+\theta)(-2+(-1+2\gamma)\theta)}{32+4(-5+\gamma)\theta}-\frac{(p-\delta\theta)(\delta\theta(2-\delta\theta)+p(-4+\delta\theta))}{2\delta^2\theta^2}$ and $\frac{\partial\pi_m^{EN}-\pi_m^{NG\text{-}D}}{\partial\gamma}=-\frac{9(-2+\theta)^2\theta}{4(8+(-5+\gamma)\theta)^2}<0$. Hence, $\pi_m^{EN}-\pi_m^{NG\text{-}D}$ is monotonically decreasing in γ. Solving $\pi_m^{EN}-\pi_m^{NG\text{-}D}=0$, we get $\gamma=\gamma_5$. Therefore, we have $\pi_m^{EN}-\pi_m^{NG\text{-}D}>0$ if $\gamma<\gamma_5$ and $\pi_m^{EN}-\pi_m^{NG\text{-}D}<0$ otherwise.

In addition, we have $\gamma_2<\bar{\gamma}$ if $p<\tilde{p}$. Here,

$$\gamma_2=\frac{\begin{array}{l}\delta^2\theta^2(-18(-4+\delta)+(-8+\delta)(4+\delta)\theta)+8p\delta\theta\big(24(-2+\theta)-\delta^2\theta+2\delta(6+\theta)\big)\\+8p^2(64-40\theta+\delta(-16+(2+\delta)\theta))\end{array}}{16p^2(-4+\delta)\theta},$$

$$\gamma_3=\frac{(-2+\delta)\left(\begin{array}{l}8p^2(8+\delta(-2+\theta)-6\theta)+(-2+\delta)\theta^2(40-2\delta(1+\delta)-16\theta+(-8+\delta)\delta\theta)\\-8p\theta(-2(-4+\delta)(-1+\delta)+(8+(-8+\delta)\delta)\theta)\end{array}\right)}{\begin{array}{l}(-2+\delta)^2\theta^2(-18(-4+\delta)+(-8+\delta)(4+\delta)\theta)-8p(-2+\delta)\theta(-6(-4+\delta)+(-8+(-4+\delta)\delta)\theta)\\+8p^2(-4(-4+\delta)+(-4+(-4+\delta)\delta)\theta)\end{array}}$$

and

$$\tilde{p}=\frac{\theta\left(\begin{array}{l}-96+48\theta+2\delta(48-14\theta+\delta(-9+(-4+\delta)\theta))\\+\sqrt{2}\sqrt{(-4+\delta)\Big(18(-4+\delta)^3-2(-576+\delta(224+\delta(48+\delta(-36+5\delta))))\theta+(-4+\delta)\Big(72+(-2+\delta)^2\delta(4+\delta)\Big)\theta^2\Big)}\end{array}\right)}{-64(-2+\theta)+4\delta(-8+(-2+\delta)\theta)}.$$

Here, $\gamma_4=\frac{8p\delta(8-5\theta)\theta+8p^2(-8+5\theta)+\delta\theta^2(-10+4\theta+\delta(-8+5\theta))}{\theta(8p^2-8p\delta\theta+\delta\theta(-18+(8+\delta)\theta))}$ and $\gamma_5=\frac{\begin{array}{l}2p^2(-8+5\theta)(-4+\delta\theta)-4p\delta\theta(-8+5\theta)(-3+\delta\theta)\\+\delta^2\theta^2(36+\theta(-20-\theta+2\delta(-8+5\theta)))\end{array}}{2\theta((-1+\delta)\delta^2\theta^3+p^2(-4+\delta\theta)-2p\delta\theta(-3+\delta\theta))}$.

□

Proof of Corollary 5.1. By calculation, we have the following results: $\frac{\partial\gamma_1}{\partial p}=0$;

$$\frac{\partial\gamma_2}{\partial p}=-\frac{64(-4+\delta)(-2+\delta)\theta(-4+\delta+2\theta)^2(4p-3(-2+\delta)\theta)(p-(-2+\delta)\theta)}{\left(\begin{array}{l}(-2+\delta)^2\theta^2(-18(-4+\delta)+(-8+\delta)(4+\delta)\theta)\\-8p(-2+\delta)\theta(-6(-4+\delta)+(-8+(-4+\delta)\delta)\theta)\\+8p^2(-4(-4+\delta)+(-4+(-4+\delta)\delta)\theta)\end{array}\right)^2}<0;$$

$$\frac{\partial\gamma_3}{\partial p}=\frac{\delta\big(-48p(-4+\delta)+18(-4+\delta)\delta\theta+4p(-6+\delta)(4+\delta)\theta-(-8+\delta)\delta(4+\delta)\theta^2\big)}{8p^3(-4+\delta)}<0;$$

$$\frac{\partial\gamma_4}{\partial p}=-\frac{288\delta(-2+\theta)^2(-2p+\delta\theta)}{(8p^2-8p\delta\theta+\delta\theta(-18+(8+\delta)\theta))^2}>0;$$

$$\frac{\partial\gamma_5}{\partial p}=\frac{9\delta^2(-2+\theta)^2\theta(p(4-\delta\theta)+\delta\theta(-3+\delta\theta))}{((-1+\delta)\delta^2\theta^3+p^2(-4+\delta\theta)-2p\delta\theta(-3+\delta\theta))^2}>0.$$

□

Proof of Corollary 5.2. By calculation, we have the following results within the feasible range:

$$\begin{array}{l}\gamma_2<\gamma_1 \text{ if } p>\underline{p} \text{ and } \gamma_2>\gamma_1 \text{ if } p<\underline{p};\\ \gamma_3<\gamma_1;\ \gamma_4<\gamma_1;\\ \gamma_5<\gamma_1 \text{ if } p<\bar{\bar{p}} \text{ and } \gamma_5>\gamma_1 \text{ if } p>\bar{\bar{p}}.\end{array}$$

Here,

$$\underline{p}=\frac{1}{4}\left(\begin{array}{l}\sqrt{2}\sqrt{\frac{(-4+\delta)(-2+\delta)^2\left(\delta(-3+\theta)^2-8(-2+\theta)^2\right)\theta^2\left(9(-4+\delta)^2-2(72+\delta(-29+3\delta))\theta+(-6+\delta)^2\theta^2\right)}{\left(\delta^2(-3+\theta)^2+28(-2+\theta)^2-2\delta(32+3\theta(-9+2\theta))\right)^2}}\\+\frac{2(-2+\delta)\theta\left(\delta^2(-3+\theta)^2+24(-2+\theta)^2-4\delta(15+\theta(-13+3\theta))\right)}{\delta^2(-3+\theta)^2+28(-2+\theta)^2-2\delta(32+3\theta(-9+2\theta))}\end{array}\right)$$

and $\bar{\bar{p}}=\frac{\delta\theta\left(-6+2\delta\theta-\sqrt{\delta\theta}\right)}{-8+2\delta\theta}$. □

Proof of Proposition 5.4. Combining Propositions 5.1–5.3, we are able to derive Proposition 5.4. □

Proof of Proposition 5.5. Comparing the equilibrium quantity Q_g under no encroachment and encroachment, we can directly derive the results in Proposition 5.5. □

Proof of Proposition 5.6. First, when the manufacturer encroaches, comparing the gray marketer's profits under no encroachment and encroachment, we have the following relationships:

When $\frac{\delta\theta}{2}<p<\frac{3\delta\theta}{4}$ and $\gamma<\gamma_4$, $\pi_g^{NG\text{-}E}>\pi_g^{EN}=0$;
When $\tilde{p}<p<\frac{\delta\theta}{2}$ and $\bar{\gamma}<\gamma<\gamma_3$, $\pi_g^{NG\text{-}E}>\pi_g^{EG\text{-}D}=0$;
When $\tilde{p}<p<\frac{\delta\theta}{2}$ and $\gamma<\bar{\gamma}$, or $p<\tilde{p}$ and $\gamma<\gamma_2$; $\pi_g^{NG\text{-}E}>\pi_g^{EG\text{-}E}>0$.

In summary, manufacturer encroachment always reduces the gray marketer's profit.

Second, when the manufacturer encroaches, comparing the retailer's profits under no encroachment and encroachment, we have the following relationships:

When $\delta\theta<p<1$ and $\gamma<\gamma_1$, we have $\pi_r^{EN}-\pi_r^{NN}=-\frac{1}{16}-\frac{(-2+\theta)(2+(-2+\gamma)\theta)^2}{2(8+(-5+\gamma)\theta)^2}<0$.
When $\frac{3\delta\theta}{4}<p<\delta\theta$ and $\gamma<\gamma_5$, we have $\pi_r^{EN}-\pi_r^{NG\text{-}D}=\frac{1}{2}\left(-\frac{(-2+\theta)(2+(-2+\gamma)\theta)^2}{(8+(-5+\gamma)\theta)^2}+\frac{(p-\delta\theta)^2(-2+\delta\theta)}{\delta^2\theta^2}\right)>0$ if $\gamma>\gamma_R$ and $\pi_r^{EN}-\pi_r^{NG\text{-}D}<0$ if $\gamma<\gamma_R$.
When $\frac{\delta\theta}{2}<p<\frac{3\delta\theta}{4}$ and $\gamma<\gamma_4$, we have $\pi_r^{EN}-\pi_r^{NG\text{-}E}=\frac{1}{32}\left(-2+\delta\theta-\frac{16(-2+\theta)(2+(-2+\gamma)\theta)^2}{(8+(-5+\gamma)\theta)^2}\right)<0$.
When $\tilde{p}<p<\frac{\delta\theta}{2}$ and $\bar{\gamma}<\gamma<\gamma_3$, we have $\pi_r^{EG\text{-}D}-\pi_r^{NG\text{-}E}=\frac{(-4+\delta+2\theta)(-2p+\delta\theta)^2}{(-4+\delta)\delta^2\theta^2}-\frac{1}{32}(2-\delta\theta)<0$.
When $\tilde{p}<p<\frac{\delta\theta}{2}$ and $\gamma<\bar{\gamma}$, or $p<\tilde{p}$ and $\gamma<\gamma_2$, we have

$$\pi_r^{EG\text{-}E}-\pi_r^{NG\text{-}E}=\frac{1}{32}\left(-2+\delta\theta+\frac{8(-4+\delta+2\theta)\left(\begin{array}{l}(-4+\delta)^2-2p(-1+\gamma)(-2+\delta)\\+2\left(-8+\gamma(-2+\delta)^2-(-5+\delta)\delta\right)\theta\end{array}\right)^2}{(-4+\delta)\left(\begin{array}{l}2(-4+\delta)^2\\+\left(-20+\gamma(-2+\delta)^2-(-8+\delta)\delta\right)\theta\end{array}\right)^2}\right)<0.$$

In summary, manufacturer encroachment increases the retailer's profit when $\frac{3\delta\theta}{4}<p<\delta\theta$ and $\gamma_R<\gamma<\gamma_5$. Otherwise, manufacturer encroachment reduces the retailer's profit. □

Acknowledgements

This work was supported by the National Key R&D Program of China [grant number 018YFB1601401], the National Natural Science Foundation of China [grant numbers 71991464/71991460, 71921001, 72201263 and 72001066] and the Fundamental Research Funds for the Central Universities [grant number WK2040000066].

Data availability statement

No new data/code were created or analysed in this study.

References

[1] V. Abhishek, K. Jerath and Z.J. Zhang, Agency selling or reselling? Channel structures in electronic retailing. *Manage. Sci.* **62** (2016) 2259–2280.

[2] R. Ahmadi and B.R. Yang, Parallel imports: challenges from unauthorized distribution channels. *Marketing Sci.* **19** (2000) 279–294.

[3] R. Ahmadi, F. Iravani and H. Mamani, Supply chain coordination in the presence of gray markets and strategic consumers. *Prod. Oper. Manage.* **26** (2017) 252–272.

[4] M.S. Altug and O. Sahin, Impact of parallel imports on pricing and product launch decisions in pharmaceutical industry. *Prod. Oper. Manage.* **28** (2019) 258–275.

[5] A. Arya, B. Mittendorf and D.E.M. Sappington, The bright side of supplier encroachment. *Marketing Sci.* **26** (2007) 651–659.

[6] B. Berman and M. Dong, Gray markets: past, present, and future. *Int. Rev. Retail Distrib. Consum. Res.* **26** (2015) 95–111.

[7] J. Chen, L. Liang, S. Mukhopadhyay and D.Q. Yao, Price or quantity? An analysis of strategies in the presence of gray markets. Nav. Res. Logistics (NRL) **70** (2023) 558–573.

[8] Danone, New formula milk created in China, for Chinese consumers. Retrieved from https://www.danone.com/stories/articles-list/new-formula-milk-created-in-china-for-chinese-consumers.html (2021).

[9] L. Ding, H. Yuan and B. Hu, Adopt or not: manufacturers' RFID decisions for gray marketing in a competitive environment. *Comput. Ind. Eng.* **151** (2021) 106957.

[10] D.F. Duhan and M.J. Sheffet, Gray markets and the legal status of parallel importation. *J. Marketing* **52** (1988) 75–83.

[11] R. Hou, W. Li, X. Lin and Y. Zhao, Impact of quality decisions on information sharing with supplier encroachment. *RAIRO-Oper. Res.* **56** (2022) 145–164.

[12] S. Huang, X. Guan and Y.-J. Chen, Retailer information sharing with supplier encroachment. *Prod. Oper. Manage.* **27** (2018) 1133–1147.

[13] H. Huang, Y. He and J. Chen, Competitive strategies and quality to counter parallel importation in global market. *Omega* **86** (2019) 173–197.

[14] F. Iravani, S. Dasu and R. Ahmadi, Beyond price mechanisms: How much can service help manage the competition from gray markets? *Eur. J. Oper. Res.* **252** (2016) 789–800.

[15] Z.-Z. Jiang, J. Zhao, Z. Yi and Y. Zhao, Inducing information transparency: the roles of gray market and dual-channel. *Ann. Oper. Res.* **329** (2023) 277–306.

[16] B. Kim and K.S. Park, Organizational structure of a global supply chain in the presence of a gray market: information asymmetry and valuation difference. *Int. J. Prod. Econ.* **175** (2016) 71–80.

[17] KPMG, Effective channel management is critica in combating the gray market and increasing technology companies's bottom line (2008).

[18] H. Li, S.X. Zhu, N. Cui and J. Li, Analysis of gray markets in differentiated duopoly. *Int. J. Prod. Res.* **54** (2016) 4008–4027.

[19] H. Li, K. Leng, Q. Qing and S.X. Zhu, Strategic interplay between store brand introduction and online direct channel introduction. *Transp. Res. Part E: Logistics Transp. Rev.* **118** (2018) 272–290.

[20] H. Li, J. Shao and S.X. Zhu, Impacts of power dynamics and gray markets on supply chains. *Int. J. Prod. Econ.* **222** (2020) 107488.

[21] K.J. Li, J. Zhang and R. Schaefer, Parallel imports of status goods: a strategic analysis of aesthetic design. *Prod. Oper. Manage.* **31** (2022) 2268–2288.

[22] B. Liu, X. Guan and Y. Wang, Supplier encroachment with multiple retailers. *Prod. Oper. Manage.* **30** (2021) 3523–3539.

[23] X. Nguyen, C.C. Chao, P. Sgro and M. Nabin, Cross-border travellers and parallel trade: implications for Asian economies. *World Econ.* **40** (2017) 1531–1546.

[24] J. Nie, Q. Wang, C. Shi and Y. Zhou, The dark side of bilateral encroachment within a supply chain. *J. Oper. Res. Soc.* **73** (2022) 811–821.

[25] A.P. Ponnachiyur Maruthasalam and G. Balasubramanian, Supplier encroachment in the presence of asymmetric retail competition. *Int. J. Prod. Econ.* **264** (2023) 108961.

[26] X. Shi, S. Yao and Y. Ma, When and how should cross-border platforms manage blockchain technology in the presence of purchasing agents? *Asia-Pac. J. Oper. Res.* **39** (2021) 21400200.

[27] X. Shi, J. Tang and C. Dong, Should a domestic firm carve out a niche in overseas markets? Value of purchasing agents. *Eur. J. Oper. Res.* **300** (2022) 85–94.
[28] A. Srivastava and A. Mateen, Supply chain contracts in the presence of gray markets. *Decis. Sci.* **51** (2019) 110–147.
[29] J. Tannenbaum, Franchisees resist poaching via software, kiosks, internet. *Wall Street J.* (1995).
[30] L. Tian, A.J. Vakharia, Y. Tan and Y.F. Xu, Marketplace, reseller, or hybrid: strategic analysis of an emerging e-commerce model. *Prod. Oper. Manage.* **27** (2018) 1595–1610.
[31] X. Wan, J. Chen and B. Chen, The manufacturer's encroachment strategy in the presence of the retailer's in-store service. *Int. J. Prod. Res.* **61** (2023) 7263–7280.
[32] J. Wang and W. Zhuo, Strategic information sharing in a supply chain under potential supplier encroachment. *Comput. Ind. Eng.* **150** (2020) 106880.
[33] M. Xue, The role of retailer's app channel in a supply chain with quality decision and different power structures. *RAIRO-Oper. Res.* **55** (2021) S885–S904.
[34] M.S. Xue and J.X. Zhang, Supply chain encroachment with quality decision and different power structures. *RAIRO-Oper. Res.* **54** (2020) 693–718.
[35] J. Yang, K.W. Li and J. Huang, Manufacturer encroachment with a new product under network externalities. *Int. J. Prod. Econ.* **263** (2023) 108954.
[36] J. Zhang, S. Li, S. Zhang and R. Dai, Manufacturer encroachment with quality decision under asymmetric demand information. *Eur. J. Oper. Res.* **273** (2019) 217–236.
[37] Y. Zhang, S.H. Zhang and H. Feng, The impacts of money-back guarantees in the presence of parallel importation. *Eur. J. Oper. Res.* **295** (2021) 170–182.
[38] Q. Zhang, X. Jiang and Y. Zheng, Blockchain adoption and gray markets in a global supply chain. *Omega* **115** (2023) 102785.
[39] S. Zheng and Y. Yu, Manufacturer encroachment with equal pricing strategy. *Transp. Res. Part E: Logistics Transp. Rev.* **152** (2021) 102346.
[40] B.R. Zheng, N. Yu, L. Jin and H.S. Xia, Effects of power structure on manufacturer encroachment in a closed-loop supply chain. *Comput. Ind. Eng.* **137** (2019) 106062.