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ONLINE REVIEWS AND SPILLOVER EFFECTS IN THE DUAL-CHANNEL SUPPLY CHAIN: A CHOICE STRATEGY OF SALES FORMAT

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Abstract. We investigate strategy choice of sales format considering online reviews and the spillover effect in a dual-channel supply chain. Generally, online and offline consumers read online reviews before purchasing, and thus offline retailers will be subjected to the spillover effect of online reviews. We develop joint product quality and service level decision models in reselling and agency selling with online reviews and the spillover effect. There are some interesting findings: first, the optimal service level and product quality increase with online reviews and spillover effect. Second, counterintuitively, the spillover effect of online reviews benefits not only the offline retailer and manufacturer but also the online retailer in some circumstances. Moreover, the win–win sales format for the manufacturer and online and offline retailers exists in some circumstances. Finally, it is interesting to find that consumers' channel preference in the agency sales format is more important than in the resales.

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1. Introduction

Online shopping has become a popular choice in people's daily lives due to the popularization of information technology and the growth of e-commerce [15]. The report shows that from 2017 to 2020, e-commerce retail sales in the US grow from 279 727 million to 374 375 million [6]. E-COMMERCE IN CHINA 2019 shows that e-commerce transactions in China reached 34.81 trillion yuan in 2019 [22]. E-tailers such as Amazon, Taobao, JingDong, and others have launched online review systems to alleviate consumers' uncertainty about products. As online reviews contain more information than the seller's product description, they have become an important information source for consumers to learn about product quality before purchasing [19]. eMarketer states that 92% of consumers read product reviews before purchase and 64% spend more than 10 min reading online reviews [10].

Consumers' willingness to pay rises for positive online reviews. According to Ye *et al.* [40], a 10% improvement in reviewers' ratings of Ctrip, the largest travel website in China, translates into a 4.4% increase in hotel sales. To attract more purchases, sellers tend to improve online ratings by improving product quality and online

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service level. On the one hand, quality improvement can raise the positiveness of customer reviews [37]. On the other hand, a good online service experience improves consumer purchase satisfaction, for example, the after-sales service expands the amount of positive additional comments [21]. As a result, the applause rate, *i.e.*, the proportion of positive reviews, can affect firms' product quality and service level decisions.

In fact, not only online consumers but also offline consumers gather information from online reviews before purchasing. A 2016 survey by Populus shows that 60% of UK internet users say online reviews influence their fashion purchase decisions, both online and offline [11]. Another survey by Power Reviews shows that 70% of consumers prefer to read product ratings and reviews when shopping in brick-and-mortar stores [32]. Such an impact of online reviews on offline purchases triggers a spillover effect. Consumers can buy products from either online retailers or physical stores. Before purchasing, online and offline consumers read online reviews: online consumers generate a quality expectation of the product based on online reviews, and offline consumers generate a quality expectation based on online reviews and their in-store experience. Then, after receiving the products, online consumers post reviews based on their experience, which positively correlated with the actual quality of the product and the level of online service.

There are two types of online sales forms in practice: reselling and agency selling. In the resales format, the manufacturer wholesales products to the online retailer, who resells them to the end consumer. In agency selling, the online retailer acts as a platform, and the manufacturer sells products directly to end consumers *via* the online platform, paying an agency fee to the online platform for each transaction. These two sales formats are widely applied in practice, *e.g.*, JD.com uses a reselling agreement when working with Huawei, Coach, and Burberry, but an agency selling agreement when working with Topsports and Sephora [20].

In recent years, online reviews have become a hot topic. However, research on the impact of the spillover effects caused by online reviews in the dual-channel supply chain is rare. Therefore, it is critical to investigate how online reviews and spillover effects affect decisions on product quality, service level, and sales format selection in a dual-channel supply chain. To address this problem, the following three sub-issues need to be considered:

(1) Considering online reviews and the spillover effect, what are the optimal product quality and service level decisions of the manufacturer and retailers in reselling and agency sales formats, respectively?
(2) How do online reviews and spillover-related parameters influence optimal policies and profits?
(3) Which sales formats are preferred by chain members?

This paper investigates the impact of online reviews and the spillover effects on product quality, service levels, and sales format selection decisions in a dual-channel supply chain. We develop a two-stage Stackelberg game model in which the manufacturer is the leader, determining wholesale price and product quality first, and the online and offline retailers are followers, determining their service level. By using the backward induction method, we obtain the optimal policies and profits of chain members in reselling and agency sales formats. Then, we examine the effects of online-reviews-relevant parameters and quality and service cost parameters on optimal policies and profits. Finally, we compare the optimal policies and profits of chain members in two sales formats to show the optimal sales format from many perspectives.

The main contributions of this paper are as follows: (1) In contrast to the previous research on online reviews focused on the role of product quality in improving online ratings, we consider that both the product quality and online service level positively affect the applause rate, and examine the manufacturer's product quality decision and the retailers' service level decisions. (2) In contrast to many studies that assume offline consumers have perfect product knowledge, we consider that offline consumers are uncertain about product quality and read online reviews before purchasing. (3) Based on the above motivation, we investigate the spillover effects caused by online reviews on the dual-channel supply chain. This topic hasn't received much attention in the literature so far.

Our work provides some interesting results. First, equilibrium product quality, wholesale price, and online and offline service levels increase with online reviews and spillover effect but decrease with product quality cost and service costs. Second, compared with the resales format, consumers' channel preference in the agency sales format is more important for the manufacturer and retailers. In the resales format, consumers' preference for the online channel does not affect optimal policies and the manufacturer's profit. However, in the agency sales format, the increase in consumers' preference for the online channel helps to improve (or reduce) the product quality, wholesale price, and online and offline service levels, depending on the marginal profit in the online channel. Third, not only the manufacturer and the online retailer but also the offline retailer may benefit from the spillover effects of online reviews on offline sales. Fourth, the optimal online sales format could be different from the perspective of the manufacturer, offline retailer, or online retailer. But the win–win choice exists in some circumstances.

The remainder of this paper is organized as follows. We review the relevant literature in Section 2 and describe the problem, notations, and assumptions in Section 3. Then, in Section 4, we develop and solve the model. In Section 5, we provide a sensitivity analysis of the optimal policies and profits to the main parameters. In Section 6, we investigate the optimal sales format. Next, Section 7 provides some managerial insight for the manufacturer and retailers. Finally, Section 8 summarizes the key findings, the managerial implications, and future research prospects.

2. Literature review

Our study builds on two streams of research: (a) supply chain decisions considering online reviews and (b) supply chain decisions with spillover effect.

2.1. Supply chain decisions considering online reviews

The first stream of literature is on supply chain decisions considering online reviews. Shin *et al.* [31] model the process of consumer learning based on Bayesian theory to study the impact of online reviews on the interaction between a dynamic pricing firm and a consumer population. They suggest that consumers could update their beliefs about the product's quality and the expected degree of misfit between the product and their needs by reading online review information. Moreover, some studies simplify the model to focus on the increased sales effect of online reviews on consumer valuation of products to investigate the role of online reviews in brand competition, consumer return, remanufacture, and so on [27, 34]. Besides, some scholars attempt to model the process of consumer rating [21, 29, 35, 37]. However, the role of online reviews in offline consumer purchasing behavior is rarely considered in the above studies. Yang *et al.* [39] investigate the impact of online reviews on the players in the dual-channel supply chain, finding that online and offline prices can be higher or lower with online reviews in the decentralized channel structure. Different from Yang *et al.* [39] that online reviews are assumed to have the same impact on both online and offline consumers, Li *et al.* [15] find that offline retailers have the incentive to display online reviews in-store, therefore, offline consumers can reduce their uncertainty about product fit rely on in-store information and online reviews. They explore the online review integration strategy of the offline retailer, finding that the offline retailer should be cognizant of its online competitor's sales format before integrating online reviews. Li *et al.* [15] and Yang *et al.* [39] showed that online reviews have an increased sales effect in the offline channel. Different from Li *et al.* [15] and Yang *et al.* [39] treat online reviews as an independent parameter, we consider the online rating to be determined by the product quality and online service level to study the impact of online reviews and its spillover effects on the manufacturer and retailers decisions.

2.2. Spillover effects in dual-channel supply chain

Our study also related to the spillover effects in dual-channel supply chain [38]. In this stream, Bell *et al.* [5] proves that physical stores generate operational spillovers to the other channels by attracting customers with a higher cost-to-serve. The spillover effects from offline to online have been investigated by many scholars. The most prevalent spillover effect from offline to online channels is showrooming. It occurs when consumers enjoy the services or advertising from physical retailers but complete their purchases at an online store [18]. However, there is also a spillover effect from online to offline channels. Zhen *et al.* [42] find that for digital products, internet sales can boost offline sales, particularly in the absence of piracy. Dong *et al.* [9] investigate the impact

Table 1. Literature summary of closely related research.

Literature	Supply Chain	Dual Channel	Agency Selling	Reselling	Online Review	Spillover Effects	Decisions		
							Pricing	Service	Quality
Ali *et al.* [1]	✓								
Mondal and Roy [23]	✓								
Paul *et al.* [28]	✓						✓		
Barman *et al.* [2]	✓						✓		
Barman *et al.* [3]	✓	✓		✓			✓		
Barman *et al.* [4]	✓						✓		✓
Yang *et al.* [39]	✓	✓		✓	✓		✓		
Li *et al.* [16]	✓	✓		✓	✓		✓		✓
Li *et al.* [15]	✓	✓	✓	✓	✓		✓		
Li *et al.* [17]	✓	✓		✓		✓	✓		
Dong *et al.* [9]	✓	✓				✓	✓		
Zhen *et al.* [42]	✓	✓	✓	✓		✓	✓		
Our study	✓	✓	✓	✓	✓	✓	✓	✓	✓

of online-to-offline spillovers on the price and quality strategies among competing firms. Moreover, Sanchez *et al.* [30] and Chae *et al.* [8] have focused on the spillover effects of online word of mouth. According to Chae *et al.* [8], the WOM of one product could spill over to affect the WOM of its competing products. Kwark *et al.* [14] investigate the spillover effects of online reviews for the substitutive and complementary products. However, researchers rarely study the spillover effects of online sales to offline sales caused by online reviews. Li *et al.* [15] prove that physical stores benefit from integrating online reviews from online channels. Different from the above studies, we consider that offline consumers read online reviews and that online reviews could spill over to affect offline sales.

2.3. Research gap

By summarizing the connections and distinctions between our work and the existing related research, we find research gaps between our work and the existing related research. Table 1 shows that there are rare studies on the quality and service level decisions and the sales format selection strategy in dual-channel supply considering the spillover effects caused by online reviews.

In reality, however, the quality of the product and the online service level are two important factors that influence online reviews. On the other hand, online reviews have spillover effects on offline sales. Thus, it is important to explore the optimal quality and service level policies in a dual-channel supply chain, considering online reviews and the spillover effect. Besides, there are two main online sales formats, namely reselling and agency selling, therefore, it is also important to investigate the choice strategy of sales formats based on the above background.

3. Problem description, notations and assumptions

3.1. Problem description

In a dual-channel supply chain with one manufacturer, one offline retailer, and one online retailer, the manufacturer sells homogeneous products with quality q through both the offline and the online retailers. The supply chain structure is shown in Figure 1.

The offline retailer and the manufacturer enter into a reselling arrangement in which the offline retailer purchases products at wholesale prices w from the manufacturer and resells them to consumers at retail prices p and service levels s_1.

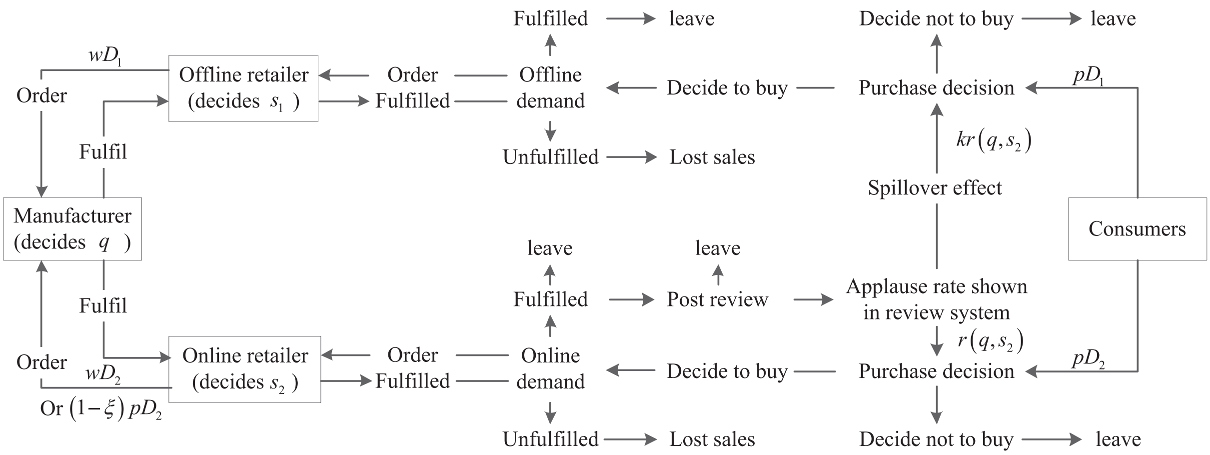

FIGURE 1. The structure and interaction process of supply chain with online reviews and spillover effects.

The online retailer and the manufacturer may enter into a reselling arrangement or an agency selling agreement. In the resales format, similarly with the offline channel, the online retailer purchases products from the manufacturer at wholesale price w and then resells them to consumers at retail price p and service level s_2. While in the agency sales format, the online retailer acts as an online platform providing a service level s_2. The manufacturer sells products directly to consumers with price p and pays an agency fee of ξ percent of sales revenue to the platform.

Consumers buy products through both online and offline channels based on online reviews. Online consumers estimate the quality of products by reading online reviews posted by early consumers. After receiving the products, they publish new online reviews based on their experience as a reference for future consumers. Offline consumers estimate product quality and make purchasing decisions based on their experience in the store (such as touch, try, salesperson's explanation, etc.). However, for some products, such as computers, cameras, mobile phones, etc., a short experience may not be able to learn all the information about the product. Therefore, some offline consumers search for online reviews about the product before purchasing. Thus, online reviews affect not only online sales but also offline sales [39].

Online reviews of products are positively correlated with product quality and online service level. As expected, improving product quality helps to increase the proportion of positive reviews [37]. On the other hand, better online service (including online retailers' speed in responding to messages, ability to handle after-sales service, etc.) helps to increase the proportion of positive comments. For example, when a consumer receives an unsatisfied product, if the seller can respond to the message promptly and propose a good solution, the consumer may not post negative comments.

The decision sequence in this paper is as follows: the manufacturer is the Stackelberg leader and decides the wholesale price w and the product quality q first; the offline and the online retailers are the followers who decide their service levels. The game between online and offline retailers is the Nash game.

3.2. Notations and assumptions

The notations involved in this paper are listed as follows:

Decision variables

s_i The service level of the retailer i, $i = 1, 2$ represent offline and online respectively;
q The quality level of the product;

w The wholesale price per unit of the product.

Parameters

p Retail price per unit of the product;
a The basic market demand of the product;
ϕ Consumers' preference for the online channel;
β Service level elasticity of the demand;
λ Applause rate elasticity of the demand;
k Spillover parameter;
b_1 Online service level elasticity of the applause rate;
b_2 Product quality elasticity of the applause rate;
r_0 Basic applause rate;
ξ Agency fee;
ν Quality cost coefficient of the manufacturer;
μ_i Service cost coefficients of the retailer i, $i = 1, 2$.

Functions

r The applause rate;
D_i Demand function, $i = 1, 2$;
π_j Profit of j, $j = M$ (manufacturer), $R1$ (offline retailer), $R2$ (online retailer).

Moreover, to facilitate model building and solving, we propose the following assumptions:

Assumption 3.1. *This paper uses a uniform pricing strategy to reduce channel conflicts, i.e., the same retail price for online and offline channels [35]. To highlight the research focus: on product quality and service level, this paper assumes the retail price is an exogenous variable determined by the wholesale price, i.e.,* $p = tw$ *and* $t > 1$ *[7].*

Assumption 3.2. *The agency fee is exogenous, as the agency fee does not change after publication [36].*

Assumption 3.3. *The manufacturer's quality investment* (vq^2) *and the offline and the online retailers' service investments* ($\mu_1 {s_1}^2$ *and* $\mu_2 {s_2}^2$) *are increasing and convex in quality level* q, *service levels* s_1 *and* s_2, *respectively [38].*

Assumption 3.4. *To simplify the model and calculation, the cost of the manufacturer product per unit product is assumed to be 0 [41].*

4. Model and equilibrium solution

4.1. Demand functions

The demands are the linear decreasing function of the retail price and the increasing function of the service level and the applause rate. Then, the demand functions in the offline and the online channels are represented by equations (1) and (2).

$$D_1 = (1 - \phi)a - p + \beta s_1 + \lambda k r \tag{1}$$
$$D_2 = \phi a - p + \beta s_2 + \lambda r. \tag{2}$$

The first part in equations (1) and (2) represents the potential market demands for the offline and online channel, in which consumers' preference for the online channel $\phi \in [0, 1]$. The second part represents the impact of price on market demands. The third part represents the impact of service level on market demands, in which $\beta > 0$ [12]. The fourth part represents the impact of online reviews on demands. As online reviews have a stronger impact on the online channel than the offline channel [18], the spillover parameter $k \in [0,1]$. The applause rate, r, can be regarded as the proportion of positive reviews.

The improvement in product quality and online service level helped increase the proportion of positive reviews. Thus, the applause rate is a linear and increasing function of the product quality and the online service level. In addition, the applause rate is also influenced by platform policies, consumer types, logistics, etc. The basic applause rate, r_0, describes the applause rate generated by these components. Thus, the applause rate is represented by equation (3).

$$r = b_1 s_2 + b_2 q + r_0. \tag{3}$$

4.2. Game model and equilibrium analysis

4.2.1. Resales format

In resales format, offline and online retailers procure products from the manufacturer with the same wholesale price w and then resell them to consumers with different service levels. For example, JD Supermarket and Tmall Supermarket, two typical resale online retailers, wholesale products from manufacturers like Huawei, Lenovo, Dyson, etc., and then sell them to consumers. According to the demand functions as shown in equations (1) and (2), and the applause rate function as shown in equation (3), the profit functions of the manufacturer, as well as offline and online retailers can be characterized by equations (4)–(6).

$$\pi_M^{\mathrm{RS}} = w(D_1 + D_2) - \nu q^2 \tag{4}$$

$$\pi_{R1}^{\mathrm{RS}} = (p - w)D_1 - \mu_1 {s_1}^2 \tag{5}$$

$$\pi_{R2}^{\mathrm{RS}} = (p - w)D_2 - \mu_2 {s_2}^2. \tag{6}$$

As the manufacturer is the Stackelberg leader, the offline and online retailers are the followers. The optimization problem of the supply chain in resales format can be characterized by Model RS.

Model RS

$$\max_{q,w} \pi_M^{\mathrm{RS}}(s_1(q,w), s_2(q,w), q, w)$$

$$\text{s.t.} \begin{cases} \max\limits_{s_1} \pi_{R1}^{\mathrm{RS}}(s_1|q,w), \\ \max\limits_{s_2} \pi_{R2}^{\mathrm{RS}}(s_2|q,w). \end{cases}$$

By using the backward induction to solve the Model RS [24–26], the optimal product quality q^{RS^*}, wholesale price w^{RS^*}, offline service $s_1^{\mathrm{RS}^*}$ and online service $s_2^{\mathrm{RS}^*}$ are summarized in Theorem 4.1.

Theorem 4.1. *In resales format, only when $\gamma_1 > 0$ there are uniquely optimal policies. The optimal product quality q^{RS^*}, wholesale price w^{RS^*}, offline service $s_1^{\mathrm{RS}^*}$ and online service $s_2^{\mathrm{RS}^*}$ are:*

$$q^{\mathrm{RS}^*} = \frac{b_2\lambda(k+1)\mu_1\mu_2(a + (k+1)\lambda r_0)}{\gamma_1} \tag{7}$$

$$w^{\mathrm{RS}^*} = \frac{2\mu_1\mu_2 v(a + (k+1)\lambda r_0)}{\gamma_1} \tag{8}$$

$$s_1^{\mathrm{RS}^*} = \frac{\beta\mu_2(t-1)v(a + (k+1)\lambda r_0)}{\gamma_1} \tag{9}$$

$$s_2^{\mathrm{RS}^*} = \frac{\mu_1(t-1)v(b_1\lambda + \beta)(a + (k+1)\lambda r_0)}{\gamma_1}. \tag{10}$$

In which, $\gamma_1 = 8\mu_1\mu_2 tv - b_2^2\lambda^2(k+1)^2\mu_1\mu_2 - 2v(t-1)(\beta^2\mu_2 + \mu_1(b_1\lambda + \beta)(b_1k\lambda + b_1\lambda + \beta)) > 0$. (The proof is in the appendix.)

By comparing the optimal service level between offline and online channels, we have Corollary 4.2.

Corollary 4.2. *In resales format, when $\mu_1 = \mu_2$, the optimal online service level is higher than the offline.*

Corollary 4.2 states that the optimal service level of the online retailer ($s_2^{\mathrm{RS}^*}$) is higher than the optimal service level of the offline retailer ($s_1^{\mathrm{RS}^*}$) when the service cost coefficient of the offline retailer and the online retailer are equal. This is because the level of the online service can directly impact online reviews, and as online reviews affect both online and offline consumers' purchase decisions, the level of the online service is much more important than offline service. This explains why many online retailers, such as JD.com and Vipshop, offer door-to-door return services to gain a good WOM.

Furthermore, substituting equations (7)–(10) into equations (1) and (2), then the optimal demand function in both channels ($D_1^{\mathrm{RS}^*}$ and $D_2^{\mathrm{RS}^*}$) and total demand function (D^{RS^*}) in resales format which are summarized in Corollary 4.3.

Corollary 4.3. *In resales format, the optimal offline demand $D_1^{\mathrm{RS}^*}$, online demand $D_2^{\mathrm{RS}^*}$ and total demand D^{RS^*} are*

$$D_1^{\mathrm{RS}^*} = (1-\phi)a + \lambda k r_0 + \frac{v\gamma_2(t-1)\left[\mu_2\beta^2 + \mu_1\lambda k b_1(\beta+\lambda b_1)\right] - 2v\gamma_2\mu_1\mu_2 t}{2\mu_1\mu_2\gamma_1} + \frac{k\lambda^2 {b_2}^2(1+k)\gamma_2}{2\gamma_1} \tag{11}$$

$$D_2^{\mathrm{RS}^*} = \phi a + \lambda r_0 + \frac{v\gamma_2\left[(t-1)(\beta+\lambda b_1)^2 - 2\mu_2 t\right]}{2\mu_2\gamma_1} + \frac{\lambda^2 {b_2}^2(1+k)\gamma_2}{2\gamma_1} \tag{12}$$

$$D^{\mathrm{RS}^*} = \frac{\gamma_2}{4\mu_1\mu_2} + \frac{\lambda^2 {b_2}^2(1+k)^2\gamma_2}{4\gamma_1}. \tag{13}$$

In which, $\gamma_2 = 2\mu_1\mu_2(a+(k+1)\lambda r_0)$.

Substituting equations (7)–(12) into equations (4)–(6), then the optimal profit of the manufacturer, as well as offline and online retailers in resales format which are summarized in equations (14)–(16).

$$\pi_M^{\mathrm{RS}^*} = \frac{v\gamma_2^2}{4\gamma_1\mu_1\mu_2} \tag{14}$$

$$\pi_{R1}^{\mathrm{RS}^*} = \frac{(t-1)v\gamma_2 D_1}{\gamma_1} - \frac{\beta^2(t-1)^2 v^2\gamma_2^2}{4\mu_1\gamma_1^2} \tag{15}$$

$$\pi_{R2}^{\mathrm{RS}^*} = \frac{(t-1)v\gamma_2 D_2}{\gamma_1} - \frac{(t-1)^2(\beta+\lambda b_1)^2 v^2\gamma_2^2}{4\mu_2\gamma_1^2}. \tag{16}$$

4.2.2. Agency sales format

In the agency sales format, the online retailer becomes a platform, the manufacturer becomes an e-seller. The manufacturer sells products to consumers through the platform and then pays an agency fee ξ to the platform. For example, some brands, like Uniqlo, Muji, Sephora, etc., sell products to consumers directly through their own Taobao online shop and pay Taobao.com agency fees. The profit functions of the manufacturer, as well as offline and online retailers, can be characterized by equations (17)–(19).

$$\pi_M^{\mathrm{AS}} = wD_1 + (1-\xi)pD_2 - \nu q^2 \tag{17}$$

$$\pi_{R1}^{\mathrm{AS}} = (p-w)D_1 - \mu_1 {s_1}^2 \tag{18}$$

$$\pi_{R2}^{\mathrm{AS}} = \xi p D_2 - \mu_2 {s_2}^2. \tag{19}$$

Similar to Model RS, the manufacturer is the Stackelberg leader, then the optimization problem of the supply chain in the agency sales format can be characterized by Model AS.

Model AS

$$\max_{q,w} \pi_M^{\mathrm{AS}}(s_1(q,w), s_2(q,w), q, w)$$
$$\text{s.t.} \begin{cases} \max\limits_{s_1} \pi_{R1}^{\mathrm{AS}}(s_1|q,w), \\ \max\limits_{s_2} \pi_{R2}^{\mathrm{AS}}(s_2|q,w). \end{cases}$$

By using the backward induction to solve the Model AS, the optimal product quality q^{AS^*}, wholesale price w^{AS^*}, offline service $s_1^{\mathrm{AS}^*}$ and online service $s_2^{\mathrm{AS}^*}$ are summarized in Theorem 4.4.

Theorem 4.4. *In agency sales format, only when $\rho_1 > 0$ there are uniquely optimal policies. The optimal product quality q^{AS^*}, wholesale price w^{AS^*}, offline service $s_1^{\mathrm{AS}^*}$ and online service $s_2^{\mathrm{AS}^*}$ are:*

$$q^{\mathrm{AS}^*} = \frac{\lambda b_2 \rho_2}{2\rho_1} \tag{20}$$
$$w^{\mathrm{AS}^*} = \frac{\rho_2 v}{\rho_1(k + (1-\xi)t)} \tag{21}$$
$$s_1^{\mathrm{AS}^*} = \frac{\beta\rho_2(t-1)v}{2\mu_1\rho_1(k + (1-\xi)t)} \tag{22}$$
$$s_2^{\mathrm{AS}^*} = \frac{\rho_2 t v \xi(b_1\lambda + \beta)}{2\mu_2\rho_1(k + (1-\xi)t)}. \tag{23}$$

In which, $\rho_1 = -b_2^2\lambda^2\mu_1\mu_2(k-\xi t+t)^2 - 2\mu_1\xi t v(b_1\lambda+\beta)(b_1k\lambda+(1-\xi)t(b_1\lambda+\beta)) - 2\beta^2\mu_2(t-1)v + 4\mu_1\mu_2 t v(1-\xi t+t) > 0$, $\rho_2 = 2a\mu_1\mu_2(k-\xi t+t)((1-\xi)t\phi - \phi + 1) + 2\lambda\mu_1\mu_2 r_0(k-\xi t+t)^2 > 0$. (The proof is in the appendix.)

By comparing the optimal service level between offline and online channels, we have Corollary 4.5.

Corollary 4.5. *In agency sales format, when $\mu_1 = \mu_2$, there exist a threshold value $\xi_0 = \beta(t-1)/[t(\beta + \lambda b_1)]$ that $s_1^{\mathrm{AS}^*} \geq s_2^{\mathrm{AS}^*}$ only if $\xi \leq \xi_0$, otherwise, $s_1^{\mathrm{AS}^*} < s_2^{\mathrm{AS}^*}$ if $\xi > \xi_0$.*

Corollary 4.5 shows that different from the resales format, in agency sales format when the service cost coefficient of the offline retailer is equal to that of the online retailer, online service level $s_2^{\mathrm{AS}^*}$ may or may not higher than offline service level s_1^*, depending on the agency fee ξ. If the agency fee ξ is large enough, online service level $s_2^{\mathrm{AS}^*}$ is higher than offline service level $s_1^{\mathrm{AS}^*}$. By simplifying the thresholds ξ_0, we find that only when the online retailer's marginal profit ($p\xi$) times the impact extent of its service level on demand ($\beta + \lambda b_1$) is higher than it is in the online channel ($\beta(p-w)$), then the online service level is higher than offline. Consequently, there are two forces driving retailers to improve their service levels: one is high marginal profit, and another is service level has a significant impact on their demand. Both are equally important.

Substituting equations (20)–(23) into equations (1) and (2), then the optimal demand function in both channels ($D_1^{\mathrm{AS}^*}$ and $D_2^{\mathrm{AS}^*}$) and total demand function (D^{AS^*}) which are summarized in Corollary 4.6.

Corollary 4.6. *In agency sales format, the optimal offline demand $D_1^{\mathrm{AS}^*}$, online demand $D_2^{\mathrm{AS}^*}$ and total demand D^{AS^*} are:*

$$\begin{aligned} D_1^{\mathrm{AS}^*} &= a(1-\phi) + \frac{\rho_2 v\big(b_1 k\lambda\mu_1\xi t(b_1\lambda+\beta) + \beta^2\mu_2(t-1)\big)}{2\mu_1\mu_2\rho_1(k+(1-\xi)t)} \\ &\quad + \frac{b_2^2 k\lambda^2\rho_2(k+(1-\xi)t) - 2\rho_2 t v}{2\rho_1(k+(1-\xi)t)} + k\lambda r_0 \end{aligned} \tag{24}$$
$$D_2^{\mathrm{AS}^*} = a\phi + \frac{\xi\rho_2 t v(b_1\lambda+\beta)^2}{2\mu_2\rho_1(k+(1-\xi)t)} + \frac{b_2^2\lambda^2\rho_2(k+(1-\xi)t) - 2\rho_2 t v}{2\rho_1(k+(1-\xi)t)} + \lambda r_0 \tag{25}$$

$$D^{\mathrm{AS}^*} = a + \frac{\rho_2 v\big(\mu_1 \xi t(b_1\lambda+\beta)(b_1(k+1)\lambda+\beta) + \beta^2\mu_2(t-1)\big)}{2\mu_1\mu_2\rho_1(k+(1-\xi)t)} + \frac{b_2^2\lambda^2(k+1)\rho_2(k+(1-\xi)t) - 4\rho_2 tv}{2\rho_1(k+(1-\xi)t)} + (k+1)\lambda r_0. \tag{26}$$

Substituting equations (20)–(25) into equations (17)–(19), then the optimal profit of the manufacturer, as well as offline and online retailers in agency sales format are summarized in equations (27)–(29).

$$\pi_M^{\mathrm{AS}^*} = \frac{4\rho_1\rho_2 v(D_2(1-\xi)t + D_1) - b_2^2\lambda^2\rho_2^2 v(k+(1-\xi)t)}{4\rho_1^2(k+(1-\xi)t)} \tag{27}$$

$$\pi_{R1}^{\mathrm{AS}^*} = \frac{\rho_2(t-1)v\big((k+(1-\xi)t)\big(2\rho_1(a(1-\phi)+k\lambda r_0) + b_2^2 k\lambda^2\rho_2\big) - 2\rho_2 tv\big)}{2\rho_1^2(k+(1-\xi)t)^2} + \frac{\rho_2^2(t-1)v^2\big(2b_1 k\lambda\mu_1\xi t(b_1\lambda+\beta) + \beta^2\mu_2(t-1)\big)}{4\mu_1\mu_2\rho_1^2(k+(1-\xi)t)^2} \tag{28}$$

$$\pi_{R2}^{\mathrm{AS}^*} = \frac{\big(2\mu_2(k+(1-\xi)t)\big(2\rho_1(a\phi+\lambda r_0) + b_2^2\lambda^2\rho_2\big) + \rho_2 v\big(\xi t(b_1\lambda+\beta)^2 - 4\mu_2 t\big)\big)}{4\mu_2\rho_1^2(k+(1-\xi)t)^2} \times \xi\rho_2 tv. \tag{29}$$

5. Sensitivity analysis

In this section, we explore the impact of key parameters, such as online service level elasticity (b_1), product quality elasticity (b_2), spillover effects (k), applause rate elasticity of the demand (λ), basic applause rate (r_0), consumer's channel preference (ϕ), service cost coefficients (μ_1 and μ_2), and quality cost coefficient (ν), on the optimal policies and profits of the manufacturer and the retailers, and discover several interest results.

5.1. Sensitivity analysis to optimal policy

Proposition 5.1 summarizes the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the optimal policies of the manufacturer and the retailers in resales format.

Proposition 5.1. *In resales format, the impact of parameters on optimal product quality q^{RS^*}, wholesale price w^{RS^*}, offline retailer's service level $s_1^{\mathrm{RS}^*}$, and online retailer's service level $s_2^{\mathrm{RS}^*}$ are as follows:*

(1) *The higher b_1, b_2, k, λ or r_0, the higher q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$;*
(2) *The higher μ_1, μ_2 or ν, the less q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$;*
(3) *ϕ does not affect q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$. (The proof is in the appendix.)*

According to Proposition 5.1-(1), when online service level or product quality has a significant impact on the applause rate, or when online reviews strongly affect market demand, then the manufacturer improves product quality, and the online retailer improves online service level to get a higher applause rate. For this reason, the manufacturer's production cost and the online channel's market size increased. Consequently, the manufacturer higher the wholesale price, and the offline retailer improves the offline service level. Surprisingly, when online reviews spillover affects offline sales strongly, the online retailer improves service level. This is because the more important online reviews are to offline consumers, the higher the offline demand caused by online reviews, leading the manufacturer to improve product quality to increase the applause rate. Proposition 5.1-(1) also indicates that when the basic-applause rate r_0 increases, the manufacturer and retailers are willing to improve product quality and service level. As a result, the manufacturer increases the wholesale price for high-quality and high-service-level products.

According to Proposition 5.1-(2), the manufacturer reduces the product quality when the quality cost coefficient (ν) increases. For this reason, the applause rate decreased, and then the market size shrank in both channels. Consequently, the retailers reduce service levels to decrease operation costs, and the manufacturer lower the wholesale price for poor-quality products. Similarly, the online retailer reduces the online service level when the online service cost coefficient (μ_2) increases, which leads to a lower applause rate. Thus, the manufacturer reduces the quality and wholesale price of the product, and the offline retailer reduces the offline service level. However, surprisingly, the higher the offline service cost coefficient (μ_1), the lower the service levels in both channels and the quality and wholesale price of the product. This is because the lower offline service level leads to lower offline demand. Then, the manufacturer reduces product quality, which reduces online sales indirectly. Thus, the online retailer reduces service levels.

Proposition 5.1-(3) indicates that in resales format, the consumer's preference for the online channel does not affect the manufacturer's, the offline retailer's, and the online retailer's optimal policies. This is because, on the one hand, the manufacturer gets the same marginal profit from both channels, resulting in a fixed revenue for the manufacturer when the consumer channel preference changes. Thus, the consumer's channel preference does not affect the manufacturer's policy. On the other hand, retailers get the same quality product at the same price, leading to a fixed service level policy when consumer channel preference changes.

Proposition 5.2 summarizes the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the optimal policies of the manufacturer and the retailers in agency sales format.

Proposition 5.2. *In agency sales format, the impact of parameters on optimal product quality q^{AS^*}, wholesale price w^{AS^*}, offline retailer's service level $s_1^{\mathrm{AS}^*}$, and online platform's service level $s_2^{\mathrm{AS}^*}$ are as follows:*

(1) *The higher the b_1, b_2, k, λ or r_0, the higher the q^{AS^*}, w^{AS^*}, $s_1^{\mathrm{AS}^*}$, and $s_2^{\mathrm{AS}^*}$;*
(2) *The higher μ_1, μ_2 or ν, the lower q^{AS^*}, w^{AS^*}, $s_1^{\mathrm{AS}^*}$, and $s_2^{\mathrm{AS}^*}$;*
(3) *There is a threshold value $\hat{\xi}$, when $\xi < \hat{\xi}$, q^{AS^*}, w^{AS^*}, $s_1^{\mathrm{AS}^*}$, and $s_2^{\mathrm{AS}^*}$ increase with ϕ, otherwise, q^{AS^*}, w^{AS^*}, $s_1^{\mathrm{AS}^*}$, and $s_2^{\mathrm{AS}^*}$ decrease with ϕ. (The proof is in the appendix.)*

According to Proposition 5.2, in agency sales format, the manufacturer raises the wholesale price and product quality, and the retailers improve service level when online reviews-related parameters increase or when quality and service costs decrease. This result is similar to the resales format. However, different from the result in the reselling case, consumers' preference for the online channel, ϕ, affects the optimal wholesale price, product quality, and service levels in agency selling. Counter-intuitively, the offline (online) retailer may decrease service level when consumers prefer more on the offline (online) channel depending on the agency fee. By simplifying the threshold value $\hat{\xi}$, we find that only if the manufacturer's marginal revenue from online sales is higher than offline sales, *i.e.*, $p(1-\xi) > w$, the manufacturer increases the product quality and the wholesale price when consumers prefer more on the online channel. For this reason, the platform and the offline retailer increase service levels because of the higher price. However, when consumers prefer more on the online channel if the marginal revenue from online sales is lower than from offline sales, *i.e.*, $p(1-\xi) \leq w$, the manufacturer reduces the product quality and the wholesale price. Then the platform and the offline retailer reduce their service levels.

5.2. Sensitivity analysis to optimal profit

Proposition 5.3 summarizes the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the manufacturer's optimal profit in resales format.

Proposition 5.3. *In resales format, the impact of parameters on the manufacturer's profit $\pi_M^{\mathrm{RS}^*}$ are as follows:*

(1) *The higher b_1, b_2, k, λ, or r_0, the higher $\pi_M^{\mathrm{RS}^*}$;*
(2) *The higher μ_1, μ_2 or ν, the less $\pi_M^{\mathrm{RS}^*}$;*
(3) *ϕ does not affect $\pi_M^{\mathrm{RS}^*}$. (The proof is in the appendix.)*

According to Proposition 5.3 the manufacturer's profit increases with online reviews-related parameters, such as online service level elasticity of the applause rate b_1, product quality elasticity of the applause rate b_2, applause rate elasticity of the demand λ, spillover effects k or basic applause rate r_0, but decreases with product and service costs, μ_1, μ_2 or ν. This is because when b_1, b_2, k, λ, or r_0 increase, the manufacturer improves the product quality and retailers improve service levels which have been shown in Proposition 5.1, and the manufacturer gains more sales as a result. On the other hand, when b_1, b_2, k, λ, or r_0 increase, the manufacturer raises the wholesale price, resulting in a higher marginal profit for the manufacturer. As a result of the increased marginal profit and sales, the manufacturer's profit rises. Similarly, an increase in μ_1, μ_2, or ν results in lower sales and lower marginal profit, reducing the manufacturer's profit decrease. Besides, in the resales format, consumers' preference for the online channel does not affect the manufacturer's profit. This is because, Proposition 5.1 shows that the manufacturer and the retailers have a fixed wholesale price, quality, and service level policy when consumer channel preference changes. Thus, the manufacturer's marginal profit and sale volume do not change with consumer channel preference.

Proposition 5.4 summarizes the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the manufacturer's optimal profit in agency sales format.

Proposition 5.4. *In agency sales format, the impacts of parameters on the manufacturer's profit* $\pi_M^{\mathrm{AS}^*}$ *are as follows:*

(1) *The higher* b_1*,* b_2*,* k*,* λ *or* r_0*, the higher* $\pi_M^{\mathrm{AS}^*}$*;*
(2) *The higher* μ_1*,* μ_2 *or* ν*, the less* $\pi_M^{\mathrm{AS}^*}$*;*
(3) *When* $\xi < \hat{\xi}$*,* $\pi_M^{\mathrm{AS}^*}$ *increases with* ϕ*, otherwise,* $\pi_M^{\mathrm{AS}^*}$ *decreases with* ϕ*. (The proof is in the appendix.)*

From Proposition 5.4, the manufacturer's profit increases with online reviews-related parameters but decreases with product and service costs in agency sales format. This result is similar to the resales format. However, different in the result in the reselling case, consumers' preference for the online channel ϕ affects the manufacturer's profit in the agency sales format depending on the agency fee ξ. Only if the agency fee is small enough that the marginal revenue from online sales is higher than offline sales, *i.e.*, $p(1-\xi) > w$, when consumers prefer more to the online channel, the manufacturer gains more demand due to the product quality and service levels increase and more marginal profit due to wholesale price increases and, as a consequent, get more profit. Otherwise, if the agency fee is large enough that the manufacturer's marginal revenue from online sales is lower than offline sales, *i.e.*, $p(1-\xi) \leq w$, his profit decreases when consumers prefer more on the online channel.

As the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the offline and the online retailers' profits are too complex, it is impossible to derive the analytical result. For this reason, we use a set of numerical examples to illustrate the impacts of b_1, b_2, k, λ, r_0, ϕ, μ_1, μ_2 and ν on the offline and the online retailers' profits. Base on Li *et al.* [18] and Song *et al.* [33], we take the following baseline parameters: $a = 30$, $\beta = 0.75$, $k = 0.8$, $\lambda = 0.5$, $t = 1.5$, $r_0 = 5$, $v = 3.2$, $\mu_1 = 1.5$, $\mu_2 = 1.2$, $b_1 = 1.2$, $b_2 = 1.8$, $\phi = 0.5$, $\xi = 0.15$. In which, the unit of ϕ, k, and ξ is 1 as these parameters are between 0 to 1, the unit of other parameters like a, r_0, β, λ, b_1, b_2, μ_1, μ_2 and ν are depended the specific industry. The numerical analysis in this paper provides conclusions on the overall impact trend. In practice, we need to adjust the parameter's value based on the actual situation of firms.

Figure 2 shows that optimal profits of the manufacturer and the offline retailer increase with online service level elasticity (b_1) and the product quality elasticity (b_2) in both sales formats, but the online retailer's profit may decrease with b_1. This is because when the online service level significantly affects the applause rate, the online retailer has to take responsibility for WOM for the dual-channel. It is unfair to the online retailer. However, the manufacturer can get revenue from both channels, so her profit is not reduced when the manufacturer takes on more responsibility for word-of-mouth (b_2 increase). Moreover, Figure 2a also indicates that the resales format is a win–win strategy when the online service level has a moderate impact on online reviews ($b_1 \in [1.5, 3]$), as the profits of the manufacturer, offline and online retailers in resales format are higher than in agency format.

Figure 3a shows that optimal profits of the manufacturer and retailers increase with basic applause rate (r_0) in both sales formats. This is because a better applause rate always benefits both channels. Figure 3b shows that the offline retailer's profit decreases with consumers' preference for the online channel (ϕ), while the online retailer's profit increases with ϕ in both sales formats, whereas the manufacturer's profit increases with ϕ in agency selling and does not change with ϕ in resales format. Moreover, the resales format is a win–win strategy when consumers' preference for the online channel is moderate (*i.e.*, $\phi \in [0.2, 0.4]$) in Figure 3b.

From Figure 4, the optimal profits of the manufacturer and the offline retailer decrease with the offline service cost coefficient (μ_1), online service cost coefficient (μ_2) and product quality cost coefficient (ν), but the online

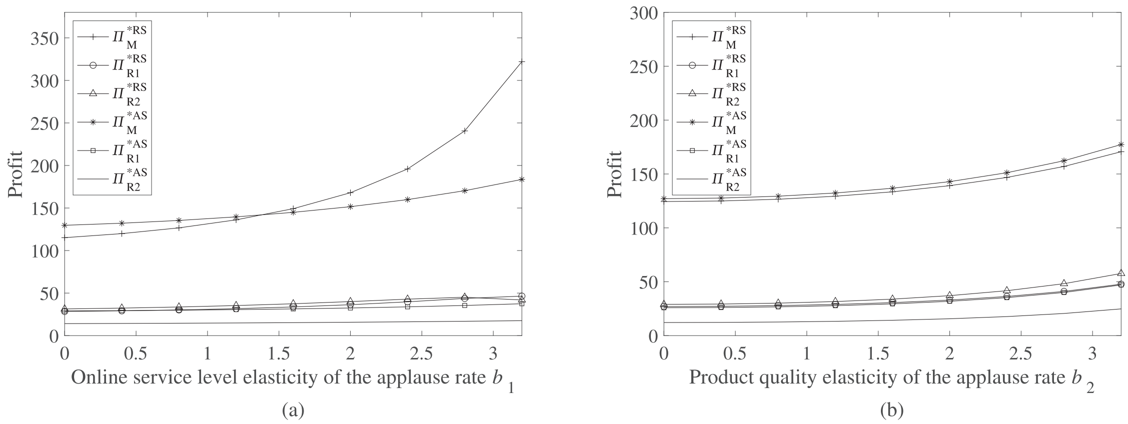

FIGURE 2. The variation of the optimal profits with b_1 and b_2.

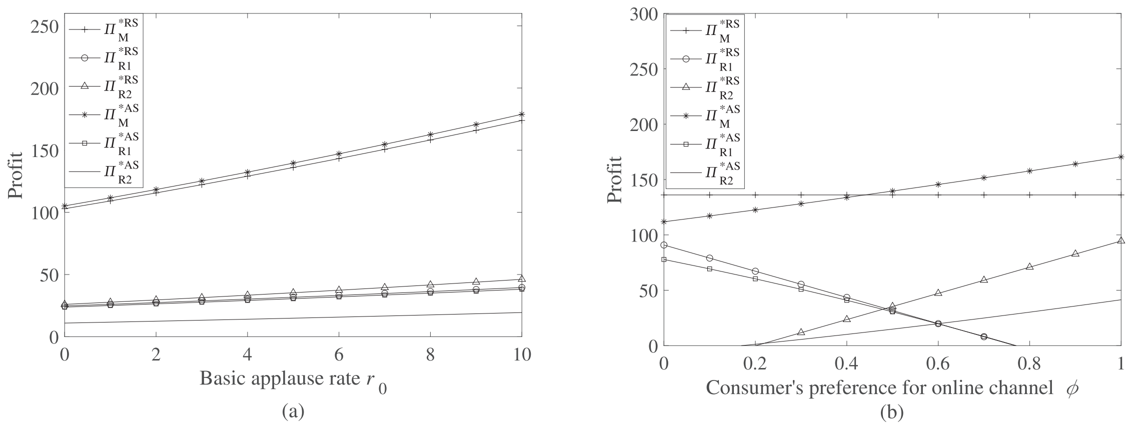

FIGURE 3. The variation of the optimal profits with r_0 and ϕ.

retailer's profit changes minimally when online with the service cost increase. This is the result of the online retailer lowering online service level. Figure 4 also indicates that the quality cost significantly impacts profits more than service costs.

Figure 5 shows the impact of the spillover parameter (k) on optimal profits with different agency fees. From Figures 5a and 5b, the optimal profits of the manufacturer and the offline retailer increase with the spillover effects (k). This is because when online reviews spillover affects offline sales strongly, the demand of the offline retailer and the manufacturer increases, leading to higher profits. Surprisingly, Figure 5c indicates that the online retailers' profit benefits from the spillover effect, although the increase in profits is very small. This is especially true when the online retailer charges a high agency fee from the manufacturer. The reason is that the manufacturer improves the product quality if the online reviews strongly affect offline sales, which has been proved in Propositions 5.1 and 5.2. The improved product quality can compensate for the costs of over-investment in online services by online retailers.

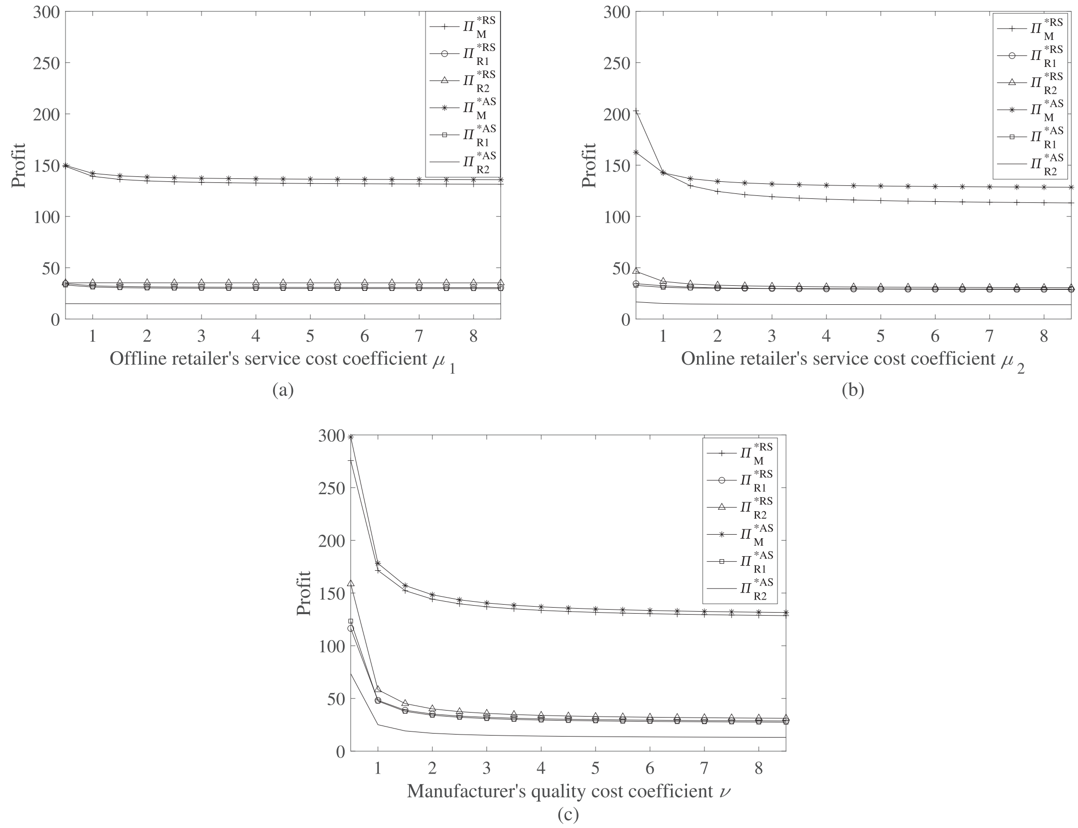

FIGURE 4. The variation of the optimal profits with μ_1, μ_2 and ν.

6. Reselling *vs.* agency selling

6.1. Optimal sales format from the perspective of firms

In this section, we compare the optimal policies, demands, and profits between the reselling and agency sales formats to examine the optimal format of the sales from the perspective of firms. The results are compiled in Propositions 6.1–6.4.

Proposition 6.1. *For the product quality, wholesale price, offline and online service levels:*

(1) $q^{\mathrm{RS}^*} > q^{\mathrm{AS}^*}$ *only if* $\xi < \hat{\xi}$ *and* $\phi < \hat{\phi}_q$, *or* $\xi > \hat{\xi}$ *and* $\phi > \hat{\phi}_q$, *otherwise,* $q^{\mathrm{RS}^*} < q^{\mathrm{AS}^*}$;
(2) $w^{\mathrm{RS}^*} > w^{\mathrm{AS}^*}$ *and* $s_1^{\mathrm{RS}^*} > s_1^{\mathrm{AS}^*}$ *only if* $\xi < \hat{\xi}$ *and* $\phi < \hat{\phi}_w$, *or* $\xi > \hat{\xi}$ *and* $\phi > \hat{\phi}_w$, *otherwise,* $w^{\mathrm{RS}^*} \leq w^{\mathrm{AS}^*}$ *and* $s_1^{\mathrm{RS}^*} \leq s_1^{\mathrm{AS}^*}$;
(3) $s_2^{\mathrm{RS}^*} > s_2^{\mathrm{AS}^*}$ *only if* $\xi < \hat{\xi}$ *and* $\phi < \hat{\phi}_s$, *or* $\xi > \hat{\xi}$ *and* $\phi > \hat{\phi}_s$, *otherwise,* $s_2^{\mathrm{RS}^*} \leq s_2^{\mathrm{AS}^*}$.

In which $\hat{\xi} = \frac{t-1}{t}$, $\hat{\phi}_q = \frac{\Gamma}{2a\gamma_1\mu_1\mu_2(k-\xi t+t)}$, $\hat{\phi}_w = \frac{\rho_1(c\gamma_1+\gamma_2 v)+\Gamma v}{2a\gamma_1\mu_1\mu_2 v(k-\xi t+t)}$, $\hat{\phi}_s = \frac{\rho_1(k+t)(c\gamma_1+\gamma_2 v)+\Gamma\xi t v}{2a\gamma_1\mu_1\mu_2\xi t v(k-\xi t+t)}$, $\Gamma = \mu_1^2\mu_2(b_1\lambda+\beta)((b_1k\lambda+b_1\lambda+\beta)(4av(k+t)+\lambda(k+t)^2(4r_0v-b_2^2c(k+1)\lambda)-4c(k-1)(t-1)tv)-4a(k+1)\xi tv(b_1\lambda+\beta)+\lambda\xi t(b_1(k+1)\lambda(k+t)+\beta(k^2+2k+t))(b_2^2c(k+1)\lambda-4r_0v)+\beta b_2^2ck(k+1)^2\lambda^2\xi t-8\beta ck\xi ttv)+\beta^2\mu_1\mu_2^2(t-1)(4v(a-ckt+3ct)+b_2^2(-c)(k+1)\lambda^2(3k-\xi t+t+2)+4\lambda r_0v(2k-\xi t+t+1))+2\mu_1^2\mu_2^2(2(1-k)t(b_2^2c(k+1)^2\lambda^2-2v(a+2ct+k\lambda r_0+\lambda r_0))-\lambda(k-\xi t+t)(b_2^2(k+1)\lambda(a-2ct)+8r_0tv))+2\beta^2c\mu_1\mu_2(t-1)v(b_1\lambda+\beta)(\beta(k\xi t-t+1)-b_1(k+1)\lambda(t-1))+2\beta ck\mu_1^2\xi(t-1)tv(b_1\lambda+\beta)^2(b_1k\lambda+b_1\lambda+\beta)-2\beta^4c\mu_2^2(t-1)^2v$. *(The proof is in the appendix).*

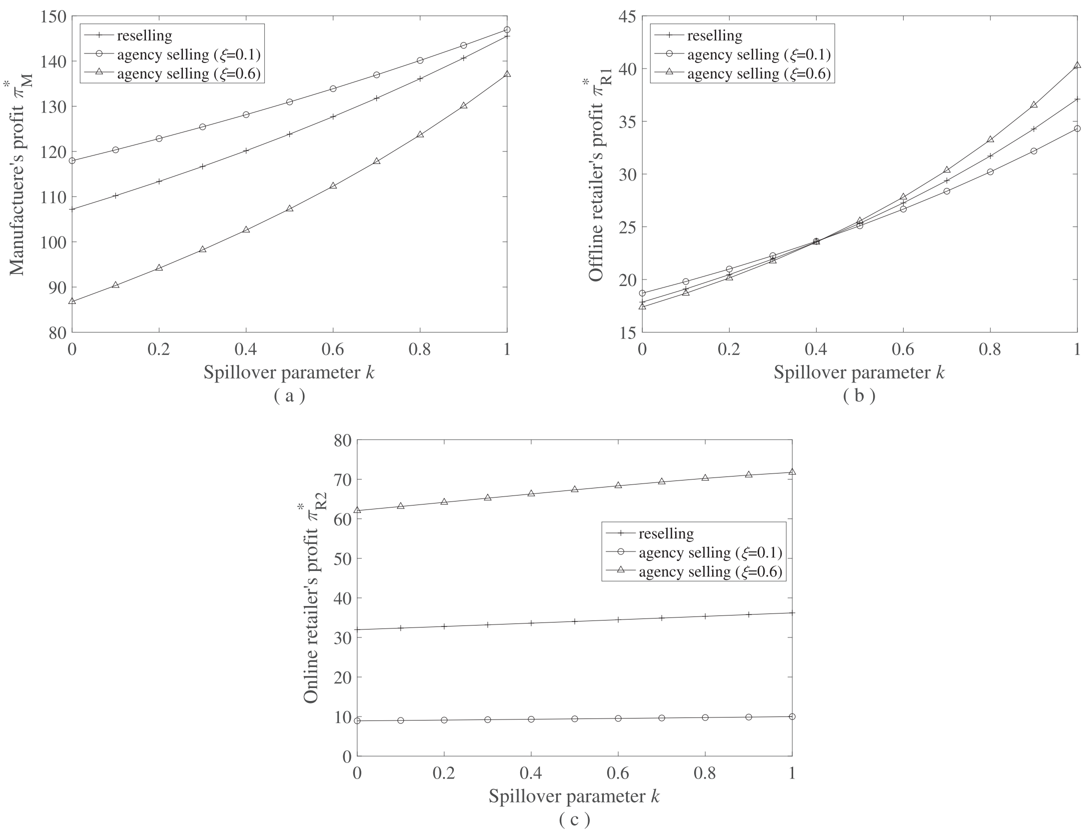

FIGURE 5. The variation of the optimal profits with k.

Proposition 6.1 indicates that when the agency fee and consumers' preference for the online channel are both small or large, the optimal product quality, wholesale price, and retail prices in the resales format are higher than those in agency sales format. This means that when the online platform has a high basic market size, consumers profit in the resales format if the platform receives a high agency fee in an agency sale, and they profit in agency sales format if the platform receives a low agent fee. However, when the online platform has a low basic market size, consumers profit in the agency sales format if the platform receives a high agency fee in an agency sale, and they profit in the resales format if the platform receives a low agent fee.

Proposition 6.2. *The manufacturer prefers resales format if* $\Gamma_M > 0$, *and agency sales format if* $\Gamma_M < 0$. *In which,* $\Gamma_M = \rho_1(\mu_1(b_1\lambda+\beta)(b_1k\lambda+b_1\lambda+\beta)(c\gamma_1(k+t)((1-\xi)t-1)+\gamma_2(t-1)v(k-\xi t+t)) - \mu_1\mu_2(\gamma_2(k-\xi t+t)(4tv-b_2^2(k+1)^2\lambda^2)+4c\gamma_1 t((1-\xi)t-1)) + \beta^2\mu_2(t-1)[c\gamma_1((1-\xi)t-1)+\gamma_2 v(k-\xi t+t)]) + \gamma_1\rho_2\{\mu_1\mu_2[4tv - b_2^2(k+1)\lambda^2(k-\xi t+t)] - \beta^2\mu_2(t-1)v - \mu_1\xi tv(b_1\lambda+\beta)(b_1k\lambda+b_1\lambda+\beta)\}$.

Proposition 6.3. *The offline retailer prefers resales format if* $\Gamma_{R1} > 0$, *and agency sales format if* $\Gamma_{R1} < 0$. *In which,* $\Gamma_{R1} = \rho_1^2\{2\mu_1\mu_2[\gamma_2(k-\xi t+t)^2(2v(a\gamma_1(1-\phi)-2c\gamma_1 t+\gamma_1 k\lambda r_0-\gamma_2 tv)+b_2^2k(k+1)\lambda^2(c\gamma_1+\gamma_2 v)) + 2c\gamma_1^2((1-\xi)t-1)((k+(1-\xi)t)(a(1-\phi)-ct+k\lambda r_0)-c(k+1)t)] + 2b_1k\lambda\mu_1(b_1\lambda+\beta)(c^2\gamma_1^2((1-\xi)t-1)((k+t)^2-\xi t(t-1))+\gamma_2(t-1)v(2c\gamma_1+\gamma_2 v)(k-\xi t+t)^2)+\beta^2\mu_2(-(t-1))(c^2\gamma_1^2(k+1)^2-(c\gamma_1+\gamma_2 v)^2(k-\xi t+t)^2)\} - 2\gamma_1^2\rho_1\rho_2\{\mu_1\mu_2(k-\xi t+t)[2av(1-\phi)+b_2^2ck(k+1)\lambda^2+2k\lambda r_0 v]+c(k+1)v(2b_1k\lambda\mu_1\xi t(b_1\lambda+\beta)+\beta^2\mu_2(t-1)-4\mu_1\mu_2 t)\} + \gamma_1^2\rho_2^2 v\{2\mu_1\mu_2[2tv-b_2^2k\lambda^2(k-\xi t+t)]-2b_1k\lambda\mu_1\xi tv(b_1\lambda+\beta)-\beta^2\mu_2(t-1)v\}$.

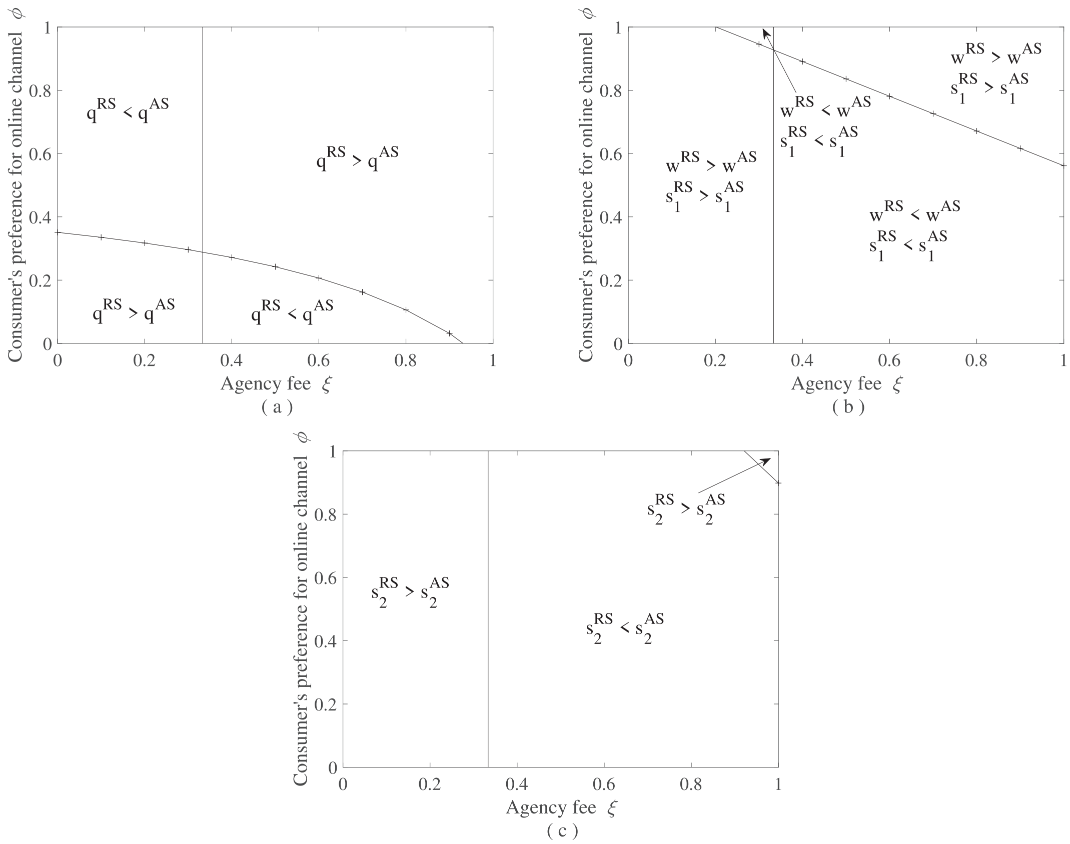

FIGURE 6. The variation of the difference in optimal policies between resale and agency sales with the changing ξ and ϕ.

Proposition 6.4. *The online retailer prefers resales format if* $\Gamma_{R2} > 0$*, and agency sales format if* $\Gamma_{R2} < 0$*. In which,* $\Gamma_{R2} = \rho_1^2\{(b_1\lambda+\beta)^2[c^2\gamma_1^2(\xi t-t+1)(\xi t((k+1)^2+(t-1)^2)-(t-1)(k+t)^2)+\gamma_2 v(t-1)^2(2c\gamma_1+\gamma_2 v)(k-\xi t+t)^2]-2\mu_2(t-1)(2c\gamma_1^2((1-\xi)t-1)((k-\xi t+t)(-a\phi+ct-\lambda r_0)+c(k+1)t)-\gamma_2(k-\xi t+t)^2(2v(a\gamma_1\phi-2c\gamma_1 t+\gamma_1\lambda r_0-\gamma_2 tv)+b_2^2(k+1)\lambda^2(c\gamma_1+\gamma_2 v)))\}+2\gamma_1^2\rho_1\rho_2\{\mu_2(t-1)[4c(k+1)tv-(k-\xi t+t)(2av\phi+b_2^2c(k+1)\lambda^2+2\lambda r_0 v)]+c\xi tv(k+1)(\xi t-2t+2)(b_1\lambda+\beta)^2\}+\gamma_1^2\rho_2^2 v[2\mu_2(t-1)(b_2^2\lambda^2\xi t-kb_2^2\lambda^2-b_2^2\lambda^2 t+2tv)+\xi tv(\xi t-2t+2)(b_1\lambda+\beta)^2]$.

Propositions 6.2–6.4 indicate that the optimal sales format for the manufacturer as well as offline and online retailers is not uniquely determined and may be either resales or agency sales, depending on the value of the specific parameters.

As the expressions for the threshold $\hat{\phi}_q$, $\hat{\phi}_w$, $\hat{\phi}_s$, Γ_M, Γ_{R1}, and Γ_{R2} are cumbersome, we explore the optimal sales format through a numerical example. Specifically, we show the variation of the difference in optimal policies and profits between resale and agency sales with the changing ξ and ϕ.

Figure 6a shows that $q^{\mathrm{RS}^*} > q^{\mathrm{AS}^*}$ when ϕ and ξ are both small (the left bottom part in Fig. 6a) or when ϕ and ξ are both large (the upper right part in Fig. 6a); $w^{\mathrm{RS}^*} > w^{\mathrm{AS}^*}$, $s_1^{\mathrm{RS}^*} > s_1^{\mathrm{AS}^*}$, and $s_2^{\mathrm{RS}^*} > s_2^{\mathrm{AS}^*}$ when ξ is small or when ϕ and ξ are both large. This suggests that: when consumers prefer more on the online channel and platform agency fees are lower, or when consumers prefer more on the offline channel and the platform agency fee is higher, the manufacturer sets the higher quality and wholesale price in agency sales format, and both the offline and online retailers set higher service level, which verifies the results in Proposition 6.1.

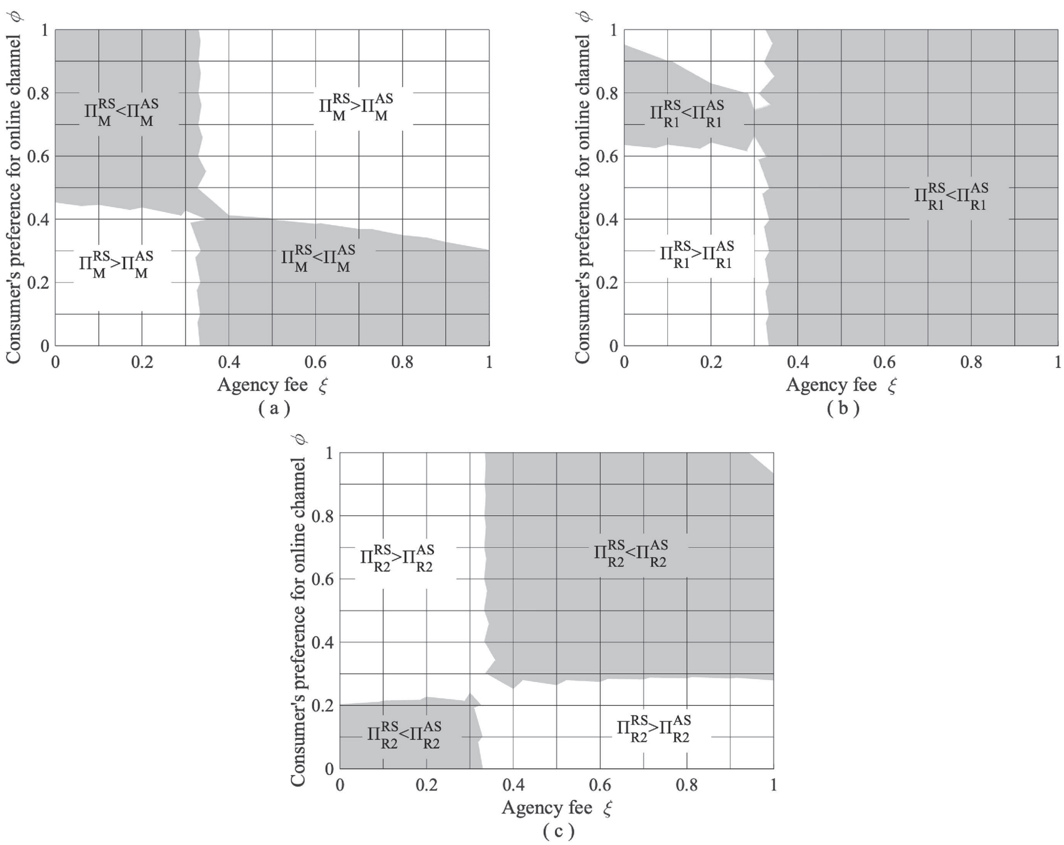

FIGURE 7. The variation of the difference in optimal profits between resale and agency sales with the changing ξ and ϕ.

Figure 7a shows that the manufacturer prefers resales format when ϕ and ξ are both small (*i.e.*, $\phi \in [0, 0.4]$ and $\xi \in [0, 0.3]$) or when ϕ and ξ are both large (*i.e.*, $\phi \in [0.4, 1]$ and $\xi \in [0.4, 1]$), otherwise, the manufacturer prefers agency sales format. This is because when ϕ and ξ are both small or when ϕ and ξ are both large, the manufacturer has more demands due to the high service levels and product quality; on the other hand, the manufacturer earns higher wholesale price. Figure 7b shows that from the perspective of the offline retailer, the optimal sales format of the online channel is reselling when ϕ and ξ are both small (*i.e.*, $\phi \in [0, 0.6]$ and $\xi \in [0, 0.3]$). Figure 7c shows that the online retailer prefers agency sales when ϕ and ξ are both small (*i.e.*, $\phi \in [0, 0.2]$ and $\xi \in [0, 0.3]$) or when ϕ and ξ are both large (*i.e.*, $\phi \in [0.3, 1]$ and $\xi \in [0.4, 1]$).

Next, Figures 8 and 9 illustrate the impact of the spillover effects and applause rate elasticity on the optimal sales format. Figures 8a and 9a show that when online reviews have a significant effect in both online and offline channels, then compared with the agency sales format, the manufacturer set a higher product quality and gain more profit in resales format. Figure 9b also shows that the offline retailer prefers the resales format when online reviews strongly spillover affect the offline channel. Figure 9c shows that the online retailer always prefers the resales format.

Figures 7 and 9 indicate in some circumstances it exists an optimal sales format for the online channel satisfies the preferences of each member of the supply chain. Such as when $\phi \in [0.2, 0.4]$ and $\xi \in [0, 0.3]$ in Figure 7, or when $k, \lambda \in [0.5, 1]$ in Figure 9 both the manufacturer and retailers prefer reselling; while when $\phi \in [0.3, 0.4]$ and $\xi \in [0.4, 0.8]$ in Figure 7, all members prefer agency selling.

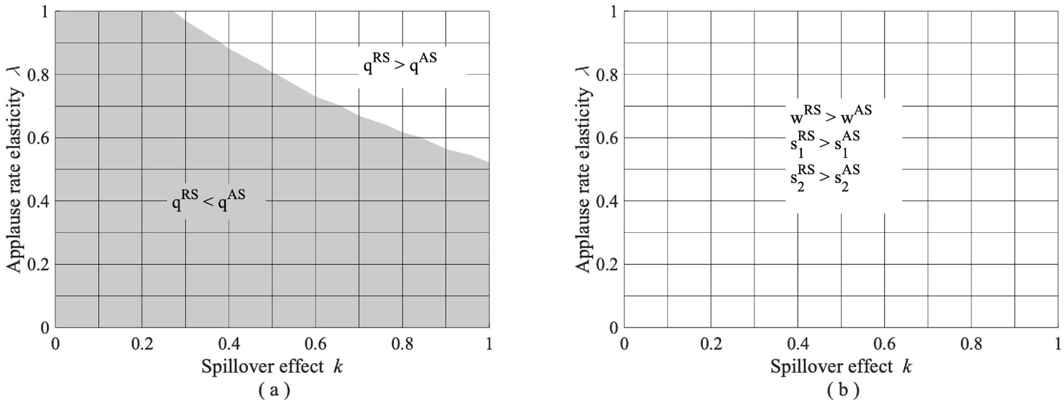

FIGURE 8. The variation of the difference in optimal profits between resale and agency sales with the changing k and λ.

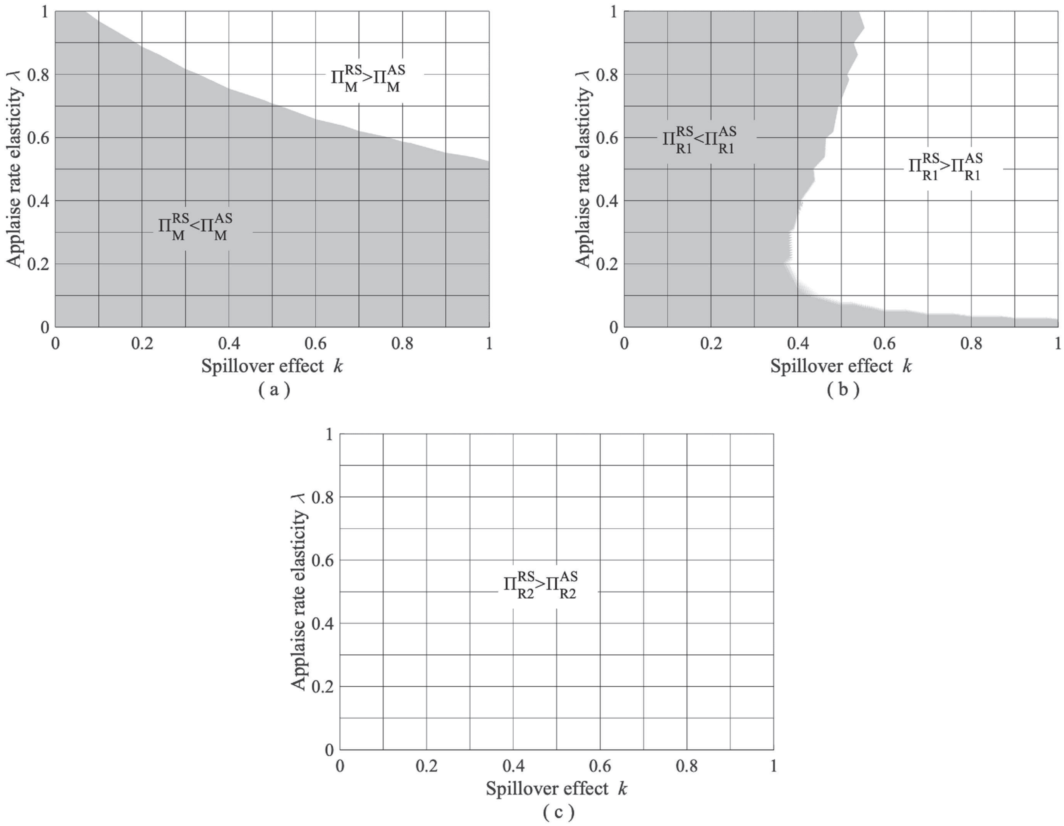

FIGURE 9. The variation of the difference in optimal profits between resale and agency sales with the changing k and λ.

6.2. Optimal sales format from the perspective of society

In this section, we compare the optimal social welfare between the reselling and agency sales formats to examine the optimal format of the sales from the perspective of society.

The social welfare is the sum of profit of supply chain members and consumer welfare as:

$$sw = \pi_M + \pi_{R1} + \pi_{R1} + U. \tag{30}$$

Following Johari and Hosseini-Motlagh [13], the consumer welfare is the difference between the maximum price that consumers are willing to buy the product and the real market price. The maximum prices that consumers are willing to buy the product from offline and the online channels are $p_1^{\max} = (1-\phi)a + \beta s_1 + \lambda kr$ and $p_2^{\max} = \phi a + \beta s_2 + \lambda r$, the real market price that consumer buy from offline and the online channels are $p_1^{\min} = (1-\phi)a - D_1 + \beta s_1 + \lambda kr$ and $p_2^{\min} = \phi a - D_2 + \beta s_2 + \lambda r$. Then, the consumer welfare as follows:

$$U = \int_{p_1^{\min}}^{p_1^{\max}} D_1 \,\mathrm{d}p + \int_{p_2^{\min}}^{p_2^{\max}} D_2 \,\mathrm{d}p = \frac{D_1^2 + D_2^2}{2}. \tag{31}$$

Subsisting equations (11)–(16), (24)–(29), and (31) into equation (30), then the optimal social welfare in resales and agency sales formats are summarized in equations (32) and (33).

$$\begin{aligned} sw^{\mathrm{RS}^*} &= D_2^{\mathrm{RS}^*}\left(\frac{a\phi + \lambda r_0}{2} - \frac{\gamma_2\big(2tv - b_2^2(k+1)\lambda^2\big)}{4\gamma_1} + \frac{\gamma_2(t-1)v\big((b_1\lambda+\beta)^2 + 4\mu_2\big)}{4\gamma_1\mu_2}\right) \\ &+ D_1^{\mathrm{RS}^*}\left(\frac{a(1-\phi) + k\lambda r_0}{2} + \frac{\gamma_2(t-1)v\big(\beta^2\mu_2 + b_1k\lambda\mu_1(b_1\lambda+\beta) + 4\mu_1\mu_2\big)}{4\gamma_1\mu_1\mu_2}\right. \\ &\left. - \frac{\gamma_2\big(2tv - b_2^2k\lambda^2(k+1)\big)}{4\gamma_1}\right) - \frac{\gamma_2^2 v\Big((t-1)^2 v\Big(\beta^2\mu_2 + \mu_1(b_1\lambda+\beta)^2\Big) - \gamma_1\Big)}{4\gamma_1^2\mu_1\mu_2} \end{aligned} \tag{32}$$

$$\begin{aligned} sw^{\mathrm{AS}^*} &= D_1^{\mathrm{AS}^*}\left(\frac{1}{2}(a(1-\phi) + k\lambda r_0) + \frac{b_2^2k\lambda^2\rho_2}{4\rho_1} + \frac{\rho_2 v\big(b_1k\lambda\mu_1\xi t(b_1\lambda+\beta) + \beta^2\mu_2(t-1) + 2\mu_1\mu_2 t\big)}{4\mu_1\mu_2\rho_1(k+(1-\xi)t)}\right) \\ &+ D_2^{\mathrm{AS}^*}\left(\frac{1}{2}(a\phi + \lambda r_0) + \frac{b_2^2\lambda^2\rho_2}{4\rho_1} + \frac{\rho_2 tv\big(\xi(b_1\lambda+\beta)^2 + 2\mu_2\big)}{4\mu_2\rho_1(k+(1-\xi)t)}\right) \\ &- \frac{\rho_2^2 v\big(b_2^2\lambda^2\mu_1\mu_2(k-\xi t+t)^2 + \mu_1\xi^2t^2v(b_1\lambda+\beta)^2 + \beta^2\mu_2(t-1)^2v\big)}{4\mu_1\mu_2\rho_1^2(-k+\xi t-t)^2}. \end{aligned} \tag{33}$$

As the expressions for equations (32) and (33) are cumbersome, we explore the optimal sales format for the society through a numerical example.

Figure 10 illustrates the impact of ϕ and ξ on the optimal sales format for the society. Figure 10 shows that when ϕ and ξ are both small (*i.e.*, $\phi \in [0, 0.2]$ and $\xi \in [0, 0.3]$) or when ϕ and ξ are both large (*i.e.*, $\phi \in [0.3, 1]$ and $\xi \in [0.4, 1]$), agency sale is best to the society. When ϕ is large but ξ is small (or ϕ is small but ξ is large), resale is best for the society. This is because when ϕ and ξ are both small (or both large), the price in the resales format is higher than that in agency sales, leading to low consumer welfare in the resales format and, consequently, lower social welfare.

7. Managerial insight and robustness check

In this section, we summarized the managerial implications based on the theoretical results in propositions and corollaries and main observations in numerical analysis. Moreover, we take a uniformly random sample of size 1000 to show the robustness of the conclusion on optimal online sales format selection strategy. Each parameter in the robustness check is from the value ranges used in the numerical study.

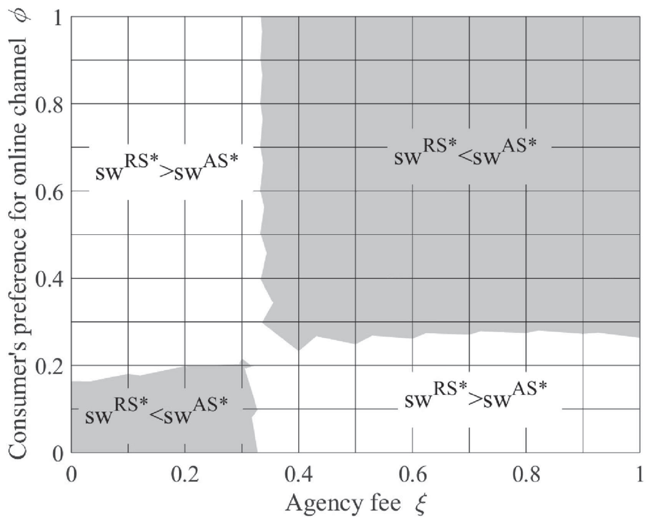

FIGURE 10. The variation of the difference in optimal social welfare between resale and agency sales with the changing ϕ and ξ.

TABLE 2. The robustness analysis of the sales format selection from the perspective of manufacturer.

	b_1		b_2		μ_1		μ_2		ν		ξ		ϕ		k	
	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
RS	13%	100%	0%	0%	0%	0%	10%	0%	0%	0%	31%	100%	90%	0%	0%	0%
AS	87%	0%	100%	100%	100%	100%	90%	100%	100%	100%	69%	0%	10%	100%	100%	100%

(1) From the perspective of consumers, regardless of sales format, consumers benefit from the high impact of service level and product quality on online reviews, from the high impact of online reviews on online and offline channels, and from the low quality and service cost. Because the manufacturer improves the product quality and retailers improve service levels. For the same reason, consumers benefit from the reselling (agency selling) format when they are more prefer the online channel and the platform charges a high (low) agency fee, or when they are more prefer the offline channel and the platform charges a low (high) agency fee.

(2) From the manufacturer's perspective, regardless of sales format, the manufacturer benefits from the high impact of service level and product quality on online reviews, the high impact of online reviews on online and offline channels, and the low quality and service cost. Additionally, the manufacturer can ignore the consumers' channel preference in the resales but not in the agency sales format.

Moreover, the optimal online sales format for the manufacturer generally tends to be agency sales. But in some circumstances, the optimal online sales format could be resales, depending on the agency fee, online service level elasticity of the applause rate, and consumer channel preference. For example, the manufacturer prefers the resales format for the online channel if online service strongly affects online reviews, the platform shares more sales revenue, or consumers prefer the offline channel more. Table 2 summarizes the proportions of optimal online sales format from the perspective of the manufacturer in the 1000 samples demonstrating that the above conclusion is robust.

TABLE 3. The robustness analysis of the sales format selection from the perspective of offline retailer.

	b_1		b_2		μ_1		μ_2		ν		ξ		ϕ		k	
	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
RS	62%	100%	100%	100%	100%	100%	69%	0%	92%	100%	69%	0%	100%	54%	14%	100%
AS	38%	0%	0%	0%	0%	0%	31%	100%	8%	0%	31%	100%	0%	46%	86%	0%

TABLE 4. The robustness analysis of the sales format selection from the perspective of online retailer.

	b_1		b_2		μ_1		μ_2		ν		ξ		ϕ		k	
	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
RS	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	69%	0%	58%	100%	100%	100%
AS	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	31%	100%	42%	0%	0%	0%

(3) From the perspective of the offline retailer, the offline retailer should proactively display online reviews in-store to benefit from the online reviews spillover effect. To get more positive reviews, offline retailers should share the cost of improving reviews with online retailers.
Moreover, the optimal online sales format from the perspective of the offline retailer generally tends to be resales. But in some circumstances, the optimal online sales format could be agency sales, depending on the agency fee, spillover effect, service cost, etc. For example, the offline retailer prefers the manufacturer to sign an agency sales agreement with the online retailer if the platform shares a high agency fee or online reviews have a low spillover effect on offline sales. Table 3 summarizes the proportions of optimal online sales format from the perspective of the offline retailer in the 1000 samples, demonstrating that the above conclusion is robust.

(4) From the online retailer's perspective, the online retailer should allow offline retailers to display online reviews in-store, as the spillover effect of online reviews on offline sales benefits the online retailer in some cases.
Moreover, the optimal online sales format for the online retailer generally tends to be resales. However, in some circumstances, the optimal online sales format could be agency sales, depending on the agency fee. For example, the online retailer prefers the agency sales format if it shares high revenue. Table 4 summarizes the proportions of optimal online sales format from the perspective of the online retailer in the 1000 samples, demonstrating that the above conclusion is robust.

8. CONCLUSIONS

This paper studies the sales format selection strategy in a dual-channel supply chain considering online reviews and spillover effects. We construct the demand function based on the fact that both online and offline consumers read online reviews before purchasing. Then, the joint product quality and service level decision models for resale and agency sales are constructed, respectively. Next, we analyze the optimal sales format of the online channel from different perspectives by evaluating the equilibrium decisions and profits in two sales formats.

We find that: (1) the manufacturer improves the product quality and the wholesale price, and retailers increase service levels only if the online service and product quality affect online reviews strongly, online reviews affect both online and offline sales strongly, or low service and product quality cost. And as a consequence, the profit of the manufacturer increases. (2) In the resales format, consumers' preference for the online channel has no impact on optimal policies and the manufacturer's profit. However, in the agency sales format, when the consumer's preference for the online channel increases, if the marginal profit in the online channel is higher (lower) than the wholesale price, the manufacturer improves (reduces) the product quality and the wholesale price, and retailers increase (reduce) their service levels. (3) Not only the manufacturer and the online retailer but also the offline retailer may benefit from the spillover effects of online reviews on offline sales. (4) The optimal sales format could be different from each perspective. In some threshold ranges, there exists an optimal sales format for the online channel that satisfies the preferences of each member of the supply chain.

There are two interesting extensions to future research. First, This paper focuses on the decentralized supply chain in which the manufacturer is the leader while two retailers are the follower. However, in a decentralized supply chain, the manufacturer and the retailers all aim to maximize their profits, resulting in a double marginal effect that lowers the overall benefits of the supply chain. Therefore, it would be interesting to study the coordination of the supply chain with the impact of online reviews and spillover effects in the future. Second, this paper focuses on the service competition between online and offline channels without considering price competition. In reality, not all products are sold at a unified price, and there may be differences in online and offline prices for some products, such as fast fashion items like clothing. Therefore, it would also be interesting to study the pricing and service competition with the impact of spillover effects on sales format selection strategy in the future.

Appendix A.

Proof of Theorem 4.1. According to equations (1)–(3), (5), and (6), taking the second-order derivatives of $\pi_{R1}(s_1)$ and $\pi_{R2}(s_2)$ with respect to s_1 and s_2, respectively, it is easily to verify that $\pi_{R1}(s_1)$ and $\pi_{R2}(s_2)$ are concave functions of s_1 and s_2, respectively. Let $\partial\pi_{R1}(s_1)/\partial s_1 = 0$ and $\partial\pi_{R2}(s_2)/\partial s_2 = 0$, we can obtain the best response function of offline and online retailers, *i.e.*,

$$s_1 = \frac{(t-1)w\beta}{2\mu_1} \tag{A.1}$$

$$s_2 = \frac{(t-1)w(\beta+\lambda b_1)}{2\mu_2}. \tag{A.2}$$

Substituting equations (A.1) and (A.2) into equation (4), we can obtain the Hesse matrix of $\pi_M(q,w)$, *i.e.*,

$$H_1 = \begin{bmatrix} -2v & \lambda b_2(1+k) \\ \lambda b_2(1+k) & \frac{(t-1)L}{\mu_1\mu_2} - 4t \end{bmatrix} \tag{A.3}$$

in which $L = \mu_1(\beta+\lambda b_1)(\beta+\lambda b_1+k\lambda b_1)+\beta^2\mu_2$.

According to equation (A.3), we find that only when $[8vt - 2vL(t-1)/(\mu_1\mu_2) - \lambda^2{b_2}^2(1+k)^2 > 0$ the Hesse matrix H_1 is negative definite and $\pi_M(q,w)$ is the joint concave function of q and w. Let $\partial\pi_M(q,w)/\partial q = 0$ and $\partial\pi_M(q,w)/\partial w = 0$, we can obtain the optimal product quality q^{RS^*} and the optimal wholesale price w^{RS^*}. Substituting q^{RS^*} and w^{RS^*} into equations (A.1) and (A.2), we can obtain the optimal offline service level $s_1^{\mathrm{RS}^*}$ and the optimal online service level $s_2^{\mathrm{RS}^*}$. □

Proof of Theorem 4.4. The proof is identical to the proof of Theorem 4.1. □

Proof of Proposition 5.1. (1) Taking the first-order derivatives of q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$ with respect to b_1, b_2, k, λ and r_0:

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial b_1} = \frac{4b_2\lambda^2(k+1)\mu_1^2\mu_2(t-1)v(a+(k+1)\lambda r_0)(2b_1(k+1)\lambda+\beta(k+2))}{2\gamma_1^2} > 0 \tag{A.4}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial b_1} = \frac{4\lambda\mu_1^2\mu_2(t-1)v^2(a+(k+1)\lambda r_0)(2b_1(k+1)\lambda+\beta(k+2))}{\gamma_1^2} > 0 \tag{A.5}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial b_1} = \frac{4\beta\lambda\mu_1\mu_2(t-1)^2v^2(a+(k+1)\lambda r_0)(2b_1(k+1)\lambda+\beta(k+2))}{2\gamma_1^2} > 0 \tag{A.6}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial b_1} = \frac{4\lambda\mu_1^2\mu_2(t-1)^2v^2(b_1\lambda+\beta)(a+(k+1)\lambda r_0)(2b_1(k+1)\lambda+\beta(k+2))}{2\gamma_1^2\mu_2} + \frac{\lambda(t-1)(\gamma_2 v)}{2\gamma_1\mu_2} > 0 \tag{A.7}$$

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial b_2} = \frac{\gamma_2\lambda(k+1)\big(2b_2^2(k+1)^2\lambda^2\mu_1\mu_2+\gamma_1\big)}{2\gamma_1^2} > 0 \tag{A.8}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial b_2} = \frac{2b_2\gamma_2\lambda^2(k+1)^2\mu_1\mu_2 v}{\gamma_1^2} > 0 \tag{A.9}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial b_2} = \frac{\mu_2 v\beta\lambda^2 b_2(t-1)(1+k)^2\gamma_2}{\gamma_1^2} > 0 \tag{A.10}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial b_2} = \frac{\mu_1 b_2 v\lambda^2(\beta+b_1\lambda)(t-1)(1+k)^2\gamma_2}{\gamma_1^2} > 0 \tag{A.11}$$

$$\begin{aligned}\frac{\partial q^{\mathrm{RS}^*}}{\partial k} &= \frac{2b_2(k+1)\lambda\mu_2\mu_1^2(a+(k+1)\lambda r_0)\big(b_2^2(k+1)\lambda^2\mu_2+b_1\lambda(t-1)v(b_1\lambda+\beta)\big)}{\gamma_1^2}\\ &\quad+\frac{b_2\lambda\mu_2\mu_1(a+2(k+1)\lambda r_0)}{\gamma_1} > 0\end{aligned} \tag{A.12}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial k} = \frac{4\mu_2\mu_1^2 v(a+(k+1)\lambda r_0)\big(b_2^2(k+1)\lambda^2\mu_2+b_1\lambda(t-1)v(b_1\lambda+\beta)\big)}{\gamma_1^2} + \frac{2\lambda\mu_2\mu_1 r_0 v}{\gamma_1} > 0 \tag{A.13}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial k} = \frac{(\beta(t-1))\frac{\partial w^{\mathrm{RS}^*}}{\partial k}}{2\mu_1} > 0 \tag{A.14}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial k} = \frac{\frac{\partial w^{\mathrm{RS}^*}}{\partial k}((t-1)(b_1\lambda+\beta))}{2\mu_2} > 0 \tag{A.15}$$

$$\begin{aligned}\frac{\partial q^{\mathrm{RS}^*}}{\partial \lambda} &= \frac{b_2(k+1)\mu_1\mu_2(a+2(k+1)\lambda r_0)}{\gamma_1} + 2b_2\lambda(k+1)\mu_1^2\mu_2(a+(k+1)\lambda r_0)\\ &\quad\times\frac{\big(b_2^2(k+1)^2\lambda\mu_2+(t-1)v((b_1k+b_1)(b_1\lambda+\beta)+b_1(b_1k\lambda+b_1\lambda+\beta))\big)}{\gamma_1^2} > 0\end{aligned} \tag{A.16}$$

$$\begin{aligned}\frac{\partial w^{\mathrm{RS}^*}}{\partial \lambda} &= \frac{2(k+1)\mu_1\mu_2 r_0 v}{\gamma_1} + 4\mu_1^2\mu_2 v(a+(k+1)\lambda r_0)\\ &\quad\times\frac{\big(b_2^2(k+1)^2\lambda\mu_2+(t-1)v((b_1k+b_1)(b_1\lambda+\beta)+b_1(b_1k\lambda+b_1\lambda+\beta))\big)}{\gamma_1^2} > 0\end{aligned} \tag{A.17}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial \lambda} = \frac{(\beta(t-1))\frac{\partial w^{\mathrm{RS}^*}}{\partial \lambda}}{2\mu_1} > 0 \tag{A.18}$$

$$\begin{aligned}\frac{\partial s_2^{\mathrm{RS}^*}}{\partial \lambda} &= \frac{\mu_1(t-1)v(ab_1+(k+1)r_0(2b_1\lambda+\beta))}{\gamma_1} + 2\mu_1^2 v(a+(k+1)\lambda r_0)(b_1\lambda+\beta)\\ &\quad\times\frac{(t-1)\big(b_2^2(k+1)^2\lambda\mu_2+(t-1)v((b_1k+b_1)(b_1\lambda+\beta)+b_1(b_1k\lambda+b_1\lambda+\beta))\big)}{\gamma_1^2} > 0\end{aligned} \tag{A.19}$$

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial r_0} = \frac{\mu_1\mu_2 b_2\lambda^2(1+k)^2}{\gamma_1} > 0 \tag{A.20}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial r_0} = \frac{2v\lambda\mu_1\mu_2(1+k)}{\gamma_1} > 0 \tag{A.21}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial r_0} = \frac{\mu_2 v\beta\lambda(t-1)(1+k)}{\gamma_1} > 0 \tag{A.22}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial r_0} = \frac{\mu_1 v\lambda(\beta+b_1\lambda)(t-1)(1+k)}{\gamma_1} > 0. \tag{A.23}$$

(2) Taking the first-order derivatives of q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$ with respect to μ_1, μ_2 and ν:

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial \mu_1} = -\frac{4b_2\beta^2\lambda(k+1)\mu_2^2(t-1)v(a+(k+1)\lambda r_0)}{2\gamma_1^2} < 0 \tag{A.24}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial \mu_1} = -\frac{4\beta^2\mu_2^2(t-1)v^2(a+(k+1)\lambda r_0)}{\gamma_1^2} < 0 \tag{A.25}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial \mu_1} = -\frac{4\beta^3\mu_1\mu_2^2(t-1)^2v^2(a+(k+1)\lambda r_0)+\beta\gamma_1\gamma_2(t-1)v}{2\gamma_1^2\mu_1^2} < 0 \tag{A.26}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial \mu_1} = -\frac{4\beta^2\mu_2(t-1)^2v^2(b_1\lambda+\beta)(a+(k+1)\lambda r_0)}{2\gamma_1^2} < 0 \tag{A.27}$$

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial \mu_2} = -\frac{4b_2\lambda(k+1)\mu_1^2(t-1)v(b_1\lambda+\beta)(a+(k+1)\lambda r_0)(b_1k\lambda+b_1\lambda+\beta)}{2\gamma_1^2} < 0 \tag{A.28}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial \mu_2} = -\frac{4\mu_1^2(t-1)v^2(b_1\lambda+\beta)(a+(k+1)\lambda r_0)(b_1k\lambda+b_1\lambda+\beta)}{\gamma_1^2} < 0 \tag{A.29}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial \mu_2} = -\frac{4\beta\mu_1(t-1)^2v^2(b_1\lambda+\beta)(a+(k+1)\lambda r_0)(b_1k\lambda+b_1\lambda+\beta)}{2\gamma_1^2} < 0 \tag{A.30}$$

$$\begin{aligned}\frac{\partial s_2^{\mathrm{RS}^*}}{\partial \mu_2} &= -\frac{4\mu_1^2(t-1)^2v^2(b_1\lambda+\beta)^2(a+(k+1)\lambda r_0)(b_1k\lambda+b_1\lambda+\beta)}{2\gamma_1^2\mu_2} \\ &\quad -\frac{(t-1)(\gamma_2 v)(b_1\lambda+\beta)}{2\gamma_1\mu_2^2} < 0\end{aligned} \tag{A.31}$$

$$\frac{\partial q^{\mathrm{RS}^*}}{\partial v} = -\frac{b_2\gamma_2\lambda(k+1)\left(b_2^2(k+1)^2\lambda^2\mu_1\mu_2+\gamma_1\right)}{2\gamma_1^2 v} < 0 \tag{A.32}$$

$$\frac{\partial w^{\mathrm{RS}^*}}{\partial v} = -\frac{b_2^2\gamma_2\lambda^2(k+1)^2\mu_1\mu_2}{\gamma_1^2} < 0 \tag{A.33}$$

$$\frac{\partial s_1^{\mathrm{RS}^*}}{\partial v} = -\frac{\beta b_2^2\gamma_2\lambda^2(k+1)^2\mu_2(t-1)}{2\gamma_1^2} < 0 \tag{A.34}$$

$$\frac{\partial s_2^{\mathrm{RS}^*}}{\partial v} = -\frac{b_2^2\gamma_2\lambda^2(k+1)^2\mu_1(t-1)(b_1\lambda+\beta)}{2\gamma_1^2} < 0. \tag{A.35}$$

(3) Taking the first-order derivatives of q^{RS^*}, w^{RS^*}, $s_1^{\mathrm{RS}^*}$, and $s_2^{\mathrm{RS}^*}$ with respect to ϕ, we find that $\partial q^{\mathrm{RS}^*}/\partial\phi = 0$, $\partial w^{\mathrm{RS}^*}/\partial\phi = 0$, $\partial s_1^{\mathrm{RS}^*}/\partial\phi = 0$, $\partial s_2^{\mathrm{RS}^*}/\partial\phi = 0$.

□

Proof of Proposition 5.3. Taking the first-order derivatives of $\pi_M^{\mathrm{RS}^*}$ with respect to b_1, b_2, r_0, ϕ, μ_1, μ_2 and ν:

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial b_1} = \frac{v^2\lambda\gamma_2^2(t-1)[(1+k)(\beta+2\lambda b_1)+\beta]}{2\mu_2\gamma_1^2} > 0 \tag{A.36}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial b_2} = \frac{vb_2\lambda^2(1+k)^2\gamma_2^2}{2\gamma_1^2} > 0 \tag{A.37}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial k} = \frac{2\mu_2\mu_1^2 v(a+(k+1)\lambda r_0)^2\big(b_2^2(k+1)\lambda^2\mu_2 + b_1\lambda(t-1)v(b_1\lambda+\beta)\big)}{\gamma_1^2} + \frac{2\lambda\mu_2\mu_1 r_0 v(a+(k+1)\lambda r_0)}{\gamma_1} > 0 \tag{A.38}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial \lambda} = \frac{2(k+1)\mu_2\mu_1 r_0 v(a+(k+1)\lambda r_0)}{\gamma_1} + 2\mu_2\mu_1^2 v(a+(k+1)\lambda r_0)^2 \times \frac{\big(b_2^2(k+1)^2\lambda\mu_2 + (t-1)v((b_1k+b_1)(b_1\lambda+\beta) + b_1(b_1k\lambda + b_1\lambda+\beta))\big)}{\gamma_1^2} > 0 \tag{A.39}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial r_0} = \frac{v\lambda(1+k)\gamma_2}{\gamma_1} > 0 \tag{A.40}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial \phi} = 0 \tag{A.41}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial \mu_1} = -\frac{v^2\beta^2\gamma_2^2(t-1)}{2{\mu_1}^2\gamma_1^2} < 0 \tag{A.42}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial \mu_2} = -\frac{v^2\gamma_2^2(t-1)(\beta+\lambda b_1)(\beta+\lambda b_1 + k\lambda b_1)}{2{\mu_2}^2\gamma_1^2} < 0 \tag{A.43}$$

$$\frac{\partial \pi_M^{\mathrm{RS}^*}}{\partial v} = -\frac{\lambda^2{b_2}^2(1+k)^2\gamma_2^2}{4\gamma_1^2} < 0. \tag{A.44}$$

□

Proof of Proposition 5.2. The proof is identical to the proof of Proposition 5.1. □

Proof of Proposition 5.4. The proof is identical to the proof of Proposition 5.3. □

Proof of Proposition 6.1. By comparing the optimal policies in Theorems 4.1 and 4.4, we can obtain equations (A.45)–(A.48).

$$q^{\mathrm{RS}^*} - q^{\mathrm{AS}^*} = b_2\lambda(t-1-\xi t)\frac{\Gamma - 2a\gamma_1\mu_1\mu_2\phi[k+t(1-\xi)]}{2\gamma_1\rho_1} \tag{A.45}$$

$$w^{\mathrm{RS}^*} - w^{\mathrm{AS}^*} = \lambda(t-1-\xi t)\frac{\rho_1(c\gamma_1+\gamma_2 v) + \Gamma v - 2a\gamma_1\mu_1\mu_2\phi v[k+t(1-\xi)]}{\gamma_1\rho_1[k+t(1-\xi)]} \tag{A.46}$$

$$s_1^{\mathrm{RS}^*} - s_1^{\mathrm{AS}^*} = \lambda\beta(t-1)(t-1-\xi t)\frac{\rho_1(c\gamma_1+\gamma_2 v) + \Gamma v - 2a\gamma_1\mu_1\mu_2\phi v[k+t(1-\xi)]}{2\gamma_1\mu_1\rho_1[k+t(1-\xi)]} \tag{A.47}$$

$$s_2^{\mathrm{RS}^*} - s_2^{\mathrm{AS}^*} = \frac{\rho_1(k+t)(c\gamma_1+\gamma_2 v) + \Gamma\xi t v - 2a\gamma_1\mu_1\mu_2\xi t\phi v[k+t(1-\xi)]}{2\gamma_1\mu_2\rho_1[k+t(1-\xi)]} \times (b_1\lambda+\beta)(t-1-\xi t). \tag{A.48}$$

As $k+t(1-\xi) > 0$, $\gamma_1 > 0$ and $\rho_1 > 0$, let $\hat{\xi} = \frac{t-1}{t}$, $\hat{\phi}_q = \frac{\Gamma}{2a\gamma_1\mu_1\mu_2(k-\xi t+t)}$, $\hat{\phi}_w = \frac{\rho_1(c\gamma_1+\gamma_2 v)+\Gamma v}{2a\gamma_1\mu_1\mu_2 v(k-\xi t+t)}$, $\hat{\phi}_s = \frac{\rho_1(k+t)(c\gamma_1+\gamma_2 v)+\Gamma\xi t v}{2a\gamma_1\mu_1\mu_2\xi t v(k-\xi t+t)}$, thus $q^{\mathrm{RS}^*} > q^{\mathrm{AS}^*}$ only if $\xi < \hat{\xi}$ and $\phi < \hat{\phi}_q$, or $\xi > \hat{\xi}$ and $\phi > \hat{\phi}_q$, otherwise, $q^{\mathrm{RS}^*} < q^{\mathrm{AS}^*}$; $w^{\mathrm{RS}^*} > w^{\mathrm{AS}^*}$ and $s_1^{\mathrm{RS}^*} > s_1^{\mathrm{AS}^*}$ only if $\xi < \hat{\xi}$ and $\phi < \hat{\phi}_w$, or $\xi > \hat{\xi}$ and $\phi > \hat{\phi}_w$, otherwise, $w^{\mathrm{RS}^*} \leq w^{\mathrm{AS}^*}$ and $s_1^{\mathrm{RS}^*} \leq s_1^{\mathrm{AS}^*}$; $s_2^{\mathrm{RS}^*} > s_2^{\mathrm{AS}^*}$ only if $\xi < \hat{\xi}$ and $\phi < \hat{\phi}_s$, or $\xi > \hat{\xi}$ and $\phi > \hat{\phi}_s$, otherwise, $s_2^{\mathrm{RS}^*} \leq s_2^{\mathrm{AS}^*}$. □

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