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MARGINAL PROFIT MAXIMIZATION ESTIMATION OF SUPPLY CHAINS BY WASTE ENERGY DECREMENT: A CASE STUDY OF THE POWER INDUSTRY

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Abstract. Economic growth and excessive fossil energy consumption have direct effects on environmental destruction and greenhouse gas increments. The existing appropriate pattern for economic performance increase as well as pollution emissions abatement is a basic issue in industry activities. In this paper, a data envelopment analysis (DEA) model is introduced for estimating the directional marginal profit maximization of supply chain divisions based on wasted energy and power losses. The purpose of this study is to estimate the directional marginal productivity in the supply chain, which enables us to find the optimal direction of efficient divisions on the frontier. This makes the allocation of resources create a marginal profit increase and the pollution emissions be abated simultaneously. Indeed, the proposed model considers the synergistic effects of each input on MP estimation in predetermined directions. The model is able to estimate the marginal profit maximization of desirable output and undesirable output decrease for each input simultaneously. The results suggested that the gas field division of one of the supply chains had fundamental capacities for energy production increments and flare gas decrements. Furthermore, the gas field division of this supply chain also had a considerable structure for the marginal profit maximization of outputs based on flare gas decreases. Additionally, the distribution lines of 0.80% supply chains provided wasted energy reduction by adding one extra unit to the line's capacity in the determined direction. Especially, there were supply chains that had investment opportunities for an acceptable abatement of power losses. This not only enables divisions to respond to fluctuations in demand as they produce more energy in critical situations like climate change but also decreases harmful emissions as wasted energy in supply chain divisions.

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1. Introduction

Excessive energy consumption and greenhouse gas emissions cause global warming, soil degradation, erosion, and environmental pollution. This is because they decrease energy resources and power plant efficiency and create a lack of sustainability and stability in production activities. The gas turbine output of gas power plants and their productivity depend on the environment's temperature [18]. Additionally, the output power of gas power plants decreases by 5%–10% for each 10° increase up to 15° [26]. Also, the efficiency results of combined cycle

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power plants indicate that environmental temperature has significant effects on energy production abatement [34]. Similarly, environmental factors such as monsoon winds and storms cause transmission airline conductor vibrations, which create broken wire and a power cut in the transmitter network. In other words, fluctuations in distribution lines have bad and destructive effects on dispatching power. The major factors that cause disturbances in the voltage of distribution networks are the installation of power sources such as electric motors, electric furnaces, and electric welding machines. In this case, the energy and power plant sectors and transmission and distribution networks should have the necessary patterns to adjust output levels when confronting increased demand based on climate change and other critical situations. In other words, the power plant sectors of supply chains face a special climate situation as they consume more fossil fuels to increase efficiency and productivity. Similarly, the transmitter and distributor lines of the electricity supply chain are meeting more power losses under climate change and needing more electricity for economic return increments. Thus, the transmitter and distributer networks should have the necessary preparation for capacity adjustments and adjust output levels by controlling variable resources.

Providing suitable patterns for examining the relationship between economic return and environmental preservation is an important issue. It is necessary for harmful emissions management and wasted energy inhibition. Hence, the existing appropriate policy for energy consumption optimization and pollution emission reduction plays an important role in the sustainability and performance efficiency of industrial sections.

The energy and power plant industries are among the most important basic infrastructures of countries in the power industry. Therefore, energy consumption optimization and pollution reduction are two of the main problems in industry activities. Comprehensive pattern collection is an essential issue to decrease greenhouse gas emissions and control power loss in energy and power plant sectors and transmission and distribution lines. Flaring gas and greenhouse gases are considered important resources for pollution gas emissions in oil and gas fields and power plant sectors. Also, nitrogen oxides (NO_x), sulfur oxides (SO_x), carbon dioxide (CO_2), and hydrocarbons (CH) provide 0.9% of environmental pollution in energy and power plant sections. Hence, the increase in capacity utilization and more energy generation should be controlled as more resource allocation to desirable outputs increases along with greenhouse gases and energy losses abatement in the power industry. Indeed, resource management plays an essential role in the direction of increased energy production, flare, and greenhouse gas decrease in energy sections. In this way, we not only prevent resource losses but also decrease the destructive effects of greenhouse gases in oil and gas fields and power plant sections. Similarly, the transmission and distribution lines need an appropriate pattern to maximize capacity utilization in a direction that provides more energy while reducing power losses. In other words, the existing applicable pattern in the direction of economic return and wasted energy decrease creates sustainability for energy and power plant sections and the transmission and distribution lines in the electricity supply chain. Lee [23] provided a theoretical foundation for Directional Marginal Productivity (DMP) supporting the meta-data envelopment analysis (DEA), which measures efficiency *via* a marginal-profit-maximizing orientation. Also, DMP investigated the differential characteristics of non-smooth piece-wise linear frontier estimates by DEA.

This study examines how supply chain divisions, especially the energy and power plant sectors and transmission and distribution lines, utilize more resources to increase capacities as well as decrease undesirable outputs. Indeed, the proposed approach indicates changes in desirable and undesirable outputs in predetermined directions based on the increase of an extra unit of input. This is because divisions enable response to demand fluctuations as more energy production in critical situations such as climate change decreases harmful emissions as wasted energy inhibition in supply chain divisions.

The proposed model applied multiple inputs to estimate the desirable output MP of supply chain divisions and undesirable output reductions simultaneously. Indeed, the proposed model considered the synergistic effects of each input to MP estimation in predetermined directions as it was able to estimate the marginal-profit maximization of desirable output while decreasing undesirable output. In this case, we estimate the marginal production of each input and calculate the marginal vector of outputs. This way, the weighted average of marginal productivity vector each input is considered as marginal production vector of desirable output and undesirable output for supply chain. Therefore, we achieve an appropriate pattern for marginal production of desirable output, emissions inhibition, and wasted energy under each input increment that creates profit information for policymakers to choose the best direction for output production, provide exact control and utilization of energy resources, and protect resources from excessive exhaustion and environmental degradation in the power industry divisions. We can control energy consumption and enhance the economic growth of supply chain divisions if two or more inputs are expanded simultaneously. In other words, we can influence the effects of each input on the estimation of marginal profit maximization and environmental degradation reduction simultaneously.

Several studies have addressed the performance evaluation or productivity improvement of an inefficient firm based on input-oriented measures, output-oriented measures, and hyperbolic measures by Färe *et al.* [12] and Kuosmanen [20]. Chambers *et al.* [5,6] and Chung *et al.* [7] measured the performance evaluation of an inefficient firm under distance function. However, only a few have discussed the productivity improvement of an efficient firm on the frontier.

Lee *et al.* [25] indicated that using reinforcement learning (RL) for resource allocation in wireless networks depends on network circumstances such as the number of users and quality-of-service requirements. Due to this dependence, the policy is hard to use in a practical system where the network circumstances are dynamically changing. They proposed that a single policy could be used in different circumstances. Moreover, it achieves a close performance to the circumstance-dependent policy for each circumstance, which learns the optimal policy for the corresponding circumstance.

Mozaffari *et al.* [27] proposed some alternative two-stage DEA network models with undesirable outputs in a full-fuzzy mode. These models were either directional or radial and were embedded within a multi-objective linear planning structure based on fuzzy triangular numbers. Their proposed models were applied to a real-life case of 15 ammonia-manufacturing units in the Iranian petrochemical sector.

Zhang *et al.* [46] proposed a new intermediate network DEA model by combining the intermediate approach with network DEA. Their new model had several methodological advantages. In addition, sustainability involves a three-stage system (*i.e.*, economic growth, environmental protection, and health promotion). Towards the holistic system, quite a few studies have evaluated its performance.

Zhu *et al.* [47] proposed a Mixed Integer Linear Program (MILP) to find the most efficient targets that were related to a measure that satisfied the strong monotonicity property. Additionally, the proposed approach was applied to real data from 38 universities involved in China's 985 university project.

The current paper presents a model based on DEA to estimate the DMP of desirable outputs while decreasing undesirable outputs. Indeed, the proposed model estimates marginal profit maximization in predetermined directions while decreasing the considerable negative effects of pollution emissions and greenhouse gases in energy and power plant sections and wasted energy inhibition in transmission and distribution lines. The paper aims to estimate an MP model for desirable and undesirable outputs based on directional distance functions (DDF) for multiple inputs and desirable and undesirable outputs. Indeed, the proposed model considers the synergistic effects of each input on MP estimation in predetermined directions. Therefore, the model is able to estimate the marginal-profit maximization of desirable output and undesirable output decrease for each input simultaneously. The proposed model describes how the allocation of one unit of each input affects the increment of desirable output and the decrement of undesirable output simultaneously. Thus, the supply chain divisions should have the necessary preparation for capacity adjustments and adjust desirable and undesirable output levels by controlling variable resources. In this way, there is a balance between economic activity and environmental preservation in industrial processes.

The remainder of this paper is organized as follows: In Section 2, a literature review of how DEA has been used to respond to demand fluctuations under undesirable outputs in the energy and power plant sectors and transmission and distribution lines is presented. Section 3 is devoted to introducing the approach for calculating proportional reallocation for obtaining MP of supply chain divisions in the presence of inputs, desirable and undesirable outputs, and sets of intermediate measures. The next section presents a case study to demonstrate the applicability of the proposed method to the Iranian power industry. Finally, the last section presents conclusions.

2. Literature review

Below are brief overviews of various studies on the weak disposability of undesirable outputs, single-desirable-output MP DEA models under undesirable outputs, the DMP, and supply chain sustainability.

2.1. Weak disposability

Shephard [35] introduced the definition of weak disposability and proposed production axioms based on it. Hailu *et al.* [16] introduced disposability conditions to their technology, called "weakly disposable", and treated detrimental variables as inputs.

Färe and Grosskoph [9] proposed the free disposability assumption of undesirable outputs. They suggested that assuming that a finite amount of input can produce an infinite amount of undesirable output is physically unreasonable. They showed that the monotonicity condition introduced by Hailu *et al.* [16] is inconsistent with physical laws and the standard axioms of production.

Färe and Grosskoph [10] also proposed a model based on DEA for the weak disposability of undesirable outputs, assuming that all firms have a uniform abatement factor. Kuosmanen [22] presented an alternative approach to undesirable emissions as outputs, imposing the assumption that these undesirable outputs are weakly disposable. He demonstrated how weakly disposable technology can result in non-uniform abatement.

2.2. Marginal productivity (MP)

Banker *et al.* [4] and Fried *et al.* [15] proposed that a piece-wise linear production function can be estimated using DEA based on collected observations. This is because the production function cannot be observed easily in practice.

Banker and Thrall [3] and Førsund *et al.* [14] developed a range of scale elasticity (SE) to explicitly support the decision-maker since DEA may not have a unique shadow price scale. Lee and Johnson [24] applied a nonparametric approach to obtain the capacity measure from a cross-sectional dataset.

Podinovski and Førsund [29] and Atici and Podinovski [2] pointed out that the derivative in the returns to scale (RTS) may not always exist. Therefore, they replaced the classical derivative with directional derivatives. They also defined left-hand and right-hand SE and gave an explicit definition of differential characteristics on a non-differentiable efficient frontier. They also proposed a directional-derivative approach to calculate elasticity measures without any simplifying assumptions. Examples of direct methods for the calculation of SE can be found in Podinovski and Førsund in Krivonozhko *et al.* [19] and Førsund *et al.* [14], and for various marginal rates in Rosen *et al.* [33] and Asmild *et al.* [1].

Lee [23] suggested that firms should select the direction *via* DMP to move in the direction of marginal profit maximization. Yang *et al.* [43] proposed research based on RTS. Yang *et al.* [45] verified the research on directional RTS. They analyzed the directional SE of production functions and the directional RTS of Chinese biological institutes based on the DEA method.

Moreover, Yang *et al.* [44] estimated directional RTS for two categories of inefficient and efficient decision-making (strongly and weakly efficient). The basic idea was to examine the ratio of the amount of change in outputs on the efficient frontier in the specified direction. It was assumed that it was caused by an increase (or decrease) in a small enough number of inputs in the specified direction.

Das *et al.* [8] investigated a hybrid energy system that entailed a photovoltaic module, wind turbine, biogas generator, and vanadium redox flow battery for supplying stable power to a remote island, Saint Martin, Bangladesh. In addition, a fuzzy decision-making technique was applied to find the optimal solution. A comparative analysis of a single objective function was compared with a multi-objective one.

Pacher *et al.* [28] developed a performance measurement model based on a two-stage network DEA technique. It aimed to measure the joint impact of sustainable operations and operational activities on the business performance of a retail company. The results showed that the additional sustainable constraints led to improved operational efficiency for some firms in the retail chain. This resulted in improved business efficiency, while for other firms, the integration of sustainable objectives decreased business efficiency.

Wang *et al.* [40] proposed the additive decomposition method, in which the overall efficiency was a weighted average of the two-stage efficiencies in a feedback system. They found that the weight of the first stage was never less than that of the second stage. This suggested that the first stage was favored, which caused a biased efficiency evaluation. Also, they built an improved feedback two-stage DEA model with constant weights and developed a heuristic method to solve it.

Wang *et al.* [41] explored whether nuclear energy can promote economic growth without increasing carbon emissions. They discussed the impact of coal, oil, natural gas, and renewable energy on economic growth and carbon emissions. Also, it was indicated that there was a positive relationship between increased oil, increased natural gas, and economic growth. However, there was a negative relationship between the increase in coal and economic growth. Meanwhile, there was a positive relationship between increased oil, increased coal, and increased carbon emissions. On the contrary, the positive relationship between increased natural gas and increased carbon emissions were not significant, and nuclear energy reduced carbon emissions more significantly than renewable energy.

Wang *et al.* [39] showed that the effect of renewable energy consumption on economic growth is positive. They also suggested that increased renewable energy consumption contributed to economic growth. In addition, this positive relationship changed as the threshold value changed. This showed that the role of increasing renewable energy consumption to promote economic development was nonlinear. In other words, if the EU countries increased their renewable energy consumption by more than a certain amount (the threshold value), the role of renewable energy consumption in promoting economic development would be more significant.

Wu *et al.* [42] extend the DEA model to consider two-sided non-homogeneous problems, handling DMU sets that have non-homogeneity in both inputs and outputs. They indicated that the overall efficiency of 38 industrial sectors in China has maintained a rising trend for five years.

2.3. Sustainability of the supply chains

A slacks-based measure network DEA model called network (NSBM) was proposed by Ton *et al.* [38]. Tavana *et al.* [37] extended Ton *et al.*'s [38] proposed epsilon-based measure (EBM) and suggested a network epsilon-based measure (NEBM). A model was proposed by Farzipoor *et al.* [13] to select a third-party logistics provider when dual-role factors and fuzzy data are present. Tajbakhsh *et al.* [36] proposed a multi-stage DEA model to evaluate the sustainability of a chain of business partners. They assessed supply chain sustainability in the banking and beverage sectors.

Pouralizadeh [30] presented a new DEA model for sustainability improvement in the electricity supply chain in the presence of dual-role factors and undesirable outputs. This model was able to identify whether increasing inputs under managerial disposability to new technology innovation could reduce undesirable production in the electricity supply chain divisions or whether the increased inputs for investment were ineffective in decreasing the number of undesirable outputs.

Pouralizadeh [31] proposed two models for sustainability assessment of the electricity supply chain *via* reduction of wasted resources and pollution emissions management. She suggested that supply chains are generally evaluated under natural and management disposability based on unified operational and environmental efficiency. Also, it was suggested that supply chain divisions with the necessary facilities and new technology to confront undesirable outputs can utilize more inputs (under managerial disposability) for more output production without increasing undesirable outputs. Those supply chain divisions that lack the adequate ability to reduce undesirable outputs should prevent the increase of undesirable outputs by using available capacities under natural disposability.

Pouralizadeh *et al.* [32] proposed a new DEA-based model to evaluate the sustainability of an electricity supply chain in the presence of undesirable outputs. They investigated a supply chain with five stages and fifteen divisions from different districts of Iran. Also, the weak disposability assumption was adopted for activity level control in production activities. The proposed model enabled the authors to determine the type and size of inputs needed to control undesirable outputs.

Kuosmanen [21] presented an approach in which undesirable emissions as outputs impose the assumption that undesirable outputs are weakly disposable. He showed how weakly disposable technology can be modeled in such a way that non-uniform abatement factors can be applied as Kuosmanen technology. Lee [23] introduced the weak disposability of Kuosmanen's convex technology with undesirable outputs.

2.4. Marginal profit maximizing for desirable and undesirable outputs

Let us suppose $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T > 0$, $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})^T > 0$, and $B_j = (b_1, b_2, \ldots, b_{hj})^T > 0$ show the column vectors of the inputs and desirable and undesirable outputs. Also, let set I represent the inputs and index $i \in I$, set J represent the desirable outputs and index $j \in J$, and Q indicate the set of undesirable outputs and index $q \in Q$ when $Q^* \subset Q$ is the subset of undesirable output investigated for DMP. The marginal profit maximization is estimated by of one specific desirable output j^* and one undesirable output q^*, given the level of one specific input i^* of one specific firm r. Let $(g^{X_{i^*}}, g^{Y_{j^*}}, g^{Bq^*})$ define the predetermined vector for the input i^*, the desirable output j^*, and the undesirable output q^* when $\sum_{j \in j^*} g^{Y_j} + \sum_{q \in Q^*} g^{B_q} = 1$, and $g^{X_{i^*}} = 0$, $g^{Y_{j^*}} \geq 0$, $g^{B_{q^*}} \geq 0$. The set K shows the firm and index $k \in K$, and the index $r \in K$ is used for the firm under consideration. Also, the column vectors of structural variables (λ) are used for connecting the input and output vectors by convex combination under variable returns to scale (VRS).

The proposed model by Lee [23] describes how a change in a single input i^* affects multiple desirable outputs $j^* \subset j$ and undesirable outputs $q^* \subset q$. Model (1) defines the DDF with undesirable outputs as follows:

$$
\begin{aligned}
&\text{Max } \eta \\
&\qquad \sum_{k=1}^{K} (\lambda_k + \mu_k) x_{i^*k} \leq x_{i^*r} \\
&\qquad \sum_{k=1}^{K} (\lambda_k + \mu_k) x_{ik} \leq x_{ir} && \forall i \neq i^* \\
&\qquad \sum_{k=1}^{K} \lambda_k y_{jk} \geq y_{jr} + \eta g^{Y_j} && \forall j \in j^* \\
&\qquad \sum_{k=1}^{K} \lambda_k y_{rk} \geq y_{jr} && \forall j \in J \backslash J^* \\
&\qquad \sum_{k=1}^{K} \lambda_k B_{qk} = B_{qr} - \eta g^{B_q} && \forall q \in Q^* \\
&\qquad \sum_{k=1}^{K} \lambda_k B_{qk} = B_{qr} && \forall q \in Q \backslash Q^* \\
&\qquad \sum_{k=1}^{K} (\lambda_k + \mu_k) = 1 && k = 1, \ldots, K \\
&\qquad \lambda_k, \mu_k \geq 0, \quad \eta \text{ is free}
\end{aligned}
\tag{1}
$$

where η is the decision variable for the efficiency estimation. The firm r is efficient, if $\eta = 0$, and inefficient, if $\eta > 0$.

Let v_i, u_j, w_q, and u_o represent the dual multipliers of input constraints, desirable and undesirable output constraints, and convex-combination constraints in model (1). The MP is one of the differential characteristics on the frontier, and the firm under evaluation is on the frontier. Therefore, $\eta = 0$, and $\sum_{i \in I} v_i x_{ir} - \sum_{j \in J} u_r y_{jr} + \sum_{q \in Q} w_q B_{qr} + u_o = 0$.

The dual model of model (1) is presented as follows:

$$\begin{aligned}
&\min\ v_{i^*}\\
&\text{s.t. } \sum_{i\in I} v_i x_{ir} - \sum_{j\in J} u_r y_{jr} + \sum_{q\in Q} w_q B_{qr} + u_o = 0\\
&\qquad \sum_{i\in I} v_i x_{ik} - \sum_{j\in J} u_r y_{jk} + \sum_{q\in Q} w_q B_{qr} + u_o \geq 0 \qquad \forall k\\
&\qquad \sum_{i\in I} v_i x_{ik} + u_o \geq 0 \qquad \forall k\\
&\qquad \sum_{j\in j^*} u_j g^{y_j} + \sum_{q\in Q*} w_q g^{B_q} = 1\\
&\qquad v_i, u_j \geq 0 \quad u_0, w_q \text{ is free.}
\end{aligned} \tag{2}$$

Lee [23] eliminated the unit of each factor for normalization as $X_i^{\max} = \max\{X_{ik}\}$, $B_q^{\max} = \max\{B_{qk}\}$, and $Y_j^{\max} = \max\{Y_{jk}\}$. By eliminating the unit of factors, the following dual model estimates the DMP $y_j, j \in j^*$ and $B_q, q \in Q^*$ with respect to X_{i^*}.

$$\begin{aligned}
&\min\ \frac{v_{i^*}}{X_{i^*}^{\max}}\\
&\text{s.t. } \sum_{i\in I} \frac{v_i x_{ir}}{X_i^{\max}} - \sum_{j\in J} \frac{u_r y_{jr}}{Y_j^{\max}} + \sum_{q\in Q} \frac{w_q B_{qr}}{B_q^{\max}} + u_o = 0\\
&\qquad \sum_{i\in I} \frac{v_i x_{ik}}{X_i^{\max}} - \sum_{j\in J} \frac{u_r y_{jk}}{Y_j^{\max}} + \sum_{q\in Q} \frac{w_q B_{qr}}{B_q^{\max}} + u_o \geq 0\\
&\qquad \sum_{i\in I} \frac{v_i x_{ik}}{X_i^{\max}} + u_o \geq 0\\
&\qquad \sum_{j\in j^*} u_j g^{y_j} + \sum_{q\in Q*} w_q g^{B_q} = 1\\
&\qquad v_i, u_j \geq 0 \quad w_q, u_0 \text{ is free.}
\end{aligned} \tag{3}$$

3. Modeling

In this study proposed a model describes how the allocation of one unit of each input affects increments of desirable output and decrements of undesirable output simultaneously. Supply chain divisions should prepare for capacity adjustments and adjust desirable and undesirable output levels by controlling variable resources. In this way, industry balances economic activity and environmental preservation.

Providing suitable patterns for examining the relationship between economic return and environmental preservation is an important issue and necessary for harmful emissions management and wasted energy inhibition. Hence, the existence of an appropriate policy for energy consumption optimization as well as pollution emission reduction plays an essential role in the sustainability and performance efficiency of industrial sections. Comprehensive pattern collection is an essential issue for the decrease of toxic and greenhouse gas emissions and power loss control in the energy and power plant sectors and transmission and distribution lines. Indeed, benefit capacity adjustment under demand fluctuations is performed according to MP estimation. This study examines how electricity supply chain divisions, especially the energy and power plant sectors and transmission and distribution lines, will respond to demand fluctuations in critical situations such as climate change, defective equipment, power losses, unauthorized uses, and excessive domestic consumption in electricity production.

3.1. DDF for efficiency assessment of supply chain under undesirable outputs

In this section, a DEA model for MP estimation of supply chain divisions is proposed. We suppose a supply chain contains an arbitrary number of suppliers, manufacturers, transmitters, distributors, and customers. Assume a supply chain consists of five stages: supplier, manufacturer, transmitter, distributor, and customer. We treat each supply chain as a DMU.

Let us consider h_s, h_m, h_t, h_d and h_c as the number of divisions in supplier, manufacturer, transmitter, distributor, and customer. The electricity supply chains are power suppliers in the power production process. They are comprised of fuel suppliers (oil and gas fields), power producers (power plants), electricity transmitters (transmission lines), power distributors (distribution lines), and final customers. These entities collaborate on power production and management in the economic sector.

The production factors of the jth supply chain (DMU) are summarized as follows:

$X_k^h = (x_{1k}^h, x_{2k}^h, \ldots, x_{mk}^h)^T > 0$: a column vector of m inputs from the hth division in the kth supply chain $h = 1, \ldots, H$, $k = 1, \ldots, K$.

$Y_k^h = (y_{1k}^h, y_{2k}^h, \ldots, y_{sk}^h)^T > 0$: a column vector of s desirable outputs from the hth division in the kth supply chain $h = 1, \ldots, H$, $k = 1, \ldots, K$.

$B_k^h = (b_{1k}^h, b_{2k}^h, \ldots, b_{sk}^h)^T > 0$: a column vector of s undesirable outputs from the hth division in the kth supply chain $h = 1, \ldots, H$, $k = 1, \ldots, K$.

$V_k^{(h,h')} = (v_{1k}^{(h,h')}, v_{2k}^{(h,h')}, \ldots, v_{Pk}^{(h,h')})^T > 0$: a column vector of P material flows or intermediate measures sent from the division h to the division h' in the kth supply chain $h = 1, \ldots, H$, $k = 1, \ldots, K$.

$s_{pk}^{(h,h')}$: the slack variables of the pth intermediate measure from the division h to division h' in the kth supply chain $(p = 1, \ldots, P)$, $(k = 1, \ldots, K)$.

$\Lambda^h = (\lambda_1^h, \lambda_2^h, \ldots, \lambda_n^h)^T$: an unknown column vector.

η_r^h : the efficiency score of the rth output from the hth division.

3.2. Directional MP modeling of the supply chain in the presence of undesirable output

Let $g = (g_{ir}^h, g_{jr}^h, g_{qr}^h)$ be the predetermined directional vector for inputs and desirable and undesirable outputs of the hth division in the rth supply chain, where η^h is the decision variable for the efficiency estimate of the hth division. If $\eta^h = 0$; then the firm under consideration r is efficient, and if, $\eta^h > 0$, it is inefficient. In this study, we considered the different weights for partners at a particular stage of the network supply chain as $\omega_h, (h = 1, \ldots, H)$, which are weights for H divisions that were defined by decision makers in production activities.

Now let us suppose $i^* \subset I$ indicates the categories of inputs considered for more utilization, and $j^* \subset J$, $Q^* \subset Q$ defines the desirable and undesirable outputs set whose marginal profit maximization is estimated *via* DMP.

We will estimate DMP by a directional vector $g = (g_{i^*r}^h, g_{j^*r}^h, g_{Q^*r}^h)$, where $g_{i^*r}^h = 0$, and $\sum_{j \in j^*} g_{jr}^h + \sum_{q \in Q*} g_{qr}^h = 1$.

Model (1) can be further developed as a network model by incorporating the set of intermediate measures for each supply chain division into an efficiency assessment of the overall supply chain *via* a marginal-profit-maximizing orientation.

$$\text{Max } \omega_h \eta^h$$

$$\sum_{k=1}^{K} \left(\lambda_k^h + \mu_k^h\right) x_{i^*k}^h \leq x_{i*r}^h \qquad i^* \subset I, \quad h = 1, \ldots, H$$

$$
\begin{aligned}
&\sum_{k=1}^{K}\left(\lambda_k^h+\mu_k^h\right)x_{ik}^h \le x_{ir}^h && \forall i \ne i^*,\quad h=1,\dots,H\\
&\sum_{k=1}^{K}\lambda_k^h y_{j*k}^h \ge y_{ik}^h+\eta^h g_{j*r}^h && j^* \subset J,\quad h=1,\dots,H\\
&\sum_{k=1}^{K}\lambda_k^h y_{jk}^h \ge y_{jr}^h && \forall j \in J\backslash J^*,\quad h=1,\dots,H\\
&\sum_{k=1}^{K}\lambda_k B_{qk} = B_{qr}^h-\eta^h g_{qr}^h && \forall q \in Q^*,\quad h=1,\dots,H\\
&\sum_{k=1}^{K}\lambda_k B_{qk} = B_{qr}^h && \forall q \in Q\backslash Q^*,\quad h=1,\dots,H\\
&\sum_{k=1}^{K}\lambda_k^h v_{pk}^{(h,h')} = \sum_{k=1}^{K}\lambda_k^{h'} v_{pk}^{(h,h')} && h=1,\dots,h_s,\quad p=1,\dots,P_s,\quad h'=h_s+1,\dots,h_m\\
&\sum_{k=1}^{K}\lambda_k^h v_{pk}^{(h,h')} = \sum_{k=1}^{K}\lambda_k^{h'} v_{pk}^{(h,h')} && h=h_s+1,\dots,h_m,\quad p=1,\dots,P_m,\quad h'=h_m+1,\dots,h_t\\
&\sum_{k=1}^{K}\lambda_k^h v_{pk}^{(h,h')} = \sum_{k=1}^{K}\lambda_k^{h'} v_{pk}^{(h,h')} && h=h_m+1,\dots,h_t,\quad p=1,\dots,P_t,\quad h'=h_t+1,\dots,h_d\\
&\sum_{k=1}^{K}\lambda_k^h v_{pk}^{(h,h')} = \sum_{k=1}^{K}\lambda_k^{h'} v_{pk}^{(h,h')} && h=h_t+1,\dots,h_d,\quad p=1,\dots,P_d,\quad h'=h_d+1,\dots,h_c\\
&\sum_{k=1}^{K}\left(\lambda_k^h+\mu_k^h\right)=1 && k=1,\dots,K,\quad h=1,\dots,H\\
&\lambda_k,\mu_k^h \ge 0, \eta \text{ free}, && k=1,\dots,K,\quad h=1,\dots,H.
\end{aligned}
\tag{4}
$$

The first category constraints that correspond to inputs are controllable and are considered discretionary. However, the second category constraints are related to inputs that are not controllable and are defined as non-discretionary inputs. The third category constraints that correspond to desirable outputs estimate their MP in predetermined directions. The fourth category constraints indicate other outputs. The fifth category constraints correspond to undesirable outputs decreasing in predetermined directions. The sixth category constraints indicate the weak disposability of other undesirable outputs. The seventh, eighth, ninth, and tenth category constraints correspond to intermediate measures. These measures are sent from supplier to manufacturer, manufacture to the transmitter, and from them to the distributor, finally to customer divisions. The last category constraints are related to variable RTS in the production process.

Let us consider $v_i^h(i=1,\dots,m)$, $u_r^h(r=1,\dots,s)$, $w_q^h(q=1,\dots,Q)$, ${B'}_p^h(p=1,\dots,P_s)$, $\bar{B}_p^h(p=1,\dots,P_m)$, $\tilde{B}_p^h(p=1,\dots,P_t)$, $\hat{B}_p^h(p=1,\dots,P_d)$ indicate the dual variables that correspond to the ith the category constraints of the input, the rth category constraints of the desirable output, and the dual variables of the category constraints related to the intermediate measures. These measures are sent from supplier to manufacturer, manufacture to the transmitter, and from them to the distributor, finally to customer divisions. The dual of model (4) is proposed as follows:

$$
\text{Min } \sum_{h=1}^{H}\omega_h\left(v_{i^*}^h\right)
$$

$$\text{s.t.}\ \sum_{i\in I} v_i^h x_{ir}^h - \sum_{j\in J} u_j^h y_{jr}^h + \sum_{q\in Q} w_q^h b_{qr}^h + \sum_{p=1}^{P} B_p' v_{pr}^{(h(s),h(m))} + u_o^h = 0 \qquad h = 1,\ldots,h_s,\quad p = 1,\ldots,p_s$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ir}^h - \sum_{j\in J} u_j^h y_{jr}^h + \sum_{q\in Q} w_q^h b_{qr}^h + \sum_{p=1}^{P} \bar{B}_p v_{pr}^{(h(m),h(t))} \\ &\quad - \sum_{p=1}^{P} B_p' v_{pr}^{(h(s),h(m))} + u_o^h = 0 \qquad h = 1,\ldots,h_m,\quad p = 1,\ldots,p_m\end{aligned}$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ir}^h - \sum_{j\in J} u_j^h y_{jr}^h + \sum_{q\in Q} w_q^h b_{qr}^h + \sum_{p=1}^{P} \tilde{B}_p v_{pr}^{(h(t),h(d))} \\ &\quad - \sum_{p=1}^{P} \bar{B}_p v_{pr}^{(h(m),h(t))} + u_o^h = 0 \qquad h = 1,\ldots,h_t,\quad p = 1,\ldots,p_t\end{aligned}$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ir}^h - \sum_{j\in J} u_j^h y_{jr}^h + \sum_{q\in Q} w_q^h b_{qr}^h + \sum_{p=1}^{P} \hat{B}_p v_{pr}^{(h(d),h(c))} \\ &\quad - \sum_{p=1}^{P} \tilde{B}_p v_{pr}^{(h(t),h(d))} + u_o^h = 0 \qquad h = 1,\ldots,h_d,\quad p = 1,\ldots,p_d\end{aligned}$$

$$\sum_{i\in I} v_i^h x_{ir}^h - \sum_{j\in J} u_j^h y_{jr}^h + \sum_{q\in Q} w_q^h b_{qr}^h - \sum_{p=1}^{P} \hat{B}_p v_{pr}^{(h(d),h(c))} + u_o^h = 0 \qquad h = 1,\ldots,h_c,\quad p = 1,\ldots,p_c$$

$$\sum_{i\in I} v_i^h x_{ik}^h - \sum_{j\in J} u_j^h y_{jk}^h + \sum_{q\in Q} w_q^h b_{qk}^h + \sum_{p=1}^{P} B_p' v_{pk}^{(h(s),h(m))} + u_o^h \le 0 \qquad k \neq r,\quad h = 1,\ldots,h_s,\quad p = 1,\ldots,p_s$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ik}^h - \sum_{j\in J} u_j^h y_{jk}^h + \sum_{q\in Q} w_q^h b_{qk}^h + \sum_{p=1}^{P} \bar{B}_p v_{pk}^{(h(m),h(t))} \\ &\quad - \sum_{p=1}^{P} B_p' v_{pk}^{(h(s),h(m))} + u_o^h \ge 0 \qquad k \neq r,\quad h = 1,\ldots,h_m,\quad p = 1,\ldots,p_m\end{aligned}$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ik}^h - \sum_{j=1}^{s} u_j^h y_{jk}^h + \sum_{q\in Q} w_q^h b_{qk}^h + \sum_{p=1}^{P} \tilde{B}_p v_{pk}^{(h(t),h(d))} \\ &\quad - \sum_{p=1}^{P} \bar{B}_p v_{pk}^{(h(m),h(t))} u_o^h \ge 0 \qquad k \neq r,\quad h = 1,\ldots,h_t,\quad p = 1,\ldots,p_t\end{aligned}$$

$$\begin{aligned}&\sum_{i\in I} v_i^h x_{ik}^h - \sum_{j\in J} u_j^h y_{jk}^h + \sum_{q\in Q} w_q^h b_{qk}^h + \sum_{p=1}^{P} \hat{B}_p v_{pk}^{(h(d),h(c))} \\ &\quad - \sum_{p=1}^{P} \tilde{B}_p v_{pk}^{(h(t),h(d))} + u_o^h \ge 0 \qquad k \neq r,\quad h = 1,\ldots,h_d,\quad p = 1,\ldots,p_d\end{aligned}$$

$$\sum_{i\in I} v_i^h x_{ik}^h - \sum_{j\in J} u_j^h y_{jk}^h + \sum_{q\in Q} w_q^h b_{qk}^h - \sum_{p=1}^{P} \hat{B}_p v_{pk}^{(h(d),h(c))} + u_o^h \ge 0 \qquad k \neq r,\quad h = 1,\ldots,h_c,\quad p = 1,\ldots,p_c$$

$$
\begin{aligned}
&\sum_{i\in I} v_i^h x_{ik}^h + u_o^h \geq 0 && k=1,\dots,K\\
&\sum_{j\in j^*} u_j^h g_i^h + \sum_{q\in Q*} w_q^h g_{bq}^h = 1 && h=1,\dots,H\\
&v_i^h, u_j^h \geq 0, w_q^h, B'_p, \bar{B}_p, \tilde{B}_p, \hat{B}_p \text{ is free,} && h=1,\dots,H.
\end{aligned}
\tag{5}
$$

For normalization, we eliminate the measuring unit of production factors as follows: $(X_i^h)^{\max} = \max\{X_{ik}^h\}$, $(Y_j^h)^{\max} = \max\{Y_{jk}^h\}$, $(B_q^h)^{\max} = \max\{B_{qk}^h\}$, $(V_p^{(h,h')})^{\max} = \max\{\, V_p^{(h,h')}\}$.

$$
\begin{aligned}
\alpha = \text{Min} &\sum_{h=1}^{H} \omega_h \frac{v_{i^*}^h}{\left(X_{ir}^h\right)^{\max}}\\
\text{s.t. } &\sum_{i\in I} \frac{v_i^h x_{ir}^h}{\left(X_{ir}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{\left(Y_{jr}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qr}^h}{\left(B_{qr}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{B'_p v_{pr}^{(h(s),h(m))}}{\left(V_p^{(h(s),h(m))}\right)^{\max}} + u_o^h = 0 && h=1,\dots,h_s,\quad p=1,\dots,p_s\\
&\sum_{i\in I} \frac{v_i^h x_{ir}^h}{\left(X_{ir}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{\left(Y_{ijr}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qr}^h}{\left(B_{qr}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{\bar{B}_p v_{pr}^{(h(m),h(t))}}{\left(V_p^{(h(m),h(t))}\right)^{\max}} - \sum_{p=1}^{P} \frac{B'_p v_{pr}^{(h(s),h(m))}}{\left(V_p^{(h(s),h(m))}\right)^{\max}} + u_o^h = 0 && h=1,\dots,h_m,\quad p=1,\dots,p_m\\
&\sum_{i\in I} \frac{v_i^h x_{ir}^h}{\left(X_{ir}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{\left(Y_{jr}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qr}^h}{\left(B_{qr}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pr}^{(h(t),h(d))}}{\left(V_p^{(h(t),h(d))}\right)^{\max}} - \sum_{p=1}^{P} \frac{\bar{B}_p v_{pr}^{(h(m),h(t))}}{\left(V_p^{(h(m),h(t))}\right)^{\max}} + u_o^h = 0 && h=1,\dots,h_t,\quad p=1,\dots,p_t\\
&\sum_{i\in I} \frac{v_i^h x_{ir}^h}{\left(X_{ir}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{\left(Y_{jr}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qr}^h}{\left(B_{qr}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{\hat{B}_p v_{pr}^{(h(d),h(c))}}{\left(V_p^{(h(d),h(c))}\right)^{\max}} - \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pr}^{(h(t),h(d))}}{\left(V_p^{(h(t),h(d))}\right)^{\max}} + u_o^h = 0 && h=1,\dots,h_d,\quad p=1,\dots,p_d\\
&\sum_{i\in I} \frac{v_i^h x_{ir}^h}{\left(X_{ir}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{\left(Y_{jr}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qr}^h}{\left(B_{qr}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{\hat{B}_p v_{pr}^{(h(d),h(c))}}{\left(V_p^{(h(d),h(c))}\right)^{\max}} + u_o^h = 0 && h=1,\dots,h_c,\quad p=1,\dots,p_c\\
&\sum_{i\in I} \frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{\left(Y_{jk}^h\right)^{\max}} + \sum_{j\in J} \frac{W_q^h b_{qk}^h}{\left(B_{qk}^h\right)^{\max}}\\
&\quad + \sum_{p=1}^{P} \frac{B'_p v_{pk}^{(h(s),h(m))}}{\left(V_p^{(h(s),h(m))}\right)^{\max}} + u_o^h \leq 0 && k\neq r,\quad h=1,\dots,h_s,\quad p=1,\dots,p_s
\end{aligned}
$$

$$\sum_{i\in I}\frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}}-\sum_{j\in J}\frac{u_j^h y_{jk}^h}{\left(Y_{jk}^h\right)^{\max}}+\sum_{j\in J}\frac{W_q^h b_{qk}^h}{\left(B_{qk}^h\right)^{\max}}$$

$$+\sum_{p=1}^{P}\frac{\bar{B}_p v_{pk}^{(h(m),h(t))}}{\left(V_p^{(h(m),h(t))}\right)^{\max}}-\sum_{p=1}^{P}\frac{B'_p v_{pk}^{(h(s),h(m))}}{\left(V_p^{(h(s),h(m))}\right)^{\max}}+u_o^h\geq 0 \qquad k\neq r,\quad h=1,\dots,h_m,\quad p=1,\dots,p_m$$

$$\sum_{i\in I}\frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}}-\sum_{j\in J}\frac{u_j^h y_{jk}^h}{\left(Y_{jk}^h\right)^{\max}}+\sum_{j\in J}\frac{W_q^h b_{qk}^h}{\left(B_{qk}^h\right)^{\max}}$$

$$+\sum_{p=1}^{P}\frac{\tilde{B}_p v_{pk}^{(h(t),h(d))}}{\left(V_p^{(h(t),h(d))}\right)^{\max}}-\sum_{p=1}^{P}\frac{B'_p v_{pk}^{(h(s),h(m))}}{\left(V_p^{(h(s),h(m))}\right)^{\max}}+u_o^h\geq 0 \qquad k\neq r,\quad h=1,\dots,h_t,\quad p=1,\dots,p_t$$

$$\sum_{i\in I}\frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}}-\sum_{j\in J}\frac{u_j^h y_{jk}^h}{\left(Y_{jk}^h\right)^{\max}}+\sum_{j\in J}\frac{W_q^h b_{qk}^h}{\left(B_{qk}^h\right)^{\max}}$$

$$+\sum_{p=1}^{P}\frac{\hat{B}_p v_{pk}^{(h(d),h(c))}}{\left(V_p^{(h(d),h(c))}\right)^{\max}}-\sum_{p=1}^{P}\frac{\tilde{B}_p v_{pk}^{(h(t),h(d))}}{\left(V_p^{(h(t),h(d))}\right)^{\max}}+u_o^h\geq 0 \qquad k\neq r,\quad h=1,\dots,h_d,\quad p=1,\dots,p_d$$

$$\sum_{i\in I}\frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}}-\sum_{j\in J}\frac{u_j^h y_{jk}^h}{\left(Y_{jk}^h\right)^{\max}}-\sum_{p=1}^{P}\frac{\hat{B}_p v_{pk}^{(h(d),h(c))}}{\left(V_p^{(h(d),h(c))}\right)^{\max}}+u_o^h\geq 0 \qquad k\neq r,\quad h=1,\dots,h_c,\quad p=1,\dots,p_c$$

$$\sum_{i\in I}\frac{v_i^h x_{ik}^h}{\left(X_{ik}^h\right)^{\max}}+u_o^h\geq 0$$

$$\sum_{j\in j^*}u_j^h g_j^h+\sum_{q\in Q*}w_q^h g_q^h=1 \qquad h=1,\dots,H$$

$$v_i^h,u_j^h\geq 0,w_q^h,B'_p,\bar{B}_p,\tilde{B}_p,\hat{B}_p \text{ is free,} \qquad h=1,\dots,H. \tag{6}$$

After measuring the ith input dual multiplier $v_{i^*}^h$ of supply chain divisions by model (6), the α_i^h is defined as follows:

$$\alpha_i^h=\frac{v_{i^*}^h}{(X_{ir}^h)^{\max}} \quad i=1,\dots,m,\quad h=1,\dots,H,\quad r\in k=1,\dots,K. \tag{7}$$

The vector of DMP with respect to desirable output Y_j^h and undesirable output B_q^h of the hth division is estimated based on increasing one extra unit of inputs as follows:

$$\frac{\partial(Y_j^h,B_{qr}^h)}{\partial X_{i^*r}^h}=\alpha_i^h\left[g_{Y_j}^h(Y_j^h)^{\max},-g_{B_q}^h(B_q^h)^{\max}\right]$$

$$\forall j\in J^*,\quad \forall q\in Q^*,\quad i=1,\dots,m,\quad h=1,\dots,H,\quad r\in k=1,\dots,K. \tag{8}$$

The proposed model considers the synergistic effects of each input on MP estimation in predetermined directions. Therefore, the model is able to estimate the marginal-profit maximization of desirable output and undesirable output decrease for each input simultaneously. Also, the approximate value of DMP vector of the overall supply chain with respect to desirable output Y_j^h and undesirable output B_q^h of supply chain divisions is calculated by model (6) as follows:

$$\frac{\partial(Y_j^h,B_{qr}^h)}{\partial X_{i^*r}^h}=\sum_{h=1}^{H}\omega_h\left[\alpha_i^h\left(g_{Y_j}^h\left(Y_j^h\right)^{\max},-g_{B_q}^h\left(B_q^h\right)^{\max}\right)\right]$$

$$\forall j \in J^*, \quad \forall q \in Q^*, \quad i = 1, \ldots, m, \quad h = 1, \ldots, H, \quad r \in k = 1, \ldots, K. \tag{9}$$

This way, the weighted average of marginal productivity vector each input is considered as marginal production vector of desirable output and undesirable output for supply chain.

4. A REAL CASE OF THE POWER INDUSTRY

In this section, the proposed model is applied to the analysis of the power industry in Iran. The dataset, the inputs, and the desirable and undesirable outputs will be described in the following subsections, and in the next subsection, the results will be presented.

4.1. The dataset

In our application, we consider 10 supply chains (or DMUs), including oil and gas fields (suppliers) supplying fuel to power stations, power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers. Two suppliers are assumed per supply chain: oil and gas companies that satisfy the fuel demand of power plants (the intermediate product) and sell fuel as the final output. In a productive process, inputs and outputs constitute the most essential factors. Therefore, the selection of input and output variables is imperative when measuring either relative technological efficiency or productivity growth. Indeed, we choose capital, labor, and energy as inputs and pollution emissions and power losses as undesirable outputs in supply chain divisions.

In the proposed model, suppliers use two inputs (capital and labor) to produce one desirable output (the amount of oil or gas sold by the supplier) and one undesirable output (flaring gas). Each manufacturer includes at least three power plants with different technologies (thermal, combined cycle, gas, hydro, wind, and solar). They use fuel, capital, and labor to produce electricity, which they sell to regional power companies. The total amount of electricity produced by power plants is considered a desirable output, while three undesirable outputs are considered for manufacturers' $\mathrm{SO_x}$, $\mathrm{NO_x}$, and $\mathrm{CO_2}$ emissions. The transmitters transfer electricity from manufacturers to distributing companies, and the capacity and length of the lines are considered inputs. The loss in the transmission lines is considered an undesirable output, while the transferred electricity from transmitters to distribution lines is a desirable output.

Distribution companies receive electricity from transmitters and dispatch it to the final consumers. They use two additional capital inputs, estimated as the capacity and length of the distribution lines. One final desirable output, the dispatched electricity to power customers, and one undesirable output, power losses, are considered for the distribution network. Finally, customers are classified as residential, agricultural, public, or industrial. They use one input and produce one desirable output and one undesirable output. Therefore, we choose inputs as capital, labor, and energy, desirable outputs as produced energy, and undesirable outputs as pollution emissions and power losses. More details concerning the parameters used to characterize this supply chain are as follows:

h_s	Numerator of divisions in the supplier level (h_s : 1, 2).
$x_{1k}^{h(s)}$	Capacity of oil (10^3 Barrels) and gas fields ($10^6\,\mathrm{m}^3$) of the h_sth supplier in the kth supply chain.
$x_{2k}^{h(s)}$	Number of employees from the h_sth supplier in the kth supply chain.
$y_{1k}^{h(s)}$	Oil (10^3 Barrels) and gas ($10^6\,\mathrm{m}^3$) sold to other companies from the h_sth supplier in the kth supply chain.
$b_{1j}^{h(s)}$	Flaring gas of oil field (10^3 barrels) and gas field ($10^6\,\mathrm{m}^3$) of the h_sth supplier in the kth supply chain.
h_m	Numerator of division in the manufacturer level (h_m : 3, 4, and 5).
$x_{1k}^{h(m)}$	Power nominal of the h_mth manufacturer in the kth supply chain (10^6 kWh).

$x_{2k}^{h(m)}$	Number of employees of the h_mth manufacturer in the kth supply chain.
$y_{1k}^{h(m)}$	The total of produced electricity of the h_mth manufacturer in the kth supply chain (10^6 kWh).
$b_{1k}^{h(m)}$	Emissions of $\mathrm{NO_x}$ harmful substances of the h_mth manufacturer in the kth supply chain (10^3 Kg/10^6 kWh).
$b_{2k}^{h(m)}$	Emissions of $\mathrm{SO_x}$ harmful substance of the h_mth manufacturer in the kth supply chain (10^3 Kg/10^6 kWh).
$b_{3k}^{h(m)}$	Emission of $\mathrm{CO_2}$ harmful substance of the h_mth manufacturer in the kth supply chain (10^3 Kg/10^6 kWh).
h_t	Numerator of the divisions in the level of the transmitters (h_t : 6, 7).
$x_{1k}^{h(t)}$	Capacity of transmission lines of the h_tth transmitter in the kth supply chain (MWa).
$x_{2k}^{h(t)}$	Length of transmission line of the h_tth transmitter in the kth supply chain (km circuit).
$y_{1k}^{h(t)}$	The transferred electricity of the h_tth transmitter in the kth supply chain (10^6 kWh).
$b_{1k}^{h(t)}$	Loss of transmission line of the h_tth transmitter in the kth supply chain (10^6 kWh).
h_d	Numerator of the division in the distributer level (h_d : 8, 9, 10, and 11).
$x_{1k}^{h(d)}$	Capacity of the distribution lines of the h_dth distributer in the kth supply chain (Mva).
$x_{2k}^{h(d)}$	Length of distribution line of the h_dth distributer in the kth supply chain (km).
$y_{1k}^{h(d)}$	The dispatched electricity of the h_tth distributer in the kth supply chain (10^6 kWh).
$b_{1k}^{h(d)}$	Percentage of losses in the distribution line of the h_dth distributer in the kth supply chain.
h_c	Numerator of the division in the customer level (h_c : 12, 13, 14, and 15).
$x_{1k}^{h(c)}$	Average cost with fuel subsidy of the h_cth customer in the kth supply chain (USD).
$y_{2k}^{h(c)}$	Sales of electricity of the h_cth customer in the kth supply chain (10^6 kWh).
$b_{1k}^{h(c)}$	Cutting off the power of the h_cth customer in the kth supply chain (minute/year).
$v_{pk}^{(h,h')}$	Material flow from the division h to division h' in the kth supply chain (10^6 kVA).

Ten supply chains (or DMUs), including oil and gas fields (suppliers), provide different fuels to power stations, power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers. All the data from the two oil and gas fields (suppliers), power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers (residential, public, agricultural, and industrial) is available on the TAVANIR website (2015). For each supply chain, we consider two suppliers: oil and gas companies that satisfy the fuel demand of power plants (intermediate products) and those that can also sell fuel as a final output.

The dataset has been collected from the power industry companies in Iran, and the reference year is 2015 (see the TAVANIR website for the detailed data [17]). Desirable output is computed as the difference between the average annual production and the amount of oil and gas delivered to power plants. Information related to the demand for fuel for power plants is collected from TAVANIR Company (2015) in the power industry. It is considered an intermediate measure from oil and gas fields to power plants. Desirable outputs of regional power companies were collected from the transmission section of TAVANIR Company in the power industry. Distribution companies receive electricity from transmitters and dispatch it to the final consumers. All of the data for distribution companies was obtained from the dispatch section of TAVANIR Company in the power

TABLE 1. The first and second suppliers' input.

DMU	Supplier 1 (division 1)		Supplier 2 (division 2)	
	Capacity of oil (10^3 Barrels)	Labor	Capacity of gas field (10^6 m^3)	Labor
	x^1_{1k}	x^1_{2k}	x^2_{1k}	x^2_{2k}
1	0.39352	1	0.33333	0.83333
2	0.94444	0.40625	1	0.83333
3	0.33333	1	0.5	0.8
4	0.5	0.971875	0.3	0.46667
5	0.19444	0.875	0.9	1
6	0.66667	0.6875	0.5	0.8
7	0.72222	0.75	0.5	0.46
8	0.61111	0.5	1	0.75
9	0.14444	0.671875	0.9	0.72667
10	1	0.78125	0.3	0.96667

TABLE 2. The first manufacturer's inputs.

DMU	Manufacturer 1 (division 3)		Manufacturer 2 (division 4)		Manufacturer 3 (division 5)	
	Power nominal (10^6 kWh)	Labor	Power nominal (10^6 kWh)	Labor	Power nominal (10^6 kWh)	Labor
	x^3_{1k}	x^3_{2k}	x^4_{1k}	x^4_{2k}	x^5_{1k}	x^5_{2k}
1	1	1	1	0.445158	0.667029	0.300451
2	0.256232	0.669461	0.674974	0.195367	0.147211	0.644216
3	0.165254	0.447227	0.37001	0.9210	0.939209	0.495493
4	0.126889	0.297305	0.559605	0.921016	0.467587	0.502003
5	0.081994	0.266467	0.124642	0.249232	0.92003	0.70681
6	0.216261	0.688623	0.214953	0.687971	0.220568	0.2003
7	0.015284	0.434132	0.542056	0.744069	1	0.222834
8	0.023586	0.448802	0.669782	0.628244	0.941451	0.325488
9	0.061243	0.449102	0.687305	1	0.396306	1
10	0.181159	0.94012	0.440498	0.212113	0.115063	0.398097

industry. Finally, customers were classified as residential, agricultural, public, or industrial. They use one input and produce one desirable output. The desirable output of customers is computed as the number of total sales of electricity to residential, public, agricultural, and industrial divisions in 2015 (see [32]). The datasets corresponding to the 10 supply chains (DMUs) under analysis are presented in Tables 1–13.

4.2. Results

In this section, we describe the results obtained by applying the proposed method. Model (6) is applied to estimate the MP of two outputs under two inputs for 15 divisions of 10 supply chains (DMUs). Also, the 11 directions were investigated as intervals between $(g^h_{Y_1}, g^h_{Y_2}) = (1, 0)$ and $(g^h_{Y_1}, g^h_{Y_2}) = (0, 1)$ to identify marginal profit maximization of supply chain divisions.

The α^h_i is computed by the objective function of model (6). Moreover, $\frac{\partial(Y^h_{j\,r}, B^h_{qr})}{\partial X^h_{i^*r}} = \alpha^h_i(g^h_{Y_j} Y^{\max}_j, -g^h_{B_q} B^{\max}_q)\,\forall j \in J^*, \forall q \in Q^*$ is the DMP of outputs in the $(g^{Y_1}_h, g^{Y_2}_h)$ direction that is esti-

Table 3. Transmitter level inputs.

DMU	Transmitter 1 (division 6)		Transmitter 2 (division 7)	
	Capacity of transmission lines (Mva)	Length of transmission line (km circuit)	Capacity of transmission lines (Mva)	Length of transmission line (km circuit)
	x_{1k}^{6}	x_{2k}^{6}	x_{1k}^{7}	x_{2k}^{6}
1	0.671576	0.592201501	0.611689547	1
2	1	0.621035944	0.12040672	0.152710968
3	0.333056985	0.588078407	1	0.621035944
4	0.403428348	0.70540969	1	0.621035944
5	0.167540416	0.193955517	0.333056985	0.588078407
6	0.343029919	0.759737918	0.12040672	0.152710968
7	0.34554144	0.393292828	0.213649996	0.304836811
8	0.263636585	0.562897596	0.375679696	0.414745164
9	0.611689547	1	0.179634732	0.256917749
10	0.263636585	0.562897596	0.188154398	0.098913435

Table 4. The distributor level inputs.

DMU	Distributor 1 (division 8)		Distributor 2 (division 9)	
	Capacity of distribution lines (Mva)	Length of distribution line (km)	Capacity of distribution lines (Mva)	Length of distribution line (km)
	x_{1k}^{8}	x_{2k}^{9}	x_{1k}^{9}	x_{2k}^{9}
1	0.686580315	0.624972953	0.270315091	0.160869061
2	1	1	0.552001895	0.381887479
3	1	1	0.726841981	0.542543724
4	0.758833377	0.191740595	0.423359394	0.172999536
5	0.079302141	0.206840592	0.587538498	0.517915183
6	1	1	0.752191424	0.304345303
7	0.320644991	0.57421718	0.342099029	0.266696332
8	0.183628514	0.798862477	1	0.477654388
9	0.686580315	0.624972953	0.448708837	0.351377496
10	0.237025289	0.550307564	0.493721867	1

mated based on an extra one-unit allocation of the ith input from the hth division in the rth the supply chain.

In this way, the proposed model provides an approach to the best choice of direction for more energy generation with less pollution emissions and energy losses in energy and power plant sections and transmitter and distributer lines. We consider desirable and undesirable outputs for 15 divisions of 10 supply chains as effective resource allocation that creates a desirable output increase and a decrease of undesirable output simultaneously under a predetermined direction. In other words, marginal profit maximization is created by controlling input levels to decrease undesirable output in an appropriate direction.

Tables 14 and 15 indicate the directional marginal productivity of supply chain divisions for two inputs in different directions. Moreover, tables show that there are divisions of supply chains that have significant marginal profit maximization under the increase of one extra unit of inputs while decreasing their undesirable outputs.

TABLE 5. The distributor level inputs.

DMU	Distributor 3 (division 10)		Distributor 4 (division 11)	
	Capacity of distribution lines (Mva)	Length of distribution line (km)	Capacity of distribution lines (Mva)	Length of distribution line (km)
	x_{1k}^{10}	x_{2k}^{10}	x_{1k}^{11}	x_{2k}^{11}
1	0.817555938	0.228087914	0.395805798	0.155358412
2	0.439390214	0.300371279	0.116662261	0.171571203
3	0.897713302	0.539232911	0.079302141	0.206840592
4	0.460781903	0.200142545	0.279760331	0.868350283
5	0.974920089	0.539232911	0.270332188	0.433417823
6	0.325547086	0.183998541	0.16688695	0.280702297
7	0.221293337	0.221822582	1	1
8	1	1	0.475372279	0.808939445
9	0.817555938	0.228087914	0.358357565	0.932459584
10	1	1	0.475372279	0.808939445

TABLE 6. The customer level inputs.

DMU	Customer 1 (division 12)	Customer 2 (division 13)	Customer 3 (division 14)	Customer 4 (division 15)
	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)
	x_{1k}^{12}	x_{1k}^{13}	x_{1k}^{14}	x_{1k}^{15}
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	0.5
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	1
9	1	1	1	0.5
10	1	1	1	1

According to Table 14, only one division of supply chain 5 has a marketable decrease of undesirable output when utilizing one extra unit of inputs. The second and fourth distributer lines of supply chain 3 provide a slight decrease of power loss under $(g_{Y_1}^3, g_{B_1}^3) = (0, 1)$ direction.

Indeed, the utilization of more than one unit from the capacity of the second distributer line decreases the wasted energy by 0.047 units. However, increasing one extra unit from the distribution line's length of distributer 4 provides a more slight reduction of the power loss under $(g_{Y_1}^3, g_{B_1}^3) = (0, 1)$ direction. Also, the gas field and the fourth distributer lines of supply chains 4 obtained the undesirable output reduction between 15 divisions, while the allocation of more than one-unit inputs to other divisions did not affect flare gas emissions or wasted energy. In other words, the increase of one extra unit of the gas field capacity of supply chain 4 provides decrease of flaring gas by 0.02 units under the $(g_{Y_1}^4, g_{B_1}^4) = (0, 1)$ direction. However, the power loss of the four distributer lines of supply chain 4 has a slight reduction under the $(0, 1)$ direction.

Table 7. The level of desirable outputs of suppliers 1 and 2 and manufacturers 1, 2, and 3.

DMU	Supplier 1 (division 1)	Supplier 2 (division 2)	Manufacturer 1 (division 3)	Manufacturer 2 (division 4)	Manufacturer 3 (division 5)
	Sold oil (10^3 Barrels)	Sold gas ($10^6\,\mathrm{mm}^3$)	Produced electricity ($10^6\,\mathrm{kWh}$)	Produced electricity ($10^6\,\mathrm{kWh}$)	Produced electricity ($10^6\,\mathrm{kWh}$)
	y_{1k}^1	y_{1k}^2	y_{1k}^3	y_{1k}^4	y_{1k}^5
1	0.042878107	0.113641925	1	0.03364106	0.064122104
2	1	0.6900842	0.961117016	0.909102556	0.090205742
3	0.221720938	0.356977886	0.406302835	0.566828191	0.568130132
4	0.653813956	0.184900351	0.279998751	0.830666795	0.361670863
5	0.112213989	1	0.095396722	0.159877478	0.696620498
6	0.574874748	0.321001534	0.479940493	0.181616247	0.199053321
7	0.420981259	0.225434678	0.014743364	0.892480848	0.900088586
8	0.391218668	0.905818346	0.031328434	1	1
9	0.14942874	0.943611201	0.159054402	0.811220613	0.306534562
10	0.63104532	0.211570684	0.414665747	0.366038283	0.108720399

Table 8. The level of desirable outputs of transmitters and distributers 1, 2, and 3.

DMU	Transmitter 1 (division 6)	Transmitter 2 (division 7)	Distributor 1 (division 8)	Distributor 2 (division 9)	Distributor 3 (division 10)
	The dispatched electricity ($10^6\,\mathrm{kWh}$)	The dispatched electricity ($10^6\,\mathrm{kWh}$)	The dispatched electricity ($10^6\,\mathrm{kWh}$)	The dispatched electricity ($10^6\,\mathrm{kWh}$)	The dispatched electricity ($10^6\,\mathrm{kWh}$)
	y_{1k}^6	y_{1k}^7	y_{1k}^8	y_{1k}^9	y_{1k}^{10}
1	1	0.140600711	1	0.061068074	0.196523643
2	0.702111903	1	0.702111903	0.42054083	0.599032663
3	0.344664193	0.969653421	0.70313936	0.421156242	0.66435251
4	0.644984209	0.318352158	0.230851485	0.901421058	0.659948435
5	0.133162769	0.715439166	0.057069758	0.310741615	1
6	0.500863544	0.301192342	0.031201161	1	0.420989452
7	0.879767022	0.167807535	0.879767022	0.170065003	0.238776041
8	0.734554814	0.530449123	0.734554814	0.537585107	0.317756351
9	0.537211931	0.228903718	0.071137883	0.231983096	0.443783629
10	0.578066119	0.105229386	0.076306566	0.807897257	0.477532731

Table 14 indicates the allocation of more than one-unit capacity by the second distributer of supply chain 5, provided a significant reduction in power losses between the 10 supply chains. Therefore, we prefer selecting $(g_5^{Y_1}, g_5^{B_1}) = (0,1)$ for the best decrease of wasted energy in the distributor 2, as the utilization of one extra unit of inputs in this direction creates approximately 0.22 units of power loss abatement, providing a significant decrease in wasted energy.

Also, according to the results, applying one unit of distributer lines resources in supply chain 6 does not create considerable effects on wasted energy abatement. Indeed, increase of one extra unit of inputs in supply chains 6 have no effect on the power loss reduction. Also, increase of one extra unit of inputs of the distributor 2 of supply chain 7 create power loss decrease when investment in direction of wasted energy decrease. On the other hand, the increase in inputs to the second distributer line of supply chain 9 creates a significant decrease

Table 9. The level of desirable outputs for distributor 4 and customers.

DMU	Distributor 4 (division 11)	Customer 1 (division 12)	Customer 2 (division 13)	Customer 4 (division 14)	Customer 5 (division 15)
	The dispatched electricity (10^6 kWh)	Sold electricity (10^6 kWh)	Sold electricity (10^6 kWh)	Sold electricity (10^6 kWh)	Sold electricity (10^6 kWh)
	y_{1k}^{11}	y_{1k}^{12}	y_{1k}^{13}	y_{1k}^{14}	y_{1k}^{15}
1	0.583443768	0.221352722	0.157976366	0.02620202	0.103555453
2	0.987189815	0.778772766	0.809577548	0.060437255	0.779537561
3	0.201092175	0.950989497	0.987088938	0.08237208	0.878795659
4	0.044896287	0.793115567	0.815401683	0.060012938	0.723104366
5	0.181283638	0.459903894	0.389603413	0.067986909	0.391640428
6	0.084952575	0.968278496	0.946065522	0.072787303	1
7	0.513294586	1	1	1	0.977445641
8	1	0.435502188	0.437993719	0.170761466	0.327450371
9	0.731343557	0.388538913	0.314668545	0.103341382	0.407762267
10	0.04452059	0.445317007	0.433864838	0.176219954	0.318409708

Table 10. The level of undesirable outputs of suppliers 1 and 2 and manufacturer 1.

DMU	Supplier 1 (division 1)	Supplier 2 (division 2)	Manufacturer 1 (division 3)		
	Flaring gas (10^3 Barrels)	Flaring gas (10^6 mm^3)	Emission of NO_x gas electricity (10^3 kg/10^6 kWh)	Emission of SO_x gas electricity (10^3 kg/10^6 kWh)	Emission of CO_2 gas electricity (10^3 kg/10^6 kWh)
	b_{1k}^1	b_{1k}^2	b_{1k}^3	b_{2k}^3	b_{3k}^3
1	0.041666667	0.411764706	1	1	1
2	1	0.941176471	0.665184708	0.176087878	0.666444408
3	0.333333333	0.5	0.517156677	0.008184919	0.519474267
4	0.75	0.382352941	0.504749297	0.504749297	0.504749297
5	0.166666667	1	0.095683513	0.001622119	0.095603478
6	0.583333333	0.5	0.564523849	0.009104754	0.56625019
7	0.583333333	0.470588235	0.014701897	0.000249241	0.014689599
8	0.5	0.941176471	0.033300362	0.007712209	0.033278589
9	0.15	1	0.202450091	0.003204131	0.203357353
10	1	0.382352941	0.51992679	0.00822876	0.522256795

in undesirable outputs in the $(0, 1)$ direction. Also, the utilization of one extra unit of capacity by the distributer 2 in this direction creates approximately 0.16 units of power loss reduction. Overall, supply chains 5 and 9 have appropriate capacities for energy loss abatement when resource allocation is increased.

Generally, we apply the direction $(0, 1)$ when consider investment for power loss decrement and pollution emissions inhibition. Similarly, Table 15 shows the change rate of desirable and undesirable outputs of gas fields in supply chain 4 under the four predetermined directions when using more than one unit of gas field resources. Table 15 shows the value α calculated from the objective function of model (6) for that supply chains have an increase in desirable output and a decrease in undesirable output for some divisions in the $(0.1, 0.9)$ direction. According to Table 15, an increase of one unit in the first distribution line capacity in supply chain 8 provides an increase of 0.26 units of produced and dispatched electricity to power consumers and a decrease of 0.003 units

TABLE 11. The level of undesirable outputs of manufacturers 2 and 3.

DMU	Manufacturer 2 (division 4)			Manufacturer 3 (division 5)		
	Emission of NO_x gas electricity (10^3 kg/10^6 kWh)	Emission of SO_x gas electricity (10^3 kg/10^6 kWh)	Emission of CO_2 gas electricity (10^3 kg/10^6 kWh)	Emission of NO_x gas electricity (10^3 kg/10^6 kWh)	Emission of SO_x gas electricity (10^3 kg/10^6 kWh)	Emission of CO_2 gas electricity (10^3 kg/10^6 kWh)
	b^4_{1k}	b^4_{2k}	b^4_{3k}	b^5_{1k}	b^5_{2k}	b^5_{3k}
1	0.018339969	0.000310917	0.018324628	0.000714848	0.000717035	0.000717035
2	0.909498611	0.909498611	0.909498611	1	1	1
3	0.560827561	0.553798112	0.56082158	0.007746835	0.025376356	0.007781596
4	0.586751499	0.00928638	0.589380973	0.005132335	0.004791391	0.005159651
5	0.159947129	0.159947129	0.015994709	0.010945121	0.375639	0.010962278
6	0.087987748	0.001491653	0.08791415	0.002824691	0.002637045	0.002839725
7	0.877619468	0.877619468	0.877619468	0.017202233	0.890922388	0.017226197
8	1	1	1	0.01861501	0.891248093	0.01864657
9	0.567179048	111.1477598	0.569624379	0.003457921	0.003228209	0.003476326
10	0.255405638	246.5806559	0.256487779	0.00218406	0.128830578	0.002185889

TABLE 12. The level of undesirable outputs of transmitters and distributors.

DMU	Transmitter 1	Transmitter 2	Distributor 1	Distributor 2	Distributor 3	Distributor 4
	Loose of power (10^6 kWh)	Loose of power (10^6 kWh)	Loss of power (%)	Loss of power (%)	Loss of power (%)	Loss of power (%)
	b^6_{1k}	b^7_{1k}	b^8_{1k}	b^9_{1k}	b^{10}_{1k}	b^{11}_{1k}
1	1	0.145000995	0.912652537	1	1	0.912010276
2	0.394160221	0.843596266	0.462427746	0.646907216	0.789551141	0.513166346
3	0.344664193	1	1	0.733891753	0.813097866	0.850995504
4	0.644984209	0.328315407	1	0.691365979	0.564385578	0.772639692
5	0.133162769	0.737829775	0.850995504	0.816365979	0.813097866	0.731535003
6	0.500863544	0.301237872	1	0.741623711	0.587932303	0.465639049
7	0.879649108	0.173059293	0.873474631	0.711984536	0.974981604	1
8	0.734554814	0.564633152	0.721258831	0.858891753	0.590875644	0.520231214
9	0.537211931	0.236067561	0.914579319	0.467139175	1	0.515735389
10	0.578066119	0.108522678	0.80539499	0.723582474	0.590875644	0.520231214

of wasted energy simultaneously in the $(g_8^{Y_1}, g_8^{B_1}) = (0.1, 0.9)$ direction, while the gas field in supply chain 4 creates a slight increase in the amount of gas sold to other companies and a reduction of 0.02 units in flare gas emissions. Also, the third distribution line of supply chain 6 creates slight changes in desirable and undesirable outputs. In this way, marginal profit maximization along with a decrease in power losses happen in supply chain 8 when using an extra unit of distribution (1 line capacity) in economic activity.

According to Table 15, there is only one division in supply chains 1, and 4, which creates the increment of desirable and the decrement of undesirable outputs in the $(0.2, 0.8)$ direction. Indeed, the utilization of more than one unit of capacity in the gas field of supply chain 4 creates an increase of 0.17 units of sold gas and a decrease of 0.024 units of flare gas emission in the $(g_4^{Y_1}, g_4^{B_1}) = (0.2, 0.8)$ direction, while the fourth distribution line of supply chain 1 provides a slight increase in dispatched electricity to power consumers and a little decrease in wasted energy in the $(0.2, 0.8)$ direction when applying more units of length of distribution lines.

Table 13. The level of undesirable outputs of customers.

DMU	Customer 1 (division 12)	Customer 2 (division 13)	Customer 3 (division 14)	Customer 4 (division 15)
	Cut off electricity (10^6 kWh)	Cut off electricity (10^6 kWh)	Cut off electricity (10^6 kWh)	Cut off electricity (10^6 kWh)
	b_{1k}^{12}	b_{1k}^{13}	b_{1k}^{14}	b_{1k}^{15}
1	1	0.736897504	0.132856652	0.46130294
2	0.93164931	1	0.081148377	0.919553226
3	0.931960686	0.998798752	0.090601482	0.849195903
4	0.934535869	0.992044009	0.079366601	0.840153421
5	0.96695592	0.845789283	0.16043469	0.811942401
6	0.943708329	0.952092311	0.079620876	0.961028007
7	0.890977227	0.919954691	1	0.85873138
8	0.922692205	0.958151594	0.406059599	0.68408507
9	0.966338694	0.808068198	0.288471233	1
10	0.92868137	0.93422555	0.412463887	0.65475959

Table 14. DMP of supply chains under $(0,1)$ normalized direction.

Division of supply chains	Objective function	DMP	Objective function	DMP
	α_1^h	$\frac{\partial(Y_1^h,B_1^h)}{\partial X_1^h}$	α_2^h	$\frac{\partial(Y_1^h,B_1^h)}{\partial X_2^h}$
Distribution 2 of supply chain 3	0.003	$(0,-0.0466)$	0	$(0,0)$
Distribution 4 of supply chain 3	0	$(0,0)$	0.0000618	$(0,-0.000962)$
Gas field of supply chain 4	0.0000653	$(0,-0.0240)$	0	$(0,0)$
Distribution 1 of supply chain 4	0	$(0,0)$	00000331	$(0,-0000515)$
Distribution 2 of supply chain 4	0	$(0,0)$	0.0000946	$(0,-0.00147)$
Distribution 3 of supply chain 4	0	$(0,0)$	0.0000316	$(0,-0.000430)$
Distribution 4 of supply chain 4	0.000353	$(0,-0.00550)$	0	$(0,0)$
Distribution 2 of supply chain 5	0.014	$(0,-0.21728)$	0	$(0,0)$
Distribution 3 of supply chain 6	0	$(0,0)$	0.0000981	$(0,-0.00133)$
Distribution 4 of supply chain 6	0	$(0,0)$	0.0000153	$(0,-0.000237)$
Distribution 1 of supply chain 7	0.000270	$(0,-0.00420)$	0.0000538	$(0,-0.000838)$
Distribution 2 of supply chain 7	0.003	$(0,-0.04656)$	0.000253	$(0,-0.00393)$
Distribution 1 of supply chain 8	0.000673	$(0,-0.0105)$	0	$(0,0)$
Distribution 1 of supply chain 9	0	$(0,0)$	0.000663	$(0,-0.00103)$
Distribution 2 of supply chain 9	0.01	$(0,-0.1552)$	0	$(0,0)$

On the other hand, supply chain 4, only DMU obtained the increment of desirable output and the decrease of undesirable outputs of the gas field division in the four directions. This was while the DMP of other supply chain divisions remained unchanged.

Finally, Table 16 shows the marginal profit maximization of desirable output and undesirable output simultaneously under predetermined directions. According to the proposed results, the undesirable output abatement of the distribution lines happened in the $(0,1)$ direction for 0.70% of supply chains. Moreover, the supply chain 9 obtained the acceptable level of wasted energy decrease in second distribution line when applying more resources in the direction of power loss reduction. Similarity, the supply chain 5 enables a decrease of approximately 0.22% of power loss only in one distribution line division. It is worth mentioning that supply chain 4 provides the marginal profit maximization of desirable output simultaneously with undesirable output reduction

Table 15. DMP of supply chain 4 under the four normalized directions.

Division of supply chains		Objective function of Supply chain division	DMP	Objective function of Supply chain division	DMP
	$(g^h_{Y_1}, g^h_{Y_2})$	α^h_1	$\frac{\partial(Y_1^{S_2}, B_1^{S_2})}{\partial X_1^{S_2}}$	α^h_1	$\frac{\partial(Y_1^{S_2}, B_1^{S_2})}{\partial X_1^{S_2}}$
Gas field of supply chain 4	$(0.1, 0.9)$	0.000726	$(0.0758, -0.0240)$	0	$(0, 0)$
Distribution 3 of supply chain 6	$(0.1, 0.9)$	0	$(0, 0)$	0.000109	$(0.0647, -0.00133)$
Distribution 1 of supply chain 8	$(0.1, 0.9)$	0.000229	$(0.262, -0.00321)$	0	$(0, 0)$
Distribution 4 of supply chain 1	$(0.2, 0.8)$	0	$(0, 0)$	0.0000255	$(0.0429, -0.000318)$
Gas field of supply chain 4	$(0.2, 0.8)$	0.0000717	$(0.171, 0.0240)$	0	$(0, 0)$
Gas field of supply chain 4	$(0.3, 0.7)$	0.0000917	$(0.292, -0.0240)$	0	$(0, 0)$
Gas field of supply chain 4	$(0.4, 0.6)$	0.000109	$(0.455, -0.0240)$	0	$(0, 0)$
Gas field of supply chain 4	$(0.5, 0.5)$	0.000103	$(0.537, -0.0189)$	0	$(0, 0)$
Gas field of supply chain 4	$(0.6, 0.4)$	0.000103	$(0.644, -0.0151)$	0	$(0, 0)$

Table 16. DMP of 10 supply chains under undesirable outputs and normalized directions.

DMU		$\frac{\partial(Y_1, B_1)}{\partial X}$		$\frac{\partial(Y_1, B_1)}{\partial X}$	$\frac{\partial(Y_1, B_1)}{\partial X}$		
Case	$(0, 1)$	$(0.1, 0.9)$	$(0.2, 0.8)$	$(0.3, 0.7)$	$(0.4, 0.6)$	$(0.5, 0.5)$	$(0.6, 0.4)$
1	$(0, 0)$	$(0, 0)$	$(0.0429, -0.0003)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
2	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
3	$(0, -0.0933)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
4	$(0, -0.093)$	$(0.076, -0.024)$	$(0.0429, -0.000318)$	$(0.292 - 0.024)$	$(0.455, -0.024)$	$(0.537, -0.019)$	$(0.64, -0.015)$
5	$(0, -0.21728)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
6	$(0, -0.00330)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
7	$(0, -0.09327)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
8	$(0, -0.0105)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
9	$(0, -0.3109)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
10	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$

in the seven predetermined directions, as the $(0.6, 0.4)$ direction is an appropriate direction for resource control and environmental preservation from harmful emissions.

4.3. Managerial insights

Overall, supply chains 5 and 9 have appropriate capacities for power loss reduction in the first and second distribution lines among the 10 supply chains. According to the obtained results, an increase of one unit investment creates a marketable decrease in wasted power, as applying one unit of line capacities in supply chains 5, and 9 creates approximately a decrease of 0.22 and 0.31 units of power losses, respectively. In this case, supply chain management enables the choice of directions in which the increment of inputs provides only a significant abatement of power losses in a determined direction. On the other hand, there is only a supply chain 4, as increases in inputs to a gas field's division create increments in gas sold to other companies and flaring gas abatement in a predetermined direction.

The increase of one unit of input to output production in the $(0, 1)$ direction implies only undesirable output decline without any increase in desirable output. In this case, the inputs apply to investment for the clean-up of harmful emissions and wasted energy inhibition in industry activity. According to Table 14, distributions 1 and 2 of supply chain 7 have the necessary ability to invest based on increasing each of the two inputs of distribution line capacity and length, while supply chains 3, 4, 5, 6, 8, and 9 are able to invest only one input for power losses. Also, two distribution lines of supply chain 7 are able to apply capacity and length of distribution lines to wasted energy, while other supply chains have the ability to invest only in one input. Supply chains 5 and 9 indicated high capabilities to protect wasted energy, but other supply chains provided a slight decrease in power loss inhibition. As a result, the policymakers of the industry sector are able to choose divisions from the supply chain that effectively decrease power losses.

Based on Table 15 results, the gas field division of the supply chain 4 had fundamental capacities for energy production increment and flare gas decrement. Furthermore, the gas field division of this supply chain also had a considerable structure for the marginal profit maximization of outputs based on flare gas decreases. In supply chain 4, only DMU obtained the increment of desirable output and the decrease of undesirable outputs of the gas field division in the four directions, while the DMP of other supply chain divisions remained unchanged. Indeed, the utilization of more than one unit of capacity in the gas field of supply chain 4 creates an increase of 0.644 units of sold gas and a decrease of 0.02 units of flare gas emission in that direction. The desirable output production of the supply chain 4 gas field in the $(0.6, 0.4)$ direction not only creates marginal profit maximization output but also can provide a slight abatement from flare gas emissions.

5. Conclusion

Rapid economic growth and excessive energy consumption cause serious damage to the environment and organisms. The extraction of natural resources and the increased utilization of fossil fuels cause contamination concentrations, greenhouse gas emissions, and environmental destruction. Therefore, the existence of an appropriate pattern of resource control in the direction of economic return increment along with wasted energy and pollution emissions reduction is a fundamental issue in industry activities. This study provides a DEA model for the estimation of marginal profit maximization while simultaneously avoiding undesirable output increment. The current paper estimates the marginal productivity of 10 supply chains and their 15 divisions under two categories of inputs in 8 normalized directions, while resource management and energy loss abatement are considered. Indeed, the proposed model, in addition to identifying outputs in different directions, enables distinguishing inputs with significant effects on economic return and less energy loss. Therefore, the choice of the right direction for production increment in supply chain divisions not only causes resource control and capacity management and has a marketable effect on economic return but also protects the environment from the negative effects of greenhouse gases and toxic emissions. This study has two empirical results in the distribution line divisions. First, the results show that there are two supply chains, and the two distribution lines have a high level of effectiveness opportunity where an increase of one unit in line capacities in a predetermined direction creates a significant decrease in energy losses and wasted energy, so these supply chains need investment for more abatement of power losses. It is worth mentioning that the utilization of one more unit of gas field capacity in a supply chain created a significant increase in the amount of gas sold to other companies and a flaring gas decrease in the six predetermined directions. Second, energy resource control and the resources' appropriate allocation in the direction of economic profit increment and decrement of greenhouse gas emissions and harmful pollutants not only provide performance productivity but also protect the environment from the harmful effects of toxic pollution. Podinovski and Førsund [29], Atici and Podinovski [2] replaced the classical derivative by directional derivatives, defined left-hand and right-hand SE. They proposed a directional-derivative approach to calculate elasticity measures without any simplifying assumptions. Rosen *et al.* [33] and Asmild *et al.* [1] measure various marginal rates. Lee [23] provided a theoretical foundation for DMP supporting the meta-DEA, which measures efficiency *via* a marginal-profit-maximizing orientation. Lee [23] used the numerical illustrations of Podinovski and Førsund [29], which include one input, one desirable output, and one undesirable output, as well as three observations. In summary, all of the abovementioned references for estimating directional marginal profit maximization do not consider the network DEA model based on multiple inputs and multiple outputs. We proposed a DEA model to estimate DMP in electricity supply chain divisions as the proposed model considers the synergistic effects of each input on MP estimation in predetermined directions simultaneously. This way, the proposed model abeles us to find inputs that increment of their one unit provide economic growth and environmental preservation. The inputs determination create resource control and prevent capacities exhaustion.

5.1. Policy implications

Climate change causes changes in temperature, the temporal pattern of precipitation, and the amount of rainfall. Moreover, temperature changes have a direct effect on energy production performance. Generally, climate change causes power plant productivity abatement as power plants' fuel consumption increases with increases in the environment's temperature. The increase in temperature reduces the efficiency of fossil power plants, as they require more fossil fuel for power production.

Similarly, environmental factors such as monsoon winds and storms cause transmission airline conductor vibrations, which create broken wire and a power cut in the transmitter network. In other words, fluctuations in distribution lines have bad and destructive effects on dispatching power. The major factors that cause disturbances in the voltage of distribution networks are the installation of power sources such as electric motors, electric furnaces, and electric welding machines. Also, thunder and lightning, rainfall, and the wetting of the wiring paths create power fluctuations on distribution lines.

In response, it is fundamentally important to adjust the output level of supply chain energy and power plant sectors, transmitters, and distribution lines by using variable resources to adjust their output levels. Generally, inputs determine and play a fundamental role in MP. The inputs' accurate selection and the resources' appropriate allocation create a desirable output increment and performance productivity as well as pollution emission management. Therefore, the energy and power plant sectors and transmission and distribution networks should have the necessary patterns to adjust output levels when confronting increased demand based on climate change and other critical situations. For capacity planning and resource allocation, we have to move toward DMP, which is essential for supply chain divisions. In this case, DMP estimates marginal profit maximization based on predetermined directions in supply chain divisions. The direction vectors indicate weights between investigated areas that can be defined by policymakers in the production process.

Supply chain divisions should have adequate decisional capacities regarding available resource utilization based on an appropriate pattern of marginal profit maximization outputs and undesirable output abatement. Indeed, the proposed model, in addition to identifying outputs in different directions, enables distinguishing inputs with significant effects on the production of more outputs. Therefore, the divisions' recognition in the electricity supply chain and choice of the right direction for production increment not only cause resource control and capacity management and have a marketable effect on economic growth but also protect the environment from the negative effects of greenhouse gases. The supply chain divisions that obtained marginal profit maximization have the necessary capacities for responsiveness to demand fluctuations in climate change and critical situations.

5.2. Limitations and future recommendations

The proposed approach has three methodological limitations in leading the directional marginal productivity of supply chain divisions. First, the source of energy is different among districts. Each region has its own essential structure and different conditions for business activity. For instance, the southern regions of Iran have noticeable energy sources and a high capacity of power plants compared to other regions. Such regional differences affect efficiency measures *via* the marginal-profit maximization orientation in each region. Second, it is possible that the proposed model is infeasible in the limited range of predetermined directions, as the marginal-profit maximization outputs in defined directions may not replace the production possibilities set when moving on the frontier. The DMP estimation can create capacity adjustments, but moving along the efficient frontier too far may be out of production possibility. To maintain feasibility, meaning that a firm remains within its original production possibility set after taking adjustments, we suggest a limited range of resource adjustments and recalculating the MP in each iterative short-distance move to ensure that the firm remains within the production possibility set due to the law of diminishing marginal returns [24]. Third, the increase of specific inputs may be impossible for policymaking because of government restrictions, such as a specialist workforce in the energy sector and the length of transmission and distribution lines. It is more difficult to increase staff and the new transmission and distribution network due to the government quota for staff numbers. Thus, the best choice is also to keep its scale constant. The problem considered in this study needs further research in the future. Similarly, the choice of right directions based on acceptable techniques to problem feasibility. In the proposed model, we considered the synergistic effects of two inputs, one desirable output, and one undesirable output in DMP estimation simultaneously. In this way, marginal-profit maximization estimates are based on the synergistic effects between the different inputs. The inputs can be considered separately, estimating the marginal production of each input, and then the total DMP estimated *via* marginal-profit maximization for each input.

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Data availability statement

`https://web.archive.org/web/20210514014836/http://amar.tavanir.org.ir` [17].

Author contribution statement

The main idea and concept design, material preparation, data collection, and model design were performed by the author. Moreover, the result analysis was written based on the results of the models running.

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